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GENERALIZED FERMAT PRIMES

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Fermat conjectured that all numbers of the form $2^{2^N} + 1$ were prime. It's one of the few times that Fermat was wrong. Although these numbers are prime for 0, 1, 2, 3, and 4, no prime has been found for N greater than 4, and, of course, a lot of work has been done in finding factors of these Fermat numbers [1].

Just to check up on Fermat's intuition, I have searched for primes of the form $b^{2^N} + 1$ where the base, b, is other than 2. In a certain sense, these might be considered one of the simplest types of primes since $b^x + 1$ can only be prime if x is a power of 2, and it is very easy to prove primality. Of course, b must be even.

I have made two tabulations — for each N I have compiled a list of the smallest b's that produce a prime (see Table 1) and I have also searched

Table 1. Smallest Base Values Yielding Generalized Fermat Primes

N	2^N	Base b for $P = b^{2^N} + 1$		
0	1	2,4,6,10,12,16,18,22, . . .	$P-1$	A6093
1	2	2,4,6,10,14,16,20,24, . . .		A5574
2	4	2,4,6,16,20,24,28,34, . . .		
3	8	2,4,118,132,140,152,208,240, . . .		
4	16	2,44,74,76,94,156,158,176, . . .		
5	32	30,54,96,112,114,132,156,332, . . .		
6	64	102,162,274,300,412,562,592,728,1084, . . .		
7	128	120,190,234,506,532,548,960,1738, . . .		
8	256	278,614,892,898,1348,1494,1574,1938, . . .		
9	512	46,1036,1318,1342,2472 (Tested up to 2728)		
10	1024	824 (Tested up to 986)		
11	2048	150 (Tested up to 958)		
12	4096	None found (Tested up to 58)		