Double Auction Markets with Stochastic Supply and Demand Schedules:

Call Markets and Continuous Auction Trading Mechanisms*

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Abstract

Performance under two double auction trading mechanisms is investigated experimentally: call markets and continuous double auctions. Bayesian Nash equilibrium models of price formation provide the focus of the investigation. These models require randomly drawn redemption values, a departure from the usual experimental practice of stationary supply and demand schedules. Both mechanisms generate high efficiency levels and small deviations in price from the competitive equilibrium level, with the continuous double auction mechanism generating higher average efficiency levels. Bayesian Nash equilibrium models better organize the data than non-strategic trading models. Deviations in behavior from fully strategic trading are characterized and discussed.

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Performance under two different double auction trading mechanisms is investigated: a call market and a continuous double auction trading mechanism. Both auctions are two-sided with several buyers and several sellers. A call market is a discrete trading mechanism in which buyers (sellers) submit a single bid (offer) in each trading period and the market clears according to well defined rules of who trades and at what prices. In a continuous double auction trades can occur at any time in the trading period, with buyers and sellers free to continuously update unaccepted bids and offers. Both trading mechanisms have wide applications in field settings and have been the subject of intense experimental study as well (see Holt, 1995, for a survey of experimental work).

This paper adds to the literature through an experimental investigation of double auction (DA) markets in which buyers' valuations and sellers' costs are randomly drawn in each trading period. Although this procedure is common practice in experimental studies of one-sided auctions, it is rarely employed in experimental studies of DA markets, as these typically involve stationary supply and demand schedules. A random value environment is natural for investigating Bayesian Nash equilibrium theories of price formation in DA markets (Satterthwaite and Williams, 1989a, 1989b; Wilson, 1987; Friedman, 1991) since traders are assumed to have incomplete information regarding each other's valuations. In contrast, with stationary supply and demand schedules, traders effectively acquire complete information regarding market clearing price and quantity after several trading periods. Arguably, random valuation procedures are also (1) more representative of field settings, since supply and demand schedules are rarely, if ever, stationary from one period to the next, and (2) provide the appropriate vehicle for examining the Hayek (1945) hypothesis - that markets are capable of achieving (close to) the competitive equilibrium (CE) price and quantity resulting from truthful revelation - as this hypothesis is intended to apply to markets where traders have incomplete information regarding the CE.

Behavior is studied in markets with two buyers and two sellers (m = 2) and in markets with eight buyers and eight sellers (m = 8). The call market studied is the buyer's bid double auction (BBDA) (Satterthwaite and Williams [SW], 1989a, 1989b) in which fully rational traders achieve near 100% efficiency with m = 8 (SW, 1989a). The continuous double auction (CDA) trading mechanism studied uses New York stock exchange rules but no specialists book. The analysis proceeds on two levels: (1) Comparisons of efficiency and price convergence between the two mechanisms and compared to outcomes in markets with stationary supply and demand schedules, and (2) Comparisons of behavior with theoretical predictions for the two mechanisms.

Both trading mechanisms achieve relatively high efficiency levels (75% or higher) even in thin markets with m = 2. The CDA achieves substantially higher average efficiency levels than the BBDA both in markets with m = 2 (87.0% versus 77.1% average efficiency) and with m = 8 (95.1% versus 88.9%). There are no significant differences in price levels, relative to the CE norm, between the two institutions, with both large and small numbers of traders. Thus, consistent with received wisdom from experiments with stationary supply and demand schedules (Holt, 1995), CDA outcomes are close to the CE level in markets with stochastic supply and demand schedules and achieve higher efficiency levels than a sealed bid trading mechanism.[1] However, unlike markets with stationary supply and demand schedules, there is no tendency for efficiency or prices to converge to the CE level with increased trader experience *within* an experimental session. Rather, if anything, the data suggest greater deviations from the CE norm in the small markets with two buyers and two sellers.

[1] As Holt notes, these comparisons ignore costs associated with gathering buyers and sellers and the greater time cost of conducting a CDA.

The key theoretical prediction for the BBDA, higher efficiency with increased numbers of traders, is satisfied, with remarkably high efficiency levels (94%) observed for experienced traders with m = 8. However, buyers tend to bid more than predicted, particularly with m = 2, and sellers do not consistently follow the dominant strategy of offering at cost (typically offering at above cost). The first result is consistent with bidding above the risk neutral Nash equilibrium (RNNE) in one-sided, first-price, private value auctions (behavior which has sometimes been attributed to risk aversion). The second result is consistent with deviations from the dominant bidding strategy in one-sided, second-price, private value auctions and in uniform price, multiple unit auctions. Deviations from the dominant bidding strategy are attributed to (i) the non-transparency of the strategy and (ii) the relatively small costs associated with the deviations. Experimental sessions in which computerized sellers follow the dominant bidding strategy of offering at cost are used to test if the limited degree of strategic buyer misrepresentation observed might be in response to sellers' errors at offering above cost. There is no evidence to support this conjecture.

Contrasting theoretical predictions of the Wilson (1987) and Friedman (1991) models of the CDA price formation process are compared with those of ZI simulations. Consistent with both the Wilson and Friedman models, there is a strong tendency for higher valued buyers and lower cost sellers to trade first. Although these propensities are significantly less than the Wilson and Friedman model predictions, they are stronger than in the ZI simulations. Further, fewer units are traded than in the ZI simulations for all market sizes. The net result is that much of the inefficiency found in markets with m = 2 results from fewer units traded than the CE level, as both the Wilson and Friedman models imply. Finally, there is a clear tendency for price changes to be negatively correlated within an auction period, which is inconsistent with both the Friedman and Wilson specifications, but is similar to what is found in ZI simulations. However, the average absolute price changes are substantially smaller than in the ZI simulations and the opportunity to arbitrage prices is quite limited.

There have been a handful of earlier studies of DA markets with random supply and demand schedules, all of which have focused on testing Bayesian Nash equilibrium models of price formation in DA markets (Cason and Friedman, 1993, 1996; for continuous double auctions; Kagel and Vought, 1993 and Cason and Friedman, 1997; for call markets).[2] In addition to reporting results from a new data set, this paper differs from these earlier reports through explicitly comparing performance across the BBDA and the CDA.[3] In addition, we employ more extreme variation in the number of traders within the CDA.[4] One important result of this manipulation is that it reveals that inefficiencies in small markets results from too *few* trades occurring relative to the CE model prediction, as the Bayesian Nash equilibrium trading models predict. In contrast, in larger markets inefficiencies result from too *many* units trading . We offer some conjectures as to the basis for these differences resulting from market size.

The plan of the paper is as follows: Section I characterizes the theoretical implications of the Bayesian Nash equilibrium models for the two trading mechanisms studied. These are contrasted

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[2]There have also been a number of hybrid implementations of random and stationary environments with buyers' and sellers' individual valuations changing from period to period but with aggregate supply and demand schedules stationary (see Aliprantis and Plott, 1991, for example) or with stationary supply and demand schedules whose intercepts shift randomly so that equilibrium price varies while equilibrium quantity is stationary (McCabe, Rassenti and Smith, 1993).

[3]Since this paper was first written, Cason and Friedman (1998) report an experiment comparing a CDA mechanism and call market mechanisms, similar to the BBDA, in markets with four buyers and four sellers.

[4]Variation in number of traders is, among other things, essential to examining the hypothesis that thicker market promote economic efficiency.

with predictions for "zero intelligence" traders (Gode and Sunder, 1993), as this provides a useful benchmark for completely non-strategic behavior. Section II outlines the experimental procedures. The results of the experiment are reported in section III. A brief concluding section summarizes our main results.

I. Theoretical Implications

A. The Buyer's Bid Double Auction Mechanism

The BBDA (SW 1989a, 1989b) is a call market where bids and offers are collected from traders, supply and demand schedules are constructed, a market clearing price is established, and trades are executed at the market clearing price. In the BBDA, each buyer (seller) draws a single redemption value, x_{2i} (x_{1i}), from a known probability distribution F_2 (F_1) defined on the interval $[\underline{x}, \overline{x}]$. In the BBDA, buyers who get to trade earn profits equal to (x_{2i} - p), where p is the market price. Sellers who get to trade earn profits equal to (p - x_{1i}). Buyers and sellers who do not trade earn zero profits.

In the BBDA, price is selected at the upper endpoint of the closed interval determining the market clearing-price, with all trade occurring at this price. This is determined as follows: All bids and offers are arranged in non-decreasing order $s_1 \leq s_2 \leq \ldots \leq s_{2m}$, where m is the number of buyers and sellers in the market (the number of buyers is assumed to equal the number of sellers). Price is set at $p = s_{m+1}$. Trade occurs among all sellers whose offers are <u>strictly less</u> than p and all buyers whose bids are <u>greater than or equal</u> to p. SW show that in the case where $s_m < s_{m+1} < s_{m+2}$, p is a market clearing price; otherwise, there is excess demand. In the case of excess demand, the available units are allocated to buyers starting with the high bidder and continuing down the list of bidders until there are tied bids and not enough units remaining to satisfy them. At this point units are assigned randomly to the remaining eligible bidders.[5]

SW (1989a) demonstrate that each seller in the BBDA has a dominant strategy to offer at cost (x_{1i}). In response to this buyers bid less than their reservation value (x_{2i}). This strategic misrepresentation causes the BBDA to be ex post inefficient. However, the amount of buyer misrepresentation decreases rapidly as market size increases: with uniform distributions of traders' redemption values, and risk neutral buyers, expected efficiency increases from 92.6% with m = 2 to 99.6% with m = 8 (SW, 1989a; where efficiency is defined as realized consumer and producer surplus as a percentage of the maximum possible consumer and producer surplus). The buyer's bid function underlying this rapid increase in efficiency, given a uniform distribution of redemption values, is

$$b_i = \frac{m}{m+1} x_{2i} \tag{1}$$

In contrast, using a simple fixed price rule, with prices set at the midpoint of the distribution functions underlying redemption values, efficiency increases at a substantially slower rate (from 78.1% with m = 2 to 85.4% with m = 8; SW, 1989a).

Figure 1 illustrates two cases of the BBDA with m = 4. In Figure 1a, buyers 4 and 3 and sellers 1 and 2 trade at a price set by seller 3's offer (note that seller 3 does not trade here). In Figure 1b, again buyers 4 and 3 and sellers 1 and 2 trade, but in this case price is set at buyer 3's bid.[6]

The BBDA provides an explicit trading procedure that achieves efficiency levels quite close

[5] The BBDA is a special case (where k = 1) of the more general k-DA mechanism described in Rustichini, Satterthwaite and Williams (1994) and SW (1993).

[6] Smith et al. (1982), using stationary supply and demand schedules, implement a sealed bid-offer trading mechanism that is close in spirit to the k-DA mechanism with k = .5 (their PQ mechanism). However, in this experiment buyers and sellers had multiple units for sale and had to submit a single bid or offer for all units, which results in important strategic differences between their game and the single unit k-DA game.

to those that could be achieved using an optimal revelation mechanism (Myerson and Satterthwaite, 1983). However, unlike optimal revelation mechanisms, the BBDA does not require changing the mechanism's rules as the underlying distribution of valuations and costs change. Further, although SW do not analyze the effects of limitations on agents rationality and information processing on auction outcomes, they argue that "... our result that all equilibrium strategies of the BBDA in a large market are close to truthful revelation suggests that cognitive limitations are unimportant in large markets." (SW, 1989a, p. 479).

B. The Continuous Double Auction Mechanism

In a CDA trades can occur any time in a trading period with prices free to vary from one trade to the next. Wilson (1987) models the CDA price formation process as a sequential equilibrium of an extensive form game in which traders privately draw a single redemption value from a commonly-known joint distribution. The model is concerned with the price formation process within a given trading period.[7] The basic idea is that traders play a waiting game, but they are impatient as a result of possible preemption of gains by other traders. At some point a trader makes a "serious" offer, one which has a positive probability of being accepted in a sequential equilibrium. If this offer is not immediately accepted, the trader making the offer steadily improves it until it is accepted, as in a Dutch auction (other traders remain passive during this process). One of the striking predictions of the model is that at any point in time transactions occur between the highest value buyer and the lowest cost seller remaining in the market. Further, inefficiencies result strictly from lost trading

[7] Easley and Ledyard (1993) were the first to offer a theoretical model designed to explain price formation in laboratory auction markets. Their model is primarily concerned with how prices and quantities settle down to competitive equilibrium levels across trading periods in markets with stationary supply and demand schedules.

opportunities as lower value buyers and higher cost sellers, who would trade in the absence of strategic considerations, fail to trade in the time allotted.

In principle, the Wilson model is capable of making very precise predictions regarding who trades and when. However, as Cason and Friedman (1993) point out, it is not practical to test these predictions, as solutions to the model are defined implicitly by a nested set of partial differential equations whose boundary conditions at each stage are derived recursively from the solution to subsequent stage partial differential equations (with some arbitrariness as to the final stage specification), and no numerical algorithms are presently available to solve the equations even for very simple value distributions and simple sets of auxiliary hypotheses. Nevertheless, the model has a number of reasonably precise qualitative implications that can be readily tested; for example, as already stated, inefficiencies result strictly from lost trading opportunities, and transactions occur between the highest value buyer and lowest cost seller remaining in the market.

Friedman (1991) develops a Bayesian game against nature model of the CDA trading process. According to Friedman, agents are boundedly rational with limited strategic capabilities. Agents carry with them reservation prices for buying and selling based on their valuations. Sellers are willing to undercut the standing market price as long as they can do so without selling below their reservation price, and they accept the market bid whenever it is greater than their reservation price; buyers behave analogously. To complete the model Friedman (1991) employs a drastic simplification: traders are assumed to ignore the impact of their own current bids and offers on subsequent bids and offers. This "game against Nature" assumption, together with Bayesian updating and auxiliary assumptions similar to Wilson's (e.g., risk neutrality), give reservation prices as solutions to the optimal stopping problem associated with current estimates of "Nature's" bid and offer generating process.

If the reservation prices associated with trader valuations were known, the Friedman model could be solved for precise bids, asks, and acceptances. Absent this, the model still has a number of distinctive qualitative implications, several of which are quite similar to the Wilson model's predictions: (1) Early transactions will be between higher value buyers and lower cost sellers, and (2) Efficiencies are close to 100%, with inefficiencies resulting from lost trading opportunities as lower value buyers and higher cost sellers, who would trade in the absence of strategic considerations, fail to trade in the time allotted. Other qualitative predictions differ from the Wilson model. The one I look at is that changes in transaction prices will be positively correlated, with this effect most pronounced for early trades. This positive correlation between price changes results from the fact that as traders' beliefs change in response to new bids and offers, their reservation prices shift, unexpectedly. These unanticipated shifts in bids and offers can be shown to result in positively correlated changes in transaction prices. In contrast, the Wilson model implies zero correlation between changes in transactions prices, since autocorrelation between prices would open up opportunities for price arbitrage that fully rational agents would exploit.[8]

C. Zero Intelligence Traders

Gode and Sunder show that for the CDA, zero intelligence (ZI) traders achieve remarkably high efficiency levels (between 95-100%), often achieving higher efficiencies then human traders in the first period of an experiment with stationary supply and demand schedules. ZI traders are completely non-strategic, with sellers offering at or above cost (but no higher than $\overline{x}$) and buyers

[8] Cason and Friedman (1993) offer an additional qualitative difference between the Friedman and Wilson models: this concerns which traders are involved in successive improvements of offers prior to completion of a transaction. I do not pursue this qualitative difference here, in part because I know of no way of reliably determining when "serious" bids and offers begin, which is what the Wilson model requires.

bidding at or below resale values (but no lower than x).[9] Bids and offers are randomly determined within these intervals, with traders repeatedly churning out bids and offers. Employing New York stock exchange rules, so that if a bid (offer) is to have standing in the market it must improve on the current market bid (offer), bids and offers are selected at random from traders. A transaction occurs anytime a new bid (offer) exceeds (is less than) the current market bid (offer), with the transaction price determined by the current market bid (offer). ZI traders are fast enough, relative to the market period time constraint, that transactions stop when there is no longer any room for mutually profitable trades to occur.

As already noted, for the CDA, ZI traders achieve remarkably high efficiency levels. However, unlike the Wilson and Friedman models, inefficiencies result from too much (rather then too little) trading. As such, extra marginal traders (those traders whose redemption values lie just beyond the CE) get to trade, or extra marginal traders displace infra marginal traders who would get to trade at a CE outcome. As in the Wilson and Friedman models, with ZI traders early transactions tend to occur between higher value buyers and lower cost sellers. But these effects are less pronounced. For ZI traders, transactions prices are independent draws from a distribution that changes over time as successful traders leave the market. Cason and Friedman (1993) show that this implies negative autocorrelation between changes in transaction prices, in contrast to the Friedman model prediction of positive autocorrelation.

Kagel and Vogt (1993) show that ZI traders achieve very low efficiency levels in the BBDA, considerably lower than achieved with a fixed price rule set at the mid-point of the interval from

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[9] These are budget constrained ZI traders. Gode and Sunder also simulate performance of unconstrained ZI traders. Without budget constraints buyers can and do bid above value and sellers can and do offer at below cost so that a number of inefficient trades occur, resulting in sharply lower efficiency levels then in the budget constrained case.

which valuations are drawn.[10] The reason for the poor efficiency outcomes (relative to behavior) in the BBDA is that the static (one-shot) nature of the game does not permit ZI traders to correct for bids and offers within an auction period when they fail to find trading partners (and ZI traders do not learn across auction periods). In contrast, the dynamic CDA permits ZI traders to correct for such "bad" bids and offers by simply adjusting them until they can find profitable trades.

ZI traders are employed as a reference point against which to evaluate behavior in CDAs since (1) they provide a useful benchmark for completely non-strategic behavior, and (2) the algorithm achieves very high efficiency levels in the CDA and, according to Cason and Friedman (1993, 1996), it predicts quite well regarding the volume of transactions, the order of transactions relative to redemption values, and the autocorrelation in transaction prices within a given market period. In contrast, for the BBDA we briefly document the extremely poor efficiency outcomes for ZI compared to real traders, and then drop the use of ZI traders as a reference point since it is clear that it is hopelessly inadequate for organizing behavior in these auctions.

II. Experimental Procedures

Redemption values were randomly drawn from a uniform distribution [0, $5.00], with new random draws in each trading period. The parameters of the distribution were read out loud as part of the instructions and were publicly posted. Each experimental session consisted of a pre-announced number of trading periods. For inexperienced subjects there were two dry runs (with no money at stake) after the instructions were read.

The goal was to recruit 16 subjects for each experimental session. Markets with equal

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[10] Under the fixed price rule traders are assumed to follow the dominant strategy of truthful revelation. In cases where the market does not clear, the computer randomly rations trade on the long side of the market.

numbers of buyers and sellers were used throughout. Auction markets with m = 2 and m = 8 were conducted under both mechanisms. In markets with m = 2 traders were randomly assigned to a new small market group in each trading period.

Six BBDA sessions were conducted, three each with live sellers and buyers and three each with live buyers and computerized sellers. In both cases two inexperienced subject sessions were conducted first, followed by a third session recruiting subjects back from the first two sessions.[11] In the sessions with computerized sellers, it was announced that "sellers" were following the dominant strategy of offering at cost.

Five CDA sessions were conducted. The first two used inexperienced subjects. This was followed by three experienced subject sessions, recruiting subjects back from the first two sessions and using subjects who had first participated in a CDA as part of a class room teaching exercise.[12] In one of these experienced subject sessions only 12 subjects arrived on time and in the other only 14 came on time. These sessions were run as planned, with the large market auctions employing m = 6 and m = 7, respectively, and the small market auctions employing m = 2 (in the session with 14 subjects, one small auction market operated with m = 3).

The BBDA auctions used a dual market bidding procedure. With dual market bidding, using the same redemption values, traders first participate in a market with m = 2, but before the market clearing price is established, they play again in a market with m = 8. Payment is made in only one

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[11] Kagel and Vogt (1993) report an earlier set of BBDA auctions with live buyers and sellers. The sessions here differ from these earlier ones through an enhanced set of instructions which included diagrams and problems to help subjects better understand the BBDA rules. There are no major differences between behavior under the two sets of instructions.

[12] The teaching exercise never discussed bidding strategies or models, but was used to illustrate how markets operate and to operationalize the concepts of consumer and producer surplus.

of the two markets, which is determined randomly after both sets of bids and offers have been submitted.[13] In determining the effect of increases in the number of traders on behavior, the dual market technique minimizes the extraneous variability resulting from changes in redemption values and the subject population. That is, it operationalizes responses to ceteris paribus changes in the number of traders since the same subjects bid with exactly the same redemption values, only with m changing. The dual market technique has been employed to good effect in one-sided auctions, reliably resulting in increased bidding with increased numbers of bidders in first-price, private value auctions (Dyer, Kagel and Levin, 1989; Battalio, Kogut and Meyer, 1990). In order to simplify instructions as much as possible, auctions with inexperienced subjects began with several periods of bidding in a single market before introducing the dual market technique.

The BBDA auctions used software developed in our laboratory for this purpose. Although there were no restrictions on bids or offers, the instructions did point out that the only way to possibly lose money was to bid (offer) above (below) valuation (cost). Following each auction period all bids and offers were reported back to traders for the market(s) they participated in, along with the underlying redemption values (individual subject IDs were suppressed). In addition, all traders learned the market price, the number of transactions, and their own earnings. Finally, in the auctions with live buyers and live sellers, halfway through the session subjects switched roles with buyers becoming sellers and vice versa.

The CDA auctions were conducted using a modified version of software developed by Shyam Sunder at Carnegie Mellon University. This software did not permit multiple market bidding (which can get rather cumbersome in a CDA), so an ABA design was employed with agents first bidding in

[13] This eliminates any possibility of portfolio effects with subjects hedging their bets between the two auctions.

a single large market, then bidding in one of several small markets, followed by bidding in a single large market. Here, too, there were no restrictions that bids (offers) had to be less (greater) than or equal to valuations (costs). However, as in the BBDA, the instructions pointed out that the only way to possibly lose money was to bid (offer) above (below) valuation (cost). The computer program provides a graphical representation of bids, offers, and transaction prices, along with a ticker tape reporting of the same information. There was no switching of roles between buyers and sellers in these auctions as the software did not readily permit it, and the software and ticker tape feedback were sufficiently unfamiliar to subjects that a number of them asked to play the same role on returning for a second session (we accommodated these requests as much as possible).[14]

For both mechanisms subjects were provided with a starting capital balance of $5.00, in lieu of a participation fee, against which any potential net losses would be subtracted. Although buyers (sellers) should never bid above (below) their redemption value under either mechanism, there were occasional mistakes in the early auction periods in the CDA which resulted in losses, and a number of bids (offers) of this sort throughout the BBDA which resulted in occasional losses. However, no subject's cash balance ever dropped much below $5.00.

III. Experimental Results

A. BBDA Auctions

Table 1 reports measures of market performance for the BBDA auctions with live buyers and live sellers. For sessions 1 and 2, which used inexperienced bidders, we have dropped the first

[14] This is not to criticize the software, which is relatively user friendly. Williams (1980) reports that inexperienced subjects in computerized CDAs make more mistakes and take longer to converge to the CE outcome than in auctions done by hand. This is attributed to subjects needing to gain familiarity with the software. In contrast, the mechanics underlying the BBDA are much easier for subjects to deal with.

several (8) auction periods (which involved bidding in a single auction market). Two efficiency measures are reported in the top part of Table 1: The first measure computes efficiency in each auction period separately and takes the average of these efficiency measures (excluding those auctions with m = 2 in which the maximum possible consumer and producer surplus was zero). The second measure sums realized consumer and producer surplus across auction periods and divides by the sum of the maximum possible consumer and producer surplus.[15] The second measure is the one employed in SW (1989a) in computing the expected increase in efficiency under the BBDA with increases in m. The first measure provides some indication of the variation in efficiency between auction periods and accentuates the improvements in efficiency resulting from increases in m.

Both measures show similar results. In all cases actual efficiency is less then predicted for idealized (risk neutral) traders. These differences are small, however, particularly with m = 2 where the average efficiency across auction periods measure is not significantly different from the BBDA prediction. Differences between realized and idealized efficiency become larger, and achieve statistical significance, with m = 8. Note, however, that realized efficiencies are closest to the BBDA prediction for experienced traders (session 3x), suggesting that experience may improve performance under this mechanism. Finally, in all three sessions, efficiency increases with increases in m, as the theory predicts, with the largest increase occurring in the session with experienced traders.

Realized efficiency is much higher than with ZI traders, who achieve consistently low efficiencies averaging .30 and .35 with m = 2 and 8 respectively (using the sum of the surplus

[15] With m = 2 there were occasional trades even when maximum possible consumer and producer surplus was zero (this results from buyers bidding above value or sellers offering below cost). These efficiency losses are not captured in the first measure, but are incorporated into the second measure.

measure).[16] Thus, efficiency is considerably closer to the level predicted for perfectly rational (risk neutral) traders then for completely non-strategic, ZI traders. Realized efficiency is also considerably higher than with a fixed price rule of $2.50 where efficiency averages .79 and .84 with m = 2 and 8, respectively (again, using the sum of the surplus measure). One way to view the fixed price rule is as a clumsy instrument that is bound to achieve 100% with increases in m, as the dominant strategy of bidding (offering) at redemption values is completely transparent.[17] In contrast, the BBDA opens up the possibility of achieving substantially higher efficiency with small numbers of traders, while at the same time creating the possibility that the trading rules are so complicated that traders fail to respond to the strategic possibilities, or respond incorrectly to them. Consequently, what this comparison shows is that although human traders do not respond in an idealized fashion to the BBDA mechanism, "mistakes" are not large enough, or frequent enough, to offset the promised improvements in efficiency relative to a fixed price rule.

The bottom part of Table 1 shows the price and quantity data underlying these efficiency results. With the exception of session 1 with m = 8, quantity traded is always below the CE prediction and prices are always above the CE prediction, so that price and quantity deviations are consistent, directionally, with the BBDA predictions. However, quantity traded is usually a little greater than the BBDA's predictions and prices are consistently, and substantially, greater than the BBDA's predictions. As shown below, the latter results from sellers' tendencies to offer at above cost and buyers' tendencies to bid closer to valuations than the BBDA predicts.

Table 2 shows individual trader's bids and offers relative to the BBDA's predictions. With

[16] All ZI calculations consist of 20 simulations for each experimental auction period.

[17] This is the assumption underlying the calculations reported. This prediction should, of course, be tested experimentally.

m = 2, buyers bid significantly more than the risk neutral BBDA predicts, as they bid an average of \$0.28 below their valuations compared to the BBDA prediction of bidding \$0.81 below valuations. Not shown in Table 2 is that with m = 8 there was a small, but statistically significant, increase in bids relative to valuations averaging \$0.08 (compared to a predicted increase of \$0.54).[18] This small increase in realized bids along with the sharp increase in predicted bids resulted in bids being much closer to the risk neutral BBDA prediction with m = 8 (average reduction in bids relative to valuations of \$0.21 versus the BBDA prediction of \$0.27 shown in Table 2).

Bidder behavior in the BBDA has a number of similarities to behavior in one-sided private value auctions. The overbidding relative to the risk neutral BBDA prediction with m = 2 is not unlike the overbidding relative to the RNNE found in one-sided, first-price sealed bid auctions. The overbidding in one-sided, private value auctions is most extreme with small numbers of bidders, in which case it is not uncommon for bidders to take home 50% or less of predicted RNNE profits (see Kagel and Roth, 1992, Table 4). Some of the overbidding in one-sided, first-price auctions may be a result of risk aversion, and some may be the result of buyers' misperceptions and/or inexperience (see Kagel, 1995, for a review of the experimental literature on this point). Some of these same forces are likely to be at work in the BBDA as well. Also, increases in bidding with increased numbers of rivals is reliably reported in one-sided, first-price private value auctions. This result is replicated in the BBDA as well.

Table 2 shows that sellers failed to follow the dominant strategy of offering at cost, usually

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[18] Treating average differences in individual subject bids with m = 2 and m = 8 as the unit of observation, there was an increase of \$0.14 in session 1 ($t = 1.99$, $df = 15$, $p < .05$, 1-tailed paired t-test), a reduction of \$0.02 in session 2 ($t = -.13$), an increase of \$0.09 in market 3x ($t = 2.18$, $df = 15$, $p < .025$, 1-tailed paired t-test), and an overall increase in bids of \$0.08 ($t = 1.42$, $df = 47$, $p < .10$, 1-tailed, paired t-test).

offering at above cost. Not reported in Table 2 is that several (6) sellers offered within $0.10 of cost in 90% or more of the auctions. In contrast, none of the buyers ever bid within $0.10 of their valuations 90% of the time or more. This suggests that the dominant bidding strategy had some (weak) drawing power in these auctions.

Deviations from the dominant offer strategy are not unexpected, as deviations from dominant bidding strategies are reported in one-sided, second-price private value auctions (Kagel, Harstad and Levin, 1987; Kagel and Levin, 1993) and in one-sided, multiple unit uniform price auctions (Cox, Smith and Walker, 1985). As with the one-sided auctions, the dominant offer strategy is far from transparent in the BBDA. Further, the expected cost of deviating from the dominant strategy was relatively small, averaging $0.05 per auction period ($0.11 conditional on selling), so that sellers were close to playing best responses.[19] As such, any trial and error search process that might be expected to help subjects find the dominant offer strategy would generate relatively weak feedback effects.[20]

Given the extent of seller misrepresentation reported, it is not clear that buyers should bid according to (1). Further, it is possible that the smaller than predicted underrevelation on the part of buyers constitutes a (possibly mistaken) strategic response to sellers offering at above cost. The BBDAs with computerized sellers were designed to investigate this last question. The answer, in short, is that bidder behavior is quite similar to auctions with live sellers. With $m = 2$ buyers bid

[19]Bidding is above the dominant strategy in one-sided buyers auctions. With symmetry this should translate into bidding below cost for sellers in the BBDA, as in both cases it increases the chances of winning an item (when you don't want to!). Similar to the results reported here, Cason and Friedman (1997) observe bidding above cost for sellers in call markets where they have a dominant strategy to bid their value. But they also find buyers bidding below value when they have a dominant strategy to bid their value, unlike the pattern generally reported in one-sided auctions.

[20] The basic reason for the low cost of deviating from the dominant offer strategy is that adoption of the strategy would have no effect on who sells for 91% of all offers. These calculations are for $m = 8$. With $m = 2$, the corresponding costs are $0.07 per auction ($0.16 conditional on trading), and adopting the dominant strategy would have no effect on who sells for 89% of all offers. Calculations are based on each seller, in turn, adopting the dominant strategy, with all others trading as they did.

significantly more than the risk neutral BBDA predicts (average buyer misrepresentation of \$0.37 compared to the BBDA prediction of \$0.82). In contrast, with m = 8 there were minimal differences between predicted and actual misrepresentation (\$0.266 actual versus \$0.272 predicted). Both patterns are quite similar to those reported earlier in Table 2 for BBDA auctions with live sellers.

B. CDA Auctions

Table 3 reports measures of market performance for the CDAs. For sessions 1 and 2, with inexperienced bidders, the first several (5) auction periods have been dropped as there is some tendency for mistakes (bidding above valuation or offering at below cost) to occur in these periods as a result of unfamiliarity with the software. As already noted, we also use ZI traders as the reference point since (i) ZI traders do such a good job of organizing much CDA trading data, and (ii) because of the absence of point predictions for both the Wilson and Friedman models.

Efficiency measures are reported in the top part of Table 3. In auctions with m = 2, average efficiency across auction periods is quite high and significantly above the level reported under the BBDA mechanism ($t = 2.11$, $p < .05$, two-tailed t-test). However, efficiency is significantly below the ZI benchmark. With m = 8, efficiency tends to increase relative to markets with m = 2 (the one exception is session 7 using the sum of the surplus measure). Using the average across auction periods measure, with m = 8 average efficiency is higher than in the BBDA auctions with live sellers ($t = 1.82$, $p < .10$, two-tailed t-test), while being essentially the same as in the ZI simulations.[21]

Although efficiency levels are quite high, unlike CDAs with stationary supply and demand schedules, there is no tendency for market efficiency to improve with replication within a given

[21]Pooling the data from sessions 9x (m = 6) and 10x (m = 7) with the m = 8 data does not change this conclusion as efficiency averages 95.86% with an even smaller standard error of the mean.

experimental session (and even some tendency for efficiency to decrease with experience in markets with m = 2). The evidence for this is twofold. First, we ran a number of different regression specifications with auction period as a right hand side variable, finding no systematic patterns in the data (no continuous improvements or drop-offs in efficiency over time). Second, for each experimental market we identified the median efficiency level for that market (100% in all cases) and counted the frequency with which efficiency deviated from the median in the first half compared to the second half of the auction within a given experimental session. We then pooled the data for the large markets ($m \geq 6$) and for the small markets (m = 2), separately, and conducted a simple binomial test for significant differences in deviations from the median. There were no significant differences between halves in the large markets (44% - 11 out of 25 - of the deviations occurred in the second half of the sessions, $p > .10$, two-tailed test). However, in the markets with m = 2, 66.7% of the deviations (18 out of 27) occurred in the second half of the sessions, which is significantly different from the 50% reference point at the 10% level (two-tailed test).

Unlike auctions with stationary supply and demand schedules, there is no reason to expect efficiency to increase over time within a given experimental session as traders must search to find mutually beneficial trades in each auction period with past realizations no help in the search process. From this perspective the apparent decrease in efficiency in the small markets is somewhat puzzling. Perhaps it is a statistical aberration. Alternatively, it could be that traders are learning to behave more strategically over time, which implies missed trading opportunities and reduced efficiency. This would be consistent with the overall pattern of more strategic play observed in markets with fewer numbers of traders (discussed in some detail below).

The bottom part of Table 3 shows the price and quantity data underlying these efficiency results. With m = 2 average quantity traded is significantly below the CE model's prediction, consistent with the Wilson and Friedman models' predictions. In contrast, with m = 2, ZI traders <u>always</u> trade more than the CE model predicts.[22] With m = 8, humans trade a little above the CE model's prediction but still below the ZI level. That is, with the increase in the number of traders, valuations are congested enough in the neighborhood of the CE that excess numbers of mutually profitable trades reliably occur. But the number of such trades is still less than in the ZI simulations.[23]

One possible explanation for why there are fewer trades than the CE level with m = 2 and excess numbers of trades with m = 8 is that agents play the game the way Wilson and Friedman suggest, but there are errors in implementing these strategies. With the increase in m there is increased congestion which, in conjunction with a constant error rate, is enough to result in more trades than the CE level or the Wilson and Friedman models (without errors) predict. This is supported by the fact that in experimental session 9x, with m = 6, in one third of all auctions (5 out of 15) there were fewer units traded than the CE model predicts, and there were no auctions with more units traded. In contrast, in the ZI simulations all deviations from the CE output level involve more units traded. This conjecture is also supported by the ZI simulations themselves. In going from ZI simulations with m = 2 to m = 8 the underlying stochastic bidding strategies have been held constant (bid a random number within one's budget constraint), but there is increased bidder congestion. The net result is that in 8.7% of the simulated auction markets with m = 2 there are units

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[22] With m = 2, in 18 out of 136 auctions the number of units traded deviated from the CE quantity. In 14 of these 18 cases <u>fewer</u> units were traded than the CE prediction. In contrast, ZI traders failed to trade at the CE quantity in 51 out of 680 simulations. *All* of these involved too many units traded. The Z statistic for the frequency of over trading in the ZI simulations versus the realized data is 7.05 ($p < .01$).

[23] Pooling the data from sessions 9x (m = 6) and 10x (m = 7) with the m = 8 data results in even fewer trades relative to the ZI benchmark as 9x generates fewer trades than even the CE benchmark ; i.e., it is closer to the m= 2 pattern than the m = 8 pattern.

traded in excess of the CE, compared to 39.4% with m = 8.

With m = 2, average absolute price deviations do not differ significantly from the ZI benchmark, and average less than \$0.06 per auction from the CE prediction. Although average absolute price deviations were \$0.11 from the CE in the corresponding BBDAs, these differences between institutions are not significant at conventional levels (t = 0.89). With m = 8, average absolute price deviations were just over \$0.12 per auction, well below the ZI benchmark and only marginally higher than the price deviations observed in the BBDA (\$0.11).[24] Overall, price deviations from the CE level are relatively small as judged by the ZI benchmark and in comparison to the BBDA auctions.[25] This is somewhat surprising, to this investigator at least, given the search process underlying the price formation process in these markets. Finally, employing the same techniques used to evaluate changes in efficiency over time to evaluate price convergence, we conclude that unlike markets with stationary supply and demand schedules, there is no tendency for prices to converge to the CE outcome across market periods within an experimental session.[26] This is hardly surprising, since each auction period essentially sets off a new search for mutually acceptable trading prices, with minimal hints as to what is the relevant equilibrium price interval based on past auction outcomes.

Tables 4 and 5 examine the pattern of transactions in CDAs. Table 5 reports rank order

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[24]Pooling data from 9x and 10x with the m = 8 data leaves these conclusions unaffected as prices averaged \$0.125 versus \$0.122 for the m= 8 sessions alone.

[25] Friedman (1992) advocates averaging transactions prices within an auction period and then computing average absolute price differences across auction periods to put the data on a more comparable footing relative to call markets, which impose a single price for all trades. This was not done here. Doing so makes very little difference when m = 2 since there is usually only one unit traded. For the larger markets this results in eliminating the difference between ZI price deviations and realized price deviations and results in marginally lower prices than observed in the BBDAs.

[26]Once again the regression analysis shows no consistent time trends. The non-parametric price deviation analysis yields median price deviations of zero in all the small markets and zero, or pennies above it, in the large markets. 52.9% (18/34) of all price deviations occur in the second half sessions with $m \geq 6$ and 63.6% (7/11) in the m = 2 sessions. Neither proportion is significant at the 10%(two-tailed)level.

correlations between the order in which transactions occurred with the ranking of traders redemption values. In both large and small markets there are statistically significant correlations indicating that the highest value buyers and lowest cost sellers tend to trade first. In markets with m = 2, the realized correlations are significantly stronger on the seller's side of the market than implied by the ZI simulations (80% of the ZI correlations fall between .16 and .82).[27] In markets with m > 2, there is essentially no difference between the strength of the experimental correlations and the ZI simulations (pooled values are virtually the same). With the notable exception of sellers' correlations in sessions 8 and 10x with m = 2, rank order correlations are well below 1.0, the predicted value of both the Wilson and Friedman models.

Table 5 reports total consumer plus producer surplus by the order in which trades occurred. In markets with m = 2, the first transaction generated an average surplus of $2.25 compared to an average of $1.97 in the ZI simulations ($t = 2.28$, $p < .03$, two-tailed t-test) and a maximum possible surplus for the units traded of $2.41 per auction. Surplus generated is below the ZI simulations for the second unit traded, but this is to be expected since with m = 2 at most two units can be traded. In the large markets, more surplus is also generated on the first unit traded than in the ZI simulations, although the difference in this case is not statistically significant at conventional levels, and is well below the maximum possible surplus. Further, average surplus decreases monotonically for later transactions and is higher then in the ZI simulations for transactions 3 and 4 (but not for 2), which is weakly consistent with the Wilson and Friedman models' predictions.

[27] This is based on 20 simulations for each small market. For sessions 8 and 10x, additional simulations were run to determine the likelihood that the sellers' rank order correlation of 1.0 resulted from an unusual distribution of costs. In session 8, 6 simulations out of 100 resulted in a rank order correlation of 1.0 and in session 10x, 2 simulations out of 100 produced this result. Thus, it is quite unlikely that the correlations for 8 and 10x were a result of chance factors alone.

Given the data in Tables 4 and 5, I conclude that there is a clear tendency for higher value buyers and lower cost sellers to trade first and for more surplus to be generated in early transactions. Both of these tendencies are stronger than in the ZI simulations, although the difference between experimental and ZI data is only statistically significant with m = 2. As such I conclude that the pattern of transactions partners is closer to the pattern predicted by the Wilson and Friedman models than a completely random trading process suggests, although there are still substantial deviations from the idealized pattern that both models suggest.[28]

One aspect of these results worth speculating about is why the trading partner pattern moves closer to the ZI prediction as m increases. One factor, already mentioned, is that as m increases the average distance between traders' redemption values becomes smaller. As a result of this congestion, given a constant noise level in actual transactions, relative to the trading processes that Friedman or Wilson formulate, one would expect to see greater deviations from the trading pattern predicted. Further, systematic increases or decreases in traders' impatience to transact can exaggerate or dampen this effect. From the data in Tables 4 and 5 it appears that whatever tendencies there are in this direction involve greater impatience to trade since in going from small to large markets (i) reductions in the realized correlation coefficients in Table 4 are more extreme, particularly for sellers, than the changes in the ZI correlations (by definition ZI traders' impatience does not change as m increases), and (ii) the increase in consumer and producer surplus on the first unit traded is somewhat larger for the ZI simulations than for the experimental data (59.2 versus 50.5).

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[28] Our conclusion in this respect differs from Cason and Friedman (1993 and 1996). They report weak evidence that higher value buyers and lower cost sellers tend to trade first and that gains from trade decrease as more units are transacted and conclude that this pattern is consistent with the one implied by the ZI algorithm. One reason our results differ from Cason and Friedman is that they did not have any data for small markets (m = 2) where the experimental data are in closer conformity to the Friedman and Wilson models' predictions.

Table 6 reports the covariance of price changes within each auction period for the large markets and the average absolute size of these price changes.[29] As noted, the ZI model implies a negative covariance for price changes of around -0.50, which is very close to what the ZI simulations yield for the sample data. For sessions 8, 10x and 11x, for which we have a reasonable number of observations, the covariance in the actual price changes is statistically significant at better then the 5% level and close to the level found in the ZI simulations. However, as the last two columns in Table 6 show, average absolute price changes are substantially larger in the ZI simulations than in the experimental data for each and every experimental session. Thus, although the negative covariances reported are consistent with the ZI benchmark (and earlier reports of negative price change covariances in Cason and Friedman, 1993)[30], the average absolute price changes are sharply lower than the ZI simulations suggest.

The negative covariances imply that price increases are consistently followed by price decreases, which is clearly inconsistent with Friedman's model. To the extent that traders can profitably arbitrage these price changes, they are also inconsistent with the Wilson specification. However, arbitrage is difficult in these markets since traders each have a single unit to trade and no re-sales were permitted. Thus, what room for arbitrage there is requires agents to hold off a transaction until after the second unit has traded, since prior to this traders do not have the information required to determine if they could profit by holding out and getting a more favorable

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[29] Cov (u_t, u_{t-1}) where $u_t = P_t - P_{t-1}$ and P_t is the price of transaction t within a given auction period. Note, under the CDA rules there was no specialists book, so that after each transaction all existing bids and offers were canceled, leaving no room for the specialists book to contribute to the negative price change correlations reported.

[30] Although the data here shows no evidence for reductions in these negative price correlations with experience, Cason and Friedman (1996) argue that a meta-analysis, using their data, ours, and some additional data, shows clear evidence of such an effect.

(expected) price. But there is a positive probability of not being able to trade after two units have transacted - with m = 8, 6% of the time there is no third unit traded and 34% of the time there is no fourth unit traded (the corresponding values are 19% and 46% if we pool the data from m = 6 and 7 with the m = 8 sessions). The net expected profit for a trader waiting to cash in on these arbitrage opportunities is approximately $0.11 for the third unit traded and -$0.12 for the fourth unit traded.[31] Thus, the arbitrage opportunities are limited and the incentives relatively small, substantially smaller than if one were competing against true ZI traders. As such I count these results as slightly favoring the Wilson price formation process compared to the Friedman model.

IV. Summary and Conclusions

This paper experimentally investigates a call market trading mechanism (the BBDA) and a CDA trading mechanism for two sided markets. The primary procedural innovation involves employing fully stochastic supply and demand schedules so that traders' redemption values, along with the CE price and quantity, vary stochastically between auction periods. Fully stochastic supply and demand schedules are necessary to test recent Bayesian Nash equilibrium models of price formation developed for these trading mechanisms and, arguably, provide the appropriate vehicle for testing the Hayek (1945) hypothesis - that markets are able to achieve close to CE price and quantity resulting from truthful revelation of redemption values.

Both auction mechanisms achieve reasonably high efficiency levels (75% or higher) even in very thin markets with m = 2. Consistent with received wisdom from experiments with stationary

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[31] As a buyer or seller there is only a 50% chance of profiting on the third unit as prices may increase or decrease. This yields an expected gain of $0.175 (50% of the average absolute price difference). From this I have subtracted half of the expected total surplus realized on the third unit traded (Table 5) after multiplying by the probability of not trading with m = 8. Similar calculations are made for the fourth unit. These are approximate, back of the envelope, calculations.

supply and demand schedules, prices and efficiency in the CDA are close to the CE level, and efficiency is higher than in the sealed bid trading mechanism (the BBDA). However, unlike markets with stationary supply and demand schedules, there is no tendency in the CDA for prices or efficiency to converge to the CE level with increased trader experience within an auction session, and some evidence suggesting reduced efficiency over time in markets with only two buyers and two sellers. The latter may be attributed to increased strategic play with increased experience in the small markets, which would result in reduced efficiency relative to the CE level.

In the BBDA, efficiency levels are consistently closer to those implied by idealized (risk neutral) traders (SW 1989a, 1989b) than to non-strategic, ZI traders or a fixed price rule. Further, as predicted, efficiency increases with increased numbers of traders. However, in small markets buyers consistently bid well above the risk neutral prediction even with computerized sellers who followed the dominant strategy of offering at cost. Further, real sellers tend to offer at above cost. The latter is largely attributed to (i) the fact that the dominant bidding strategy is far from transparent, and (ii) the relatively small cost associated with deviating from it.

In the CDA, with small markets inefficiencies consistently resulted from too few rather then too many trades relative to the CE level. This is consistent with strategic bidding models of the price formation process (Wilson, 1987, Friedman, 1991) and contrary to the implications of the ZI (Gode and Sunder, 1993) trading algorithm. Markets with larger numbers of buyers and sellers (m = 8) consistently produce more trades than the CE level, but fewer trades than implied by the ZI algorithm. Early transactions consistently produced more total surplus than later transactions as the Wilson and Friedman models suggest. Further, the amount of surplus generated in early trades is consistently greater than implied by the random trading process underlying the ZI algorithm, with these differences being most pronounced in markets with fewer traders. I conjecture that the closer conformity of behavior to non-strategic trading models with larger numbers of traders is caused by congestion effects in conjunction with random variation in bidding strategies that are not accounted for in the Wilson and Friedman models.

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Table 1
Measures of Market Performance: BBDA Efficiency

BBDA Auction Number (No. Auction Periods)[a]	Efficiency: Average Across Auction Periods (standard error of mean)				Sum of Surplus Across Auction Periods			
	m = 2		m = 8		m = 2		m = 8	
	Actual	Predicted	Actual	Predicted	Actual	Predicted	Actual	Predicted
1 (16)	.752 (.076)	.780 (.056)	.832 (.047)	.989 (.005)	.843	.924	.871	.993
2 (16)	.786 (.052)	.795 (.054)	.875 (.047)	.998 (.002)	.870	.939	.885	1.00
3x (25)	.774 (.047)	.784 (.046)	.935 (.019)	.996 (.002)	.890	.942	.940	.996
Average	.771 (.033)	.786 (.029)	.889 (.021)	.995 (.002)	.871	.936	.906	.996

	Quantity Deviations from CE Prediction (standard error of mean)				Price Deviations from CE Prediction (standard error of mean)			
	m = 2		m = 8		m = 2		m = 8	
	Actual	Predicted	Actual	Predicted	Actual	Predicted	Actual	Predicted
1 (16)	-.132 (.066)	-.340 (.066)	.125 (.180)	-.438 (.128)	.229 (.072)	.031 (.026)	.151 (.046)	.015 (.010)
2 (16)	-.226 (.064)	-.264 (.061)	-.625 (.155)	-.063 (.063)	.079 (.031)	.020 (.015)	.136 (.042)	.000 (.000)
3x (25)	-.179 (.051)	-.295 (.052)	-.080 (.114)	-.240 (.087)	.051 (.023)	.020 (.012)	.061 (.035)	.006 (.005)
Average	-.179 (.034)	-.299 (.034)	-.175 (.091)	-.246 (.058)	.111 (.025)	.023 (.010)	.107 (.023)	.007 (.004)

[a] With m = 2 there are 4 auction markets operating simultaneously in each auction period.
[b] Negative numbers indicate fewer trades than the CE prediction with truthful revelation.
[c] Positive numbers indicate prices are higher than the CE prediction with truthful revelation.

Table 2
Individual Subject Behavior in the BBDA

	Buyers' Misrepresentation[a]: Mean Values Across Subjects (standard error of the mean)				Sellers' Misrepresentation[b]: Mean Values Across Subjects (standard error of the mean)	
	m = 2		m = 8		m = 2	m = 8
BBD Auction Number	Actual	Predicted	Actual	Predicted	Actual	Actual
1	.242 (.199)	.800 (.039)	.099 (.197)	.267 (.013)	.222 (.113)	.144 (.120)
2	.326 (.045)	.765 (.034)	.342 (.124)	.255 (.011)	.351 (.079)	.391 (.083)
3x	.271 (.052)	.853 (.029)	.184 (.064)	.284 (.010)	.164 (.098)	.073 (.121)
Average	.280 (.078)	.806 (.028)	.208 (.080)	.269 (.007)	.245 (.056)	.203 (.065)

[a] Buyers' misrepresentation = (resale value - bid).
[b] Sellers' misrepresentation = (offer - cost); truthful revelation is a dominant strategy for sellers.

Table 3
Measures of Market Performance: CDA

	Efficiency							
CDA Auction Number (No. Auction periods)[a]	Average Across Auction Periods (standard error of mean)				Sum of Surplus Across Auction Periods			
	m=2		m=8		m=2		m=8	
	Actual	ZI	Actual	ZI	Actual	ZI	Actual	ZI
7 (32/7)	.825 (.077)	.962 (.009)	.939 (.042)	.955 (.006)	.963	.957	.942	.956
8 (32/11)	.923 (.047)	.968 (.009)	.977 (.016)	.951 (.004)	.941	.965	.978	.951
9x[b] (24/15)	.899 (.060)	.946 (.015)	.952 (.002)	.958 (.005)	.939	.936	.952	.958
10x[c] (16/17)	.927 (.040)	.967 (.013)	.989 (.009)	.954 (.004)	.902	.959	.977	.959
11x (32/17)	.804 (.068)	.965 (.010)	.938 (.005)	.954 (.004)	.898	.954	.959	.954
Average[d]	.870 (.028)	.962 (.005)	.950 (.026)	.953 (.025)	.932	.954	.961	.953

CDA Auction Number (No. Auction periods)	Quantity Deviations from CE Prediction[e] (standard eror of mean)				Price Deviation from CE Prediction Using Individual Transactions (standard error of mean)			
	m=2		m=8		m=2		m=8	
	Actual	ZI	Actual	ZI	Actual	ZI	Actual	ZI
7 (32/7)	-.125 (.110)	.142 (.032)	.143 (.340)	.343 (.043)	.106 (.070)	.065 (.023)	.097 (.046)	.300 (.021)
8 (32/11)	0.0 (.053)	.081 (.023)	.363 (.152)	.436 (.038)	.073 (.048)	.024 (.007)	.131 (.045)	.297 (.017)
9x (24/15)[b]	-.130 (.072)	.087 (.026)	-.333 (.126)	.277 (.026)	.018 (.013)	.041 (.015)	.130 (.057)	.308 (.016)
10x (16/17)[c]	0.0 (0.0)	.013 (.013)	.118 (.118)	.350 (.028)	0.0 (0.0)	.032 (.016)	.127 (.056)	.263 (.014)
11x (32/17)	-.143 (.085)	.086 (.024)	.118 (.146)	.426 (.031)	.049 (.033)	.060 (.017)	.127 (.037)	.226 (.011)
Average[d]	-.085 (.036)	.087 (.012)	.200 (.107)	.413 (.021)	.054 (.020)	.046 (.077)	.122 (.024)	.263 (.009)

[a] (x/y) = number of auction periods with m=2/ number of auction periods ~~with~~ m=8.
[b] In large markets m=6.
[c] In large markets m=7.
[d] Average for large markets is for m=8 only.
[e] Negative numbers indicate fewer trades than the CE prediction with truthful revelation.

Table 4
Rank Order Correlations: Transaction Number and Ranking of Buyer Valuations and Seller Costs

Experimental Session	Small Markets (prob=0)				Large Markets (prob=0)			
	Buyers[b]		Sellers[b]		Buyers[b]		Sellers[b]	
	Actual	ZI[c]	Actual	ZI[c]	Actual	ZI[c]	Actual	ZI[c]
7 (m=8)[a]	0.25 (0.23)	0.36 (0.06)	0.50 (0.01)	0.59 (0.05)	0.51 (<0.01)	0.33 (0.05)	0.37 (0.06)	0.28 (0.04)
8 (m=8)[a]	0.76 (<0.01)	0.56 (0.04)	1.00 (<0.01)	0.59 (0.05)	0.40 (<0.01)	0.33 (0.02)	0.18 (0.24)	0.33 (0.03)
9x (m=6)[a]	0.24 (0.26)	0.46 (0.07)	0.62 (<0.01)	0.53 (0.05)	0.34 (0.04)	0.44 (0.04)	0.33 (0.04)	0.42 (0.03)
10x (m=7)[a]	0.42 (0.02)	0.64 (0.05)	1.00 (<0.01)	0.49 (0.05)	0.26 (0.04)	0.39 (0.03)	0.31 (0.01)	0.33 (0.02)
11x (m=8)[a]	0.52 (<0.01)	0.56 (0.05)	0.76 (<0.01)	0.56 (0.05)	0.24 (0.05)	0.25 (0.03)	0.44 (<0.01)	0.37 (0.03)
Pooled	0.46 (<0.01)	0.52 (0.03)	0.81 (<0.01)	0.55 (0.02)	0.34 (<0.01)	0.35 (0.02)	0.36 (<0.01)	0.35 (0.01)

[a] Number of buyers and sellers in large market. Small markets m=2.
[b] Buyers' valuations ranked from 1 to m starting with highest valuation.
Sellers' costs ranked from 1 to m starting with lowest cost.
[c] Mean of ZI simulations with standard error of mean in parentheses.

Table 5
Consumer and Producer Surplus as a Function of Transaction Rank[a]
(values are in cents)

Experimental Session	Small Markets: Transaction Order				Large Markets: Transaction Order							
	1		2		1		2		3		4	
	Actual	ZI	Actual	ZI	Actual	ZI	Actual	ZI	Actual	ZI	Actual	ZI
7 (m=8)[b]	261.7	205.9	151.6	137.2	259.3	255.8	284.1	239.0	239.1	216.8	148.0	165.6
	t=1.63 (0.10)		t=0.22 (0.83)		t=0.09 (0.93)		t=1.10 (0.27)		t=0.53 (0.60)		t= -0.37 (0.71)	
8 (m=8)[b]	234.2	219.0	48.0	93.2	268.9	251.2	208.5	232.6	226.6	197.7	167.6	180.1
	t=0.66 (0.51)		t= -1.38 (0.18)		t=0.54 (0.59)		t= -0.74 (0.46)		t=0.88 (0.38)		t= -0.32 (0.75)	
9x (m=6)[b]	247.8	208.7	142.7	135.7	278.5	258.6	211.0	207.4	201.4	156.1	ND	
	t=1.63 (0.10)		t=0.13 (0.89)		t=0.73 (0.47)		t=0.11 (0.92)		t=1.03 (0.30)			
10x (m=7)[b]	168.7	171.4	79.0	170.8	254.8	261.6	220.9	221.8	170.3	196.4	204.6	160.3
	t= -0.08 (0.93)		t= -0.88 (0.42)		t= -0.27 (0.79)		t=0.04 (0.97)		t= -1.01 (0.31)		t=1.60 (0.11)	
11x (m=8)[b]	201.4	172.7	97.6	110.2	307.3	264.7	197.7	240.0	237.3	201.7	187.0	165.9
	t=1.14 (0.25)		t= -0.26 (0.79)		t=1.61 (0.11)		t= -1.55 (0.12)		t=1.25 (0.21)		t=0.67 (0.50)	
Average	225.4	197.1	104.9	121.6	275.9	256.3	217.4	223.7	212.6	192.7	178.4	159.5
	t=2.28 (0.03)		t= -0.67 (0.51)		t=1.52 (0.13)		t= -0.45 (0.66)		t=1.33 (0.18)		t=1.13 (0.26)	
Maximum possible[c]	240.5 (12.8)		65.8 (20.6)		388.0 (7.6)		252.7 (12.4)		154.5 (11.9)		94.1 (17.6)	

ND - no data
[a] T-tests for difference between actual and ZI surplus. Probability t=0 in parentheses (2-tailed, t-test).
[b] Number of buyers and sellers in large market. Small markets m=2.
[c] Maximum possible is conditional on the number of units actually traded. Standard errors of the mean reported in parentheses.

Table 6
Within Auction Period Price Changes: CDA, Large Markets

Experimental Session[a]	Covariance of Price Changes		Average Absolute Price Changes[c] (standard error price change)	
	Actual (prob=0)	ZI[b] (standard error of mean)	Actual	ZI[b]
7 (26)	-.287 (0.34)	-.445 (0.04)	25.9 (19.2)	91.8 (73.9)
8 (46)	-.623 (<0.01)	-.506 (0.03)	35.2 (36.0)	92.4 (72.8)
9x (16)	-.014 (0.97)	-.532 (0.03)	16.1 (51.4)	93.4 (75.6)
10x (84)	-.370 (0.02)	-.487 (0.03)	36.0 (14.5)	86.4 (71.0)
11x (60)	-.525 (<0.01)	-.502 (0.03)	40.4 (52.3)	87.9 (69.9)

[a] Number of price change observations in parentheses (actual data).
[b] Average over 20 simulations for each experimental session.
Covariance of price changes = $Cov[U_t, U_{t-1}]$ where $U_t = P_t - P_{t-1}$ and P_t is the price of transaction t for a given action period.
[c] Values are in cents.

Figure 1
Price and Quantity Determination in BBDA