TECHNICAL MEMORANDUM

Moist Processes Lecture Note Series

The parametrization of cloud cover

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1 Introduction

When considering the approach to model clouds in general circulation models (GCMs), there are a number of zero order issues that require attention, in addition to the representation of the complex warm phase and ice phase microphysics processes that govern the growth and evolution of cloud and precipitation particles.

Unlike cloud resolving models (CRMs) or large-eddy models (LEMs), which, having grid resolutions finer than O(1km), aim to resolve the motions relevant for the clouds under consideration, GCMs must additionally consider macroscopic geometrical effects. Claiming to resolve cloud scale motions allows CRMs and LEMs to make the assumption that each grid scale is completely cloudy if condensate is present. This approach is clearly not adequate for GCM size grid scale of O(100km) for which clouds are a subgrid-scale phenomenom, (although some schemes such as Ose, 1993; Fowler et al., 1996, have indeed adopted this approach).

GCMs must therefore consider cloud geometrical effects. To reduce the fractal cloud to a tractable low dimensional object, GCMs usually reduce the problem to the specification of the:

- horizontal fractional coverage of the gridbox by cloud,
- vertical fractional coverage of the gridbox by cloud,
- sub-cloud variability of cloud variables in both the horizontal and vertical, and the
- overlap of the clouds in the vertical column

The above list is far from exhaustive, and implicitly neglects interactions between adjacent GCM columns (for example, how cloud affects solar fluxes in adjacent columns at low sun angles), probably a safe assumption for grid-scales exceeding 10km or so (Di Giuseppe and Tompkins, 2003)

In fact, most GCMs further simplify the above list (i) by assuming clouds fill GCM grid boxes in the vertical and (ii) by neglecting many of the consequences of sub-cloud fluctuations of cloud properties. Both of these are considerable simplifications. Although vertical GCM grids are much finer than the horizontal resolution, the same is of course also true of cloud processes. Using O(50) levels in the vertical implies that some cloud systems or microphysical related processes are barely if at all resolved, such as tropical thin cirrus (Dessler and Yang, 2003), or the precipitation melting layer (Kitchen et al., 1994), which can have important implications (Tompkins and Emanuel, 2000). Likewise, many authors have highlighted the biases that can be introduced when sub-cloud fluctuations are neglected, due to the strong nonlinearity of cloud and radiative processes (Cahalan et al., 1994; Barker et al., 1999; Pincus and Klein, 2000; Pomroy and Illingworth, 2000; Fu et al., 2000; Rotstayn, 2000; Larson et al., 2001).

Nevertheless, the zero order primary task of cloud schemes, in addition to representing the microphysics of clouds, is to predict the horizontal cloud coverage. It is clear that a utopian perfect microphysical model will render poor results if combined with an inaccurate predictor of cloud cover, due to the incorrect estimate of in-cloud liquid water. This lecture presents the general approaches used to date in GCMs for this task.

2 Background

The first thing to realize is that fractional cloud cover can *only* occur if there is horizontal subgrid-scale variability in humidity and/or temperature (controlling the saturation mixing ratio, q_s). If temperature and humidity

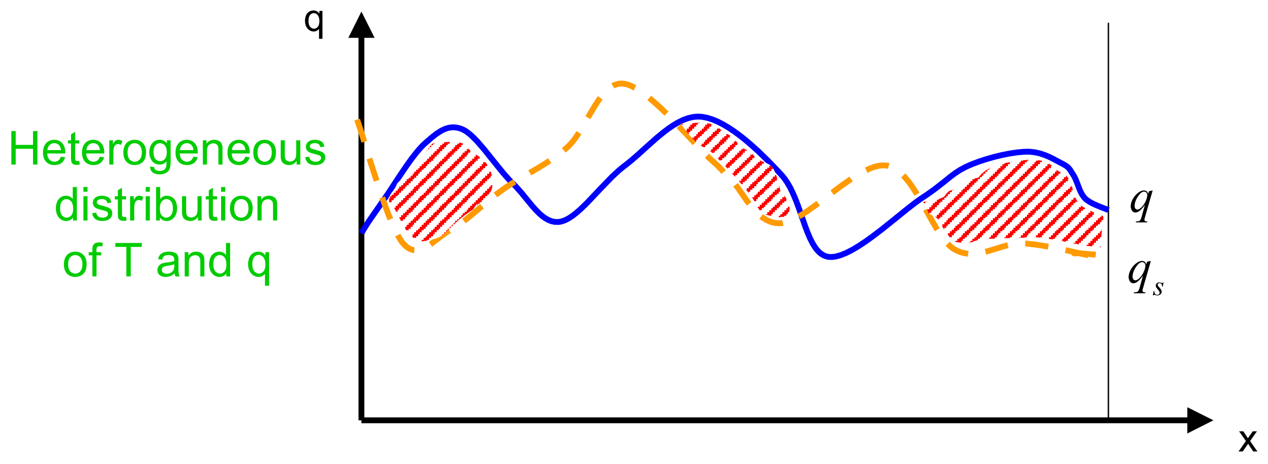

Figure 1: Schematic showing that partial cloud cover in a gridbox in only possible if temperature or humidity fluctuations exist. The blue line shoes humidity and the yellow line saturation mixing ratio across an arbitrary line representing a gridbox. If all supersaturation condenses as cloud then the shaded regions will be cloudy.

are homogeneous, then either the whole grid box is subsaturated and clear, or supersaturated and cloudy[1].

This is illustrated schematically in Fig. 1. Fluctuations in temperature and humidity may cause the humidity to exceed the saturated value on the subgrid scale. If it assumed that all this excess humidity is immediately converted to cloud water (and likewise that any cloud drops evaporate instantly in subsaturated conditions), then it is clear that the grid-mean relative humidity ($\overline{RH}$, where the overline represents the gridbox average) must be less unity if the cloud cover is also less than unity, since within the cloudy parts of the gridbox $RH = 1$ and in the clear sky $RH < 1$. Generally speaking, since clouds are unlikely when the atmosphere is dry, and since RH is identically 1 when $C = 1$, there is likely to be positive correlation between RH and C.

The main point to emphasize is that, *all* cloud schemes that are able to diagnose non-zero cloud cover for $\overline{RH} < 1$ (i.e. any scheme other than an "all-or-nothing" scheme) must make an assumption concerning the fluctuations of humidity and/or temperature on the subgrid-scale, as in Fig. 1. Either (i) they will *explicitly* give the nature of these fluctuations, most usually by specifying the probability density function (PDF) for the total water at each gridcell, or (ii) they will *implicitly* assume knowledge about the time-mean statistics of the fluctuations (i.e. the actual PDF at each grid point is maybe not known).

It is important to recall, when trying to categorize the seemingly diverse approaches to cloud cover parametrization, that *this central fact ties all approaches together.*

3 Relative humidity schemes

Relative humidity schemes are called such because they specify a diagnostic relationship between the cloud cover and the relative humidity.

In the last section we saw that subgrid-scale fluctuations allow cloud to form when $\overline{RH} < 1$. RH schemes formulize this by setting a critical RH (denoted RH_{crit}) at which cloud is assumed to form, and then increase C

[1]For simplicity, throughout this text we ignore the subtle complication of the ice phase, where supersaturations are common (Heymsfield et al., 1998; Gierens et al., 2000; Spichtinger et al., 2003)

according to a monotonically increasing function of RH, with C=1 identically when RH=1.

One commonly used function was given by Sundqvist et al. (1989):

$$C = 1 - \sqrt{\frac{1-RH}{1-RH_{crit}}} \tag{1}$$

Thus it is apparent that RH_{crit} defines the magnitude of the fluctuations of humidity (the humidity variance). If RH_{crit} is small, then the subgrid humidity fluctuations must be large, since cloud can form in dry conditions.

It is clear that one of the drawbacks of this type of scheme is that the link between cloud cover and local dynamical conditions is vague. Convection will indeed produce cloud if its local moistening effect is sufficient to increase RH past the critical threshold, but it is apparent that a grid cell with 80% RH undergoing deep convection is likely to have different cloud characteristics than a gridcell with 80% RH in a frontal stratus cloud. RH schemes simply state that, averaged across all conditions across the globe, a gridcell with X% RH will have Y% cloud cover.

This lack of differentiation between different local conditions lead some authors to augment their RH schemes. The ECHAM4 climate model Roeckner et al. (1996) augments the cloud cover in the presence of a strong temperature inversion to improve the representation of stratocumulus.

Other authors augment their schemes by using additional predictors to RH. The Slingo (1980, 1987) scheme was used operationally in ECMWF until its replacement by the Tiedtke (1993) scheme in 1995, and as of April 2005 is still used in the Tangent linear and adjoint computations of the 4D-Var inner-loops (although the aim is to replace this in the future by Tompkins and Janisková, 2004). The basic form for the mid-level cloud cover (C_{mid}) is given as

$$C^*_{mid} = \left(\frac{RH - RH_{crit}}{1 - RH_{crit}}\right)^2, \tag{2}$$

but Slingo modifies this according to an additional predictor, the vertical velocity at 500 hPa (ω_{500}), thus

$$C_{mid} = C^*_{mid}\frac{\omega_{500}}{\omega_{crit}}, \tag{3}$$

if $0 > \omega_{500} > \omega_{crit}$ while the cloud cover is set to zero if subsidence is occurring ($\omega_{500} > 0$).

Likewise Xu and Randall (1996) used a cloud resolving model (CRM) to derive an empirical relationship for cloud cover based on the two predictors of RH and cloud water content:

$$C = RH^p\left[1 - exp\left(\frac{-\alpha_0\overline{q_l}}{(q_s - q_v)^\gamma}\right)\right], \tag{4}$$

where γ, α_0 and p are 'tunable' constants of the scheme, with values chosen using the CRM data. One weakness of such a scheme is, of course, this dependence on the reliability of the CRM's parametrizations, in particular the microphysics scheme. Additionally, it is unlikely that the limited set of (convective) cases used as the training dataset used would encompass the full range of situations that can naturally arise, such as cloud in frontal systems for example.

While these latter schemes use additional predictors for cloud cover, we shall still refer to them as "relative humidity" schemes, since the common and central predictor in all cases is RH. It is doubtful if any of the schemes could be reasonably simplified by replacing the RH dependence with a fixed value.

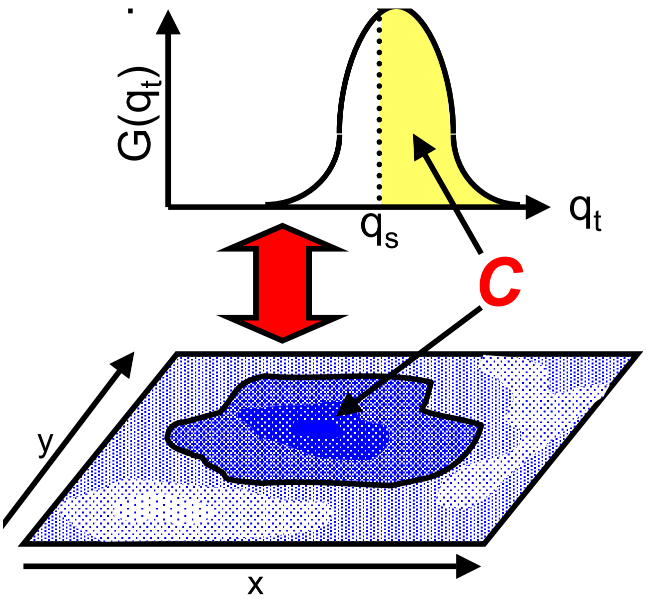

Figure 2: Schematic showing the statistical scheme approach. Upper panel shows an idealized PDF of total water (q_t). The vertical line represents the saturation mixing ratio $q_t = q_s$, thus all the points under the PDF to the right of this line are cloudy. The integral of this area translates to the cloudy portion of the gridbox, marked on the lower part of the figure, with darker shading schematically representing high total water values.

4 Statistical schemes

Instead of describing the spatial and temporal mean statistics of the humidity fluctuations such as the *RH* schemes, another group of schemes take a different approach, *by specifying the underlying distribution of humidity (and/or temperature) variability at each grid box*. This is shown schematically in Fig. 2. If the PDF form for total water q_t is known, then the cloud cover is simply the integral over the part of the PDF for which q_t exceeds q_s:

$$C = \int_{q_s}^{\infty} G(q_t) dq_t, \tag{5}$$

Likewise, the cloud condensate is given by

$$\overline{q_l} = \int_{q_s}^{\infty} (q_t - q_s) G(q_t) dq_t. \tag{6}$$

As always we are assuming that all supersaturation is immediately condensed as cloud. Here we are also ignoring temperature fluctuations for simplicity, but these can be included, as outlined in appendix A.

The main tasks of the statistic scheme is therefore to give a suitable form for the PDF of total water fluctuations, and to derive its defining moments.

4.1 Defining the PDF

Various distributions have been used, many of which are symmetrical. Smith (1990) uses a symmetric triangular PDF, diagnosing the variance based on a critical *RH* function at which cloud is determined to form, later modified by Cusack et al. (1999). This PDF has been subsequently adopted by Rotstayn (1997) and Nishizawa (2000). LeTreut and Li (1991) use a uniform distribution, setting the distribution's variance to an arbitrarily

Table 1: PDF forms used in statistical cloud schemes. In the summary column, the key is: U=unimodal, B=Bimodal, S=Symmetric, Sk=Skewed.

PDF Shape	Summary	Reference
Double Delta	U,S	Ose (1993); Fowler et al. (1996)
Uniform	U,S	LeTreut and Li (1991)
Triangular	U,S	Smith (1990); Rotstayn (1997); Nishizawa (2000)
Polynomial	U,S	Lohmann et al. (1999)
Gaussian	U,S	Bougeault (1981); Ricard and Royer (1993); Bechtold et al. (1995)
Beta	U,sk	Tompkins (2002)
Log-normal	U,sk	Bony and Emanuel (2001)
Exponential	U,Sk	Bougeault (1981); Ricard and Royer (1993); Bechtold et al. (1995)
Double Gaussian/Normal	B,Sk	Lewellen and Yoh (1993); Golaz et al. (2002)

defined constant. A Gaussian-like symmetrical polynomial function was used by Lohmann et al. (1999) with variance determined from the subgrid-scale turbulence scheme following Ricard and Royer (1993), who investigated Gaussian, exponential and skewed PDF forms. Bechtold et al. (1992) based their scheme on the Gaussian distribution, which was modified in Bechtold et al. (1995) to a PDF linearly interpolated between Gaussian and exponential distributions. Bony and Emanuel (2001) have introduced a scheme that uses a generalized Log-Normal distribution. Lewellen and Yoh (1993) detail a parameterization that uses a Bi-normal distribution that can be skewed as well as symmetrical and is bimodal, although a number of simplifying assumptions were necessary in order to make the scheme tractable. Likewise Golaz et al. (2002) also give a bimodal scheme. These forms are summarized in table 1 and drawn *schematically* in Fig. 3.

Examples of PDFs measured in the literature are shown in Fig. 4. Although it is difficult to theoretically derive a PDF form, since the q_t distribution is the result of a large number of interacting processes, therefore forcing the use of empirical methods, it is possible to use physically-based arguments to justify certain functional forms. For example, in the absence of other processes, large-scale dynamical mixing would tend to reduce both the variance and the asymmetry the distribution. Therefore, the Gamma and Lognormal distributions would be difficult to use since they are always positively skewed, and only tend to a symmetrical distributions as one of their defining parameters approaches infinity. Bony and Emanuel (2001) attempt to circumnavigate this by switching between Lognormal and Gaussian functions at a threshold skewness value.

Another problem that distributions such as the Lognormal, Gamma, Gaussian and Exponential suffer from is that they are all unbounded functions. Thus, if these functional forms are used, the maximum cloud condensate mixing ratio approaches infinity, and part of the grid cell is always covered by cloud. Precautionary measures, such as the use of a truncated function, can be taken, but this increases the number of parameters required to describe the distribution, and again introduces undesirable discreteness. Moreover, functions such as the Gaussian function or the polynomial used by Lohmann et al. (1999) are also negatively unbounded, implying that part of the gridcell has negative water mass. The choice of function must also involve a fair degree of pragmatism, since in addition to providing a good fit to the available data, it must also be sufficiently simple and of few enough degrees of freedom to be of use in a parameterization scheme. For example, Larson et al. (2001) were able to provide good fits to their aircraft data using a 5-parameter double Gaussian function, but it is unclear how these parameters would be determined in a GCM cloud scheme. The Beta distribution used by Tompkins (2002) is bounded and can provide both symmetrical and skewed distributions, but has the disadvantage of an upper limit on the skewness when the distribution is restricted to a sensible bell-shaped regime, and that the form is not mathematically as simple as alternative unimodal distributions.

Considering the question of whether a unimodal distribution is necessary, we refer to a number of observational

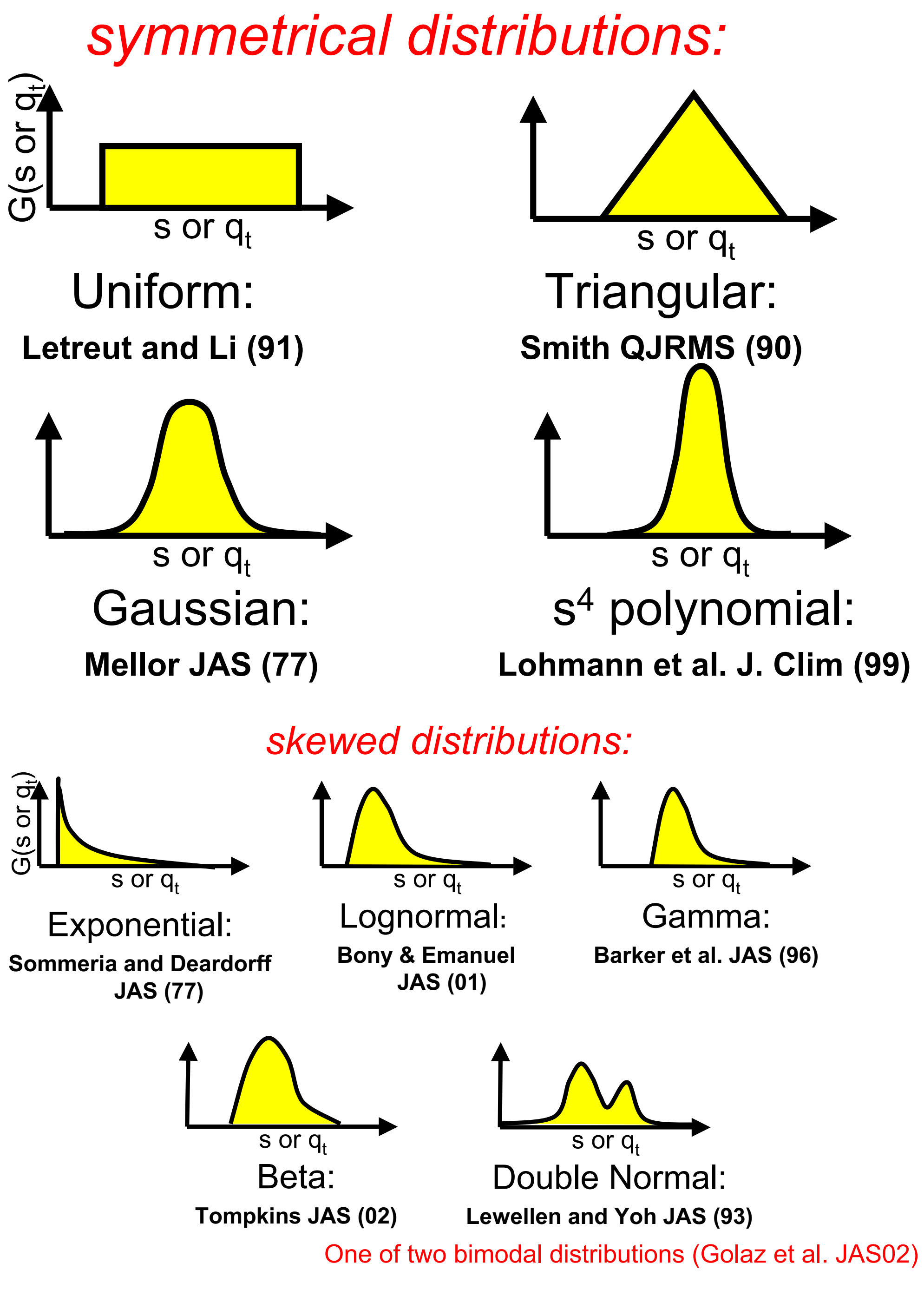

Figure 3: Schematic of PDF forms for $G(q_t)$ used to date, divided into symmetrical and skewed categories. The papers referred to are: LeTreut and Li (1991); Smith (1990); Mellor (1977); Lohmann et al. (1999); Sommeria and Deardorff (1977); Bony and Emanuel (2001); Barker (1996); Tompkins (2002); Lewellen and Yoh (1993). Note that Barker (1996) is not describing a cloud scheme, but a corrective mechanism for radiative biases that assumed this distribution for in-cloud water fluctuations.

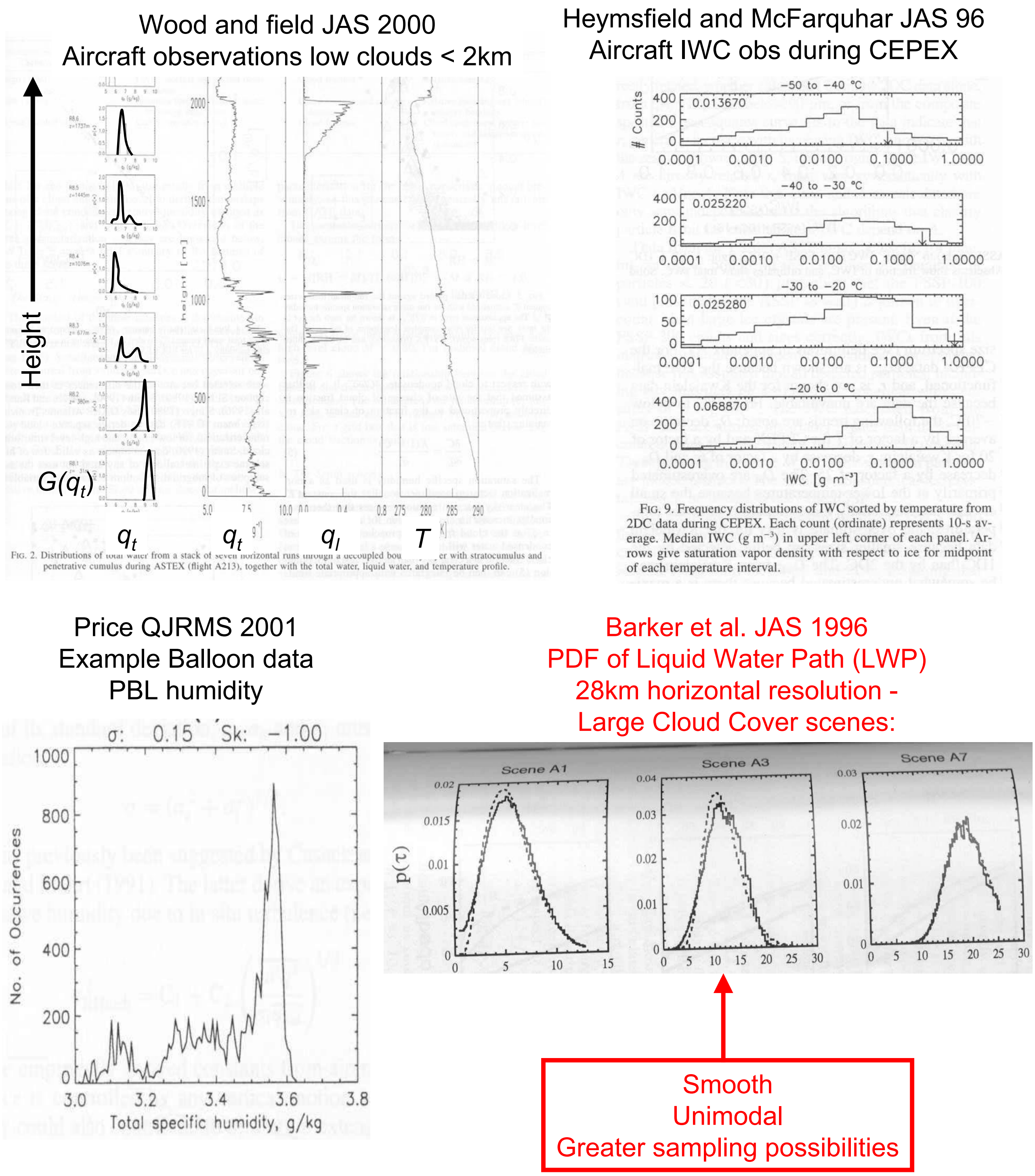

Figure 4: Reproduction of LWP, ice water content and total water PDFs from various observational studies. The papers referred to are: Wood and Field (2000); Heymsfield and McFarquhar (1996); Price (2001); Barker et al. (1996).

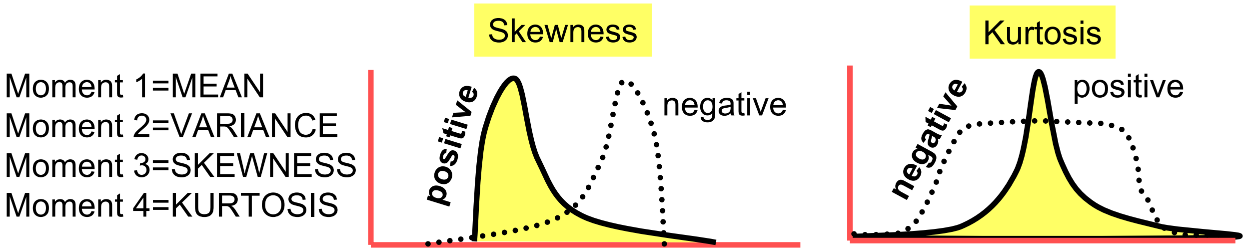

Figure 5: Schematic illustrating the 3rd and 4th moments; skewness and kurtosis

studies. Some of the data from the following studies is shown for illustrative purposes in Fig. 4. Ek and Mahrt (1991) examined PBL relative humidity variability in a limited number of flight legs, and assumed a unimodal Gaussian fit for their distribution. Recently, Wood and Field (2000) studied flight data from both warm and cold clouds and reported unimodal distributions of q_t, but also observing more complex distributions, giving some weakly and strongly bimodal examples. Davis et al. (1996) reported uni- or bi-modal skewed distributions in liquid water content from flight data in marine stratocumulus clouds. Larson et al. (2001) have also examined flight data for PBL clouds and found that mainly unimodal or bimodal distributions occurred. They reported that PDFs that included positive or negative skewness were able to give an improved fit the data. Price (2001) used tethered balloon data of PBL humidity collected during a three year period, finding that roughly half of the data could be classified as symmetrical or skewed unimodal. A further 25% of the data could be regarded as multimodal.

Although many of the above studies reported a significant frequency of occurrence of distributions classed as bi- or multi-modal, these distributions often possessed a single principle distribution peak, as in the example given by Price (2001), and thus a unimodal distribution could still offer a reasonable approximation to these cases. This also applies to the flight data examples shown in Heymsfield and McFarquhar (1996) taken in ice clouds. Additionally, as stated in the introduction, the bimodal and multimodal distributions may be exaggerated in both flight and balloon data. Satellite data on the other hand can give a more global view at relatively high spatial resolutions. Two such studies have been reported by Wielicki and Parker (1994) and Barker et al. (1996) who used Landsat data at a resolution of 28.5 meters to examine liquid water path in a large variety of cloud cover situations. They reported unimodal distributions in nearly or totally overcast scenes, and exponential-type distributions in scenes of low cloud fraction, as expected since in these cases only the tail of the q_t distribution is detected. Note that the analysis of LWP is likely to lead to much smoother (and thus more unimodal) PDFs due to the vertical integration.

In summary, it appears that in the observational data available conducted over a wide variety of cloud conditions (although rarely in ice-clouds), approximate unimodality was fairly widespread, and that a flexible unimodal function can offer a reasonable approximation to the observed variability of total water. That said, a significant minority of cases are very likely to be better modelled using a bimodal distribution like those advocated by Lewellen and Yoh (1993) and Golaz et al. (2002).

4.2 Setting the PDF moments

The second task of statistical schemes is to define the higher order moments of the distribution. If the distribution is simple, such as the uniform distribution, then it is defined by a small number of parameters. In the case of the uniform distribution, one could specify the lower or upper bounds of the distribution; two parameters are required. Equivalently, one could give the first two distribution moments: namely the mean and the

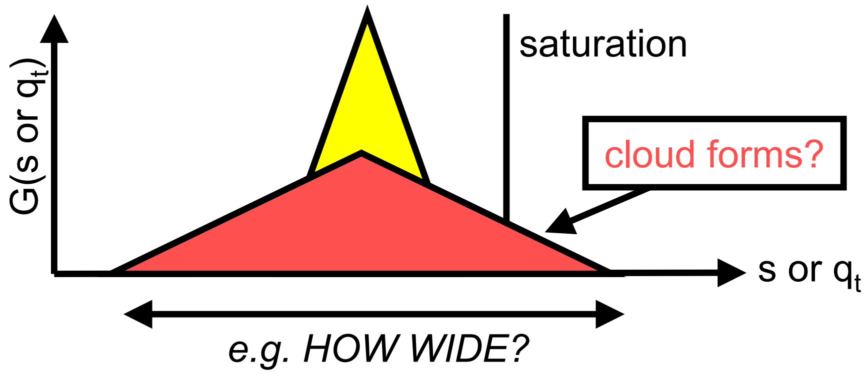

Figure 6: Even if the mean total water is correct, if the incorrect distribution width is diagnosed, for example the narrow yellow distribution, then clear sky conditions will prevail when in fact partial cloud cover exists (pink triangle).

variance. Likewise, more complicated PDFs that require 3 parameters can be uniquely defined using the first three moments: mean, variance and skewness; four-parameter distributions need the fourth moment of kurtosis (describing the PDF 'flatness', see schematic in Fig. 5), and so on.

It is clear to see why the accurate specification of the moments is important. The schematic of Fig. 6 shows that, even if the distribution mean is correct, diagnosing a variance that is too small (i.e. the distribution is too narrow) will lead to the incorrect prediction of clear sky conditions.

Some schemes diagnostically fix the higher order moments of the distribution, such as the variance. However, it is clear that this is not an ideal approach, since by having a fixed distribution width (for example), the PDF (and thus cloud properties) are not able to respond to local dynamical conditions. The fixed width (and higher order moments) are then equivalent to the specification of the critical relative humidity at which cloud is assumed to form in the *RH* schemes. Indeed Smith (1990) actually sets the width of the triangular distribution in that scheme in terms of a RH_{crit} parameter.

To illustrate this with a specific example, let us consider the uniform distribution adopted by LeTreut and Li (1991). The PDF for a typical partially cloudy grid box is shown in Fig. 7. Considering the humidity, it is assumed that no supersaturation exists as is usual, and thus in the cloudy portion, $q_v = q_s$. Thus the grid-mean humidity can be written as:

$$\overline{q_v} = Cq_s + (1-C)q_e \tag{7}$$

where q_e is the humidity in the 'environment' of the cloud; the cloud-free part of the gridbox. From the uniform distribution shape, it is possible to define q_e in terms of a critical *RH* for cloud formation RH_{crit}:

$$q_e = q_s(1-(1-C)(1-RH_{crit})). \tag{8}$$

The definition of *RH* is $\overline{q_v}/q_s$, which substituting the definitions above gives

$$RH = 1-(1-RH_{crit})(1-C)^2, \tag{9}$$

which can be rearranged to give

$$C = 1-\sqrt{\frac{1-RH}{1-RH_{crit}}}. \tag{10}$$

This is recognised to be the relative humidity scheme used by Sundqvist et al. (1989)! Thus it is seen that a so-called statistical scheme with fixed moments can be reduced to a *RH* scheme, or likewise that *RH* schemes do

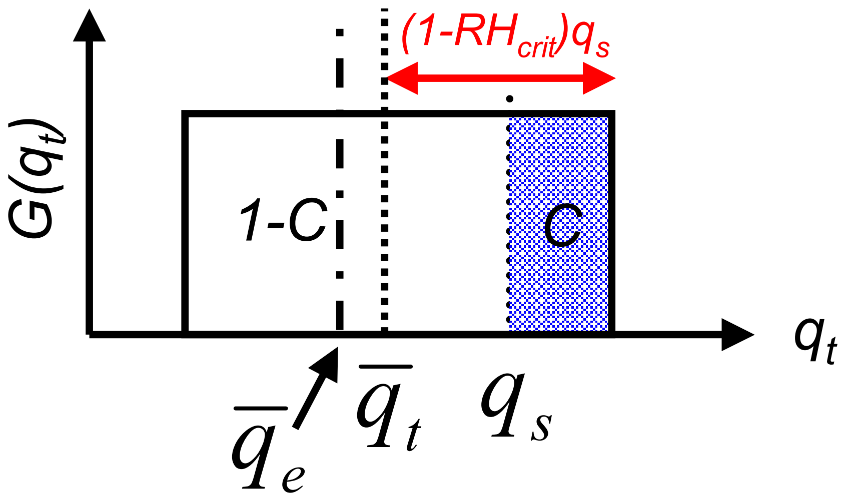

Figure 7: Graphical aid to the derivation of the cloud cover as a function of the RH when the total water is assumed to be uniformly distributed. If cloud begins to form at RH_{crit} then the width of the distribution is $2q_s(1-RH_{crit})$. See text for details.

not need to rely on ad-hoc relationships, but can be derived consistently with an assumed underlying PDF of total water. This point was fully appreciated by Smith (1990), whose work actually provides the *RH*-formulation associated with the triangular distribution in its appendix.

The example of the Smith (1990) scheme also raises another interesting point. Since the scheme was based on the linear *s* variable (see appendix below), that aims to take temperature fluctuations into account, it is often claimed that the Smith scheme (and related schemes) include the effect of temperature fluctuations. However, there is nothing in the scheme that specifically accounts for the *separate* effect of temperature fluctuations and their correlation with humidity (see Tompkins, 2003, for more discussion). By fixing the width of the distribution, the scheme simply defines a 'net' effect of temperature and humidity fluctuations combined. The point is that one could write the scheme purely in terms of humidity fluctuations, and arrive to the same relationship, which is witnesses by the *RH* derivation contained in the appendix of that paper. This in turn implies that such schemes are not, in fact, taking temperature into account in any meaningful way.

In summary, it is important to stress that there is not a clear distinction between the so-called '*RH* schemes' and statistical schemes. If a time-invariant variance is used in a statistical scheme, it can be reduced to a *RH*-type formulation and we have seen how the *RH* scheme of Sundqvist et al. (1989) can be derived by assuming a uniform distribution for total water, and likewise that the Smith (1990) scheme also reduces to an equivalent *RH* formulation.

4.3 Diagnostic versus Prognostic schemes

At this point we pause to consider the merits or otherwise of prognostic versus diagnostic cloud schemes. Shakespeare summarizes the issue well in Act III of Hamlet:

"To be (prognostic) or not to be (prognostic), that is the question. Whether 'tis nobler in the mind to suffer, the slings and arrows of outrageous closure assumptions, or to take arms against a sea of authors, (convinced that diagnostic cloud schemes are the best), And by opposing, end them?"

Hamlet is antagonizing over the issue of whether to implement a diagnostic or a prognostic approach in his cloud scheme. By this, we mean whether or not to include a prognostic equation for the central parameters of the scheme in question. In the case of the statistical schemes this is likely (but not necessarily) to imply a memory (a prognostic equation) for the higher order moments such as variance, where as in the Tiedtke Scheme approach outlined below the prognostic variable is the cloud cover itself.

Irrespective of the variable in question, the underlying question is always whether *the variable has a fast equilibrating timescale relative to the timestep of the model*. Let us take the case of turbulence (Lenderink and Siebesma, 2000). The prognostic equation for variance is:

$$\frac{d\sigma^2(q_t)}{dt} = -2\overline{w'q_t'}\frac{dq_t}{dz} - \frac{\sigma^2(q_t)}{\tau} \tag{11}$$

The two terms on the RHS represent the creation of variance due to a turbulent flux of humidity occurring in the presence of a humidity gradient, and a dissipation term modelled by a Newtonian relaxation back to isotropy with a timescale of τ. This equation is highly simplified by the neglect of both turbulent and large-scale flow transport of variance, and also the horizontal gradient terms, but it serves its illustrative purpose.

It would be possible to introduce a prognostic predictive equation for total water variance along these lines. However, if the dissipative timescale τ is very short compared to the model timestep, then a very good approximation could be obtained by assuming $\frac{d\sigma^2(q_t)}{dt} = 0$, giving

$$\sigma^2(q_t) = -2\tau\overline{w'q_t'}\frac{dq_t}{dz}. \tag{12}$$

A diagnostic approach has the advantage that it simplifies implementation, and saves computational cost and memory. The simplification does not imply that the local cloud properties are independent of the local dynamics; a scheme based on eqn. 12 can not be reduced to a *RH* scheme, since the variance in each gridbox is related to the local turbulent flux. Note also that now, with such an approach, one can sensibly include the contribution of temperature fluctuations due to turbulence, as done by Ricard and Royer (1993).

For examples of this kind of approach, examine the diagnostic schemes in the literature that are described by Bougeault (1982); Ricard and Royer (1993); Bechtold et al. (1995); Lohmann et al. (1999); Chaboureau and Bechtold (2002). These schemes mostly restrict their concern diagnostic relationships for variance to the influence of turbulence. For example, above the boundary layer, Lohmann et al. (1999) imposed a fixed width distribution to compensate for the lack of consideration of other processes.

It is thus apparent that for generalized cloud situations, that include the evolution of clouds such as large-scale cirrus, which may evolve over many hours or even days, it will normally be necessary to resort to implementing a prognostic approach.

4.4 A prognostic statistical scheme

The first attempt to implement a fully prognostic statistical scheme into a GCM was made by Tompkins (2002). This modelled the total water fluctuations using a Beta distribution,

$$G(t) = \frac{1}{B(p,q)} \frac{(t-a)^{p-1}(b-t)^{q-1}}{(b-a)^{p+q-1}} \qquad (a \le t \le b) \tag{13}$$

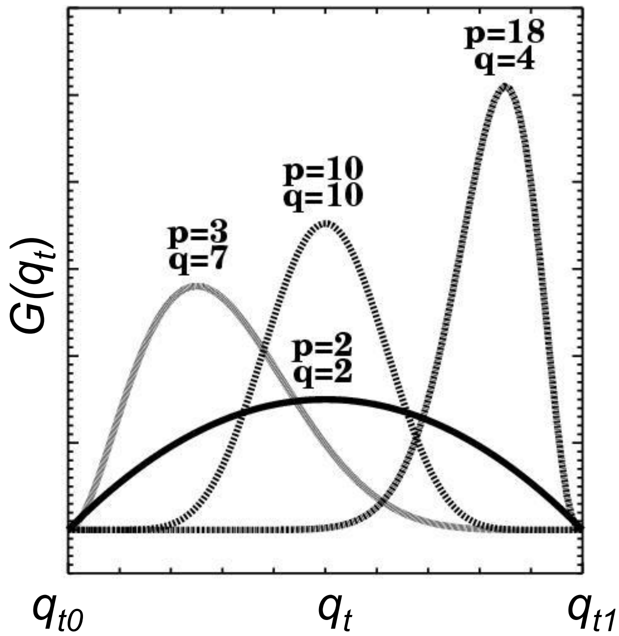

Figure 8: Examples of the Beta distribution for various shape parameters.

where a and b are the distribution limits and p and q are shape parameters (Fig. 8)[2] and the symbol B represents the Beta function, and can be defined in terms of the Gamma function, Γ, as follows:

$$B(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}. \tag{14}$$

The skewness (ς) of the distribution is related to the difference between the two shapes parameters p and q,

$$\varsigma = \frac{2(q-p)}{p+q+2}\sqrt{\frac{p+q+1}{pq}}, \tag{15}$$

and thus if $p = q$ the distribution is symmetrical, but also both positive and negatively skewed distributions are possible. As p and q tend to infinity the curve approaches the Normal distribution. The standard deviation of the distribution is given by

$$\sigma(t) = \frac{b-a}{p+q}\sqrt{\frac{pq}{p+q+1}} \tag{16}$$

Although this distribution is a 4-parameter function, using a simplification such as imposing $(p-1)(q-1) =$constant can reduce it to a three parameter distribution (Tompkins used the less satisfying p=constant closure, which unnecessarily restricted to distribution to positive skewness regimes), specified uniquely by the mean, variance and skewness of total water.

Tompkins (2002) attempted to introduce two additional prognostic equations to predict the evolution of the PDF shape. Once the distribution shape is known, (i.e. distribution limits a and b and the shape parameters p and q) the cloud cover can be obtained from

$$C = 1 - I_{\frac{q_s-a}{b-a}}(p,q), \tag{17}$$

[2]the original notation is repeated, but please note that the shape parameter q is not to be confused with mixing ratio, q_v.

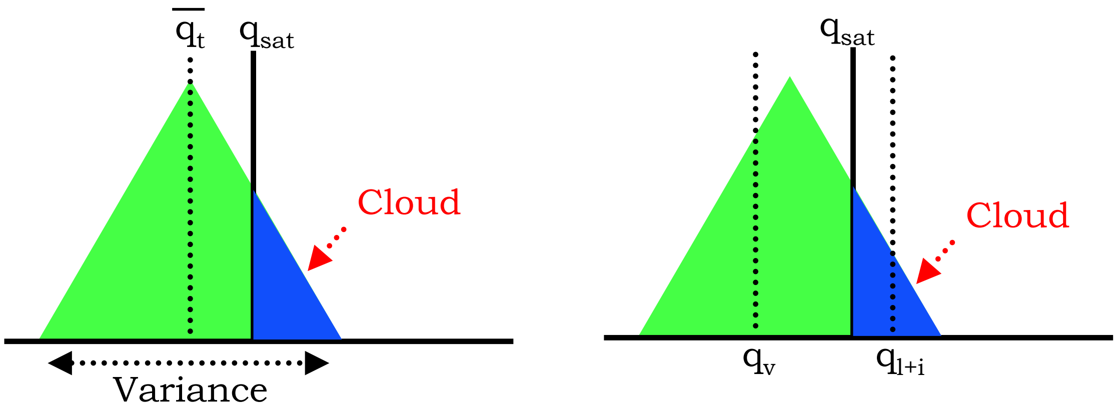

Figure 9: Schematic of the two ways of specifying the triangular distribution. Left Panel: The distribution mean and variance is given. Right Panel: The mean vapour and cloud water (ice+liquid) are given. In both cases the distribution is uniquely specified and the cloud cover can be diagnosed.

where I_x is the incomplete Beta function ratio defined as

$$I_x(p,q) = \frac{1}{B(p,q)} \int_0^x t^{p-1}(1-t)^{q-1}dt, \tag{18}$$

subject to the limits $I_0(p,q) = 0$ and $I_1(p,q) = 1$.

Tompkins (2002) then attempted to parametrize the sources and sinks of variance and skewness separately from physical processes such as convection, turbulence, microphysics and so on. However, there is one complication that requires consideration, and is summarized by the following equation for cloud water q_c:

$$\overline{q} = (b-a)\frac{p}{p+q}\left(1 - I_{\frac{(q_s-a)}{(b-a)}}(p+1,q)\right) + (a-q_s)\left(1 - I_{\frac{(q_s-a)}{(b-a)}}(p,q)\right), \tag{19}$$

This is simply eqn. 6, with the Beta distribution substituted for $G(q_t)$. This tells us that if the distribution moments are known, then the cloud water is uniquely defined. Why is this a cause for concern? The reason is that *most cloud schemes already implement a separate prognostic equation for cloud liquid/ice water*. In other words, in partially cloudy conditions, if distribution moments *and* the cloud liquid water are given from the respective prognostic equations, then the problem is potentially over-specified. To clarify this we can re-examine the simple 2-parameter triangular distribution in Fig. 9. The figure shows that the 2-parameter distribution can be uniquely defined by giving either the mean and variance, or the mass mixing ratios of vapour and cloud water separately.

Thus a decision must be reached concerning the prognostic equation set to be used. The first option is to use water vapour and cloud water separately to implicitly derive the variance (right panel of Fig. 9). The advantage of this approach is that one does not need to explicitly derive complex variance source/sink terms, such as the impact of microphysics on variance. If, over a timestep, the microphysics reduces the cloud water (for example by autoconversion to snow, or by settling out of the gridbox) then this implicitly renders a narrowing of the distribution. Additionally it is much easier to ensure conservation of cloud water (presuming the numerics employed are designed to ensure conservation of prognostic quantities). The disadvantage is that *the information is only available in partially cloudy conditions*. In clear sky conditions one only knows the distribution mean, since $q_c = 0$ identically (see schematic of Fig. 10). Likewise in overcast conditions, where $q_v = q_s$. In these situations, the loss of information requires supplementary ad-hoc assumptions to be made, to close the system. For example, one could resort to assuming a fixed distribution width in clear-sky conditions, thus returning to

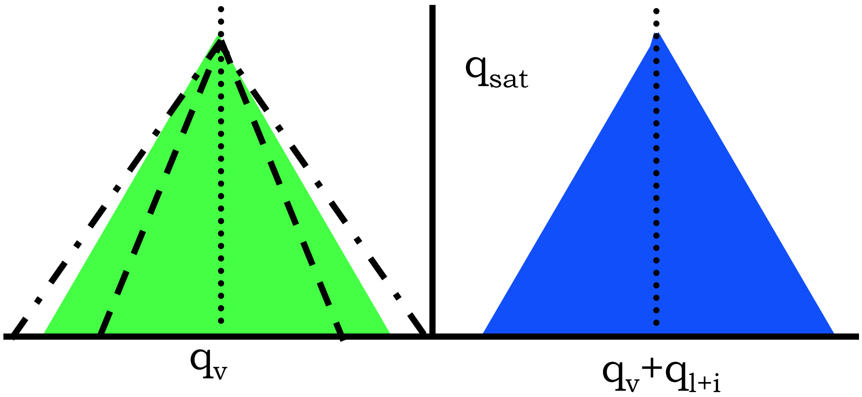

Figure 10: Schematic of the problem that arises if distribution width is derived from separate prognostic equations for vapour and cloud water. The curve is not uniquely defined for overcast (blue PDF) or clear sky (green PDF) conditions. For example, for the clear-sky case, there are any number of possible variances (width) of the distribution that give the correct mean water vapour and zero cloud water. Two examples are marked: a wider distribution (dot-dashed) or narrower (dotted).

cloud formation at a specified (RH_{crit}). We will see below that this issue arises once again in the Tiedtke (1993) scheme, which resorts to such a solution.

The second approach is to abandon the separate cloud water prognostic variable in favour of a prognostic variance equation. This has the advantage that the distribution is *always* known, even in clear sky or overcast conditions. The disadvantage is that all sources and sinks must now be parametrized in terms of variance sources and sinks. For turbulence (Deardorff, 1974), and perhaps convective sources and sinks(Lenderink and Siebesma, 2000; Klein et al., 2005), this is relatively straight-forward. However, for the microphysical processes the problem quickly becomes complicated. For simple autoconversion terms A (the rate of conversion from liquid to rain), it is possible to derive the sink of variance[3]

$$\frac{d\sigma^2(q_t)}{dt} = \overline{A'q'_t} = \int A'(q_t)q'_t G(q_t)dq_t, \tag{20}$$

which analytically tractable for simple forms of A and $G(q_t)$. Nevertheless, we can imagine more complicated scenarios, such as ice settling handled by a semi-Lagrangian advection scheme, allowing settling from any particular gridbox to other all levels below it. Trying to parametrize this equivalently in terms of variance sources and sinks is difficult. Moreover, by abandoning the prognostic equation for ice, any inaccuracies in the handling of such a process via a variance equation are likely to manifest themselves in a (potentially severe) compromising of the cloud mass conservation.

Tompkins (2002) tried to provide a solution for this dilemma by implementing a hybrid scheme. In partially cloudy conditions variance is derived directly from the cloud water and vapour prognostic equations. In clear sky and overcast conditions, the variance is prognosed using a subset of source and sinks terms, including turbulence, dissipation, and a highly simplified sink term due to microphysics, which is necessary in overcast conditions. The reader is referred to Tompkins (2002) for details of these source and sinks terms, although

[3] Note once again the care that must be taken with regard to the numerics with long timesteps. Since autoconversion terms tend to be nonlinear they usually reduce the variance. Even if this equation is integrated implicitly for stability, the limit for long timesteps will be zero, which is unrealistic for partially cloudy conditions since the precipitation process does not affect the clear sky part of the domain. Thus instead one should integrate this term implicitly for the cloudy portion [cld] of the gridcell and then combine the result with the clear sky [clr] variance thus: $\overline{\sigma^2(q_t)} = C(q_s^2 + \sigma^2(q_t)[cld]) + (1-C)(q_v^2[clr] + \sigma^2(q_t)[clr]) - \overline{q_t^2}$

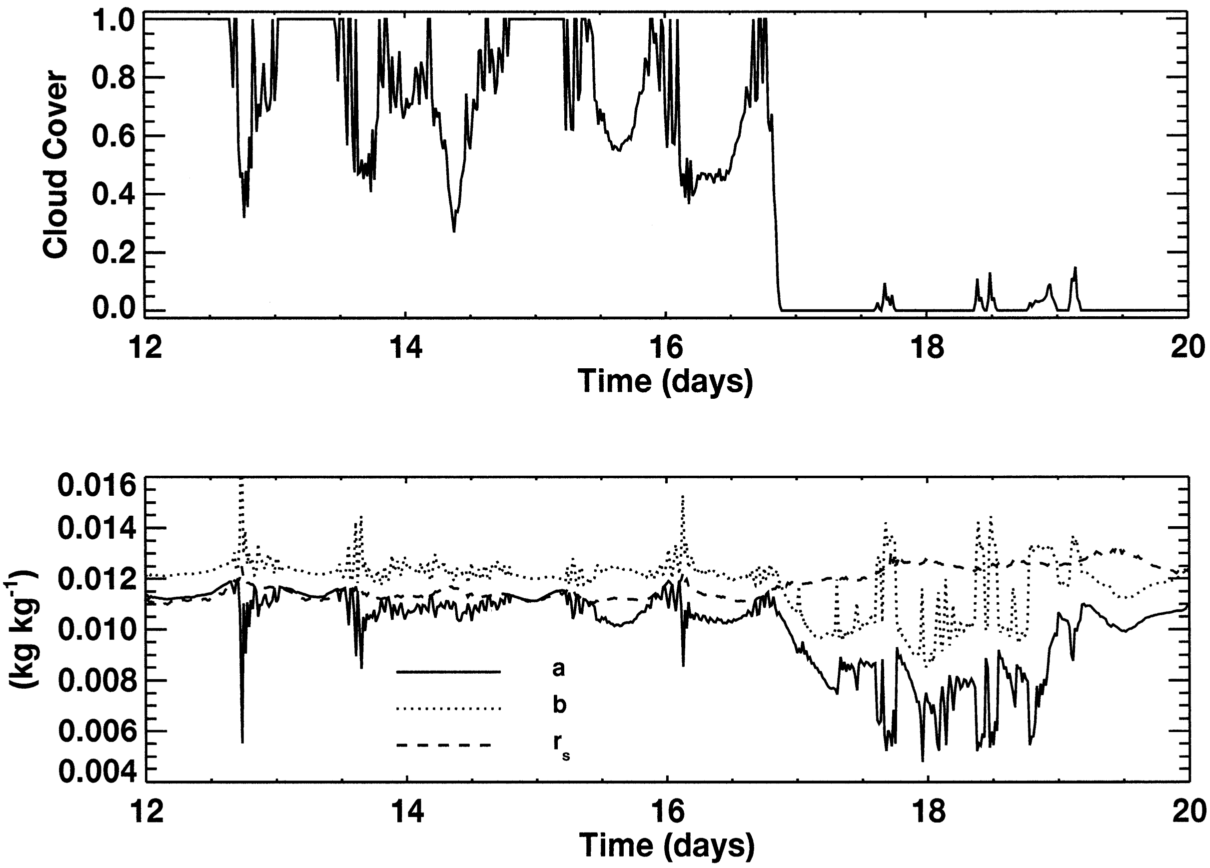

Figure 11: Figure taken from Tompkins (2002) showing evolution of the boundary layer at a gridpoint subject to stratocumulus cloud. The upper panel shows the cloud cover, while the lower shows the total water distribution minimum (a), maximum (b) in addition to q_s (marked r_s in the plot, according the notation used in that paper). In the earlier period, the scene is overcast and the whole of the PDF is moister than q_s. In this case the increase in variance from turbulence breaks up the cloud deck intermittently. In the latter period instead the gridbox is relatively dry, and turbulence instead creates small cloud coverage; representing the cloud capped thermals known as 'fair weather cumulus'.

it should be noted that some of these, in particular the skewness budget terms from microphysics and deep convection, have been justifiably criticized by Klein et al. (2005) for their ad hoc nature. Nevertheless, the inclusion of even a reduced set of variance sources/sinks, especially from turbulence, is able to reproduce the observations of turbulence increasing or decreasing variance according the mean humidity gradients, and coincidently creating cloud or breaking-up an overcast cloud deck (Fig. 11).

5 The ECMWF prognostic cloud cover scheme

The aim here is not to describe the Tiedtke (1993) scheme in full; since this information is adequately covered by other course notes, the online documentation, and the publications of Tiedtke (1993); Gregory et al. (2000); Jakob (2000). Instead this section briefly places the Tiedtke scheme into the context of the cloud scheme family, in particular the statistical schemes.

The Tiedtke scheme chooses a different set of prognostic equations for cloud scheme, namely: water vapour, cloud water and cloud cover. We saw in the last section how the former two, vapour and cloud could be used to equivalently specify the mean and variance of total water, *in partially cloudy conditions*. The Tiedtke scheme takes this approach a step further by adding a third predicted equation, giving a memory for the cloud cover.

We also learned in the previous section that such an approach has some advantages, since it greatly simplifies some of the source and sink derivations. A good example is the link to the convection scheme. The convection scheme provides a mass of detrained cloudy air, which is then simply added directly to the respectively cloud water and cover equations, without recourse to distribution functions.

This is not to say that the Tiedtke scheme does not use assumptions concerning the underlying distributions to derive some of the sources and sinks of the prognostic equations. For example, the source of cloud water and cloud cover from a gridbox cooling is derived assuming the clear sky humidity observes a uniform distribution (note that an error in the original derivation of Tiedtke (1993) was corrected by Jakob (2000)). The assumption leads to a cloud fraction source of

$$\frac{\partial C}{\partial q_s} = -\frac{(1-C)^2}{2(q_s - q_v)} \tag{21}$$

This direct translation of PDF moment sources and sinks into consistent cloud cover and water sources and sinks is discussed in far greater detail in Wang and Wang (1999); Gregory et al. (2002); Larson (2004). Note that the Tiedtke (1993) scheme does not parametrize all sources and sinks consistently with an underlying distribution. For example, the horizontal subgrid-scale eddies act to homogenize the total water field and will reduce the width of the distribution. Thus if $\overline{q_t} < q_s$ then the cloud cover will reduce as a result, while with $\overline{q_t} > q_s$ dissipation will increase cloud cover. The Tiedtke scheme instead always reduces cloud cover, in conflict with any possible humidity distributions.

For the most part, *if* the Tiedtke scheme uses an underlying distribution assumption, it is usually that the clear sky humidity fluctuations are distributed uniformly, while the cloudy portion is homogeneous (described by a delta function). It is thus clear that the scheme is not reversible. If a gridbox is subjected to an equal magnitude cooling followed by warmed over two consecutive timesteps, and all other processes (e.g. precipitation) are neglected, there is a net creation of cloud, as illustrated in Fig. 12.

This issue is important since it implies that such a scheme could be more sensitive to increases in model horizontal resolution. While the time-mean statistics of vertical velocity (which cool or warm a gridbox adiabatically) could be resolution invariant, it is likely that temporal variability will increase with higher model resolutions which resolve a greater portion of the velocity spectrum. If the cloud scheme contains aspects such as outlined here, where the passing of a wave leads to net cloud creation, then a high resolution model may be expected to have higher mean cloud cover, and other associated changes to the mean climate.

While the Tiedtke scheme has many merits and has proved to be very effective in its prediction of cloud characteristics despite its simplicity (e.g. Hogan et al., 2001), we should point out some potential drawbacks of the approach. One issue concerns self-consistency. With a pure statistical scheme approach, where the PDF moments are predicted, the cloud water and cover are constrained to be consistent with each other, since they are both derived from the same distribution[4]. This is not true for the Tiedtke scheme, and it is not unusual for cloud water and cover to be inconsistent, with only one of the two fields non-zero for instance.

Another point relates to the earlier statistical scheme discussion concerning the choice of prognostic variables. Since the cloud cover and cloud mass only describe *integrated* statistics of the PDF, we are again faced with the loss-of-information dilemma in clear and overcast gridboxes. Once again in clear skies, the Tiedtke scheme only provides the mean humidity, and no information concerning the humidity variability is available. This

[4] note that we have not touched on the issue of representing ice-phase processes with such schemes, in which significant and long lasting supersaturations are permitted. In this case it is likely that a separate ice variable is mandatory, in which case the self-consistency argument may no longer be valid.

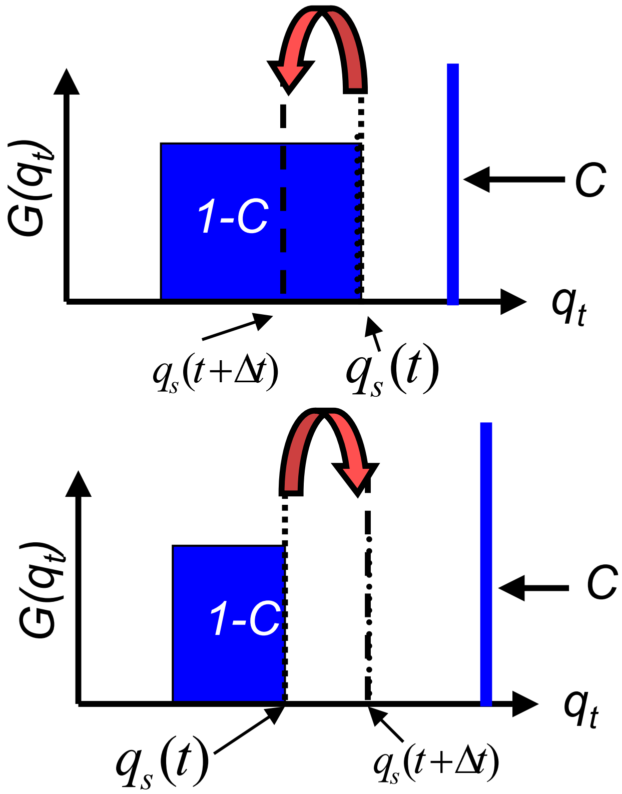

Figure 12: Schematic showing one reversibility issue with Tiedtke scheme. The upper panel shows the impact of a cooling applied to a partially cloudy gridbox. The cooling reduces q_s and thus condenses cloud water, increasing the cloud fraction by the area contained between the dotted and dashed lines. It is assumed no rain is produced and over the subsequent timestep the gridbox is subjected to an equal and opposite warming. One would expect the gridbox should return to the original state, but due to the assumption that the cloud is well mixed and homogeneous, this is not the case (lower panel). The non-reversibility is not necessarily a problem per se; it is consistent with the assumption that the in-cloud homogenization occurs on fast timescales compared to the model timestep. If this were true in the atmosphere then the non-reversibility of the scheme would be realistic. Observations indicate that clouds are far from the homogeneous entities that the Tiedtke cloud scheme and the bulk mass flux convection schemes assume!

is the reason why, despite its 'prognostic' credentials, the Tiedtke scheme again is forced to resort to the implementation of a fixed critical relative humidity for cloud formation; the variance is clear sky regions is fixed.

6 Summary

In summary, this lecture has tried to summarize the various approaches to diagnosing the proportion of a grid box covered by cloud in global models. The main point is that partial coverage can occur if and only if subgrid-scale fluctuations of humidity and temperature exist. All cloud schemes that predict partial cloud cover therefore implicitly or explicitly make assumptions concerning the magnitude and distribution of these fluctuations; the total water probability density function (PDF).

We discussed simple diagnostic schemes that use *RH* as their main or only predictor for cloud cover. We then discussed statistical schemes that explicitly specify the humidity PDF. We showed that if the moments of such schemes are time-space invariant, then the cloud cover deriving from statistical schemes can be written as diagnostic *RH* form. In other words, rather than using ad hoc relationships, one can derive a *RH*-scheme to be consistent with an underlying PDF.

It was pointed out that knowing the PDF for humidity and cloud fluctuations gives vital extra information that can be used to correct biases in nonlinear processes such as precipitation generation or interaction with radiation.

More complex statistical schemes were then discussed which attempt to predict the sources and sinks of the distribution moments, so that the PDF can realistically respond to the various relevant atmospheric processes. The lecture dwelled on the choice of the prognostic variables, in particular whether it is preferable to predict the PDF moments themselves, or instead to predict integrated and direct cloud quantities such as the cloud liquid water. Advantages and potential drawbacks of each approach were discussed; which were essentially that by directly predicting the cloud variables, one ensures their conservation, and processes such as microphysics are far easier to handle, but that in clear sky or overcast conditions there is lack of information so that diagnostic/fixed assumptions have to be made concerning the subgrid distribution in these situations. These assumptions may also lead to a lack of "reversibility" in this approach.

It was pointed out that the Tiedtke scheme is essentially a manifestation of the second approach, where both cloud water *and* cloud cover are predicted, and where often an underlying assumption concerning the humidity and cloud distribution is made to derive the sources and sinks of these prognostic variables. We highlighted that, while proven successful in NWP, the scheme suffers from the same drawback of requiring the implementation of a fixed (independent of local dynamical conditions) RH_{crit} for cloud formation and a lack of reversibility.

A Appendix: Temperature variability

In this document fluctuations of temperature were explicitly ignored. Since water vapor perturbations can be correlated with temperature perturbations, which alter the local saturation vapor pressure, it may also be necessary to consider temperature variability.

To this end, it has been useful to form a variable, s, defined as[5]

$$s = a_l(q_t' - \alpha_l T_l') \tag{22}$$

where q_t' is the fluctuation of the total water mixing ratio, q_t, equal to the sum of the vapor (q_v), cloud ice (q_i) and liquid cloud water (q_l) mixing ratios, and T_l' is the liquid water temperature fluctuation ($T - \frac{L}{C_p}q_l$), an analogue to moist static energy. The fluctuations are defined about the mean thermodynamic state, $\overline{T_l}$, and the constants are defined as $\alpha_l = \frac{\partial q_s}{\partial T}(\overline{T_l})$ and $a_l = [1 + \frac{L}{c_p}\alpha_l]^{-1}$, where q_s is the saturation vapor mixing ratio, L is the latent heat of vaporization and c_p is the specific heat of dry air. Physically, s describes the distance between the thermodynamic state to the linearized saturation vapor mixing ratio curve, as illustrated in Fig. 13.

Defining $s_s = a_l(q_s - \overline{q_t})$ the cloud condensate mass q_c $(= q_l + q_i)$ is given by $q_c = s - s_s$, providing $s > s_s$. Assuming that any supersaturation efficiently condenses to cloud, it is possible to express the cloud fraction C as

$$C = \int_{s_s}^{\infty} G(s)ds \tag{23}$$

where $G(s)$ is the PDF of s.

[5] Once again, the commonly used notation is repeated here, but this variable is not to be confused with the more common use for s, which is the dry static energy

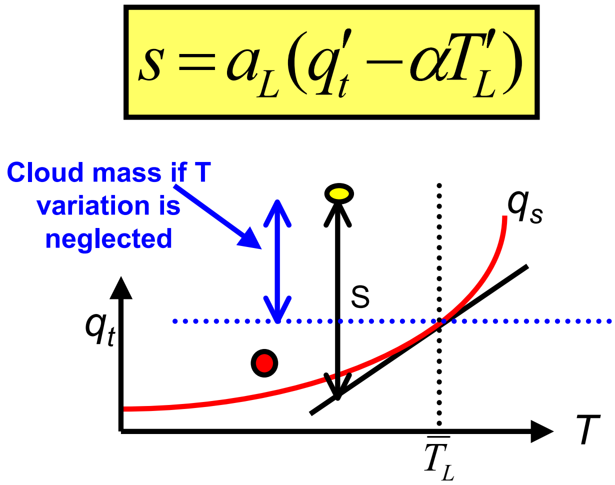

Figure 13: Schematic showing the definition of s, in terms of q_t. See text for details.

The variance of s, and therefore the associated liquid water and cloud cover, depends on the correlation between T_l and q_t perturbations in addition to their respective magnitudes:

$$\sigma^2(s) = a_l^2(\overline{q_t'^2} + \alpha_l \overline{T_l'^2} - 2\alpha_l \overline{q_t' T_l'}). \tag{24}$$

This aspect was disregarded by many previous statistical schemes, which were formulated in terms of s, but simply set the variance to a fixed or arbitrary value. In such schemes it is not known whether cloud is a result of temperature perturbations, water perturbations, or a combination of the two, as was discussed in the main text. Instead, some parametrizations such as Ricard and Royer (1993) have calculated temperature perturbations separately that result from turbulence, since the turbulence scheme can provide the various correlations ($\overline{q_t'^2}$, $\overline{T_l'^2}$ and $\overline{q_t' T_l'}$) separately.

Temperature fluctuations are likely to be smaller in magnitude than total water fluctuations, especially in the tropics where gravity waves remove lateral fluctuations of virtual temperature on fast timescales (Bretherton and Smolarkiewicz, 1989). The recent observational studies of Price and Wood (2002); Tompkins (2003) also seem to confirm that temperature fluctuations, while significant, are less important than humidity fluctuations, even in the lower troposphere in midlatitudes.

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