

McGill University
ECON 763
Financial econometrics
Final exam

No documentation allowed
Time allowed: 3 hours

- 30 points
1. Answer by TRUE, FALSE or UNCERTAIN to each one of the following statements, and justify briefly your answer (maximum: one page per statement).
 - (a) If a stochastic process has finite second moments, it cannot have finite third moments.
 - (b) The sample autocorrelations of a nonstationary process do not have finite moments.
 - (c) A strictly stationary process is also weakly stationary.
 - (d) Convergence in probability does not imply convergence in mean of order two.
 - (e) The Wold decomposition does not hold for deterministic processes.
 - (f) The autocorrelation function of a second-order stationary process is positive semidefinite.
 - (g) The autocorrelations and the partial autocorrelations of a stationary AR(1) process are identical.
 - (h) The spectral density of a stationary AR(1) model is not constant.
 - (i) A sign test for the median of independent random variables remains valid if the observations are heteroskedastic.
 - (j) The normal distribution is not stable.

20 points 2. Consider the following ARMA model:

$$X_t = 0.5 X_{t-1} + u_t - 0.25 u_{t-1} \quad (1)$$

where $\{u_t : t \in \mathbb{Z}\}$ is an *i.i.d.* $N(0, 1)$ sequence. Answer the following questions.

- (a) Is this model stationary? Why?
- (b) Is this model invertible? Why?
- (c) Compute:
 - i. $E(X_t)$;
 - ii. $\gamma(k)$, $k = 1, \dots, 8$;
 - iii. $\rho(k)$, $k = 1, 2, \dots, 8$.
- (d) Graph $\rho(k)$, $k = 1, 2, \dots, 8$.
- (e) Find the coefficients of $u_t, u_{t-1}, u_{t-2}, u_{t-3}$ and u_{t-4} in the moving average representation of X_t .
- (f) Compute the first two partial autocorrelations of X_t .
- (g) If $X_{10} = 1$ and assuming the parameters of the model are known, can you compute the best linear forecasts of X_{10}, X_{11}, X_{12} and X_{13} based on X_{10} (only)? If so, compute these.
- (h) If $X_{10} = 1, u_{10} = 2, u_9 = 1, u_8 = 0.99, u_7 = 1.2$, and assuming the parameters of the model are known, can you compute the best linear forecasts of X_{11}, X_{12} and X_{13} based on the history of the process up to X_{10} ? If so, compute these.

15 points

3. Let X_1, X_2, \dots, X_T be a time series where X_1, X_2, \dots, X_T have continuous distributions.

- (a) Propose a method for testing the hypothesis that X_1, X_2, \dots, X_T are independent and identically distributed (i.i.d.) without any assumption on the existence of the moments for X_1, X_2, \dots, X_T .
- (b) If X_1, X_2, \dots, X_T have common median m_0 , describe a procedure for testing whether these observations are independent without assuming identical distributions.
- (c) Consider the “median regression” model:

$$y_t = x_t' \beta + u_t, \quad t = 1, \dots, T, \quad (2)$$

where $x_t, t = 1, \dots, T$, are $k \times 1$ fixed vectors and the the disturbances $u_t, t = 1, \dots, T$, are independent with median zero and continuous distributions. Propose procedures for testing hypotheses of the form $H_0 : \beta = \beta_0$ and build confidence sets for β .

20 points

4. Let $R_{it}, i = 1, \dots, n$, be returns on n securities for period t , and \tilde{R}_{Mt} the return on a benchmark portfolio ($t = 1, \dots, T$). The (unconditional) CAPM which assumes time-invariant *betas* can be assessed by testing:

$$\bar{\mathcal{H}}_{E1} : \bar{a}_i = 0, \quad i = 1, \dots, n. \quad (3)$$

in the context of the MLR model

$$r_{it} = \bar{a}_i + \beta_{it} \tilde{r}_{Mt} + \varepsilon_{it}, \quad \beta_{it} = \bar{\beta}_i + \sum_{j=1}^q d_{ji} z_{jt}, \quad (4)$$

$t = 1, \dots, T, i = 1, \dots, n$, where $r_{it} = R_{it} - R_{ft}$, $\tilde{r}_{Mt} = \tilde{R}_{Mt} - R_{ft}$, R_{ft} is the riskless rate of return, z_{1t}, \dots, z_{qt} represent q conditioning variables upon which the “betas” β_{it} may depend, and ε_{it} is a random disturbance, such that

$$V_t \equiv (\varepsilon_{1t}, \dots, \varepsilon_{nt})' = JW_t, \quad t = 1, \dots, T, \quad (5)$$

where J is an unknown, non-singular matrix and the distribution of the vector $w = \text{vec}(W)$, $W = [W_1, \dots, W_T]'$ is either: (i) known (hence, free of nuisance parameters), or (ii) specified up to an unknown finite dimensional nuisance-parameter (denoted ν).

- Put the model (4) in matrix notation.
- On assuming that the vectors W_1, \dots, W_T are i.i.d. $N[0, I_n]$, describe the likelihood ratio test for \mathcal{H}_E , and discuss how this test could be implemented.
- Propose a procedure for testing whether the errors W_1, \dots, W_T are i.i.d. $N[0, I_n]$.
- If another distribution is assumed for w (such as a heavy-tailed distribution), discuss how such a test could be implemented.

15 points

5. Consider a time series of asset returns R_t , $t = 1, \dots, T$, which are i.i.d. according to stable distribution, with characteristic function

$$\ln \int_{-\infty}^{\infty} e^{ist} d\mathbf{P}(S < s) = \begin{cases} -\sigma^\alpha |t|^\alpha [1 - i\beta \operatorname{sign}(t) \tan \frac{\pi\alpha}{2}] + i\mu t, & \text{for } \alpha \neq 1, \\ -\sigma |t| [1 + i\beta \frac{\pi}{2} \operatorname{sign}(t) \ln |t|] + i\mu t, & \text{for } \alpha = 1. \end{cases} \quad (6)$$

- (a) Discuss the interpretation of the different parameters μ , σ , α and β .
(b) Why are stable random variables called “stable”?
(c) On assuming that $\beta = 0$, propose a method for testing

$$H_0(\alpha_0) : \alpha = \alpha_0. \quad (7)$$

- (d) On assuming that $\beta = 0$, discuss how a confidence set for α could be built.