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March 11, 2015

**McGill University**
**ECON 763**
**Financial econometrics**
**Mid-term exam**

No documentation allowed
Time allowed: 1.5 hour

20 points 1. Consider a process that follows the following model:

$$X_t = \sum_{j=1}^{m} [A_j \cos(\nu_j t) + B_j \sin(\nu_j t)] \, , \; t \in \mathbb{Z} \, ,$$

where $\nu_1, \ldots, \nu_m$ are distinct constants on the interval $[0, \, 2\pi)$ and $A_j, B_j, \; j = 1, \, \ldots \, , m$, are random variables in $L_2$ , such that

$$\begin{aligned} E(A_j) &= E(B_j) = 0 \, , \, E(A_j^2) = E(B_j^2) = \sigma_j^2 \, , \; j = 1, \, \ldots \, , \, n \, , \\ E(A_j A_k) &= E(B_j B_k) = 0, \text{ for } j \neq k \, , \\ E(A_j B_k) &= 0, \, \forall j, \, k \, . \end{aligned}$$

(a) Show that this process is second-order stationary.

(b) For the case where $m = 1$, show that this process is deterministic

[Hint: consider the regression of $X_t$ on $\cos(\nu_1 t)$ and $\sin(\nu_1 t)$ based two observations.]

50 points 2. Consider the following models:

$$X_t = 0.5 \, X_{t-1} + u_t - 0.25 \, u_{t-1} \tag{1}$$

where $\{u_t : t \in \mathbb{Z}\}$ is an $i.i.d. \; N(0,1)$ sequence. For each one of these models, answer the following questions.

(a) Is this model stationary? Why?

(b) Is this model invertible? Why?

(c) Compute:

i. $E(X_t)$;
ii. $\gamma(k)$, $k = 1, \ldots, 8$;
iii. $\rho(k)$, $k = 1, 2, \ldots, 8$.

(d) Graph $\rho(k)$, $k = 1, 2, \ldots, 8$.

(e) Find the coefficients of $u_t$, $u_{t-1}$, $u_{t-2}$, $u_{t-3}$ and $u_{t-4}$ in the moving average representation of $X_t$.

(f) Find the autocovariance generating function of $X_t$.

(g) Find and graph the spectral density of $X_t$.

(h) Compute the first two partial autocorrelations of $X_t$.

30 points 3. Let $X_1, X_2, \ldots, X_T$ be a time series.

(a) Define:

i. the sample autocorrelations for this series;
ii. the partial autocorrelations for this series.

(b) Discuss the asymptotic distributions of these two sets of autocorrelations in the following cases:

i. under the hypothesis that $X_1, X_2, \ldots, X_T$ are independent and identically distributed (i.i.d.);
ii. under the hypothesis that the process follows a moving average of finite order.

(c) Describe how you would identify the process described in equation (1) in question 2.

(d) Propose a method for testing the hypothesis that $X_1, X_2, \ldots, X_T$ are independent and identically distributed (i.i.d.) wothout any assumption on the existence of moments for $X_1, X_2, \ldots, X_T$.