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ICE-COVER GROWTH RATES AT NEARSHORE LOCATIONS IN THE GREAT LAKES

Kenneth M. Hinkel

Great Lakes Environmental Research Laboratory
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Malcolm Baldrige, Secretary

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John V. Byrne, Administrator

Environmental Research Laboratories

George H. Ludwig, Director

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ICE-COVER GROWTH RATES AT NEARSHORE LOCATIONS IN THE GREAT LAKES*

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Ice thickness data from 32 nearshore locations around the Great Lakes were correlated to accumulated degree-days of frost over a 6- to 11-year period. A simple parastatistical model was used to compute ice-cover growth coefficients that reflect the relative impact of site-specific factors and processes on ice growth for each ice measurement site. In addition, two data sets were used to illustrate the inhibiting influence of snow on ice growth. Statistical parameters generated for each site and data set were used to summarize the degree of predictive accuracy. For 27 sites, a weighted R^2 value of 0.82 was achieved with an average standard error of estimate of 6.95 cm. As an additional test, the site-specific ice growth coefficients were applied to unpublished ice thickness data for the abnormally cold winters of 1977-78 and 1978-79. For these two seasons, the average standard error of estimate was 5.39 cm.

1. INTRODUCTION

In the mid-1960's, during a period of rapid economic growth, it became increasingly apparent that accurate ice forecasting in the Great Lakes would benefit the commercial, recreational, and industrial sectors of the United States and Canada. The ability to predict ice-cover formation, maximum expected thickness, and breakup at nearshore locations would assist in the siting and design of harbors, power plants, and breakwalls to achieve the desired engineering effect at minimal cost.

Any predictive technique requires sufficient input data to both develop and test the model. In an effort to address a shortage of information on ice-cover thickness, the Lake Survey District of the U.S. Army Corps of Engineers initiated a data gathering program in 1966; this was continued by the NOAA Great Lakes Environmental Research Laboratory until 1979. Observers recorded the thickness, stratigraphy, and snow cover at numerous nearshore locations around the Great Lakes on a weekly basis. This data set was to be used in developing a model to predict ice growth at these locations, and to enhance understanding of the complicated process of ice-cover formation and decay.

The purpose of this paper is twofold. First, a theoretically derived equation for determining ice thickness was used to develop a statistical model to compute the rate of ice growth at the lake sites. Air temperature data were used to drive the model. The predicted results could then be compared to the ice thickness observed and the degree of simulation accuracy

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determined. Secondly, it can be shown that the rate of ice growth is strongly influenced by site-specific characteristics other than air temperature. Since the rate of ice growth and decay are functions of many interacting processes, it is necessary to recognize that sophisticated forecasting algorithms require more detailed input data than are presently available.

Traditionally, two ice simulation techniques have been used: heat budget analysis and simple statistical analysis. Heat budget analysis has been used in studies by Scott and Ragotzkie (1961), Polyakova (1966), Williams (1965), Bilello (1968), and Dutton and Bryson (1960) and involves the gathering of detailed, site-specific hydrological, meteorological, and ice related parameters. These input data are used to solve the mass and energy equations. Because of the large number of observations and computations, this process is greatly facilitated by computer technology. Since this technique accounts for most of the-variables influencing ice growth, it is employed whenever the input data are available. It is especially useful as a river ice forecasting technique where accurate simulation is necessary and data gathering programs are economically justified. Greene's (1981) model of ice growth and breakup on the St. Lawrence was sufficiently sensitive to simulate the unusually early 1981 breakup.

The second technique uses simple statistical methods to relate average ice growth to available information. Usually, air temperature is the independent variable used to predict ice thickness over an area represented by the temperature data. Often, it is used for inland lakes and ponds, as in the studies by Bile110 (1980) and Andrews (1968), and it has been used by Ferguson and Cork (1972) to forecast ice formation on the Niagara River.

Assel (1976) used several multiple regression techniques on a portion of the ice thickness data set used in this study. Accumulated air temperatures from 12 weather stations were used as the independent variable to compute ice growth at 24 nearshore locations in the Great Lakes, with ice thickness data collected from 3-8 winters. This study is an extension of Assel's initial efforts. Many of his concepts and methodological techniques have been applied to a larger, edited data set.

For this analysis, a simple linear regression was performed on data from each site. In accordance with this statistical technique and the theoretical model, the regression coefficient (slope) values produced for each of the Great Lakes sites should represent the relative influence of site-specific variables on ice growth.

2. THE THEORETICAL BASIS OF THE ICE GROWTH EQUATION

To discuss the analytical model used in this study, it is first necessary to describe ice growth as it relates to relevant energy fluxes.

Figure 1 clarifies terms and schematically illustrates the operational system by representing the lake water, ice cover, and air. For the purpose of the ensuing discussion, energy amounts will be defined in conventional terms such that any energy transfer away from the surface, in either direction, will be a negative quantity, with units of joules per square meter. In this one-dimensional model, it is assumed that the thermal energy moves only in a vertical direction through materials that are homogeneous.

If an ice slab is present and a linear temperature gradient through this ice is assumed, the gradient can be defined as $(T_o - T_s)/Z$, where T_o is the temperature at the base of the ice slab, T_s is the temperature at the ice/air interface, and Z is the ice thickness. In this study, T_s will be assumed equal to the air temperature as recorded at the nearest weather station.

The governing equation describing the increase in ice thickness as a function of the heat conducted through the ice and snow layers is

$$\rho_i \lambda \frac{\partial Z_i}{\partial t} = \frac{(T_o - T_s)}{\left(\frac{Z_i}{K_i} + \frac{Z_s}{K_s}\right)} \,, \tag{1}$$

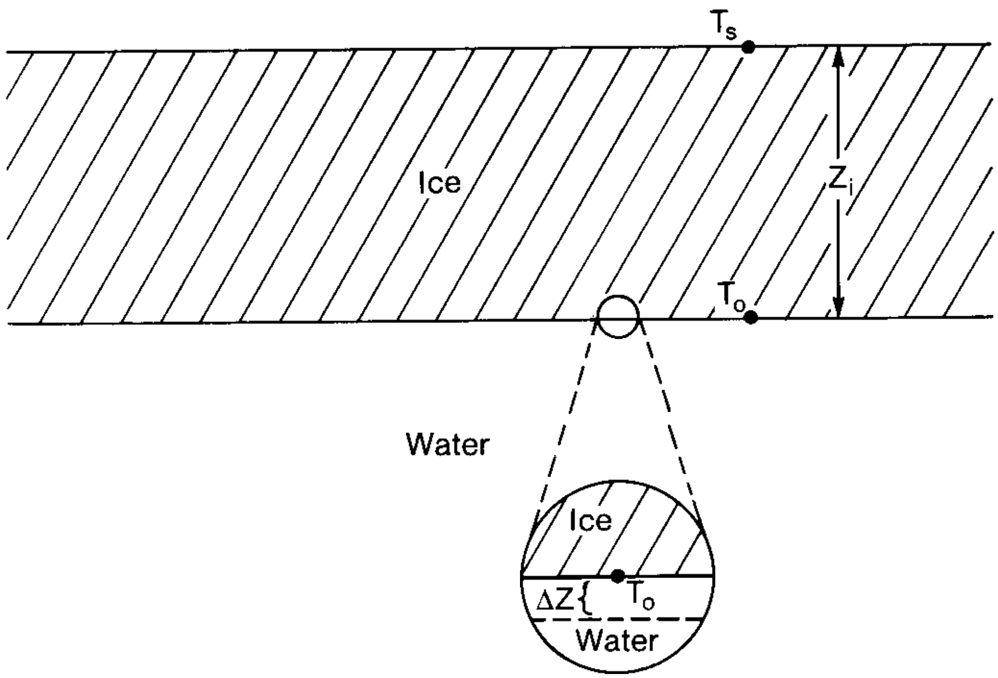

FIGURE 1--Lake ice *stratigraphy*.

where ρ_i = density of ice (kg m^{-3}),
λ = latent heat of fusion (J kg-l),
Z_i = ice thickness (m),
Z_s = snow thickness (m),
K_i = thermal conductivity of ice (W m^{-1} $°C^{-1}$),
K_s = thermal conductivity of snow (W m-1 $°C^{-1}$),
T_o = temperature at ice/water interface ("C)
T_s = surface temperature ("C), and
t = duration of constant T_s (s).

If there is no snow, and one ignores the effect of cooling the ice slab, an estimate of ice thickness can he obtained by the classic Stefan solution to equation (1)

$$Z_i^2 = 2K_i(T_o - T_s)t/(\rho_i\lambda) \tag{2}$$

This analytical method is often used as a first approximation for predicting ice growth on still, shallow lakes and ponds (Outcalt, 1980). In these cases, the ice sheet forms when the temperature of the water is isothermal throughout, at a temperature of maximum density. The surface loses heat to the surrounding cooler air, and crystallization begins.

If it is assumed that the temperature of the air above the ice sheet is equal to the temperature at the surface of the ice ($T_a = T_s$) and that $T_a < 0°C$, equation (2) can be written

$$Z_i = \sqrt{\frac{2K_i t^*}{\rho_i\lambda}\left(DDF\right)}\ , \tag{3}$$

where t* = number of seconds in a day, and DDF = degree-days of frost (°C) defined as

$$DFF = 0.0 - \frac{(T_{max} + T_{min})}{2}$$

where T_{max} = maximum daily air temperature (°C),
T_{min} = minimum daily air temperature (°C).

Assuming the temperature at a nearby weather station closely depicts the conditions at the lake site, we have a measure of the thermal regime of the air on a daily basis, and the accumulated degree-days of frost (ADDF) can be used as a surrogate indication of the heat input-output at the site, commencing with freeze-over.

Using the values discussed above and summarized in table 1, we can generate equation (4) by substitution

$$Z_i(\mathrm{cm}) \cong 3.47\ (\mathrm{ADDF})^{1/2}\ . \tag{4}$$

If, for example, the average daily temperature for a 7-day period was −15°C, the ADDF would he 105. Theoretically, the maximum ice thickness would have been 35.6 cm if initiated on day 1 of this period.

The maximum theoretical ice growth parameter (3.47) can be compared to slope coefficients generated from site-specific regression analyses to indicate the relative impact of locally significant physical processes over the 6- to ll-year period. However, it should be noted that the predicted relationship in equation (4) normally exceeds the slope coefficient produced for the individual sites and serves to highlight some of the drawbacks of the Stefan solution. This formula accounts for the heat loss needed to freeze that additional thickness. It does not, however, consider the heat reaching

TABLE 1.--*Physical constants and descriptions*

Symbol	Definition	Units
λ	Latent heat of fusion of ice	3.4×10^5 J kg^{-1}
ρ_i	Density of ice (black)	920 kg m^{-3} (Ager, 1962)
t^*	Time conversion factor	86,400 s day^{-1}
K_i	Black ice, thermal conductivity	2.18 w m^{-1} °C^{-1} (Sellers, 1965)
K_{si}	Snow-ice, thermal conductivity	1.90 w m-l °C^{-1} (Yen, 1981)
K_s	Snow thermal conductivity	0.41-1.55 W m^{-1} °C^{-1} (Williams and Gold, 195'3)
C_i	Volumetric heat capacity of ice	1.93×10^6 J m^{-3}

the base of the ice sheet from the underlying water. For shallow sites, this heat would be minimized. Should water currents be present under the ice as a result of underground springs, incoming streams, or normal lake conditions, heat would be transferred to the system, decreasing the rate of ice growth.

Essentially, equation (4) applies to the growth of "black ice," a type of ice that forms on a calm water surface in the absence of snow. The individual crystals just below the surface layer are aligned parallel to the direction of heat flow, and are thus perpendicular to the lake surface. Often termed congelation ice, the process of its formation is analogous to that of cooling volcanic magma in that crystallization is orderly and systematic (Shumskii, 1952). The values generally cited in the literature, which quantify the physical properties of ice as determined by laboratory experiments, most closely approximate this form of ice. A degree of error is introduced, however, owing to the chemical and organic impurities often found in the interstitial spaces and variation in the formational environments.

Thus, the Stefan solution can be employed only for extremely simplified conditions. On the Great Lakes, pure black ice stratigraphy is rare since some snow is usually present. However, the pure black ice model is a useful departure point for consideration of the effects of snow cover.

3. THE INFLUENCE OF SNOW COVER

When ice is growing, snowfall can have three important influences on the growth rate. In relative terms, a light snowfall on a thick slab of ice acts as a thermal insulator, inhibiting the transfer of heat through the ice by conduction. Conversely, a heavy snowfall on a thin ice sheet causes stress fractures to develop and allows lake water to saturate the snow, resulting in a net increased ice thickness due to the formation of snow-ice (Adams and Roulet, 1980). In addition, the high albedo of snow prevents the ice from absorbing shortwave radiation that would raise the internal temperature of the ice.

Figure 2 depicts an ice slab covered with snow. Assuming that the heat flow through the ice (Q_i) is equal to the heat flow through the snow layer (Q_s) and that heat energy is transferred from the ice to the snow without undergoing a phase change, one can define the following relationships:

$$Q_i = K_i \left(\frac{T_o - T_i}{Z_i} \right) \tag{5}$$

$$Q_s = K_s \left(\frac{T_i - T_s}{Z_s} \right), \tag{6}$$

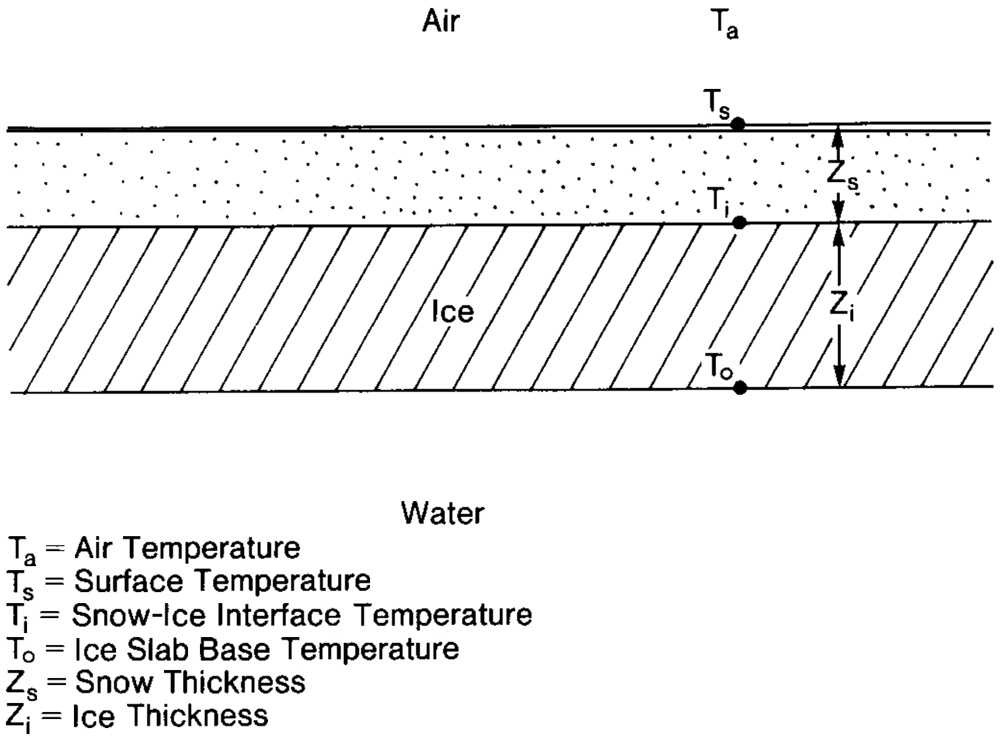

FIGURE 2.--*Lake ice with snow* cover.

where T_i = temperature at the ice/snow interface,
K_s = thermal conductivity of snow.

Setting $Q_i = Q_s$ and $T_o = 0.0$, substituting the appropriate equation, and solving for the temperature at the ice/snow interface (T_i) yields:

$$T_i = \frac{Z_i K_s T_s}{Z_i K_s + Z_s K_i} . \tag{7}$$

If steady-state conditions are assumed, the magnitude of the thermal disturbance at T_i will be dampened by the snow to a degree determined by the depth of the snow cover and the thermal conductivity of the pack.

However, K_s is a function of the snow-cover density. Because snow is continually undergoing metamorphic change and altering its density, this value is difficult to estimate. Analytically, the equation is sensitive to changes in K_s, with consequent results for T_i.

According to Williams and Gold (1958). reasonable values for K_s, determined as a function Of naturally occurring snow densities, range from 0.41 to 1.55 w m-l $°C^{-1}$. Given the following data, equation (7) can be used to solve for Ti with the extreme K_s values

$$Z_i = 10.0 \text{ cm}$$

$$Z_s = 5.0 \text{ cm}$$

$$T_s = -20°C$$

$$K_i = 2.18 \text{ W m}^{-1} °C^{-1}$$

if Ks = 0.41 W m^{-1} $°C^{-1}$, then $T_i = -5.5°C$;

if K_s = 1.55 w m-l $°C^{-1}$, then $T_i = -11.7°C$.

This example serves to illustrate the insulating properties of the snow cover, which dampens the effect of the thermal disturbance. The direct influence of the snow cover on ice growth can be approximated by using the finite difference form of equation (1)

$$Z_i^{n+1} = Z_i^{n} + (\Delta t/\rho_i\lambda)\,\frac{(T_o - T_s)}{\left(\dfrac{Z_s^{\,n}}{K_s} + \dfrac{Z_i^{\,n}}{K_i}\right)}, \tag{8}$$

where n = time increment of At,
t = seconds in increment.

Applying equation (8) over 1 day to the previous data yields

	5-cm snow	10-cm snow
if Ks = 0.41 W m^{-1} $°C^{-1}$, then Zi^{n+1} =	13.3 cm	11.9 cm
if Ks = 1.55 W m^{-1} $°C^{-1}$, then Zi^{n+1} =	17.1 cm	15.0 cm

If no snow cover existed, Zi^{n+1} = 22.0 cm.

Thus, it is apparent that the snow cover moderates the thermal disturbance to a degree determined by the depth of the snow and its thermal properties.

However, Adams and Roulet (1980) have suggested that a heavy snowfall on a relatively thin ice sheet has an opposite effect in that it can effectively increase ice thickness. When the weight of the snow exceeds the shear strength of the ice, fractures develop along the crystal boundaries. The additional "eight of the snowcauses the slab to be submerged to a depth where buoyant equilibrium is reached. The flood water saturates the snow to a height determined by the thickness of the ice and the mass of the snow. The refreezing process begins at the surface of this flood water and advances toward the top of the original ice, expelling water in front of the freezing plane because of volumetric expansion. The random crystalline structure of the snow becomes consolidated in the forming ice, as do air bubbles, and results in the characteristic white color of snow-ice.

If the system is open, so that internally generated pressures can be vented horizontally, ice growth proceeds through the flood waters until froze*. However, since the axes of the ice crystals are no longer vertically aligned, the density and conductivity of this material is altered.

Equation (8) can be altered to compute snow-ice growth if it is assumed that there has been flooding to form a slush layer. To simplify computations, it will also be assumed that the thickness of the slush plus the snow layer will be equal to the pre-flood snow thickness and that no capillary action occurs within the pack.

$$\Delta Z_{si} = \left(\frac{\Delta t}{\phi \rho_{si} \lambda}\right) \frac{\left(T_o - T_s\right)}{\left(\frac{Z_s}{K_s} + \frac{Z_{si}}{K_{si}}\right)} \tag{9}$$

where ϕ = porosity of slush,
ρ_{si} = density of snow-ice, 890 kg m^{-3}.

The following data can be used as an example of the effect of snow-ice growth on overall thickness:

Z_i (initial) = 10 cm

Average daily temperature = -15°C

Snow depth = 20 cm

Snow density = 350 kg m^{-3}

Snow porosity = 0.65.

Should the ice crack and hydrostatic equilibrium be achieved at 6 cm above the ice surface, the net ice thickness, at the end of a 7-day period, is greater than if the ice had not cracked and flooded.

	Black ice growth (cm) (no flooding)		snow-ice growth (cm) (flooding)		
	New ice	Total	Snow-ice	New black ice	Total
Day 0		10.0			10.0
Day 4	2.9	12.9	6.0		16.0
Day 7	2.2	15.1	6.0	2.8	18.8

It should be noted that snow-ice can also form as the result of rainfall, an extreme melt event, and flooding via an incoming stream (Gow and Langston, 1977). On the open lakes, the first two processes contribute significantly to snow-ice formation; the volume of snow-ice resulting from the last mechanism would be relatively negligible.

4. THE DEGREE-DAY LINEAR MODEL

The degree-day linear model was used in this study for several reasons that essentially relate to the quality of the data and the stated purpose of this inquiry.

The relationship between the actual model and the simple linear regression technique used to test the model is unique in that they have the same mathematical form. The degree-day model states that $Z_i = b\sqrt{ADDF}$ and the linear regression has the form $y = a + bx$. The correspondance between the two equations can be seen if x is set equal to $\sqrt{ADDF}$ and y is set equal to the observed ice thickness. The y-intercept value, a, should pass through the origin if the technique for assigning the initial day of freeze-over is valid. Of primary importance is the computed slope parameter, b, which summarizes the relationship between air temperature and ice growth at a particular site. If the input data are normally distributed, the regression parameters that indicate the "goodness of fit" can also be used to evaluate the theoretical model as it applies to an actual situation.

If our model assumptions are correct, then the slope coefficient (b) for a site, represented by the first term in equation (4), should remain stable over time, assuming that all other input variables remain unchanged. Obviously, this assumption is not valid, for each winter constitutes an interaction, to varying degrees, of many processes. I" a sense, the rate of ice growth at any particular point in time and space reflects the cumulative influence of the meteorological and hydrological variables and the ice sheet thermal properties. If it is assumed that air temperature plays a dominant role in ice formation and growth, that it adequately represents the surface temperature, and that the effect of all other site specific variables tends to average out over the 6- to ll-year study period, a good estimate of ice growth rates for any location can be made with this single variable.

The site-specific variables (which include such factors as wind speed, water depth, temperature, current velocity, and sheltering effects) will vary over space, and in many cases, one variable may counteract the effect of another. For example, an unusual winter may result in an increase in shortwave radiation received at a site, but heavy winter snowfalls may act as a buffer and reduce the influence of the incoming solar radiation. Over a period of time, however, these influences should average out to give a general idea of how ice grows during a normal season.

Several algorithms were developed to isolate data observations, based on a specified set of criteria. Despite the fact that the observations are drawn from the entire 6- to ll-year record for that station, they should produce a linear pattern when ice thickness is the y axis and $\sqrt{ADDF}$ is the x axis. Some error will be introduced in assigning the ADDF because of the necessity of estimating the initial date of freeze-over.

In summary, this model can be used if it is kept in mind that the results are produced by a statistical technique, not by an accurate simulation of the physical processes. There are advantages and disadvantages to this approach, but it is in keeping with the expressed purpose. Since no serious attempt will be made to evaluate the impact of other influences, it is desirable to have some summarizing parameter to describe the rate of ice growth at that location. An intra-site comparison of the slope values may reveal the existence of some spatial pattern.

5. THE DATA SETS

The data set used in the regression analysis is actually a composite of two extensive sets that were subsequently merged: ice stratigraphy measurements and average daily temperatures. Since the results of any predictive model are only as valid as the quality of the input, each of these sets will be described. A discussion of the general limitations will help to qualify the results.

5.1 Ice Observation Data

The ice data gathering program was initiated in 1966. On a weekly basis, commencing with freeze-over, paid observers recorded the date, total ice thickness, stratigraphy of the ice, amount of snow cover, and occurrence of any significant meteorological or anthropogenic event, such as the passing of an icebreaking ship. This information was coded for over 100 sites around the Great Lakes and published by Sleator (1978).

The quality of the data is affected by the following factors:

1. Many researchers have indicated that ice growth is strongly influenced by extremely localized factors, such as the existence of water currents, upwellings, depth to the basin floor, and average wind velocity (Bates and Brown, 1979). The observers were required to visit the same spot on a weekly and yearly basis in an effort to standardize these influences. It would be difficult, however, to locate the same spot in undifferentiated terrain. In addition, the presence or absence of snow drifts can significantly alter the depth of the ice cover within a few meters.

2. As with most attempts to monitor nature, the act of ice measurement may alter the process being observed. If a heavy snow cover is present when the ice is bored, the weight on the ice causes lake water to gush to the surface and flood the immediate vicinity. Freezing of this slush results in snow-ice, which may or may not have been measured during the subsequent observations.

3. Ice measurements were taken in areas where observers were available, usually near cities and towns. Settlements are often located "ear river outlets, which introduce currents and thermal effluents into the neighboring lake. Also, air-blown particles are associated with human activities. These can act as nuclei for water droplets, and influence precipitation patterns on a local scale. Dust or ash deposited on the snow and/or ice reduces the albedo of the surface and can alter rates of growth and decay (Mellor, 1964). Given the occurrence of political and economic events during the collection of the ice stratigraphy data, such as the enactment of the Clean Air and Water Bill and the shift to more polluting fossil fuels following the OPEC oil embargo, a degree of undeterminable spatial and temporal variability can be postulated for sites located near population and industrial centers.

4. I" many cases, the record is incomplete. Boring ice with a hand auger is demanding labor, and the far-flung network of sites prohibited efficient administration. I" a few cases, there were serious inconsistencies in the data, causing some measurements to be dropped. Despite these drawbacks, this data set is the most comprehensive gathered for Great Lakes nearshore locations.

For this study, three criteria were used to select 32 stations from the 100 described in Sleator (1978). Of primary importance was the number of seasons for which ice thickness data were available. Eleven years was the maximum length of record. A minimum of 6 years was required. Preliminary examination of the data revealed that stations with shorter records yielded an insufficient number of observations for the various regression analyses. Secondly, in some instances, measurements were not taken until long after the ice had formed, Although these data may be useful for developing ice decay models, it was deemed inappropriate to include these s&son records in an ice growth study, since no formation period could be defined. Finally, an important consideration for including a station for analysis was the availability of accurate and complete air temperature data. Although this restraint will be further discussed in the next section, it is sufficient to note that those sites without associated winter air temperature data are useless for this study. The ice measurement sites selected are listed in table 2.

TABLE 2.--Ice *measurement sites and representative weather stations*

Ice station number	Location	Name of site	Number of years of record	Weather station	Distance from site (km)	Distance from coast (km)
108	N46-28/W084-28	Mosquito Bay	9	Sault Ste. Marie wso, Mich.	la	
109	N46-31/W084-36	Gros Cap Lighthouse	10	Sault Ste. Marie wso, Mich.	22	
114	N46-32/W085-01	Tahquamenon Bay	a	Whitefish Point, Mich.	29	1
120	N46-25/W086-39	South Bay--Munising	9	Munising, Mich.	1	1
123	N46-46/W088-28	Lanse Bay--Keweenaw Bay	9	Baraga 5 WNW, Mich.	16	13
127	N47-02/W088-31	Portage Lake--Keweenaw waterway	10	Houghton FAA AP, Mich.	6	10
129	N46-35/W090-55	Chequamegon Bay--Ashland	10	Ashland Experimental Farm, Wis.	5	4
132	N46-46/W092-06	Duluth Harbor	11	Duluth Harbor Station, Minn.	1	
138	N47-57/W089-39	Grand Portage Bay	9	Grand Portage Ranger Station, Minn.	1	1
152	N46-30/W084-37	Point Iroquois	a	Sault Ste. Marie WSO, Mich.	24	
200	N44-34/W087-55	Green Bay	9	Green Bay WSO AP, Wis.	14	13
209	N45-45/W087-03	Little Bay de Noc--Escanaba	7	Escanaba, Mich.	1	
214	N45-23/W085-00	Little Traverse Bay--Petosky	a	Petosky. Mich.	1	
218	N43-15/W086-20	Muskegon Lake--Snug Harbor	6	Muskegon WSO AP, Mich.	a	5
220	N45-06/W087-36	Menominee	6	Marinette, Wis.	1	1
302	N46-13/W084-10	Lake Munuscong	9	Dunbar FES, Mich.	16	1
303	N46-06/W084-03	Raber Bay	11	Dunbar FES, Mich	14	1
304	N46-01/W084-41	St. Martin Bay	a	Cheboygan, Mich.	43	1
306	N45-46/W084-43	Mackinaw City	10	Cheboygan, Mich.	22	1
308	N45-03/W083-26	Thunder Bay--Alpena	10	Alpena Water Treatment Plant, Mich.	1	
309	N44-02/W083-36	Point Lookout--Saginaw Bay	10	East Tawas, Mich.	30	
310	N43-59/W083-49	Wigwam Bay--Saginaw Bay	6	East Tawas, Mich.	45	
314	N46-00/W083-54	De Tour--St. Marys River	6	De Tour Village, Mich.	3	
400	N42-08/W080-08	Marine Lake--Erie Harbor	9	Erie WSO AP, Pa.	a	1
402	N41-55/W083-19	Brest Bay	8	Monroe Sewage Treatment Plant, Mich.	1	
406	N42-45/W078-53	Buffalo Harbor	7	Buffalo WSFO AP. N.Y.	19	16
408	N42-33/W082-52	Marblehead--Catawba Island	7	Put-in-Bay(Perry Monument), Ohio	13	
410	N42-40/W082-43	Lake St. Clair--New Baltimore	10	Mt. Clemens AF Base. Mich.	14	1
500	N43-12/W077-31	Irondequoit Bay--Rochester	10	Rochester WSO AP. N.Y.	21	19
502	N43-39/W076-11	North Harbor	9	Oswego East, N.Y.	30	1
503	N43-52/W076-13	Henderson Harbor	6	Watertown FAA AP, N.Y.	22	3
504	N44-05/W076-21	Wilson Bay	9	Watertown FAA AP, N.Y.	27	3

WSD = Weather Service Office. FAA = Federal Aviation Administration. AP = Airport.
FES = Forest Experimental Station. WSFO = Weather Service Forecast Office. AF = Air Force.

5.2 Air Temperature Data

This study attempts to describeice growth as a function of air temperature. Air temperature records were acquired from the Climatological Data Monthly Bulletins for the various states bordering the Great Lakes (NOAA, 1966-78). When deciding which weather station should be considered representative of the lake site, the following considerations were employed:

1. The proximity of the weather station to the lake measuring site was the prime criterion for accepting or rejecting temperature data as representative. In some cases, the linear distance exceeded 30 km. (See table 2.) This was especially true in the sparsely settled Lake Superior region. The upper limit for inclusion was arbitrarily set at 50 km; weather stations located beyond this distance were considered unrepresentative of the site.

2. Often, weather stations were located inland at ranger outposts Or airports. Obviously, the meteorological conditions within a forest or at an air field will not be the same as those on a large lake. However, these were the only data available. For this study, no effort will be made to adjust these data to account for distance or inland conditions.

3. Often, the air temperature record at a station was incomplete. In these instances, the year was excluded from the study.

4. In several cases, two or three ice measuring sites ware located near each other. When this occurred, it was necessary to assign the same temperature data as representative of all the ice thickness sites, or to exclude them from the study. In view of the stated purpose, it was considered appropriate to select the former alternative.

Since most of the ice observations were taken at l-week intervals, it. is possible to sum the daily degree-days to obtain a weekly DDF value (WDDF). This statistic reflects the magnitude of the thermal departure from 0°C over the time interval, which can then be correlated to the observed ice thickness recorded for that period.

A computer program was designed to merge the temperature and ice thickness data sets, compute the WDDF between each observation, and accumulate the WDDF to create a new value, accumulated DDF (ADDF). This parameter quantifies the number of degree-days accumulated since the onset of ice growth.

Assigning the starting day of ice growth for the season was often difficult. The records did not specifically identify freeze-over, although in some cases estimates were made by the observers. Usually, the first recording occurred when the ice slab was sufficiently thick to support the observer's weight. The problem can best be illustrated with an example. The data set shows that on January 1, 1969, no ice was recorded at Raber Bay, but 7 days later, on January 8, there was 12 cm of lake ice. During this period, 105 DDF were accumulated. Given the nature of the information, however, it is impossible to determine the exact date of freeze-over and, thus, to compute the appropriate DDF. However, because any error will be carried over the entire season and averaged over a 6- to 11-year period, and will be reduced as a result of the square root transformation, it was decided to use the first day after the last observance of open water as the date of freeze-over. Referring to the previous example, it would be assumed that ice growth began on January 2, and all summations for the season would begin on that date.

Preliminary analyses revealed that degree-days of thaw (DDT) had a disruptive influence on the regression technique. In the initial regression analyses, ice thickness was consistently overestimated for those observation dates where DDT were simply subtracted from ADDF. This implies that DDT have a greater effect on the change in the ice thickness than the same number of DDF.

It was considered undesirable to include any observation that was preceeded by a thaw event of sufficient magnitude to either (1) result in accumulated DDT's over the time interval, or (2) result in a decrease in ice thickness. In a sense, the determination of the end of the growing season for any site occurred whenever the ice thickness decreased over one observation interval, regardless of the increased ice thickness later in the season. This became the working definition assigning the growth period.

6. RESULTS

It would have been desirable, as a first step, to isolate those observations whose stratigraphy was pure black ice with no snow cover and compare the site growth coefficients to the maximum theoretical value (3.47) computed by equation (4). However, only 218 of the 3,850 cases (5.6 percent) reported in Sleator (1978) for these 32 sites met these criteria. These observations were unevenly distributed between the sites and were judged to be insufficient to make significant statistical statements. A less direct method for determining the relative impact of snow-free conditions on ice growth rates will be described later in this paper.

6.1 Data Set 1

For the initial analysis, ice thickness observations for the 32 sites were included if they met the following requirements: (1) the ice slab was covered with snow, (2) the stratigraphy could be mixed, with black and/or snow-ice being represented, and (3) no earlier thaw event had occurred in that winter of sufficient magnitude to result in ice thickness reduction relative to the previous week.

The resulting data set, called data set 1, contained 625 observations.

To determine the site-specific slopes for data set 1, individual simple linear regression analyses were performed on all sites with more than four observations. This limit was arbitrarily selected as the minimum number of observations necessary for testing, and this rule was maintained throughout the study. The results are summarized in table 3, where b is the slope

TABLE 3.--*Regression results* **for data** *set 1 (snow-covered cases)*

Site number	Name of site	(N) Number of cases	(b) Regression slope	(SD) Standard deviation	(R^2) Coefficient of determination	(SEE) Standard error of estimate
108	Mosquito Bay	18	1.48	0.22	0.74	4.84
109	Gros Cap Lighthouse	30	1.75	0.25	0.63	8.27
114	Tahquamenon Bay	14	2.18	0.30	0.81	7.59
120	South Bay--Munising	27	2.73	0.25	0.83	7.36
123	Lanse Bay--Keweenaw Bay	45	1.78	0.19	0.67	7.22
127	Portage Lake--Keweenaw waterway	48	1.65	0.10	0.84	5.55
129	Chequamegon Bay--Ashland	53	2.56	0.16	0.83	9.13
132	Duluth Harbor	7	2.64	0.47	0.87	7.30
152*	Point Iroquois	21	1.18	0.32	0.64	7.68
200	Green Bay	29	1.69	0.27	0.58	10.84
209*	Little Bay de Noc, Escanaba	22	1.87	0.43	0.48	II.78
214	Little Traverse Bay--Petosky	11	2.40	0.50	0.72	5.71
218	Muskegon Lake, Snug Harbor	9	3.42	0.39	0.92	3.62
220	Menominee	25	2.86	0.21	0.89	5.30
302	Lake Munuscong	32	2.57	0.17	0.88	6.47
303	Raber Bay	40	2.37	0.29	0.63	11.88
304	St. Martin Bay	26	2.03	0.24	0.74	7.20
306	Mackinaw City	16	3.24	0.62	0.66	10.02
308	Thunder Bay--Alpena	5	1.73	0.57	0.76	3.58
309	Point Lookout--Saginaw Bay	13	3.15	0.52	0.77	8.21
314	De Tour--St. Marys River	13	1.33	0.37	0.54	8.50
400	Marine Lake--Erie Harbor	13	2.80	0.23	0.93	4.60
402	Brest Bay	9	2.84	0.19	0.97	3.66
406*	Buffalo Harbor	5	4.17	0.89	0.88	5.66
408	Marblehead--Catawba Island	12	2.86	0.48	0.78	8.38
410	Lake St. Clair--New Baltimore	10	2.54	0.21	0.95	2.87
500	Irondequoit Bay--Rochester	29	2.25	0.23	0.78	5.12
502	North Harbor	22	2.20	0.25	0.80	6.01
503	Henderson Harbor	7	2.95	0.39	0.92	6.17
504	Wilson Bay	14	3.94	0.47	0.85	8.89
Weighted average		577	2.31	0.26	0.79	7.74

* = Excluded sites; see text.

coefficent of the regression line, SD refers to the standard deviation around the regression line, and R^2 is the coefficient of determination.

The column heading SEE refers to the standard error of estimate of the variation of the y values around the regression line. It is defined as

$$SEE = \sqrt{\frac{\sum(\hat{y}_i - y_i)^2}{n}} , \qquad (10)$$

where $\hat{y}_i$ = the computed y value for x, based on the slope and a-intercept values,

y_i = actual y value associated with x for the case i, i.e., the observed ice thickness.

This is a useful summarizing statistic, since it indicates the degree to which the actual and predicted ice thicknesses are related. Larger SEE values reflect a greater divergence between the observed and expected values over the range of x.

Ultimately, two data sets will be used, and a quantitative method of comparing the overall predictive abilities would be desirable. To this end, a weighted average of the regression results was computed for b, R^2, SD, and SEE for each station. The weighted average, A, is defined as

$$A = \sum_{i=1}^{z} \frac{x_i \times n_i}{n} \qquad (11)$$

where x_i = b, R^2, SD, or SEE values for station i,

i-z = stations represented in data set,

n = number of observations.

Before computing weighted averages for the regression parameters, it is necessary to eliminate some of the sites from each data set for the following reasons:

1. In those instances where the R^2 value was less than 0.50, the correlation is considered to be insignificant; those site observations will not be included in the calculation of the summarizing parameters of that particular data set.

2. Some sites have computed growth coefficients that exceed the maximum theoretical value derived earlier (3.47). Recognizing that the values for Ki and density can fluctuate in the natural system, a 20-percent leeway will be allowed for the maximum value of b; i.e., (3.47 × 1.20) ≅ 4.15. Any computed growth coefficient in excess of this value will be excluded from

the computation of A and is considered to indicate either factors operating beyond the scope of those considered in this paper or inaccurate data.

In the tables, those sites that will not be included have been identified by an asterisk (*).

The weighted averages are summarized at the bottom of table 3 for the 27 stations that met the stated criteria in data set 1.

6.2 Data Set 2

In an effort to acquire a slope value that would accurately reflect both the snow and snow-free conditions on any ice stratigraphy, an expanded data set was created. The only constraint imposed on the file search was that no melt event shall have occurred of such magnitude that the actual ice thickness was reduced.

Data set 2 is composed of 843 observations, or 21.9 percent of the original data set. Since a significant number of snow-free cases have been introduced, it is expected that the overall slope parameter will increase. These values, though based on less stringent criteria, should also produce more realistic results when predicting ice growth under the variety of conditions experienced at that site during the years of record. The regression results for the 32 sites are listed in table 4.

In accordance with established procedures, weighted values were again computed for this data set; they are listed at the bottom of table 4.

A summary of these weighted parameters for the two data sets should prove to be useful for the discussion below.

	n	b	R^2	SD	SEE
Data set 1	577	2.31	0.79	0.26	7.74
Data set 2	843	2.45	0.81	0.23	7.01

Clearly, the close correspondence of values reflects the fact that data set 1 is a subset of data set 2.

It should be noted that no a-values (y-intercepts) have been reported for the regression results. Theoretically, the a-value should be 0.0 if the date of freeze-over is known and the data are normally distributed over the range of x. However, this was not the case. In data set 2, a-values ranged from 3.5 to -24.0 cm, with a mean of 7.0 cm. Seven of the a-values were positive, and 60 percent were within 7 cm of 0.0. Such results suggest that the previously described method of establishing the date of freeze-over consistently caused an overestimation of the accumulated degree-days.

TABLE 4.--*Regression results for data set 2 (combination of snow and snow-free conditions)*

Site number	Name of site	(N) Number of cases	(b) Regression slope	(SD) Standard deviation	(R^2) Coefficient of determination	(SEE) Standard error of estimate
108	Mosquito Bay	18	1.48	0.22	0.74	4.84
109	Gros Cap Lighthouse	39	1.90	0.23	0.65	9.20
114	Tahquamenon Bay	16	2.31	0.25	0.85	7.57
120	South Bay--Munising	29	2.73	0.24	0.82	7.33
123	Lanse Bay--Keweenaw Bay	49	1.91	0.18	0.71	7.30
127	Portage Lake--Keweenaw Waterway	48	1.65	0.10	0.84	5.55
129	Chequamegon Bay--Ashland	60	2.64	0.14	0.85	8.93
132	Duluth Harbor	47	2.12	0.17	0.77	9.11
138	Grand Portage Bay	20	2.87	0.51	0.64	8.68
152	Point Iroquois	22	1.68	0.25	0.68	8.17
200	Green Bay	31	2.03	0.16	0.85	6.45
209	Little Bay de Noc, Escanaba	30	2.92	0.29	0.78	8.92
214	Little Traverse Bay--Petosky	11	2.91	0.50	0.72	5.71
218	Muskegon Lake, Snug Harbor	14	2.90	0.19	0.95	3.43
220	Menominee	31	2.73	0.16	0.90	5.52
302	Lake Munuscong	36	2.61	0.17	0.88	6.37
303	Raber Bay	41	2.38	0.15	0.86	7.26
304	St. Martin Bay	31	2.03	0.15	0.86	5.62
306	Mackinaw City	21	3.13	0.37	0.79	7.20
308	Thunder Bay--Alpena	11	1.96	0.25	0.87	4.03
309	Point Lookout--Saginaw Bay	22	3.41	0.29	0.88	7.65
310	Wigwam Bay--Saginaw Bay	6	3.25	I.16	0.66	5.28
314	De Tour--St. Marys River	17	2.38	0.39	0.71	9.82
400	Marine Lake--Erie Harbor	19	2.54	0.21	0.90	5.19
402	Brest Bay	19	3.16	0.22	0.92	4.89
406	Buffalo Harbor	5	4.17	0.89	0.88	5.66
408	Marblehead--Catawba Island	16	3.03	0.38	0.82	7.75
410	Lake St. Clair--New Baltimore	16	2.49	0.29	0.84	5.26
500	Irondequoit Bay--Rochester	32	2.09	0.21	0.76	5.38
502	North Harbor	37	2.31	0.19	0.82	6.01
503	Henderson Harbor	14	2.72	0.25	0.91	5.99
504	Wilson Bay	35	3.87	0.27	0.87	7.67
Weighted average		843	2.45	0.23	0.81	7.01

An effort was made to adjust the data by solving for the x-intercept of the site regression analysis. The square of the x-intercept was subtracted from all the x values in the site data sat (where "a" was less than 0.0). A least squares regression analysis was performed on the adjusted data set for 11 sites representing the range of slope values, y-intercepts, and sample sizes. In no case did the adjusted slope differ significantly from the original site slope at the 95-percent confidence level. The a-values were reduced in magnitude, with a range of from 4.19 to -10.12 cm, but 10 of the 11 a-values were negative.

Finally, for purposes of comparison, the slope was forced through the points 0,0 and $\bar{x},\bar{y}$ for the 11 adjusted site data sets. In all cases, the resulting slope was significantly shallower, the R^2 value decreased, and the SEE increased.

In this paper, the emphasis is on incremental ice growth as a function of site characteristics that vary over space. The impact of the a-values cannot be evaluated at this time in terms of predicting the thickness of ice at the site. It would appear that, if the date of freeze-over can be accurately established, the effect of the a-value on the computed ice thickness will be minimal.

7. DISCUSSION OF RESULTS

The overall effect of introducing snow-free cases to the accurate computation of the ice growth coefficient (b) is of primary importance. It is expected that the individual and overall h values will increase to reflect the inhibiting impact of the snow cover. It should be noted, however, that both pure black and mixed stratigraphy snow-free observations have been added in data set 2. The fact that snow-ice exists within the ice profile indicates that snow was present on the slab earlier in the season.

Table 5 quantifies the relative changes that occurred to the b value as a result of adding the snow-free cases. The average change in slope at any station represented in both data sets was about 1.08 percent. This value, though small, does support the notion that growth rates increase when the snow cover is removed. Similarly, the weighted average computation for b increases from 2.31 to 2.45 with the addition of 266 snow-free cases.

Any regression technique that maximizes the R^2 value should be more powerful for predicting ice growth. For most sites, an increase in the number of cases had the hoped-for effect of increasing the R^2 value while reducing the SD and SEE. However, one is then faced with a problem exemplified by station 410. The results of the regression analyses performed on each data set for station 410 are summarized below.,

	N	b	R^2	SD	SEE
Data set 1	10	2.54	0.95	0.21	2.87
Data set 2	16	2.49	0.84	0.29	5.26

TABLE 5.--*Relative influence of snow-free cases on computed ice growth coefficients*

site number	+ΔN	b_2/b_1	site number	+ΔN	b_2/b_1
108	0"	1.00	303	1	1.00
109	9	1.09	304	5	1.00
114	2	1.06	306	5	0.97
120	2	1.00	308	6	1.13
123	4	1.07	309	9	1.08
127	0"	1.00	310	6*	
129	7	1.03	314	4	1.79
132	40	0.80	400	6	0.91
138	20*		402	10	1.11
152	1	1.42	406	0*	1.00
200	2	1.20	408	4	1.06
209	8	1.56	410	6	0.98
214	0*	1.00	500	3	0.93
218	5	0.85	502	15	1.05
220	6	0.95	503	7	0.92
302	4	1.02	504	21	0.98

Total +ΔN = 218 Average b_2/b_1 = 1.076

* = Not used in computation of average.
+ΔN = Number of additional cases introduced into data set 2.

In this case, by increasing the number of observations, the R^2 value is reduced and the SD and SEE are increased. This is contrary to the desired results. Note, however, that the slope has not been altered significantly, which implies that the additional data were evenly distributed about the regression line. Although the small sample size prohibits a definitive statement, it does support the notion that the slope parameter is a good first estimate of the rate of ice growth at a particular site.

Increasing the sample size had an additional effect. It now became possible to estimate a slope for those stations where previously no slope could be estimated because of the small sample size. Station 138 with n = 20 and station 310 with n = 6 are represented for the first time. The validity of station 310 is questionable, given the large SD. The results are considered to be without sufficient foundation to justify the "se of b as a predictive parameter.

One can quantify the influence of the two differing slope coefficients by substituting the b value for 3.47 in equation (4). For example, the following ice thicknesses have been computed for station 123 by "sing a range of accumulated degree-days (ADDF).

			ADDF			
Data set	N	b	500	750	1000	1500
1	45	1.78	39.8 cm	48.7 cm	56.3 cm	68.9 cm
2	49	1.91	42.7 cm	52.3 cm	60.4 cm	74.0 cm

Clearly, the divergence of the predicted values for ice thickness increases over time. Given that the ADDF rarely exceeds 1000 for any of these sites, there is roughly 7-percent uncertainty in the computed ice thickness, depending on the value of b used. This uncertainty increases with wider divergence between the two slope coefficients, such that if the difference exceeds 0.35, there is greater than 15-percent uncertainty in the computed ice thickness.

For the purpose of establishing significant representative slope coefficients for the individual sites, the following criteria will be applied to data set 2:

1) More than 10 observations are required.

2) The R2 value will be greater than 0.60.

3) The quotient value of SD/b will be less than 0.15.

Table 6 lists the resulting 27 stations and the weighted average parameters.

TABLE 6.--*Summary of sites with significant regression results*

Site number	Name of site	(N) Number of cases	(b) Regression slope	(SD) Standard deviation	(R^2) Coefficient of determination	(SEE) Standard error of estimate
108	Mosquito Bay	18	1.48	0.22	0.74	4.84
109	Gros Cap Lighthouse	39	1.91	0.23	0.65	9.20
114	Tahquamenon Bay	16	2.31	0.25	0.85	7.57
120	South Bay--Munising	29	2.73	0.24	0.82	7.33
123	Lanse Bay--Keweenaw Bay	49	1.91	0.18	0.71	7.30
127	Portage Lake--Keweenaw Waterway	48	1.65	0.10	0.84	5.55
129	Chequamegon Bay--Ashland	60	2.64	0.14	0.85	8.93
132	Duluth Harbor	47	2.12	0.17	0.77	9.11
152	Point Iroquois	22	1.68	0.26	0.68	8.17
200	Green Bay	31	2.03	0.16	0.85	6.45
209	Little Bay de Noc, Escanaba	30	2.92	0.29	0.78	8.92
218	Muskegon Lake, Snug Harbor	14	2.90	0.19	0.95	3.43
220	Menominee	31	2.73	0.16	0.90	5.52
302	Lake Munuscong	36	2.61	0.17	0.88	6.37
303	Raber Bay	41	2.38	0.15	0.86	7.26
304	St. Martin Bay	31	2.03	0.15	0.86	5.62
306	Mackinaw City	21	3.13	0.37	0.79	7.20
308	Thunder Bay--Alpena	11	1.96	0.25	0.82	4.03
309	Point Lookout--Saginaw Bay	22	3.41	0.29	0.88	7.65
400	Marine Lake--Erie Harbor	19	2.54	0.21	0.90	5.19
402	Brest Bay	19	3.16	0.22	0.92	4.89
408	Marblehead--Catawba Island	16	3.03	0.38	0.82	7.75
410	Lake St. Clair-New Baltimore	16	2.49	0.29	0.84	5.26
500	Irondequoit Bay--Rochester	32	2.09	0.21	0.76	5.38
502	North Harbor	37	2.31	0.19	0.82	6.01
503	Henderson Harbor	14	2.72	0.25	0.91	5.99
504	Wilson Bay	35	3.87	0.27	0.87	7.67
Weighted averages		184	2.42	0.20	0.82	6.95

In an effort to determine the distribution characteristics of the site slope values, these slopes were standardized and converted to Z-scores using the formula (x_i -:)/SD.

It is interesting to note that, when descriptive measures of central tendency were computed for the b values as a set ($N = 27$), they closely approximated those obtained using a weighted average; this tends to validate the averaging technique. Both mean slope values are nearly equal to the median (2.49).

	$\bar{x}$	SD
Weighted average	2.42	0.20
Central tendency	2.47	0.56

When the slope values were converted to Z-scores, and classified on the basis of the computed standard deviations from the mean (0.56), the graphic representation as a histogram closely approximated a normal curve, though somewhat peaked and skewed to the right. However, a chi-square test conducted at the 95-percent confidence level indicates that the data are normally distributed. Only one slope value (site 504, b = 3.87) lies beyond 2 SD's.

It would be useful to have one value that could be used as a general growth rate constant for ice nearshore locations in the Great Lakes. Clearly, the mean value could be used in this capacity.

Using 2.42 as the mean value, it would be interesting to observe how much the predicted ice thickness would diverge from the true ice thickness over time at those sites with the highest and lowest growth coefficients [sites 504 (3.87) and 108 (1.48), respectively]. The following table indicates the result of those determinations, employing $Z(cm) = b\sqrt{ADDF}$.

	ADDF					
b	500	750	1000	1250	1500	Acm at 1500
1.48	33.1 cm	40.5 cm	46.8 cm	52.3 cm	57.3 cm	-36.4
2.42	54.1 cm	66.3 cm	76.5 cm	85.6 cm	93.7 cm	-
3.87	86.5 cm	106.0 cm	122.4 cm	136.8 cm	150.0 cm	56.3

A similar calculation can be undertaken for the range of sites, as defined in terms of their Z-scores, using a standard deviation value of 0.56.

SD units	ADDF b	500	750	1000	1250	1500	Δcm at 1500
(-1)-(-2)	1.30	29.1 cm	35.6 cm	41.1 cm	46.0 cm	50.3 cm	-43.4
0-(-1)	1.86	41.6 cm	50.9 cm	58.8 cm	65.8 cm	72.0 cm	-21.7
x	2.42	54.1 cm	66.3 cm	76.5 cm	85.6 cm	93.7 cm	-
0-1	2.98	66.6 cm	81.6 cm	94.2 cm	105.4 cm	115.4 cm	21.7
1-2	3.54	79.2 cm	97.0 cm	112.0 cm	125.2 cm	137.1 cm	43.4

It is clear that 65 percent of the sites (± 1 SD) located in areas that average only 750 ADDF during the winter will be within 15 cm of the mean thickness at the end of the ice growth period. Those sites that have Z-scores in excess of 1 SD and/or are exposed to more severe winters will diverge more from the mean thickness.

Before discussing the spatial attributes of the growth coefficients, it is of value to compare the regression results listed in table 6 with those reported by Assel (1976). The study reported here found a weighted mean SEE of 6.95 cm when using the linear relation between ice thickness and the square root of ADDF as presented in equation (4).

For Assel (1976), the best estimating equation (SEE of 6.99 cm) was multilinear. Ice thickness was computed as a function of ADDF and thawing degree-days accumulated during the prior warm season.

One important difference between the two estimating equations is the role played by ADDF. Assel began DDF accumulation with the first occurrence of freezing air temperatures. This study, owing to the use of the Stefan equation, begins DDF accumulation with the formation of the ice cover, a date substantially later than that used by Assel.

Although this model cannot claim an overall improvement in predictive ability, it is based on a larger number of winter seasons (6-11 years as compared with 3-8 years), and it suggests ways of incorporating site-specific information and physical processes to learn more about the effects of snow cover on ice growth rates.

Having generated ice growth coefficients for those sites that are considered statistically significant, it would be desirable to determine whether any spatial patterns exist. Low growth values, over a 6- to 11-year time period, suggest that either (1) heat is consistently entering the system without being accounted for, or (2) deep or continual snow accumulation is acting as an insulator. In both cases. should these conditions persist through time and reflect the "normal" microenvironment of the site, an intersite comparison of growth rates should reveal similarities of site properties and situations that may help isolate those factors that significantly influence ice growth.

Table 7 lists the stations where slopes were determined to accurately represent the relationships between ice thickness and air temperature; these are arranged in order of increasing slope values.

Ideally, it would have been desirable to classify the site-specific slope values by using additional input variables, such as water depth and temperature, current and wind speed and direction, volume and temperature of incoming streams, and average incoming solar radiation for each site. Unfortunately, this technique requires interval data, which is unavailable for most of these sites.

The slope values were correlated to the depth of the water beneath the site on the assumption that this variable would be a good surrogate measure of the heat flow to the base of the ice sheet, a process not accounted for by the ice growth model. Depth of water was used as the independent variable to perform a simple linear regression. The results were not significant, and no linear trends are apparent.

A binary system was used to record various data for each site. Sites possessing that characteristic to the required degree were indicated with an (X) in the appropriate column. This information (columns 3-6) was gleaned from nautical and topological maps and augmented by the ice observation reports.

The interval data were used to generate a value to reflect the average snow cover present at each site. This parameter was determined by summing all recorded snow thickness observations and dividing by the number of observations in data set 2 for that site (column 1). The percentage of snow-free conditions for each site was defined as the ratio between snow-free and snow-covered observations.

The slope values, when mapped by rank, produced no apparent spatial trends (fig. 3). With several possible exceptions, the data in table 7 proved to be of little value in explaining varying ice growth rates over space.

Note that the lowest growth rate (station 108) is located in an area where currents are likely to exist: Mosquito Bay at the head of the St. Marys River. The relatively low growth coefficient may reflect the dampening influence of the currents on the growth.

Ice growth rates for sites located in Lake Superior tend to be lower than the average, while those for the other Great Lakes appear to be randomly dispersed. Referring to the data in column 1 of table 8, which represent the average snow cover thickness, it does not appear that heavy snowfall is inhibiting ice growth at these locations. In fact, those sites with relatively deep

TABLE 7.--*Summary of site characteristics and parameters*

Slope	Rank	Station number	Station name and location	Snow[1]	%[2]	Depth[3]	Winds[4]	Currents[5]	City[6]
1.48	1	108	Mosquito Bay	8.6			X	X	
1.65	2	127	Portage Bay. Keweenau	1.4		X		X	X
1.68	3	152	Point Iroquois	2.7	21	X	X	X	
1.90	4	109	Gros Cap Lighthouse	8.9		X	X	X	
1.91	5	123	Lanse Bay. Keweenau	8.8		X	X		
1.96	6	308	Thunder Bay, Alpena	2.1	36		X		X
2.03	7	200	Green Bay	9.7			X		X
2.03	8	304	St. Martin Bay	13.5	23	X			
2.09	9	500	Irondequoit Bay. Rochester	6.8		X			X
2.12	10	132	Duluth Harbor	3.4	26	X	X		X
2.31	11	114	Tahquamenon Bay	8.9			X	X	
2.31	12	502	North Harbor	6.9	30		X		
2.38	13	303	Raber Bay	17.6	12	X			
2.49	14	410	Lake St. Clair	4.1	31		X		
2.54	15	400	Marine Lake, Erie Harbor	7.2					
2.61	16	302	Lake Munuscong	11.0	11		X		
2.64	17	129	Chequamegon Bay, Ashland	12.4			X		X
2.72	18	503	Henderson Harbor	6.4	29	X			
2.73	19	120	South Bay, Munising	9.1		X	X		X
2.73	20	220	Menominee	6.5		X		X	X
2.90	21	209	Little Bay de Noc, Escanaba	14.3		X	X		X
2.92	22	218	Muskegon Lake, Snug Harbor	5.2					X
3.03	23	408	Marblehead, Catawba Island	8.9			X		
3.13	24	306	Mackinaw City	15.2	38		X	X	X
3.16	25	402	Brest Bay	4.1			X		
3.41	26	309	Point Lookout, Saginaw Bay	5.4	36	X	X		
3.87	27	504	Wilson Bay	4.9	29		X		

[1]Sum of snow cover thickness ÷ number of observations = average snow present (centimeters).

[2]Percent of snow-free cases.

[3]Depth of water below site; (X) = greater than 6 m.

[4]Site exposed to lake winds; (X) = yes.

[5]Site affected by water currents or river drainage; (X) = yes.

[6]Site associated with city (>5,000 people) or heavy industry; (X) = yes.

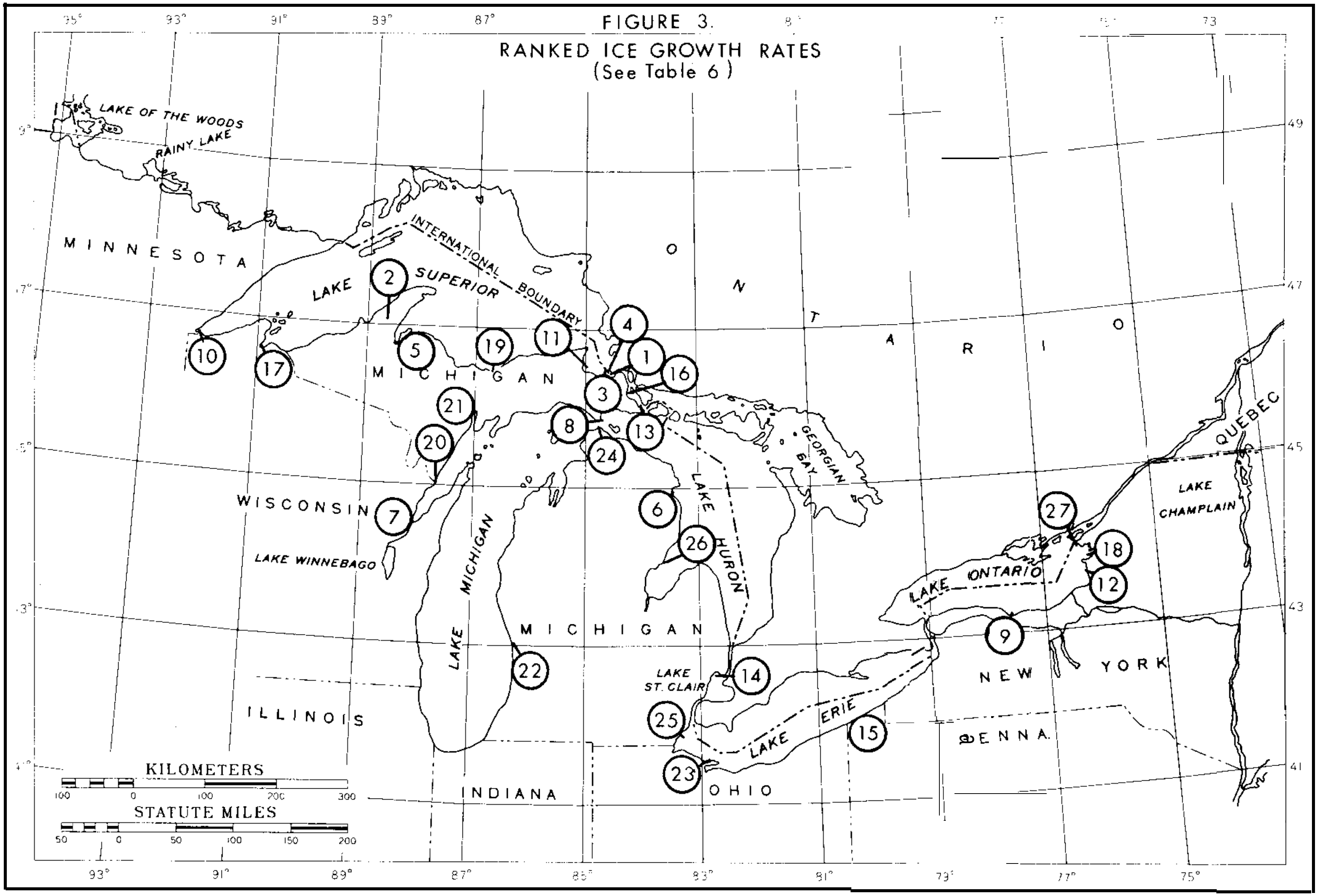

FIGURE 3.--*Ranked ice growth rates. (See table 6.)*

TABLE 8.--*Winter severity index, 1967-79*
(standard deviation units)

Weather station	81-year $\bar{X}$ (ADDF) °C	SD(°C)	Season ending in												
			1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979
Duluth	1267	232	0.547	-0.388	0.134	0.724	0.922	1.582	-0.276	0.651	0.125	0.095	0.862	0.557	1.828
Sault Ste. Marie	1008	196	0.337	0.153	-0.352	1.026	0.939	0.719	-1.321	0.342	-0.393	0.031	1.444	0.688	1.122
Green Bay	790	202	0.104	-0.248	0.045	1.054	1.287	1.005	-0.936	-0.262	-0.376	-0.317	2.025	1.628	1.871
Milwaukee	500	175	0.246	-0.274	-0.177	0.794	1.280	0.657	-0.326	-0.229	-0.554	0.611	2.143	1.549	1.737
Muskegon	370	150	-0.200	0.107	0.200	1.027	0.727	0.273	-0.647	0.327	-0.873	0.820	2.233	1.738	1.934
Alpena	670	179	0.749	0.369	0.140	1.257	0.816	0.709	-0.598	0.268	-0.553	0.251	1.419	1.289	1.196
Detroit	325	146	-0.411	-0.075	-0.397	0.870	0.144	-0.486	-1.123	-0.658	-1.479	-0.336	1.304	2.078	1.369
Toledo	307	155	-0.019	0.368	0.516	1.452	0.503	-0.090	-0.658	0.348	-0.671	0.432	2.903	3.112	1.514
Cleveland	246	145	-0.697	0.883	0.145	1.000	0.593	-0.303	-1.014	-1.021	-0.966	0.372	2.931	1.808	1.023
Buffalo	361	142	-0.042	0.810	0.218	1.239	0.423	-0.239	-0.711	-0.232	-1.493	0.007	1.887	1.757	1.027
Rochester	364	143	-0.580	0.741	-0.126	1.224	0.853	-0.329	-1.056	-0.455	-1.399	-0.042	1.371	1.287	1.047
Regional mean			0.003	0.222	0.031	1.061	0.772	0.318	-0.788	-0.084	-0.785	-0.085	1.873	1.590	1.424

From Assel (1980) and DeWitt et al. (1980).

Note: Positive values reflect colder-than-normal conditions.

average snow cover (sites 304, 303, 302, 129, 209, and 306) are associated with the entire range of growth rates. This tends to support the notion that a heavy snow cover can cause two opposite results, as discussed previously.

In addition, the snow cover on Lake Superior is "ore likely to experience prolonged periods of thermal-constructive metamorphism than other sites, owing to its latitude and continentality, producing a hoar layer at the ice/snow interface and effectively inhibiting the transfer of heat to the surface.

Also, it should be recalled that it was necessary to use the same. weather station data for several sites. (See table 2.) Note that stations 108, 109, and 152, all located within 6 km of each other, had very similar ice growth rates. Temperature data for these three sites came from Sault Ste. Marie, located 24 km away.

In the case of stations 503 and 504, different (yet significant) growth rate values were produced by use of the same air temperature data. Both sites were located about 25 km from the Watertown Weather Station, in opposite directions. Thus, it appears that this particular methodology had no adverse effect on the results.

8. APPLICATION TO FIELD DATA

The results of these analyses were based on observations for the winters of 1966-67 to 1976-77. Data available in Assel (1980) and DeWitt et al. (1980) can be used to quantify the severity of these winters on a regional scale relative to an 81-year average. ADDF were used to define sites in terms of their standard deviation from the 81-year mean for each year of record. By averaging these site values for each year, a regional measure of the winter's severity can be generated. (See table 8.) Positive standard deviation values represent colder-than-normal conditions.

It is clear that, in general, the period 1966-77 experienced winters that were more severe than normal, with 7 of 11 years colder than normal, 2 of them exceeding 1 SD. The winter of 1976-77, with a regional value of 1.873 SD, is ranked by Kahlbaum and Keyes (in DeWitt et al., 1980) as the fifth most severe winter on the Great Lakes since 1779.

To determine how well the site-specific growth rates predict ice thickness for years not included in the data sets, a comparison of simulated and observed ice thickness was made for the winters of 1977-78 and 1978-79. Ice stratigraphy data gathered during the ice-observation program but not published in Sleator (1978) were used for the comparison.

Note from table 8 that these two winters experienced below-normal thermal conditions at every site for both of these years. Regionally, the standard deviation values are about 1.5. Thus, the predictive slope values will be tested under abnormal conditions. Since these parameters were themselves computed from a data set that, overall, contained a preponderance of colder-than-normal years, this should not pose a serious problem. In fact, if the simulation accuracy is as good as that determined for the significant growth rate in data set 2, it would tend to validate the use of the slope as a predictive tool.

The previously described methodology was used to merge the new ice observation data with temperature data from the nearest weather station. However, there were too few observations per site for the 2 test years to generate a new b value. Of the 34 site records, 25 had fewer than 10 observations. Therefore, it was decided to use the general slope value computed previously for that particular site to predict ice thickness with the appropriate ADDF, and compare the expected result with that observed in the field.

Initially, the formula $\hat{Y} = b\sqrt{ADDF}$ was applied to the yearly data. However, it was noticed that in some cases the computed $\hat{Y}$ were consistently high or low throughout the season. This error is related to the problem of accurately identifying the initial date of freeze-over. To overcome this problem, the regression line was forced to pass through the point representing the mean of x and the mean of y, and a y-intercept value (a) was computed. In a sense, the magnitude of the y-intercept indicates the accuracy of the initial estimate of freeze-over. As can be seen in table 9, only 7 of the 34 site years deviated from 0 by more than 10 cm.

Using this technique, the modified formula for estimating ice growth would be

$$\hat{Y} = a + b\sqrt{ADDF}\ , \qquad (12)$$

where a = y-intercept for that site during that year.

To summarize the accuracy of the fit of the general site growth rate. the SEE was computed, using n-l degrees of freedom. Resulting standard errors are listed in table 9.

To determine the relative predicting accuracy, a weighted average was again used to determine the general SEE. The 1978-79 winter for station 408 was excluded because it contained only three observations. The resulting SEE value of 5.39 compares well to that computed for the significant sites in data set 2 (6.95).

It is clear that, if the initial date of freeze-over can be identified, a reasonably accurate estimation of ice thickness can be made at these sites.

9. SUMMARY

Nearshore sites are located in the contact zone between three different ecosystems--the land, the water, and the atmosphere. The effect of one upon the other causes these sites to be complex in terms of the operating processes and difficult to model over time and space.

TABLE 9.--*Application of computed ice growth parameters to 1977-78 and 1978-79 observed ice thickness data*

site number		General slope value	a (y-intercept) (cm)	Number of cases	Standard error of estimate (cm)
108	78	1.48	1.006	9	3.98
109	78	1.91	9.201	9	3.59
	79	1.91	10.47	7	6.64
120	78	2.73	-22.075	4	8.13
	79	2.73	-6.03	8	7.16
123	78	1.91	1.22	11	3.96
	79	1.91	0.58	9	7.29
127	78	1.65	-5.81	17	3.22
	79	1.65	-4.75	11	5.87
129	78	2.64	-1.82	11	7.67
	79	2.64	-16.79	14	11.84
132	78	2.12	-2.55	7	5.74
152	78	1.68	6.96	8	3.16
	79	1.68	8.85	6	4.96
200	78	2.03	0.88	7	8.96
220	78	2.73	0.59	10	4.18
	79	2.73	-8.84	10	5.46
302	78	2.61	-15.08	9	3.80
	79	2.61	-17.75	8	6.42
303	78	2.38	-8.83	9	4.72
	79	2.38	-12.31	7	6.80
304	78	2.03	0.45	11	6.23
308	78	1.96	-8.90	5	9.52
309	79	3.41	-14.95	6	5.48
400	78	2.54	-5.92	9	4.45
402	78	3.16	-8.29	7	5.13
	79	3.16	-5.36	6	3.61
408	78	3.03	0.004	6	4.04
	79*	3.03	-8.61	3	6.02
410	79	2.49	-8.59	6	4.57
500	78	2.09	-2.83	9	2.48
	79	2.09	-9.13	9	2.67
502	78	2.31	7.70	15	5.10
	79	2.31	0.56	9	2.57
TOTAL				292	
AVERAGE					A = 5.39

*Excluded site.

This paper presents the results of an initial attempt to analyze a large data set composed of ice stratigraphy and associated air temperatures. Statistically significant site-specific growth coefficients were generated for 27 nearshore locations by a simple parastatistical model.

The results indicate that rates of ice growth are strongly influenced by the snow cover and dynamic site-specific factors. Individual site growth coefficients summarize the cumulative effect of these processes over a 6- to 11-year period. When applied to unpublished ice stratigraphy data for two abnormally cold winters, a reasonable degree of correlation between observed and expected ice thickness was achieved on a regional scale.

At a smaller scale, the allowable estimating error will depend on the intended application. Engineering projects, for example, may require detailed input data. In these instances, site-specific data gathering programs must be developed to measure those variables that are known or suspected to influence ice growth significantly. Although this study was not designed to pinpoint those variables, it did serve to illustrate the relative impact of these factors on ice growth.

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