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CHAPTER 25

Investment, Taxation, and Econometric Policy Evaluation: Some Evidence on the Lucas Critique

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Summary

The aggregate investment schedule may be used to study the impact of various policy measures, such as changes in corporate tax rates, depreciation allowances, and investment tax credits. Its parameters should be invariant with respect to the policy changes themselves, a point forcefully stressed by Lucas (1976). On the impact of investment tax credits, Lucas makes two predictions: first, if the model is implemented under an assumption of static expectations (versus rational expectations) and estimated from a period during which policy rules changed appreciably, it will exhibit parameter instability; second, the impact of tax credits is likely to be heavily underestimated. This chapter presents empirical evidence on both these effects by studying a version of the Hall-Jorgenson model estimated from US data (1956–1972). For this purpose, we use recursive stability analysis, an exploratory methodology that makes very weak assumptions on the form of the instability to be detected and provides indications on the direction of prediction errors. The main finding is a discontinuity associated with the first imposition of the tax credit (1964–1966); further, this shift led to underprediction of investment. The results thus support Lucas's hypothesis.

25.1 Introduction

The stability over time of the aggregate investment schedule has great importance for macroeconomic policy. In particular, one may use this relationship to study the impact of various policy measures, such as changes in nominal corporate tax rates, changes in depreciation allowances, investment tax credits, and the like. An ingenious formulation of an investment function making possible such studies is due to Hall and Jorgenson (1967). This model was employed, for example, by Gordon and Jorgenson (1976) to study the impact of investment tax credits in the United States over the period 1960–1985.

It is easy to understand that the model used for such policy simulations should exhibit a good stability over time. In particular, the parameters should be invariant with respect to the policy changes themselves, a point forcefully stressed by Lucas (1976). This author argues that parameters in econometric relationships reflect economic agents' decision rules: since the latter integrate knowledge about policies, changes in policy rules are likely to induce shifts in the parameters. Lucas describes three cases where such phenomena could be observed: the first one deals with income transfers and the aggregate consumption function, the second one concerns studies of the effect of investment tax credits with the help of the Hall-Jorgenson model of investment demand, while the third one is based on the Phillips curve. In this chapter, we provide empirical evidence on this issue by considering the second example.

In this case, Lucas argues that the effect of a change in the rate of an investment tax credit depends crucially on expectations concerning future changes in this rate: the impact of a change in the tax credit differs, depending on expectations about future changes of the tax credit. In other words, the response coefficient to a change in the rate of the tax credit depends on expectations about future changes of this rate. In particular, after developing a simple investment model, Lucas shows that the impact of a given change may be substantially bigger if it is viewed as transitory rather than permanent (once-and-for-all) [1]. Consequently, if an investigator assumes that changes in the investment tax credit are viewed as permanent by the relevant economic agents, while the latter in fact view it as transitory, he may appreciably underestimate the impact of the tax credit. Thus, to forecast accurately the effect of a proposed change in the tax credit, it is important

1. to make correct assumptions concerning expectations on future changes in the tax credit that will follow a proposed change;

2. to specify and estimate the model under correct expectational assumptions over the historical period used for estimation.

Note here that Hall and Jorgenson (1967), as well as Gordon and Jorgenson (1976), assumed that changes in tax rates were viewed as permanent.

To get evidence on the Lucas critique, we shall reexamine the same model and data as Gordon and Jorgenson (1976). Over the sampling period used for the estimation of their investment function (1956–1972), five major changes in the tax credit took place.

The tax credit was originally introduced in 1962 to stimulate investment. Then "the effectiveness of the tax credit was increased substantially in 1964 with the repeal of the Long Amendment [2]. The investment tax credit was suspended in 1966–1967 and repealed in 1969 in order to reduce the level of investment. The tax credit was re-enacted in 1971 to stimulate investment expenditures" [3]. These events suggest that policy regime changes took place over the period considered and, from Lucas' argument, we should observe parameter instability in the Gordon-Jorgenson model (unless expectations effectively obeyed the scheme implicitly assumed by Hall and Jorgenson). Further, since it is argued that the assumption of static expectations should lead to underestimating the impact of the tax credit, we also expect that the introduction of the investment tax credit be associated with underpredictions of investment expenditures.

To study such general effects, we need an exploratory methodology that is sensitive to a wide variety of possible structural changes and capable of providing information on the timing of structural change. Further, it should give indications on the direction of prediction errors associated with the use of a model. An attractive procedure of this type consists of estimating the model recursively (adding one observation at a time) and examining a number of resulting statistics. This approach was first formalized by Brown *et al.* (1975); a systematization as well as a number of extensions were provided by Dufour (1979, 1982, 1986). [For further work along those lines, see also Hackl (1980)]. Because it is especially well adapted to our objectives, this is the approach we will follow to study the Lucas effects.

In Section 25.2, we present the investment model that will be analyzed. In Section 25.3, we describe succinctly the methodology used and define the main statistics considered. In Section 25.4, we present the empirical results. In Section 25.5, we summarize our findings and conclusions.

25.2 The Model

The model studied by Gordon and Jorgenson (1976) is based on quarterly data and has the form

$$IPDE58_t = \alpha + \delta K_t + \sum_{i=0}^{6} \beta_i V_{t-i} + u_t \tag{25.1}$$

where $IPDE58_t$ is real investment (1958 dollars) in producers' durable equipment (during period t), K_t is gross beginning-of-period real capital stock of producers' durable equipment, V_t is a proxy for the desired capital stock defined as

$$V_t = (PGNP_{t-2})(GNP58_{t-1}/C_{t-2} \tag{25.2}$$

$GNP58_t$ is the real gross national product (1958 dollars), which, like $IPDE58_t$, is seasonally adjusted and measured at annual rates; $PGNP_t$ is the GNP price deflator, C_t is the rental cost of capital, and u_t is a random disturbance. The cost of capital C_t is given by the expression

$$C_t = PIPDE_t[0.138 + R_t(1-U_t)][1 - U_t Z_t - TC_t + Y_t Z_t TC_t U_t]/(1-U_t) \quad (25.3)$$

where $PIPDE_t$ is the price deflator for investment in producers' durable equipment, 0.138 is the depreciation rate on producers' durable equipment as calculated by Christensen and Jorgenson (1969), U_t is the nominal corporate tax rate, R_t is the interest rate on new issues of high-grade corporate bonds, Z_t is the present discounted value of depreciation allowances, TC_t is the effective tax credit and Y_t equals 1.0 during those years in which the Long Amendment applied and zero otherwise.

In order to estimate this model, Gordon and Jorgenson (1976) used a second-degree Almon polynomial lag structure constrained to pass through zero after seven periods. This imposes the restrictions

$$\beta_i = a_0 - a_1 i - a_2 i^2 \ , \quad i = 0, 1, \ldots, 7 \quad (25.4)$$

with $\beta_7 = a_0 - 7a_1 - 49a_2 = 0$, so that there are effectively only two free parameters in the distributed lag on V_t. Under these conditions, the equation to be estimated is

$$IPDE58_t = \alpha + \delta K_t + a_1 W_{1t} + a_2 W_{2t} + u_t \quad (25.5)$$

where

$$W_{1t} = \sum_{i=0}^{6}(7-i)V_{t-i} \quad \text{and} \quad W_{2t} = \sum_{i=0}^{6}(49-i^2)V_{t-i} \ . \quad (25.6)$$

Furthermore, since the original Durbin-Watson statistic was 0.7554, a first-order autoregressive transformation was used (with $\hat{\rho} = 0.6223$, estimated by the Cochrane-Orcutt method). The following equation, based on the period 1956/I–1972/IV, was finally obtained:

$$\begin{aligned} IPDE58_t = & \underset{(1.522)}{-9.656} + \underset{(0.0163)}{0.0572}\, K_t + \underset{(0.00071)}{0.00181}\, V_t + \underset{(0.00033)}{0.00218}\, V_{t-1} \\ & + \underset{(0.00019)}{0.00233}\, V_{t-2} + \underset{(0.00031)}{0.00228}\, V_{t-3} + \underset{(0.00038)}{0.00202}\, V_{t-4} + \underset{(0.00036)}{0.00156}\, V_{t-5} \\ & + \underset{(0.00023)}{0.00088}\, V_{t-6}, \quad R^2 = 0.9577, \quad DW = 1.9788, \quad SE = 1.0150. \end{aligned} \quad (25.7)$$

The sample 1956/I–1972/IV represents effective observations, not including those observations that are "lost" because of the presence of lagged explanatory variables and the autoregressive transformation. The standard errors are given in parentheses. R^2 is the multiple correlation coefficient, DW is the Durbin-Watson statistic, and SE is the standard error of the regression (all for the transformed model).

This model is based on a static-expectations assumption [see equation (25.2)]. By contrast, in his theoretical argument, Lucas (1976) considers a tax credit that follows a Markovian scheme, which includes as special cases both a permanent credit (i.e., the probability that the tax credit will disappear is zero) and a frequently imposed but always transitory credit (i.e., the probability that the tax credit will disappear soon is high). Assuming rational expectations on the part of investors, he then shows that the impact of the tax credit can be much bigger if it is viewed as transitory rather than permanent. Indeed, under reasonable values of the model parameters, the ratio of the actual to predicted effect may be in the range of 4 to 7.

In this chapter, we study the stability over time of the above model. For this purpose, we use an "exploratory" methodology aimed at being sensitive to a wide variety of instability patterns. It is based on estimating recursively the model under study and considering associated paths of coefficient estimates and prediction errors. An especially interesting aspect of this approach for our problem is that it can give us information on the timing of parameter shifts and the direction of prediction errors, two issues for which Lucas's conjecture has implications. In the next section, we give a succinct description of the methodology employed.

On the basis of this approach, we shall present (in section 25.4) the results of three different recursive estimation experiments with the same data as Gordon and Jorgenson (1976). In the first one, we simply estimate equation (25.5) recursively by ordinary least squares. In the second one, we take into account the fact that Gordon and Jorgenson (1976) made a correction for "autocorrelation" (which, however, may only be an *ad hoc* response to a parameter instability problem) and we study how the conclusions are affected after making such a correction. We thus estimate recursively the transformed model

$$IPDE58_t(\hat{\rho}) = \alpha(1-\hat{\rho}) + \delta K_t(\hat{\rho}) + a_1 W_{1t}(\hat{\rho}) + a_2 W_{2t}(\hat{\rho}) + \varepsilon_t^* \qquad (25.8)$$

where $\hat{\rho} = 0.6223$, $IPDE58_t(\hat{\rho}) = IPDE58 - \hat{\rho}\, IPDE58_{t-1}$, $K_t(\hat{\rho}) = K_t - \hat{\rho} K_{t-1}$, etc. [See Dufour (1982, Section 2.5) for a discussion of this procedure. Note that ρ is not recursively estimated.]

Finally, in the third experiment, we try to deal with an extra difficulty: since the capital stock K_t is a function of past investment, K_t cannot, strictly speaking, be taken as independent of the disturbance vector. The regressor K_t may be viewed as a form of lagged dependent variable, and the tests performed in the two first experiments cannot be considered exact. A suggested in Dufour (1982, Section 2.5), we get rid of the troublesome regressor $K_t(\hat{\rho})$ by subtracting $\hat{\delta} K_t(\hat{\rho})$ on both sides of (25.6) where $\hat{\delta}$ is the estimate of δ based on the full sample. We thus consider the regression

$$\mathrm{IPDE58}_t(\hat{\rho}) - \hat{\delta} K_t(\hat{\rho}) = \alpha(1-\hat{\rho}) + a_1 W_{1t}(\hat{\rho}) + a_2 W_{2t}(\hat{\rho}) + \varepsilon_t^* \qquad (25.9)$$

where $\hat{\delta} = 0.0572$ and $\hat{\rho} = 0.6223$, and perform the recursive estimation experiment on the remaining coefficients. Of course, this third experiment loses some of the advantages of "recursivity" (δ is not estimated recursively), which may lead to a loss of power. But it appears necessary in the present circumstances as a way of cross-checking the results obtained without taking into account the presence of a lagged dependent variable.

25.3 Methodology

In this section, which draws heavily on Dufour (1986, Section 2.3), we sketch the main features of recursive stability analysis and define the main statistics used. For a detailed description and more complete bibliography, the reader is referred to Dufour (1982).

Let us consider the following regression model:

$$y_t = x_t'\beta + u_t, \quad t = 1, \ldots, T \tag{25.10}$$

where y_t is an observation on the dependent variable, x_t is a $K \times 1$ column vector of nonstochastic regressors, β is a $K \times 1$ vector of regressor coefficients, u_t is a disturbance term that follows a normal distribution with mean zero and variance σ^2. Assume also that the disturbances $u_1, \ldots, u_T$ are independent.

We wish to investigate the constancy of the regression coefficient β over different observations. In other words, we consider the alternative model

$$y_t = x_t'\beta_t + u_t, \quad t = 1, \ldots, T \tag{25.11}$$

and wish to test the null hypothesis H_0: $\beta_1 = \ldots = \beta_T \equiv \beta$.

When the data have a natural order (e.g., time), a simple way to investigate the stability of regression coefficients is to estimate the model recursively. Using the first K observations in the sample to get an initial estimate of β, we gradually enlarge the sample, adding one observation at a time, and reestimate β at each step. We get in this way the sequence of estimates

$$b_r = (X_r'X_r)^{-1}X_r'Y_r, \quad r = K, \ldots, T, \tag{25.12}$$

where $X_r' = (x_1, \ldots, x_r)$ and $Y_r = (y_1, \ldots y_r)'$. We assume here that $\text{rank}(X_r) = K$, $r = K, \ldots, T$. A computationally efficient algorithm allows one to get this sequence easily [see Brown *et al.* (1975, p. 152)]. It is intuitively clear that the examination of this sequence of estimates can provide information on possible instabilities of the regression coefficients. We see also that two different permutations of the data usually yield different sequences of estimates. However, when the data are ordered (e.g., by time), it appears that the most easily interpretable results will be obtained by putting the data either in their original time order or in the reverse order. In the first case, one gets "forward recursive estimates"; and in the second case, "backward recursive estimates". In this chapter we will concentrate our attention on forward recursive estimation.

Recursive estimates are a descriptive device reflecting the influence of different observations in a sequential updating process. However, recursive estimates are strongly autocorrelated, even under the null hypothesis of stability, and the analysis of their behavior remains delicate from a statistical point of view. One can show easily that recursive estimates follow a "heteroscedastic random walk"; see Dufour (1982, eq.24). Thus the observation of a "trend" must be interpreted with great care. Consequently, it is important to have statistics that are easier to interpret. For this purpose, we look at the associated sequences of prediction errors. Namely, we consider the sequences of prediction errors

$$v_{kr} = y_r - x_r' b_{r-k}, \quad r = K + k, \ldots, T \tag{25.13}$$

where $1 \leq k \leq T - K$. Since these have different variances, it is convenient to standardize them and to compute

$$w_{kr} = \frac{v_{kr}}{d_{kr}}, \quad r = K + k, \ldots, T \tag{25.14}$$

where $d_{kr} = [1 + x_r'(X_{r-k}' X_{r-k})^{-1} x_r]^{\frac{1}{2}}$. We call the sequence w_{kr}, $r = K + k, \ldots, T$, "k-steps-ahead recursive residuals" — or simply "k-step recursive residuals". Depending on whether the sample is in forward or backward order, we get "forward" or "backward recursive residuals". It is easy to verify that, under the null hypothesis of stability,

$$E(w_{kr}) = 0, \quad E(w_{kr}^2) = \sigma^2.$$

Further, when $k = 1$, one has

$$E(w_{1r} w_{1s}) = 0, \quad r \neq s$$

so that the sequence $w_{1r}, r = K + 1, \ldots, T$, is a normal white noise. For $k \geq 2$, the sequence w_{kr}, $r = K + k, \ldots, T$, is dependent but only up to lag $k - 1$ [see Dufour (1982, pp. 41–44)].

It is not difficult to determine how relatively simple forms of structural change will affect the behavior of prediction errors (or recursive residuals). For example, a sudden shift in the coefficients at some point t_0 will, in many circumstances, lead to an increase in the size of prediction errors and/or a tendency to either overpredict or underpredict the dependent variable (for $t \geq t_0$); a systematic drift in one or several of the coefficients will often lead to a systematic tendency to over- or underpredict; etc. Thus, we will first use the sequences of standardized prediction errors to perform an exploratory analysis and search for patterns indicative of possible structural shifts. For this purpose, it is especially useful to look at several "clues". The simple statistical properties of the one-step recursive residuals (forward or backward) designate them as the basic instrument of analysis for that search. However, we will find instructive to compare these with the k-step recursive residuals ($k \geq 2$): since the latter are forecasts using estimates from a sample further away in time, they may exhibit better-defined and more recognizable patterns; they can also help to identify possible breakpoints.

When interpreting and comparing various sets of residuals, it is always useful to recall that all recursive residuals have the same standard error σ (under the null hypothesis). Interpretation will generally be easier if we scale the residuals with an estimate of σ. Since the most natural estimate is the one obtained from the full sample (i.e., the standard error of the regression), one computes

$$\tilde{w}_{kr} = \frac{w_{kr}}{\hat{\sigma}}, \qquad r = K + k, \ldots, T \tag{25.15}$$

where

$$\hat{\sigma}^2 = \frac{(Y_T - X_T b_T)'(Y_T - X_T b_T)}{(T - K)} \quad .$$

This procedure can also help display the recursive residuals, for in most practical situations, it will bring all residuals in a convenient scale — not too close to zero and well inside the interval $(-10, +10)$ [4].

Though the first purpose of the instruments we defined is exploratory, it is important to assess the statistical significance of what is observed. Because one-step recursive residuals have such simple statistical properties, we will use these for this assessment. Roughly speaking, we expect two main types of effects to result from structural shifts: tendencies either to over- or underpredict the dependent variable and discontinuities in the size of the prediction errors. Consequently, we will compute a number of simple statistics aimed at being sensitive to these characteristics. Statistics especially sensitive to the sign of prediction errors include the CUSUM test originally suggested by Brown *et al.* (1975), a Student t-test and the corresponding Wilcoxon signed-rank test, run tests based on the number of runs and the length of longest run, and serial dependence tests. Statistics sensitive to discontinuities in the size of prediction errors include the CUSUM of squares test suggested by Brown *et al.* (1975) and heteroscedasticity tests. We now define succincty the various test statistics.

If we let $w_t \equiv w_{1t}$, $t = K + 1, \ldots, T$, the CUSUM test is based on considering the cumulative sums

$$W_r = \frac{1}{\hat{\sigma}} \sum_{j=K+1}^{r} w_j, \qquad r = K + 1, \ldots, T \tag{25.16}$$

where $\hat{\sigma}^2 = S_T/(T - K)$ and $S_T = \sum_{t=K+1}^{T} w_t^2$. The null hypothesis H_0 is rejected at level α if $C \equiv \max_{K+1 \leq r \leq T} |\tilde{W}_r| \geq c_\alpha$, where

$$\tilde{W}_r = \frac{W_r}{\{\sqrt{T-K} + 2[(r-K)/\sqrt{T-K}]\}} \tag{25.17}$$

and c_α depends on the level of the test ($c_{0.01} = 1.143$, $c_{0.05} = 0.948$, $c_{0.10} = 0.850$). In other words, H_0 is rejected if the graph of W_r crosses either one of two straight lines determined by the level of the test. The t-test is based on the standard Student's t-statistic to test the hypothesis that the recursive residuals have zero mean against a systematic tendency to over- or underpredict. It is based on the statistic

$$\tilde{t} = \frac{\sqrt{T-K}\,\bar{w}}{s_w} \tag{25.18}$$

where

$$\bar{w} = \sum_{t=K+1}^{T} \frac{w_t}{T-K}, \qquad s_w^2 = \sum_{t=K+1}^{T} \frac{(w_t - \bar{w})^2}{T-K-1} \quad ; \tag{25.19}$$

under the null hypothesis, $\tilde{t}$ follows a Student's t-distribution with $T-K-1$ degrees of freedom. The Wilcoxon signed-rank test is based on the statistic

$$S = \sum_{t=K+1}^{T} u(w_t)\, R_t^+ \tag{25.20}$$

where

$$\begin{aligned} u(z) &= \begin{cases} 1, & \text{if } z \geq 0 \\ 0, & \text{if } z < 0 \end{cases} \\ R_t^+ &= \sum_{i=K+1}^{T} u(|w_t| - |w_i|) \ . \end{aligned} \tag{25.21}$$

We may view it as a robust alternative to the t-test; its distribution (for $T \leq 50$) has been extensively tabulated by Wilcoxon *et al.* (1973, pp. 171–259). For $T > 50$, one can use the standardized form $X' = [S - E(S)]/\sigma(S)$, where $E(S) = n(n+1)/4$, $\sigma(S) = [n(n+1)(2n+1)/24]^{\frac{1}{2}}$ and $n = T - K$. Under H_0, S' is approximately $N(0,1)$.

An intuitively attractive way of looking at the sequence of the recursive residuals consists of observing runs of overpredictions (or underpredictions), as defined by the sequence $u(w_t)$, $t = K+1, \ldots, T$. Two simple tests are then obtained by considering the number of runs R in this sequence or by observing the length of the longest run in the sequence. The distribution of the number of runs R is obtained by noting that $R - 1$ follows a binominal distribution $\text{Bi}(T-K-1, \frac{1}{2})$; a small number of runs suggests that the model has a tendency to overpredict or underpredict. Besides, an especially long run of negative or positive errors of prediction suggests the presence of a shift. The probability of getting at least one run of a given length or greater may be computed from formulas (135) to (138) in Dufour (1982).

In a large number of cases, structural change leads to situations where the means of the cross products $Z_{kt} \equiv w_t w_{t+k}, t = K+1, \ldots, T-k$ (where $1 \leq k \leq T-K-1$) differ from zero [see Dufour (1982, pp. 52-55)]. This suggests testing whether Z_{kt} has mean zero: we can do this by using "serial dependence tests" not corrected for the mean (for such tests are more accurately viewed in this context as location tests rather than serial dependence tests). We will consider two types of statistics for doing this: the modified von Neumann ratio

$$VR = \frac{(n-1)^{-1}\sum_{t=K+1}^{T-1}(w_{t+1}-w_t)^2}{n^{-1}\sum_{t=K+1}^{T} w_t^2} \tag{25.22}$$

where $n = T - K$, and rank Wilcoxon-type serial dependence tests based on statistics of the form

$$S_k = \sum_{t=K+1}^{T-k} u(Z_{kt})\, R_{kt}^{+}, \qquad k = 1, 2, \ldots \tag{25.23}$$

where $R_{kt}^{+} = \sum_{i=K+1}^{T-k} u(|Z_{kt}| - |Z_{ki}|)$. VR provides an exact parametric test of the null hypothesis $E(w_t w_{t+1}) = 0, t = K+1, \ldots, T-1$; for a table, see Theil (1971, pp. 728–729). Each statistic S_k is distributed like the Wilcoxon signed-rank statistic to test the zero median hypothesis; it gives an exact test of the null hypothesis $E(w_t w_{t+k}) = 0$, $t = K+1, \ldots, T-k$, where $k \geq 1$ [see Dufour (1981)]. Further, under $H_0, E(S_k) = n_k(n_k+1)/4$, and $\sigma(S_k) = [n_k(n_k+1)(2n_k+1)/24]^{1/2}$, where $n_k = T - K - k$.

The CUSUM of squares test is based on considering the statistic

$$S_r = \frac{\sum_{j=K+1}^{r} w_j^2}{\sum_{j=K+1}^{T} w_j^2}, \qquad r = K+1, \ldots, T \tag{25.24}$$

The null hypothesis is rejected at level α if

$$S \equiv \max_{1 \leq j \leq T-K-1} |S_{K+j} - \frac{j}{T-K}| \geq d_\alpha \tag{25.25}$$

where d_α is obtained by entering Table 1 of Durbin (1969) at $\alpha/2$ and $n = (\frac{1}{2})(T-K)-1$ if $T-K$ is even, or by interpolating linearly between $n = (\frac{1}{2})(T-K)-(3/2)$ and $n = (\frac{1}{2})(T-K)-(\frac{1}{2})$ if $T-K$ is odd. We do not use heteroscedasticity tests in this paper and so do not need to define them here.

Whenever possible we will report the marginal significance level (p-value) of each statistic. Of course, a test significant at a very low level provides stronger evidence against the null hypothesis. Note also that any of the tests suggested above can be applied to a subset of the one-step recursive residuals, provided this subset is suggested by *a priori* considerations (e.g., dates of policy changes).

25.4 Recursive Stability Analysis of Investment Demand

As indicated in Section 25.2, the first experiment consists of estimating equation (25.5) recursively by ordinary least squares (1956/I–1972/IV) [5]. The recursive estimates obtained are listed in *Table 25.1* and graphed in *Figure 25.1(a–d)*; the corresponding recursive residuals (1, 2, 3, 4, and 8 steps ahead) are listed in *Table 25.2*, with a number of test statistics in *Table 25.3*, and they are graphed in *Figure 25.2(a–d)* [6].

When we look at the recursive estimates, we distinguish four main phases:

1. The first phase (beginning to 1961/I) is characterized by relatively large fluctuations (including some "weird" values, especially at the very beginning, which is not surprising for, at the beginning, the estimations are based on few observations) and by a rough trend (upward for α and a_1, downward for δ and a_2).
2. The second phase (1961/II–1963/III) is one of relative stability exhibiting no clear trend, expect for δ which increases after 1962/IV.
3. The third phase (1963/IV–1966/IV) displays well-defined trends (downward for α and a_2, upward for δ and a_1) during which all coefficients change sign.
4. Finally, during the fourth phase (1967/I–1972/IV), a_1 and a_2 move in directions opposite to the ones they followed in the previous phase, while α and δ seem stable.

Thus, the fourth quarters of 1963 and 1966 appear to be breakpoints.

When we examine one-step-ahead recursive residuals, we find no systematic tendency to over- or underpredict over the full period (as indicated by the global location tests in *Table 25.3*). However, we can observe a run of 13 consecutive underpredictions from 1963/IV to 1966/IV, a very surprising outcome if the model is correct: under the null hypothesis of stability, the probability of getting at least one run of this length or more is 0.0065. The total number of runs of either over- or underpredictions (16) is extremely small in relation to the sample size, and there is strong evidence of serial dependence (at least up to a distance of 3 quarters). Indeed, the trajectory of the one-step recursive residuals has several striking features. The first period (beginning to 1963/III) exhibits a tendency to overpredict (negative residuals). This phenomenon is also indicated by the CUSUM test [see *Figure 25.2(f)*]. Note also that the CUSUM of squares test is not significant (at level 0.05). Next, we note a long run of 13 consecutive underpredictions (1963/IV–1966/IV), a "breakpoint" between 1966/IV and 1967/I, another run of 9 underpredictions (1967/IV–1969/IV), while the sequel of the series looks relatively "random". We can also observe that two-, three-, and four-steps-ahead recursive residuals display basically the same pattern. The form of the pattern is in fact clearer from the latter than from the one-step residuals.

It is interesting to compare the trajectory of the one-step recursive residuals with the movement of the effective investment tax credit [7]. The long run of underpredictions starts in 1963/IV, which roughly coincides with the repeal of the Long Amendment (1964/I),

Table 25.1: Gordon-Jorgenson model: forward recursive estimates (OLSQ): 1956/I–1972/IV.

Quarter [a]	α	δ	a_1	a_2
56.04	-326.102	1.6091	-0.0183925	0.0018862
57.01	-355.362	1.7910	-0.0199375	0.0020607
57.02	-200.678	0.9335	-0.0113759	0.0011276
57.03	-258.690	1.2004	-0.0145302	0.0014536
57.04	103.729	-0.2716	0.0044718	-0.0004420
58.01	51.225	-0.3009	0.0038135	-0.0004790
58.02	-201.634	0.5238	-0.0078796	0.0006041
58.03	-213.437	0.5234	-0.0076991	0.0005612
58.04	-133.600	0.1898	-0.0028328	0.0000649
59.01	-121.596	0.1442	-0.0021972	0.0000024
59.02	-127.719	0.1655	-0.0024678	0.0000277
59.03	-86.004	0.0339	-0.0010597	-0.0000916
59.04	-57.819	-0.0476	-0.0005529	-0.0001204
60.01	-40.011	-0.0949	-0.0004649	-0.0001113
60.02	-39.388	-0.0965	-0.0004630	-0.0001109
60.03	-27.361	-0.1262	-0.0004526	-0.0000987
60.04	-12.912	-0.1608	-0.0005411	-0.0000723
61.01	7.845	-0.2074	-0.0007922	-0.0000198
61.02	20.961	-0.2326	-0.0011519	0.0000372
61.03	27.251	-0.2398	-0.0011481	0.0000465
61.04	29.245	-0.2407	-0.0011379	0.0000489
62.01	29.312	-0.2407	-0.0011375	0.0000489
62.02	28.384	-0.2413	-0.0011478	0.0000481
62.03	28.268	-0.2420	-0.0011981	0.0000533
62.04	28.814	-0.2319	-0.0007962	0.0000129
63.01	30.011	-0.2123	-0.0007552	0.0000171
63.02	31.475	-0.1941	-0.0011388	0.0000686
63.03	31.800	-0.1914	-0.0012565	0.0000832
63.04	31.227	-0.1943	-0.0010485	0.0000580
64.01	30.255	-0.1965	-0.0007345	0.0000205
64.02	28.943	-0.1961	-0.0004098	-0.0000180
64.03	27.405	-0.1945	-0.0001861	-0.0000452
64.04	26.252	-0.1911	-0.0000746	-0.0000586
65.01	23.723	-0.1807	0.0002025	-0.0000908
65.02	21.435	-0.1694	0.0004589	-0.0001199
65.03	17.259	-0.1464	0.0008914	-0.0001683
65.04	12.713	-0.1186	0.0013065	-0.0002138
66.01	7.462	-0.0837	0.0016886	-0.0002547
66.02	2.236	-0.0461	0.0019594	-0.0002822
66.03	-2.754	-0.0090	0.0022482	-0.0003115
66.04	-6.225	0.0172	0.0025062	-0.0003381
67.01	-3.042	-0.0074	0.0022301	-0.0003095
67.02	-2.278	-0.0128	0.0020597	-0.0002909
67.03	-1.595	-0.0184	0.0019768	-0.0002822
67.04	-2.542	-0.0093	0.0019180	-0.0002744
68.01	-5.921	0.0244	0.0015286	-0.0002263
68.02	-6.344	0.0287	0.0014691	-0.0002190
68.03	-7.195	0.0375	0.0013446	-0.0002039
68.04	-7.587	0.0415	0.0012932	-0.0001976
69.01	-8.738	0.0531	0.0011811	-0.0001834
69.02	-8.968	0.0554	0.0011641	-0.0001812
69.03	-9.197	0.0578	0.0011459	-0.0001788
69.04	-9.346	0.0592	0.0011719	-0.0001815
70.01	-8.954	0.0553	0.0010426	-0.0001677
70.02	-8.798	0.0536	0.0009942	-0.0001625
70.03	-9.123	0.0576	0.0009975	-0.0001622
70.04	-7.915	0.0421	0.0012129	-0.0001890
71.01	-7.838	0.0410	0.0012349	-0.0001916
71.02	-8.278	0.0469	0.0010713	-0.0001723
71.03	-8.479	0.0495	0.0009754	-0.0001612
71.04	-9.088	0.0572	0.0006227	-0.0001205
72.01	-9.427	0.0612	0.0004061	-0.0000956
72.02	-9.405	0.0609	0.0004209	-0.0000973
72.03	-9.213	0.0593	0.0005416	-0.0001110
72.04	-9.204	0.0593	0.0005455	-0.0001114

[a] End-of-sample quarter. All samples start in 56.01 (1956/I).

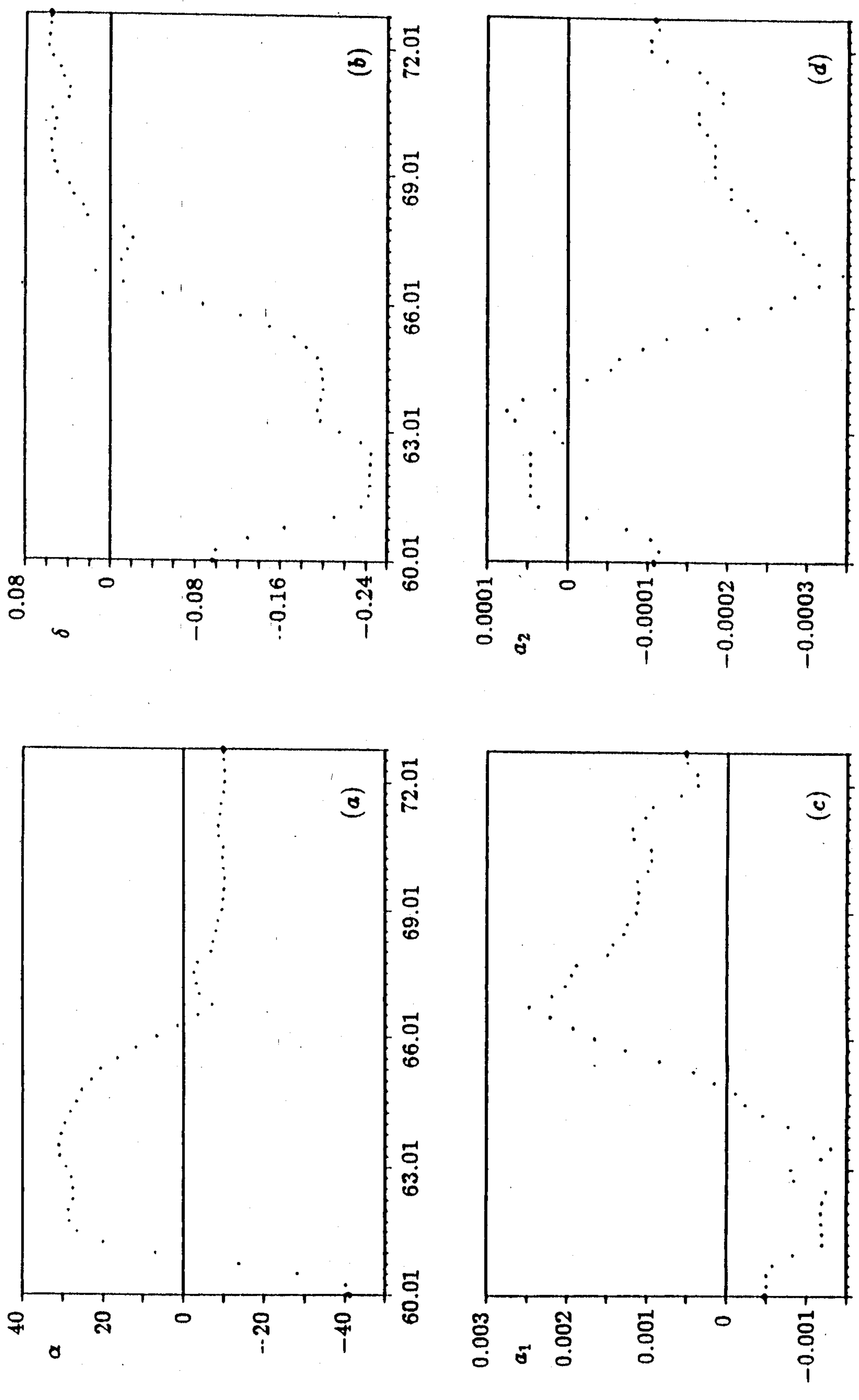

Figure 25.1: Gordon-Jorgenson model (OLSQ): recursive estimates.

Table 25.2: Gordon-Jorgenson model: forward recursive residuals (OLSQ): 1956/I–1972/IV

Quarter [a]	*1 step*	*2 steps*	*3 steps*	*4 steps*	*8 steps*
57.01	0.0352	-	-	-	-
57.02	-0.3443	-0.0826	-	-	-
57.03	0.1202	-0.2410	-0.0586	-	-
57.04	-0.9358	-0.3520	-0.4835	-0.1767	-
58.01	-2.0437	-1.9434	-0.7658	-0.6970	-
58.02	-0.9692	-1.6723	-1.4633	-1.1100	-
58.03	-0.7965	-1.2545	-2.1070	-2.3049	-
58.04	-0.8318	-1.0136	-0.6006	-1.4623	-0.2768
59.01	-0.1621	-0.7309	-0.8453	0.0888	-0.5647
59.02	0.1215	-0.0394	-0.6529	-0.7163	-0.3496
59.03	-1.0660	-0.6810	-0.5661	-0.9969	-0.6585
59.04	-1.1178	-1.5424	-1.0406	-0.8184	0.2308
60.01	-0.9078	-1.3542	-1.7216	-1.1940	0.2001
60.02	-0.0415	-0.4826	-1.0236	-1.4752	-1.1135
60.03	-1.0557	-0.9519	-1.2886	-1.6988	-1.3356
60.04	-1.6579	-1.9499	-1.7102	-1.9269	-1.2509
61.01	-3.0500	-3.4503	-3.5545	-3.0574	-2.0987
61.02	-2.4494	-3.3145	-3.6992	-3.7670	-3.0522
61.03	-1.5073	-2.1589	-3.1013	-3.5108	-3.0344
61.04	-0.5886	-1.0080	-1.7293	-2.7538	-2.7488
62.01	-0.0238	-0.1942	-0.6676	-1.4533	-2.5886
62.02	0.3981	0.3744	0.1668	-0.3661	-3.0006
62.03	0.1264	0.2424	0.2210	-0.0026	-3.0570
62.04	-1.2763	-1.0963	-0.8944	-0.8431	-3.4240
63.01	-1.8985	-2.2543	-1.9576	-1.6653	-2.9185
63.02	-1.5525	-2.1554	-2.4872	-2.1761	-2.2537
63.03	-0.2502	-0.8525	-1.5101	-1.8747	-1.3751
63.04	0.3969	0.2401	-0.4329	-1.1003	-0.9530
64.01	0.6896	0.7948	0.5781	-0.1357	-0.7935
64.02	0.9716	1.1661	1.2154	0.9388	-0.7791
64.03	1.6153	1.8299	1.9545	1.9152	-0.5585
64.04	1.0617	1.4188	1.6490	1.7873	-0.2733
65.01	1.8532	2.0483	2.3979	2.5852	1.0665
65.02	1.3797	1.7851	1.9928	2.3620	1.9435
65.03	2.3894	2.6599	3.0484	3.2200	3.2393
65.04	2.3603	2.8922	3.1595	3.5466	3.9159
66.01	2.5607	3.0896	3.6277	3.8720	4.6526
66.02	2.3817	2.9778	3.5237	4.0661	5.0238
66.03	2.2171	2.8001	3.4335	3.9931	5.1704
66.04	1.5839	2.2018	2.8415	3.5181	5.1804
67.01	-1.5789	-0.9150	-0.0800	0.7362	3.2344
67.02	-0.5451	-1.1402	-0.4052	0.4580	3.4158
67.03	-0.5047	-0.7066	-1.3239	-0.5287	2.7265
67.04	0.4544	0.2537	0.0078	-0.6620	2.5274
68.01	1.6167	1.6684	1.4435	1.1773	2.5613
68.02	0.2516	0.9135	1.0190	0.8428	1.4060
68.03	0.6852	0.7289	1.3712	1.4368	0.9470
68.04	0.4210	0.6192	0.6681	1.3293	0.2713
69.01	1.6063	1.6606	1.7926	1.7423	1.6934
69.02	0.3751	0.7129	0.7986	0.9801	1.4135
69.03	0.4108	0.4787	0.8364	0.9210	1.7631
69.04	0.3736	0.4438	0.5115	0.8600	1.7502
70.01	-1.2635	-1.1376	-1.0360	-0.9323	-0.0573
70.02	-0.5683	-0.8777	-0.7516	-0.6496	0.2055
70.03	0.9227	0.7655	0.4169	0.5001	1.1474
70.04	-2.7727	-2.5033	-2.5670	-2.7944	-1.6355
71.01	-0.1711	-0.8527	-0.6282	-0.7284	-0.5813
71.02	1.0118	0.9240	0.1605	0.3545	0.2081
71.03	0.4975	0.7913	0.7025	-0.0633	-0.0538
71.04	1.8427	1.8979	2.1286	1.9701	1.1664
72.01	1.3878	1.9639	2.0065	2.2322	1.3999
72.02	-0.1190	0.3230	0.9883	1.1032	0.6790
72.03	-1.3400	-1.3280	-0.8496	-0.1343	-0.3332
72.04	-0.0745	-0.2912	-0.3096	0.0560	0.9355

[a] Scaled recursive residuals are not reported: the standard error of the transformed regression is $\hat{\sigma} = 1.015$ (based on the sample 1956/I–1972/IV).

Table 25.3: Gordon-Jorgenson model: test statistics (OLSQ), based on the 64 one-step-ahead recursive residuals.

Type	*Indicator*						*Result*	*p-values* [a]
Global location tests [b]	t-test						0.619	0.9506
	Number of positive residuals						32	1.0000
	Wilcoxon test						1053	0.9307
Runs tests [c]	Number of runs						16	0.000019
	Length of longest run						13	0.0065
Serial correlation tests [d]	Modified von Neumann ratio						0.6779	< 0.002
	Rank tests							
	k	Signed-rank tests			Sign tests			
		S_k	S'_k	*p-value*	S_k	S'_k	*p-value*	
	1	1735	4.977	0.00000065	48	4.158	0.000038	
	2	1421	3.116	0.0018	41	2.540	0.0151	
	3	1284	2.431	0.0150	37	1.664	0.1237	
	4	1091	1.296	0.1951	33	0.7746	0.5190	
	5	1041	1.177	0.2390	33	0.9113	0.4350	
	6	1095	1.854	0.0637	35	1.576	0.1480	
	7	1058	1.839	0.0659	35	1.722	0.1112	
	8	983	1.509	0.1313	32	1.069	0.3497	
	9	1015	2.053	0.0401	33	1.483	0.1770	
	10	958	1.856	0.0635	32	1.361	0.2203	
	11	877	1.430	0.1528	31	1.236	0.2717	
	12	807	1.075	0.2825	30	1.109	0.3317	

[a] Marginal significance levels.

[b] See Dufour (1982, Section 4.3). The tests are two-sided.

[c] See Dufour (1982, Section 4.5). The tests are one-sided: $P[R \leq 16] = 0.000019$ and $P[L \geq 16] = 0.0065$, where R = number of runs (of +'s or -'s) and L = length of the longest run.

[d] S_k is a rank statistic for testing serial dependence [see Dufour (1982, Section 4.6)], k being the lag used; test; $S'_k = (S_k - E_0(S_k))/Std_0(S_k)$. We consider here two-sided tests (against positive or negative serial dependence). For a more complete theory of these tests, see Dufour (1981).

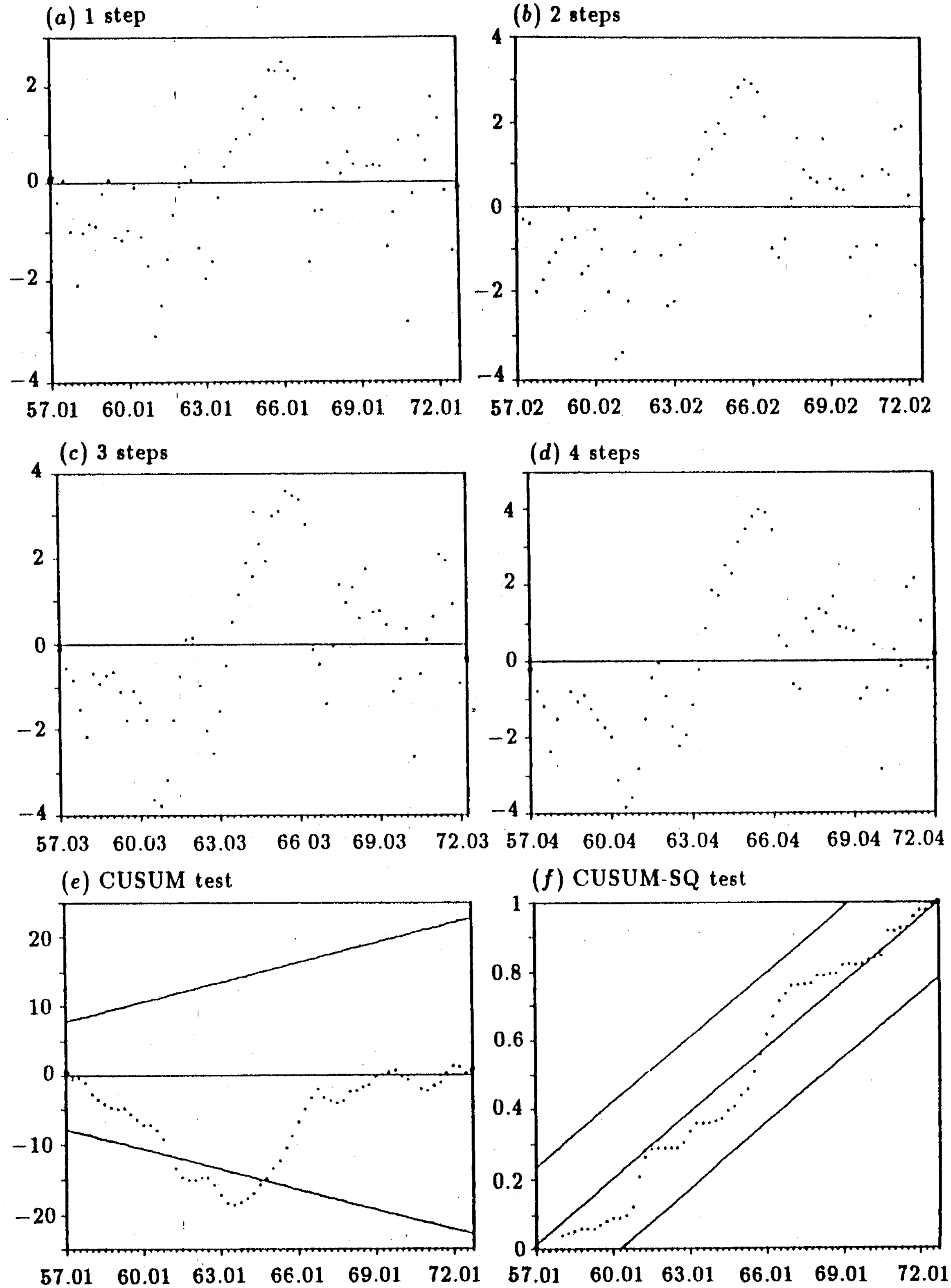

Figure 25.2: Gordon-Jorgenson model (OLSQ): recursive residuals and CUSUM tests. Significance boundaries in (*e*) and (*f*) correspond to tests of level 0.05.

Table 25.4: t-statistics for subperiods (OLSQ).

Period [a]	t	*p-value*
1962/I–1966/III	2.553	0.0200
1964/I–1966/III	8.834	0.00000251
1967/II–1969/I	1.724	0.128
1971/II–1972/IV	1.127	0.303
Remainder [b]	−3.790	0.000705

[a]1962/I–1966/III corresponds to the first application of the tax credit; 1964/I–1966/III is the same period after the repeal of the Long Amendment; 1967/II–1969/I corresponds to the second application and 1971/II–1972/IV to the third one.

[b]1957/I–1961/IV, 1966/IV–1967/I, 1969/II–1971/I.

and lasts as long as the effective tax credit is nonzero (up to 1966/IV). The suspension of the tax credit (1967/I) is associated with a discontinuity in the same series, while the following run of underpredictions (1967/IV–1969/IV) can be related to the reimposition of the tax credit (1967/II–1969/I).

On this issue, it is also instructive to compute t-statistics to test the null hypothesis of a zero mean (based on the one-step-ahead recursive residuals) for each of the subperiods corresponding to the different phases of the tax credit. This is justified by the fact that the (one-step-ahead) recursive residuals are i.i.d.$N(0,\sigma^2)$ under the null hypothesis [see Dufour (1982, Section 4.3)]. The results of these calculations are given in *Table 25.4*. From the latter, it is remarkable that each period where the effective tax credit is nonzero corresponds to a positive t-statistic (indicating a tendency to underpredict), while the period over which it does not apply yields a negative t-statistic. This effect is especially strong for the first application of the tax credit after the repeal of the Long Amendment.

Thus, if we estimate recursively equation (25.5) by ordinary least squares, we find several clues of instability. In particular, the results point to the presence of a substantial shift associated with the first imposition of the investment tax credit, especially after the repeal of the Long Amendment. Furthermore, this shift induced systematic underprediction of the level of investment expenditures over the corresponding period. On the other hand, the two other applications of the tax credit are not associated with statistically significant effects, even though the t-statistics are also positive.

Consider now the results of a similar experiment applied to equation (25.8), i.e., model (25.5) after correction for autocorrelation (using $\hat{\rho} = 0.6223$). The recursive estimates are listed in *Table 25.5* and graphed in *Figures 25.3(a–d)*; the recursive residuals are listed in *Table 25.6* with a number of test statistics in *Table 25.7*, and they are graphed in *Figures 25.4(a–d)* [8]. When we look at the recursive estimates, we can still observe the same basic phases: first (1957/I–1961/I), wide fluctuations with rough trends (upward for α and a_2, downward for δ and a_1); second (1961/II–1963/IV), a period of relative stability with no general trend (expect for δ which starts to increase near 1961/IV); third (1963/IV–1966/IV), well-defined trends for all coefficients (downward for α and a_2, upward

Table 25.5: Gordon-Jorgenson model: forward recursive estimates (data transformed with $\hat{\rho} = 0.6223$): 1956/I–1972/IV.

Quarter	α	δ	a_1	a_2
56.04	-363.656	-1.9131	0.0519153	-0.0071130
57.01	-271.063	1.2423	-0.0022213	0.0000800
57.02	-306.493	1.1325	0.0015707	-0.0004430
57.03	-313.996	1.4091	-0.0048950	0.0003493
57.04	255.175	-0.7712	0.0072497	-0.0006241
58.01	-391.011	1.1731	0.0056639	-0.0010445
58.02	-312.995	0.9128	0.0062321	-0.0010437
58.03	-252.689	0.7487	0.0023925	-0.0005566
58.04	-259.507	0.8018	-0.0006244	-0.0002177
59.01	-277.221	0.8733	-0.0013764	-0.0001448
59.02	-273.910	0.8607	-0.0013323	-0.0001475
59.03	-103.182	0.2313	-0.0007210	-0.0000952
59.04	-50.171	0.0343	-0.0014466	0.0000221
60.01	-38.357	-0.0089	-0.0016395	0.0000519
60.02	-53.803	0.0468	-0.0015769	0.0000337
60.03	-23.015	-0.0706	-0.0018095	0.0000797
60.04	-0.459	-0.1558	-0.0021968	0.0001378
61.01	33.432	-0.2782	-0.0028205	0.0002315
61.02	41.104	-0.3030	-0.0031011	0.0002691
61.03	42.370	-0.3038	-0.0030304	0.0002635
61.04	40.573	-0.3014	-0.0030468	0.0002626
62.01	38.703	-0.2985	-0.0030512	0.0002605
62.02	36.644	-0.2961	-0.0030638	0.0002588
62.03	35.078	-0.2750	-0.0023387	0.0001821
62.04	36.499	-0.2411	-0.0019230	0.0001498
63.01	40.351	-0.2223	-0.0024909	0.0002265
63.02	41.286	-0.2195	-0.0026724	0.0002494
63.03	37.019	-0.2210	-0.0018309	0.0001472
63.04	33.500	-0.2135	-0.0012767	0.0000814
64.01	30.380	-0.2017	-0.0009162	0.0000394
64.02	26.554	-0.1819	-0.0006697	0.0000114
64.03	23.462	-0.1666	-0.0008041	0.0000256
64.04	23.063	-0.1643	-0.0007949	0.0000246
65.01	15.427	-0.1174	-0.0003890	-0.0000193
65.02	13.657	-0.1057	-0.0002985	-0.0000288
65.03	4.147	-0.0414	0.0001009	-0.0000698
65.04	-1.534	-0.0006	0.0003055	-0.0000897
66.01	-8.201	0.0494	0.0004660	-0.0001035
66.02	-12.363	0.0820	0.0005219	-0.0001067
66.03	-16.481	0.1144	0.0006648	-0.0001197
66.04	-17.838	0.1252	0.0007345	-0.0001264
67.01	-5.792	0.0266	0.0000840	-0.0000639
67.02	-8.587	0.0465	0.0008241	-0.0001450
67.03	-7.345	0.0350	0.0008335	-0.0001475
67.04	-7.942	0.0409	0.0007639	-0.0001390
68.01	-11.173	0.0732	0.0003778	-0.0000914
68.02	-8.527	0.0467	0.0006748	-0.0001282
68.03	-9.310	0.0544	0.0006067	-0.0001196
68.04	-9.254	0.0539	0.0006092	-0.0001199
69.01	-10.988	0.0698	0.0006009	-0.0001170
69.02	-10.243	0.0626	0.0006111	-0.0001191
69.03	-10.426	0.0645	0.0006022	-0.0001178
69.04	-10.709	0.0673	0.0006918	-0.0001274
70.01	-9.662	0.0556	0.0004967	-0.0001075
70.02	-9.905	0.0586	0.0005126	-0.0001088
70.03	-11.118	0.0743	0.0003393	-0.0000866
70.04	-7.688	0.0284	0.0011028	-0.0001802
71.01	-8.888	0.0446	0.0007806	-0.0001413
71.02	-9.601	0.0541	0.0005443	-0.0001132
71.03	-9.385	0.0514	0.0006417	-0.0001245
71.04	-10.134	0.0596	0.0002408	-0.0000785
72.01	-10.252	0.0607	0.0001823	-0.0000719
72.02	-9.901	0.0584	0.0003461	-0.0000903
72.03	-9.456	0.0571	0.0004711	-0.0001039
72.04	-9.656	0.0572	0.0004686	-0.0001039

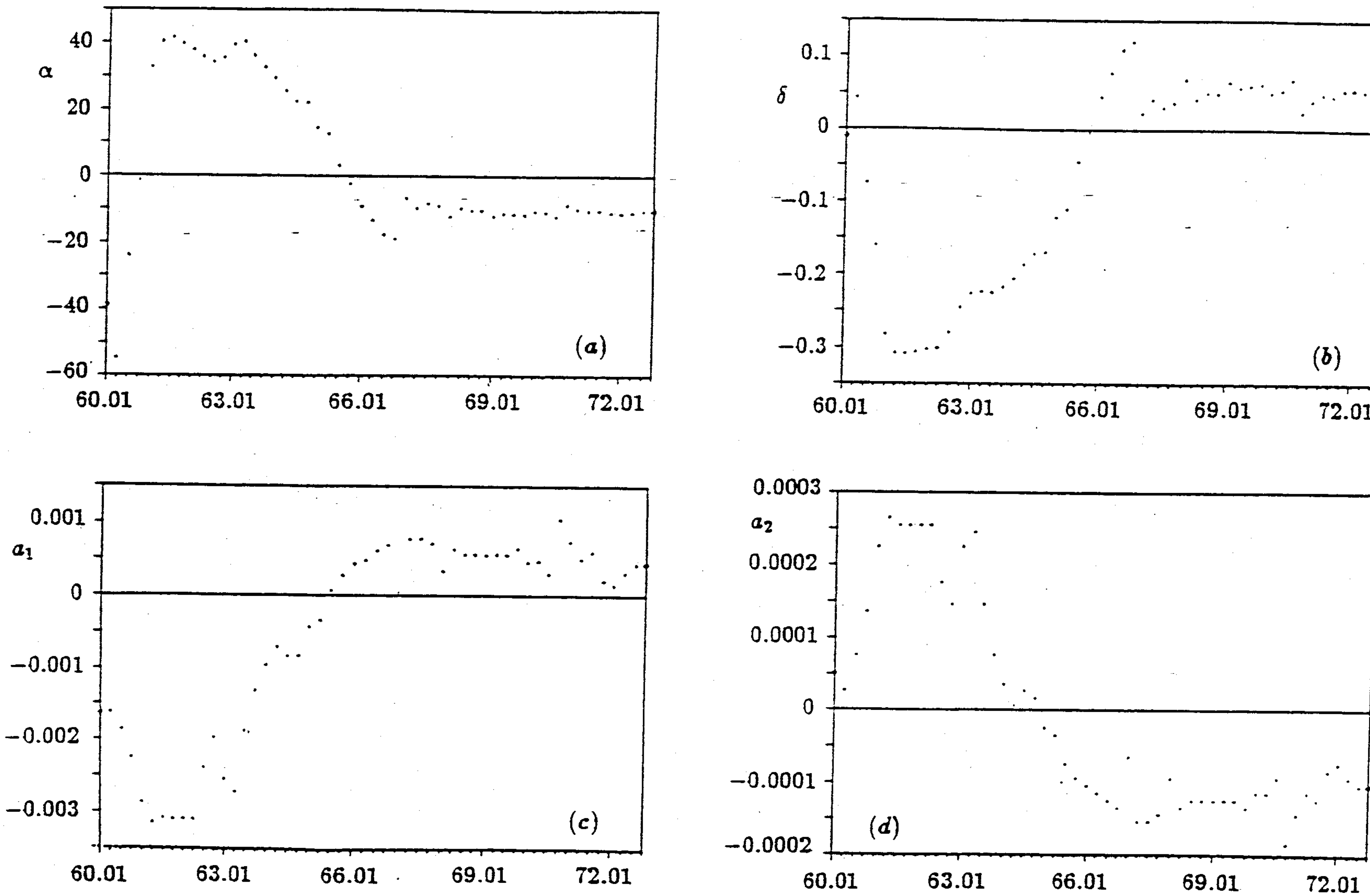

Figure 25.3: Gordon-Jorgenson model ($\hat{\rho} = 0.6223$): recursive estimates.

Table 25.6: Gordon-Jorgenson model: forward recursive residuals (data transformed with $\hat{\rho} = 0.6223$): 1956/I–1972/IV [a]

Quarter	*1 step*	*2 steps*	*3 steps*	*4 steps*	*8 steps*
57.01	0.8617	–	–	–	–
57.02	-0.2876	0.7745	–	–	–
57.03	0.5832	0.0882	0.8594	–	–
57.04	-1.1129	-0.4005	-0.4743	0.7274	–
58.01	-1.9398	-2.1876	-0.9812	-0.7629	–
58.02	0.1589	-1.0499	-0.4120	-0.1960	–
58.03	0.5489	0.5060	-0.7196	-0.6797	–
58.04	0.5562	0.7814	0.7972	0.0316	0.7833
59.01	0.2795	0.5693	0.7879	0.7170	-0.1665
59.02	-0.0302	0.1087	0.4062	0.6159	0.0048
59.03	-1.4740	-1.1866	-0.9079	-0.5339	-0.4487
59.04	-0.6864	-1.4794	-1.2503	-1.0678	0.4201
60.01	-0.2317	-0.5886	-1.4267	-1.2044	-0.6091
60.02	0.4567	0.2821	-0.1955	-1.1910	-0.7522
60.03	-1.1254	-0.8059	-0.8034	-1.0566	-1.1056
60.04	-1.0778	-1.4389	-1.0643	-1.0100	-1.3586
61.01	-2.1211	-2.3610	-2.6153	-2.0790	-1.9059
61.02	-0.6305	-1.1889	-1.4830	-1.8146	-2.0349
61.03	-0.1769	-0.3097	-0.9042	-1.2230	-1.2533
61.04	0.2439	0.1839	0.0216	-0.6485	-0.9146
62.01	0.2705	0.3254	0.2559	0.0762	-0.9703
62.02	0.3446	0.4043	0.4581	0.3720	-1.4314
62.03	-0.5286	-0.4090	-0.3113	-0.2141	-1.3980
62.04	-1.4905	-1.5515	-1.3492	-1.1775	-1.8167
63.01	-1.0779	-1.5502	-1.6256	-1.3894	-1.1316
63.02	-0.2264	-0.6454	-1.1347	-1.2500	-0.6952
63.03	0.8557	0.6718	0.1440	-0.3490	-0.0831
63.04	0.6493	0.9670	0.7573	0.2130	-0.0957
64.01	0.5692	0.7803	1.0892	0.8612	-0.1237
64.02	0.6716	0.8141	1.0016	1.2811	-0.1755
64.03	1.0488	1.1659	1.2715	1.4139	0.0405
64.04	0.0779	0.3026	0.4656	0.6190	0.2725
65.01	1.2389	1.2223	1.4180	1.5533	1.6100
65.02	0.2578	0.5865	0.5869	0.8015	1.5434
65.03	1.5437	1.5541	1.8458	1.7981	2.2194
65.04	0.8328	1.2251	1.2443	1.5695	1.9277
66.01	0.9921	1.1928	1.5967	1.5917	2.1479
66.02	0.6097	0.8805	1.1017	1.5327	1.9872
66.03	0.6486	0.8110	1.0981	1.3219	1.9889
66.04	0.2372	0.4550	0.6430	0.9605	1.9735
67.01	-2.4115	-2.1238	-1.6915	-1.3304	0.2121
67.02	1.0585	-0.0779	0.0356	0.3018	1.5654
67.03	-0.2880	0.2444	-0.8673	-0.6599	0.6505
67.04	0.1035	0.0075	0.1772	-0.6717	0.4132
68.01	0.8737	0.8043	0.6673	0.8197	0.5047
68.02	-1.0001	-0.5752	-0.4352	-0.5072	-0.6355
68.03	0.4017	0.0870	0.4180	0.4110	-0.3358
68.04	-0.0358	0.0552	-0.2357	0.1098	-0.7244
69.01	1.3803	1.3494	1.4046	1.0719	1.2657
69.02	-0.6068	-0.3531	-0.3529	-0.2463	-0.1877
69.03	0.1533	0.0395	0.2996	0.2842	0.3840
69.04	0.3481	0.3692	0.2434	0.5176	0.5684
70.01	-1.3258	-1.1676	-1.1161	-1.2200	-1.0088
70.02	0.3041	-0.0354	0.0668	0.0978	0.3577
70.03	1.2032	1.2403	0.9179	0.9743	1.0602
70.04	-3.4415	-3.0240	-2.8938	-3.1034	-2.4678
71.01	1.4386	0.4556	0.7541	0.7997	0.4413
71.02	0.9690	1.3068	0.2586	0.5726	0.4643
71.03	-0.3259	-0.0517	0.3485	-0.6952	-0.4223
71.04	1.4282	1.2599	1.4814	1.8241	0.9441
72.01	0.2908	0.6649	0.5394	0.7795	0.4317
72.02	-1.0287	-0.9480	-0.5379	-0.6084	-0.5938
72.03	-1.4314	-1.5655	-1.4939	-1.1499	-1.4101
72.04	0.6363	0.5052	0.4022	0.4314	0.9185

[a] Scaled recursive residuals are not reported: the standard error of the transformed regression is $\hat{\sigma} = 1.015$ (based on the sample 1956/I–1972/IV).

Table 25.7: Gordon-Jorgenson model ($\hat{\rho} = 0.6223$): test statistics. [a]

Type	*Indicator*	*Result*	*p-values*
Global location tests	t-test	−0.1203	0.9042
	Number of positive residuals	38	0.1686
	Wilcoxon test	1126	0.5652
Runs tests	Number of runs	29	0.2250
	Length of longest run	14	0.0032
Serial correlation tests	Modified von Neumann ratio	1.967	> 0.10
	Rank tests		

k	Signed-rank tests			Sign tests		
	S_k	S'_k	*p-value*	S_k	S'_k	*p-value*
1	1161	1.047	0.2949	35	0.8819	0.4500
2	1103	0.8869	0.3751	36	1.270	0.2529
3	1114	1.210	0.2262	36	1.408	0.2000
4	789	−0.9276	0.3536	26	−1.033	0.3663
5	897	0.0906	0.9278	32	0.6509	0.6029
6	1126	2.094	0.0362	36	1.838	0.0869
7	1092	2.109	0.0349	37	2.252	0.0331
8	787	−0.0897	0.9285	30	0.5345	0.6889
9	870	0.8379	0.4021	28	0.1348	1.0000
10	710	−0.2798	0.7796	26	−0.2720	0.8919
11	578	−1.217	0.2235	24	−0.6868	0.5831
12	696	0.0638	0.9492	25	−0.2774	0.8899

[a] Number of residuals: 64

Table 25.8: t-statistics for subperiods ($\hat{\rho} = 0.6223$).

Period	t	*p-value*
1962/I–1966/III	2.178	0.0429
1964/I–1966/III	6.066	0.0000812
1967/II–1969/I	1.130	0.256
1971/II–1972/IV	0.194	0.853
Remainder [a]	−1.839	0.0762

[a] 1957/I–1961/IV, 1966/IV–1967/I, 1969/II–1971/I.

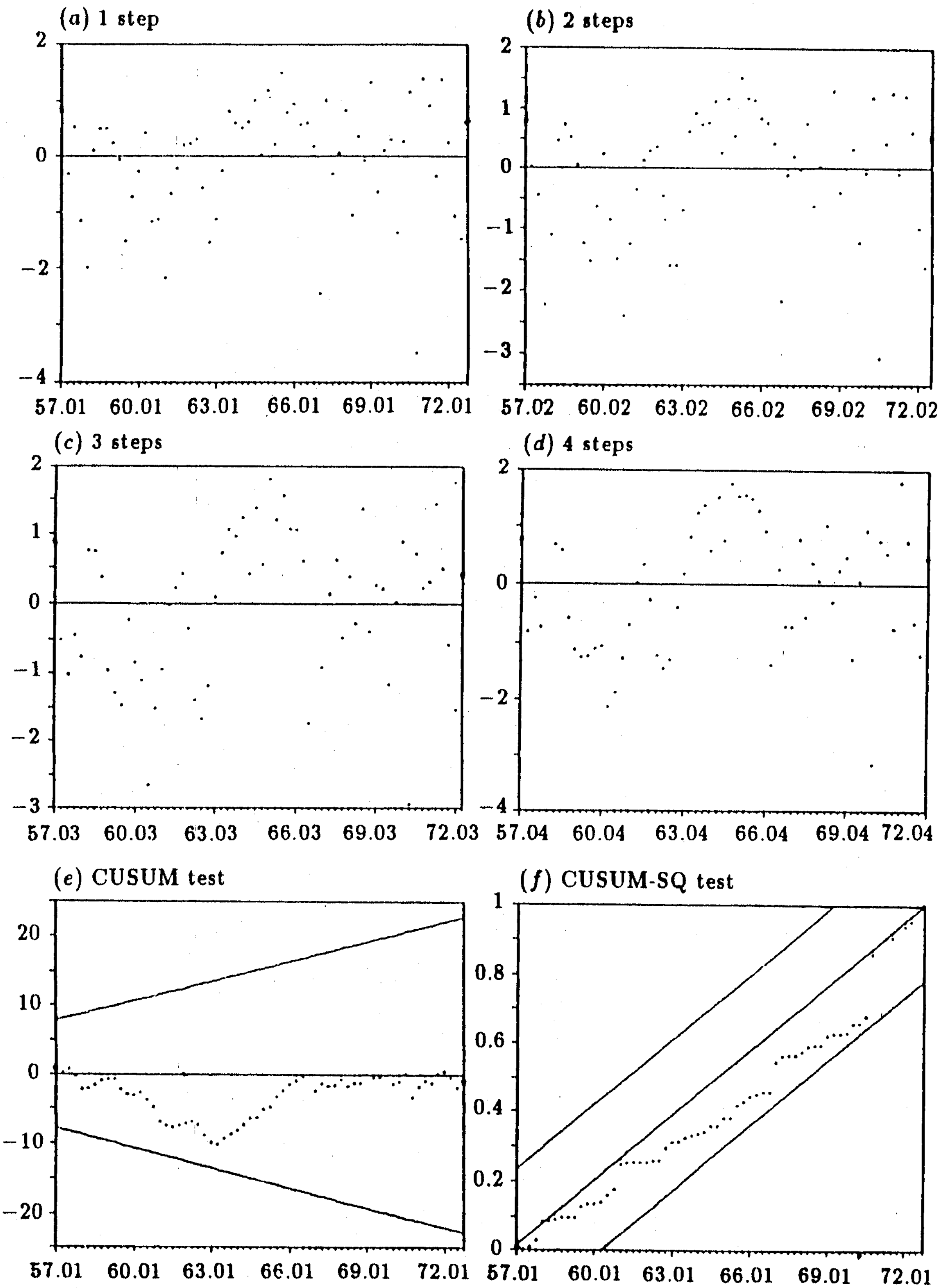

Figure 25.4: Gordon-Jorgenson model ($\hat{\rho} = 0.6223$): recursive residuals and CUSUM tests.

for δ and a_1) during which all coefficients change sign; fourth (1967/I–1972/IV), a period during which all coefficients seem to stabilize. On the other hand, the one-step recursive residuals [*Figure 25.4(a)*] appear more "random" than without the tranformation [compare *Figures 25.2(a)* and *25.4(a)*]. Global location tests and serial dependence tests are not significant at standard levels (say, 0.10). Nevertheless, we can still observe a tendency to overpredict in the earlier period (up to 1963/II) as well as a run of 14 consecutive underpredictions from 1963/III to 1966/IV followed by a sudden drop (1967/I) [9]. The (1967/IV–1969/IV) run of underpredictions disappears. These observations are confirmed when we look at several-steps-ahead recursive residuals [*Figures 25.4(b–d)*]. We thus continue to find signs of instability, especially in association with the first application of the tax credit (after the repeal of the Long Amendment).

The t-statistics for the separate subperiods corresponding to the different applications of the tax credit are reported in *Table 25.8*. As in the first experiment, the t-statistic for periods where the tax credit was in force are positive, while for the rest of the sample the t-statistic is negative. Moreover, the t-statistic for the first application period is significant (at level 0.04) and very strongly significant (at level 0.00008) if the period where the Long Amendment applied is excluded.

Finally, to take into account the fact that K_t is a form of lagged dependent variable, let us consider the result of estimating recursively equation (25.9). The recursive estimates are listed in *Table 25.9* and graphed in *Figures 25.5(a–c)*; the recursive residuals are listed in *Table 25.10*, with a number of test statistics in *Table 25.11*, and they are graphed in *Figures 25.6(a–d)*. From the recursive estimates, we still observe the same four phases: first (1956/IV–1961/I), wide fluctuations with rough trends (upwards for α and a_2, downward for a_1); second (1961/II–1963/II), a period of relative stability; third (1963/III–1966/IV), a clear trend (downward for α and a_2, upward for a_1); fourth (1967/I–1972/IV), a period where all coefficients seem to stabilize. On the basis of the one-step recursive residuals [*Figure 25.6(a)*], we find now that none of the test statistics in *Table 25.11* nor the CUSUM and CUSUM of squares tests [*Figures 25.6(e)* and (f)] are significant (at level 0.05). In particular, the longest-run test is not conclusive. [Two residuals in the middle of the longest run previously observed (1963/III–1966/IV) are now below the zero line.] Nevertheless, several-steps-ahead recursive residuals [*Figures 25.6(b–d)*] do not seem to be affected in the same way and exhibit basically the same pattern as in the previous experiment; in particular, two- and three-steps-ahead recursive residuals contain continuous runs of underpredictions covering the period 1963/III–1966/IV. Indeed, the similarity between *Figures 25.4(a)* and *25.6(a)* (showing one-step-ahead recursive residuals) is striking: we still note a tendency to overpredict up to 1963/II and a tendency to underpredict over the period 1963/III–1966/IV, while the rest looks relatively "random". If we compute t-statistics over the seperate subperiods corresponding to the seperate phases of the tax credit, we find results analogous to the ones obtained before (see *Table 25.12*). The t-statistic attached to 1962/I–1966/III (first application of the tax credit) is positive and significant at level 0.04 while, for the period 1964/I–1966/III (after the repeal of the Long Amendment), it is significant at level 0.00065. Note again the contrast between the application periods of the tax credit (which yield positive t-statistics) and the remainder of

Table 25.9: Gordon-Jorgenson model: forward recursive estimates (data transformed with $\hat{\rho} = 0.6223$, capital subtracted): 1956/I–1972/IV.

Quarter	α	a_1	a_2
56.03	-269.178	0.0255443	-0.0033785
56.04	-533.474	0.0279611	-0.0041437
57.01	44.434	0.0029566	-0.0002796
57.02	-9.307	0.0057092	-0.0006864
57.03	68.847	-0.0004134	0.0001406
57.04	4.559	0.0061358	-0.0007076
58.01	-63.294	0.0073405	-0.0009702
58.02	-81.517	0.0053041	-0.0007791
58.03	-75.324	0.0038777	-0.0006091
58.04	-67.076	0.0003639	-0.0002038
59.01	-65.305	-0.0011968	-0.0000274
59.02	-66.868	-0.0023877	0.0001015
59.03	-60.603	-0.0013351	-0.0000030
59.04	-55.332	-0.0013686	0.0000111
60.01	-51.703	-0.0014485	0.0000271
60.02	-55.690	-0.0015420	0.0000297
60.03	-43.791	-0.0013385	0.0000303
60.04	-31.604	-0.0014409	0.0000654
61.01	-11.526	-0.0016787	0.0001307
61.02	-4.316	-0.0020039	0.0001809
61.03	-2.846	-0.0019123	0.0001735
61.04	-3.729	-0.0019261	0.0001733
62.01	-4.591	-0.0019339	0.0001726
62.02	-5.986	-0.0019498	0.0001717
62.03	-5.767	-0.0018010	0.0001555
62.04	-0.979	-0.0014920	0.0001302
63.01	4.346	-0.0020012	0.0001969
63.02	5.020	-0.0020984	0.0002089
63.03	0.472	-0.0012382	0.0001047
63.04	-1.676	-0.0007826	0.0000500
64.01	-2.772	-0.0005481	0.0000218
64.02	-3.606	-0.0004066	0.0000045
64.03	-4.652	-0.0005419	0.0000177
64.04	-4.448	-0.0005549	0.0000195
65.01	-5.903	-0.0002692	-0.0000150
65.02	-5.859	-0.0002796	-0.0000137
65.03	-7.456	0.0000680	-0.0000553
65.04	-8.199	0.0002511	-0.0000771
66.01	-9.084	0.0004542	-0.0001013
66.02	-9.629	0.0005739	-0.0001156
66.03	-10.370	0.0008547	-0.0001482
66.04	-10.814	0.0010835	-0.0001745
67.01	-8.858	-0.0001366	-0.0000350
67.02	-9.637	0.0007289	-0.0001329
67.03	-9.487	0.0005972	-0.0001179
67.04	-9.458	0.0006046	-0.0001187
68.01	-9.734	0.0005181	-0.0001096
68.02	-9.439	0.0005908	-0.0001171
68.03	-9.544	0.0005848	-0.0001167
68.04	-9.523	0.0005818	-0.0001163
69.01	-9.996	0.0007138	-0.0001318
69.02	-9.832	0.0006639	-0.0001260
69.03	-9.892	0.0006779	-0.0001276
69.04	-9.988	0.0008207	-0.0001437
70.01	-9.773	0.0004730	-0.0001046
70.02	-9.809	0.0005361	-0.0001117
70.03	-9.967	0.0006536	-0.0001251
70.04	-9.564	0.0005583	-0.0001137
71.01	-9.678	0.0005456	-0.0001125
71.02	-9.787	0.0004912	-0.0001066
71.03	-9.701	0.0005681	-0.0001150
71.04	-10.012	0.0002613	-0.0000814
72.01	-10.088	0.0002035	-0.0000750
72.02	-9.845	0.0003521	-0.0000912
72.03	-9.461	0.0004706	-0.0001038
72.04	-9.656	0.0004686	-0.0001039

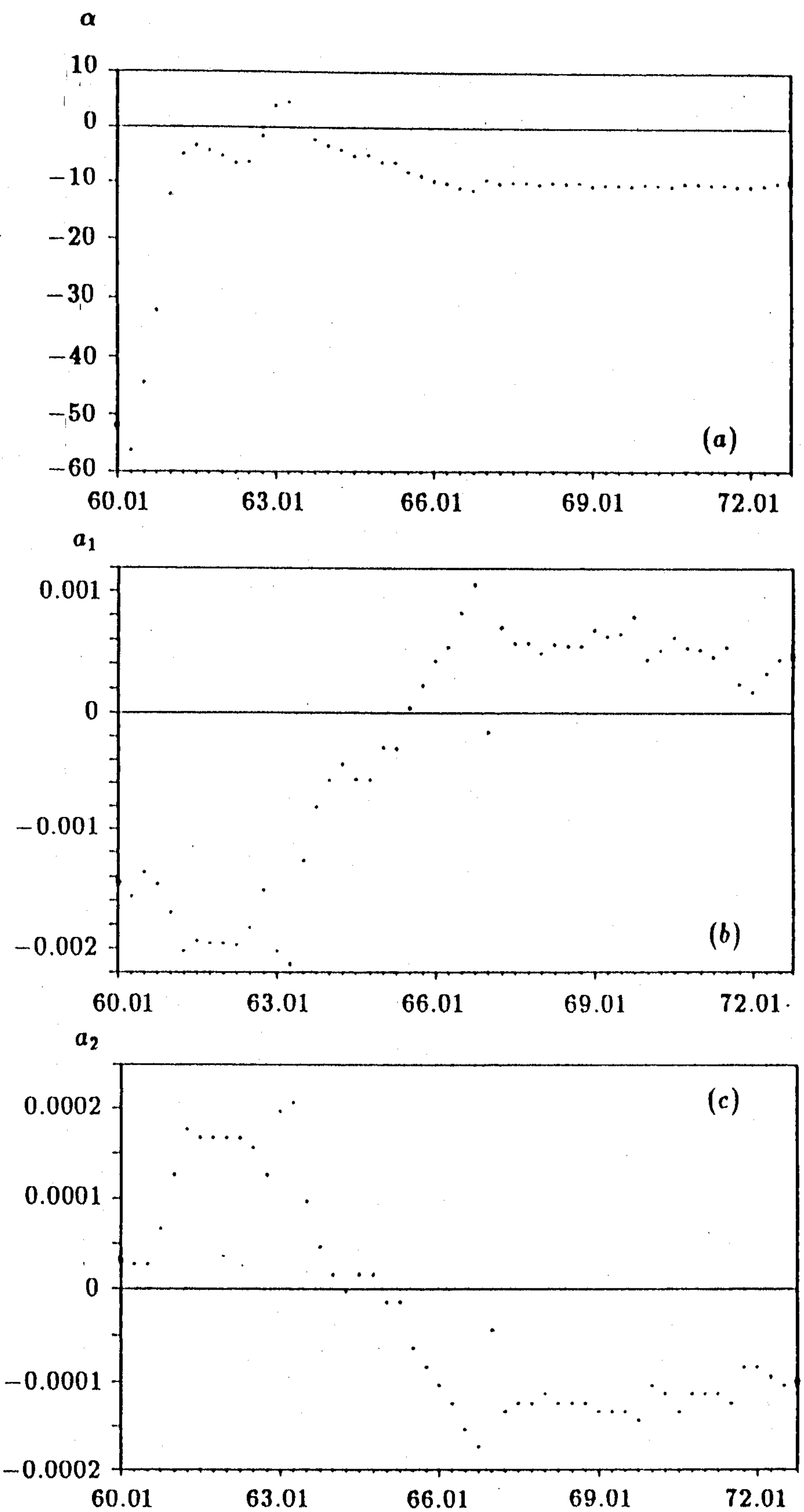

Figure 25.5: Gordon-Jorgenson model ($\hat{\rho} = 0.6223$, capital subtracted): recursive estimates.

Table 25.10: Gordon-Jorgenson model: forward recursive residuals (data transformed with $\hat{\rho} = 0.6223$, capital subtracted): 1956/I–1972/IV

Quarter	*1 step*	*2 steps*	*3 steps*	*4 steps*	*8 steps*
56.04	-0.4199	-	-	-	-
57.01	0.8541	0.1812	-	-	-
57.02	-0.2369	0.7132	0.1499	-	-
57.03	0.6522	0.1718	0.8587	0.3168	-
57.04	-1.0829	-0.2235	-0.3217	0.6768	-
58.01	-1.9935	-2.2629	-0.9657	-0.7152	-
58.02	-0.8332	-1.6755	-1.9926	-0.7974	-
58.03	0.1999	0.0907	-0.6741	-1.2200	0.2449
58.04	0.6683	0.6189	0.6749	0.1889	0.7389
59.01	0.6272	0.9089	0.8025	0.8915	0.0256
59.02	0.7184	0.9022	1.1223	0.9815	0.5414
59.03	-0.7690	-0.4986	-0.2149	0.2369	0.0179
59.04	-0.5613	-0.6971	-0.5808	-0.4781	0.1685
60.01	-0.3321	-0.4330	-0.5716	-0.4750	-0.0714
60.02	0.3859	0.3018	0.1742	-0.0535	0.4872
60.03	-1.2749	-1.1330	-1.1798	-1.2770	-0.8328
60.04	-1.4013	-1.6667	-1.5027	-1.5372	-1.4847
61.01	-2.5474	-2.8018	-3.0353	-2.8012	-2.7459
61.02	-1.0226	-1.5398	-1.8255	-2.0898	-2.1254
61.03	-0.2222	-0.4303	-1.0725	-1.4242	-1.6397
61.04	0.1430	0.0745	-0.1657	-0.8660	-1.3921
62.01	0.1549	0.1869	0.1104	-0.1498	-1.3990
62.02	0.2718	0.3031	0.3316	0.2379	-1.6604
62.03	-0.1166	-0.0304	0.0190	0.0664	-1.5225
62.04	-1.2664	-1.1632	-0.9974	-0.8782	-1.8958
63.01	-0.9177	-1.3088	-1.2424	-1.0481	-1.0927
63.02	-0.1178	-0.4768	-0.8837	-0.8700	-0.5227
63.03	0.8714	0.7339	0.2743	-0.1395	0.0708
63.04	0.5483	0.8849	0.7324	0.2609	0.0214
64.01	0.3913	0.5752	0.9175	0.7550	-0.0325
64.02	0.4291	0.5246	0.6941	1.0191	-0.1177
64.03	0.9357	1.0010	1.0654	1.1851	0.0794
64.04	-0.1538	0.0407	0.1380	0.2461	0.1134
65.01	0.9715	0.9208	1.0886	1.1629	1.3162
65.02	-0.0314	0.1821	0.1455	0.3315	1.0884
65.03	1.3211	1.2844	1.4728	1.4017	1.7198
65.04	0.6624	0.9118	0.8809	1.0904	1.3294
66.01	0.9107	1.0193	1.2699	1.2271	1.5876
66.02	0.6385	0.7855	0.8994	1.1596	1.4354
66.03	0.7928	0.8899	1.0479	1.1668	1.5086
66.04	0.4721	0.6282	0.7343	0.9060	1.4917
67.01	-2.3454	-2.1507	-1.9133	-1.7587	-0.9133
67.02	1.0167	0.1404	0.2867	0.4837	1.1923
67.03	-0.3686	0.1141	-0.6157	-0.4467	0.2265
67.04	-0.0860	-0.0986	-0.0645	-0.2878	0.1369
68.01	0.8886	0.8772	0.8678	0.8781	0.9148
68.02	-0.9809	-0.9008	-0.9049	-0.9173	-0.9136
68.03	0.3398	0.2651	0.3357	0.3271	0.1463
68.04	-0.0626	-0.0383	-0.1070	-0.0417	-0.3491
69.01	1.4165	1.4069	1.4279	1.3582	1.5590
69.02	-0.5310	-0.4060	-0.4097	-0.3839	-0.4400
69.03	0.2155	0.1735	0.2908	0.2849	0.2953
69.04	0.4084	0.4258	0.3655	0.5220	0.5205
70.01	-1.2921	-1.1593	-1.1369	-1.1864	-1.0270
70.02	0.3034	0.0739	0.1626	0.1793	0.2792
70.03	1.3505	1.3714	1.2230	1.2684	1.3395
70.04	-3.5859	-3.5319	-3.5113	-3.5892	-3.4442
71.01	1.0081	0.9063	0.9443	0.9528	0.9026
71.02	0.7913	0.8227	0.7326	0.7538	0.7647
71.03	-0.4334	-0.3792	-0.3406	-0.4102	-0.3727
71.04	1.3976	1.3088	1.3699	1.4116	1.4399
72.01	0.3310	0.5957	0.5165	0.5822	0.4998
72.02	-1.0090	-0.9369	-0.6372	-0.7017	-0.6821
72.03	-1.4293	-1.5536	-1.4869	-1.2252	-1.2691
72.04	0.6363	0.5065	0.4086	0.4396	0.6492

Table 25.11: Gordon-Jorgenson model ($\hat{\rho} = 0.6223$, capital subtracted): test statistics. [a]

Type	*Indicator*					*Result*	*p-values*
Global location tests	t-test					−0.4535	0.6502
	Number of positive residuals					35	0.6201
	Wilcoxon test					1112	0.7962
Runs tests	Number of runs					34	0.6460
	Length of longest run					7	0.3892
Serial correlation tests	Modified von Neumann ratio					1.974	> 0.10
	Rank tests						
	k	Signed-rank tests			Sign tests		
		S_k	S'_k	*p-value*	S_k	S'_k	*p-value*
	1	1101	0.4079	0.6833	31	−0.2500	0.9007
	2	1117	0.7462	0.4556	35	0.8819	0.4500
	3	1106	0.9079	0.3639	30	−0.2540	0.8991
	4	818	−0.9158	0.3598	25	−1.408	0.2000
	5	866	−0.3607	0.7183	25	−1.291	0.2451
	6	1100	1.623	0.1046	34	1.172	0.2976
	7	1092	1.831	0.0671	36	1.838	0.0868
	8	738	−0.7032	0.4820	25	−0.9272	0.4270
	9	973	1.427	0.1534	33	1.336	0.2288
	10	748	−0.1843	0.8538	26	−0.4045	0.7877
	11	639	−0.8912	0.3728	25	−0.5443	0.6835
	12	806	0.8012	0.4230	28	−0.4121	0.7838

[a]Number of residuals: 64.

Table 25.12: t-statistics for subperiods ($\hat{\rho} = 0.6223$, capital subtracted).

Period	t	*p-value*
1962/I–1966/III	2.197	0.0414
1964/I–1966/III	4.697	0.000653
1967/II–1969/I	0.957	0.370
1971/II–1972/IV	0.105	0.920
Remainder [a]	−1.944	0.0613

[a]1956/IV–1961/IV, 1966/IV–1967/I, 1969/II–1971/I.

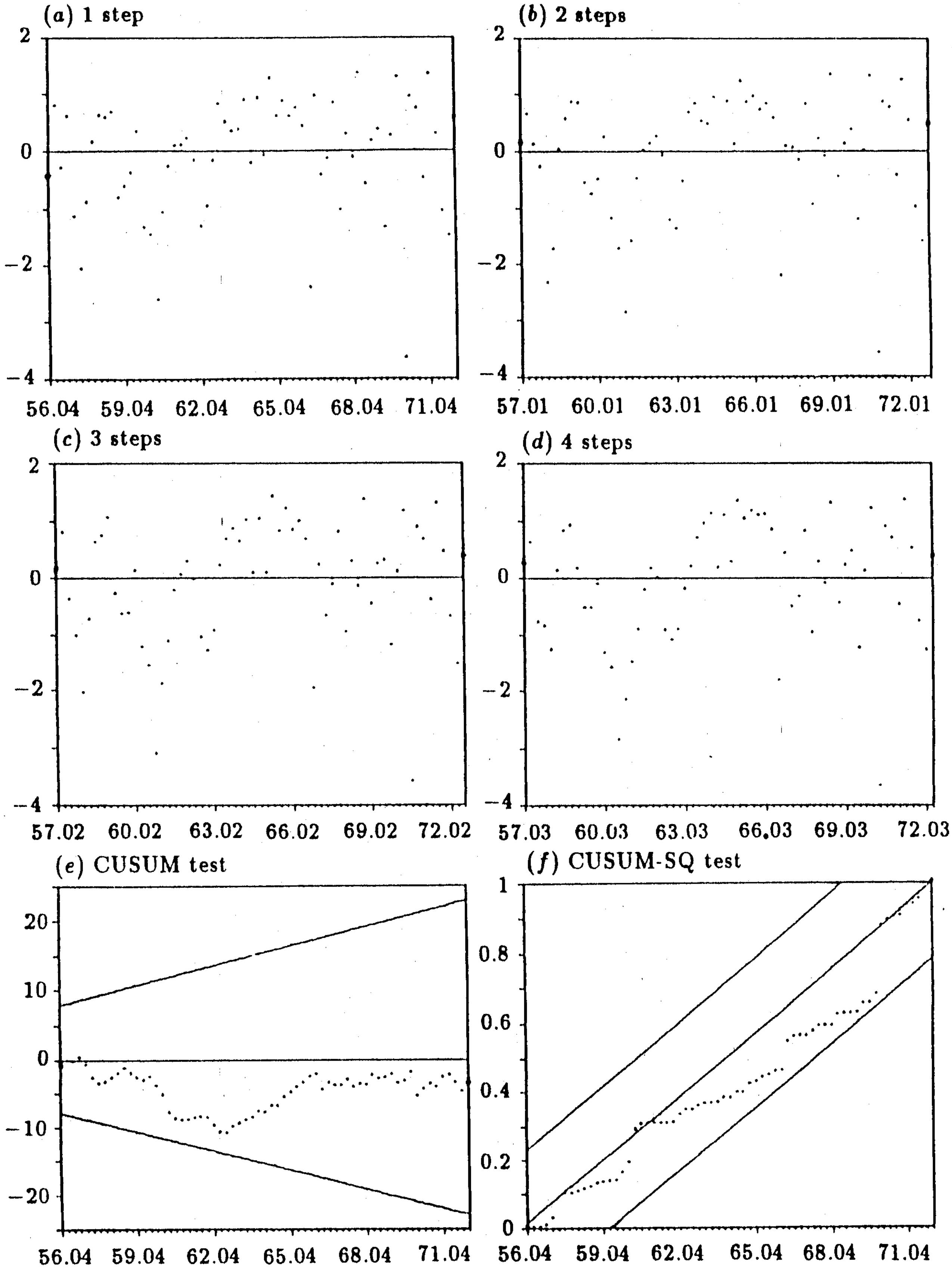

Figure 25.6: Gordon-Jorgenson model ($\hat{\rho} = 0.6223$, capital subtracted): recursive residuals and CUSUM tests.

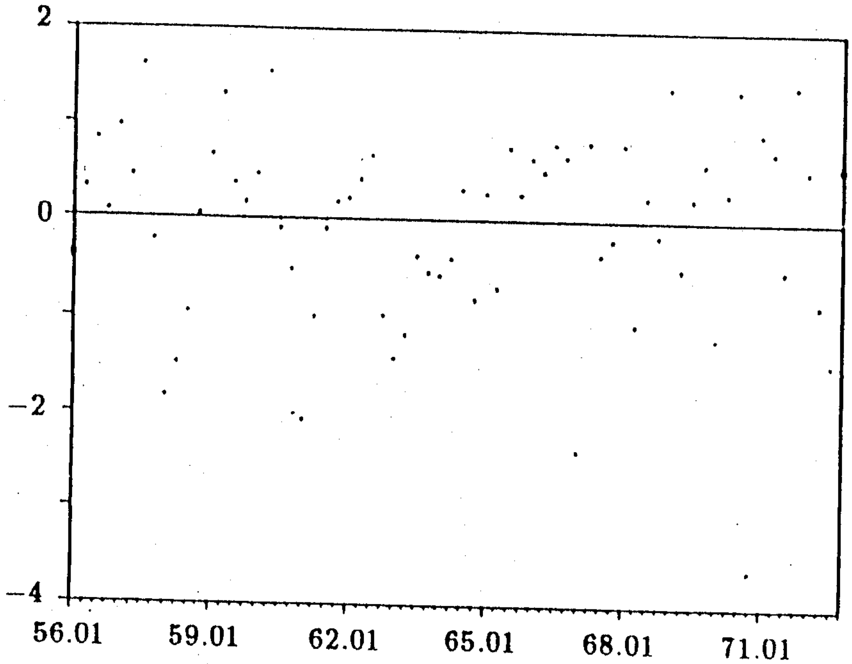

Figure 25.7: Gordon-Jorgenson model ($\hat{\rho} = 0.6223$): generalized least squares residuals.

the sample (which yields a negative t-statistic).

Although the evidence is less strong than for the two previous experiments, we continue to observe a phenomenon of underprediction associated with the first imposition of the tax credit, and this especially after the repeal of the Long Amendment. For the two other applications of the tax credit, we do not observe significant effects, although the corresponding t-statistics are positive and thus indicate a tendency to underpredict.

25.5 Conclusion

The results obtained in this recursive stability analysis are not as clear and definite as those we got, for example, for the demand for money during the German hyperinflation [Dufour (1986)]. They are confused, in particular, by the presence of a regressor (the capital stock) which contains lagged values of the dependent variable. Nevertheless, one feature remains constant throughout the three experiments performed: there appears to be a discontinuity associated with the introduction of the first investment tax credit (1962/I–1966/III), especially after the repeal of the Long Amendement (1964/I). Furthermore, the discontinuity is a type that leads to underprediction of investment, a behavior in contrast with the performance of the model before 1962 (where we find a tendency to overpredict). This phenomenon of underprediction is in agreement with Lucas's forecast. There is also some indication of a tendency to overpredict investment over the two other

Table 25.13: Effective investment tax credit (1961–1972).

Quarter	*Tax credit*	*Y*
61.01	0.0%	0
61.02	0.0%	0
61.03	0.0%	0
61.04	0.0%	0
62.01	3.1%	1
62.02	3.5%	1
62.03	3.9%	1
62.04	4.3%	1
63.01	4.7%	1
63.02	5.1%	1
63.03	5.5%	1
63.04	5.6%	1
64.01	5.6%	0
64.02	5.6%	0
64.03	5.6%	0
64.04	5.6%	0
65.01	5.6%	0
65.02	5.6%	0
65.03	5.6%	0
65.04	5.6%	0
66.01	5.6%	0
66.02	5.6%	0
66.03	5.6%	0
66.04	0.0%	0
67.01	0.0%	0
67.02	5.6%	0
67.03	5.6%	0
67.04	5.6%	0
68.01	5.6%	0
68.02	5.6%	0
68.03	5.6%	0
68.04	5.6%	0
69.01	5.6%	0
69.02	0.0%	0
69.03	0.0%	0
69.04	0.0%	0
70.01	0.0%	0
70.02	0.0%	0
70.03	0.0%	0
70.04	0.0%	0
71.01	0.0%	0
71.02	4.0%	0
71.03	5.0%	0
71.04	5.6%	0
72.01	5.6%	0
72.02	5.6%	0
72.03	5.6%	0
72.04	5.6%	0

periods where the tax credit was in force (1967/II–1969/I and 1971/II–1972/IV). This is suggested by the signs of the corresponding t-statistics, but the effects appear too small to be considered significant.

On the whole, the evidence we found is quite consistent with the type of instabilility suggested by Lucas (1976), even though it appears difficult to qualify this evidence as being very "strong". Of course, one could try to explain the instability detected by a misspecification other than the one pointed out by Lucas (e.g., the Almon lag scheme used may be wrong). In any event, whatever the "true" problem may be, it is certainly useful to know about its existence.

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Notes

[1] Lucas (1976) assumes the tax credit follows a Markovian scheme (which includes as special cases both a permanent credit and a frequently imposed but always transitory credit) and shows that the impact of the tax credit on investment can be much bigger if it is viewed as transitory rather than permanent. Indeed, under reasonable values of the parameters, the ratio of the actual to predicted effect may be in the range of 4 to 7.

[2] The Long Amendment forbade firms to use for depreciation purposes the part of the cost of a capital asset financed by the tax credit.

[3] Gordon and Jorgenson (1976, p.278). We list in *Table 25.13* the "effective tax credit" (1961–1972) as measured by these authors. The "effective tax credit" could be nonzero longer than the nominal credit because, even after the credit was suspended, firms could still use a credit to which they were entitled but did not use when it was in force.

[4] Though scaled recursive residuals are similar to t-statistics, one can check easily that they do not generally follow Student t-distributions. Note also that $\tilde{\sigma}^2$ tends to overestimate σ^2 when structural change is present [see Dufour (1982, pp. 60–61)]: clearly, this can make a number of important residuals look "small" and should be discounted when interpreting the results.

[5] Of course, given that K_t is a form of lagged dependent variable and if disturbance are autocorrelated, least squares coefficient estimates could be inconsistent. Nevertheless, the appearance of "autocorrelation" may be a symptom of an instability problem and thus an experiment without such a correction seems indicated. In any case, this will allow us to illustrate how a misspecification can lead to a parameter instability phenomenon in a recursive estimation experiment.

[6] Eight-steps-ahead recursive residuals are not graphed. The test statistics in *Table 25.3*, as well as those in *Tables 25.7* and *25.11*, are based on forward one-step-ahead recursive residuals. We report systematically three categories of tests (general tests, runs tests, and serial dependence tests) that can be compared and cross-checked [see Dufour (1982, Section 4)].

[7] For a listing of the variables TC_t (effective tax credit rate) and U_t (dummy for Long Amendment) from 1961/I to 1972/IV, see *Table 25.13*.

[8] Recursive residuals obtained in this way do not enjoy exactly their convenient theoret-

ical properties (for the true value of ρ is unknown). However, if $\hat{\rho}$ is consistent estimate of ρ, we can still expect it will fall in the neighborhood of the true value of ρ and thus provide approximately valid test statistics. But this is not guaranteed. In view of this difficulty, we performed some sensitivity analysis by considering models transformed by different values of ρ inside a grid in the neighborhood of ρ. In all cases we obtained essentially the same conclusions. For further discussions of this problem, see Dufour (1982; Section 2.5).

[9] It is interesting to compare the residuals in *Figure 25.4(a)* (recursive) with the corresponding generalized least squares residuals in *Figure 25.7* and to see how more revealing recursive residuals are.