Observing Global Ocean Circulation with SEASAT Altimeter Data

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Abstract A nondynamic algorithm for correcting the geocentric radius of the SEASAT orbit has been developed. This scheme reduces the satellite altitude error to a few decimeters and requires only weak a priori knowledge of sea surface undulations. Application has been made to a single three-day period during which SEASAT took global altimeter measurements of sea surface topography with geographic resolution of 900 km at the equator. The resulting corrected SEASAT ephemeris then enabled computation of a sea surface largely free of the 2 to 7 m error that would otherwise arise from error in the ephemeris distributed with the SEASAT data. Subsequent subtraction of GEM L2 geoid heights from this sea surface has yielded ocean dynamic heights in reasonable qualitative and quantitative agreement with values obtained from oceanographic data. The fact that results of this quality could be obtained from only one three-day arc of SEASAT data demonstrates the potential of satellite altimetry for determining global ocean circulation. The results also show that the solution to the problem of ephemeris error is to be found by considering both dynamic and nondynamic orbit determination methods.

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Introduction

A satellite-borne altimeter observes profiles of the ocean surface. For SEASAT, the precision of this measurement is about 5 cm (Lorell et al., 1980). However, what is required is data on the sea surface shape relative to the geocenter (or equivalently to the ellipsoid), not relative to the trajectory of the satellite from which altimeter measurements are taken. Thus it was recognized long ago (Marsh and Douglas, 1971) that orbit determination error would create a serious problem for the interpretation of altimeter data at any level of interest to oceanography, if not geodesy. For example, the SEASAT geophysical data records distributed by the Jet Propulsion Laboratory contain two independently derived estimates of spacecraft altitude above the ellipsoid. These ephemerides differ by up to 7 m (Marsh and Williamson, 1980), an amount that is many times the deviation of the sea surface from the geoid caused by geostrophic currents or any other large-scale oceanographic phenomenon.

None of the above is meant to imply that SEASAT and GEOS-3 altimeter data are useless for geodesy or oceanography. The problem of relative ephemeris error has been completely solved for investigations involving profiles of altimeter data up to several thousand kilometers in length. Over this scale, the error can be treated as a linear trend (Douglas and Gaborski, 1979; Anderle and Hoskin, 1977) to the level of a few centimeters. Thus it is possible to take a series of intersecting altimeter profiles over a region several thousand kilometers on a side and use the discrepancy of sea surface height at the intersections to solve simultaneously for a trend in each profile that effectively eliminates ephemeris error. This scheme reached its peak of application in the computation of a mean altimetric sea surface for the Northwest Pacific Ocean by Marsh and Cheney (1982). After solving for a linear trend for each profile, they obtained root-mean-square (rms) discrepancy of sea surface height at intersections of ascending and descending SEASAT altimeter profiles of only 12 cm.

Although effective for many applications, the linear trend approximation to satellite ephemeris error has limitations. The length over which ephemeris error can be treated as linear is limited, and any sea surface based on such a solution will have an overall erroneous tilt and bias. This occurs because the simultaneous solution for profile trends is singular unless one or more profiles are held unadjusted for trend, creating an error in the final surface from the error in the unadjusted profile(s). But since satellite ephemeris error is very long wavelength, *mesoscale* information is unaffected. GEOS-3 altimeter data especially have shown how effectively altimeter data can be used for mesoscale investigations (Cheney and Marsh, 1981; Douglas and Cheney, 1981; Douglas et al., 1983).

Reduction of Global-Scale Satellite Ephemeris Error

The discussion in the previous section establishes that intersections of altimeter profiles offer a powerful constraint, although not a total solution, for correcting the ephemeris of a satellite. At first glance it may seem surprising that the intersections on a global basis do not entirely solve the problem of ephemeris error, because they are so numerous and well distributed geographically. As an example, consider the groundtrack of SEASAT over the three-day period September 20–22, 1978, shown in Figure 1. During this time SEASAT was following a repeating groundtrack every three days, giving the uniform coverage (900 km at the equator) shown. The number of intersections (over the ocean surface) or "crossovers" during this period is about 1150. If the time sampling of these crossover points were ideal, they could be used to determine radial ephemeris error for frequencies up to 13 cycles per orbit. However, peculiarities of crossover time distribution for SEASAT introduce a narrow band of frequencies near 1.02 cycles per orbital revolution such that crossover points cannot determine the radial orbit error, that is, singular frequencies exist. All lower frequencies, and higher frequencies up to 13 cycles per revolution, can be extracted from a crossover analysis. (For a detailed discussion, see the Appendix.) It is ironic (but not unexpected) that most of the ephemeris error occurs at a frequency of once per revolution (Marsh and Williamson, 1980), so near to the singular frequency.

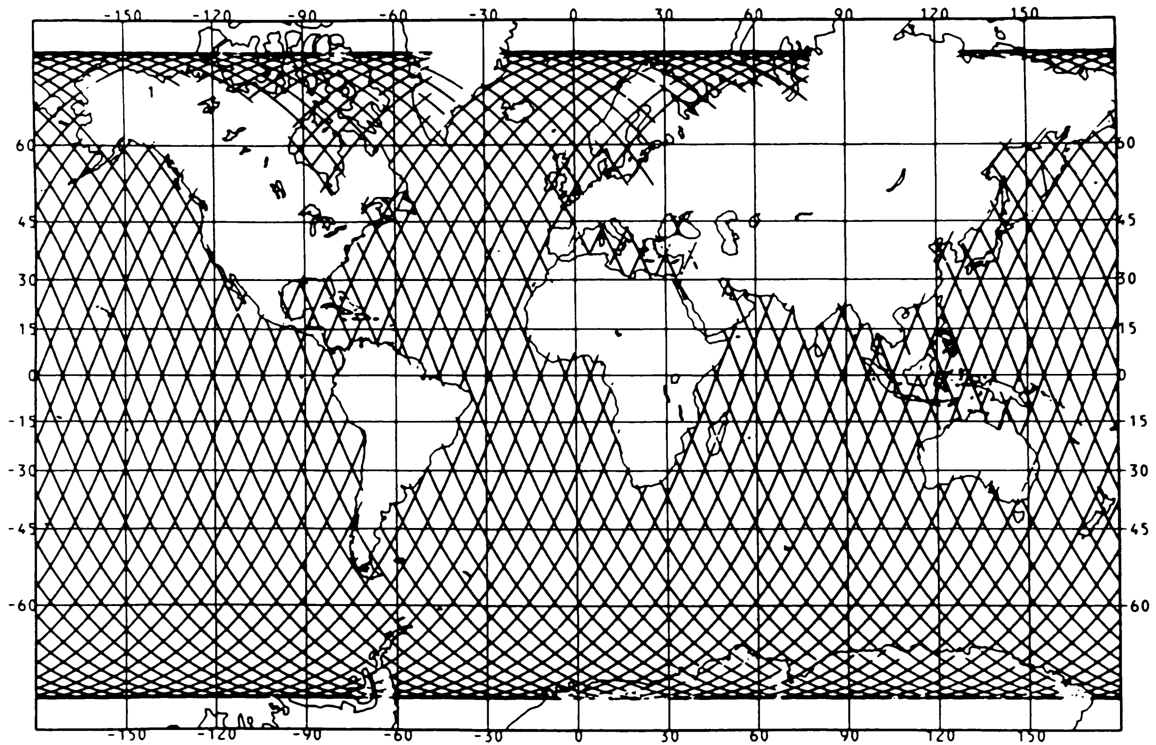

FIGURE 1. SEASAT satellite altimeter tracks for September 20–22, 1978. The resolution is about 900 km at the equator.

In order to determine the dominant once per revolution error of the ephemeris, additional information is required. The information can be added in several ways. Cloutier (1981) constrains the allowable change in the correction to the ephemeris between crossings to avoid the singularity that results from using crossovers alone. His model for the error is a sequence of trends between crossovers, so that the number of unknowns in his scheme is equal to twice the number of crossovers. While Cloutier (1981) has demonstrated that his method is capable of reducing crossover discrepancies essentially to zero, he has not shown that the method produces the correct sea surface height map. In fact, his scheme is insensitive to the average radius of the satellite's orbit, and is also unable to correct for any systematic error in the origin of the satellite ephemeris relative to the mass center of the earth.

From the arguments given above, it appears that there are two quantities that cannot be determined from a crossover adjustment alone. These are the bias in the orbit and the location of the mass center of the earth. Errors in the x, y, and z coordinates of the mass center appear as errors in the (1, 0), (1, 1), and (1, −1) spherical harmonic components of the dynamic height. (For the geoid, these harmonics are, of course, zero.) Our solution to this problem is to use residuals between altimeter-determined surface heights and heights from a geoid model along *with* the crossover differences, and solve by least squares for the Fourier series coefficients. (The highest frequency we solve for is twice per revolution.) The addition of the surface height data removes the singularity at 1.02 cycles per revolution and other possible singularities caused by poor sampling. We will show below that because the surface heights used in our solution are so weakly weighted (1 : 100) compared to crossovers, an error in the surface height model of even several meters does not propagate more than 20 cm into our final altimetric sea surface.

For the reasons given above, our model for the SEASAT altitude ephemeris error is a Fourier series whose independent variable is time. The coefficients must be determined by least squares, because the data are not equally spaced. For example, large data gaps occur while the satellite is over land. (An extended exposition of the method is presented in the Appendix.)

The solution involves two kinds of observations. These are the crossover difference values, each a function of two times, and the surface heights implied by the altimeter data, each of the latter a function of one time. The use of altimeter heights as data necessitates a model for the sea surface, and both measurement types require a tidal model. Fortunately, the SEASAT data records contain tidal information accurate to perhaps 10 cm in the broad ocean areas, so that further consideration of tides is unnecessary. The real model problem is, of course, the sea surface. But in fact any of the recent *satellite* geoids produced at Goddard Space Flight Center will serve as a model for sea surface profiles at the level of a few meters, enabling altimeter-derived surface heights to be used jointly with crossover differences in a solution for the Fourier coefficients in the series for the ephemeris error. (Of course, crossover differences are independent of the static part of sea surface topography and the geoid.) Given that such a solution is possible, it must now be shown that the error of the sea surface model does not propagate to a significant degree into the ephemeris correction.

Since our solution uses two kinds of data—surface heights and differences of surface heights at crossovers—the question of the proper weight for each must be addressed. Clearly, crossovers should have a much heavier weight than surface heights. In the least squares formulation the appropriate weights are the inverse of the variances of the measurement types. Since crossover differences are not a function of surface heights, their weight will be much greater than that of the height data. Crossovers with a precision of the order of a decimeter have already been obtained in a regional solution (Marsh and Cheney, 1982). This is about one-twentieth of the rms error of a geoid model approximation to the sea surface that ignores dynamic topography. Thus, the relative weight of crossovers to surface heights in a combined solution is 1:400, although, since surface heights are about 10 times more numerous than crossovers in our solution, the final relative weight is closer to 1:100. The effect of this is that only a fraction of the error in the sea surface model used to determine the correction to the SEASAT altitude ephemeris propagates into the final corrected altimetric sea surface. This can be seen by comparing the sea surfaces obtained by using different models in the least squares determination of the ephemeris error. We will show later that when our solution for dynamic heights is iterated, the final results are nearly unaffected by the use or nonuse of dynamic heights in the orbit correction step, demonstrating that only a fraction of the model error propagates into the final surface.

Results for Ocean Dynamic Heights

Figure 2, based on a compilation of dynamic heights prepared by Levitus (1982) from all available hydrographic data and referenced to 2000 dbars, shows the general features of the large-scale circulation. The heights in Figure 2 were smoothed by a two-dimensional Gaussian filter to approximately 20°. In this section we apply our algorithm to a three-day SEASAT data set to see if these main features of the general circulation can be observed.

Applying our correction algorithm to the Doppler ephemeris on the SEASAT Geophysical Data Record for September 20–22, 1978 reduced the rms crossover difference from 1.8 m to 28 cm. This crossover *difference* result suggests (but does not prove) that the accuracy of our corrected ephemeris is of the order of 20 cm. But the corrected orbit is only half of the picture, because ocean dynamic heights are the difference of the corrected altimetric sea surface and the ocean geoid. C. A. Wagner (private communication, 1982) estimates that the accuracy of the GEM L2 geoid model used in this paper is about 20 cm at a resolution of 20°, so dynamic heights obtained from our corrected SEASAT data set and GEM L2 should be accurate to about the 30-cm level at this scale, good enough to see the main circulation features. Figure 3 shows our results for dynamic height obtained from corrected SEASAT data and GEM L2, and also the current directions implied by geostrophy. Comparison with Figure 2 shows that the main features of the general circulation are visible in our map, although with some important

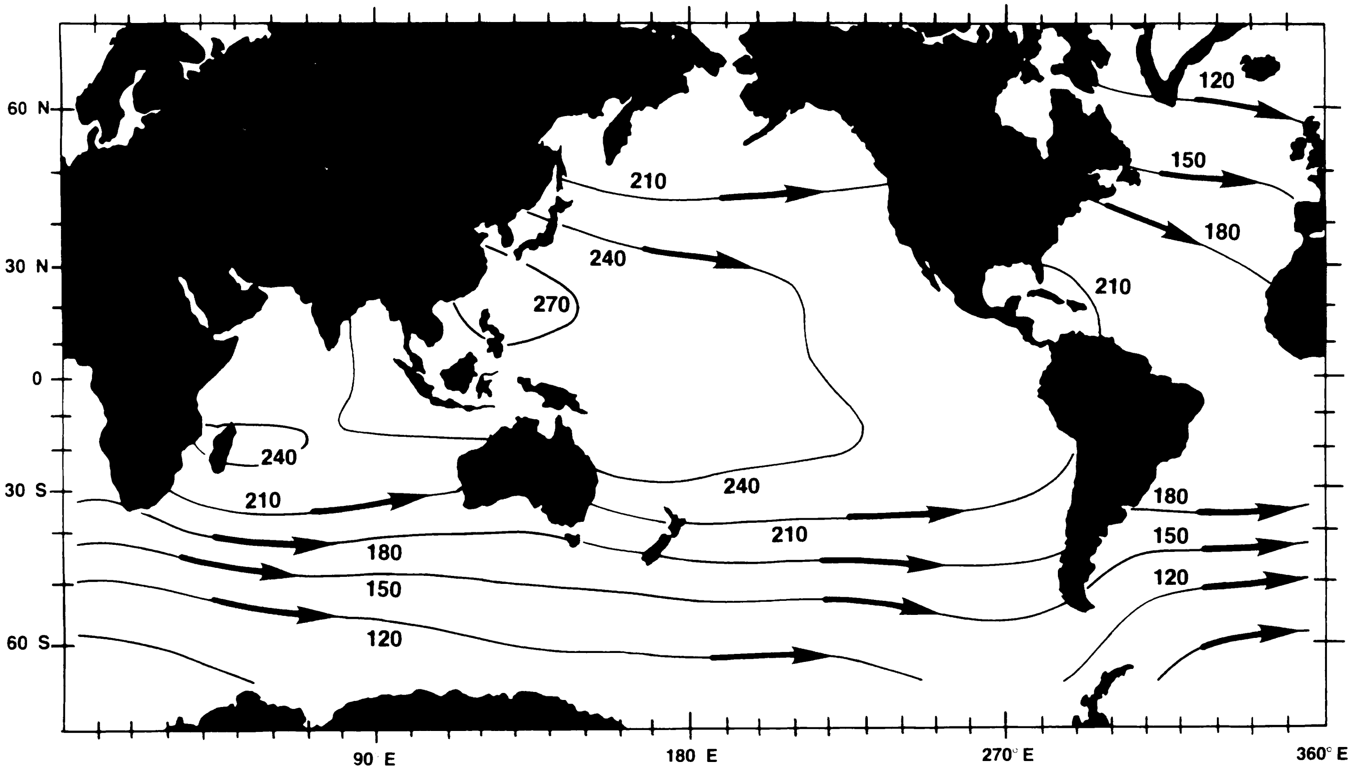

FIGURE 2. Smoothed ocean dynamic heights (cm) derived from Levitus (1982) referred to 2000 dbars.

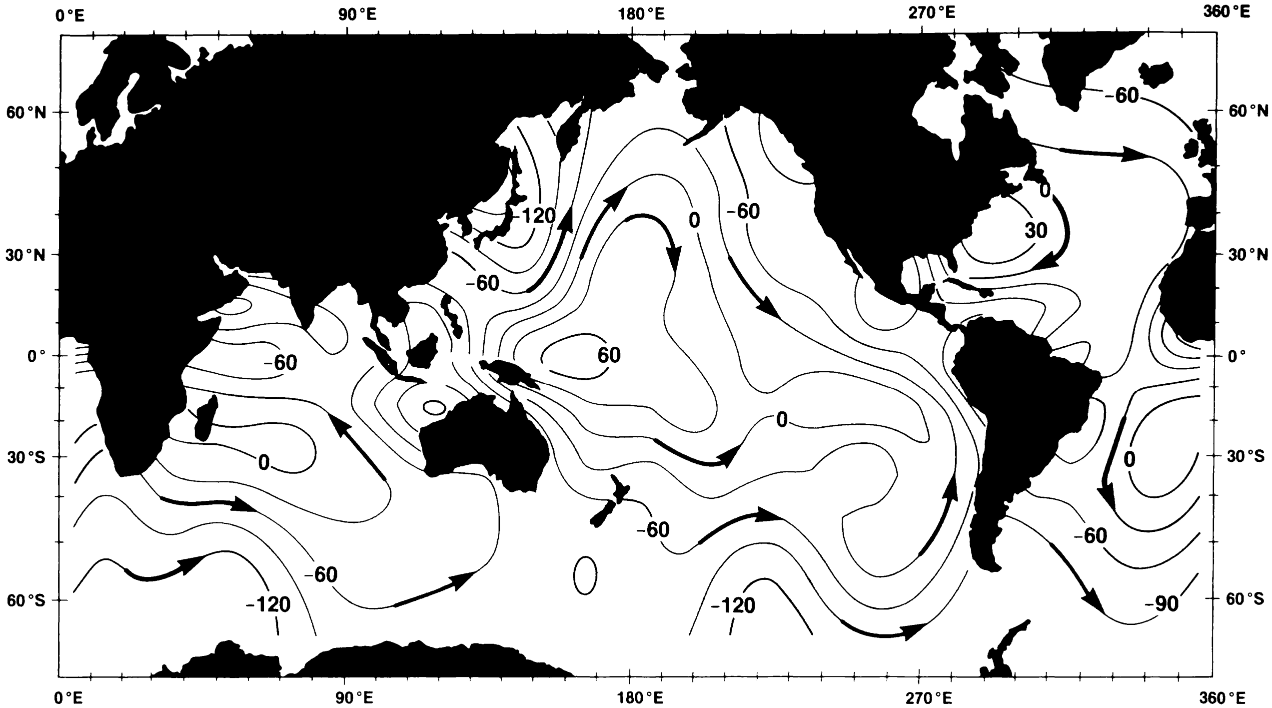

FIGURE 3. Smoothed ocean dynamic heights (cm) obtained from ephemeris-corrected SEASAT data and the GEM L2 geopotential model. The gyres have the right direction, but there are large distortions arising from error of the geopotential.

distortions, probably caused by geoid error. (Of course, only the slopes in Figures 2 and 3 are significant.) It is interesting to note that if the 1° resolution gravimetric geoid on the SEASAT data tapes is used instead of GEM L2 to compute dynamic height, the results do not resemble the real situation. The gravimetric geoid adequately models short-wavelength features such as trenches and ridges, but has errors at the scale of ocean basins greater than the dynamic topography; that is, more than 2 m. In contrast, GEM L2 produces satisfactory results when used to compute dynamic heights, at least at the resolution considered here.

The results shown in Figure 3 were computed in 10° blocks, because the resolution of the three-day arc of data was about this figure. Contouring was done by a two-dimensional Gaussian filter with a half-width of 10°, resulting in a further smoothing to about the 20° level. In spite of this smoothing, our dynamic height map appears to show a large amount of detail. This detail should not be taken seriously at this point. The main significance of our result is that we have shown that both an algorithm for correcting the satellite ephemeris and a geoid model exist that are good enough to detect features of the general circulation from a single three-day arc of SEASAT data. It is also important to remember that no a priori dynamic height information was used to prepare Figure 3.

Discussion

Figure 3 can be used as a rough means of evaluating the accuracy of the various quantities that went into it. These are the corrected SEASAT ephemeris and the GEM L2 geoid. The fact that our dynamic heights produce basin-wide gyres in the proper direction and of about the right magnitude certainly indicates that the combined error of GEM L2 and our SEASAT sea surface is less than the magnitude of the dynamic topography. The largest distortion in Figure 3 is in the north Pacific, where the flow appears to veer sharply northward and then abruptly south at the date line. In addition, the Antarctic Circumpolar Current shown in Figure 3 is much more complex than that shown in Figure 2. The dynamic height map of Tai and Wunsch (1983) prepared from Rapp's SEASAT/GEOS-3 surface and geoid heights computed from GEM 9 also shows the former feature, as does the new map of dynamic height prepared by Cheney and Marsh (1982) from the entire SEASAT/GEOS-3 data set and geoid heights computed from another GEM gravity model. Thus the apparent northward excursion of the north Pacific current may reflect an inaccuracy common to all GEM models. Since they share a large fraction of their data, an error common to all of them would not be surprising.

It was mentioned earlier that our procedure could be iterated. In the first step when the ephemeris was corrected, no dynamic height information was used. But Figure 3 provides at least an estimate of dynamic height, so it is reasonable to recorrect the SEASAT ephemeris by using the GEM L2 geoid plus our first estimate of dynamic height as the model for the ocean surface against which altimeter heights are compared, rather than GEM L2 geoid heights alone. When this iteration is performed, the changes are small, rarely exceeding 20 cm and usually less. Since the range of dynamic heights exceeds a meter, this verifies our contention that our method is not unduly sensitive to the initial reference surface used to correct the orbit. The results of the iteration also tend to confirm our earlier suggestion that most of the distortions of dynamic height seen in Figure 3 are caused by geoid model error.

There is another area of improvement in addition to the iteration on dynamic heights. It is preferable to begin the process with the most accurate trajectory possible, but the satellite ephemeris distributed with the SEASAT altimeter data was computed several years ago and is now obsolete. A recomputation of the SEASAT ephemerides would thus be valuable.

Conclusions

An algorithm has been developed that enables computation of a correction to the geocentric radius of SEASAT that reduces altitude error of the satellite to perhaps 20 cm, allowing computation of a global sea surface to comparable accuracy. Subtraction of geoid heights computed for the GEM L2 geopotential model then reveals ocean dynamic heights that resemble the actual ocean, at least qualitatively. Improvements can be made with improved geopotential models and SEASAT ephemerides. It has been determined that crossovers cannot be used alone to correct the SEASAT ephemeris because of a singularity arising from the crossover distribution. In addition, in this paper we have used just one of the many possible constraints (crossovers) that might be used in the reduction of altimeter data to determine ocean circulation. The best solution will come when all possible constraints, including hydrodynamic ones, are used.

Appendix. The Once per Revolution Singularity

Consider radial orbit error $O(t)$, which depends on time only. The difference between the orbit error at two times t_j and t_k can be measured at a crossover point, because the permanent sea surface topography nearly cancels in the subtraction. This difference function D is

$$D(t_j, t_k) = O(t_k) - O(t_j) \tag{1}$$

We want to know what part of $O(t)$ or $O(\omega)$ can be reconstructed from perfect crossover data. The orbit error functions in the time and frequency (ω) domain are related by the Fourier transformation

$$O(\omega) = \int_{-\infty}^{\infty} O(t)e^{-i2\pi\omega t}\, dt \tag{2}$$

$$O(t) = \int_{-\infty}^{\infty} O(\omega)e^{i2\pi\omega t}\, d\omega \tag{3}$$

Recovery of $O(t)$ from $D(t_j, t_k)$ depends primarily on how t_j and t_k are chosen. For SEASAT in its three-day repeat orbit, the time differences between t_j and t_k take on only 45 discrete values. This is shown in Figure 4, where the number of crossover points is plotted against the time spacing $\Delta t = t_k - t_j$ (this plot

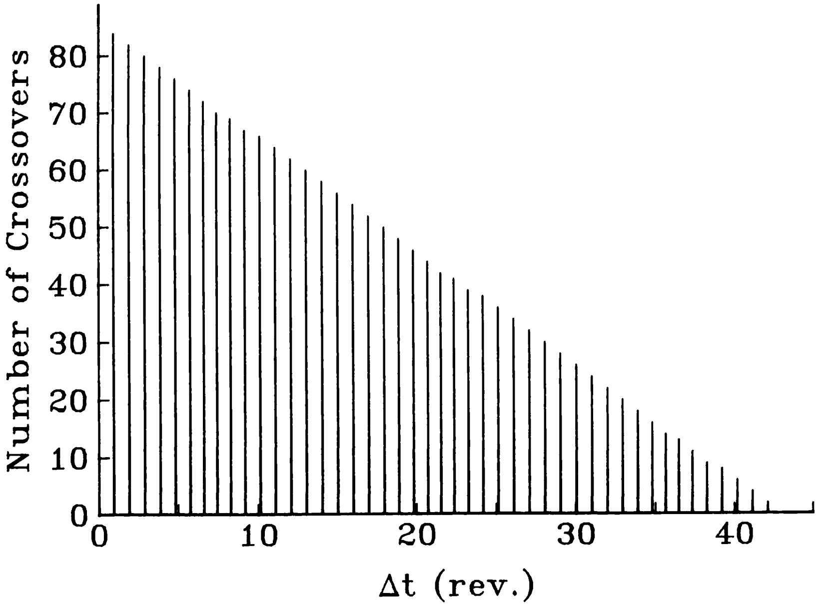

FIGURE 4. Histogram of crossover time differences for the three-day SEASAT repeat period. Note the regularity of the distribution. It is this distribution that causes the singularity near once per revolution.

includes crossover points over continents). Goad et al. (1980) also noticed this regular pattern. What is more surprising is that each Δt_ℓ (for $\ell = 1, 2, \ldots, 45$) is nearly an integral multiple of $\Delta t_1 = .98$ rev. This regularity in crossover time differences introduces a singular frequency of $1/\Delta t_1$ (i.e., 1.02 cycles per revolution).

To understand the origin of the singular frequency, consider the simplest case, where the time difference at every crossover point is the same (Δt). If we use Equation (3), Equation (1) becomes

$$O(t_k) - O(t_j) = \int_{-\infty}^{\infty} O(\omega)(e^{i2\pi\omega t_k} - e^{i2\pi\omega t_j})\, d\omega \qquad (4)$$

Since Δt is constant, we can rewrite t_j and t_k as

$$t_j = t - \Delta t/2, \qquad t_k = t + \Delta t/2 \qquad (5)$$

where t lies halfway between t_j and t_k. This leads to a simplification of Equation (4):

$$O(t + \Delta t/2) - O(t - \Delta t/2) = \int_{-\infty}^{\infty} (e^{i\pi\omega\Delta t} - e^{-i\pi\omega\Delta t}) O(\omega) e^{i2\pi\omega t}\, d\omega \quad (6)$$

If we use Equation (1) and notice that $2i \sin(\pi\omega\Delta t) = (e^{i\pi\omega\Delta t} - e^{-i\pi\omega\Delta t})$, Equation (6) becomes

$$D(t, \Delta t) = 2i \int_{-\infty}^{\infty} \sin(\pi\omega\Delta t) O(\omega) e^{i2\pi\omega t}\, d\omega \quad (7)$$

From Equation (7) it is clear that there are an infinite number of singular frequencies. They occur whenever the sine term vanishes; that is, $\omega\Delta t = n$, where n is any integer. At these frequencies, changes in $O(\omega)$ have no effect on the crossover function $D(t, \Delta t)$. Therefore, even if $D(t, \Delta t)$ was known exactly for all time, orbit errors with frequencies of $n/\Delta t$ could not be recovered. It is easy to understand why the zero frequency orbit error ($n = 0$) cannot be recovered from crossover differences. The nature of the nonzero frequencies, which are singular points of Equation (7), is not obvious unless one examines the crossover sampling times.

Now consider the actual situation where there are 45 discrete Δt values. In this case one could divide the crossover data into 45 sets, each with a constant Δt_ℓ. Equation (7) becomes

$$D(t, \Delta t_\ell) = 2i \int_{-\infty}^{\infty} \sin(\pi\omega\Delta t_\ell) O(\omega) e^{i2\pi\omega t}\, d\omega \quad (8)$$

for $\ell = 1, 2, \ldots, 45$. For simplicity we assume that $D(t, \Delta t_\ell)$ can be measured continuously for all time t (in reality $D(t, \Delta t_\ell)$ is sampled at regular intervals of $\Delta t_\ell/2$). Under this assumption the 45 equations in Equation (8) are redundant, unless none of their singular points coincide. The function $O(\omega)$ is only indeterminate when the sine functions in all of the 45 equations go to zero simultaneously or when

$$\omega\Delta t_\ell = n \quad (9)$$

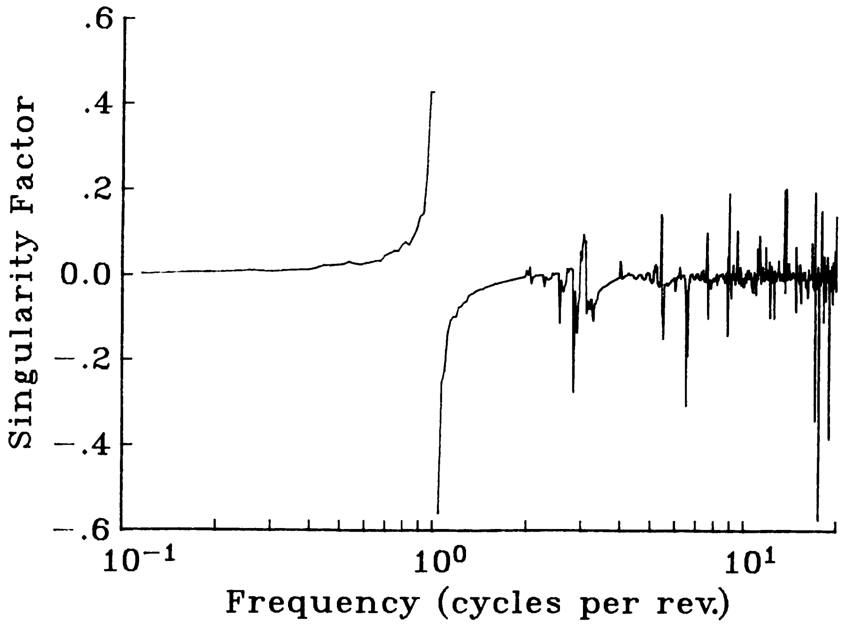

FIGURE 5. Singularity factor computed from Equation (8). Ephemeris error can be recovered from crossovers at frequencies less than once per revolution, but there is a singularity very near once per revolution that prevents determination of the dominant satellite ephemeris error from crossovers alone.

for every $\ell = 1, 2, \ldots, 45$. Unfortunately, this situation occurs for SEASAT in its three-day repeat orbit. This is because all of the 45 Δt's are nearly multiples of Δt_1:

$$\Delta t_\ell \cong \ell \Delta t_1 \tag{10}$$

The singular frequencies of Equation (8) are

$$\omega = n/(\Delta t_1) \tag{11}$$

Each of the 45 sets of singular frequencies coincide at these frequencies. Only the $n = 0$ and $n = \pm 1$ singularities are important, however, because Equation (10) is only approximate.

A crude measure of the singularity S of the system of Equations (8) is given by

$$S(\omega) = \left[\sum_{\ell=1}^{45} N_\ell \sin(\pi \omega \Delta t_\ell) \right]^{-1} \tag{12}$$

where N_ℓ is the number of crossover points with time differences Δt_ℓ. For the three-day repeat pattern there are 1925 crossover points, so at nonsingular frequencies the value of $S(\omega)$ should be roughly 1/1000. This function is plotted in Figure 5. As expected, $S(\omega)$ is about 10^{-3} for frequencies less than 0.5 cycles per revolution. At 1.023 cycles per revolution (i.e., $1/\Delta t_1$), $S(\omega)$ has a value of 4.7, indicating that less than one of the 1925 crossover points contributes information about $O(\omega)$ at this frequency. With real data that contain errors, there is a band of frequencies ranging from 0.88 to 1.14 cycles per revolution, where the inverse problem defined by Equation (8) is unstable.

Acknowledgment

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