

**McGill University**  
**ECON 763**  
**Financial econometrics**  
**Final exam**

No documentation allowed  
Time allowed: 3 hours

- 30 points
1. Answer by TRUE, FALSE or UNCERTAIN to each one of the following statements, and justify briefly your answer (maximum: one page per statement).
- (a) If a stochastic process has finite fourth moments, it also has finite third moments.
  - (b) The sample autocorrelations of a nonstationary process do not have finite moments.
  - (c) A strictly stationary process is also weakly stationary.
  - (d) Convergence in probability does not imply almost sure convergence.
  - (e) The Wold decomposition does not hold for deterministic processes.
  - (f) The autocorrelation function of a second-order stationary process is negative semidefinite.
  - (g) The autocorrelations and the partial autocorrelations of a stationary MA(1) process are identical.
  - (h) The spectral density of a stationary AR(1) model is constant.
  - (i) A sign test for the median of independent random variables remains valid if the observations are heteroskedastic.
  - (j) The normal distribution is not stable.

- 20 points
2. Consider the following ARMA model:

$$X_t = 1.0 + 0.5 X_{t-1} + u_t - 0.5 u_{t-1} \quad (1)$$

where  $\{u_t : t \in \mathbb{Z}\}$  is an *i.i.d.*  $N(0, 1)$  sequence. Answer the following questions.

- (a) Is this model stationary? Why?
- (b) Is this model invertible? Why?
- (c) Compute:
  - i.  $E(X_t)$ ;
  - ii.  $\gamma(k)$ ,  $k = 1, \dots, 8$ ;
  - iii.  $\rho(k)$ ,  $k = 1, 2, \dots, 8$ .
- (d) Graph  $\rho(k)$ ,  $k = 1, 2, \dots, 8$ .
- (e) Find the coefficients of  $u_t$ ,  $u_{t-1}$ ,  $u_{t-2}$ ,  $u_{t-3}$  and  $u_{t-4}$  in the moving average representation of  $X_t$ .
- (f) Compute the first two partial autocorrelations of  $X_t$ .
- (g) If  $X_{10} = 1$  and assuming the parameters of the model are known, can you compute the best linear forecasts of  $X_{10}$ ,  $X_{11}$ ,  $X_{12}$  and  $X_{13}$  based on  $X_{10}$  (only)? If so, compute these.
- (h) If  $X_{10} = 1$ ,  $u_{10} = 2$ ,  $u_9 = 1$ ,  $u_8 = 0.99$ ,  $u_7 = 1.2$ , and assuming the parameters of the model are known, can you compute the best linear forecasts of  $X_{11}$ ,  $X_{12}$  and  $X_{13}$  based on the history of the process up to  $X_{10}$ ? If so, compute these.

15 points

3. Let  $X_1, X_2, \dots, X_T$  be a time series where  $X_1, X_2, \dots, X_T$  have continuous distributions.
- (a) Propose a method for testing the hypothesis that  $X_1, X_2, \dots, X_T$  are independent and identically distributed (i.i.d.) without any assumption on the existence of the moments for  $X_1, X_2, \dots, X_T$ .
  - (b) If  $X_1, X_2, \dots, X_T$  have common median  $m_0$ , describe a procedure for testing whether these observations are independent without assuming identical distributions.
  - (c) Consider the “median regression” model:

$$y_t = x_t' \beta + u_t, \quad t = 1, \dots, T, \quad (2)$$

where  $x_t$ ,  $t = 1, \dots, T$ , are  $k \times 1$  fixed vectors and the the disturbances  $u_t$ ,  $t = 1, \dots, T$ , are independent with median zero and continuous distributions. Propose procedures for testing hypotheses of the form  $H_0 : \beta = \beta_0$  and build confidence sets for  $\beta$ .

20 points

4. Let  $R_{it}$ ,  $i = 1, \dots, n$ , be returns on  $n$  securities for period  $t$ , and  $\tilde{R}_{Mt}$  the return on

a benchmark portfolio ( $t = 1, \dots, T$ ). The (unconditional) CAPM which assumes time-invariant *betas* can be assessed by testing:

$$\mathcal{H}_E : a_i = 0, \quad i = 1, \dots, n, \quad (3)$$

in the context of the MLR model

$$r_{it} = a_i + \beta_i \tilde{r}_{Mt} + \varepsilon_{it}, \quad t = 1, \dots, T, \quad i = 1, \dots, n, \quad (4)$$

where  $r_{it} = R_{it} - R_{ft}$ ,  $\tilde{r}_{Mt} = \tilde{R}_{Mt} - R_{ft}$ ,  $R_{ft}$  is the riskless rate of return and  $\varepsilon_{it}$  is a random disturbance, such that

$$V_t \equiv (\varepsilon_{1t}, \dots, \varepsilon_{nt})' = JW_t, \quad t = 1, \dots, T, \quad (5)$$

where  $J$  is an unknown, non-singular matrix and the distribution of the vector  $w = \text{vec}(W)$ ,  $W = [W_1, \dots, W_T]'$  is either: (i) known (hence, free of nuisance parameters), or (ii) specified up to an unknown finite dimensional nuisance-parameter (denoted  $\nu$ ).

- (a) Put the model (4) in matrix notation.
- (b) On assuming that the vectors  $W_1, \dots, W_T$  are i.i.d.  $N[0, I_n]$ , describe the likelihood ratio test for  $\mathcal{H}_E$ , and discuss how this test could be implemented.
- (c) Propose a procedure for testing whether the errors  $W_1, \dots, W_T$  are i.i.d.  $N[0, I_n]$ .
- (d) If another distribution is assumed for  $w$  (such as a heavy-tailed distribution), discuss how such a test could be implemented.

15 points

5. Consider a time series of asset returns  $R_t$ ,  $t = 1, \dots, T$ , which are i.i.d. according to stable distribution, with characteristic function

$$\ln \int_{-\infty}^{\infty} e^{ist} d\mathbf{P}(S < s) = \begin{cases} -\sigma^\alpha |t|^\alpha [1 - i\beta \text{sign}(t) \tan \frac{\pi\alpha}{2}] + i\mu t, & \text{for } \alpha \neq 1, \\ -\sigma |t| [1 + i\beta \frac{\pi}{2} \text{sign}(t) \ln |t|] + i\mu t, & \text{for } \alpha = 1. \end{cases} \quad (6)$$

- (a) Discuss the interpretation of the different parameters  $\mu$ ,  $\sigma$ ,  $\alpha$  and  $\beta$ .
- (b) Why are stable random variables called “stable”?
- (c) On assuming that  $\beta = 0$ , propose a method for testing

$$H_0(\alpha_0) : \alpha = \alpha_0. \quad (7)$$

- (d) On assuming that  $\beta = 0$ , discuss how a confidence set for  $\alpha$  could be built.