NOAA Technical Memorandum ERL GLERL-90

GREAT LAKES PRIMARY PRODUCTION MODEL—METHODOLOGY AND USE

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ATMOSPHERIC ADMINISTRATION / Environmental Research Laboratories

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GREAT LAKES PRODUCTION MODEL—METHODOLOGY AND USE

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ABSTRACT. The Great Lakes Production Model (GLPM) estimates *in situ* integral daily production, accounting for diel variations in surface irradiance and depth variations in photosynthesis-irradiance parameters, algal biomass, and light extinction. A comparison of integral production estimates in the northern Gulf of Mexico and Lake Michigan obtained using the GLPM with those obtained by *in situ* and simulated *in situ* techniques indicated good agreement. In an effort to obtain estimates of primary production at sites where P-I parameters are not available, a version of the GLPM was designed to run in a monte carlo mode. A model sensitivity analysis indicates that the model is most sensitive to changes in two input parameters: the light extinction coefficient and algal biomass. Model framework and background are presented, input terms are defined, and example output is displayed.

1. INTRODUCTION

Phytoplankton are the dominant primary producers in large bodies of water such as the world's oceans or the Laurentian Great Lakes, where the littoral zone is confined to a very narrow region. Since the 1950s, phytoplankton production, or photosynthesis, has been measured in the Great Lakes using a variety of techniques, e.g. oxygen evolution, pH increases, and uptake of $^{14}CO_2$ (Putnam and Olson, 1961; Saunders et al., 1962; Verduin, 1962). The most widely used technique has been the ^{14}C technique pioneered by Steeman-Neilsen (1952). This technique allows for relatively accurate estimates of photosynthesis in very oligotrophic waters.

Early application of the ^{14}C technique has focused on the use of *in situ* incubations to estimate *in situ* primary production (Vollenweider, 1969). In many cases, particularly in small lakes, this is still the method of choice (Wetzel and Likens, 1991). However in large bodies of water, it is logistically difficult to use *in situ* incubations to estimate primary production; their large size prohibits the wide-spread application of *in situ* incubations. To alleviate this logistical difficulty and yet still provide for *in situ* estimates, solar-stimulated incubations have been used (Lohrenz et al., 1992b). These experiments are very similar to *in situ* experiments except they take place in a shipboard incubator that simulates *in situ* irradiance and temperature. This type of incubation holds much promise for large bodies of water.

Although *in situ* and solar-stimulated *in situ* incubations can provide estimates of *in situ* production, they provide very limited predictive power because they are a cummulative measure of all variables (e.g. light, temperature, nutritional status, biomass), and thus, provide little insight into the possible effect of changes in light, temperature, or nutrients on rates of primary production. To provide for an *in situ* estimate of primary production in large bodies of water and yet still provide some predictive power, many investigators have employed a mechanistic modeling approach based on a few input parameters, e.g. chlorophyll, photosynthesis-irradiance parameters (P-I), incident irradiation, etc. (Fee, 1972; Jitts et al., 1976; Harrison et al., 1985; Herman and Platt, 1986).

The Great Lakes Production Model (GLPM) estimates *in situ* integral daily production, accounting for diel variations in surface irradiance and depth variations in P-I parameters, algal biomass, and light

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GLERL Contribution No. 993

extinction. The strength of the GLPM is that it accepts discrete measurements of biological and environmental parameters and generates a nearly continuous estimate of primary production in both space and time. In addition, by using a monte carlo approach, the model can be used to (1) predict the range of primary production estimates based on variance associated with certain input parameters, and (2) obtain estimates of primary production at sites where P-I parameters are not available. This technical memo provides documentation for the modeling approach used to estimate phytoplankton production in the Great Lakes since 1983 (Fahnenstiel and Scavia, 1987). Model framework and background are presented, input terms are defined, and example output is displayed.

2. MODEL DEVELOPMENT

Fee (1972, 1973) developed a simple, easily-implemented mathematical approach to compute integral daily phytoplankton production in water bodies. Fee's approach incorporates diel variations in surface irradiance, depth variations in the photosynthesis vs. light (P vs. I) response, and an estimate of vertical light extinction. The water column is divided into a number of discrete vertical intervals in which photosynthesis vs. irradiance measurements are made. Within each depth interval, the instantaneous rate of photosynthesis is determined by interpolating the P vs. I values, using the irradiance at the mid-point of the depth interval. This irradiance is calculated from the time-dependent surface value using an estimate of underwater light extinction. This process is repeated for each time increment and for each discrete depth interval, and the results are summed to generate daily primary production.

Platt et al. (1980) developed an empirical equation that describes the rate of photosynthesis by phytoplankton as a continuous function of available light. The earlier relationships of Jassby and Platt (1976) and Platt and Jassby (1976) between photosynthesis and light were extended to include the range of light intensities above the threshold of photoinhibition. The basic form of the P-I equation is

$$P^B = P_S^B \bullet \left(1 - e^{-\alpha I / P_S^B}\right) \bullet e^{-\beta I / P_S^B} \qquad (1)$$

where P^B is the specific photosynthetic rate at irradiance I, normalized to chlorophyll biomass (mg C•mg chl^{-1}•h^{-1}), P_S^B is the saturated rate of photosynthesis in the absence of photoinhibition (same units as P^B), α is the initial linear slope at low irradiances (mg C•mg chl^{-1}•Einst^{-1}•m^2), I is the depth-specific irradiance (Einst•m^{-2}•h^{-1}), and β is the negative slope at high irradiances (same units as α). The maximum photosynthetic rate at light saturation, P_M^B, is related to P_S^B by the following equation:

$$P_M^B = P_S^B \bullet \left[\frac{\alpha}{(\alpha+\beta)}\right] \bullet \left[\frac{\beta}{(\alpha+\beta)}\right]^{\frac{\beta}{\alpha}} \qquad (2)$$

Values of the photosynthetic parameters (namely P_M^B and α), and their response to environmental factors, have been widely reported for marine (Platt and Jassby, 1976; Côté and Platt, 1983; Gallegos et al., 1983; Harrison and Platt, 1986; Harding et al., 1986; Harding et al., 1987; Lohrenz et al., 1992a, 1994a, 1994b) and freshwater environments (Fee, 1972; Heyman, 1986; Fee et al., 1987, Fahnenstiel and Scavia, 1987; Fahnenstiel et al., 1989; Makarewicz, 1991).

Multiplying the specific rate of photosynthesis, P^B, by the algal biomass concentration, B (mg chl•m^{-3}), results in an estimate of the rate of primary production (mg C•m^{-3}•h^{-1}). The GLPM combines the integral approach of Fee (1973) with the empirically-based P-I relationship of Platt et al. (1980) to estimate *in situ* daily water column production, as in

$$P = \int_z \int_t B \bullet P^B dtdz \tag{3}$$

where P is the daily integral water column primary production (mg C•m^{-2}). Because B and P^B are not continuous functions (i.e., they generally represent a number of discrete points in space and time), an analytical solution to equation (3) is not possible, and numerical methods must be employed. Equation (3) may be approximated as

$$P = \sum_z \sum_t B(z) \bullet P^B(z,t)\, \Delta t\, \Delta z \tag{4}$$

where $B(z)$ and $P^B(z,t)$ represent discrete values of algal biomass and specific rate of photosynthesis, respectively, at depth z and time t. A non-linear least squares estimation package (e.g., IMSL, SYSTAT) can be used to fit Equation 1 to measured P-I data in order to determine values for the P-I parameters. Photosynthesis-irradiance measurements are determined using a photosynthetron. Fahnenstiel et al. (1989) fully describes techniques to measure *in situ* photosynthetic rates for a range of irradiances. Linear interpolation was used to estimate the P-I parameters and algal biomass at depths for which measurements were not available.

Furthermore, because surface irradiance values measured in air are subject to reflectance at the water surface, Fresnel's Equation and Snell's Law are used to estimate the proportion of light transmission across the air-water interface as a function of the solar zenith angle (Kirk, 1983):

$$r = \frac{\sin^2(\theta_a - \theta_w)}{2\sin^2(\theta_a + \theta_w)} + \frac{\tan^2(\theta_a - \theta_w)}{2\tan^2(\theta_a + \theta_w)} \tag{5a}$$

$$\frac{\sin\theta_a}{\sin\theta_w} = 1.33 \tag{5b}$$

$$\cos\theta_a = \sin\varepsilon \tag{5c}$$

$$\sin\varepsilon = \sin\gamma\sin\delta - \cos\gamma\cos\delta\cos\tau \tag{5d}$$

$$\delta = 0.39637 - 22.9133\cos\varphi + 4.02543\sin\varphi - 0.3872\cos 2\varphi + 0.052\sin 2\varphi \tag{5e}$$

where r is reflectance, θ_a is zenith angle, θ_w is the angle to the downward vertical of the transmitted beam in water, δ is solar declination, φ is date expressed as an angle, ε is solar elevation, γ is latitude, and τ is time of day expressed as an angle.

Finally, irradiance at depth is calculated as follows:

$$I_z = (1 - r) \bullet I_s \bullet e^{-kPAR \bullet z} \tag{6}$$

where, I_Z is the photosynthetically active irradiance (PAR) at depth z (Einst•m^{-2}•h^{-1}), r is reflectance, I_S is the surface photosynthetically active irradiance measured in air (same units as I_Z), $kPAR$ is the underwater light extinction coefficient of photosynthetically active irradiance (m^{-1}), and z is depth (m).

The model's numerical integration scheme uses a time step of 1 hour and a depth step of 0.1 m. Input to the GLPM includes station location and water column depth, simulation date, hourly values of surface irradiance (although, the model can accommodate less frequent surface irradiance measurements), depth-varying chlorophyll measurements, depth-varying P-I parameters, and depth-varying (where appropriate) kPAR. By reducing the model's simulation time to less than 24 hours, the GLPM can accommodate diel variations in P-I parameters, algal biomass, or light extinction.

Model output includes a nearly-continuous profile of the instantaneous rate of production (mg C•m^{-3}•t^{-1}) vs. depth, the mean water column production rate (mg C•m^{-3}•t^{-1}), and the integral daily production (mg C•m^{-2}•t^{-1}) summed over the entire water column, where t is the simulation duration (typically 1 day). Model output also includes the minimum, maximum, and average non-zero irradiance levels at specific depths, the complete set of input parameters, and a comprehensive plot generated using DISSPLA subroutines. The source code for the GLPM is written in HP-UNIX FORTRAN, and is available upon request.

3. MODEL USE

An earlier simplified version of the GLPM has been used to estimate daily production at an offshore site in Lake Michigan (Fahnenstiel and Scavia, 1987) and to evaluate the impact of internal waves on fixed-depth primary production estimates in Lake Michigan (Fahnenstiel et al., 1988). In addition, a version of the GLPM has been applied to the estuarine environment in the northern Gulf of Mexico (Lohrenz et al., 1992a, 1994a). A comparison of integral production estimates in the northern Gulf of Mexico obtained using the GLPM with those obtained by *in situ* and simulated *in situ* techniques indicated good agreement (r^2=0.65, n=12, P=0.002) (Lohrenz et al., 1992a). Similar good agreement between *in situ* and model estimates were found for samples from Lake Michigan (Fahnenstiel and Scavia, 1987).

Here, as a typical example of the model's use, we estimate integral primary production at an outer bay master station in Saginaw Bay, Lake Huron. The input parameters were measured/collected/calculated during a sampling cruise in May 1992 (Station 20, May 29, 1992) and are listed in Table 1. This simulation was part of a study to determine the impact of zebra mussels on algal production and biomass in Saginaw Bay. Output from the GLPM (Figures 1 and 2) includes the profile of the instantaneous daily production rate vs. depth (mg C•m^{-3}•d^{-1}), the mean water column daily production (76.96 mg C•m^{-3}•d^{-1}), and the integral daily production summed over the entire depth (1277.50 mg C•m^{-2}•d^{-1}). In addition, all of the input parameters are displayed.

The irregular shape of the rate profile indicates the competing and opposite influences of increasing (with depth) P-I parameters and algal biomass, and decreasing (with depth) light levels on primary production estimates over the water column depth. These influences are most evident in the top 6 m, which account for over 85% of the daily water column production for this station on this sampling date.

4. MONTE CARLO MODE

In an effort to generate a range of primary production estimates based on variance associated with certain input parameters and to obtain estimates of primary production at sites where P-I parameters are not available, a version of the GLPM was designed to run in a monte carlo mode. This mode provides the potential for estimating daily integrated productivity when only chlorophyll, incident irradiation, and the extinction coefficient are measured. Because of the use of modern instrument packages, all three of these parameters are readily measured on most research ships. In the monte carlo mode, up to 1000 model simulations are performed at a particular site using sets of random and independent values of P_S^B, α, and β, generated from their respective environmental distributions. The uncertainty in the P-I parameters leads to a distribution of primary production estimates. The resulting estimates of depth-specific and integral primary production are then pooled to determine their mean and variance.

As an example of its use, the GLPM was run in monte carlo mode to estimate primary production at Station 20 in Saginaw Bay on May 29, 1992. The input data is the same as in Table 1 except that here it was assumed that the P-I parameters were unavailable. Instead, 200 monte carlo simulations were generated using P-I parameters randomly selected from the pooled distributions of the 1992 and 1993 P-I parameters for the entire bay. A Lilliefors test demonstrated that a lognormal model provided a good fit to the pooled 1992 and 1993 P_S^B and α distributions (Table 2). The monte carlo model uses a routine from IMSL to randomly select values of P_S^B and α from lognormal distributions, characterized by the mean and standard deviation of their respective underlying normal distributions (Table 2). Only 16% of the β values for 1992 and 1993 were non-zero; therefore, β was set equal to zero for the monte carlo simulations. Sample output (Figures 3 and 4) includes the number of simulations, profiles of the mean and standard deviation of the instantaneous daily production rates vs. depth (1259.76 and 352.75 mg $C{\bullet}m^{-3}{\bullet}d^{-1}$, respectively), and the mean and standard deviation of the daily primary production summed over the entire depth (mg $C{\bullet}m^{-2}{\bullet}d^{-1}$). In addition, all of the input parameters and the mean and standard deviation of the randomly selected P-I parameters are displayed.

This exercise was repeated for all of the Saginaw Bay sites during seven cruises in 1992 and five cruises in 1993. Again, the P-I parameters were randomly selected from the pooled distributions of the 1992 and 1993 P-I parameters for the entire bay (Table 2). It is important to note that Saginaw Bay is highly variable in terms of trophic status and productivity. The inner bay is considered eutrophic, whereas some outer bay stations near the interface with Lake Huron are oligotrophic. In 1992 and 1993, P_S^B ranged from 1.18 to 8.12 mg $C{\bullet}mg\ chl^{-1}{\bullet}hr^{-1}$ throughout the bay, α ranged from 2.85 to 19.13 mg $C{\bullet}mg\ chl^{-1}{\bullet}Einst^{-1}{\bullet}m^2$, and primary productivity ranged from 0.011 to 1.92 g $C{\bullet}m^{-2}{\bullet}d^{-1}$. Thus, by combining all P-I data from all stations during 2 years to generate a single set of distributions from which the monte carlo simulations were sampled, our example probably represents the worst case scenario for evaluating the usefulness of the monte carlo approach for estimating integral production.

Integral production values for Saginaw Bay estimated by the monte carlo model compared well to those estimated by the original model using the measured P-I parameters (Figure 5). The resulting regression equation was highly significant ($P<0.001$) and yielded a slope and intercept of 0.94 and 0.024 respectively ($r^2=0.79$, $n=99$). Given the tremendous variability in Saginaw Bay, this relationship suggests that the monte carlo technique may provide reasonably accurate estimates of integral production at sites where measured P-I parameters are unavailable. Approximately 52% of monte carlo estimated production estimates are within 20% of the observed estimates using measured P-I parameters, and approximately 80% of monte carlo estimates are within 40% of the observed estimates. The overall root-mean-square-error of the monte carlo estimates is 0.16 g $C{\bullet}m^{-2}{\bullet}d^{-1}$.

5. EMPIRICAL RELATIONSHIPS

Input data requirements can be further simplified by using empirical relationships that estimate the light extinction coefficient based on values of more-easily measured and more-readily available parameters. Kirk (1983) and Bukata et al. (1988) present relationships for kPAR as a function of secchi depth for Great Lakes waters, and Bukata et al. (1988) and Baker and Baker (1976) present relationships for beam attenuation as a function of light transmission and kPAR as a function of beam attenuation. Using these principles, we determined similar and comparable empirical relationships for kPAR in Saginaw Bay (Table 3).

6. SENSITIVITY ANALYSIS

A simple sensitivity analysis was performed to examine the relative importance of various input parameters on model output. Measured input parameters from the 1992 and 1993 Saginaw Bay data sets were independently varied by +/- 25%, a value roughly equal to the coefficient of variation of the P-I parameters in the bay. The analysis included only sites where all input parameters were non-zero. The effects of these individual changes on model output were averaged across sites and are listed in Table 4. The largest changes in production resulted from changes in the extinction coefficient and chlorophyll concentration. A 25% decrease in extinction coefficient produced a 26% increase in production, and a 25% increase in extinction coefficient produced an 18% reduction in production. Changes in chlorophyll produced equal changes in primary production, i.e. a +/- 25% change in chlorophyll concentration resulted in a corresponding +/- 25% change in productivity. All other parameter changes produced <20% change in model production. The model was least sensitive to changes in surface irradiance and β.

A 25% change in the two important photosynthetic parameters, P_S^B and α, produced productivity changes from 10 to 16%. The increases in productivity produced by the chlorophyll increase and k decrease were not significantly different from each other (paired sample t-test, $p>0.05$); however these productivity increases were significantly different than those caused by changes in all other parameters (P_S^B, α, incident irradiation, and β; $P<0.05$). Given the large variability of chlorophyll concentrations and extinction coefficients in the Great Lakes relative to photosynthetic parameters, these data support the monte carlo approach for estimating integral primary productivity when resources are limited and P-I parameters are unavailable.

7. REFERENCES

Baker, A.L, and K.K. Baker. Estimation of planktonic wind drift by transmissometry. *Limnology and Oceanography* 21(3):447-452 (1976).

Bukata, R.P., J.H. Jerome, and J.E. Bruton. Relationship among secchi disk depth, beam attenuation coefficient, and irradiance attenuation coefficient for Great Lakes waters. *Journal of Great Lakes Research* 14(3):347-355 (1988).

Côté, B., and T. Platt. Day-today variations in the spring-summer photosynthetic parameters of coastal marine phytoplankton. *Limnology and Oceanography* 28:320-344 (1983).

Fahnenstiel, G.L., and D. Scavia. Dynamics of Lake Michigan phytoplankton: primary production and growth. *Canadian Journal of Fisheries and Aquatic Sciences* 499-508 (1987).

Fahnenstiel, G.L., D. Scavia, G.A. Lang, J.H. Saylor, G.S. Miller, and D.J. Schwab. Impact of inertial period internal waves on fixed-depth primary production estimates. *Journal of Plankton Research* 10:77-87 (1988).

Fahnenstiel, G.L., J.F. Chandler, H.J. Carrick, and D. Scavia. Photosynthetic characteristics of phytoplankton communities in Lakes Huron and Michigan: P-I parameters and end-products. *Journal of Great Lakes Research* 15:394-407 (1989).

Fee, E.J. A numerical model for the estimation of integral production and its application to Lake Michigan. Ph. D. thesis, Univ. Wisconsin-Milwaukee. 169 pp. (1972).

Fee, E.J. Modelling primary production in water bodies: a numerical approach that allows vertical inhomogeneities. *Journal Fisheries Research Board of Canada* 30:1469-1473 (1973).

Fee, E.J., R.E. Hecky, and H.A. Welch Phytoplankton photosynthesis parameters in central Canadian lakes. *Journal of Plankton Research* 9:305-316 (1987).

Gallegos, C.L., T. Platt, W.G. Harrison, and B. Irwin. Photosynthetic parameters of arctic marine phytoplankton: vertical variations and time scales of adaptation. *Limnology and Oceanography* 28:698-708 (1983).

Harding, L.W., B.W. Meeson, and T.R. Fischer. Phytoplankton production in two east coast estuaries: photosynthesis-light functions and patterns of carbon assimilation in Chesapeake and Delaware Bays. *Est. Coastal and Shelf Science* 23:773-806 (1986).

Harding, L.W., T.R. Fisher, and M.A. Tyler. Adaptive responses of photosynthesis in phytoplankton: specificity to time-scale of change in light. *Biol. Ocean.* 4:403-437 (1987).

Harrison, W. G., T. Platt, and M.R. Lewis. The utility of light-saturation models for estimating marine primary productivity in the field: A comparison with conventional “simulate” *in situ* methods. *Canadian Journal of Fisheries and Aquatic Sciences* 42:864-872 (1985).

Harrison, W.G., and T. Platt. Photosynthetic-irradiance relationships in polar and temperate phytoplankton populations. *Polar Can.* 36:1364-1369 (1986).

Herman, A.W. and T. Platt. Primary production profiles in the ocean: estimation from chlorophyll/light model. *Oceanologica Acta* 9:31-40 (1986).

Heyman, U. The response of photosynthetic parameters to environmental factors in Siggeforasjön, Sweden. *Arch. Hydrobiol.* 106:155-175 (1986).

Jassby, A.D., and T. Platt. Mathematical formulation of the relationship between photosynthesis and light for phytoplankton. *Limnology and Oceanography* 21:540-547 (1976).

Jitts, H. R., A. Morel, and Y. Saijo. The relation of oceanic primary production to available photosynthetic irradiance. *Australian Journal of Marine Freshwater Research* 27:441-454 (1976).

Kirk, J.T.O. *Light and Photosynthesis in Aquatic Ecosystems*. Cambridge Univ. Press. Cambridge, MA (1983).

Lohrenz, S.E., D.G. Redalje, G.L. Fahnenstiel, M.J. McCormick, G.A. Lang, K. Prasad, X. Chen, D.A. Atwoood, and B. Chen. Phytoplankton rate processes in coastal waters of the northern Gulf of Mexico and relationships to environmental parameters. Proceedings of NECOP Workshop, June 26-27, 1994, Baton Rouge, LA (1994a).

Lohrenz, S.E., G.L. Fahnenstiel, and D.G. Redalje. Spatial and temporal variations of photosynthetic parameters in relation to environmental conditions in coastal waters of the northern Gulf of Mexico. *Esturaries* 17(4):779-795 (1994b).

Lohrenz, S.E., G.L. Fahnenstiel, D.G. Redalje, and G.A. Lang. Regulation and distribution of primary production in the northern Gulf of Mexico. Proceedings of NECOP Workshop, October 2-4, 1991, Chauvin, LA (1992a).

Lohrenz, S.E., D.A. Wiesenburg, C.R. Rein, R.A. Arnone, C.D. Taylor, G.A. Knauer, and A.H. Knap. A comparison of *in situ* and simulated *in situ* methods for estimating oceanic primary production. *Journal of Plankton Research* 14:201-221 (1992b).

Makarewicz, J.C. Photosynthetic parameters as indicators of ecosystem health. *Journal of Great Lakes Research* 17:333-343 (1991).

Platt, T., C.L. Gallegos, and W.G. Harrison. Photoinhibition of photosynthesis in natural assemblages of marine phytoplankton. *Journal of Marine Research* 38:678-701 (1980).

Platt, T., and A.D. Jassby. The relationship between photosynthesis and light for natural assemblages of coastal marine phytoplankton. *Journal of Phycology* 12:421-430 (1976).

Putnam, H.D., and T.A. Olson. Studies on the productivity and plankton of Lake Superior. Rept. School Public Health, Univ. Minn. 24 pp. (1961).

Saunders, G.W., F.B. Trama, and R.W. Bachman. Evaluation of a modified ^{14}C technique for shipboard estimation of photosynthesis in large lakes. Univ. Michigan, Great Lakes Res. Div. Publ. 8, 61 pp. (1962).

Steeman-Nielsen, E. The use of radioactive carbon (^{14}C) for measuring organic production in the sea. *J. Cons. Inst. Explor. Mer.* 18:117-140 (1952).

Verduin, J. Energy flow through biotic systems of western Lake Erie, p. 107-121. In *Great Lakes Basin*, AAAS. Publ. No. 71 (1962).

Vollenweider, R. A. A manual on methods for measuring primary production in aquatic environments. IBP Handbook No. 12. Blackwell Sci. Publ. Oxford (1969).

Wetzel, R. G. and G. E. Likens. *Limnological Analyses*. 2nd Ed., Springer-Verlag, New York (1991).

Table 1.--Great Lakes Production Model input parameters for Station 20, Saginaw Bay, May 29, 1992.

Parameter	Value
Station Number:	20
Latitude:	44˚ 07.56´ N
Longitude:	83˚ 30.00´ W
Station Depth:	16.6 m
Cruise Number:	3
Simulation Date:	May 29, 1992
Simulation Duration:	24 hrs
Extinction Coefficient:	0.588 m^{-1}

Chlorophyll *a* Profile:

Depth (m)	Chl *a* (mg m^{-3})
1	4.914
8	6.050

Photosynthesis-Irradiance Parameters:

Depth (m)	P_S^B (mg C•mg chl^{-1}•hr^{-1})	α (mg C•mg chl^{-1}•$Einst^{-1}$•m^2)	β (mg C•mg chl^{-1}•$Einst^{-1}$•m^2)
1	3.580	10.092	0
8	4.840	10.719	0

Surface Irradiance (μEinst•m^{-2}•sec^{-1}):

Hour	Par	Hour	PAR	Hour	PAR	Hour	PAR
1	0.	7	94.42	13	1745.56	19	695.55
2	0.	8	388.33	14	1782.78	20	326.94
3	0.	9	767.78	15	1717.22	21	86.42
4	0.	10	1118.89	16	1557.78	22	2.89
5	0.	11	1401.11	17	1322.50	23	0.
6	0.	12	1612.78	18	1021.94	24	0.

Table 2.--Parameters used to define the underlying normal distributions of the pooled 1992 and 1993 P-I parameters, P_S^B and α, for Saginaw Bay.

Parameter	Mean	Standard Deviation	n	Lilliefors Probability
ln (P_S^B)	1.213	0.379	99	0.249
ln (α)	2.318	0.387	99	0.321

Table 3.--Empirical relationships relating extinction coefficient (kPAR) to secchi depth (SECCHI) and light transmission (TRANS) for Saginaw Bay. Units: kPAR in m^{-1}, Secchi in m, TRANS in volts (range=0-5, 5=100% transmission).

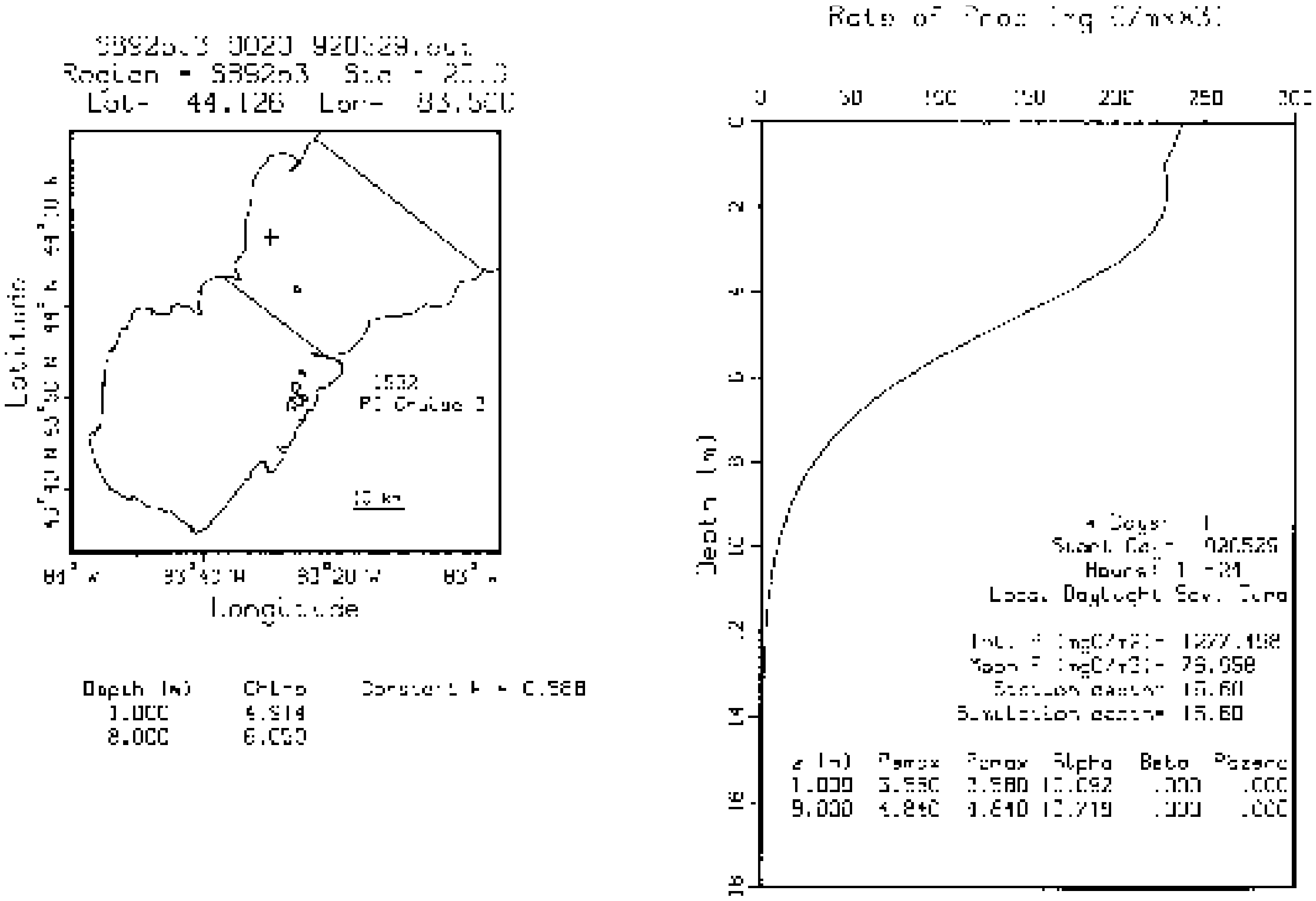
Relationship	P	r^2	n
<u>1991 data</u>			
$kPAR = 1.167 - 0.643 \cdot \ln(TRANS)$	<0.001	0.900	56
$kPAR = 1.171 \cdot SECCHI^{-1}$	<0.001	0.943	83
<u>1992 data</u>			
$kPAR = 1.214 - 0.736 \cdot \ln(TRANS)$	<0.001	0.916	78
$kPAR = 1.408 \cdot SECCHI^{-1}$	<0.001	0.957	84
<u>1993 data</u>			
$kPAR = 1.369 - 0.825 \cdot \ln(TRANS)$	<0.001	0.803	56
$kPAR = 1.365 \cdot SECCHI^{-1}$	<0.001	0.949	60

Table 4.--Effect on model output of independently varying model input parameters. Values represent mean ratio of resultant production to original production at all Saginaw Bay sites in 1992 and 1993 where input parameters were non-zero (n=16). Mean of original production estimates is 486.0 mg $C \cdot m^{-2} \cdot d^{-1}$.

	Parameter					
Change in Parameter	P_S^B	α	β	kPAR	Chl	Suface Light
-25%	0.841	0.87	1.007	1.256	0.750	0.881
+25%	1.135	1.101	0.993	0.824	1.250	1.094

```
SAGINAW BAY DATA, GLPM CALCULATION
Region = SB92p3   Station = 20    Lat =  44.126  Lon = -83.500
Station Depth (m) =    16.60000
Simulation Depth (m) =    16.60000

First Day (YYMMDD) = 920529    No. days =   1
From Hour  1   to Hour 24     = 24 Total Hours


 Using depth-variable P-I parameters
  -Linearly interpolated between depths
      Depth        PSMAX        PBMAX        ALPHA         BETA        PBZERO
       (m)         (mg C/       (mg C/    (mg C-m^2/    (mg C-m^2/     (mg C/
                 mg Chl-hr)   mg Chl-hr)   mg Chl-E)     mg Chl-E)   mg Chl-hr)
      1.0000       3.5800       3.5800      10.0920       0.0000       0.0000
      8.0000       4.8400       4.8400      10.7190       0.0000       0.0000

 Using depth-variable Chl-a concentrations
  -Linearly interpolated between depths
     Depth (m)   Chl (ug/l)
      1.0000       4.9140
      8.0000       6.0500

 Using constant k (per m) =  0.5881000

 Incident Solar Radiation (uE/m2/sec)
  Local Daylight Savings Time
     920529  0.0000000E+00  0.0000000E+00  0.0000000E+00  0.0000000E+00
 0.0000000E+00  0.0000000E+00   94.41666       388.3333       767.7778
  1118.889       1401.111       1612.778       1745.556       1782.778
  1717.222       1557.778       1322.500       1021.944       695.5555
  326.9445       86.41666       2.891667      0.0000000E+00  0.0000000E+00

 Incident Solar Radiation (uE/m2/sec) corrected
 for Fresnel"s Eq and Snell"s Law
  Local Daylight Savings Time
     920529  0.0000000E+00  0.0000000E+00  0.0000000E+00  0.0000000E+00
 0.0000000E+00  0.0000000E+00   38.15732       305.0094       703.5377
  1073.897       1364.482       1577.624       1709.614       1746.483
  1681.863       1523.823       1287.926       980.8500       637.3583
  256.7926       34.92422      0.0000000E+00  0.0000000E+00  0.0000000E+00

Integral prod (mg C/m2) over each depth interval and instantaneous prod
rate (mg C/m3) at bottom 0.05 m of each interval.  Both values have been
integrated over the above-specified time interval.  Also included are
the ave, min and max non-zero light values (uE/m2/s) at the surface, 0.05 m,
and bottom 0.05 m of each interval

 Day=      920529
   From    To (m)  Integral P     Depth(m)  Prod Rate   Light-Avg   Min     Max
                                    0.00                994.82    34.92  1746.48
                                    0.05    237.010     966.00    33.91  1695.88
    0.00    3.00   681.363          2.95    211.522     175.51     6.16   308.11
    3.00    6.00   440.358          5.95     82.690      30.07     1.06    52.78
    6.00    9.00   126.744          8.95     17.540       5.15     0.18     9.04
    9.00   12.00    24.286         11.95      3.075       0.88     0.03     1.55
   12.00   15.00     4.215         14.95      0.529       0.15     0.01     0.27
   15.00   16.60     0.532         16.55      0.206       0.06     0.00     0.10

    0.00   16.60  1277.499

Prod-int  = integral prod over sim depth over time period (mg C/m2)
Prod-mean = mean prod = prod-int/sim depth (mg C/m3)
SurfPAR   = Total surface PAR (corrected) over time'  period (E/m2)

  Day     From  To (m)   Prod-int   Prod-mean   SurfPAR
920529    0.00   16.60   1277.499     76.958     53.720
```

Figure 1.--Printed output from Great Lakes Production Model.
Simulation corresponds to Station 20, Saginaw Bay, May 29, 1992.



```
Depth (m)    Chl-a      Constant K = 0.588
  1.000      4.914
  8.000      6.050
```

```
               # Days:   1
          Start Date =   920529
             Hours: 1 - 24
      Local Daylight Sav. Time

   Int. P (mgC/m2)= 1277.498
  Mean P (mgC/m3)=  79.958
     Station depth= 15.60
  Simulation depth= 15.60

z (m)   Pamax   Pbmax   Alpha   Beta   Pozero
1.000   3.550   3.580  10.092   .000    .000
9.000   4.840   3.640  10.719   .000    .000
```

Figure 2.--Graphical output from the Great Lakes Production Model. Simulation corresponds to Station 20, Saginaw Bay, May 29, 1992.

```
    MONTE CARLO SIMULATION    SB92pi3-20  -920529.M-C

SAGINAW BAY DATA, GLPM CALCULATION
Region = SB92p3   Station = 20    Lat =  44.126  Lon = -83.500
Station Depth (m) =    16.60000
Simulation Depth (m) =    16.60000

First Day (YYMMDD) = 920529    No. days =   1
From Hour  1   to Hour 24     = 24 Total Hours


 P-I parameters were randomly selected from
 lognormal distributions defined by the following data
    ln(PSMAX)          ln(ALPHA)       ln(BETA)
  mg C/mg Chl-hr  mg C-m^2/mg Chl-E  mg C-m^2/mg Chl-E
   Mean      SD      Mean      SD      Mean      SD
  1.2130   0.3785   2.3183   0.3873   0.0000   0.0000

 Using depth-variable Chl-a concentrations
  -Linearly interpolated between depths
     Depth (m)   Chl (ug/l)
      1.0000       4.9140
      8.0000       6.0500

 Using constant extinction coefficient
    k (per m) =  0.5881000

 Incident Solar Radiation (uE/m2/sec)
  Local Daylight Savings Time
     920529  0.0000000E+00  0.0000000E+00  0.0000000E+00  0.0000000E+00
 0.0000000E+00  0.0000000E+00   94.41666       388.3333       767.7778
  1118.889       1401.111       1612.778       1745.556       1782.778
  1717.222       1557.778       1322.500       1021.944       695.5555
  326.9445       86.41666       2.891667      0.0000000E+00  0.0000000E+00

 Incident Solar Radiation (uE/m2/sec) corrected
 for Fresnel"s Eq and Snell"s Law
  Local Daylight Savings Time
     920529  0.0000000E+00  0.0000000E+00  0.0000000E+00  0.0000000E+00
 0.0000000E+00  0.0000000E+00   38.15732       305.0094       703.5377
  1073.897       1364.482       1577.624       1709.614       1746.483
  1681.863       1523.823       1287.926       980.8500       637.3583
  256.7926       34.92422      0.0000000E+00  0.0000000E+00  0.0000000E+00


 Day=      920529

NUMBER OF MONTE CARLO SIMULATIONS =         200

 The following values represent the mean and stdev
 of the randomly selected PI values.
  Depth         PSMAX             ALPHA             BETA
   (m)     mg C/mg Chl-hr  mg C-m^2/mg Chl-E  mg C-m^2/mg Chl-E
            Mean      SD      Mean      SD      Mean      SD
  0.0000   3.7917   1.5170  11.5987   4.6967   0.0000   0.0000


RESULTS OF MONTE CARLO SIMULATION
INTEGRAL PROD (mg C/m2) OVER TOTAL DEPTH

  Day     From  To (m)     MEAN       STDEV       MIN        MAX
920529    0.00   16.60   1259.755    352.752    392.743   2521.871
```

Figure 3.--Printed output from Great Lakes Production Model run in monte carlo mode.
Simulation corresponds to Station 20, Saginaw Bay, May 29, 1992.

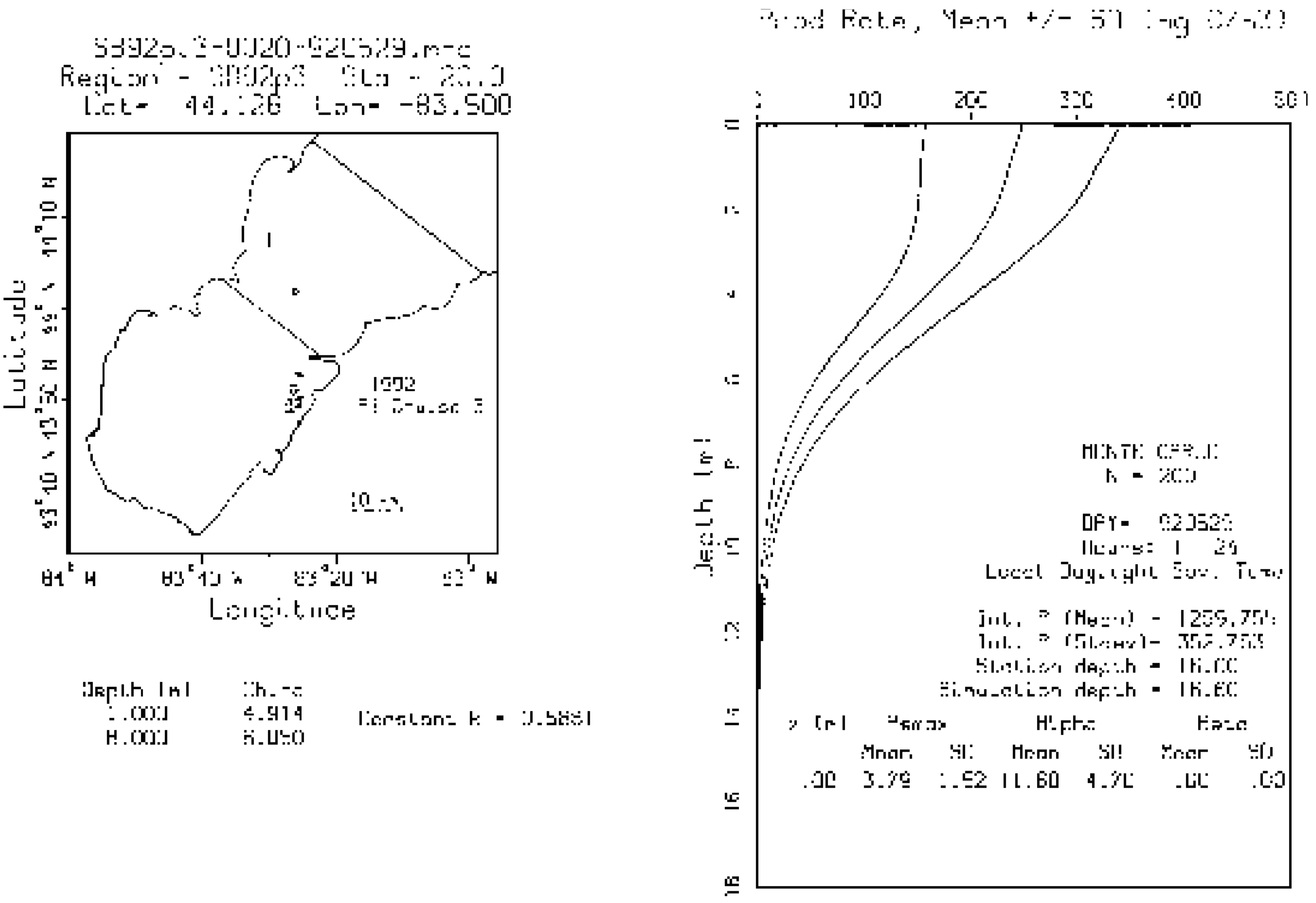


Figure 4.--Graphical output from the Great Lakes Production Model run in monte carlo mode. Simulation corresponds to Station 20, Saginaw Bay, May 29, 1992.

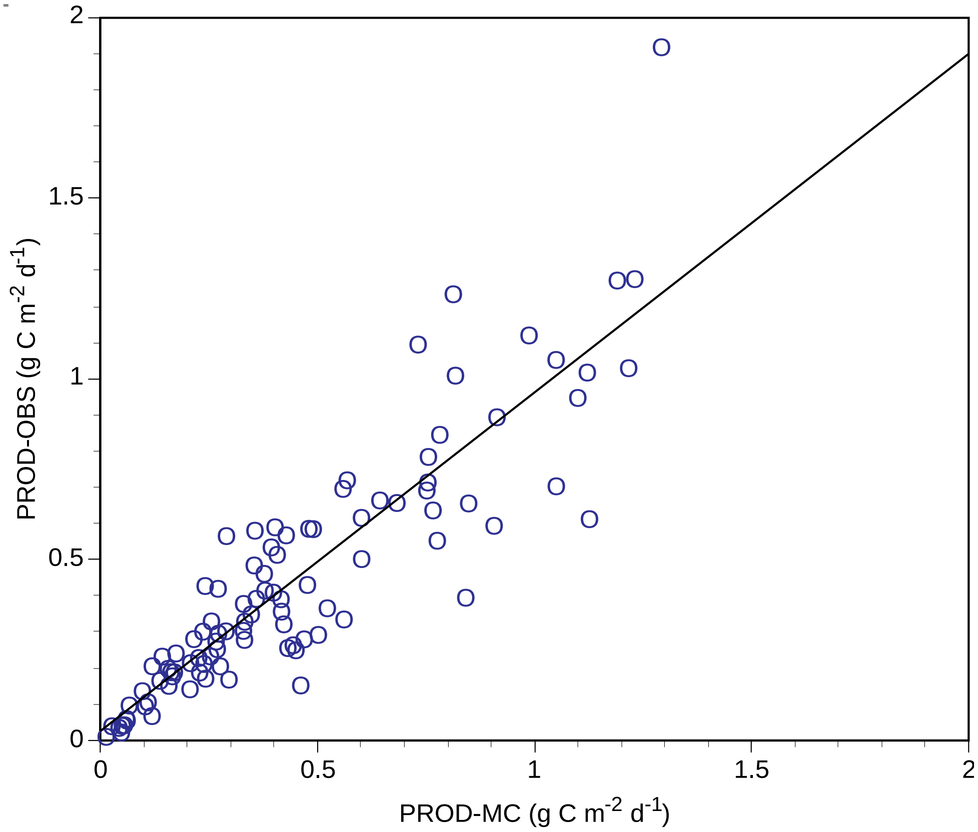


Figure 5.--Comparison of 1992 and 1993 integral production values estimated by the original model using measured P-I parameters (PROD-OBS, g C•m^{-2}•d^{-1}) vs. those estimated by the monte carlo model using the pooled distributions of 1992 and 1993 P-I parameters (PROD-MC, g C•m^{-2}•d^{-1}).