Newsletter No. 18 **October 2002**

It looks as though the centuries old-problem of deciding whether a number is prime has at last been solved. The proof is striking in its simplicity and the event is even more remarkable in that the solvers include two undergraduates. Manindra Agrawal and his two students Neeraj Kayal and Nitin Saxtena from the Indian Institute of Technology in Kanpur have devised an algorithm that ascertains whether a number is prime in a reasonable amount of time. The problem with previous methods was that the time taken to determine whether a number was prime escalated alarmingly as the size of the number increased.

Prime numbers are divisible only by themselves and one. So 2, 17, 43 and 101 are prime but not 28 (divisible by 2) or 51 (divisible by 3). One way of checking whether a number is prime is to divide it by all the primes less than the number's square root. If it cannot be divided by 11 then it is not divisible by any multiple of 11. This test is simple but becomes onerous as the number gets larger. In about 250 BC the Greek astronomer Eratosthenes devised a method he called his 'sieve' for determining if a number was prime. However, it too is tedious for large numbers. Other sieves have been devised, notably that due to the Indian mathematician Sundaram in the 1930s (see below), but today computers are used although, for very large numbers, even that is not practical.

Number theorists have for a long time thought that there might be an algorithm that would solve the problem in a reasonable time. The search had intensified over the past few decades because prime numbers are important for public key cryptography. What no-one expected though was that the solution would be so simple. The significance of the Indians' work is that undergraduates could solve such a long-standing problem. It makes you wonder what other long-standing problems might be susceptible to simple methods. If you're interested in looking at the algorithm it is currently posted on the institute's website at www.cse.iitk.ac.in/primality.pdf

> ***Dialectic mathematics invites contemplation. Algorithmic mathematics invites action.***
>
> Peter Henrici

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WHAT'S NEW ON THE NZMATHS SITE THIS MONTH?

Bright Sparks

This month we'll have a new area of our site up and running. It's an interactive puzzle area for students and, by the end of the January, we hope to have seven new interactive problems there with special features for New Zealand teachers and students. The area is called Bright Sparks and can be accessed by the 'ball' on the Home page.

Our first puzzle is that old chestnut the Frogs problem. Can you move three green frogs from one set of lily pads to where the three brown frogs are and vice versa? So what is different, new and useful about our version of the problem? After all, if you look on the web you'll find the frogs' puzzle all over the place in one guise or another. Well, first of all the graphics are better than any other site we've seen. Second, the problem starts with three frogs on each side and builds that number up. But if the student is having trouble, the program slips the student back to just two frogs a side. If the student is successful, then we ask the student for the number of moves it will take to make the transition if there are n frogs on each side.

But we won't leave teachers in the lurch. There are two means of support that will be available by the end of January next year. First there will be notes available for you in our staff seminar series (see the Info Centre block on our Home page). These will provide full solutions (as well as the normal suggestions for ways to use the puzzles in a seminar to the teachers in your school or to your syndicate). But second there will be two forms of access to the Bright Sparks section of the site. One of these is complete open access so that anyone can try the puzzles from anywhere in the world. This is available now for Frogs. The other is closed access for New Zealand teachers only. If you apply to us for a password you will be able to choose from the puzzles and choose from the levels of the puzzles so that you can personalize the parts of the puzzles that you want your students to try. In addition we'll provide a mentoring system. The point here is that while we know whether a student can do a problem or not by seeing what they have achieved on screen, we don't know if the student really understands what is going on. Are they just guessing patterns or do they have a deeper knowledge of what's going on? Can they explain it? To try to fathom this, we'll be encouraging your students to send us their reasons behind their on-screen efforts.

All the puzzles that we will add to the site will have the features that the Frogs' puzzle has. They will involve animation, they will involve generalizations, and they will provide the opportunity for your class to write in and show us how well they understand what has been happening on screen. We hope that you and your students will get a lot out of this new area of the site. Let us know how it goes. We'll let you know as each puzzle is put on the site and we'll let you know as soon as we have the special closed access ready.

Believe it or not

Recently I met a maths colleague in a hardware shop buying a piece of equipment (well, it was a fancy electric saw, if you must know!). He found a young assistant to help him and the conversation went like this:

Y.A.	That will be $170 sir.
M.C.	Does that include G.S.T.?
Y.A.	Oh yes, sir.
M.C.	(Obviously something was bothering him) What was the original price?
Y.A.	(Frowning) Er … $160, sir, which means $20 G.S.T. but we're only charging you $10 so saving you $10 (smiling with satisfaction).
M.C.	So I'm paying $170 but you're only getting $150?
Y.A.	That's right sir.
M.C.	But if you'd only charged me $150 in the first place there would only be $18.75 G.S.T. to pay which would mean a total bill of $168.75.
Y.A.	Um …. yee …. es.
M.C.	That's all right then. You get the same and I pay $1.25 less, so that's what we'll do!

Sundaram's Sieve

In the table below, the first row is composed of the terms of the arithmetic sequence 4, 7, 10, …. with first term 4 and common difference 3. This sequence is also the first column. Succeeding rows are then completed so that each consists of an arithmetic sequence with the common difference in successive rows being the odd integers 3, 5, 7, 9, ….

4	7	10	13	16	19	22	25	………
7	12	17	22	27	32	37	42	………
10	17	24	31	38	45	52	59	………
13	22	31	40	49	58	67	76	………
16	27	38	49	60	71	82	93	………
…	…	…	…	…	…	…	…	………

The remarkable property of this table is that if N occurs in it, then 2N + 1 is not a prime number but if N does not occur in it then 2N + 1 is a prime number.

Web Links
You may not realise that we are regularly reviewing other web sites. To access these click on the Links jigsaw piece on our Home page. We've decided that it might be useful to put some of the best sites that we've found in the newsletter. So here is the first of these.

Site title: Internet Projects for Introductory Statistics

URL: http://www.awlonline.com/weiss/i_iprojects/index.htm

Purpose of the site: This site is designed to help you understand statistics by analysing real data and interacting with graphical demonstrations of statistical concepts.

New Zealand context:
This site provides useful information for teachers looking for real-life statistical situations to use in their teaching. The information provided could be used as the basis for planning a statistical study, suitable for either a series of small group teaching episodes or a classroom focus. Each situation is described with background information, data displays and information of the statistical concepts involved. Topics covered include the Titanic Disaster, Space Shuttle Challenger and Global Warming.

New and Different:
Each situation described is real and has been used to clearly illustrate the statistical concepts involved. For example, the *Titanic Disaster* provides students with survival statistics and asks them to question whether women and children were really saved first and whether the first class passengers were saved unfairly.

The site is easy to find your way around as the *Navigating the Site* section sets out clearly how the site is structured. The *References* section contains links to other sites, which could be used for follow up work.

Curriculum References
Mathematics, Levels: 3+
Strand: Statistics

Collective Nouns
In an email the other day mathematicians were referred to as a stream of mathematicians or a set of mathematicians. Is there a correct collective noun for

mathematicians? Can you think of something appropriate? We'd like to hear from you.

Solution to August's problem
The problem was to find how many soccer matches Action United drew.

Apart from the number of points for winning (2), drawing (1) and losing (0) we were given three pieces of information. Firstly that 40 teams competed in two divisions. Secondly that 156 more matches were played in one division than the other. From these we can ascertain how many teams competed in each division.

Two approaches are feasible. One uses algebra the other systematic 'trial and error'. Most people use the second approach and it can be very quick. For example, suppose there were 30 teams in one division (call it A) and 10 in the other (call it B). The teams in each division play every other team in their division once. So, in division A, each of the 30 teams play 29 others giving a total of 30 ✕ 29 ÷ 2 = 435 matches. Can you see why we have to divide 30 ✕ 29 by 2? Repeat the process for division B and check to see if the two results differ by 156. They don't, so try other values. You could draw up a table and zoom in on the correct answer, something like this….

Trial	Division A		Division B		Difference between Number of matches
	No. teams	No. matches	No. teams	No. matches	
1	30	435	10	45	390 too high!
2	25	300	15	105	195 still to high!
3	22	231	18	153	78 far too low!
4	24	276	16	120	156 spot on!

O.K. so division A has 24 teams and division B 16 but we don't know yet in which division Action United plays. Let's look at the other information that was given in the problem. Action United gained 20 points for the season. We know it lost two matches, so suppose it won **w** and drew **d**.

If Action United played in division A with 24 teams it would have played 23 teams and
w + **d** = 21. If Action United was in division B then it would have played 15 matches and **w** + **d** = 13.

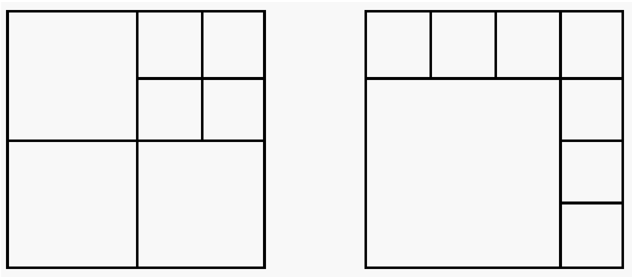
Now, each win is worth two points and a draw one, so the number of points gained by Action United is 2**w** + **d**. Now since 2**w** + **d** = 20, **w** + **d** can't be 21. So **w + d** = 13.

If **w + d** = 13 and 2**w** + **d** = 20, then **w** = 7. This gives **d** = 6.

Action United drew six matches.

Our August winner is Evan Jones . Congratulations! That's two in a row!

Solution to September's problem
A bit of playing around with paper and scissors may convince you that it is possible to cut a square into just about every number of squares above five. How can we be absolutely sure? These diagrams show that it is possible to cut a square into seven and eight squares.

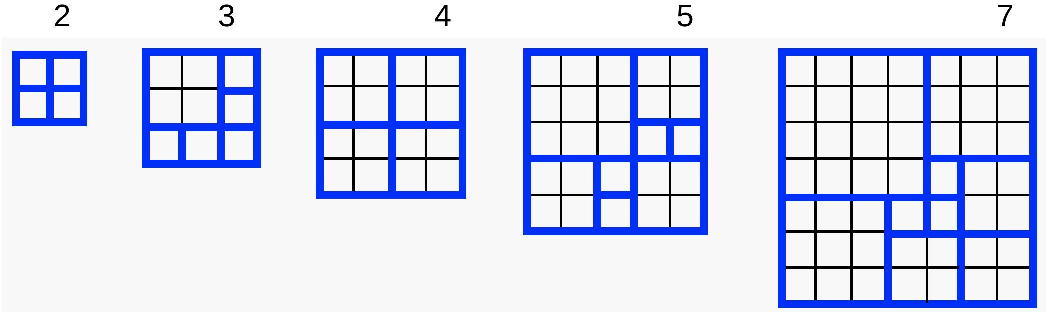


Six squares have already been shown. Can you see how seven squares have been obtained from four? Using one of the squares and quartering it adds three squares so the four become seven. We can use this method to add three squares to any total. Now we are already able to cut the square into six, seven or eight squares and three more than any of these and three more than any of those …. and so on. Hence we can cut a square into any number of squares apart from two, three and five.

We're sorry to say that we had no answers for this one. Why not try it with your class?

Problem of the month

I thought this month that we'd follow up the square dissection problem with another. What is the minimum number of squares into which a square can be cut? This type of problem is often called 'Mrs. Perkin's quilt' after an example of one named by the great puzzlist Henry Dudeney. Before we look at a specific problem let's investigate a bit by looking at squares with sides up to seven, particularly the two by two, three by three, four by four, five by five and seven by seven squares.



Minimum number of squares:

4 6 4 8 9

Playing around with various squares we can see that the minimum number of cut squares for an even sided square is four since we can cut by halving each side. A little more investigation will show that only prime sided squares are likely to cause complications.

O.K. Here's your problem (two really). What is the minimum number of squares into which (a) an eleven sided square (b) a thirteen sided square can be cut?

Remember that each month we give a petrol voucher to one of the correct entries. Please send your solutions to derek@nzmaths.co.nz and remember to include a postal address so we can send the voucher if you are the winner.

Finally, enjoy your teaching and remember, as Roy R. Behrens wrote in his book *Design in the Visual Arts* ,

> ***"It is a pleasant irony that one learns more from teaching than one learns from being taught."***