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BOAS

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NORTHWESTERN UNIVERSITY

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DEPARTMENT OF MATHEMATICS

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19 May 1974

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A72055
A53177
A259978
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Dear Dr. Sloane:

Thank you for the information. I don't see how I failed to find sequence 1385 in your Handbook.

By way of apology I offer you some sequences that seem not to be either in the Handbook or Supplement 1. These came up in connection with a problem in elementary number theory, but some of them seem reasonably interesting apart from the context in which I encountered them. Number (10) was not easy to guess from its first 3 terms. I give only a few terms of the sequences that can be extended easily.

(1) 1,4,16,36,... (even squares) A16742
(2) 1,2,4,6,10,12,16,18,22, 28,... (primes - 1) A6093
(2a) 1,3,4,6,8,12,14,18,20,... (primes + 1) A8864
(3) 1,2,2,1,2,4,2,2,3,4,4,2,2,6,1,...
(number of proper divisors of composite numbers) A144925
Yes (4) ~~1,~~9,10,15,16,21,22,25,26,27,...
(composite n with composite n-1) A5381
(5) 1,2,2,3,2,2,1,2,2,... (number of proper divisors of terms in (4)) A259977
A5382 Yes (6) ~~1,~~3,7,19,31,37,... (primes p with 2p-1 prime) A5382
Yes (7) ~~1,~~5,7,11,15,23,... (2p+1 with p prime) 2. PRIME + 1 A72055
Also 2. PRIME -1 (8) ~~1,~~5,7,11,23,... (primes in number (7)) A5385
(9) ~~1,~~15,27,35,39,... (composites in (7)) A53177
(10) ~~1,~~35,95,119,143,... (entries in (9) that are prime to 3) A259978
(11) 1,2,5,10,17,26,37,... (squares + 1) A2522
(12) ~~1,~~4,9,25,49,... (squares of primes) A1248
(13) ~~1,~~5,10,26,50,122,...(1+squares of primes) A66872
(14) ~~1,~~10,26,122,362,1682,3722,5042,6242,7922,10202,...
(integers that are both 1+ square of prime and twice a prime) A259979
(15) ~~1,~~12,18,20,32,44,... (1+prime, with 4 proper divisors)
(16) 1,2,2,2,4,4,2,6,7,4,4,10,6,6,6,4,6,10,6,4,8, A259980
6,6,21,2,6,18,6,4,18,10,8,10,10,12,12,5,16,22, A259981
14,10,2,12,21,12,20,4,10,22,10,2,12,20,14,24,8,...
(curious sequence, explained below)
(17) ~~1,~~2,4,6,7,10,6,6,4,6,10,8,6,6,21,6,18,18,10,10,10,...
(a subsequence of (16)) A259983

The problem originates in the rather silly observation that 64/16 = 4/1 ("cancel" the 6's). I once asked what happens in base n, i.e. when is (nx+y)/(nz+x)=y/z? There are no nontrivial instances* of the cancellation phenomenon when n is prime (so see 1322); sequence (16) gives the number of instances of the phenomenon for each composite n. When n-1 is prime the only instances have x=n-1 and the number of them is the number of proper divisors of n (hence (3),(4),(5),(17) are of interest; (17) is the subsequence of (16) corresponding to the entries in (4)). I possess considerably more terms of (16). The other sequences arise if one tries to account for various peculiarities of (16), which was generated by a computer program.

In another context altogether, I would once have saved considerable time if I could have identified 1,9,5,3,6 quickly. A259982

Sincerely yours,

R.P. Boas

* I exclude $\frac{22}{22}$, etc.

Bell Laboratories

600 Mountain Avenue
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Phone (201) 582-3000

August 28, 1974

Professor R. P. Boas
Department of Mathematics
Northwestern University
Evanston, Illinois 60201

Dear Professor Boas:

I'm shocked that I can't find any trace of having thanked you for your long letter of May 19, containing many new and interesting sequences. My correspondence is usually a mess, but this is going too far. Anyway, many thanks for the letter, and also for a postcard drawing my attention to a sequence in Math. Comp. (Barrodale, Vol. 20, 1966, pp. 318-322.) But I think Barrodale only gives bounds on what he calls M(k), so that is not really a good sequence?

In your letter you say you have many more terms of (16): 1,2,2,2,4,4,2,6,7,... . If it would be possible for you to send me about 70 terms this would enable me to fill up two lines in the entry in the next supplement to my book. Also what reference should I give for this sequence? Has it been published?

You end your letter with the cryptic remark that once you would have saved considerable time if you could have identified 1,9,5,3,6 quickly. I give up! Please explain.

With best wishes,

MH-1216-NJAS-mv

N. J. A. Sloane

New seq

BOAS

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TELEPHONE (312) 492-3298

14 September 1974

Dear Dr. Sloane:

Thank you for your letter of 28 August, which was waiting for me when I returned from vacation.

You are right about Barrodale; it serves me right for not checking the reference before sending it on.

seq

As to 1,2,2,2,4,4,2,6,7,..., I attach the first 84 entries. These are the numbers of distinct integral solutions (x,y,z) of $(bx+y)/(bz+x) = y/z$ for the first 84 composite integers b (not counting 1 as composite). There are 0 solutions for prime b, so you could also make the sequence 1,0,0,1,0,2,0,2,2,4,0,4,... These were copied from computer print-outs that also listed the individual solutions (x,y,z). Beyond this I don't have a systematic list, but if you should want more I can run the program for additional values of b (it takes around a second per entry on our machine in this range, with the current version of the program). A259981

The structure of the sequence is rather mysterious. I know, for example, that the entry 2 occurs when and only when b = 8 or 9 or b = 2q with q an odd prime, but it is a deep unsolved problem whether there are infinitely many of these. When b is composite and b-1 is prime, the entry is the number of proper factors of b. The entry is odd if and only if b is an even square. Most of this stuff is unpublished and likely to remain so, so you will have to label it "personal communication".

As to 1,9,5,3,6, the terms 9,5,3,6 are the leading decimal digits of 2^{-20}. The story on this one is that I was asked to referee a note on the harmonic series, dealing with the fact (known for at least 60 years) that if you discard the reciprocals of the integers whose decimals contain a zero, the resulting series converges. The question then arises, what is its sum? (Very likely Bell Labs knew the answer already, but I didn't think to ask.) A259982

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Somebody with more courage than wisdom undertook to program this, and I was shown the alleged partial sums at intervals of 10^5 from 10^5 to $8\text{x}10^5$. I happened to notice that the last two differences were both 0.05631. This suggested that the computer was adding0.05631 x 9^{-5} = 9.536 x 10^{-7} at each step, and I went to bed worried about what this number was. In the middle of the night I woke up with the realization that it <u>had</u> to be the appropriate negative power of 2, and that the machine was adding the large terms first.

Incidentally, in case Bell Labs doesn't already know it, the sum of the series turns out to be between 23.103428 and 23.103474. The sum of the terms larger than 10^{-6} is less than 10, so the remainder approaches zero very slowly; indeed, so slowly that no conceivable machine could get within 1 of the sum by direct addition.

Sincerely yours,

R.P. Boas

R.P. Boas

A2808

#1322 A 259981

[redacted]

Base	Entry	Base	Entry	Base	Entry
4	1	55	16	102	6
6	2	56	22	104	6
8	2	57	14	105	46
9	2	58	6	106	14
10	4	60	10	108	10
12	4	62	2	110	6
14	2	63	12	111	18
15	6	64	21	112	24
16	7	65	12	114	6
18	4	66	20	115	18
20	4	68	4		
21	10	69	10		
22	6	70	22		
24	6	72	10		
25	6	74	2		
26	4	75	12		
27	6	76	20		
28	10	77	14		
30	6	78	24		
32	4	80	8		
33	8	81	24		
34	6	82	8		
35	6	84	10		
36	21	85	26		
38	2	86	6		
39	6	87	6		
40	18	88	18		
42	6	90	10		
44	4	91	28		
45	18	92	16		
46	10	93	10		
48	8	94	6		
49	10	95	6		
50	10	96	30		
51	12	98	4		
52	12	99	24		
54	6	100	37		

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December 2, 1974

Professor R. P. Boas
Northwestern University
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Evanston, Illinois 60201

Dear Professor Boas:

Thank you very much for your letter of 14 September. (Things have been a bit hectic this semester.) The sequence you sent will go into the next Supplement to the H'book, but this won't be finished for a few months.

You mentioned one problem that depends on there being a zero in the decimal expansion of the number. The enclosed note deals with another. For example, it seems very likely that every power of 2 above 2^{15} contains a zero when written in base 3. Have you any idea how one might prove this?

With best wishes.

Yours sincerely,

MH-1216-NJAS-mv

N. J. A. Sloane

Enc.
As above

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9 December 1974

Dear Dr. Sloane:

Thank you for the reprint and for the intriguing problem. Unfortunately I haven't any idea of how one might attack it. Does it have an application to telephones?

I presume 3^n always has zeros in base 2 when $n > 1$, but how many must it have?

Sincerely yours,

Ralph Boas

Scan A 5381
Boas letter etc
etc

8 pages

a lot of sequences