Behavior in Multi-Unit Demand Auctions: Experiments with Uniform Price and Dynamic Vickrey Auctions *

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Abstract

We experimentally investigate the sensitivity of bidders demanding multiple units of a homogeneous commodity to the demand reduction incentives inherent in uniform price auctions. There is substantial demand reduction in both sealed bid and ascending price clock auctions with feedback regarding rivals' drop-out prices. Although both auctions have the same normal form representation, bidding is much closer to equilibrium in the ascending price auctions. We explore the behavioral process underlying these differences along with dynamic Vickrey auctions designed to eliminate the inefficiencies resulting from demand reduction in the uniform price auctions.

Key words: multi-unit demand auctions, uniform price auction, dynamic Vickrey auction, demand reduction, experiment.

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Spurred by the recent FCC spectrum auctions, theoretical research in multi-unit demand auctions reveals two distinctly different behavioral forces at work in auctions of this sort. In uniform price auctions, such as employed in Treasury bill auctions or in the recent FCC spectrum auctions, when bidders have non-increasing demand for homogeneous goods there is an incentive to *reduce* demand on some units in an effort to win other units at more favorable prices (see, for example, Ausubel and Cramton, 1996 and Englebrecht-Wiggans and Kahn, 1998). In contrast, when there are complementarities between items so that the value of a package of items exceeds the sum of its parts there are incentives for agents to bid *above* the value they place on any individual item (see, for example, Krishna and Rosenthal, 1996). Indeed, the recent FCC spectrum auctions have provided examples of both types of incentives: In the nationwide narrowband auction bidders appear to have had non-increasing demands, while in the broadband MTA auction there appear to have been complementarities between items (for analysis of the FCC auctions, see Cramton, 1995; McAfee and McMillan, 1996; Ausubel, Cramton, McAfee and McMillan, 1997).

In this paper we experimentally investigate the sensitivity of bidders to the demand reduction possibilities inherent in uniform price auctions when bidders have non-increasing demand for multiple units. Demand reduction reduces seller's revenue and introduces economic inefficiencies as buyers with lower valued units earn items in place of higher valued buyers.[1] We compare behavior under two standard uniform price auction rules (winning bidders pay the highest

[1]Treasury bill auctions are often considered the canonical example of multi-unit demand auctions in which bidders have non-increasing demands. Policy debates regarding the optimal structure of Treasury bill auctions reveal a long history of confusion by a number of prominent economists regarding the incentive effects of uniform price auction rules (see Ausubel and Cramton, 1996).

rejected bid price): (1) a sealed bid auction and (2) an ascending price, English clock auction in which bidders receive information regarding rivals' drop-out prices. In the experiment, both auctions promote demand reduction, thereby demonstrating that the incentives for such behavior are reasonably transparent even for relatively naive bidders. However, although theory predicts that in our experimental design the two auctions will yield the same prices and allocations, bidding is closer to the equilibrium outcome in the ascending price auction. To understand the mechanism underlying these differences we create two additional auction institutions that do not exist in field settings - a uniform price clock auction with *no* feedback about rivals' drop-out prices and a sealed bid auction in which the critical drop-out information used in the clock auctions is provided exogenously. The results of these treatments indicate that the closer conformity to equilibrium outcomes in the clock auctions results from both the information inherent in observing others' drop out prices and the ability of the clock to provide this information in a highly salient way.

We also compare bidding in the uniform price auctions with a dynamic Vickrey/Ausubel auction (Ausubel, 1997). Theoretically, the Vickrey auction eliminates any incentive for demand reduction, thereby promoting full efficiency and, in a number of plausible settings, including our experiment, raises greater expected revenue than the uniform price auction (Maskin and Riley, 1989; Ausubel and Cramton, 1996). Experimentally, the dynamic Vickrey auction eliminates the demand reduction found in the uniform price auctions, thereby improving economic efficiency. However, it raises *less* average revenue than in uniform price sealed bid auctions.

Behavior is studied in the simplest possible setting while still preserving the essential strategic elements of more complicated auctions of this sort: A human subject with flat demand for two units of a homogeneous commodity competes against different numbers of rivals demanding a single unit of the commodity. In both the uniform price and the Vickrey auction the role of single unit buyers is played by computers whose bids are equal to their private values (a dominant strategy for single unit buyers in these auctions). With independent private values drawn from a uniform distribution and with supply of two units, the equilibrium prediction for the "large" bidder in the uniform price auction is to bid her value on unit 1 and to bid sufficiently low on unit 2 to insure that this bid does not affect the market price. This holds irrespective of the value of the item or the number of computer rivals. In contrast, in the Vickrey auction the "large" bidder should bid his value on both units. Thus, the experimental design yields clear differences in behavior between the dynamic Vickrey and uniform price auction rules in an environment free from the strategic uncertainties inherent in interactions between human bidders (e.g., problems of learning best responses given rivals' out-of-equilibrium bids).

There is little earlier experimental work on multi-unit demand, independent private value (IPV) auctions against which to directly compare our results. Early work by Miller and Plott (1985) compared the revenue raising effects of uniform price versus pay-your-bid auctions. In the Miller and Plott design the supply at the market clearing price exceeded the total demand of any individual bidder so that truthful revelation was a dominant strategy. Alsemgeest, Noussair and Olson (1998) examine a private value uniform price clock auction in which 4 units are supplied and each of 3 bidders demands up to two units of the item. There is some incentive for demand reduction on the lower valued unit, which they observe in their data. However, they do not solve for the equilibrium bid function and do not compare behavior against any benchmark calculations or alternative institutions. Multi-unit demand auction experiments with super-additive values do not directly address the issues of concern here as super-additivity largely eliminates the incentive for demand reduction (see for example, Ledyard, Porter and Rangel, 1997; Plott, 1997; Isaac and James, 1997; Brenner and Morgan, 1997).[2] Our results are, however, directly related to two other branches of the experimental literature.

First, experiments investigating the strategic equivalence of single unit second-price and English clock auctions show that bids are typically above value in the second-price auctions, but converge quickly to the dominant strategy prediction in the clock auctions (Kagel, Harstad, and Levin,1987; Kagel and Levin, 1993). These differences in behavior have been attributed to the fact that (i) any time you bid above your value and win in the English auction you necessarily lose money, while this is not the case in the second-price auctions, and (ii) the real time nature of the clock auction induces learning without actually having to lose money, since comparisons of the standing price with resale values should alert bidders that they are bound to lose money if they win with a price exceeding their value (Kagel et al., 1987; Kagel, 1995). These conjectures have not been followed by any systematic experimental investigations that we are aware of. It does, however, suggest that bidding will be closer to equilibrium in our multi-unit demand uniform price clock auctions than in the sealed bid auctions.

Second, there is evidence from continuous double oral auction experiments that, under some parameter values, when a subset of sellers have market power, they withhold supply in order to raise prices and profits (Holt, Langan, and Villamil, 1986; Davis and Williams, 1991). That is, when it is economically profitable to practice supply reduction, sellers do so in an institution that otherwise

[2]Earlier experimental work in markets with superadditive values include Grether, Isaac, and Plott (1989) and Banks, Ledyard, and Porter (1989). There have also been a number of experimental studies of multi-unit auctions in which all agents have unit demands. Burns (1985) reports a multi-unit demand auction with single units auctioned off sequentially.

promotes highly competitive behavior.[3] This is essentially the same process at work as in the uniform price auctions investigated here.

The plan of the paper is as follows: Section I develops the theoretical predictions of the different auction institutions. Section II outlines our experimental design. Results of the experiment are reported in Section III. We close with a brief summary of our major results and some thoughts regarding the boarder implications of our findings.

I. Theoretical Considerations

We investigate bidding in IPV auctions with (n+1) bidders and 2 *indivisible* identical objects for sale, where $n > 2$. Bidders 1,2,...,n, demand only one unit valuing it at $V_1, V_2, \ldots, V_n$, respectively. Let $v_1, v_2, \ldots, v_n$ be the realizations of $V_1, V_2, \ldots, V_n$ and assume, without loss of generality, that $v_1 \,\$ v_2 \,\$, \ldots, \$ v_n$. The $(n+1)^{th}$ bidder, *h*, demands two units of the good, placing the same value V_h on both units. Bidders' values are drawn *iid* from a *uniform* distribution on the interval [0,V].

Uniform Price Sealed Bid Auctions: In the uniform price sealed-bid auction each bidder simultaneously submits sealed bids for each of the units demanded. These are ranked from highest to lowest, with the two highest bids each winning an item and paying a price equal to the third highest bid. For bidders demanding a single unit there is a dominant strategy to bid their value, v_i , as in the single unit Vickrey auction.

It is also a dominant strategy for bidder *h* to make her higher bid (which we will refer to as her bid on unit 1) equal to her value, v_h. This too follows from single round deletion of dominated

[3] Efficiency losses from this supply reduction are minimal, in part because sellers tend to dump "withheld" units in the final seconds of trading. Holt (1995) provides a general review of experiments with market power, along with some cautionary comments regarding the generality of the results reported for continuous double auctions.

strategies, just as in the Vickrey auction. Further, as the derivation below shows, in our design the optimal bid for *h* on unit 2 is zero.[4]

Let $V_{(k)}$ denote the k^{th} order statistic of $V_1, V_2, \ldots, V_n$ and $F_{(k)}$ its distribution function. Let $b_2(v_h)$ denote h's bid on the second unit. We calculate the expected payoff of *h* who observes v_h and bids b. To compute the expected value of bidding b one needs to consider three regions:

Region 1: $V_{(1)} \leq b$, where *h* wins both units and earns $2\int_0^b (v_h - p)\,dF_{(1)}(p)$.

Region 2: $V_{(2)} \leq b < V_{(1)}$, where *h* wins one unit, sets the market price, and earns $(v_h - b)[F_{(2)}(b) - F_{(1)}(b)]$.

Region 3: $b < V_{(2)} < V_n$, where *h* wins one unit, does not set the market price and earns $\int_b^{v_h} (v_h - p)\,dF_{(2)}(p)$.

We differentiate with respect to b and collect terms from the three regions to obtain the following first order condition (FOC) for a maximum:

(1) $(v_h - b)\, f_{(1)}(b) - [F_{(2)}(b) - F_{(1)}(b)] \leq 0$

where $f_{(k)} \geq 0$ is the derivative of $F_{(k)}$. To calculate the FOC note that

$$F_{(2)}(b) - F_{(1)}(b) = \binom{n}{n-1}[1 - F(x)]\,[F(x)]^{n-1}$$

and $$f_{(1)}(b) = n\binom{n-1}{0}[F(x)]^{n-1} f(x).$$

Substituting these expressions into (1), canceling terms and regrouping, yields

(2) $(v_h - b)\, f(b) - [1 - F(b)] \leq 0$,

with inequality only if the optimal bid is zero. With $F(\cdot)$ a uniform distribution with support

[4]We thank Lawrence Ausubel and Peter Cramton for their generous help with this derivation. These results are independent of the distribution underlying v_h, a fact that can also be exploited experimentally.

$[0, V]$, $(v_h\text{-}b)f(b)-[(1-F(b))]=[(v_h\text{-}b)-(V\text{-}b)]/V=(v_h\text{-}V)/V< 0$ for all $\forall\, v_h < V$. Thus, for our design,

(3) $\quad b_2(v_h) = 0, \ \forall\, v_h.$

Note that $b_2(v_h)$ is independent of *n*, the number of rivals demanding a single unit. Further, as the appendix to our working paper demonstrates (Kagel and Levin, 1999), risk aversion does not affect the dominant strategy of single unit bidders or *h*'s bid on unit 1, nor the optimal bidding strategy for unit 2. The extreme outcome of bidding zero on unit 2 rests critically on the supply of 2 units and the use of a common uniform distribution for single unit bidders.[5]

Identifying the optimal level of demand reduction in the sealed-bid auction is, without doubt, a complicated task for most people. As such we would expect partial demand reduction, $b_2 \in (0, v_h]$, to be more likely. This would improve efficiency and raise price relative to equilibrium since b_2 may turn out to be the second highest bid.

Uniform Price English Clock Auctions: The English clock version of the uniform price auction starts with a price of zero, with price increasing continuously thereafter. Bidders start out actively bidding on all units demanded, choosing what price to drop out of the bidding. Dropping out is irrevocable so a bidder can no longer bid on a unit she has dropped out on.[6] The drop-out price which equates the number of *remaining* active bids to the number of items for sale establishes the market price. All of the remaining units earn a profit equal to their value less the market price. All other units earn zero profit.

[5]The appendix to our working paper shows that with more units for sale and with general $F(\cdot)$ (1) there will be some demand reduction, (2) $b_2(v_h)$ will be independent of *n*, and (3) in cases where $b_2(v_h) \geq 0$ for a risk neutral bidder, b_2 will be strictly lower for a risk averse bidder.

[6]Given that the fixed strategy bidding strategy of the computers, the irrevocable exit rule has no impact on the equilibrium outcome. However, we plan to conduct additional experiments where the irrevocable exit rule may have some theoretical bite.

Posted on each bidder's screen at all times is the current price of the item, the number of items for sale, and the number of units actively bid on, so that *h* can tell at exactly what price a rival has dropped. Further, there is a brief pause in the forward progress of the clock following a drop-out during which *h* can drop out as well. Drop-outs during the pause are recorded as having dropped at the same price, but are indexed as having dropped later than the drop-out that initiated the pause.

Bidders i = 1, ..., *n* demanding a single unit have a dominant strategy to remain active until the price equals their value v_i, as does bidder *h* with respect to unit 1. Although in general *h*'s optimal dropping price for unit 2 in a dynamic auction will be different than in a static sealed-bid auction, in our design there are no *effective* differences: *h* drops at p Θ (0, v_2] which has *exactly* the same consequences as dropping out at 0.[7]

The fact that *h*'s optimal dropping price is anywhere between 0 and v_2 may (and does) introduce a significant difference in actual bidding and performance compared to the sealed bid version of the uniform price auction. Consider the clock version of the uniform price auction and for concreteness suppose that V = 100 and that v_h = 90 with a supply of 2 units. Suppose that *h* has no formal understanding of the optimal bidding strategy and so decides to remain active on both units, which is also optimal, as long as v_2 has not dropped out. Once v_2 drops out, say at p = 50, *h* has two alternatives: To drop at 50 herself, thereby earning one unit with a sure profit of 40, or remain active in an effort to win two units. In the latter case there are two events to consider: (1) v_1 drops prior to p = $v_{h,}$ in which case *h* can expect to earn a profit of 40 (20 per unit as the expected dropping price of v_1,

[7]For a formal proof, see the appendix to Kagel and Levin (1999).

given that $v_1 \in [v_2, v_h]$, is half way between 50 and 90) or (2) $v_1 \geq v_h \geq 90$, in which case the expected profit for h is zero. Thus, dropping at p = v_2 dominates waiting and trying to win two units, and is consistent with equilibrium.

However, as the above analysis suggest, the optimal bidding strategy is considerably simpler and more transparent than the *ex ante* calculations involved in the sealed bid auction. Further, in all likelihood h does not even need to make any formal computations to learn to play the equilibrium strategy under a wide range of circumstances. First, any time h wins an item when bidding above v_h she *must* lose money as a consequence. This should help promote learning to avoid this mistake. Second, for $v_2 \leq v_h$, the closer v_2 is to v_h the higher the probability (and the more transparent) the bad outcome (event 2 above) from continuing to bid on both units. This should promote equilibrium bidding on unit 2 even for bidders incapable of making the more sophisticated expected value calculation of the return from continuing to bid.

Other Uniform Price Auctions Investigated: Two additional uniform price auction institutions are investigated. The first is an ascending price clock auction like the one just described, but without any feedback regarding the number of units actively bid on or the drop-out prices of computer rivals, until the auction has ended, just as in the sealed bid version of the auction.[8] The second is a sealed bid auction in which v_2 is announced prior to the start of the auction. Thus, we make available what we believe to be the crucial information bidders use in coming closer to optimal outcomes in the clock auctions, but do so in a sealed bid format. Further, in this treatment there is no discussion of how

[8] To our knowledge eliminating information feedback in clock auctions have never been tried before.

bidders might use the information in v_2 to ease the computational difficulties inherent in determining how much to bid. It's simply there for them to figure out how to use. This treatment is implemented using two different procedures, with the prominence and saliency of v_2 increased substantially between procedures. These procedures are described in some detail along with the data analysis.

Dynamic Vickrey Auctions: We also investigate Ausubel's (1997) dynamic version of the multiple-unit Vickrey auction with feedback regarding rivals' drop-out prices. However, unlike the uniform price auction, winning bidders do not pay a common price, but rather the price at which they have "clinched" an item. This eliminates the incentive for demand reduction for bidder *h*.

Clinching works as follows: With 2 objects for sale, suppose at a given price, p, bidder *h* still demands 2 units, but the aggregate demand of all *other* bidders just dropped from 2 to 1. Then, in the language of team sports, bidder *h* has just clinched winning an item no matter how the auction proceeds. At this point, the auction temporarily stops, with bidder *h* awarded one item at the price, p, that assured clinching the item. The auction then continues with the supply reduced from 2 to 1, and with *h*'s demand reduced to one unit. This process repeats itself until all units are allocated. In this way the auction sequentially implements the rule that each bidder pays the amount of the kth highest rejected bid other than her own for the kth object won, as the Vickrey mechanism requires.

Under the Vickrey mechanism bidders have incentive for full demand revelation as the price bidder *h* pays on unit 2 has no effect on the price paid for unit 1. Thus, in equilibrium, the Ausubel auction insures full efficiency. Further, for our case of flat demands with valuations drawn iid from the same uniform distribution, the seller's expected revenue is higher as well (Maskin and Riley, 1989;

Ausubel and Cramton, 1996). [9]

II. Experimental Design

Valuations were drawn iid from a uniform distribution with support [0, $7.50]. Bidders with single unit demands were represented by computers programmed to follow the dominant bidding strategy. Bidders *h* were drawn from a wide cross-section of undergraduate and graduate students at the University of Pittsburgh and Carnegie-Mellon University.[10] Each *h* operated in her own market with her own set of computer rivals. *h*s knew they were bidding against computers, the number of computers, and the computers' bidding strategy.

The use of computer rivals has a number of advantages in a first foray into this area: *h*s face all of the essential strategic tradeoffs involved in IPV multi-unit demand auctions but in a very "clean" environment. There is no strategic uncertainty regarding other bidders' behavior and no issues of whether or not "common knowledge" assumptions are satisfied. Further, in anticipation of some "crazy" bidding types (see below) we can aggregate the data as we wish, distinguishing between "good" and "bad" players, without having to disentangle the effects of the latter's behavior on the former.

A supply of two units creates a stark and simple contrast between bidding in the uniform price

[9]Numerical analysis establishes that with the uniform distribution expected profit for bidder *h* is higher under Vickrey compared to the uniform price auction and expected earnings of unit demand bidders are less under Vickrey. We do not pursue these implications in the data analysis as they are secondary to our main concerns.

[10]Students were recruited through fliers posted throughout both campuses, advertisements in student newspapers, and electronic bulletin board postings.

auctions and in the dynamic Vickrey auction.[11] We varied the number of computer rivals (n = 3 or 5) to test the predicted invariance of outcomes to this manipulation.

All clock auctions employed a "digital" clock with price increments of $0.01 per second. In clock auctions with feedback, following each computer drop-out there was a brief pause of 3 seconds. Drop-outs by *h* during these pauses counted as dropping out at the same price, but later than the computer's drop-out. *h* could drop out on a single unit by hitting any key. Hitting the number 2 key, or hitting a second key during the pause, permitted *h* to drop out on both units at the same price. The uniform price clock auction with no feedback maintained the pause in the price following *h* dropping out, but eliminated the pause or any other information feedback following a computer drop out.

In the sealed bid auctions subjects submitted unit 1 bids first, with unit 2 bids restricted to be the same or lower than the unit 1 bid. This requirement for unit 2 bids was characterized as a convention, and since subjects were free to bid any non-negative value for unit 1, it in no way constrained their bidding strategy.

Instructions were read out loud with subjects having copies to read as well. The instructions included examples of how the auctions worked as well as indicating some of the basic strategic considerations inherent in the auctions. Examples illustrated losses could result from bidding above value on a unit, after which we noted:

> "Any time it is **necessary** to bid above your value in order to earn an item, you don't want to earn it! You can only lose money compared to the alternative of bidding your value and not

[11]Several sealed bid uniform price sessions were conducted with a supply of 3 units, but are not reported here. Equilibrium predictions are more complicated for this case (see our working paper and Ausubel and Cramton, 1996). Results from these sessions are similar to those reported with supply of 2 units; i.e., some limited demand reduction.

earning the item." (underlining and emphasis in the original).

Use of explicit advice of this sort was motivated by bids above value observed in single unit, second-price, private value auctions (Kagel and Levin, 1993; Kagel, Harstad and Levin, 1987; Cox, Smith and Walker, 1985). In our design, bids above value represent strictly dominated strategies. The focus of the present study is on the effect of different auction rules on demand reduction on unit 2. Thus, we hoped that our instructions would "move" subjects quicker beyond the "nuisance" outcome of bidding above value.[12]

The uniform price auctions provided examples illustrating cases in which more aggressive bidding on unit 2 was profitable, as well as cases where it reduced total earnings. We pointed out to bidders that:

> "...with our uniform price rule earning 2 instead of 1 units almost always increases the price you pay on your first unit (the exception is the unlikely event that 2 or more computers have the same value). The net result is that in some cases it will be profitable to increase your bid on the second unit (example 1') and in some cases it will not be profitable to increase your bid on the second unit (examples 2' and 3')."

For the Vickrey clock auctions, examples were used to illustrate how clinching worked, both in cases where it produced positive profits and in cases where bidding above value produced negative profits.[13]

The uniform distribution from which values were drawn was set with an eye on the expected

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[12]This is, of course, not the only way to deal with this issue. We could have required subjects to bid their value on unit 1, or not permitted them to bid above their value on unit 1. One disadvantage of these options is that for comparative purposes we would have wanted to do the same thing in the clock auctions. But here we were pretty sure from the earlier single unit auctions that subjects would not bid above value, so that it would be interesting (and shocking) if they did so in the more complicated multiple-unit setting. Thus, our procedures reflect a desire to both permit this last possibility while maintaining comparability with the sealed bid procedures.

[13]In this case there was no warning about the dangers of bidding above value since the whole point of the treatment was to see if subjects would bid optimally, and past experience with single unit Vickrey auctions had demonstrated that bidding above value was the mistake subjects were most likely to make.

cost to *h* of deviating from the equilibrium bidding strategy in the uniform price auctions. Table 1 shows the results of numerical calculations where *h* bids her value on unit 1 and bids a proportion of her value on unit 2; i.e., $b_1 = v_h$, $b_2 =$ á v_h (0 # á # 1.0). The average expected (opportunity) cost of full demand revelation (á = 1) on unit 2 with $n = 3$ is $0.37 per auction, 33.2% of maximum possible earnings ($0.74 conditional on winning an item). The cost with $n = 5$ is $0.18 per auction, 33.1% of maximum possible earnings ($0.53 conditional on winning an item). The overall payoff function is relatively flat for small deviations from á = 0. However, what this masks is that the opportunity costs were considerably higher when bidders stood a real chance of winning an item.[14] The impact of changes in á on average market efficiency and revenues is also reported in Table 1.

Uniform price auction sessions began with 3 dry runs to familiarize bidders with the procedures, followed by 25 auctions played for cash with the number of computers fixed throughout. The dynamic Vickrey auctions also employed 3 dry runs, followed by 27 periods played for cash, with the number of computer rivals switched from 3 to 5 (session 9) or from 5 to 3 (session 10) mid-way through the "wet" runs.[15] At the start of each auction both *h* and the computers received new valuations. At the conclusion of each auction bids were ranked from highest to lowest along with the corresponding valuations. Winning bids were identified, prices were posted, profits were calculated, and cash balances were updated.

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[14]For example, with $n = 3$ if *h*'s value is $5.63 (the expected value of v_1) the opportunity cost per auction of á = 1 more than doubles compared to the cost reported in the text.

[15]There are two reasons for these differences in procedures: (i) watching the session unfold, it was clear that behavior was close to optimal very early on in session 9 and (ii) when they do clinch it introduces a severe censoring problem (you automatically drop out of the bidding, so true reservation prices are not observed). This is particularly pronounced with respect to unit 1 bids. With more rivals, bidders are less likely to clinch an item, and when the do clinch it is with a higher v_h, both of which reduce the censoring problem.

Bidders were given starting capital balances of $5. Positive profits were added to this balance and negative profits subtracted from it. End-of-experiment balances were paid in cash. Expected profits were sufficiently high that no participation fee was provided.[16] Inexperienced subject sessions lasted between 1.5 and 2 hours.

Table 2 provides a partial summary of the experimental treatment conditions. In addition to the treatments outlined so far there are two additional treatments: First, it was suggested that the standard sealed bid auctions had two strong pro-equilibrium features - the explicit advice against bidding above value and the restriction that unit 2 bids be less than or equal to unit 1 bids. As such, for a more complete understanding of behavior we conducted a session without these two elements. Second, we report data for uniform price sealed bid auctions using experienced bidders. Study of experienced bidders focuses on this treatment since bidding is relatively far from equilibrium for inexperienced bidders.

III. Experimental Results

A. Standard Sealed Bid Uniform Price Auctions with Inexperienced Bidders

Figures 1-3 provide scatter diagrams of unit 1 bids (top panels) and unit 2 bids (bottom panels) over the last 12 auctions for each session.[17] The first thing to notice is the large number of bids <u>above</u>

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[16]In those few cases where end-of-experiment earnings were below $2.00, a token $2.00 payment was provided.

[17]Our primary focus throughout is the last 12 auction periods, reporting behavior after subjects have had a chance to familiarize themselves with the auction rules and for behavior to settle down. Results are robust to the precise definition of “more experienced” behavior - last 10 or last 15 periods.

value for unit 1, particularly in sessions 2 and 3.[18] This occurred in spite of our examples showing how such bids could result in negative profits, and our advice against bidding above value. Bids above value replicate results reported in earlier single unit, second-price auctions (Kagel and Levin, 1993; Kagel, Harstad and Levin, 1987; Cox, Smith and Walker, 1985). In many cases, as in these earlier studies, bidders do not lose money as a consequence of bidding above value: in the auctions reported here, 56.2% of all unit 1 bids greater than value earned non-negative profits with $n = 3$, 67.5% with $n = 5$. Thus, there is plenty of room for what psychologists call adventitious reinforcement - appearing to gain advantage as a consequence of bidding above value.[19] Categorizing bids within 5¢ of value as equal to value (thereby accounting for rounding off of bids relative to value and distinguishing between bids that are very close to value versus those that are further away), a substantially larger proportion of bids equal value here (55.0% of all unit 1 bids) than in earlier single unit, second-price auctions (29.5% of all bids; Kagel and Levin, 1993).[20] This, no doubt, reflects the impact of our examples and advice against bidding above value, advice not provided in the earlier single unit auctions.

Unit 2 bids are scattered all over, with relatively few bids equal to 0 as optimality requires. However, although demand reduction is far from complete, there is a wholesale shift in the distribution of unit 2 bids relative to unit 1 bids in the predicted direction; 61.4% of all unit 2 bids were more than 5¢ below value versus 11.8% of all unit 1 bids. One might argue that part of this shift can be accounted

[18]The seemingly large variation in unit 1 behavior across sessions is accounted for by the multiple (12) observations per subject. Bids for representative individual subjects are reported in Figure 4 below.

[19]The casual reader should not be too hard on our subjects for overbidding on unit 1. At a conference on auctions at the University of Maryland, one participant intimately familiar with the recent spectrum auctions remarked that this behavior reminded him of at least one of the spectrum bidders.

[20]Second-price auctions data is with 5 bidders over the last 10 auctions.

for by the requirement that $b_2 \# b_1$. Note, however, that this did not prevent subjects from bidding the same on both units, or just a penny or two less on unit 2. Further, as will be shown below, comparable levels of demand reduction are found in sessions where the requirement that $b_2 \# b_1$ was eliminated. Thus, this shift can be attributed to genuine demand reduction.

Table 3 summarizes the data contained in Figures 1-3 and our analysis of *h*'s bids compared to equilibrium predictions.

Hidden behind the aggregate data are systematic differences in individual bidding patterns. Graphs of individual bids, in conjunction with expected profit calculations, indicate four typical patterns[21]:

1. A third of all bidders (34.1%; 15/44) consistently bid above value on unit 1 over a large range of values and, more often than not, bid above value on unit 2 as well. Expected earnings for these bidders were lower than if they had bid their value for both units.[22] Bids for a single representative subject from this group are shown in Figure 4a.

2. A small percentage of bidders (18.2%; 8/44) effectively bid optimally, bidding close to value on unit 1 and close to zero on unit 2, with opportunity costs of 5% or less of maximum possible earnings over the last 12 auctions. Data for a single representative subject from this category is reported in Figure 4b.

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[21]Expected profit calculations employ Monte Carlo simulations using actual values and bids for *h* in conjunction with 100 (independent) draws for the computer rivals in each auction period. Averaging over all subjects expected profit calculations do not differ much between the MC simulations and the realized random draws for the computer rivals. However, for individual subjects, differences between the two expected profit measures do, occasionally, differ substantially.

[22]Earnings for these bidders averaged -11.7¢ per auction versus 74.4¢ per auction for optimal play.

3. Another small percentage (13.6%; 6/44) bid close to their value on unit 1, with very little or no demand reduction on unit 2. The opportunity cost of such a bidding strategy is 33% of maximum expected profit. These bidders acted as if full demand revelation is optimal. Representative data from one such bidder is reported in Figure 4c.

4. The remaining bidders (34.1%;15/44) typically bid their value on unit 1 and exhibited some, but far from complete, demand reduction on unit 2. Opportunity costs for these bidders average about half (18.3%) of maximum possible earnings. Data for a representative subject from this group is shown in Figure 4d.

Table 4 calculates actual and predicted efficiency and revenue over the last 12 auctions. The data are presented in two formats: (i) including all subjects and (ii) excluding those subjects who consistently bid above value on unit 1 (all category 1 subjects above).[23] Efficiency is defined as the sum of the values of the two units sold in an auction as a percentage of the sum of the two *highest* values in that auction. With all subjects included, actual efficiency is about the same as predicted efficiency, as the efficiency losses resulting from bidding above value on unit 1 just offset the efficiency gains resulting from over-revelation of demand on unit 2. Dropping subjects who consistently bid above value on unit 1, efficiency losses are half the level predicted due to the tendency to over-reveal demand on unit 2.

With all subjects included, actual revenue is consistently and substantially above predicted revenue (close to $1 per auction above predicted revenue for the pooled data). These higher than predicted revenues, although not as large once we drop subjects who consistently bid above value on

[23]Note, this alternative measure excludes only category 1 bidders (and all the data for these bidders) and excludes no data for any other bidders. We employ this alternative measure for the convenience of readers who (unlike ourselves) believe that category 1 bidders are "crazies" unlikely to be observed in field settings.

unit 1, are still substantial due to the tendency to over-reveal demand on unit 2 (more than 60¢ per auction above predicted revenue).

Note, that in computing revenue and efficiency and comparing across experimental treatments there is no pretense that the same results will emerge in environments where all bidders are human. As already noted computer rivals were employed to minimize possible complications associated with learning against human rivals who may be playing out-of-equilibrium strategies, and this may affect different institutions differently. Nevertheless, we believe the data to be suggestive of what will be observed in interactive settings, and can provide a benchmark against which to compare outcomes with all human bidders.

B. Uniform Price Clock Auctions with Feedback

Figures 5 and 6 report bids for the two uniform price clock auction sessions. Graphs of unit 1 bids use several different symbols to characterize bids relative to value: Circles represent prices of winning bids. These are, of course, censored since we do not know how high subjects would have been willing to bid. Squares represent observed drop-outs in cases where bidders dropped at or below v_h. These are almost entirely along the 45º line, with only occasional drop-outs significantly below value. For drop-outs above value triangles represent potentially harmful over-bids and diamonds represent harmless over-bids. Dropping out above value is potentially harmful when the drop-out price is greater than the third-highest computer value, so that had one of the two remaining computers dropped out, the bidder would have lost money. In contrast, harmless over-bids involve dropping out prior to the third highest computer dropping out, in which case there is no chance of losing money as a result of staying in the auction this long.

As the data show, most unit 1 bids satisfy optimal bid requirements: Few harmful or potentially harmful bids above value and few drop-outs below value. This too replicates earlier single unit demand experiments where bidding is close to the dominant strategy in English clock auctions (Kagel, Harstad, and Levin, 1987).

As the theory predicts, the vast majority of unit 2 bids are in the interval $[0, v_2]$. The graphs capture this fact by distinguishing between unit 2 drop-outs that occurred at or below v_2 so that they had no effect on the market price (squares) and unit 2 bids that affected the market price - winning bids (circles) and drop-outs above v_2 (+*s*). The contrast with the sealed bid auctions is striking: (1) Virtually no one won two units here, 1.7% of all auctions, compared with 15.3% in the sealed bid auctions and (2) 11.4% of all unit 2 bids affected market price here (see Table 5) compared with a pivotal bid rate of 31.8% in the sealed bid auctions. Thus, unit 2 bids were much closer to optimal in the clock than in the sealed bid uniform price auctions.

Table 5 summarizes the data reported in Figures 4 and 5. The primary contribution of Table 5 is to distinguish between cases where optimal; unit 2 bids simply involved avoiding losses (cases when $v_2 > v_h$) and where optimality required more sophisticated reasoning ($v_h > v_2$). Even in the latter case there is very little bidding above v_2 (26.2% for the pooled data).

A closer look at the data in cases where $v_h > v_2$ indicates that the likelihood of dropping out after v_2 is an increasing function of how much higher v_h is relative to v_2. This is confirmed through fitting the following random effect probit regression to the data:

$$\text{Prob}\,(d_{2it} > v_{2it} \mid v_{hit} > v_{2it}) = \underset{(0.523)}{-0.061} + \underset{(0.080)}{0.201}\,(v_{hit} - v_{2it}) - \underset{(1.360)}{0.986}\,\text{PFREQ}_{it} + u_{it}$$

where d_{2it} is the dropout price on unit 2 of bidder i in period t, $PFREQ_{it}$ is a variable measuring the past frequency with which bidder i was faced with a situation where ($v_h \geq v_2$), and $u_{it} = \eta_i + \varepsilon_{it}$ where η_i is a subject specific error term assumed constant across auctions and ε_{it} is an auction period error term, both of which are assumed to be normally distributed with the usual properties. Standard errors of the estimates are reported in parentheses. Neither the constant or the PFREQ variable is statistically significant at conventional levels. However, the variable (v_h - v_2) is positive and significant at better than the 5% level. This no doubt reflects the fact that the closer v_2 is to v_h the more transparent it is to bidders that stopping the auction provides higher profits than trying to win both items. Finally, note that *h* rarely won two units (1.7% of all auctions). This indicates that in those cases where $b_2 > v_2$, as the clock price ticked up and profits on unit 1 shrank, bidders consistently reversed their decision to try and earn two units, suggesting that the force of the logic underlying the equilibrium prediction became increasingly obvious as price came closer to v_h.[24]

Looking at individual subject data reveals three typical bidding patterns:

1. A small percentage of subjects - 6.7% (2/30) - consistently bid above value on unit 1 and are responsible for virtually all such bids. Interestingly, these few subjects consistently bid below value on unit 2. All remaining subjects consistently dropped out on unit 1 when the price reached their value.[25]

2. Some 43.3% (13/30) always bid optimally for unit 2 as they never bid more than 5¢ above

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[24]An alternative explanation to this heuristic is that bidders are risk loving. This explanation is, however, totally inconsistent with observed behavior in single unit auctions (see Kagel, 1995, for a review of this literature).

[25]This percentage is considerably less than the corresponding percentage of subjects in the sealed bid auctions (15/44; Z = 2.77, p < .01, 2-tailed test).

v_2. This is far more than the number of bidders who were playing close to equilibrium in the sealed bid auctions (6/44; Z = 2.87, $p < .01$, 2-tailed test). Of these, only 2 consistently dropped out at $p = 0$ or close to it.

3. The remaining bidders, 50.0% (15/30), occasionally bid above v_2, thereby affecting the market price, employing the rule-of-thumb characterized in the probit regression.

Table 6 reports average revenue and efficiency over the last 12 uniform price clock auctions. Efficiency is slightly less than predicted ($p < .10$, 2-tailed Wilcoxin signed rank test). This results from the occasional bids above and below value on unit 1 which resulted in efficiency losses, with virtually no unit 2 wins to offset these efficiency losses. In contrast, actual revenue is consistently and significantly higher than predicted ($p < .01$, 2-tailed Wilcoxin singed rank test). This is a direct result of the minority of unit 2 bids above v_2.

C. Understanding the Closeness to Optimal Outcomes in Clock versus Sealed Bid Auctions

Its clear from the data that bidders are much closer to the optimal outcome in the clock compared to the sealed bid version of the uniform price auction even though both auctions have the same normal form representation.[26] In conducting experiments we are not simply interested in "grading" economic theories or subjects' behavior. If experiments are to aid in understanding behavior it is essential to identify the behavioral principles underlying the outcomes reported, since it's these principles that are likely to generalize to more complicated settings both inside and outside the lab. This section explores the factors underlying the differences reported.

[26]As an alert reader has pointed out that this is not correct on one dimension - efficiency. We discount this, however, since it is an artifact resulting from offsetting errors in the sealed bid auctions (overbidding on unit 1 and not enough demand reduction on unit 2).

One possibility, suggested by a reader of an earlier draft of this paper, is that the clock auction improves performance due to better learning opportunities: Subjects, in effect, make many more bidding decisions than in the sealed bid auctions, as they must decide at each price whether to stay in or drop out, and are likely to get better at it as a consequence. Alternatively, the differences may result from a combination of two factors. First, a breakdown in "procedure invariance." That is, the different procedure used to elicit unit 1 bids induce different choices in the sealed bid and clock auctions.[27] Possible reasons for this were discussed earlier in reviewing differences in behavior in single unit auctions. Second, as argued in section I, the information released in the course of observing the computers' drop-out prices simplifies the unit 2 decision problem relative to the sealed bid auction.

To sort out between these possibilities we introduced two additional experimental treatments. First, we conducted a clock auction with no feedback regarding computer drop outs. This treatment directly challenges the experience argument. If the experience of repeatedly deciding whether to stay in or drop out of the auction is, *by itself*, primarily responsible for the superior performance of the clock auction, then we should observe a significant movement towards equilibrium in a clock auction with no feedback. Second, since v_2 is the most important information signal *hs* can observe without trivializing the problem, we conducted sealed bid auctions with v_2 announced prior to the bidding. If the reduction in the complexity of the decision problem associated with knowing v_2 is primarily responsible for the improved performance, then behavior in these auctions should be closer to what is observed in the

[27]Perhaps the most notable breakdown in procedure invariance in the economics literature consists of the preference reversal phenomena, whereby theoretically equivalent ways of eliciting individual preferences do not produce the same preference ordering. For a review of this literature see Camerer (1995).

clock auctions with feedback. [28]

Figure 7 reports aggregate data for the last 12 clock auctions with no feedback. The picture here is, indeed, worth a thousand words. Unit 1 bids primarily lie on or above the 45º line, and unit 2 bids are generally on or below the 45º line, with very few bids at zero or close to it, much like the data reported earlier for the sealed bid auctions. Table 7, which reports bids relative to values using the same format used to analyze the standard sealed bid auctions (see Table 3), reinforces the conclusion drawn from the figure. Although with n = 3, the overall frequency with which $b_1 > v_h$ (+ 5¢) is greater than in the standard sealed bid auctions, the differences are not significant after accounting for the repeated measures problem associated with using 12 auctions for each bidder.[29] We conclude that the clock *by itself* does not move behavior towards equilibrium.

Table 8 reports bids relative to value for the sealed bid auctions with v_2 announced. We report data for the two sessions separately as there were some small, but significant, differences in procedures between them.

Session 7 provided v_2 prior to bidding, but paid little attention to establishing its prominence: v_2 was reported several spaces to the right of where v_h values were reported and bids were entered. This placement, and the fact that we (purposely) did not explain the role of v_2, meant that subjects could easily ignore v_2. The fact that many of them did is indicated by the high frequency of unit 1 bids greater

[28]With v_2 announced, it should be clear to bidders that if $v_2 > v_h$ bidding above v_2 will result in losses. However, if $v_2 < v_h$, the situation is essentially the same as in the clock auction with feedback: A bidder knows she wants to win one unit at any price, but whether it is more profitable to win one or two units depends on v_1, which is unknown. In exploring these issues we confine our attention to auctions with n =3.

[29]Looking at individual subjects and counting the number who bid above v_h 50% of the time or more, there are no significant differences between the two treatments (8/18 subjects here versus 17/44 in the sealed bid auctions; Z = 0.42).

than v_2 when $v_2 > v_{h,}$ which guarantees negative profits (22.1% here versus 7.8% for the clock auctions with feedback), and the high frequency of unit 2 bids greater than v_2 when $v_2 > v_h$, which also guarantees losses (9.8% here versus 1.8% in the clock auctions). Using individual subjects as the unit of observation, these differences from the clock auctions with feedback are statistically significant for both unit 1 and unit 2 bids, but are quite similar to behavior in the standard sealed bid auctions and in the clock auctions with no feedback.[30]

The results of session 7 suggest that a number of bidders essentially ignored the information inherent in announcing v_2. In contrast, in the clock auctions with feedback, the procedures effectively force bidders to pay attention to v_2, and to "understand" the useful information embedded in it. The price clock is located right below a bidder's resale values, with drop-out prices reported right next to these resale values, and the number of computer rivals remaining reported just above the resale values. Thus, anyone looking at the clock and at their resale values must observe drop-out prices and/or the number of computer rivals remaining, and realize the information value of v_2. For example, take someone bidding above v_2 with $v_h < v_2$. Any time such a bid is successful at earning an item the bid *must* earn a negative profit, with all of the information necessary to establish the logical relationship between v_h, v_2 and sensible bids prominently displayed and difficult to ignore. In contrast, in the

[30]Although in the standard sealed bid auctions and the clock auction with no feedback bidders could not determine when $v_2 > v_h$, we can conduct these calculations after the fact. For unit 1 bids, the number of subjects who *never* bid above v_2 conditional on $v_2 > v_h$ was 56.5% (35/62) in the standard sealed bid auctions and in the clock auctions with no feedback compared to 40% (8/20) who never did so in session 7 ($Z = 1.28$, $p = .20$, 2-tailed test). In contrast, in the clock auctions with feedback 66.7% (20/30) never bid above v_2 conditional on $v_2 > v_h$, which is significantly more than in session 7 ($Z = 1.86$, $p < .08$, 2-tailed test). Further, those bidding above v_2 conditional on $v_2 > v_h$ in the clock auctions with feedback typically did so only once (8/10 cases), whereas the majority were repeat offenders in the standard sealed bid auctions and the clock auction with no feedback (14/27 cases), as well as in session 7 (8/12 cases).

standard sealed bid auction, or in a sealed bid auction with v_2 announced but effectively ignored, bidding above v_h more often than not *does not* result in losses, making it substantially harder to figure out that bidding above value in order to win a unit can only generate losses. Similarly, in the clock auction with feedback, when $v_h > v_2$, if *h* has not dropped out on her two units prior to v_2 dropping, it is immediately obvious that dropping at that point will stop the auction, resulting in positive profits. There is no comparable guide available to aid bidders in either the standard sealed bid auction, the clock auction without feedback, or in the sealed bid auction with v_2 announced but ignored.

Session 8 explores these ideas by adjusting the sealed bid procedures so that bidders would have more trouble ignoring the presence of v_2 or the information it contains. This was done as follows: (1) the v_2 value was placed just below where resale values were reported and subjects entered their bids (prominently centered just below the space allocated for entering bids on both values), (2) v_2 and its value were reported in yellow in contrast to all other values (reported in white), with the yellow color coding for v_2 carried over to the listing of bids and resale values that appeared following each auction period, and (3) bidders were required to record v_2 along with their resale values, bids and profits in their record sheets throughout the session.

Of course, there is no guarantee that these simple changes in procedures will be sufficient to elevate v_2 to anything approaching the prominence achieved in the clock auction with feedback. But apparently it goes a long way to achieving this outcome as evidenced by the data for session 8. First, bidding above v_2 when $v_h < v_2$, which guarantees negative profits, was reduced to levels similar to the clock auctions with feedback: 3.5% here versus 7.8% for unit 1 bids in the clock auctions with feedback, and 1.2% here versus 1.8% for unit 2 bids in the clock auctions with feedback.[31] Second, the percentage of subjects always reducing demand on unit 2 bids to no more than 5¢ above v_2, which effectively satisfies the theory's requirement for total demand reduction, is comparable to the level reported in the clock auctions with feedback (42.9% here versus 43.3% in the clock auctions with feedback).

Nevertheless, there was substantially less demand reduction on unit 2 bids over the last 12 auctions in session 8 as (i) in 50% of all cases where $v_h > v_2$, $b_2 > v_2$ (+ 5¢) here compared to 26.2% of all such cases for the clock auctions with feedback and (ii) in 17.1% of all cases where $v_h > v_2$ bidders won two units here compared to 1.4% for the clock auctions with feedback. What is the reason for these differences in behavior on these two important dimensions? In the clock auction, once the price is greater than v_2, with each tick of the clock bidders are reminded that there is a tradeoff between winning one unit at a lower price versus possibly winning two units at a higher price. Even then it takes some experience for bidders to get it right in the clock auctions with feedback: In the first 13 clock auctions played for cash, in 40.1% of all cases where $v_h > v_2$, $b_2 > v_2$ (+ 5¢) and in 11.7% of all such cases bidders won two units rather than one. This is much less demand reduction than in the last 12 clock auctions with feedback, and much more comparable to the levels of demand reduction in the last 12 auctions in session 8. So the clock enhances experience argument seems to have some

[31]However, the impact of v_2 on these beneficial outcomes was not nearly as fast as in the clock auctions: In the first 13 auctions played for cash, the rate of bidding above v_2 when $v_h < v_2$ was 19.1% and 8.5% for unit 1 and unit 2 bids here, compared to 5.7% and 1.3% in the clock auctions with feedback. These early rates in session 8 are quite comparable to the values reported over the last 12 auctions for session 7 with v_2 announced. The comparisons pool the data for n = 3 and 5 for the clock auctions with feedback since there are no real differences along these dimensions in the data.

validity with respect to its impact on demand reduction for unit 2 bids.

We began this section by inquiring why bidding is so much closer to predictions in the clock auctions with feedback versus the standard sealed bid auctions, even though both auctions have the same normal form representation. The picture that emerges is that a number of factors *interact* to generate the differences between auction formats. The clock with feedback provides more than just information regarding v_2. It effectively forces bidders to recognize that when $v_h < v_2$ they don't want to win an item. This eliminates elementary mistakes (earning negative profits). The clock with feedback also makes it clear that once price exceeds v_2, and v_h is greater than v_2, that dropping out will stop the auction and lock in a certain profit. But this alone is not enough to induce bidders to consistently take the correct action. For many, it takes some practice before they get it right. And the clock, by repeatedly forcing bidders to decide whether to stay in or get out, appears to enhance this experience effect.

D. Dynamic Vickrey/Ausubel Auctions

Figures 7 and 8 report bids for the two dynamic Vickrey (Ausubel) auction sessions. Bids on items clinched are reported as prices paid and represented by circles. These bids are heavily censored. In cases where no item was won we report observed drop-out prices. Drop-out prices at or below value are represented by squares. For drop-out prices above value we distinguish between potentially harmful drop-outs where subjects were bidding above value and the next (unknown) computer drop-out would have resulted in negative profits (triangles) and those that occurred before there was any chance to lose money (diamonds).

There are three types of mistakes that can be made in the Ausubel auction: winning an item at a price above value (earning negative profits), bidding above value on an item when the next computer drop-out guarantees clinching the item (a potentially harmful overbid), and dropping out below value (potential opportunity costs). Table 9 organizes the data from the dynamic Vickrey auctions in terms of these three types of mistakes. In calculating the percentage of potentially harmful overbids and drop outs below value, we employ a 5¢ allowance and, given the severe censoring problem, exclude units won from the base.

For both unit 1 and unit 2 bids there are only a handful of items won at prices above value (a maximum of 4.5% for unit 1 bids with n = 3). For unit 1 bids with n = 5 there were relatively few potentially harmful overbids (4.6%) and drop-outs at prices below value (8.3%). The result is that 87.0% of all uncensored unit 1 bids were within 5¢ of value. For unit 2 bids with n = 5 there was a very modest reduction in the percentage of potentially harmful overbids (down to 1.5%), and a modest increase in the percentage of bids more than 5¢ below v_h (up to 13.3%), with 85.2% of all uncensored unit 2 bids within 5¢ of value. There were substantially more unit 1 bid mistakes for the n = 3 case: 26.8% potentially harmful overbids and 18.3% bids below value, so that only 54.9% of the uncensored unit 1 bids were within 5¢ of v_h. In this case there was a relatively large reduction in the percentage of the unit 2 bids that were potentially harmful (down to 3.2%), and a modest increase in the percentage of bids below value (up to 29.8%), with 66.9% of all uncensored unit 2 bids within 5¢ of value. The difference in performance between n = 5 and n = 3 is the result of a few more sub-optimal bidders in one session compared to the other session.[32]

[32]Comparing the data for both sessions in periods 5-16, when bidders had accumulated some experience, but were competing against different numbers of computer rivals confirms this: The n = 5 group when competing against 3 computer rivals had 13.2% of all $b_1 < v_h$ (-5¢) and 19.7% of all $b_2 < v_h$ (-5¢). In contrast, the n = 3 group when

Unit 2 bids here contrast sharply with the demand reduction observed in both the uniform price clock auctions with feedback and the standard uniform price sealed bid auctions. In the clock auctions with feedback there is near universal demand reduction on unit 2 bids, with two units won in less than 1% of all auctions. In the standard sealed bid auctions 61.4% of all unit 2 bids exhibited some degree of demand reduction (were more than 5¢ below value). In contrast, in the dynamic Vickrey only 21.2% of all unit 2 bids were more than 5 cents below value. Thus, the dynamic Vickrey auction eliminates much of the demand reduction on unit 2 bids found in the uniform price auctions, as the theory predicts.

As with the other auctions, deviations from optimality were commonly associated with the same individuals. Five of 27 subjects accounted for most of the clinched items above value, as well as most of the potentially harmful overbids (73.3% of all bids exceeding value by more than 5¢) .[33] Four subjects bid more than 5¢ below value on unit 2 in 50% or more of the last 12 auctions, accounting for 58.2% of all such bids.

Table 10 reports revenue and efficiency over the last 12 auctions with clinching. Efficiency is predicted to be 100% so that actual efficiency has nowhere to go but down relative to this. Efficiency losses resulted from either individuals bidding above value on unit 1 or dropping out too soon on unit 2. Over half of all individuals (63.0%; 17/27) achieved 100% efficiency in the last 12 auctions; with 77.8% (21/27) averaging better than 99% efficiency. Average actual revenue is within pennies of predicted revenue, with those dropping out too soon canceling out those bidding above value. The

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competing against 5 computer rivals had 19.8% of all $b_1 < v_h$ (-5¢) and 29.3% of all $b_2 < v_h$ (-5¢).

[33]These subjects each had 3 or more potentially harmful unit 1 bids that exceeded value by 5¢ or resulted in clinching an item. One of these subjects had participated in a sealed bid uniform price pilot session where she consistently bid above value as well.

implication is that average predicted revenue is a pretty good proxy for actual revenue in comparing dynamic Vickrey auctions with uniform price auctions, and that average actual efficiency is somewhat less, but not too much less, than the 100% predicted.

One convenient way to measure closeness of actual behavior to equilibrium predictions across auction institutions is to compare bidders' actual earnings relative to predicted earnings.[34] For this measure of performance there is a clear ranking of our three primary institutions: Earnings are furthest from the maximum predicted in uniform price sealed bid auctions (only 13.6% of all subjects averaging within 5% of maximum possible profits over the last 12 auctions). Next in performance is the uniform price clock auction with feedback (46.5% of all subjects averaging within 5% of maximum possible profits). Earnings are closest to the maximum in the dynamic Vickrey auctions (85.2% of all subjects averaging within 5% of maximum possible profits). Z statistics using individual subjects as the unit of observation show all three of these differences to be statistically significant at better than the 1% level.

Like the uniform price c
lock auction with feedback, the dynamic Vickrey auction benefits from the clock procedure with feedback to prevent overbidding. However, unlike the uniform price clock auction, the dynamic Vickrey auction encourages non-strategic bidding (full demand revelation), something that bidders are naturally inclined to even in the uniform price auctions. Thus, the closer to optimal performance observed in the dynamic Vickrey auction may result, in part, from an institution that accommodates

[34]Recall that in our design closeness to equilibrium and closeness to maximum payoffs (best response) are one in the same since computer rivals all play their Nash strategies. Comparative measures of performance in choice space suffer from differences in "target size" (e.g., dynamic Vickrey makes point predictions; uniform price clock auctions permit an interval for unit 2 bids), which greatly complicates making comparisons.

itself to bidders' natural tendencies rather than any adjustments on bidders' part to the strategic requirements of the institution.

E. Further Analysis of Standard Sealed Bid Uniform Price Auctions

This section explores the effects of modifications in procedures on behavior of inexperienced bidders in the sealed bid auctions and experienced bidder behavior in these auctions.

E.1. Modified Procedures: In the uniform price sealed bid auctions we advised subjects not to bid above their values in order to earn an item and required that b_2 # b_1. Motivation for this advice was intended to speed up equilibrium outcomes on unit 1 bids. The restriction on unit 2 bids was intended as a "convention" and explained to subjects as such.[35] However, a number of readers have suggested that the restriction might promote demand reduction, a pro-equilibrium outcome we had not intended. Reported below is a session in which these two elements were dropped - both the advice against bidding above value and the requirement that b_2 # b_1.

Table 11 reports results from this treatment (for ease of comparison we repeat the earlier results from Table 3). In analyzing the data from the modified treatment we follow the convention of classifying the higher of the two bids as the unit 1 bid. This is natural since the values underlying both bids are the same and the ranking of bids to determine winners and prices paid is based strictly on the amount bid.[36]

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[35]The instructions read "You are free to bid whatever you think will bring you the most earnings. However, for programming purposes we have adopted the convention that the bid for the second unit listed on your computer screen must be less than or equal to the bid on the first unit listed."

[36] Subjects apparently treat this as a convention as well, since in 60.0% (12/20) of all cases all bids, or nearly all bids (11 out of 12) where such that b_1 $ b_2 or vice versa. Using a two thirds or more rule (8 out of 12 auctions) for "nearly all" increases the percentage to 95.0% (19/20 subjects).

The primary impact of the modified procedures is, as anticipated, to reduce the frequency of equilibrium bidding on unit 1 (27.7% of all unit 1 bids under the modified procedures vs 57.4% under the original procedures; $p < .05$, one-tailed Mann-Whitney test).[37] Although bidding above and below value both increased absent the advice against bidding above value, bidding above value accounts for most of the change. In contrast, the effect on unit 2 bids is not nearly as pronounced. There is essentially the same overall frequency of demand reduction (62.1% without the requirement that $b_2 \# b_1$ vs 61.6% with the requirement). Further, there is a small reduction in the frequency of equilibrium unit 2 bids ($b_2 = 0$) under the modified procedures and a small increase in the frequency of bidding above value on unit 2, but neither of these differences is significant at the 10% level or better in a Mann-Whitney test. The overall effect is that the number of subjects effectively playing equilibrium (category 2 in section III. A) is 10% (2/20) with the modified procedures versus 18.2% (8/44) under the original procedures, which difference is *not* significant ($Z < 1.00$). Finally, in terms of earnings, the difference between the two treatments is small, with average profit 4.5¢ less per auction under the modified procedures (a little under 5% of maximum expected profit).

E.2 Experienced Bidder Behavior in Standard Uniform Price Sealed Bid Auctions: The relatively poor performance of subjects in the sealed bid auctions raises the question of whether bidders would have done much better with more experience. Three experienced subject sessions were conducted to explore this issue. In the first two sessions (12 and 13) we purposely did not invite back subjects who were bidding substantially and consistently above value on unit 1 so that the sample selection is

[37] All Mann-Whitney tests reported use average subject values as the unit of observation to avoid the repeated measures problems.

somewhat biased.[38] In the third session (14), everyone was invited back.

Before looking at experienced bidders it is important to note that there were some adjustments towards equilibrium within the inexperienced subject sessions. Under our original procedures, where subjects were provided with advice against biding above value, there are clear and systematic increases in the frequency of equilibrium bidding on unit 1 from 41.8% of all bids in the first 13 auctions played for cash to 56.0% of all bids in the last 12 auctions.[39] In contrast, under the modified procedures (session 11) there is basically no change in the pattern of unit 1 bids. Under our original procedures, there is essentially no change in the frequency of demand reduction with respect to unit 2 bids, but under the modified procedures demand reduction grows from 49.6% in the first 13 auctions to 62.1% in the last 12, an increase of 25.2% (15/19 increasing, $p < .05$, one-tailed sign test). Finally, the frequency of total demand reduction ($b_2 = 0$) nearly doubles in all treatments from the first 13 auctions to the last 12 auctions (81.3% of all the change cases, $p < .01$, one-tailed sign test).[40]

Experienced bidder data are reported in Table 12. The first row in each session shows bid patterns in the last 12 auctions as inexperienced bidders for those subjects who returned. Each experienced subject session began with a number of auctions with the same number of computer rivals as in the inexperienced subject session. The second row reports the data for the last 12 of these

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[38]Note, some of these subjects returned nevertheless, either because of recruiting mistakes or because they were told about the session by other subjects, and no one was turned away at the door. All experienced subject sessions were conducted within one or two weeks of the corresponding inexperienced subject sessions.

[39]75.9% (22/29) of all bidders who changed increased the frequency of equilibrium bidding ($p < .01$, one-tailed sign test). The sign test drops those subjects (14/43) showing no change. Half of these subjects were already playing the dominant strategy 100% of the time, with the other half playing it 0% of the time.

[40]All of the no change bidders (31/63) failed to exercise full demand reduction in *any* auction period.

auctions. This was followed by a series of auctions in which the number of computer rivals was changed from 3 to 5 or from 5 to 3. The third row reports data for the last 12 auctions from each of these treatments. Finally, in sessions 13 and 14 there was a brief reversion back to bidding in markets with the same number of computer rivals the session began with. The fourth row reports these data.[41]

Subjects in session 12 (original procedures, experience with $n = 3$) show no systematic change in bid patterns with the notable exception of the sharp increase in complete demand reduction ($b_2 = 0$) at the end of the $n = 5$ treatment to 42.2% of all unit 2 bids. This is a result of three bidders clearly "getting it," practicing total demand reduction all the time, or whenever it was likely to make a difference. The overall effect is an increase in the number of bidders classified as playing equilibrium (category 2 in section III. A) from 4 at the beginning of the session to 7 at the end (out of 16 bidders).

In session 13 (original procedures, experience with $n = 5$) the most notable change is the sharp increase in the overall frequency of demand reduction by the end of the $n = 3$ treatment (a 27.1% increase), and a near doubling of the incidence of total demand reduction.[42] Further, by the end of the $n = 3$ treatment, the average frequency of demand reduction surpasses the level observed in session 12 (88.2% versus 69.3% under the $n = 3$ treatment in session 12).[43] Further, demand reduction remains

[41]Session 12 has a last series of auctions with $n = 3$, but with quantity supplied increased to 3, a treatment explored in pilot sessions. In all cases the treatments (and their length) were planned in advance, but since these experienced subject sessions were intended to be exploratory in nature, the treatments are somewhat uneven in nature.

[42]7 out of 8 bidders increased the frequency of $b_2 < v_h$ ($p < .05$, one-tailed sign test; the 4 no change bidders were already at 100% $b_2 < v_h$). 7 of 9 increased the frequency of total demand reduction ($p < .10$, one-tailed sign test; the 3 no change bidders never bid 0 on unit 2). The changes with respect to unit 1 bids are not as consistent across subjects (5 out of 9 increased their play of the dominant strategy, $p > .10$).

[43]This difference just misses being significant at the 10% level (one-tailed, Mann-Whitney test).

virtually unchanged on reverting back to the 5 computer rival treatment. The increased demand reduction during the n = 3 treatment may be accounted for by the fact that for any given bidding strategy, the frequency of unit 2 bids affecting the market price increases with fewer computer rivals. This, in turn, appears to have set off a new round of adjustments in bidding strategies. That is, bidders may have settled into a routine which was disrupted (in a favorable way) by the change in the number of computer rivals. Further, bidding does not revert back to the old pattern when the number of computer rivals increase since there is no reason to abandon a good thing.

The overall effect of all this for session 13 is that 90.9% (10/11) of all bidders earned higher profits in the last n = 5, experienced bidder treatment compared to the last 12 auctions as inexperienced bidders ($p < .01$, one-tailed sign test).[44] And 72.7% (8/11) earned higher profits in the last n = 5 experienced bidder treatment compared to the first n = 5 experienced bidder treatment (p = .15, one-tailed sign test). In absolute terms the results are equally dramatic. At the end of the inexperienced subject session 1 of these 12 bidders was earning within 5% of maximum predicted profit, with 2 of 12 hitting this criteria when first returning as experienced bidders. At the end of the experienced subject session 8 of 12 satisfied this criteria, impressive improvements in equilibrium play by any measure.

The most dramatic changes for experienced bidders with the modified procedures (session 14) occurred between sessions rather than within the experienced subject session. There was some reduction in the frequency of bidding above value on unit 1 from 60.6% as inexperienced bidders to

[44]The one no change bidder was playing equilibrium as an inexperienced bidder.

40-45% as experienced bidders, with these changes channeled into both more bids equal to value and more bids below value. Further there was a marked increase in the frequency of demand reduction with respect to unit 2 going from inexperienced to experienced bidders (59.1% to 75-80%). Although these changes were relatively uneven across individuals, the net effect is a sharp reduction in the difference between average actual profits and expected profits from optimal play of 37¢ per auction in the last 12 auctions as inexperienced compared to experienced bidders ($p < .15$, one-tailed Wilcoxon signed rank test).[45]

The results of this section can be summarized as follows: Inexperienced bidders in uniform price sealed bid auctions undergo some convergence towards equilibrium play within an experimental session. Returning bidders show even closer convergence to equilibrium play. The most dramatic improvement here occurred in session 13 following the switch form 5 to 3 computer rivals. We conjecture that the increased incentive to demand reduction in auctions with 3 computer rivals motivated bidders to further readjust their bidding strategies. In contrast, in auctions where subjects have experience with 3 computer rivals, there is no comparable jolt to abandoning established strategies in switching to 5 computer rivals, so that continued adjustments to equilibrium are non-existent or more gradual in nature.

One final result worth discussing comes from two uniform price sealed bid auctions with bidders whose prior experience was with uniform price clock auctions or dynamic Vickrey auctions. The most dramatic difference between these "clock" bidders and those whose prior experience was with uniform

[45]This just misses statistical significance at the 10% level: test statistic = 39, critical value = 40.

price sealed bid auctions is the substantially higher conformity to the dominant bidding strategy for unit 1 bids as experienced bidders: 91.7% versus 70.5% ($p < .05$, one-tailed, Mann-Whitney test).[46] However, these differences in prior experience have essentially no impact on overall levels of demand reduction as experienced bidders (76.0% versus 77.4%). The net result is a relatively high percentage of "clock" subjects earning within 5% or maximum predicted profits - 44.4% (16/36) with $n = 3$ and 58.3% (21/36) with $n = 5$. Finally, although the data show that bidders with uniform price clock experience come closer, on average, to optimal predicted profits compared to bidders with prior experience with the dynamic Vickrey auction, these differences are not significant at conventional levels for $n = 5$ and only marginally significant for $n = 3$ ($p < .10$, one-tailed, Mann-Whitney test).

IV. Summary and Conclusions

The present experiment explores behavior in multi-unit demand auctions when bidders have non-increasing demand for homogeneous units. Our auctions are the simplest possible while still capturing the essential strategic tradeoffs involved in the different institutions under this demand structure: A single individual demands 2 units and competes against varying numbers of rivals with single unit demands represented by computers who follow the dominant strategy of bidding their value. With supply of 2 units, in the uniform price auctions bidders demanding two units maximize expected earnings by bidding their value on unit 1 and bidding so as *not* to win unit 2. In contrast, in the dynamic Vickrey auction there should be full demand revelation on both units, thereby increasing both expected efficiency

[46] Comparisons are based on the last 12 auctions with $n = 3$ in sessions 12 and 13 versus the last 12 auctions with $n = 3$ in the "cross-over" sessions. The latter employed a structure similar to session 13: 15 periods for cash with $n = 5$, followed by 23 periods with $n = 3$, followed by 10 periods with $n = 5$. Comparable differences are found in the last 10 auctions with $n = 5$.

and revenue compared to the uniform price auctions.

As the theory predicts, we observe clear and unambiguous demand reduction in the uniform price auctions, with demand reduction sharply limited in the dynamic Vickrey auction. This demonstrates that even relatively unsophisticated bidders are sensitive to the strategic implications of these different auction institutions.

An ascending bid uniform price clock auction with feedback regarding drop out prices generates outcomes that are closer to equilibrium than does a uniform price sealed bid auction, even though both auctions have the same normal form representation. We explore the basis for these differences by conducting ascending bid clock auctions with no feedback about drop-out prices, and sealed bid auctions in which bidders are provided with the critical drop-out information we hypothesize they employ in the clock auctions with feedback. Outcomes in the clock auction with no feedback are essentially the same as those reported in the sealed bid auctions. This rules out a simple "clock enhances learning" hypothesis to explain the differences. Sealed bid auctions with the second highest computer value announced begin to approach behavior in the clock auctions with feedback once the environment is structured so that the information inherent in announcing the computer's value is more salient. However, there is not as much demand reduction on unit 2 bids as in the clock auctions with feedback. This rules out the hypothesis that simply providing bidders with the relevant information to reduce the computational complexity of the problem will help them to get it right. The picture that emerges is that two factors *interact* to generate the differences between auction formats. The clock with feedback provides more than just information regarding the second highest computer value. It both provides the information and effectively induces bidders to pay attention to the information, and to recognize the role the information can play as an aid in the decision making process. Finally, in the clock auctions with feedback, this information is not absorbed immediately as it takes some practice before bidders get it right. And by repeatedly forcing bidders to decide whether to stay in or get out, the clock appears to enhance this experience effect.

Our investigation of the role of the clock and information feedback on bidding in the uniform price auctions has potential implications for the effectiveness of alternative forms of the Vickrey auction. Our results suggest that the dynamic Vickrey auction with feedback will outperform a sealed bid Vickrey auction, or a dynamic Vickrey auction without feedback. Preliminary results from an ongoing experiment confirm this prediction (Kagel, Kinross, and Levin, in preparation). For private value auctions, the primary contribution of the Ausubel version of the Vickrey auction is that it provides an English clock analogue for the multiple unit demand case. Consequently, if our preliminary results supporting the superior performance of a dynamic Vickrey auction with feedback hold up, the dynamic Vickrey/Ausubel auction would represent a real contribution to the applied implementation literature.

Bearing in mind that it is always treacherous to extrapolate laboratory results to field settings, given the many differences between the two environments, the behavioral regularities observed in our auctions provide some potential implications for auction design in field settings.[47] The uniform price sealed bid auction generated efficiency losses relative to the ascending bid Vickrey auction, but more revenue than Vickrey (in contrast to the theory which predicts *less* revenue). Further, dropping subjects who consistently bid above value on unit 1 as showing first order “irrationality” that, arguably,

[47]As noted earlier, this is, perhaps, particularly treacherous in the present case since all human interactions in different institutions might “set of” different adjustment processes, resulting in behavior converging to a different outcome. Nevertheless, we believe the data are suggestive of likely outcomes with all human bidders.

one would not expect to observe in field settings, the uniform price sealed bid auction raised about the same revenue as the dynamic Vickrey auction, with minimal efficiency losses relative to Vickrey. The latter results from the tendency to bid less strategically than the theory predicts, thereby overrevealing demand on unit 2. As a result, there is a potential tradeoff between revenue and efficiency, unanticipated theoretically, between the dynamic Vickrey auction and the uniform price sealed bid auction.[48]

[48]Some economists have pointed out that such tradeoffs between revenue and efficiency are relevant from a broader policy perspective given that alternative sources of revenue (namely taxes) create efficiency distortions (Rothkopf and Harstad, 1994). On the other hand, to the extent that a uniform price sealed bid auction raises more revenue than the more efficient Vickrey auction through promoting irrational overbidding, it may in the long run have negative economic consequences through promoting reneging on bids, tying up government assets in court proceedings, and delays in new technologies coming on line.

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Table 1
Earnings, Efficiency and Revenue Effects of Proportionate Bidding Strategies in Uniform Price Auctions

Bid proportion[a]	Earnings per auction (dollars)		Efficiency (percentage)		Revenue per auction (dollars)		Frequency of earning two items	
á	*n* = 3	*n* = 5	*n* = 3	*n* = 5	*n* = 3	*n* = 5	*n* = 3	*n* =5
0.0	1.112	0.529	96.90	98.62	6.010	8.566	0.000	0.000
0.2	1.110	0.529	96.99	98.62	6.037	8.567	0.002	0.000
0.4	1.094	0.528	97.50	98.68	6.188	8.583	0.017	0.002
0.6	1.026	0.518	98.40	98.95	6.515	8.673	0.054	0.013
0.8	0.928	0.474	99.45	99.53	6.989	8.909	0.126	0.055
1.0	0.743	0.354	100.0	100.0	7.492	9.270	0.248	0.165

[a] Assumes $b_1 = v_h$ for unit 1 and $b_2 = \acute{a} v_h$ for unit 2

TABLE 2
Experimental Sessions

Institution	Session	Number of Computers	Number of Subjects
Uniform Price- Standard Sealed Bid	1	3	14
	2	3	15[e]
	3	5	15
Uniform Price- Clock with Feedback	4	3	14
	5	5	16
Uniform Price- Clock with No Feedback	6	3	18
Uniform Price- Sealed Bid with v_2 Announced	7	3	20
	8	3	14
Dynamic Vickrey/Ausubel	9	3 per 1-13 5 per 14-27	14
	10	5 per 1-13 3 per 14-27	13
Uniform Price - Standard Sealed Bid with Modified Procedures	11	3	20
Uniform Price - Standard Sealed Bid with Experienced Bidders	12	3 per 1-13 5 per 14-34	16[b]
	13	5 per 1-13 3 per 14-39 5 per 40-49	12[c]
	14	3 per 1-13 5 per 14-34 3 per 35-46	11[d]

Supply = 2 units in all sessions.

All sessions had starting capital balances of $5 except for session 1 which had $3 starting balance.

Sessions 1-11 employed inexperienced bidders.

[a] One subject with large negative cash balance left before session ended.
[b] Subjects from Sessions 1 and 2
[c] Subjects from Session 3
[d] Subjects from Session 11

Table 3

Bidding in Standard Uniform Price Sealed Bid Auctions (last 12 auctions)								
Number of Computer Rivals	Unit 1 bids			Unit 2 bids				
	Bid frequencies relative to v_h			Bid Frequencies relative to v_h			Frequency $b_2 = 0$[b]	Frequency b_2 is pivotal[c]
	$b_1 > v_h$	$b_1 = v_h$[a]	$b_1 < v_h$	$b_2 > v_h$	$b_2 = v_h$[a]	$b_2 < v_h$		
$n = 3$	26.5% (89/336)	57.4% (193/336)	16.1% (54/336)	15.5% (52/336)	22.9% (77/336)	61.6% (207/336)	22.6% (76/336)	31.3% (105/336)
$n = 5$	42.8% (77/180)	53.3% (96/180)	3.9% (7/180)	21.7% (39/180)	17.2% (31/180)	61.1% (110/180)	13.9% (25/180)	30.0% (54/180)
Equilibrium Outcome	$b_1 = v_h$			$b_2 = 0$			100%	0%

[a] Bidding within 5¢ of value

[b] Bids # 5¢

[c] Pivotal bids exceed the 2nd highest computer value, thereby directly impacting on the market price.

Table 4
Revenue and Efficiency: Standard Uniform Price Sealed Bid Auctions
(mean values with standard error of the mean in parentheses)

All Subjects

Session (number of computer rivals)	Efficiency (percentage)			Revenue (dollars)		
	Actual	Predicted	Difference (actual less predicted)	Actual	Predicted	Difference (actual less predicted)
1 (*n* = 3)	98.29 (0.723)	97.30 (0.253)	1.006 (0.756)	6.864 (0.325)	5.938 (0.115)	0.926 (0.224)
2 (*n* = 3)	95.36 (0.917)	96.72 (0.293)	-1.352 (0.994)	7.236 (0.345)	6.017 (0.095)	1.210 (0.351)
3 (*n* = 5)	98.19 (0.831)	98.46 (0.164)	-0.271 (0.869)	9.441 (0.218)	8.649 (0.086)	0.792 (0.177)
Pooled	97.30 (0.511)	97.51 (0.176)	-0.207 (0.516)	7.884 (0.244)	6.913 (0.204)	0.972** (0.148)
Excluding Subjects who consistently Bid Above Value on Unit 1						
Session						
1 (*n* =3)	99.04 (0.257)	97.35 (0.292)	1.690 (0.449)	6.682 (0.338)	5.900 (0.129)	0.781 (0.226)
2 (*n* = 3)	97.37 (0.629)	96.39 (0.451)	0.984 (0.512)	6.595 (0.299)	6.157 (0.146)	0.438 (0.229)
3 (*n* = 5)	99.42 (0.123)	98.40 (0.238)	1.019 (.288)	9.102 (0.242)	8.631 (0.123)	0.471 (0.125)
Pooled	98.70 (0.253)	97.41 (0.233)	1.280** (0.250)	7.409 (0.276)	6.818 (0.242)	0.643** (0.119)

+ Significantly different from zero at the 10% level, two-tailed, Wilcoxin ranked sign test using average subject values as the unit of observation.

* Significantly different from zero at the 5% level, two-tailed, Wilcoxin ranked sign test using average subject values as the unit of observation.

Table 5

Bidding in Uniform Price Clock Auctions with Feedback (last 12 Auctions)				
		Unit 1 bids	Unit 2 bids	
Number of Computer Rivals		$b_1 \$ v_2$	$b_2 > v_2$[a]	Frequency b_2 is pivotal[b]
$n = 3$	$v_2 > v_h$	6.5% (6/93)	0.0% (0/93)	10.1% (17/168)
	$v_h \$ v_2$	___	22.7% (17/75)	
$n = 5$	$v_2 > v_h$	8.7% (11/126)	3.2% (4/126)	12.5% (24/192)
	$v_h \$ v_2$	___	30.3% (20/66)	
Pooled	$v_2 > v_h$	7.8% (17/219)	1.8% (4/219)	11.4% (41/360)
	$v_h \$ v_2$	___	26.2% (37/141)	
Equilibrium Outcome		0%	0%	0%

[a] In the case of the $v_h \$ v_2$ we employ a 5¢ "allowance"; $b_2 \# v_2 + .05$.
[b] Pivotal bids exceed the 2nd highest computer value, thereby directly impacting on the market price.
v_2 = second highest computer bid.

Table 6

Revenue and Efficiency in Uniform Price Clock Auctions with Feedback
(mean values with standard error of the mean in parentheses)

Session (number of computer rivals)	Efficiency			Revenue		
	Actual	Predicted	Difference	Actual	Predicted	Difference
1 (*n* = 3)	97.4 (0.537)	97.8 (0.377)	-0.392 (0.476)	5.99 (0.247)	5.67 (0.149)	0.320 (0.169)
2 (*n* = 5)	98.3 (0.538)	99.2 (0.220)	-0.885 (0.533)	9.13 (0.227)	8.93 (0.233)	0.201 (0.052)
Pooled	97.87 (0.382)	98.53 (0.242)	-0.656+ (0.357)	7.67 (0.334)	7.41 (0.332)	0.257** (0.083)

+ Significantly different from zero at the 10% level, two-tailed, Wilcoxin ranked sign test using average subject values as the unit of observation.

** Significantly different from zero at the 1% level, two-tailed, Wilcoxin ranked sign test using average subject values as the unit of observation.

Statistical tests restricted to the pooled data.

Table 7

Bidding in Uniform Price Clock Auctions with No Feedback (last 12 auctions)								
Number of Computer Rivals	Unit 1 bids			Unit 2 bids				
	Bid Frequencies relative to v_h			Bid Frequencies relative to v_h			Frequency $b_2 = 0$[b]	Frequency b_2 is pivotal[c]
	$b_1 > v_h$	$b_1 = v_h$[a]	$b_1 < v_h$	$b_2 > v_h$	$b_2 = v_h$[a]	$b_2 < v_h$		
$n = 3$	43.1% (93/216)	38.9% (84/216)	18.1% (39/216)	22.7% (49/216)	15.7% (34/216)	61.6% (133/216)	5.1% (11/216)	41.2% (89/216)
Equilibrium Outcome	$b_1 = v_h$			$b_2 = 0$			100%	0%

[a] Bidding within 5¢ of value
[b] Bids # 5¢
[c] Pivotal bids exceed the 2nd highest computer value (by more than 5¢), thereby directly impacting on the market price.

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Table 8

Bidding in uniform price sealed bid auctions with v_2 announced (last 12 auctions)				
Number of Computer Rivals		Unit 1 bids	Unit 2 bids	Frequency b_2 is pivotal[b]
		$b_1 \; \$ \; v_2$	$b_2 > v_2$[a]	
Session 7 $n = 3$	$v_2 > v_h$	22.1% (27/122)	9.8% (12/122)	29.2% (70/240)
	$v_h \; \$ \; v_2$	-------	49.2% (58/118)	
Session 8 $n = 3$	$v_2 > v_h$	3.5% (3/86)	1.2% (1/86)	25.0% (42/168)
	$v_h \; \$ \; v_2$	---------	50.0% (41/82)	
Equilibrium Outcome		0%	0%	0%

[a] In the case of the $v_h \; \$ \; v_2$ we employ a 5¢ "allowance"; $b_2 \; \# \; v_2 + 0.05$.
[b] Pivotal bids exceed the 2nd highest computer value, thereby directly impacting on the market price.
v_2 = second highest computer bid.

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Table 9
Bidding in Dynamic Vickrey/Ausubel Auctions

Number of Computer Rivals	**Unit 1 Bids**			**Unit 2 bids**		
	Clinch at $p > v_h$	$b_1 > v_h$ & $b_1 > v_3$ [a,b]	$b_1 < v_h$ [a,c]	Clinch at $p > v_h$	$b_2 > v_h$ & $b_2 > v_2$ [a,d]	$b_2 < v_h$ [a,e]
$n = 3$	4.5% (7/156)	26.8% (19/71)	18.3% (13/71)	1.3% (2/156)	3.2% (4/124)	29.8% (37/124)
$n = 5$	1.8% (3/168)	4.6% (5/108)	8.3% (9/108)	0.6% (1/168)	1.5% (2/135)	13.3% (18/135)
Equilibrium Outcome	0%	0%	0%	0%	0%	0%

[a] Base excludes all clinched units.
[b] $b_1 > v_h + 5¢$.
[c] $b_1 < v_h - 5¢$.
[d] $b_2 > v_h + 5¢$.
[e] $b_2 < v_h - 5¢$.

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Table 10

Revenue and Efficiency in Dynamic Vickrey/Ausubel Auctions

(mean values with standard error of the mean in parentheses)

Session (number of computer rivals)	Efficiency (percentage)			Revenue (dollars)		
	Actual	Predicted	Difference	Actual	Predicted	Difference
1 (*n* = 5)	99.9 (0.073)	100	-0.113 (0.073)	9.21 (0.204)	9.18 (0.200)	0.032 (0.021)
2 (*n* = 3)	98.6 (0.749)	100	-1.42 (0.749)	6.77 (0.216)	6.76 (0.219)	0.009 (0.216)
Pooled	99.26 (0.378)	100	-0.742 (0.377)	8.03 (0.280)	8.01 (0.278)	0.021 (0.102)

Table 11

Bidding in Standard Uniform Price Sealed Bid Auctions: Effects of "Pro-Equilibrium" Procedures (last 12 auctions)								
Procedures	Unit 1 bids			Unit 2 bids				
	Bid frequencies relative to v_h			Bid Frequencies relative to v_h			Frequency $b_2 = 0$[b]	Frequency b_2 is pivotal[c]
	$b_1 > v_h$	$b_1 = v_h$[a]	$b_1 < v_h$	$b_2 > v_h$	$b_2 = v_h$[a]	$b_2 < v_h$		
Modified	48.3% (116/240)	26.7% (64/240)	25.0% (60/240)	26.7% (64/240)	11.3% (27/240)	62.1% (149/240)	15.8% (38/240)	34.2% (82/240)
Original	26.5% (89/336)	57.4% (193/336)	16.1% (54/336)	15.5% (52/336)	22.9% (77/336)	61.6% (207/336)	22.6% (76/336)	31.3% (105/336)
Equilibrium Outcome	$b_1 = v_h$			$b_2 = 0$			100%	0%

[a] Bidding within 5¢ of value

[b] Bids # 5¢

[c] Pivotal bids exceed the 2nd highest computer value, thereby directly impacting on the market price.

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Table 12
Effects of Experience on Bidding in Standard Uniform Price Sealed Bid Questions: Inexperienced Bidders

Treatment (session number)	Auctions	Unit 1 bids			Unit 2 bids				
		Bid frequencies relative to v_h			Bid frequencies relative to v_h			Frequency $b_2 = 0$	Frequency b_2 is pivotal
		$b_1 > v_h$	$b_1 = v_h$	$b_1 < v_h$	$b_2 > v_h$	$b_2 = v_h$	$b_2 < v_h$		
Original									
n=3 (102)	First 13	36.6% (128/350)	41.1% (144/350)	22.3% (78/350)	24.6% (86/350)	14.0% (49/350)	61.4% (215/350)	14.0% (49/350)	37.1% (130/350)
	Last 12	26.5% (89/336)	57.4% (193/336)	16.1% (54/336)	15.5% (52/336)	22.9% (77/336)	61.6% (207/336)	22.6% (76/336)	31.3% (105/336)
n=5 (3)	First 13	47.2% (92/195)	43.1% (84/195)	9.7% (19/195)	19.5% (38/195)	17.4% (34/195)	63.1% (123/195)	5.1% (10/195)	25.6% (50/195)
	Last 12	42.8% (77/180)	53.3% (96/180)	3.9% (7/180)	21.7% (39/180)	17.2% (31/180)	61.1% (110/180)	13.9% (25/180)	30.0% (54/180)
Modified (11)	First 13	48.5% (126/260)	27.7% (72/260)	23.8% (62/260)	29.2% (76/260)	21.2% (55/260)	49.6% (129/260)	6.5% (17/260)	38.5% (100/260)
	Last 12	48.3% (116/240)	26.7% (64/240)	25.0% (60/240)	26.7% (64/240)	11.3% (27/240)	62.1% (149/240)	15.8% (38/240)	34.2% (82/240)

[a] Bidding within 5¢ of value
[b] Bids # 5¢
[c] Pivotal bids exceed the 2nd highest computer value, thereby directly impacting on the market price.

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Figure Captions

Figures 1-3: Scatter diagram of bids relative to value for bidder *h* in last 12 auctions of uniform price sealed bid sessions. Left panel: Unit 1 bids. Right panel: Unit 2 bids.

Figure 4: Individual subject bids in uniform price sealed bid auctions for different types of bidders (see text). Circles are unit 1 bids, squares are unit 2 bids.

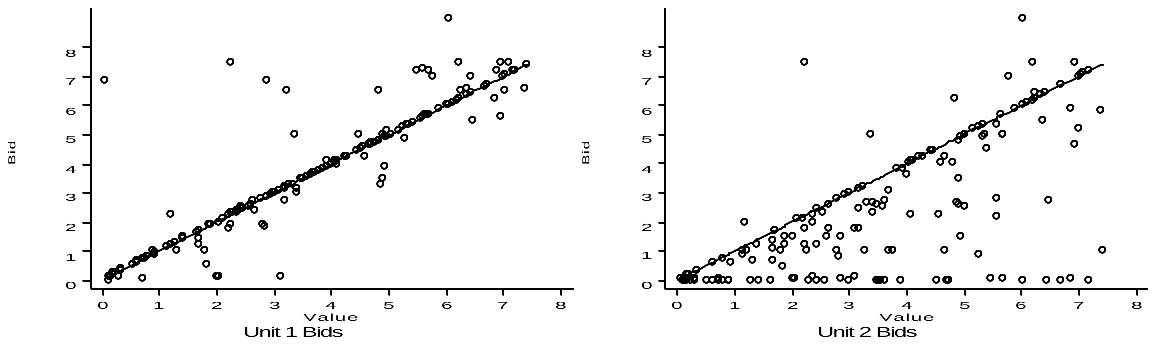
Figures 5-6: Scatter diagram of bids relative to value for bidder *h* in last 12 auctions of uniform price clock auctions with feedback on drop-out prices.

Top panel: Unit 1 bids. Circles are winning bids (these are censored). Squares are drop-outs at prices at or below resale value. Triangles are potentially harmful bids above resale value. Diamonds are harmless bids above resale value.

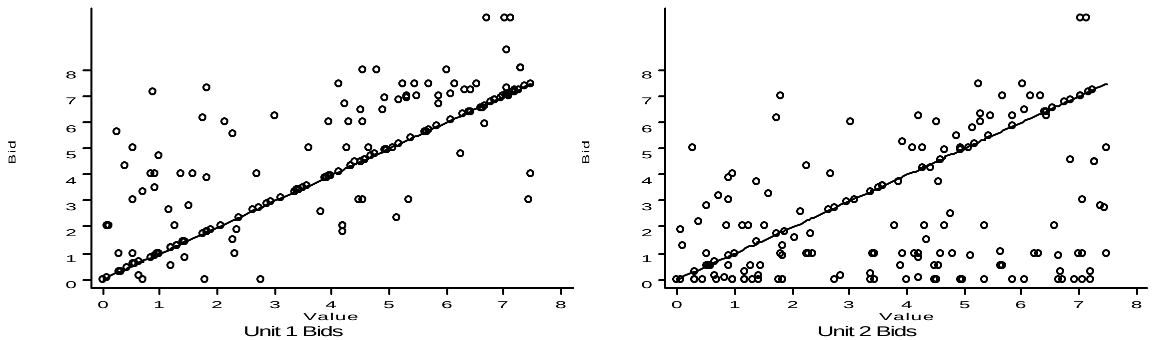
Bottom panel: Unit 2 bids. Circles are winning bids. Squares are drop-outs at or below v_2 (optimal bids). Drop outs that are pivotal are +'s. Diamonds are harmless bids above resale value.

Figure 7: Scatter diagram of bids relative to value for bidder *h* in last 12 auctions of uniform price clock auctions *without* feedback on drop-out prices.

Figures 8-9: Scatter diagram of bids relative to value for bidder *h* in last 12 auctions of Vickrey/Ausubel auctions. Top panel: Unit 1 bids. Bottom panel: Unit 2 bids. Circles are winning bids (these are censored). Squares are drop-outs at prices at or below resale value. Triangles are potentially harmful bids above value. Diamonds are harmless bids above value.



Session 1: Uniform Price Sealed Bid Auctions (n=3)



Session 2: Uniform Price Sealed Bid Auctions (n=3)

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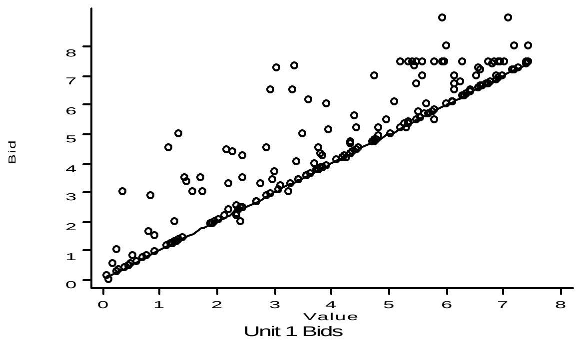

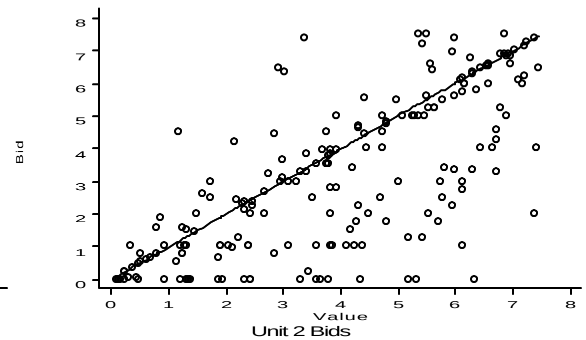

Session 3: Uniform Price Sealed Bid Auctions (n=5)

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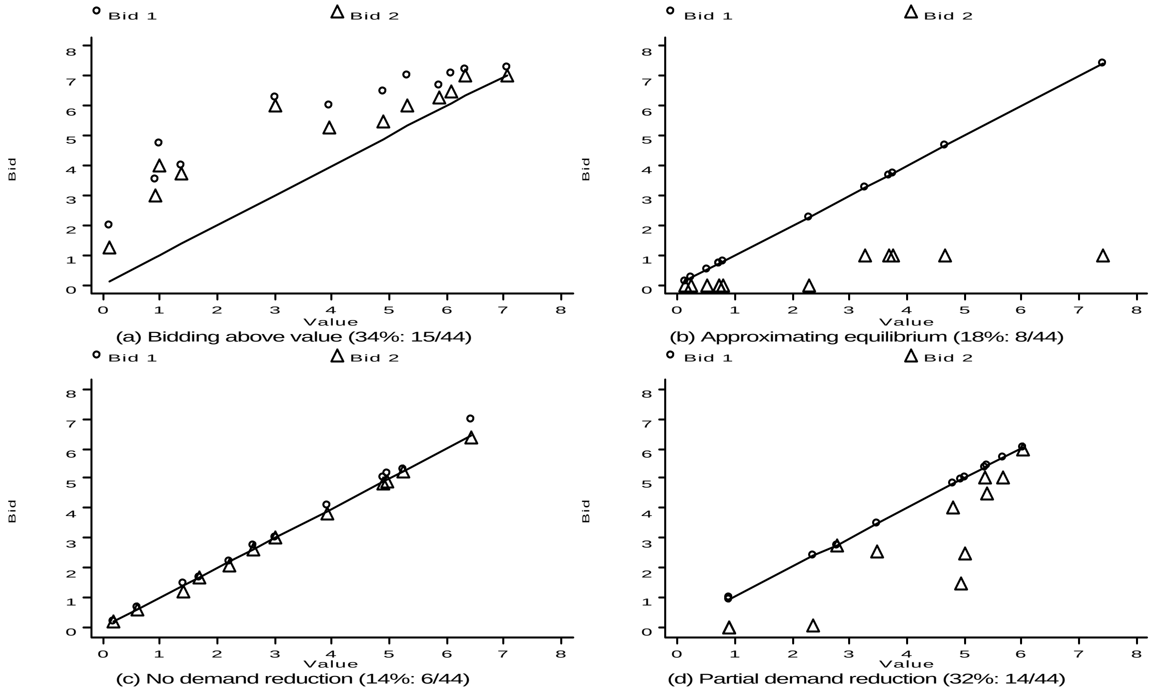
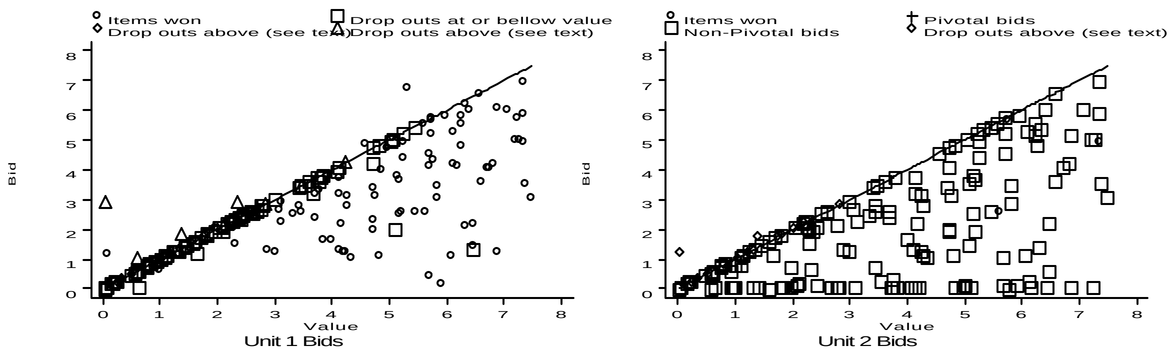
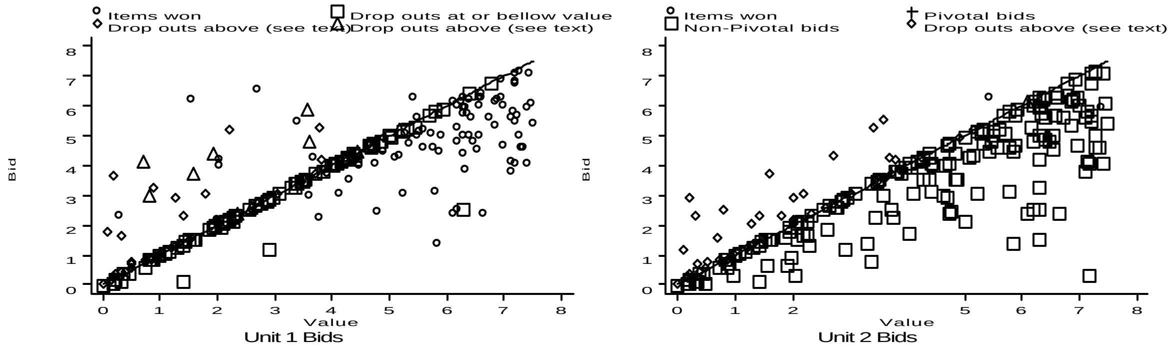


Individual subject bid patterns



Session 4: Uniform Price Clock Auction (n=3)



Session 5: Uniform Price Clock Auctions (n=5)

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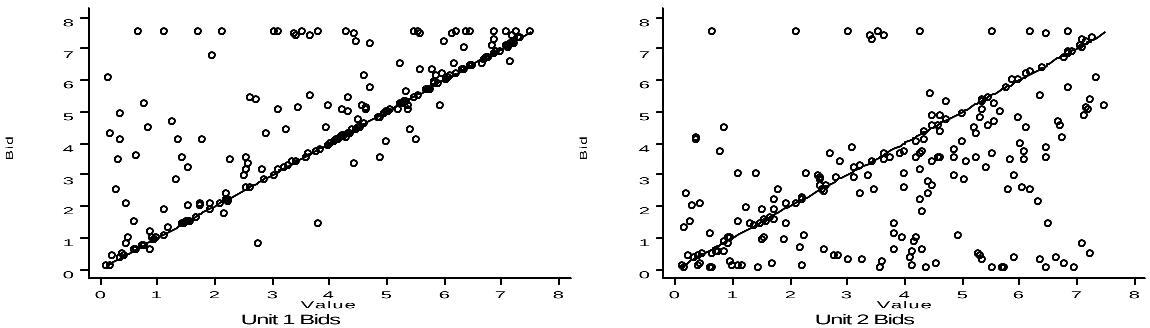

Session 6: Uniform Price Clock Auctions Without Drop Out Information
(n=3)

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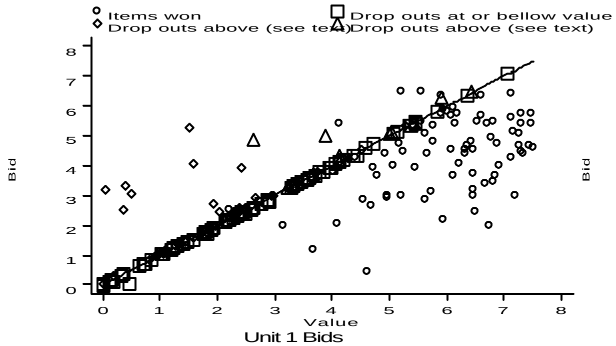

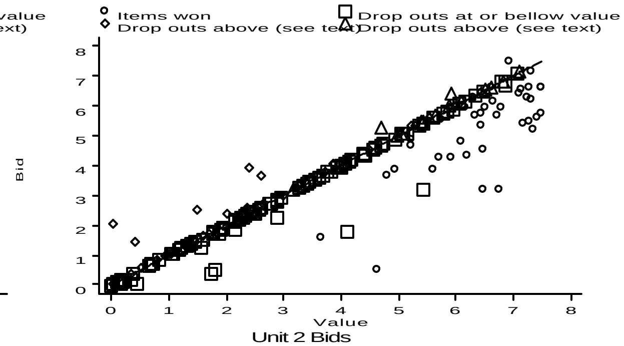

Session 9: Dynamic Vickrey/Ausubel Auctions (n=5)

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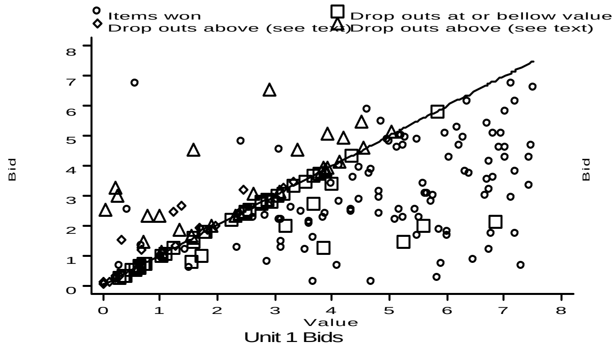

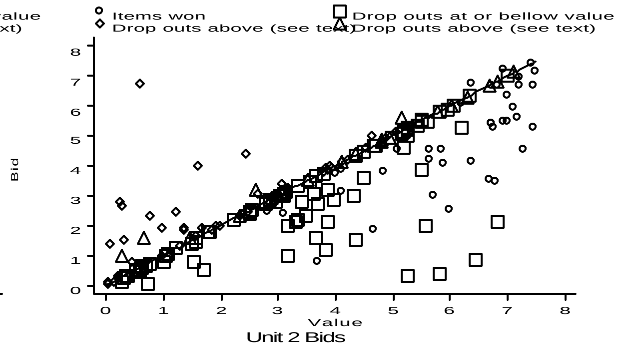

Session 10: Dynamic Vickrey/Ausubel Auctions (n=3)

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