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May 7, 1990

A5555-A5573 and A2057 A3517-9 A90749

Neil J.A. Sloane,
A.T.& T. Bell Laboratories 2C-376,
600 Mountain Avenue,
Murray Hill NJ 07974, U.S.A.

Dear Neil,

This is a second edition of my 90-04-30 letter, containing a few more members of the various suggested sequences.

I've just written a paper which contains an infinity of sequences, not all of which are in the Bible, so here is a concordance to my apocrypha.

In Fig. 1, cols. $x = 0$ & 1 are Catalan numbers (S. 577). Cols. 2, 4, 6, 8, 10, 12 are S. 1130, 1602, 1866, 1981, 2^{11}, 2104, and are dignified with the label of Laplace coefficients, though a mere trigonometer like myself would probably define them by my equation (3). Now the odd columns are almost as worthy of note (as I write it occurs to me that there must be a corresponding trig formula) and indeed the indefatigable Sloane does record $x = 3$ as S. 1415, although on this occasion he did get fatigued before completing the canonical two lines:

A5555

S. 1415: 1, 4, 14, 48, 165, 572, 2002, 7072, 25194, 90440, 326876, 1188640, 4345965, 15967980, 58929450, 218349120, 811985790, 3029594040, 11338026180, 42550029600, 160094486370, 603784920024, ... A2057

I offer $x = 5$, 7, 9 (which is not *quite* $\mathcal{S}_n^{(4)}$), and $x = 11$ as being worthy of consideration for edition 2 (which, if I have anything to do with it, will have everything beginning from, if not with, zero — in a sense to be revealed, though not in this letter). Notice that $w(2n+1, 2y+1) = w_n(y)$.

$x = 5$: 1, 6, 27, 110, 429, 1638, 6188, 23256, 87210, 326876, 1225785, 4601610, 17298645, 65132550, 245642760, 927983760, 3511574910, 13309856820, 50528160150, 192113383644, ... A3517

$x = 7$: 1, 8, 44, 208, 910, 3808, 15504, 62016, 245157, 961400, 3749460, 14567280, 56448210, 218349120, 843621600, 3257112960, 12570420330, 48507033744, ... A3518

$x = 9$: (cf. S. 2018) 1, 10, 65, 350, 1700, 7752, 33915, 144210, 600875, 2466750, 10015005, 40320150, 161280600, 641886000, 2544619500, 10056336264, ... A3519

$x = 11$: 1, 12, 90, 544, 2907, 14364, 67298, 303600, 1332045, 5722860, 24192090, 100975680, 417225900, 1709984304, ... A90749

The diagonals may also be worthy of note,and alternate members thereof are also the diagonals of Fig. 3, $w(2n+1, 2y+1) = w_n(y)$, as I now see I've already said. Indeed, $w(n, n) = 1$, while $w(n, n-2)$ and $w(n, n-4)$ are S. 173 & 522, and I offer:

A5555

$w(n, n-6)$: 5, 14, 28, 48, 75, 110, 154, 208, 273, 350, 440, 544, 663, 798, 950, 1120, 1309, 1518, 1748, 2000, 2275, 2574, 2898, 3248, 3625, 4030, 4464, 4928, 5423, 5950, 6510, 7104, 7733, 8398, ...

A5555

A5556

$w(n, n-8)$: 14, 42, 90, 165, 275, 429, 637, 910, 1260, 1700, 2244, 2907, 3705, 4655, 5775, 7084, 8602, 10350, 12350, 14625, 17199, 20097, 23345, 26970, 31000, 35464, 40392, 45815, 51765, 58275, 65379, 73112, 81510, ...

A5557

$w(n, n-10)$: 42, 132, 297, 572, 1001, 1638, 2548, 3808, 5508, 7752, 10659, 14364, 19019, 24794, 31878, 40480, 50830, 63180, 77805, 95004, 115101, 138446, 165416, 196416, 231880, ...

The totals in Figs. 1 & 3 are S. 294 & 1144, of course.

A5558

The columns of Fig. 5 are not in Sloane:

$w'_n(0)$: 1, 1, 3, 6, 20, 50, 175, 490, 1764, 5292, 19404, 60984, 226512, 736164, 2760615, 9202050, 34763300, 118195220, 449141836, 1551580888, 5924217936, 20734762776, 79483257308, 281248448936, 1081724803600, 3863302870000, ...

A5559

$w'_n(1)$: 1, 2, 8, 20, 75, 210, 784, 2352, 8820, 27720, 104544, 339768, 1288287, 4294290, 16359200, 55621280, 212751396, 734959368, 2821056160, 9873696560, 38013731756, 134510127752, 519227905728, 1854385377600, 7174705330000, ...

A5560

$w'_n(2)$: 1, 3, 15, 45, 189, 588, 2352, 7560, 29700, 98010, 382239, 1288287, 5010005, 17177160, 66745536, 232092432, 901995588, 3173688180, 12342120700, 43861998180, 170724392916, 611947174608, 2384209771200, ...

A5561

$w'_n(3)$: 1, 4, 24, 84, 392, 1344, 5760, 19800, 81675, 283140, 1145144, 4008004, 16032016, 56632576, 225059328, 801773856, 3173688180, 11392726800, 44986664800, 162594659920, 641087516256, 2331227331840, 9183622822400, ...

A5562

$w'_n(4)$: 1, 5, 35, 140, 720, 2700, 12375, 45375, 196625, 715715, 3006003, 10930920, 45048640, 164105760, 668144880, 2441298600, 9859090500, 36149998500, 145173803500, 534239596880, 2136958387520, 7892175863000, ...

while the first two diagonals are trivial and S. 173, and the next few may be worth entering (I've just realized that my notation has become ambiguous round about here; must try to cure it):

A5563

$w'_n(n-2)$: 3, 8, 15, 24, 35, 48, 63, 80, 99, 120, 143, 168, 195, 224, 255, 288, 323, 360, 399, 440, 483, 528, 575, 624, 675, 728, 783, 840, 899, 960, 1023, 1088, ... $n^2 - 1$ ...

A5564

$w'_n(n-3)$: 6, 20, 45, 84, 140, 216, 315, 440, 594, 780, 1001, 1260, 1560, 1904, 2295, 2736, 3230, 3780, 4389, 5060, 5796, 6600, 7475, 8424, 9450, 10556, ... $\frac{1}{2}n(n+1)(n-2)$ ...

A5565

$w'_n(n-4)$: 20, 75, 189, 392, 720, 1215, 1925, 2904, 4212, 5915, 8085, 10800, 14144, 18207, 23085, 28880, 35700, 43659, 52877, 63480, 75600, 89375, 104949, 122472, 142100, 163995, 188325, 215264, 244992, 277695, 313565, 352800, ... $\frac{1}{4}n^2(n+1)(n-3)$ ...

The sums of the rows of Figs. 5 & 6 are the same:

A5566

u'_n: 1, 2, 6, 18, 60, 200, 700, 2450, 8820, 31752, 116424, 426888, 1585584, 5889312, 22084920, 82818450, 312869700, 1181952200, 4491418360, 17067389768, 65166397296, 248817153312, 953799087696, 3656229836168, 14062422446800, 54086240180000, ...

The diagonals of Fig. 6 are S. 432, twice S. 1587 (!), and

A5567

$w''_n(n-4)$: 10, 70, 308, 1092, 3414, 9834, 26752, 69784, 176306, 434382, 1048812, 2490636,

Zintaly

5833006, 13500754, 30933368, 70255008, 158335434, 354419190, 788529700, 1744831060, 3841983110, 8422163130, 18387829488, ...

and others too numerous to mention. The first few cols. may be worth noting:

A5568 cols 0 & 1: 1, 2, 10, 70, 588, 5544, 56628, 613470, 6952660, 81662152, 987369656, 12228193432, 154532114800, 2980262214000, 38892421892700, ...

A5569 col 2: 4, 34, 308, 3024, 31680, 349206, 4008004, 47530912, 579058896, 7215393640, 91644262864, 1183274479040, 15497363512800, 205519758825150, ...

while from Fig. 7 I might be able to sell you:

A5570 $W_n(n-2)$: 17, 50, 99, 164, 245, 342, 455, 584, 729, 890, 1067, 1260, 1469, 1694, 1935, 2192, 2465, 2754, 3059, 3380, 3717, 4070, 4439, 4824, 5225, 5642, 6075, 6524, 6989, 7470, 7967, 8480, ... $(n-1)(8n+1)$ ...

A5571 $W_n(n-3)$: 76, 288, 700, 1376, 2380, 3776, 5628, 8000, 10956, 14560, 18876, 23968, 29900, 36736, 44540, 53376, 63308, 74400, 86716, ...

$W_n(n-4)$: 354, 1605, 4569, 10318, 20180, 35739, 58835, 91564, 136278, 195585, 272349, 369690, 490984, 639863, 820215, 1036184, 1292170, 1592829, ...

$W_n(n-5)$: 1704, 8824, 28476, 72128, 156624, 305208, 548548, 925760, 1485432, 2286648, 3400012, 4908672, 6909344, 9513336, 12847572, 17055616, 22298696, ...

A5572 $W_n(0)$: 1, 4, 17, 76, 354, 1704, 8421, 42508, 218318, 1137400, 5996938, 31940792, 171605956, 928931280, 5061593709, ...

A5573 W_n: 1, 5, 26, 139, 758, 4194, 23460, 132339, 751526, 4290838, 24607628, 141648830, 817952188, 4736107172, 27487711752, 159864676803, ...

Best wishes, and E&OE as we used to say in the East. One E was that a term in the last sequence was misprinted as 132331: unfortunately this is in the paper as well!

Yours sincerely,

Richard.

Richard K. Guy
Emeritus Professor of Mathematics

RKG/rkg

P.S.: I have an electronic copy of all this, which can be updated, emailed, or whatever. — R.