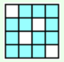
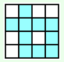
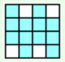
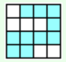
Number of n×n polyominoes (= number of n×n lake patterns of (n+2)×(n+2) squares)

Walter Trump, 2016-01-24

Transformations T: **J** = Idendity, **P** = point reflection (= Rotation by 180°),
A1 = Reflection with respect to a horizonal axis, **A2** = Reflection with respect to a vertical axis, **D1** = Reflection with respect to the primary diagonal,
D2 = Reflection with respect to the secondary diagonal, **R1** = Rotation by +90° (clockwise), **R2** = Rotation by –90°.

– We also count the obvious invariance under idendity.
– Number of polyominoes of certain symmetry type: a (8 invariances), $b = b_1 + b_2 + b_3$ (4 invariances), $c = c_1 + c_2 + c_3$ (2 invariances), d (1 invariance)

	Invariant to all T	Invariant to A1, A2, P, J	Invariant to D1, D2, P, J	Invariant to R1, R2, P, J	Invariant to A1, J	Invariant to D1, J	Invariant to P, J	Invariant to J	free polyominoes	fixed polyominoes (enumerated by Iwan Jensen)
n	a	b_1	b_2	b_3	c_1	c_2	c_3	d	$N_{free} = a+b+c+d$	$N_{fixed} = a + 2b + 4c + 8d$
1	1	-	-	-	-	-	-	-	1	1
2	1	-	-	-	-	1	-	-	2	5
3	3	1	1	-	5	6	1	7	24	111
4	3	1	5	2	17	75	7	941	1051	7943
Sample										
5	17	47	32	10	1692	1336	304	234610	238048	1890403
6	17	47	204	68	9039	42153	5189	195228256	195284973	1562052227
7	163	3890	1961	643	4665997	2437706	527675	577162256538	577169894573	4617328590967
8	163	3890	20529	5517	47418324	267667751	31112790	6200685777996227	6200686124225191	49605487608825311
9	2753	943825	350984	98250	153577520552					1951842619769780119767
10	2753	943825	6819896	1476342						282220061839181920696642671
11	84731	780983748	218032349	50125363						150134849621798165832163223922131
12	84731	780983748	8147822769	1436792525						293909551918134914019004192289440616787
13	4879497	2308661855735								
14	4879497	2308661855735								

For n = 6, 7, 8 N_{free} and d was calculated using N_{fixed}: $N_{free} = (N_{fixed} + 7a + 6b + 4c) / 8$
The other entries were calculated independent from N_{fixed} .
This investigation was inspired by Craig Knecht.

Relations between certain numbers in the polyominoe table

Let $a(y)$ be the value of a for n=y. Same for $b_1(y)$, $b_2(y)$, …

Propositions (for positive integers k, m, n), (*easy to prove*)

1) $N_{free}(n) = a(n) + b(n) + c(n) + d(n)$
2) $N_{fixed}(n) = a(n) + 2 \cdot b(n) + 4 \cdot c(n) + 8 \cdot d(n)$
3) $N_{free}(n) = (N_{fixed}(n) + 7a(n) + 6b(n) + 4c(n)) / 8$
4) $a(2k) = a(2k - 1)$
5) $b_1(2k) = b_1(2k - 1)$
6) $N_{fixed}(k) = a(2k) + 2b_1(2k)$
7) $a(2k) = a(k) + 2b_2(k) + 2c_2(k)$
8) $c_1(n) = (N_{rect}(n,m) - N_{fixed}(m)) / 2$ with $m = \lceil \frac{n}{2} \rceil$ *See table below*.

   (This equation was used for calculating $c_1(9)$ and for confirming $c_1(n)$ with n<9.)

Fixed rectangle n×m polyominoes

n	$m = \lceil \frac{n}{2} \rceil$	number of fixed polyominoes $N_{rect}(n,m)$
2	1	1
3	2	15
4	2	39
5	3	3495
6	3	18189
7	4	9339937
8	4	94844591
9	5	307156931507