EXCERPTED FROM

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A NEW KIND OF SCIENCE

SECTION 9.12

Evolution of Networks

regardless of the path one chooses, the overall form of causal network will be essentially the same. And what this means is that on a sufficiently large scale, the universe will appear to have a unique history, even though at the level of individual events there will be considerable arbitrariness.

If there is not enough convergence in the multiway system it will still be possible to get stuck with different types of strings that never lead to each other. And if this happens, then it means that the history of the universe can in effect follow many truly separate branches. But whenever there is significant randomness produced by the evolution of the multiway system, this does not typically appear to occur.

So this suggests that in fact it is at some level not too difficult for multiway systems to reproduce our everyday perception that more or less definite things happen in the universe. But while this means that it might be possible for there to be arbitrariness in the causal network for the universe, it still tends to be my suspicion that there is not—and that in fact the particular rules followed by the universe do in the end have the property that they always yield the same causal network.

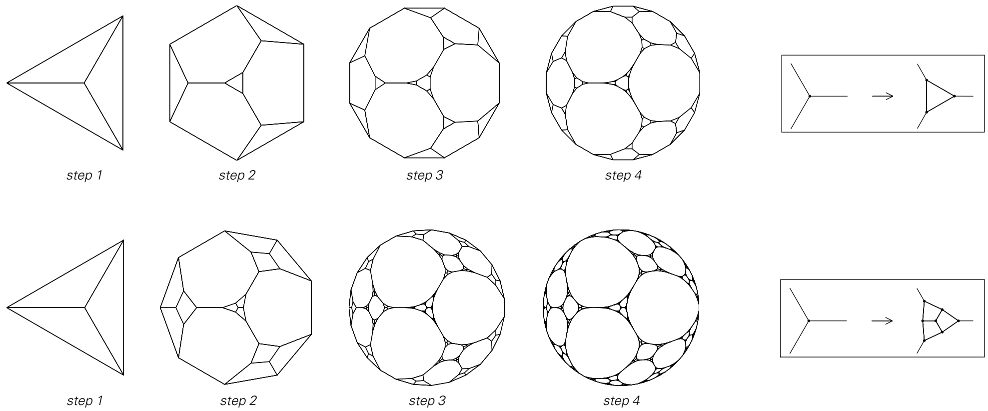
Evolution of Networks

Earlier in this chapter, I suggested that at the lowest level space might consist of a giant network of nodes. But how might such a network evolve?

The most straightforward possibility is that it could work much like the substitution systems that we have discussed in the past few sections—and that at each step some piece or pieces of the network could be replaced by others according to some fixed rule.

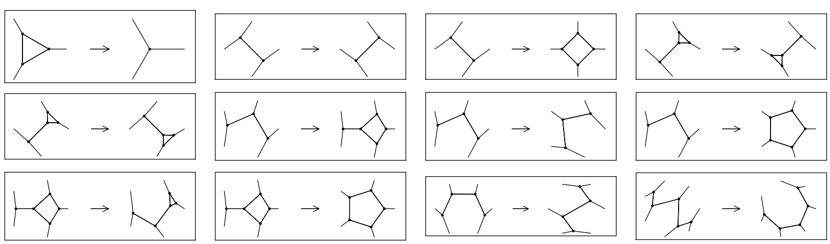
The pictures at the top of the facing page show two very simple examples. Starting with a network whose connections are like the edges of a tetrahedron, both the rules shown work by replacing each node at each step by a certain fixed cluster of nodes.

This setup is very much similar to the neighbor-independent substitution systems that we discussed on pages 83 and 187. And just as in these systems, it is possible for intricate structures to be produced, but the structures always turn out to have a highly regular nested form.



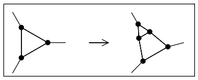
Network evolution in which each node is replaced at each step by a fixed cluster of nodes. The resulting networks have a regular nested form. The dimensions of the limiting networks are respectively *Log[2, 3] ≃ 1.58* and *Log[3, 7] ≃ 1.77*.

So what about more general substitution systems? Are there analogs of these for networks? The answer is that there are, and they are based on making replacements not just for individual nodes, but rather for clusters of nodes, as shown in the pictures below.



Examples of rules that involve replacing clusters of nodes in a network by other clusters of nodes. All these rules preserve the planarity of a network. Notice that some of them cannot be reversed since their right-hand sides are too symmetrical to determine which orientation of the left-hand side should be used.

In the substitution systems for strings discussed in previous sections, the rules that are given can involve replacing any block of elements by any other. But in networks there are inevitably some restrictions. For example, if a cluster of nodes has a certain number of connections to the rest of the network, then it cannot be replaced by a cluster which has a different number of connections. And in addition, one cannot have replacements



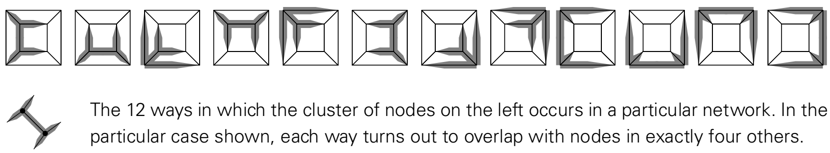
A replacement whose outcome orientation cannot be determined.

like the one on the left that go from a symmetrical cluster to one for which a particular orientation has to be chosen.

But despite these restrictions a fairly large number of replacements are still possible; for example, there are a total of 419 distinct ones that exist involving clusters with no more than five nodes.

So given a replacement for a cluster of a particular form, how should such a replacement actually be applied to a network? At first one might think that one could set up some kind of analog of a cellular automaton and just replace all relevant clusters of nodes at once.

But in general this will not work. For as the picture below illustrates, a particular form of cluster can in general appear in many overlapping ways within a given network.

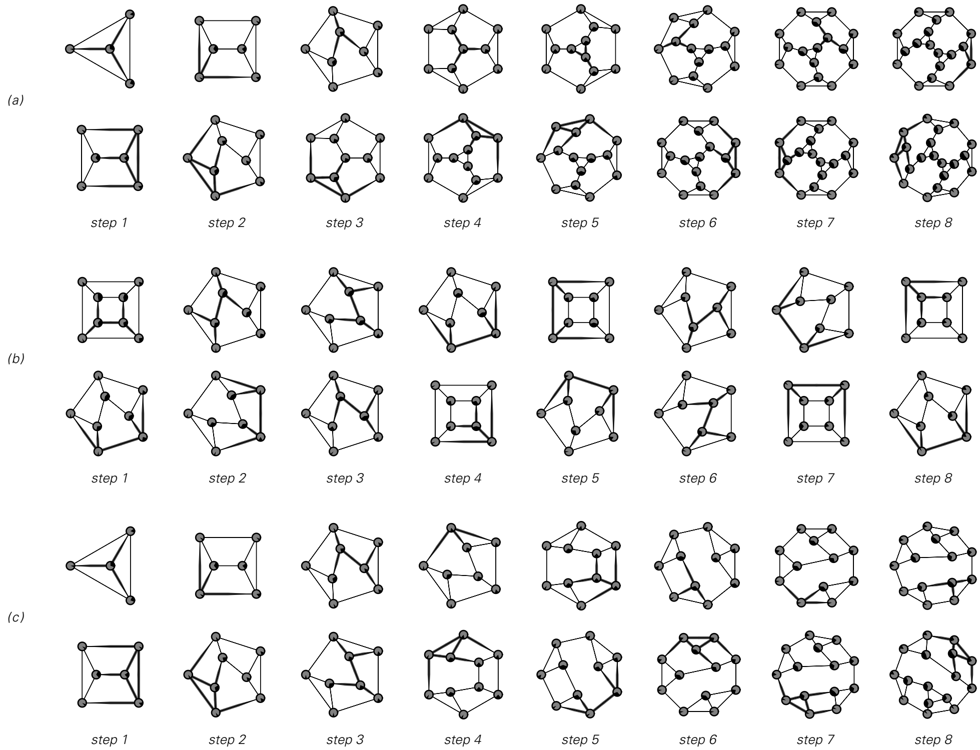


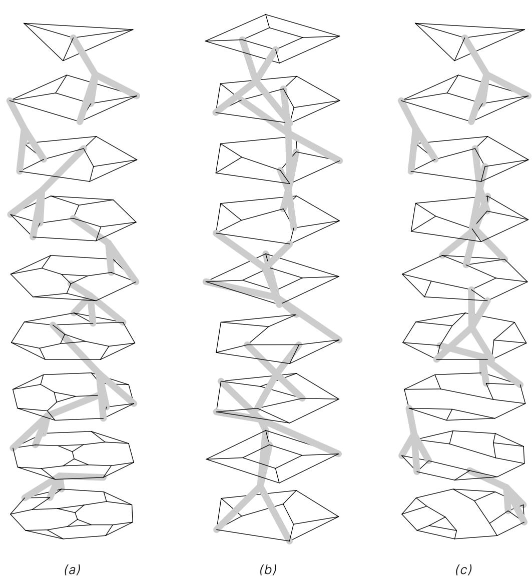
The 12 ways in which the cluster of nodes on the left occurs in a particular network. In the particular case shown, each way turns out to overlap with nodes in exactly four others.

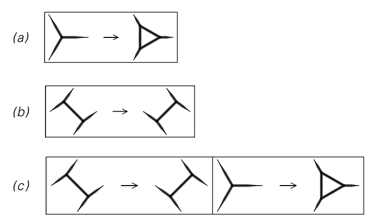
The issue is essentially no different from the one that we encountered in previous sections for blocks of elements in substitution systems on strings. But an additional complication is that in networks, unlike strings, there is no immediately obvious ordering of elements.

Nevertheless, it is still possible to devise schemes for deciding where in a network replacements should be carried out. One fairly simple scheme, illustrated on the facing page, allows only a single replacement to be performed at each step, and picks the location of this replacement so as to affect the least recently updated nodes.

In each pair of pictures in the upper part of the page, the top one shows the form of the network before the replacement, and the bottom one shows the result after doing the replacement—with the cluster of nodes involved in the replacement being highlighted in both cases. In the 3D pictures in the lower part of the page, networks that arise on successive steps are shown stacked one on top of the other, with the nodes involved in each replacement joined by gray lines.







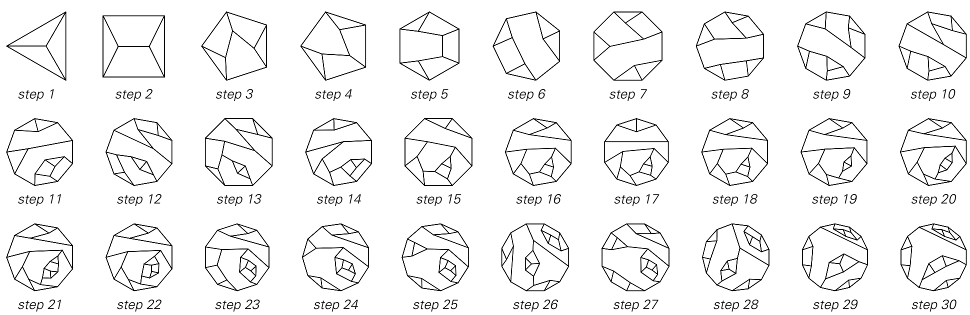
Examples of the evolution of networks in which a single cluster of nodes is replaced at each step according to the rules shown. Each pair of pictures above represents the state of the network before and after each replacement. The nodes affected by the replacement are highlighted in both cases. The location at which the replacement is performed is determined by requiring that it involve the oldest possible nodes in the network. The nodes in the pictures above are drawn with a "clock." The angle of the beginning of the black sector in the clock indicates when the node was created, while the angle of its end represents the current step, so that older nodes have larger black sectors.

Inevitably there is a certain arbitrariness in the way these pictures are drawn. For the underlying rules specify only what the pattern of connections in a network should be—not how its nodes should be laid out on the page. And in the effort to make clear the relationship between networks obtained on different steps, even identical networks can potentially be drawn somewhat differently.

With rule (a), however, it is fairly easy to see that a simple nested structure is produced, directly analogous to the one shown on page 509. And with rule (b), obvious repetitive behavior is obtained.

So what about more complicated behavior? It turns out that even with rule (c), which is essentially just a combination of rules (a) and (b), significantly more complicated behavior can already occur.

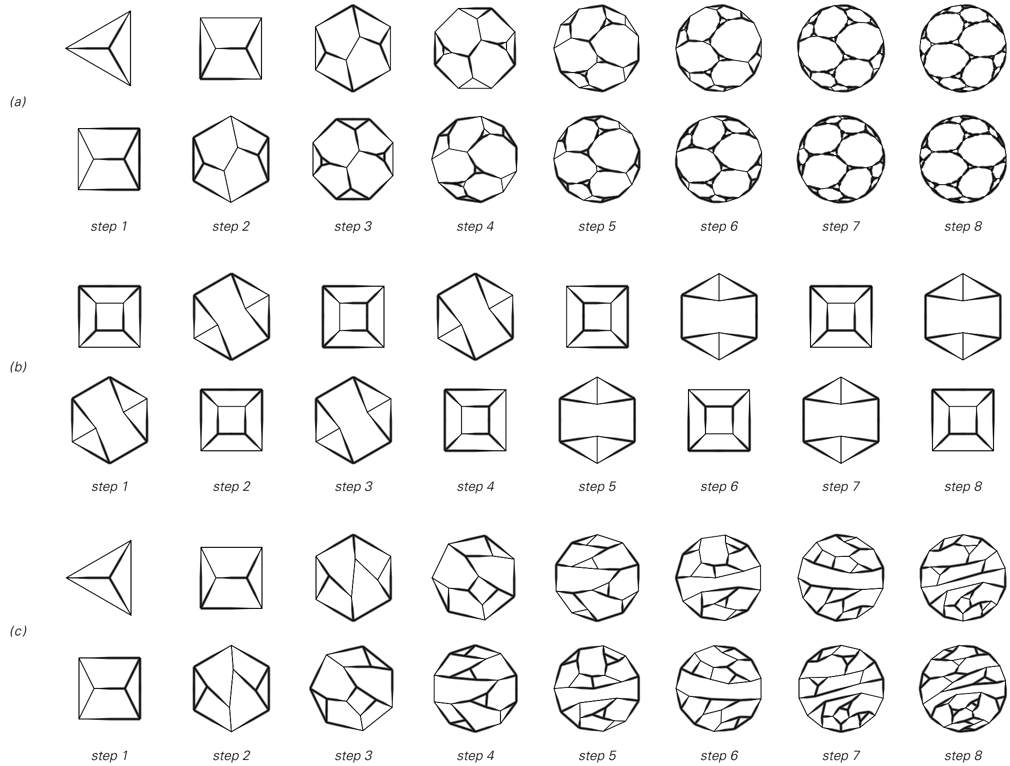
The picture below shows a few more steps in the evolution of this rule. And the behavior obtained never seems to repeat, nor do the networks produced exhibit any kind of obvious nested form.

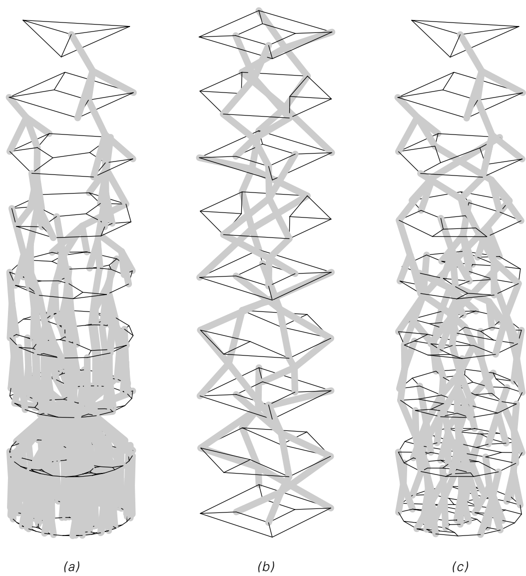


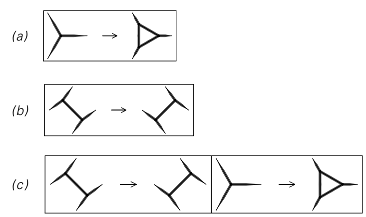
More steps in the evolution of rule (c) from the previous page. The number of nodes increases irregularly (though roughly linearly) with successive steps.

What about other schemes for applying replacements? The pictures on the facing page show what happens if at each step one allows not just a single replacement, but all replacements that do not overlap.

It takes fewer steps for networks to be built up, but the results are qualitatively similar to those on the previous page: rule (a) yields a nested structure, rule (b) gives repetitive behavior, while rule (c) produces behavior that seems complicated and in some respects random.

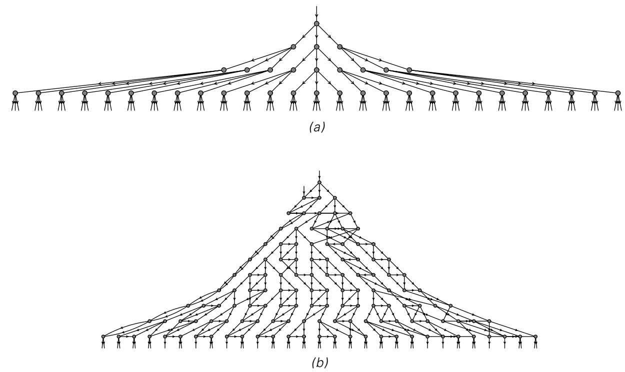


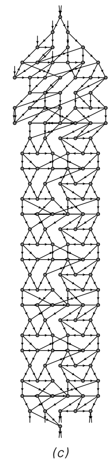




Examples of network evolution according to the same basic underlying rules as on page 511, but now with all possible clusters of nodes that do not overlap being replaced at each step.

Just as for substitution systems on strings, one can find causal networks that represent the causal connections between different updating events on networks. And as an example the pictures below show such causal networks for the evolution processes on the previous page.



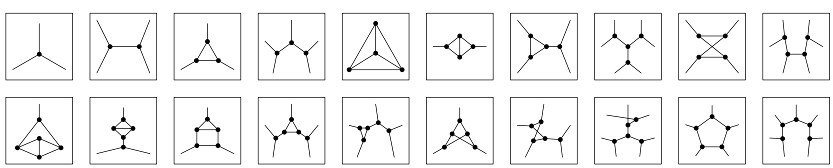


Causal networks that represent the relationship between updating events for the network evolution processes shown on the previous page.

In the rather simple case of rule (a) the results turn out to be independent of the updating scheme that was used. But for rules (b) and (c), different schemes in general yield different causal networks.

So what kinds of underlying replacement rules lead to causal networks that are independent of how the rules are applied? The situation is much the same as for strings—with the basic criterion just being that all replacements that appear in the rules should be for clusters of nodes that can never overlap themselves or each other.

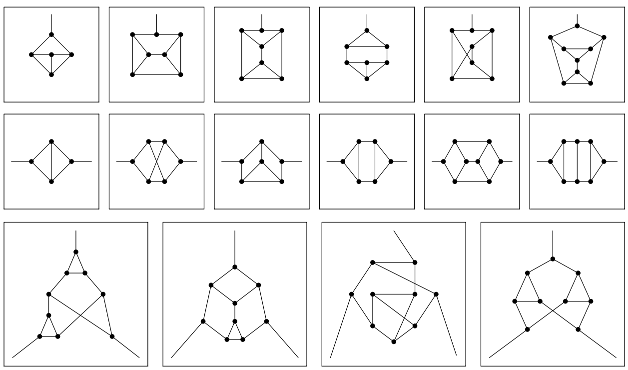
The pictures below show all possible distinct clusters with up to five nodes—and all but three of these already can overlap themselves.



All possible distinct clusters containing up to five nodes, with planarity not required.

But among slightly larger clusters there turn out to be many that do not overlap themselves—and indeed this becomes common as soon as there are at least two connections between each dangling one.

The first few examples are shown below. And in almost all of these, there is no overlap not only within a single cluster, but also between different clusters. And this means that rules based on replacements for collections of these clusters will have the property that the causal networks they produce are independent of the updating scheme used.



The simplest clusters that have no overlaps with themselves—and mostly have no overlaps with each other. Replacements for sets of clusters that do not overlap have the property of causal invariance.

One feature of the various rules I showed earlier is that they all maintain planarity of networks—so that if one starts with a network that can be laid out in the plane without any lines crossing, then every subsequent network one gets will also have this property.

Yet in our everyday experience space certainly does not seem to have this property. But beyond the practical problem of displaying what happens, there is actually no fundamental difficulty in setting up rules that can generate non-planarity—and indeed many rules based on the clusters above will for example do this.

So in the end, if one manages to find the ultimate rules for the universe, my expectation is that they will give rise to networks that on a small scale look largely random. But this very randomness will most likely be what for example allows a definite and robust value of 3 to emerge for the dimensionality of space—even though all of the many complicated phenomena in our universe must also somehow be represented within the structure of the same network.