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RESOURCE SHED DEFINITIONS AND COMPUTATIONS

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Resource Shed Definitions and Computations

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ABSTRACT. When we consider a location with a material (e.g., water, pollutant, sediment) passing through it, we can ask: Where did the material come from and how long did it take to reach the location? We can quantify the answer by defining the areas contributing to this location during various time periods as *resource sheds*. Various kinds of resource sheds and their source material distributions are rigorously defined and properties derived. For watershed hydrology, we compute resource sheds and their source material distributions with a spatially distributed hydrology model by tracing material departing from a *cell* (say 1 km²) over one time interval and arriving at the watershed mouth in another time interval. This requires modeling all cells, but only tracing contributions from one at a time. By then combining these simulations for all cell loadings, we construct a map of the contributions over the entire watershed for specific departure and arrival time intervals. We then combine results of several sets of simulations to determine the source distribution for any time period and infer resource sheds from these mappings. For lake circulation, we discuss the construction of resource sheds and their source distributions in the lake, by using lake circulation models to drive particle tracers in reverse time, and subsequent correction. We present examples for the Maumee River watershed in northern Ohio, discuss methods of computation reduction, discuss linkage with a lake circulation model to construct joint resource sheds in Lake Erie, and suggest areas of extension for the future.

1. INTRODUCTION

Accurately defining the spatial boundary of resource distribution and transport is essential to the understanding and management of resource dispersion over multiple spatial and temporal scales. Since the early twentieth century the *watershed* (land draining into a stream or lake at a given location) has been widely used as a basic unit in hydrologic research (Chow et al. 1988; U.S. Environmental Protection Agency 1995). A watershed is a hydrologic system that consists of a boundary, an internal structure (drainage areas, vegetation, river channels, lakes, reservoirs, etc.), inputs (water and materials), and outputs (water, services, and pollutants, etc) (Chow et al. 1988). As a hydrologic system, the boundary, structures, inputs (precipitation, pollutants, etc.), outputs (evapotranspiration, outflow, sediments, etc.), and processes of the watershed change over time and space. Its boundary, for example, is a continuous surface in space enclosing the watershed structure of interest (Chow et al. 1988), ranging from a small tributary (with an area of a few hectares) to a continental river basin (such as the Mississippi River Basin of a few million km²). Since the goal of water resources management is to develop and implement policies, processes, technologies, and organizations for understanding, distributing, and improving the movement and characteristics of water resources to meet the multiple needs of human societies and ecosystems in a socially responsible, economically viable, and environmentally sustainable way (He et al. 2005), the watershed is recognized as a natural unit for managing the water resources and associated ecosystem (U.S. Environmental Protection Agency 1995; National Research Council 1999; Bruckhorst and Reeve 2006).

Over the years, researchers have extended the watershed concept to water resources and atmospheric studies. Michel (2000), for example, applied the concept of *hydrocommons* (defined as hybrid basins created by linking water sending and receiving basins by conveyance systems such as storage reservoirs and aqueducts) in analyzing the linkages between transbasin water transfers, water quality, and watershed ecosystems. Creation of the hydrocommons removes the natural boundaries of both sending and receiving basins, results in altered hydrology, water quality, ecosystems, economies and land use patterns in both the sending and receiving watersheds (Michel 2000). Others have used *hydroregion* in understanding the role of regional hydrologic influences in fish species richness (Oswood et al. 2000; Santoul, et al. 2004). In atmospheric research, the airshed has been used to define the areas in which sources of pollutant emissions are located, which are transported and deposited in metropolitan areas or regions, for monitoring and forecasting air quality and implementing management programs such as *ozone action* days (Tullar and Suffet 1975; Chang and Cardelino 2000; Morawska et al. 2002; Ellis et al. 2006).

Similarly, ecologists have sought to spatially delimit otherwise intuitive ecological units such as ecosystems (Cousins 1990). Where changes in ecological conditions are abrupt, such as at the land-water interface of a coast for example, ecosystem boundaries appear distinct. Yet the phenomenon of ecological subsidy, or donor-controlled supply of resources supporting food webs spatially distinct from source areas, blurs otherwise distinct ecosystem boundaries (Polis et al. 1997). Embracing this common feature of food webs, Power and Rainey (2000) proposed the explicit delineation of ecological subsidy in space through the *resource shed*, or “source areas for resources consumed by individuals during their lifetimes”. Hence processes, rather than physical landscape features alone, can define geographic areas of ecological relevance to systems under study. Indeed, Power and Rainey (2000) observed that landscape features such as topography, coupled with physical forcing variables, can determine the shape of resource sheds. In the present study, resource sheds are determined by the movement of water, and hence relevant to water-borne materials. Moreover, we generalize the definition of resource shed to encompass source areas from which materials are derived for an individual, population, or location, over a specified time period, during a specified season. In this definition materials can include nutrients, organic matter, sediments, organism propagules, prey items, or pollutants; i.e. anything transportable by water. Hence a resource shed is a geographic area from which originate materials that subsidize food webs, or that are donated to a location, at a specific time over a specific time period. Applying this concept, Ben-David et al. (2005) investigate the variable resource sheds created by the movements and behavior of river otters and their impacts on nutrient cycling, changes in productivity and in community structure and function, and landscape heterogeneity of terrestrial communities.

While the concepts of resource shed and watershed are similar, they have some differences as well. First, the boundary of a watershed is delineated by elevation and flow direction, following the gravity principle (water flows downhill), and therefore are relatively more stable (e.g. the 8-digit hydrologic unit codes defined by the U.S. Geological Survey for the U.S. watersheds were developed in the 1970s). The boundary of a hydrological resource shed, however, is delineated by the contributing sources of water and materials to a river or lake (landscape features) during hydrologic events (physical forcing variables). Thus hydrological resource sheds have a more dynamic border, or rather a border relevant to a specific time period during which physical forcing variables operate (e.g. the resource sheds of water or sediments in a watershed change from one storm event to another). Second, the concept of watershed emphasizes temporal distribution

of water and materials within a given space, and the concept of resource shed focuses on both temporal and spatial distribution of water and materials within a changing space. Third, the watershed concept has been in existence for over 100 years and is understood, accepted, and used worldwide while the resource shed concept is relatively new and not yet well developed. For example, what are the theoretical principles for defining resource sheds? What are appropriate spatial and temporal scales of resource sheds? How do we integrate geographic information systems and spatial modeling techniques with currently available computing technology to define and model resource sheds? Despite these challenges, we believe the concept of resource sheds provides a new way of displaying, understanding, and discovering the transport and distribution of water and materials and has the potential of helping resource managers better track and manage source loadings in a study area.

In this paper, we define resource sheds and propose a set of analysis procedures to determine them. We first describe various resource shed definitions mathematically and then derive one from the other. We also describe various resource shed distributions (source density of material of interest over the resource shed) and then derive one from the other. We present methodologies for computing hydrological resource sheds within watersheds. We apply the methods to several examples in the Maumee River watershed and discuss estimation issues. Finally we show how to extend resource sheds and resource shed distributions defined for a location in a lake into contributing watersheds about the lake by linking results from the different settings.

2. RESOURCE SHEDS

2.1 Definitions

Consider a location with material passing through it (e.g., water, pollutant, sediment). Where did the material come from and how long did it take to reach the location? We can quantify the answer by computing the areas contributing to this location during various time periods. We define such areas as *resource sheds*. More specifically, let $Y(\tau, t)$ represent the set of all locations where materials departing at time τ arrive at a location of interest at time t ; see Figure 1. (Note, upper case letters are used exclusively to denote sets and lower case letters are used to represent

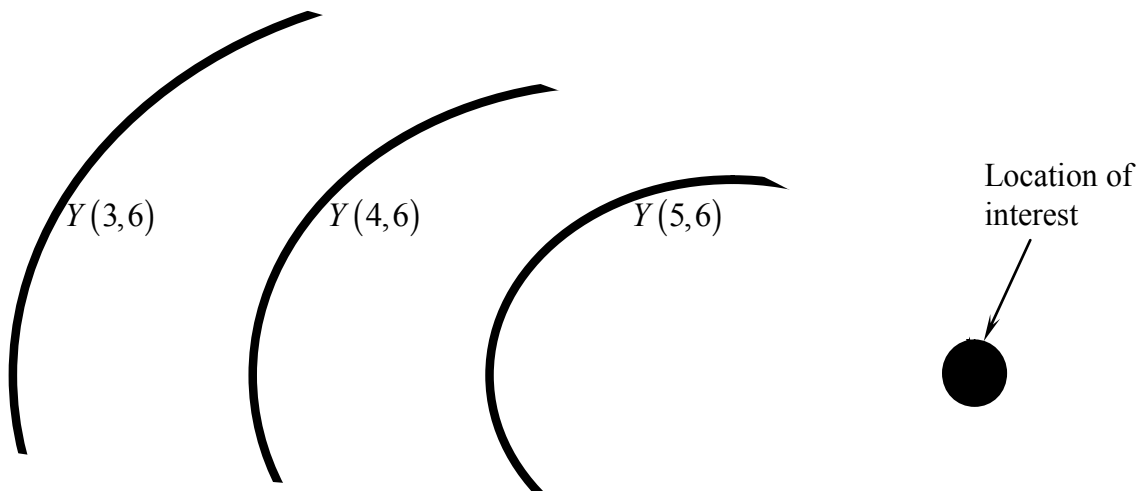


Figure 1. Example Locus (Lines) of Selected $Y(\tau, t)$.

functions or scalars.) Figure 1 shows $Y(\tau, t)$ as simple non-intersecting lines for a given time, t , for clarity.

We now can define a resource shed a little more rigorously as:

$$R(a, b, c, d) = \bigcup_{a \leq \tau < b} \bigcup_{c \leq t < d} Y(\tau, t) \quad (1)$$

where the operator, \cup , represents the union of two or more sets. $R(a, b, c, d)$ is the set of all locations (resource shed) where materials departing during time interval $[a, b)$ arrive at the location of interest during time interval $[c, d)$. While the definition applies for all time intervals, note that for $a > b$ or $c > d$ or $a > d$, $R = \emptyset$ (the empty set).

For convenience, consider only discrete time in intervals of δ and define

$$V_{i,j} = R((i-1)\delta, i\delta, (j-1)\delta, j\delta) \quad (2)$$

Then $V_{i,j}$ is the resource shed where materials departing during the time interval of length δ prior to time i (the i^{th} time interval) arrive at the location of interest during the j^{th} time interval. Also define

$$S_{i,j} = R((i-1)\delta, j\delta, (j-1)\delta, j\delta) \quad (3)$$

$$T_{i,j} = R((i-1)\delta, j\delta, (i-1)\delta, j\delta) \quad (4)$$

where $S_{i,j}$ is the resource shed where materials departing during time intervals i, \dots, j {corresponding to time interval $[(i-1)\delta, j\delta)$ } arrive at the location of interest during the j^{th} time interval and $T_{i,j}$ is the resource shed where materials departing during time intervals i, \dots, j arrive at the location of interest also during time intervals i, \dots, j . Figure 2 illustrates some of these definitions. Figure 2 and all remaining figures portray mutually exclusive $V_{i,j}$, $i = j, j-1, j-2, \dots$ for clarity. However, all resource shed definitions and derivations herein apply without loss of generality for the general case of overlapping $V_{i,j}$. Note that

$$S_{i,j} = \bigcup_{m=i,j} V_{m,j} \quad (5)$$

$$T_{i,j} = \bigcup_{m=i,j} S_{i,m} \quad (6)$$

We expect that resource sheds for any time period to fully enclose (contain) those for included time periods, for a given location of interest and time. Since

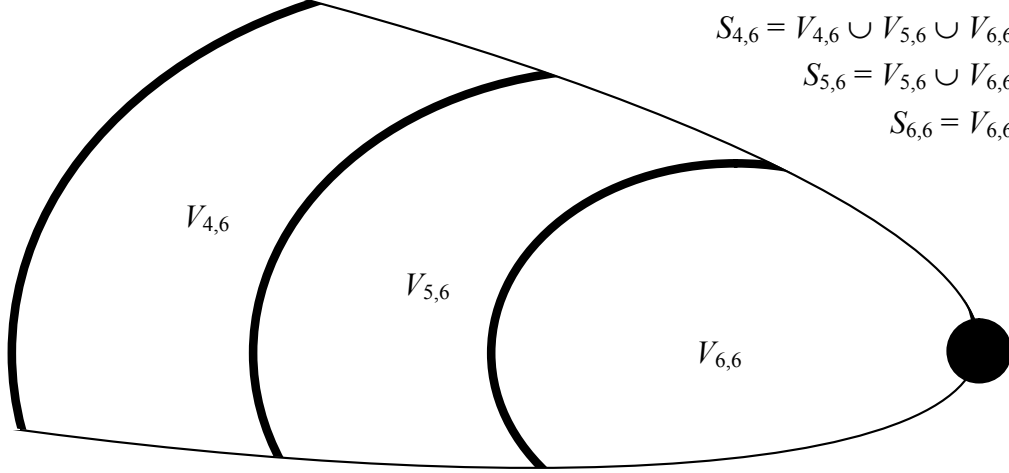


Figure 2. Example Resource Sheds for Figure 1 ($\delta = 1$).

$$\bigcup_{m=i,j} V_{m,j} \subset \bigcup_{m=k,j} V_{m,j}, \quad k \leq i \quad (7)$$

where “ $A \subset B$ ” represents the statement “set A is contained in set B ;” then by (5)

$$S_{i,j} \subset S_{k,j}, \quad k \leq i \quad (8)$$

Equations (8) and (6) indicate

$$\begin{aligned} \bigcup_{m=i,j} S_{i,m} \subset \bigcup_{m=i,j} S_{k,m} \subset \bigcup_{m=k,j} S_{k,m}, \quad k \leq i \\ \Rightarrow T_{i,j} \subset T_{k,j}, \quad k \leq i \end{aligned} \quad (9)$$

where “ \Rightarrow ” denotes “implies.”

2.2. A Classical Example

Note that the classical travel-time isochronal map for a watershed (Linsley et al. 1982 p280) is an example of our definitions. It is built to estimate a watershed’s time-area histogram, which is then used further to estimate the unit hydrograph for a watershed. For example, consider the mouth of a watershed at time 0 with outflow resulting from the application of a unit depth of water over the entire watershed. If we determine the travel times from all locations in the watershed to the mouth and plot them, we have the classical hydrological travel time isochronal map shown on the left side of Figure 3 for an arbitrary watershed. Each isochrone represents $Y(\tau, 0)$; for example, the 3δ isochrone represents $Y(-3, 0)$. Then, resource sheds ($V_{i,0}$) for water departing during the i^{th} time interval $i = 0, -1, -2, \dots$, and arriving at the watershed mouth during the 0^{th} time interval are those areas with travel times within $[-i\delta, (-i+1)\delta)$; the shaded area in Figure 3 shows $V_{-3,0}$. By identifying similar resource sheds for other time intervals from the isochronal map, we can build the time-area histogram of classical hydrology, shown on the right side of

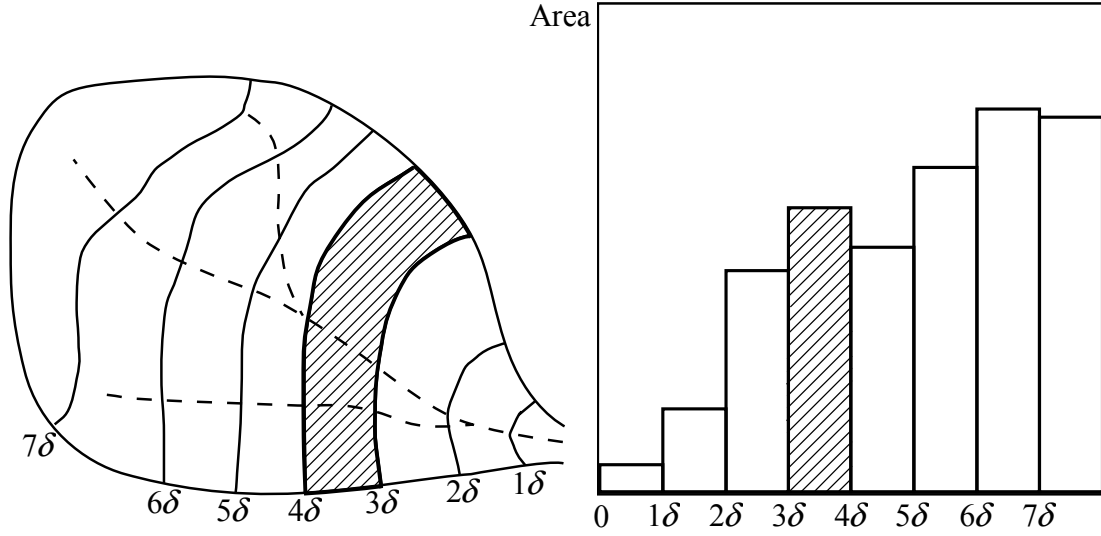


Figure 3. Isochronal Travel Time Map and Resultant Time-Area Histogram.

Figure 3, which can be transformed into an estimate of a unit hydrograph by routing through a hypothetical reservoir. The shaded bar in the time-area histogram corresponds to the shaded resource shed in the isochronal map.

3. MATERIAL DENSITIES

While these definitions refer to the spatial extent of a contributing area, they do not address the differences between parts of a contributing area. Some parts of an area may supply more material to our location of interest than other parts. Represent the material's areal density rate of change with departure and arrival times (mass/area/time/time) of material departing location ω at time τ and arriving at the location of interest at time t as

$$h(\omega, \tau, t) \quad \forall \quad \omega \in Y(\tau, t) \quad (10)$$

where the expression, “ $\forall \omega \in A$,” represents the statement “for all ω within set A .” Integrating over departure and arrival times, we have the spatial density of material (mass/area), f , at location ω departing during $[a, b)$ and arriving during $[c, d)$,

$$f(\omega, a, b, c, d) = \int_a^b \int_c^d h(\omega, \tau, t) dt d\tau \quad \forall \quad \omega \in R(a, b, c, d) \quad (11)$$

Note that for physically relevant situations,

$$\begin{aligned} f(\omega, a, b, c, d) &> 0 \quad \forall \quad \omega \in R(a, b, c, d) \\ &= 0 \quad \forall \quad \omega \notin R(a, b, c, d) \end{aligned} \quad (12)$$

(We omit the second line of the above equation in the following function definitions, presuming it is understood.) Again for convenience, consider only discrete time intervals of δ and define

$$x_{i,j}(\omega) = f(\omega, (i-1)\delta, i\delta, (j-1)\delta, j\delta) \quad \forall \omega \in V_{i,j} \quad (13)$$

Then $x_{i,j}(\omega)$ is the areal density of material at location ω departing during the i^{th} time interval and arriving during the j^{th} time interval. Define

$$z_{i,j}(\omega) = f(\omega, (i-1)\delta, j\delta, (j-1)\delta, j\delta) \quad \forall \omega \in S_{i,j} \quad (14)$$

$$w_{i,j}(\omega) = f(\omega, (i-1)\delta, j\delta, (i-1)\delta, j\delta) \quad \forall \omega \in T_{i,j} \quad (15)$$

where $z_{i,j}(\omega)$ is the density of material at location ω departing during time intervals i, \dots, j and arriving during the j^{th} time interval and $w_{i,j}(\omega)$ is the density of material at location ω departing during time intervals i, \dots, j and arriving also during time intervals i, \dots, j . Figure 4 illustrates an example resource shed distribution based on Figure 2; notice there are variations within the resource shed on how much material is contributed. A resource shed boundary can be discerned from a resource shed mapping. It corresponds to the border between areas with positive contributions and those with zero contributions. Note that

$$z_{i,j}(\omega) = \sum_{m=i,j} x_{m,j}(\omega) \quad \forall \omega \in S_{i,j} \quad (16)$$

$$w_{i,j}(\omega) = \sum_{m=i,j} z_{i,m}(\omega) \quad \forall \omega \in T_{i,j} \quad (17)$$

Similar to resource sheds, we expect resource shed distributions for any time period to enclose (be greater than) those for included time periods, for a given location of interest and time. Since

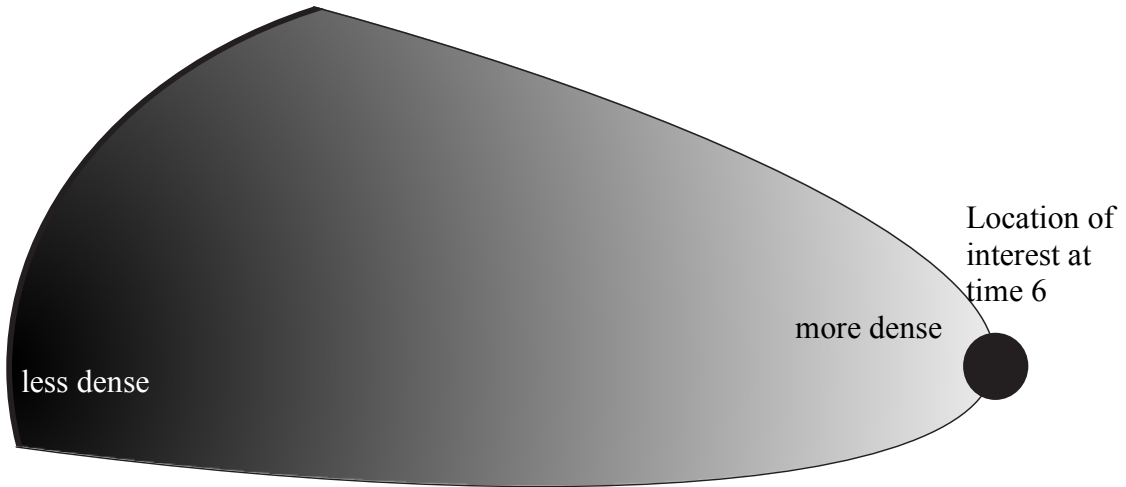


Figure 4. Example Resource Shed Distribution, $z_{4,6}(\omega) \quad \forall \omega \in S_{4,6}$, for Figure 2.

$x_{i,j}(\omega)$, $z_{i,j}(\omega)$, and $w_{i,j}(\omega)$ are strictly positive within their encompassing resource sheds, then similar to (7) and (8), for all ω

$$\begin{aligned} \sum_{m=i,j} x_{m,j}(\omega) &\leq \sum_{m=k,j} x_{m,j}(\omega), \quad k \leq i \\ \Rightarrow z_{i,j}(\omega) &\leq z_{k,j}(\omega), \quad k \leq i \end{aligned} \quad (18)$$

Equations (18) and (17) indicate for all ω

$$\begin{aligned} \sum_{m=i,j} z_{i,m}(\omega) &\leq \sum_{m=i,j} z_{k,m}(\omega) \leq \sum_{m=k,j} z_{k,m}(\omega), \quad k \leq i \\ \Rightarrow w_{i,j}(\omega) &\leq w_{k,j}(\omega), \quad k \leq i \end{aligned} \quad (19)$$

4. ESTIMATING RESOURCE SHEDS

4.1 Hydrological Resource Sheds

In a watershed, resource sheds can be computed from spatially distributed watershed hydrology models; material placed anywhere in the watershed will appear at the watershed outlet (mouth) over a period of time. We calculate the material appearing at the mouth at any time, contributed by a specific area. We consider every *cell* c (say 1 km²) of a watershed surface by tracing the material departing the cell over time interval i and computing the amount arriving at the mouth in time interval j , $\bar{x}_{i,j,c}$

$$\begin{aligned} \bar{x}_{i,j,c} &= \int_{A_c} x_{i,j}(\omega) d\omega, \quad \forall A_c \in V_{i,j} \\ &= \int_{A_c} x_{i,j}(\omega) d\omega, \quad c = 1, \dots, v_{i,j}/a \end{aligned} \quad (20)$$

where A_c is the set of all locations comprising cell c , $v_{i,j}$ is the area of $V_{i,j}$, and a is the area of A_c , $c = 1, \dots, v_{i,j}/a$ (all cells have the same area); the units of $\bar{x}_{i,j,c}$ are mass. This requires modeling all cells, but tracing contributions from only one at a time. Since it involves no more computation, we simulate material movement from each cell c during time intervals i, \dots, j to the mouth in time interval j $\bar{z}_{i,j,c}$; by (16)

$$\begin{aligned} \bar{z}_{i,j,c} &= \int_{A_c} z_{i,j}(\omega) d\omega = \int_{A_c} \left[\sum_{m=i,j} x_{m,j}(\omega) \right] d\omega, \quad \forall A_c \in S_{i,j} \\ &= \sum_{m=i,j} \int_{A_c} x_{m,j}(\omega) d\omega, \quad \forall A_c \in S_{i,j} \\ &= \sum_{m=i,j} \bar{x}_{m,j,c}, \quad \forall A_c \in S_{i,j} \\ &= \sum_{m=i,j} \bar{x}_{m,j,c}, \quad c = 1, \dots, s_{i,j}/a \end{aligned} \quad (21)$$

where $s_{i,j}$ is the area of $S_{i,j}$. Similarly we can combine results of several sets of simulations as in (17) to determine the source distribution of departing material in the watershed going through the mouth in any specified time period

$$\begin{aligned}\bar{w}_{i,j,c} &= \int_{A_c} w_{i,j}(\omega) d\omega, \quad \forall A_c \in T_{i,j} \\ &= \sum_{m=i,j} \bar{z}_{i,m,c}, \quad c = 1, \dots, t_{i,j}/a\end{aligned}\tag{22}$$

where $t_{i,j}$ = area of $T_{i,j}$. (We infer resource sheds from these mappings as described previously).

4.2 Material Fractions

We can also calculate relative fractions as well as absolute amounts; they will be useful later as source density estimates. Represent the material fraction arriving in time interval j that departed in time interval i from an area A_c as $p_{i,j,c}$. Represent the material fraction arriving in time interval j that departed during time intervals i, \dots, j from area A_c as $q_{i,j,c}$. Finally, represent the material fraction arriving during time intervals i, \dots, j that departed during time intervals i, \dots, j from area A_c as $u_{i,j,c}$. Note that

$$\begin{aligned}p_{i,j,c} &= \bar{x}_{i,j,c} / g_j, \quad c = 1, \dots, v_{i,j}/a \\ q_{i,j,c} &= \bar{z}_{i,j,c} / g_j, \quad c = 1, \dots, s_{i,j}/a \\ u_{i,j,c} &= \bar{w}_{i,j,c} / \sum_{m=i,j} g_m, \quad c = 1, \dots, t_{i,j}/a\end{aligned}\tag{23}$$

where g_j is the total material arriving at the location of interest in time interval j . Note that

$$\begin{aligned}q_{i,j,c} &= \frac{1}{g_j} \sum_{m=i,j} \bar{x}_{m,j,c} = \frac{1}{g_j} \sum_{m=i,j} g_j p_{m,j,c} = \sum_{m=i,j} p_{m,j,c} \\ u_{i,j,c} &= \frac{1}{\sum_{m=i,j} g_m} \sum_{m=i,j} \bar{z}_{i,m,c} = \frac{1}{\sum_{m=i,j} g_m} \sum_{m=i,j} g_m q_{i,m,c} = \sum_{m=i,j} \gamma_{i,j,m} q_{i,m,c}\end{aligned}\tag{24}$$

where $\gamma_{i,j,m} = g_m / \sum_{k=i,j} g_k$, which is the fraction of all material arriving at the location of interest during time intervals i, \dots, j that arrived in time interval m ($i \leq m \leq j$).

4.3 Lake Resource Sheds

In a lake, resource sheds may be computed with a lake circulation model in which particles (tracers) are defined at a given location of interest and the model is run backward in time from a specified time. The locations of the particles at earlier times define the extent of the resource

shed. By releasing the same number of particles each day in the backward simulation, one can estimate resource sheds for various prior periods to the date in question. The density of the particle distribution over the resource shed defines the fraction of material contributed. The example in Figure 5 shows only a small sampling for clarity.

The total material arriving at the location of interest in time interval j is

$$g_j = \sum_{k=1, N} \sum_{m=-\infty, j} \bar{x}_{m,j,k} \quad (25)$$

where N = number of cells in the lake (watershed). By rewriting the first line of (23) after substituting (25),

$$p_{i,j,c} = \frac{\bar{x}_{i,j,c}}{\sum_{k=1, N} \sum_{m=-\infty, j} \bar{x}_{m,j,k}} \quad (26)$$

We have been using (26) in resource shed developments to this point. However, the estimate of the material fraction arriving in time interval j that departed in time interval i from an area A_c that is determined from this backward time particle tracing is:

$$\hat{p}_{i,j,c} = \frac{\bar{x}_{i,j,c}}{\sum_{k=1, N} \bar{x}_{i,j,k}} \quad (27)$$

So backward time particle tracing where the same number of particles are released each day in the backward simulation gives only the spatial distribution of each time interval's contributions to the location of interest during the last time interval, but not the spatial-temporal distribution. Therefore,

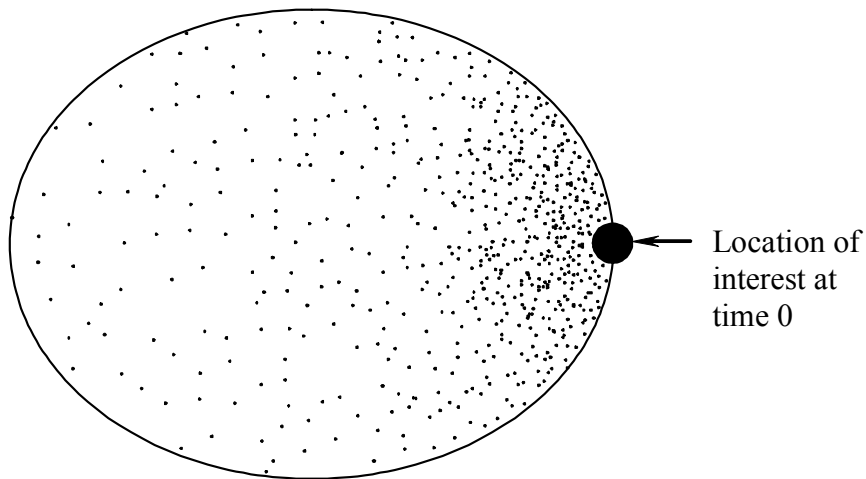


Figure 5. Example Estimating Resource Shed Distribution by Tracking Particles Backward.

$$p_{i,j,c} = \frac{\bar{x}_{i,j,c}}{g_j} = \frac{\hat{p}_{i,j,c} \sum_{k=1,N} \bar{x}_{i,j,k}}{g_j} = \phi_{i,j} \hat{p}_{i,j,c} \quad (28)$$

where $\phi_{i,j}$ = fraction of all material arriving during time interval j that originated over all cells during time interval i . The use of the estimate in (27) for $p_{i,j,c}$ in (24) implies the use of $\phi_{i,j} = 1$ (constant); the same number of particles arrive on day j originating on day i for all i . For example, the backward time particle tracing with the same number of particles released each day in the backward simulation uses $\phi_{i,j} = \frac{1}{7}$ for each day of a 7-day simulation. But constant $\phi_{i,j}$ is unreasonable. We expect that $\phi_{i,j} \rightarrow 0$ as $i \rightarrow -\infty$ and that $\phi_{i,j} < \phi_{m,j}$ for $i < m < k < j$ where k is the date/time of the peak contribution. Also note that

$$\sum_{i=-\infty,j} \phi_{i,j} = 1 \quad \forall \quad j \quad (29)$$

We would have information on $\phi_{i,j}$ indirectly in a forward simulation that uses a loading pattern with specified dynamic inputs and outputs from all rivers and channels. If we use the backward simulation, then we have to supply $\phi_{i,j}$ either from additional small forward simulations or other considerations. For example, we could use the reversed 2-parameter gamma distribution as a function of time lag only,

$$\phi_{i,j} = \int_{j-i}^{j-i+1} \frac{1}{\beta \Gamma(\alpha)} \left[\frac{x\delta}{\beta} \right]^{\alpha-1} e^{-\frac{x\delta}{\beta}} \delta dx \quad (30)$$

where δ = time increment length, $\Gamma(\bullet)$ is the gamma function, and α and β are parameters. Or we could use the reversed 1-parameter gamma distribution ($\beta = \delta$),

$$\phi_{i,j} = \int_{j-i}^{j-i+1} \frac{1}{\Gamma(\alpha)} x^{\alpha-1} e^{-x} dx \quad (31)$$

where α is taken as a function of time (season?).

Furthermore,

$$\begin{aligned} q_{i,j,c} &= \sum_{m=i,j} p_{m,j,c} = \sum_{m=i,j} \phi_{m,j} \hat{p}_{m,j,c} \\ u_{i,j,c} &= \sum_{n=i,j} \gamma_{i,j,n} q_{i,n,c} = \sum_{n=i,j} \gamma_{i,j,n} \sum_{m=i,n} \phi_{m,n} \hat{p}_{m,n,c} \end{aligned} \quad (32)$$

With information on $\phi_{i,j}$, we can calculate resource shed densities for material originating over time interval (i, j) and arriving over the j^{th} time interval. With information on $\gamma_{i,j,n}$ and $\phi_{i,j}$,

we can calculate resource shed densities for material originating over time interval (i, j) and arriving over the same time interval.

5. EXAMPLE MAUMEE RESOURCE SHEDS

5.1 Hydrological Model

We use the Great Lakes Environmental Research Laboratory's Distributed Large Basin Runoff Model (DLBRM) to model watershed outflow. It represents each cell (1 km^2) of a watershed's area as a cascade of moisture storages or *tanks*, each modeled as a linear reservoir, where tank outflows are proportional to tank storage; see Figure 6. Snow melt is proportional to snow accumulation and air temperature each day when temperatures are above freezing and infiltration into the top soil tank is a function of tank saturation (variable area infiltration where zero area corresponds to complete saturation). It computes potential evapotranspiration from heat available during the day, which is proportional to the exponential of air temperature, and computes actual evaporation or evapotranspiration for each tank from the potential and the moisture content of the tank; potential and actual evapotranspiration are therefore non-complementary, appropriate for small areas. Each tank has lateral flows between cells: an upstream flow into the tank and a downstream flow out of the tank for all moisture storages: surface zone, upper soil zone, lower soil zone, and groundwater zone. Each cell's inflow hydrographs must be known before its outflow hydrograph can be modeled and the DLBRM arranges calculations by flow network to assure this. It is implemented to minimize the number of pending hydrographs in storage and the time required for them to be in storage. It uses the same routing network for lateral flows between all surface, upper soil zone, lower soil zone, and groundwater zone storages.

Each cell's sub model (shown in Figure 6) has 15 parameters. It is calibrated for all cells by systematically searching the 15 spatial-average-parameter space by using gradient search techniques to minimize the root mean square error between modeled and actual basin outflow. The spatial variations of each parameter is the same as a function of selected watershed characteristics; for example the upper soil zone capacity from cell to cell is proportional to measurements taken from the field and the percolation coefficient (linear reservoir coefficient) varies proportional to measured permeability of the upper soil zone in the field. See Croley and He (2005, 2006) and Croley et al. (2005) for DLBRM details and Croley (2002) for the earlier lumped-parameter model which preceded the DLBRM. To speed up calibrations, we preprocess all meteorology for all watershed cells and preload it into computer memory.

Figure 7 shows example animation frames for the Maumee watershed in northwest Ohio. The watershed outlet is to the northeast; see the right map column in Figure 7 showing surface flows. In Figure 7, a storm peaks on September 1, 1950 and its pattern and intensity are obvious from the 1st column in Figure 7. The figure also shows that evapotranspiration is much higher on September 1st than on other days (see the middle column) and that surface response is higher too (see the right column). The surface outflow network is seen as most extensive during the storm peak. The Maumee is a very flashy watershed that responds quickly to surface supply. Note that while the spatial structure of the precipitation is revealed in the 1st column of Figure 7, the 2nd column reveals the spatial structure of the watershed as well as of the precipitation supply. These spatial variations mimic upper soil zone permeability. Likewise, the 3rd column reveals the spatial structure of the drainage network.

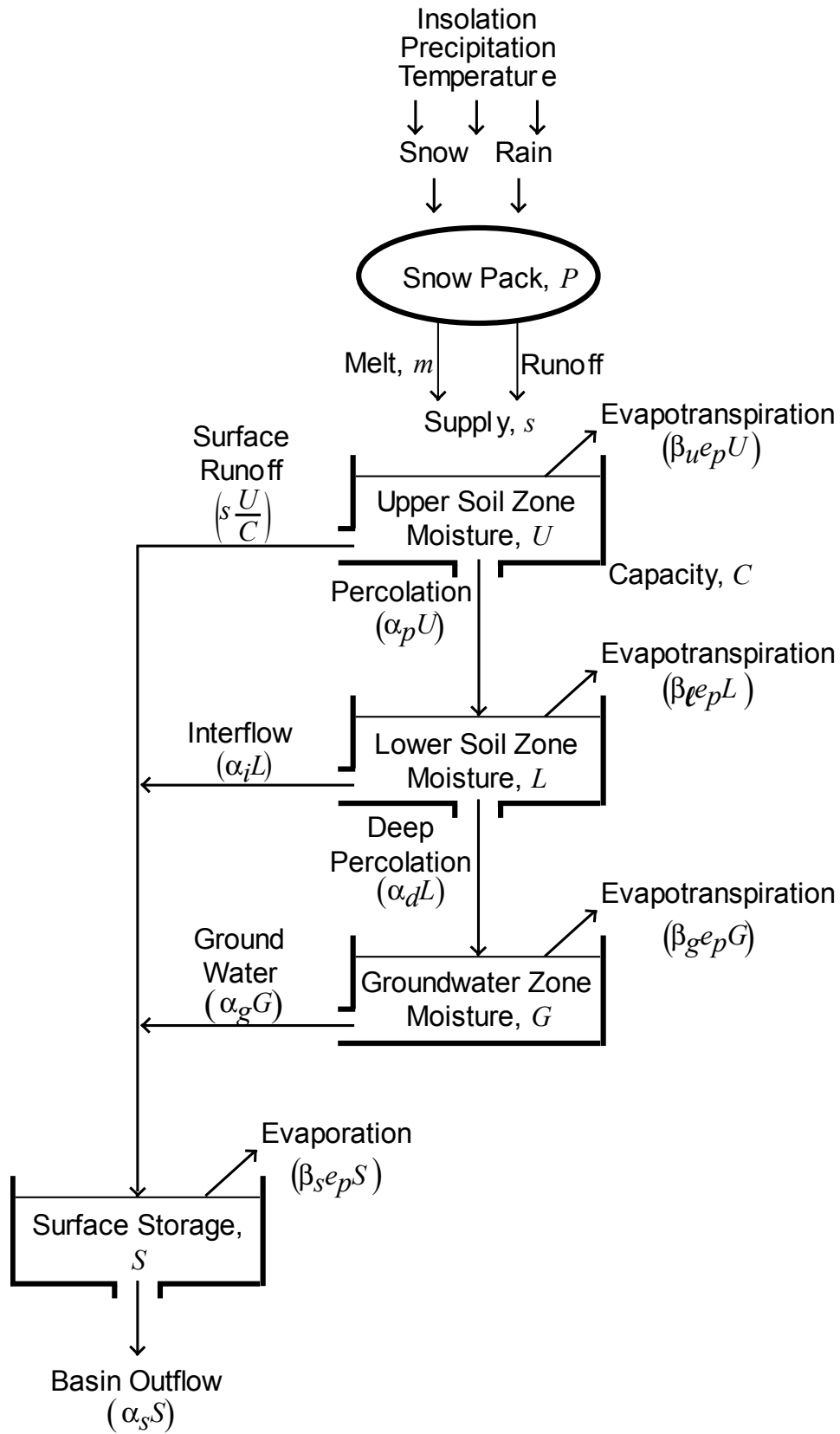


Figure 6. DLBRM Schematic for One Cell.

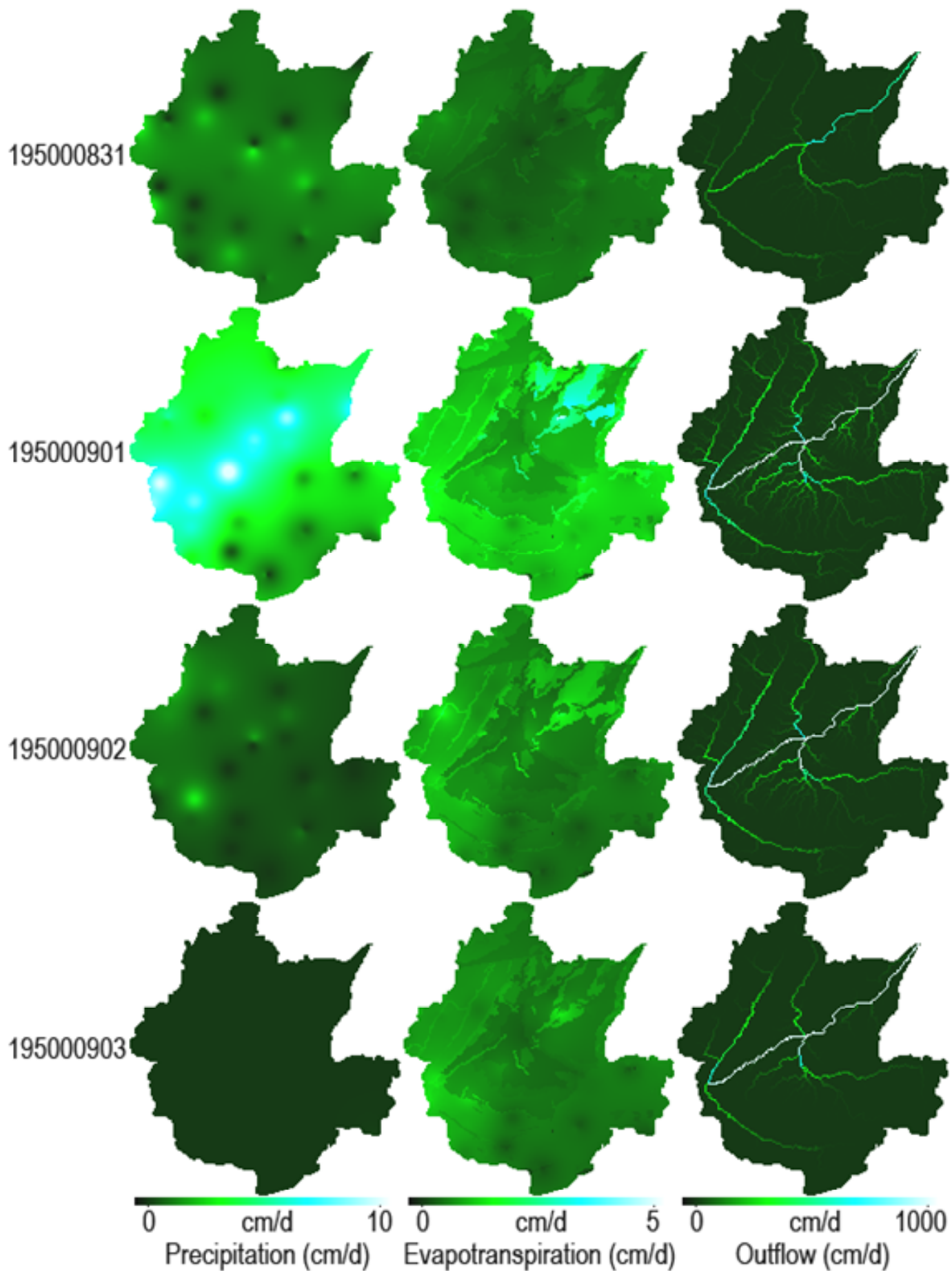


Figure 7. Maumee River Watershed Model Animation for August 31—September 3, 1950

5.2 Example Resource Sheds

Figure 8 shows example resource sheds for the Maumee River on January 1, 1950 from 1, 7, and 31 days of previous loading. In Figure 8, the brightest areas correspond to cells contributing about 0.015% of the total flow on January 1, 1950. The darkest areas are close to zero. Note several things about Figure 8. The southern and western ridgelines are prominent as is a line to the north that marks the boundary between Ohio and Michigan (AB in Figure 8). This boundary reflects the differences in the two States' definitions of some soil properties and so is an artifact of data standard differences. Point C in Figure 8 identifies the mouth of the watershed. The first map in Figure 8 shows a little response from the previous day's light rain near the mouth of the watershed. The second and third maps show most response along the edges of the watershed furthest from the mouth. Inspection of rainfall maps shows there is not much rainfall over the prior 4 days but there is a large amount 5 days prior in the southwest area. Also, spatially uniform rainfall fell over the entire watershed 7 days prior, 11-13 days prior, 15 days prior, 21-22 days prior, and 29 days prior. We can see the bright spot corresponding to the large peak 5 days earlier; the area closer to the mouth is relatively dark in the second and third maps because the only supplies there (seven or more days earlier) had already run off and are not part of the flow on this day. Note also that what happens prior to 7 days changes the picture very little (compare the last two maps). This is because the response of the watershed to supply is quick, on the order of 1 to 6 or 7 days, depending on location within the watershed. Most all supplies falling more that 6 or 7 days ago have already runoff and do not form a part of the flow on this day.

5.3 Average Resource Shed vs. Resource Shed of Average Meteorology

There are 17,541 1-km² cells in the Maumee River watershed. The DLBRM requires 0.2—0.4 seconds on today's desktop computers to simulate 1 day's hydrology from all of these cells, for

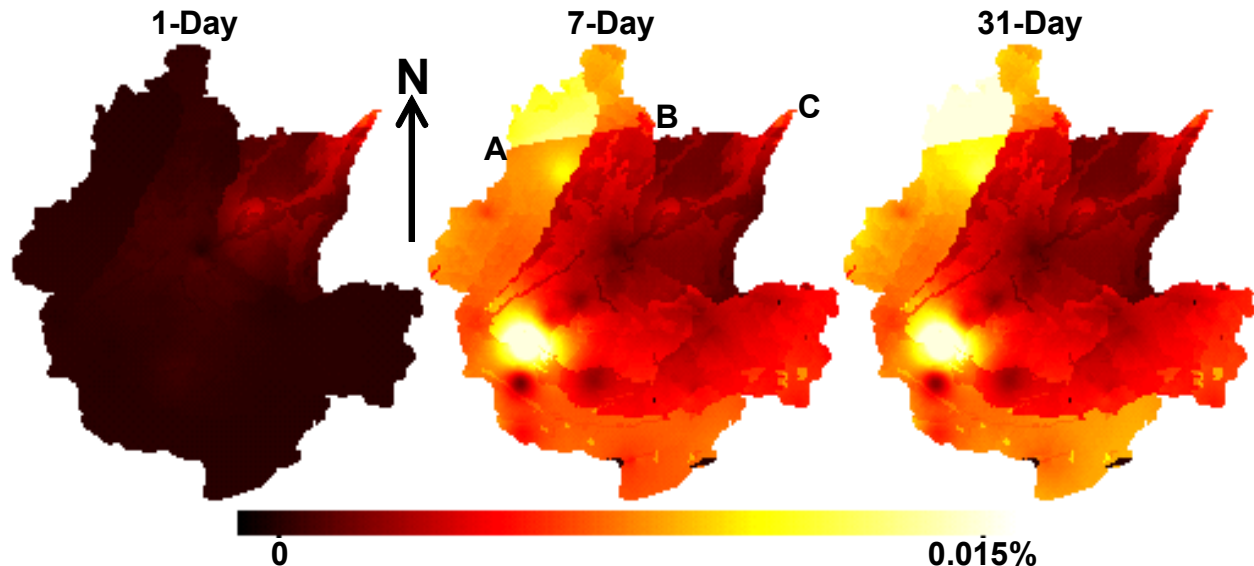


Figure 8. Maumee Resource Sheds on January 1, 1950 from prior days of loading:

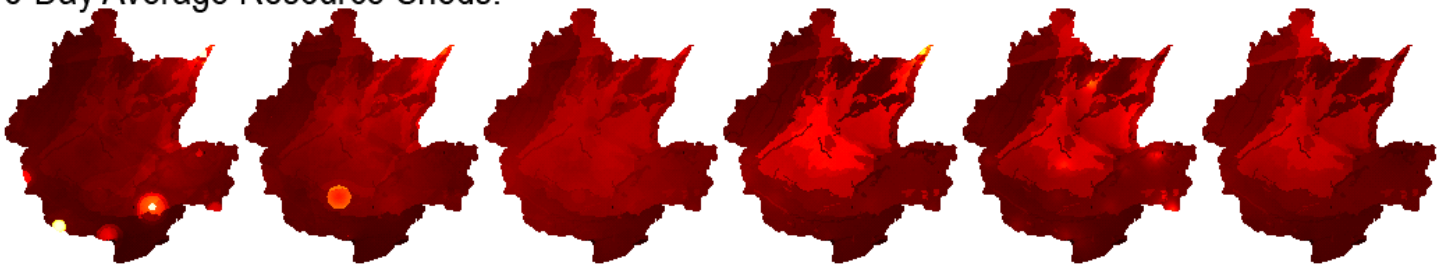
- 1-day: $Q_{January\ 1,\ 1950,\ January\ 1,\ 1950,\ c}, \quad c = 1, \dots, 17,541;$
- 7-day: $Q_{December\ 26,\ 1949,\ January\ 1,\ 1950,\ c}, \quad c = 1, \dots, 17,541;$
- 31-day: $Q_{December\ 2,\ 1949,\ January\ 1,\ 1950,\ c}, \quad c = 1, \dots, 17,541.$

any loading pattern. With 17,541 loading patterns (one tracing each cell's contribution), a one-day simulation of loadings requires 1—2 hours of computation. For 1-, 2-, ..., 10-day simulations for one date there are 55 simulation days, which require 2—5 days of computation.

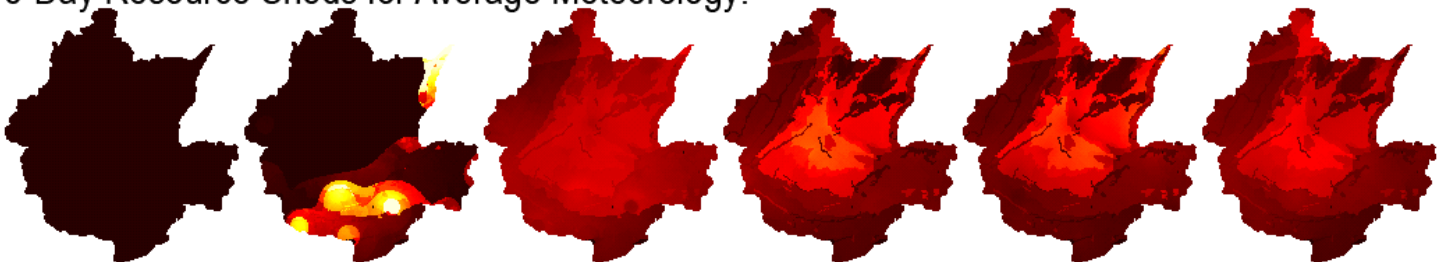
A parallel study requires average resource sheds for May 31 and August 31 for each contribution period from 1 through 31 days for linking to resource sheds in a lake that extend back into its watersheds. To calculate Maumee resource sheds over the historical record of 1950—1999 for averaging for these two dates and 31 contribution periods would require 49,600 simulation days or 5.5—11 years of computation, which is clearly impractical. Rather than calculate resource sheds for historical meteorology and then averaging them, we averaged the meteorology over all years for each day of the year and then calculated resource sheds based on this (1 year of) average meteorology. This required only 2% of the above: 992 simulation days or 40—80 days of computation. This is practical but begs the question: how well does the resource shed of average meteorology approximate the average resource shed (or how linear is the watershed model)?

To answer this question, we calculated resources sheds for selected dates for each year of record (1950—1999) for various contribution periods using about 20 machines to reduce the computation period. We averaged over the historical record and compared the resource sheds; Figure 9 shows selected average resource sheds and the resource sheds of average meteorology throughout the annual cycle. Figure 9 reveals two non-linearities in the model with respect to air temperature. The first occurs over winter, when air temperatures are close to or below freezing; the average resource shed (top Figure 9) reflects that snowmelt occurs in some years. However, average air temperature over the winter is zero and resource sheds calculated for an averaged winter date will have no snow melt (bottom Figure 9). This is because the model calculates snow-

6-Day Average Resource Sheds:



6-Day Resource Sheds for Average Meteorology:



Jan 15

Mar 01

Mar 15

Jun 01

Sep 01

Oct 15



Figure 9. Maumee River Six-Day Average Resource Sheds for Selected Times of the Year.

melt proportional to snow accumulation and temperature each day when temperatures are above freezing and zero when below freezing. There is a consequent over-accumulation of snow in the model applied to the average meteorology resulting in overly high snowmelt; see bottom Figure 9 for March 1. The second non-linearity occurs mostly in the summer. Then exponent of the average air temperature (used to calculate heat available for evapotranspiration) is less than the average exponent of the air temperatures and there is less potential (and actual) evapotranspiration in Figure 9 bottom than top. Therefore more of the moisture in the watershed ends up as outflow in the resource shed of the average meteorology (Figure 9 bottom) than in the average resource shed (Figure 9 top). However, the differences are slight so that the resource shed of the average meteorology approximates the average resource shed for April—October.

6. COMPUTATION REDUCTION

6.1 Estimation, Interpolation, and Resolution

To reduce computations we can simulate for a sample of cells distributed uniformly over the watershed; see Figure 10. In Figure 10, the three-digit codes beneath each resolution example refer to the horizontal and vertical offsets and the spacing, respectively. For example in the first resolution in the upper left corner of Figure 10, the code refers to a 4-cell spacing with 0 cell offsets in both the horizontal and vertical directions. For a selected resolution we interpolate between sampled cells to estimate values at the other cells. As computer resources become available, we can simulate again for a separate sample of cells and add them to the previous sample to increase resolution, reducing interpolation errors. Alternatively, we could simulate for all cells of the watershed at a coarser resolution. However, the disadvantage of that approach is that we would not be able to combine with another simulation to improve resolution; instead we would have to simulate for all cells of the watershed at a finer resolution. The third example in the upper right corner of Figure 10 has every 4 out of 16 cells computed, or 1 of 4, which is used in Figure 11.

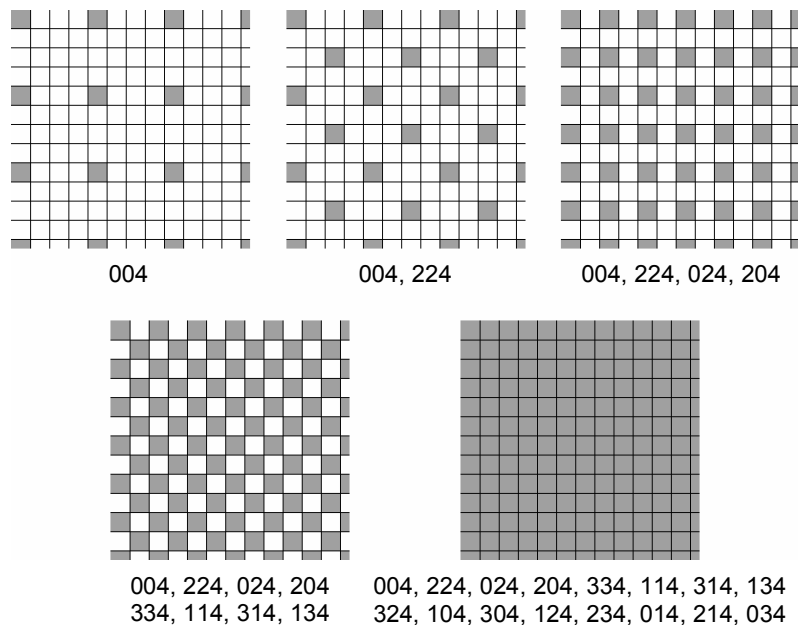


Figure 10. Resource Shed Distribution Map Computation Resolutions.

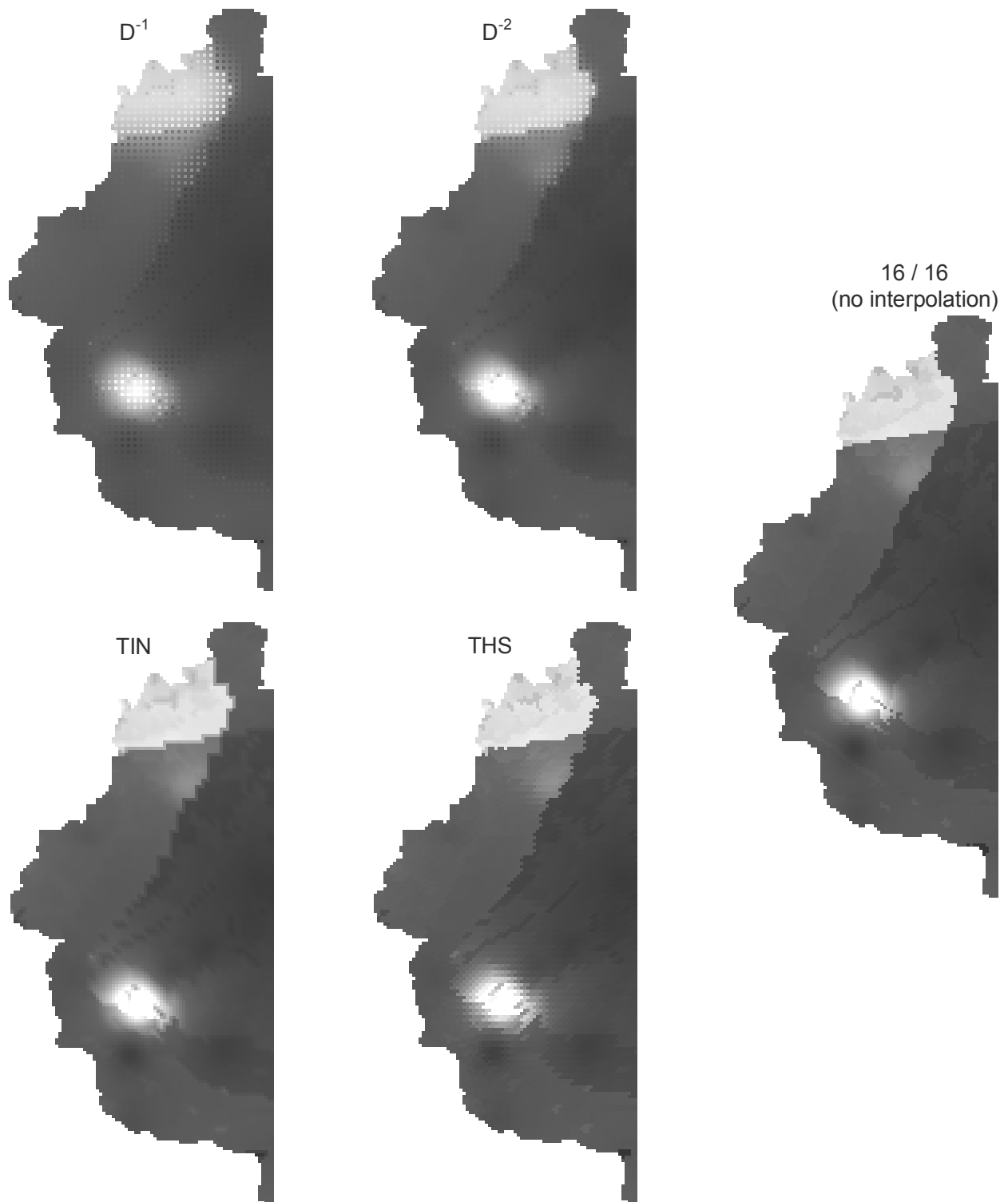


Figure 11. Western Maume Resource Shed Distribution Map Interpolations and Resolutions.

We considered several interpolation schemes, as shown in Figure 11 for the western Maume watershed. At all resolutions, the inverse distance (D^{-1}) and inverse distance squared (D^{-2}) interpolations produce artifacts at the scale of the resolution (which are barely discernable at the scale

used in Figure 11). The inverse distance squared gives better definition to the features than the inverse distance at all resolutions. The triangular irregular network with linear interpolation (TIN) does not produce the artifacts of the two inverse distance methods and gives better definition to features in the watershed. It also gives the most detailed picture of structure of the three methods at all resolutions. Finally, the Thiessen method appears probably the most aesthetically pleasing since the contrast is better at all resolutions, giving the illusion of more detail. It also paints a true picture of the resolution being used. However, it can be argued that even though the Thiessen interpolation appears crisper than the corresponding TINs, it may not represent the actual surface as well. That appears to be the case here: see the bright spot in the Southwest section of the watershed; the first three interpolations appear to capture the double tail lying to the southeast of the bright spot, while the Thiessen does not (compare with the right-most picture where all cells are present and no interpolation is required). On the other hand, the northwest head of the bright spot seems best represented in the Thiessen picture.

6.2 Ordering Computations

Computation reduction may also be achieved through computation ordering. Consider the calculation of a resource shed, $q_{i,j,c}$ for specific values of i and j for all c , $c = 1, \dots, s_{i,j}/a$. There are $j-i+1$ days to simulate. (For the DLBRM Maumee example, recall that each simulation day requires 1—2 hours of computation on today’s desktop computers.) If we simply repeat the calculation for every prior period from one to 31 days, there are $1 + 2 + \dots + 31 = 496$ simulation days required for one end date, j . For 365.25 end dates in an average year, there are $496 \times 365.25 = 181,164$ simulation days required per year. However, we could utilize more information from one simulation with the hydrology model. Consider that one simulation from day i to day j actually gives us (for all c) $q_{i,j,c}$, $q_{i,j-1,c}$, $q_{i,j-2,c}$, \dots , $q_{i,i,c}$. Therefore, if we do $365.25 \times 31 = 11,323$ simulation days per year, we can calculate the same information as above with only 6.25% of the effort.

Finally, there is interest in resource shed calculations in near real time to aid in detection of watershed source areas responsible for pollutant transport to receiving waters on an event basis. This would aid in the management of watershed runoff for limiting harmful algal blooms or in the closing of beaches in the receiving waters. It would also enable more effective land use management within a watershed. Consider a watershed application where it is deemed sufficient each day to estimate the 31 resource sheds corresponding to prior periods of 1 through 31 days. This would require 496 simulation days each day by the first method outlined previously (or about 496—992 hours of computation each day) which is clearly impractical in near real time. The second method outlined previously does not save computation since one 31-day simulation yields 31 resource sheds all ending on different days; we would again require 496 simulation days to calculate the 31 resource sheds of different durations all ending on the same day.

However, by saving all internal model moisture and tracer storages at the end of the day from the model for each of the first 30 durations (1-day through 30-day), the next day we could make a one-day simulation (for the 1-day resource shed) and use the saved internal storages (from yesterday’s 1-day through 30-day simulations) as initial conditions in 30 one-day simulations to calculate the 2-day through 31-day resource sheds. By again saving all internal storages at the end of the day, we could repeat for the next day and so forth. We need only compute 31 1-day simu-

lations (31 simulation days or about 31—62 hours of computation per day). There are several ways to reduce computations further: change the hydrological model cell size from 1 km² to 2 km × 2 km (4 km²) to reduce the computations to one sixteenth of the 1 km² model, use the 1 km² model cell size but use less than full resolution and interpolate (e.g., reducing resolution by half and interpolating would reduce computations by half), and use multiple processors (e.g., a ten-processor machine would do one-tenth of the total computations on each processor in parallel, reducing computations to one tenth). Any or a combination of these methods could reduce computations; for example, using a 10-processor machine at half resolution and interpolating for 1-km² model cell size reduces computation time by 20. That comes to about 1.5—3 hours of computation per day, which is practical in near real time.

7. LINKING RESOURCE SHEDS

7.1 Definitions for the Joint Resource Shed

We desire to (eventually) extend a lake's resource shed back into a contributing watershed, requiring the joining of resource sheds, estimated with different techniques, through a point (the mouth of a watershed). See Figure 12 where the superscripts denote Lake (L) and Watershed (W). The joint resource shed for material departing in time interval i and arriving at the location of interest (in the lake) in time interval j , $V_{i,j}^C$, is

$$\begin{aligned} V_{i,j}^C &= V_{i,j}^L, & i > k_j \\ &= V_{i,j}^L \cup \bigcup_{m=i,k_j} V_{i,m}^W & i \leq k_j \end{aligned} \quad (33)$$

where k_j is the last time interval for material leaving the watershed arriving at the location of interest in the lake in time interval j . Since $V_{i,k_j}^W = \emptyset \quad \forall i > k_j$, we can write (33) simply as

$$V_{i,j}^C = V_{i,j}^L \cup \bigcup_{m=i,k_j} V_{i,m}^W \quad (34)$$

Summing (34),

$$\begin{aligned} \bigcup_{n=i,j} V_{n,j}^C &= \bigcup_{n=i,j} V_{n,j}^L \cup \bigcup_{n=i,j} \bigcup_{m=n,k_j} V_{n,m}^W = \bigcup_{n=i,j} V_{n,j}^L \cup \bigcup_{n=i,k_j} \bigcup_{m=n,k_j} V_{n,m}^W \\ &= \bigcup_{n=i,j} V_{n,j}^L \cup \bigcup_{m=i,k_j} \bigcup_{n=i,m} V_{n,m}^W \end{aligned} \quad (35)$$

Then by (5), we can construct the resource shed with material departing during time intervals i, \dots, j and arriving during time interval j , $S_{i,j}^C$ (noting that $S_{i,k_j}^W = \emptyset \quad \forall i > k_j$)

$$S_{i,j}^C = S_{i,j}^L \cup \bigcup_{m=i,k_j} S_{i,m}^W \quad (36)$$

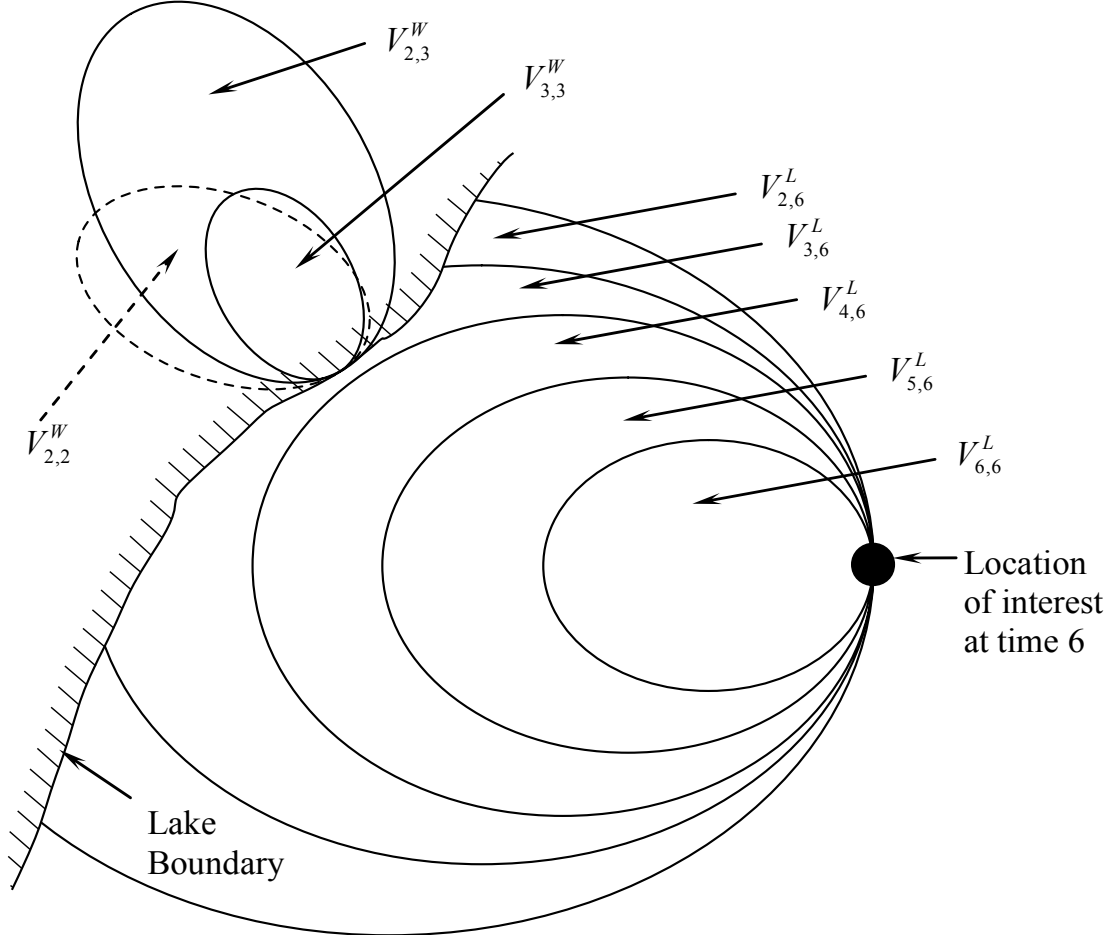


Figure 12. Example Lake Resource Shed Just Touching Watershed Mouth 3δ Ago.

Likewise, summing (36)

$$\bigcup_{n=i,j} S_{i,n}^C = \bigcup_{n=i,j} S_{i,n}^L \cup \bigcup_{n=i,j} \bigcup_{m=i,k_n} S_{i,m}^W \quad (37)$$

or

$$T_{i,j}^C = T_{i,j}^L \cup \bigcup_{n=i,j} \bigcup_{m=i,k_n} S_{i,m}^W \quad (38)$$

The joint resource shed distributions for material departing in time interval i and arriving in time interval j are

$$\begin{aligned} \bar{x}_{i,j,c}^C &= \bar{x}_{i,j,c}^L, & c = 1, \dots, v_{i,j}^C / a, & \quad i > k_j \\ &= \bar{x}_{i,j,c}^L + \sum_{m=i,k_j} \alpha_{m,j} \bar{x}_{i,m,c}^W, & c = 1, \dots, v_{i,j}^C / a, & \quad i \leq k_j \end{aligned} \quad (39)$$

where $\alpha_{m,j}$ (calculated with the lake circulation model) is the fraction of material leaving the watershed in time interval m that arrives at the location of interest in the lake in time interval j . (Since $\alpha_{m,j}$ is applied to $\bar{x}_{i,m,c}^W$, $c = 1, \dots, v_{i,m}^W / a$, $i \leq k_j$, the assumption is that watershed outflow is fully mixed.) Note that the watershed and lake do not overlap (intersect only at a point); therefore one of the two terms on the right side of (39) will be zero for each $c = 1, \dots, v_{i,j}^C / a$. Because $\alpha_{m,j} = 0 \quad \forall m > k_j$, we write simply

$$\bar{x}_{i,j,c}^C = \bar{x}_{i,j,c}^L + \sum_{m=i,j} \alpha_{m,j} \bar{x}_{i,m,c}^W \quad c = 1, \dots, v_{i,j}^C / a \quad (40)$$

By summing (40) over consecutive time intervals i, \dots, j , analogous to (35), one can demonstrate

$$\bar{z}_{i,j,c}^C = \bar{z}_{i,j,c}^L + \sum_{m=i,j} \alpha_{m,j} \bar{z}_{i,m,c}^W \quad c = 1, \dots, s_{i,j}^C / a \quad (41)$$

Similarly, summing (41), analogous to (37), gives

$$\bar{w}_{i,j,c}^C = \bar{w}_{i,j,c}^L + \sum_{n=i,j} \sum_{m=i,n} \alpha_{m,n} \bar{z}_{i,m,c}^W \quad c = 1, \dots, t_{i,j}^C / a \quad (42)$$

Finally, looking at fractions of material moved, by (23) and (40)

$$\begin{aligned} p_{i,j,c}^C &= \frac{\bar{x}_{i,j,c}^C}{g_j^L} = \frac{\bar{x}_{i,j,c}^L}{g_j^L} + \frac{1}{g_j^L} \sum_{m=i,j} \alpha_{m,j} \bar{x}_{i,m,c}^W \\ &= p_{i,j,c}^L + \sum_{m=i,j} \frac{\alpha_{m,j}}{g_j^L} g_m^W p_{i,m,c}^W \\ &= p_{i,j,c}^L + \sum_{m=i,j} \beta_{m,j} p_{i,m,c}^W, \quad c = 1, \dots, v_{i,j}^C / a \end{aligned} \quad (43)$$

where $\beta_{m,j} = \alpha_{m,j} g_m^W / g_j^L$ which is the fraction of material arriving at the location of interest in the lake in time interval j that came from the watershed mouth in time interval m . Likewise

$$q_{i,j,c}^C = q_{i,j,c}^L + \sum_{m=i,j} \beta_{m,j} q_{i,m,c}^W \quad c = 1, \dots, s_{i,j}^C / a \quad (44)$$

$$u_{i,j,c}^C = u_{i,j,c}^L + \sum_{n=i,j} \gamma_{i,j,n}^L \sum_{m=i,n} \beta_{m,n} q_{i,m,c}^W \quad c = 1, \dots, t_{i,j}^C / a \quad (45)$$

where $\gamma_{i,j,n}^L = g_n^L / \sum_{k=i,j} g_k^L$, which is the fraction of all material arriving at the location of interest in the lake during time intervals i, \dots, j that arrived in time interval n ($i \leq n \leq j$).

7.2 Example Calculation Algorithm

Suppose we want to calculate source material distributions for the 7-day resource shed for May 31 for Lake Erie and the Sandusky River watershed. [We define this resource shed distribution as meaning the areal distribution of all material (water) that moved through our location of interest in the lake in the 7-day period ending May 31, (May 25—31).] Suppose that lake circulation modeling gave us the information in Table 1 on the fraction of material arriving at the location of interest in the lake in time interval j that came from the watershed mouth in time interval i , $\beta_{i,j}$.

Table 1. $\beta_{i,j}$, $i = 25, \dots, 31$, $j = 25, \dots, 31$, May, Sandusky

$i \setminus j$	25	26	27	28	29	30	31
25	0	0	0	0.01	0.07	0.09	0.10
26		0	0	0	0.02	0.03	0.06
27			0	0	0	0.05	0.08
28				0	0	0	0.04
29					0	0	0
30						0	0
31							0

Shaded cells are produced by the lake circulation model.

Un-shaded cells are zeroes.

Furthermore, the fraction of the 7-day flow volume arriving at the location of interest in the lake each day of May 25—31 was supplied by the lake circulation modelers in Table 2.

Table 2. $\gamma_{25,31,n}^L$, $n = 25, \dots, 31$, May, Sandusky

n	25	26	27	28	29	30	31
$\gamma_{25,31,n}^L$	0.241	0.217	0.193	0.157	0.096	0.060	0.036

From watershed model calculations, we have the resource shed distributions over the Sandusky watershed for material arriving at the watershed mouth in time interval j that came from cell 1 in time intervals i, \dots, j , $\bar{z}_{i,j,1}^W$ in Table 3.

Table 3. $\bar{z}_{25,j,1}^W$, $j = 25, \dots, 31$, May, Sandusky

j	25	26	27	28	29	30	31
$\bar{z}_{25,j,1}^W$	2.10e-5	3.22e-5	2.11e-5	1.86e-5	2.09e-5	1.59e-5	2.66e-5

The total arriving at the watershed mouth in time interval j , g_j^W , is given in Table 4.

Table 4. Daily Watershed Outflow g_j^W , $j = 25, \dots, 31$, May, Sandusky.

j	25	26	27	28	29	30	31
g_j^W	9.80e-2	1.12e-1	1.05e-1	9.42e-2	9.21e-2	8.29e-2	9.61e-2

As in (23), divide each column in Table 3 by the corresponding entry in Table 4 to calculate the fraction of material departing cell 1 in time intervals i, \dots, j arriving at the watershed mouth in time interval j , $q_{i,j,1}^W$, shown in Table 5.

Table 5. $q_{25,j,1}^W = \bar{z}_{25,j,1}^W / g_j^W$, $j = 25, \dots, 31$, May, Sandusky

j	25	26	27	28	29	30	31
$q_{25,j,1}^W$	2.14e-4	2.88e-4	2.01e-4	1.97e-4	2.27e-4	1.92e-4	2.77e-4

The convolution of watershed resource shed distributions with lake resource shed distributions, as shown in (45), gives the total resource shed distribution over both the watershed and the lake for water departing its origin in time intervals i, \dots, j and arriving at the location of interest in the lake during the same period for cell c , $u_{i,j,c}^C$, as either the resource shed distribution over the lake, $u_{i,j,c}^L$, when c is a cell in the lake, or a convolution of watershed resource shed distributions arriving at the watershed mouth in time interval m , $m = i, \dots, j$ departing its origin in time intervals i, \dots, m , $q_{i,m,c}^W$, when c is a cell in the watershed. For $c = 1$ (in the watershed), the 7-day total resource shed distribution (within the watershed) ending on May 31 ($i = 25$, $j = 31$) is found by first computing the terms, $\beta_{m,n} q_{25,m,1}^W$, $n = 25, \dots, 31$, $m = 25, \dots, n$. From Tables 1 and 5, we build the shaded part of Table 6. Then we sum each column to calculate $\sum_{m=25,n} \beta_{m,n} q_{25,m,1}^W$, $n = 25, \dots, 31$ in the next row. We place $\gamma_{25,31,n}$ entries from Table 2 in the next row for $n = 25, \dots, 31$, multiply these two rows together to calculate $\gamma_{25,31,n} \sum_{m=25,n} \beta_{m,n} q_{25,m,1}^W$, $n = 25, \dots, 31$ in the final row, and finally sum the final row for $n = 25, \dots, 31$, to get $\sum_{n=25,31} \gamma_{25,31,n} \sum_{m=25,n} \beta_{m,n} q_{25,m,1}^W$ in the bottom row right column, in black. Thus 6.86e-6 or 0.000686% of the flow through the location of interest in the lake during May 25—31 came from cell 1 in the Sandusky watershed during the same period.

To summarize, for a given location in the lake and a given month, the procedure to calculate $u_{i,j,c}^C$ over the watershed ($c = 1, \dots, n_W$ where $n_W =$ the number of cells in the watershed) for $i = 1, 25$, and 31 and $j = 31$ (the 31-day resource shed, the 7-day resource shed, and the 1-day resource shed, respectively for the 31st) given $\bar{z}_{i,m,c}^W$, $c = 1, \dots, n_W$, $m = i, \dots, j$, g_m^W , $m = i, \dots, j$, $\beta_{m,n}$, $n = i, \dots, j$, $m = i, \dots, j$, and $\gamma_{i,j,n}^L$, $n = i, \dots, j$ is shown with pseudo code given in Table 7.

Table 6. $\beta_{m,n} \bar{q}_{25,m,1}^W$, $n = 25, \dots, 31$, $m = 25, \dots, n$, May, Sandusky and additional computations.

$m \setminus n$	25	26	27	28	29	30	31	$\sum_{n=25,31}$
25	0× 2.14e-4	0× 2.14e-4	0× 2.14e-4	0.01× 2.14e-4	0.07× 2.14e-4	0.09× 2.14e-4	0.10× 2.14e-4	
26		0× 2.88e-4	0× 2.88e-4	0× 2.88e-4	0.02× 2.88e-4	0.03× 2.88e-4	0.06× 2.88e-4	
27			0× 2.01e-4	0× 2.01e-4	0× 2.01e-4	0.05× 2.01e-4	0.08× 2.01e-4	
28				0× 1.97e-4	0× 1.97e-4	0× 1.97e-4	0.04× 1.97e-4	
29					0× 2.27e-4	0× 2.27e-4	0× 2.27e-4	
30						0× 1.92e-4	0× 1.92e-4	
31							0× 2.77e-4	
$\sum_{m=25,n} \beta_{m,n} q_{25,m,1}^W$	0	0	0	2.14e-6	2.07e-5	3.80e-5	6.26e-5	
$\gamma_{25,31,n}$	0.241	0.217	0.193	0.157	0.096	0.060	0.036	
$\gamma_{25,31,n} \sum_{m=25,n} \beta_{m,n} q_{25,m,1}^W$	0	0	0	3.36e-7	1.99e-6	2.28e-6	2.25e-6	6.86e-6

7.3 Example Lake Erie—Maumee River Linked Resource Sheds

We can do an example coupling of Lake Erie and Maumee River resource sheds and their distributions by using (44) and (45) with $q_{i,j,c}^W$ calculated as $q_{i,j,c}$ in (32) and with $\phi_{i,j}$ in (32) calculated with (31) for $\alpha = 30$ days. Shown in Figure 13 are coupled resource shed densities for material originating over the indicated periods ending on August 31 and passing through the location of interest over that same period. They were calculated by using the hydrology model for the Maumee watershed and the Princeton Ocean Model applied to Lake Erie circulation at the Great Lakes Program of the University of Buffalo. (Note that both the lake and watershed resource sheds are calculated from daily average meteorology over 1983—2002). Lake Erie is depicted with a constant color not to be confused as part of the spectrum of resource shed values. The location of interest is Western Basin Lake Erie Site 835 just adjacent to the tip of the arrow in Figure 13. There we see that on August 31, the 1-day resource shed does not extend into the watershed (flow from Maumee actually takes about two days to reach the location of interest at this time of year). The latter resource sheds do extend into the Maumee watershed and intensify as we look over longer prior periods.

Table 7. Watershed—Lake Resource Shed Distribution Convolution Algorithm Pseudo Code.

```

! FROM LAKE CIRCULATION MODEL OUTPUT FOR GIVEN LOCATION & MONTH:
! INPUT BETA(M, N), M = I,...,J, N = I,...,J
! INPUT OR CALCULATE GAMMA(I, J, N), N = I,...,J
! FROM WATERSHED HYDROLOGY MODEL OUTPUT FOR GIVEN MONTH:
! INPUT ZW(I, M, C), I = 1,...,31, J = 1,...,31, C = 1,...,NW
! INPUT GW(M), M = I,...,J

```

```

J = 31
DO I = 1, 25, 31      ! The 31-day, 7-day, & 1-day resource sheds
  DO C = 1, NW       ! NW = number of watershed cells
    A1 = 0           ! dummy accumulator variable
    DO N = I, J
      A2 = 0         ! dummy accumulator variable
      DO M = I, N
        A2 = A2 + BETA(M, N) * ZW(I, M, C) / GW(M)
      END DO
      A1 = A1 + A2 * GAMMA(I, J, N)
    END DO
    UC(I, J, C) = A1
  END DO
END DO

```

Shaded denotes non-executable comments.

8. SUMMARY

A resource shed is defined as the area contributing material, over one time interval, passing through a location of interest over another time interval. We looked at definitions and concepts that preceded the resource shed concept and discussed interpretations in several fields. We also contrasted the concept with watersheds. Following this introduction, we provided rigorous general definitions of resource sheds and derived expressions for: 1) resource sheds where materials departing during one time interval arrive at the location of interest during a later time interval, 2) resource sheds where materials departing during a sequence of time intervals arrive at the location of interest during the last of those time intervals, and 3) resource sheds where materials departing during a sequence of time intervals arrive at the location of interest during that same sequence of time intervals. We also expressed each of these resource sheds in terms of each other and showed that definition 2 resource sheds are contained in others with a longer departing time for the same arrival interval. Likewise, definition 3 resource sheds are contained in others with earlier start times and the same end times. These definitions of resource sheds as areas of source materials were illustrated with the classical travel-time isochronal map for a watershed, used to build a watershed's time-area histogram for use in deriving a synthetic unit hydrograph.

We then provided rigorous general definitions of resource shed distributions and derived expressions for the distributed amount of source material within a resource shed, for all three definitions of same. We again expressed each of the resource shed distributions in terms of each other and showed that resource shed distributions are contained by others with earlier start times, analogous to resource sheds themselves, for both definitions 2 and 3. We discussed the estimation of resource shed distributions (corresponding to all three definitions) within a watershed

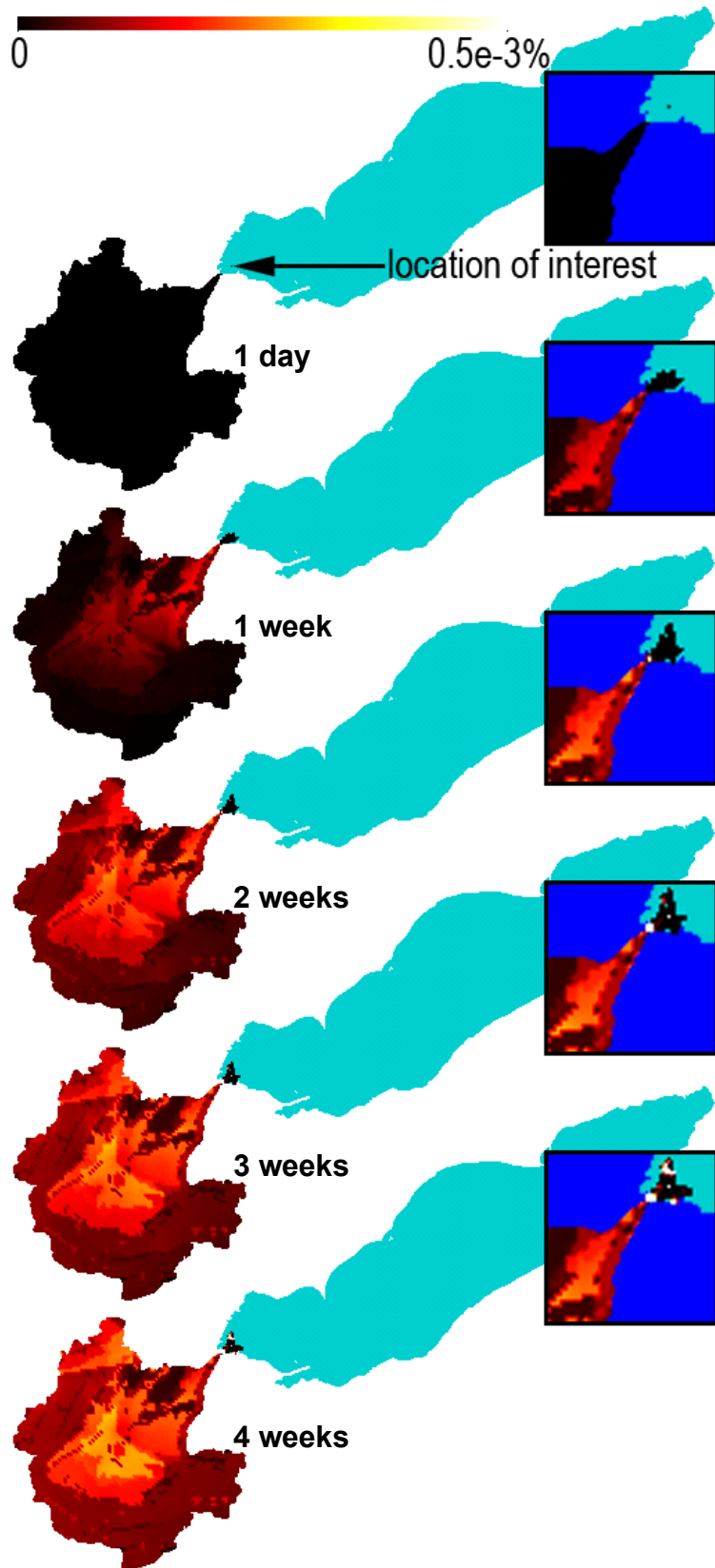


Figure 13. Maumee—Erie Linked Resource Shed for Average August 31 Conditions for Site 835

through the use of a spatially distributed hydrology model, in which the watershed area consists of discrete cells, by first defining resource shed distributions in terms of discrete space and showing their interrelationships with each other, presenting the distributions in terms of relative fractions of the material passing through the location of interest, introducing a specific hydrologic model for use on the Maumee River watershed in Ohio, and calculating example resource shed distributions for material passing through the watershed outlet on January 1, 1950 for 3 different periods of previous loadings.

Next, we used resource shed distributions to observe the non-linearity of the hydrology model and its relative impact in different seasons of the year. We did this by comparing resource sheds computed from average meteorology to the average of resource sheds calculated from actual meteorology. We found two non-linearities associated with air temperature: a major one associated with snow melt during the winter and a lesser one associated with evaporation during the summer and fall. We concluded that resource sheds calculated from average meteorology are only slightly different from average resource sheds for April—October and may be used in studies linking average resource sheds between a watershed and receiving lake waters, for example.

The large amount of computation sometimes required in the construction of resource sheds and their distributions encourages methods to reduce calculations. We can calculate resource shed distributions over only a sample of watershed cells and then interpolate for the other cells. We explored alternate resolution trade-offs and a few spatial interpolation technique trade-offs. Also, we can order resource shed computations, when more than one resource shed is computed, to save on computations. Ordering makes sense when contiguous dates and durations are considered or in near real time where yesterday's computations are extended one day in today's.

Finally, we considered the joining of a resource shed beginning in a lake and progressing (as we consider longer prior periods of flow for a given date) into the adjoining watersheds that contribute to the lake. We derived definitions and relations for the combined resources shed and the combined resource shed distribution for all three definition types identified above. We depicted the joint resource shed distribution for the Lake Erie—Maumee watershed system in an example application.

It should be possible to extend this work in several ways. Currently, we are linking hydrological watershed resource sheds and their distributions with lake resource sheds generated via different methods, as exemplified here. We plan to produce, for the World Wide Web, a dynamic linking of several Lake Erie watersheds with the lake to produce joint resource sheds and their distributions associated with about 35 locations of interest in the lake and extending into the tributary watersheds. Also, as we add material transport capabilities to our hydrologic model, we will produce resource sheds and their distributions for nutrients, sediment, insecticides, and microbes that may be of more direct use in prediction of harmful algal blooms and beach closings than simply water transport. Finally, in support of these latter predictions, we are also building a daily near real time generator of resource sheds and their distributions and plan to move to hourly as the model develops further.

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APPENDIX OF NOTATION

The following symbols are used in this paper (upper case letters denote sets and lower case letters denote functions or scalars):

A_c = set of all locations comprising cell c ;

a = area of A_c , $c = 1, \dots, v_{i,j}/a$ (all cells have the same area);

c = watershed surface cell number;

$f(\omega, a, b, c, d)$ = spatial density of material (mass/area) at location ω departing during $[a, b)$ and arriving at LOI during $[c, d)$;

g_j = total material arriving at LOI in time interval j ;

g_j^W = g_j , defined over watershed for LOI at mouth of watershed;

$h(\omega, \tau, t)$ = areal density rate of change with departure and arrival times of material (mass/area/time/time) departing location ω at time τ and arriving at LOI at time t ;

k_j = last time interval for material leaving watershed to arrive at LOI in lake in time interval j ;

LOI = denotation of “location of interest;”

$p_{i,j,c}$ = material fraction arriving at LOI in time interval j that departed in time interval i from area A_c ;

$p_{i,j,c}^C$ = $p_{i,j,c}$ defined over watershed and lake for LOI in lake;

$p_{i,j,c}^L$ = $p_{i,j,c}$ defined over lake for LOI in lake;

$p_{i,j,c}^W$ = $p_{i,j,c}$ defined over watershed for LOI at mouth of watershed;

$\hat{p}_{i,j,c}$ = material fraction arriving at LOI in time interval j that departed in time interval i from area A_c , determined from backward time particle tracing where the same number of particles are released each day in the backward simulation;

$q_{i,j,c}$ = material fraction arriving at LOI in time interval j that departed during time intervals i, \dots, j from area A_c ;

$q_{i,j,c}^C$ = $q_{i,j,c}$ defined over watershed and lake for LOI in lake;

$q_{i,j,c}^L$ = $q_{i,j,c}$ defined over lake for LOI in lake;

$q_{i,j,c}^W$ = $q_{i,j,c}$ defined over watershed for LOI at mouth of watershed;

$R(a,b,c,d)$ = set of all locations (resource shed) where materials departing during time interval $[a,b)$ arrive at LOI during time interval $[c,d)$;

$S_{i,j}$ = set of all locations (resource shed) where materials departing during time intervals i, \dots, j arrive at LOI during time interval j ;

$S_{i,j}^C$ = $S_{i,j}$ defined over watershed and lake for LOI in lake;

$S_{i,j}^L$ = $S_{i,j}$ defined over lake for LOI in lake;

$S_{i,j}^W$ = $S_{i,j}$ defined over watershed for LOI at mouth of watershed;

$s_{i,j}$ = area of $S_{i,j}$;

$s_{i,j}^C$ = area of $S_{i,j}^C$;

$T_{i,j}$ = set of all locations (resource shed) where materials departing during time intervals i, \dots, j arrive at LOI also during time intervals i, \dots, j ;

$T_{i,j}^C$ = $T_{i,j}$ defined over watershed and lake for LOI in lake;

$T_{i,j}^L$ = $T_{i,j}$ defined over lake for LOI in lake;

$T_{i,j}^W$ = $T_{i,j}$ defined over watershed for LOI at mouth of watershed;

t = time;

$t_{i,j}$ = area of $T_{i,j}$;

$t_{i,j}^C$ = area of $T_{i,j}^C$;

$u_{i,j,c}$ = material fraction arriving at LOI during time intervals i, \dots, j that departed during time intervals i, \dots, j from area A_c ;

$u_{i,j,c}^C$ = $u_{i,j,c}$ defined over watershed and lake for LOI in lake;

$u_{i,j,c}^L$ = $u_{i,j,c}$ defined over lake for LOI in lake;

$u_{i,j,c}^W$ = $u_{i,j,c}$ defined over watershed for LOI at mouth of watershed;

$V_{i,j}$ = set of all locations (resource shed) where materials departing during time interval i arrive at LOI during time interval j ;

$V_{i,j}^C$ = $V_{i,j}$ defined over watershed and lake for LOI in lake;

$V_{i,j}^L$ = $V_{i,j}$ defined over lake for LOI in lake;

$V_{i,j}^W$ = $V_{i,j}$ defined over watershed for LOI at mouth of watershed;

- $v_{i,j}$ = area of $V_{i,j}$;
 $v_{i,j}^C$ = area of $V_{i,j}^C$;
 $w_{i,j}(\omega)$ = areal density of material at location ω departing during time intervals i, \dots, j and arriving at LOI also during time intervals i, \dots, j ;
 $\bar{w}_{i,j,c}$ = mass of material in watershed surface cell c departing during time intervals i, \dots, j and arriving at LOI also during time intervals i, \dots, j ;
 $\bar{w}_{i,j,c}^C$ = $\bar{w}_{i,j,c}$ defined over watershed and lake for LOI in lake ;
 $\bar{w}_{i,j,c}^L$ = $\bar{w}_{i,j,c}$ defined over lake for LOI in lake ;
 $\bar{w}_{i,j,c}^W$ = $\bar{w}_{i,j,c}$ defined over watershed for LOI at mouth of watershed ;
 $x_{i,j}(\omega)$ = areal density of material at location ω departing during time interval i and arriving at LOI during time interval j ;
 $\bar{x}_{i,j,c}$ = mass of material in watershed surface cell c departing during time interval i and arriving at LOI during time interval j ;
 $\bar{x}_{i,j,c}^C$ = $\bar{x}_{i,j,c}$ defined over watershed and lake for LOI in lake ;
 $\bar{x}_{i,j,c}^L$ = $\bar{x}_{i,j,c}$ defined over lake for LOI in lake ;
 $\bar{x}_{i,j,c}^W$ = $\bar{x}_{i,j,c}$ defined over watershed for LOI at mouth of watershed ;
 $Y(\tau, t)$ = set of all locations where materials departing at time τ arrive at LOI at time t ;
 $z_{i,j}(\omega)$ = areal density of material at location ω departing during time intervals i, \dots, j and arriving at LOI during time interval j ;
 $\bar{z}_{i,j,c}$ = mass of material in watershed surface cell c departing during time intervals i, \dots, j and arriving at LOI during time interval j ;
 $\bar{z}_{i,j,c}^C$ = $\bar{z}_{i,j,c}$ defined over watershed and lake for LOI in lake ;
 $\bar{z}_{i,j,c}^L$ = $\bar{z}_{i,j,c}$ defined over lake for LOI in lake ;
 $\bar{z}_{i,j,c}^W$ = $\bar{z}_{i,j,c}$ defined over watershed for LOI at mouth of watershed ;
 α = gamma distribution parameter ;
 $\alpha_{m,j}$ = fraction of material leaving watershed in time interval m that arrives at LOI in lake in time interval j ;

β = gamma distribution parameter;

$\beta_{m,j}$ = fraction of material arriving at LOI in lake in time interval j that came from watershed mouth in time interval m ;

$\Gamma(\bullet)$ = gamma function;

$\gamma_{i,j,m}$ = fraction of all material arriving at LOI during time intervals i, \dots, j that arrived in time interval m ;

$\gamma_{i,j,m}^L$ = fraction of all material arriving at LOI in lake during time intervals i, \dots, j that arrived in time interval m ;

δ = length of time interval used in discrete-time resource shed definitions; i.e., time interval i is $[i - \delta, i)$;

τ = time;

$\phi_{i,j}$ = fraction of all material arriving at LOI during time interval j that originated over all cells during time interval i ;

\forall = denotation of range; e.g., “ $\forall \omega \in A$,” denotes “for all ω within set A ;”

\cup = operator representing union of sets;

\emptyset = the empty set;

\subset = denotation of inclusion; i.e., “ $A \subset B$ ” denotes “set A is contained in set B ;”

\in = denotation of inclusion; e.g., “ $\forall \omega \in A$,” denotes “for all ω within set A ;” and

\Rightarrow = denotation of implication; e.g., “ $A \subset B$ and $B \subset C \Rightarrow A \subset C$ ” denotes “set A is contained in B and B is contained in C implies A is contained in C .”

APPENDIX OF PROPER GRAY SCALE RENDERING OF FIGURE PAGES FOR PRINTING

The following pages correspond to pages in this report that appear in color but are re-rendered in proper gray scale for those requiring them. Some color printers may not show the detail in the color figures but may show it properly in gray scale. For best results, print the following gray scale figure pages in color to preserve the 24-bit grayscale. Direct conversion of the color figures to gray scale will distort the maps and usually convert to 8-bit grayscale.

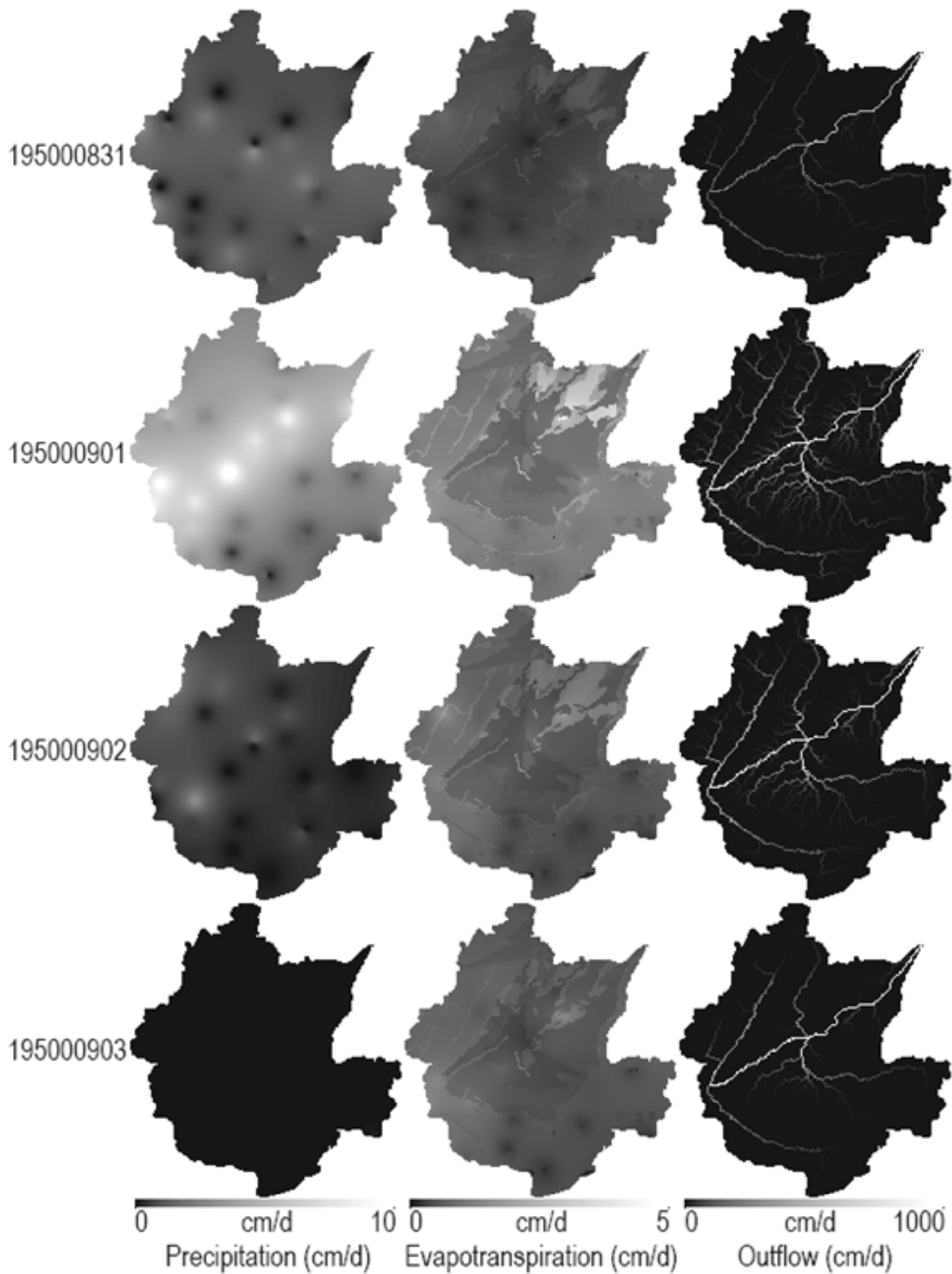


Figure 7. Maumee River Watershed Model Animation for August 31—September 3, 1950

5.2 Example Resource Sheds

Figure 8 shows example resource sheds for the Maumee River on January 1, 1950 from 1, 7, and 31 days of previous loading. In Figure 8, the brightest areas correspond to cells contributing about 0.015% of the total flow on January 1, 1950. The darkest areas are close to zero. Note several things about Figure 8. The southern and western ridgelines are prominent as is a line to the north that marks the boundary between Ohio and Michigan (AB in Figure 8). This boundary reflects the differences in the two States' definitions of some soil properties and so is an artifact of data standard differences. Point C in Figure 8 identifies the mouth of the watershed. The first map in Figure 8 shows a little response from the previous day's light rain near the mouth of the watershed. The second and third maps show most response along the edges of the watershed furthest from the mouth. Inspection of rainfall maps shows there is not much rainfall over the prior 4 days but there is a large amount 5 days prior in the southwest area. Also, spatially uniform rainfall fell over the entire watershed 7 days prior, 11-13 days prior, 15 days prior, 21-22 days prior, and 29 days prior. We can see the bright spot corresponding to the large peak 5 days earlier; the area closer to the mouth is relatively dark in the second and third maps because the only supplies there (seven or more days earlier) had already run off and are not part of the flow on this day. Note also that what happens prior to 7 days changes the picture very little (compare the last two maps). This is because the response of the watershed to supply is quick, on the order of 1 to 6 or 7 days, depending on location within the watershed. Most all supplies falling more that 6 or 7 days ago have already runoff and do not form a part of the flow on this day.

5.3 Average Resource Shed vs. Resource Shed of Average Meteorology

There are 17,541 1-km² cells in the Maumee River watershed. The DLBRM requires 0.2—0.4 seconds on today's desktop computers to simulate 1 day's hydrology from all of these cells, for

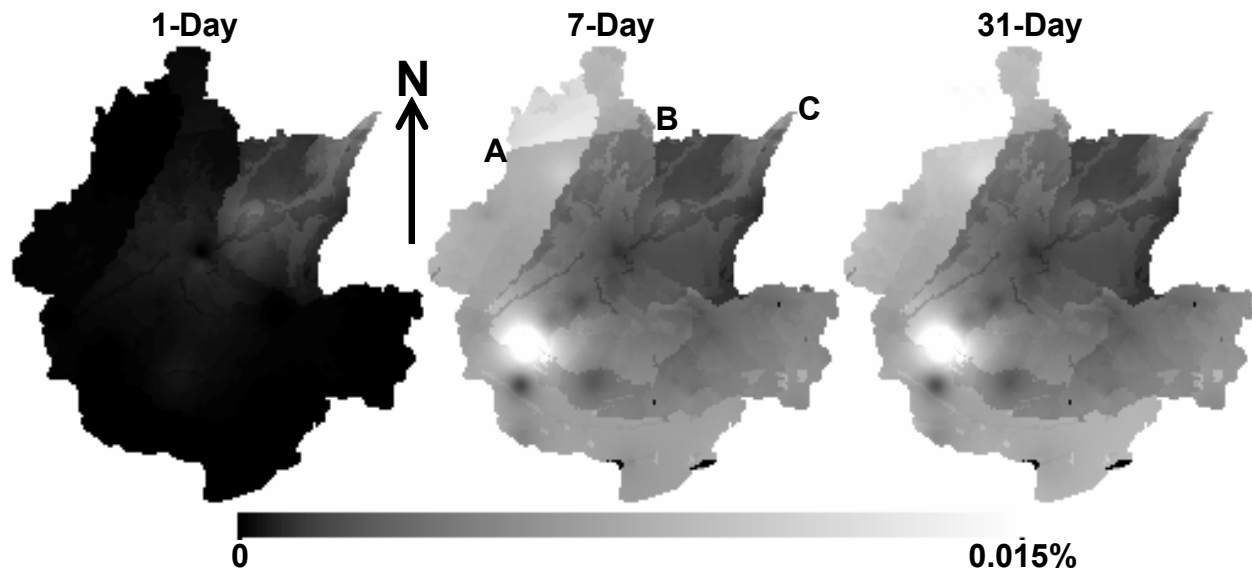


Figure 8. Maumee Resource Sheds on January 1, 1950 from prior days of loading:

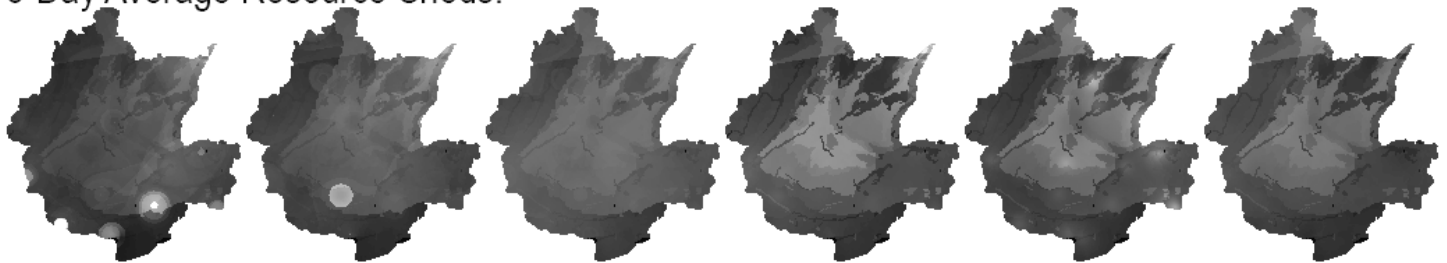
- 1-day: $Q_{January\ 1,\ 1950,\ January\ 1,\ 1950,\ c},\ c = 1, \dots, 17,541;$
- 7-day: $Q_{December\ 26,\ 1949,\ January\ 1,\ 1950,\ c},\ c = 1, \dots, 17,541;$
- 31-day: $Q_{December\ 2,\ 1949,\ January\ 1,\ 1950,\ c},\ c = 1, \dots, 17,541.$

any loading pattern. With 17,541 loading patterns (one tracing each cell's contribution), a one-day simulation of loadings requires 1—2 hours of computation. For 1-, 2-, ..., 10-day simulations for one date there are 55 simulation days, which require 2—5 days of computation.

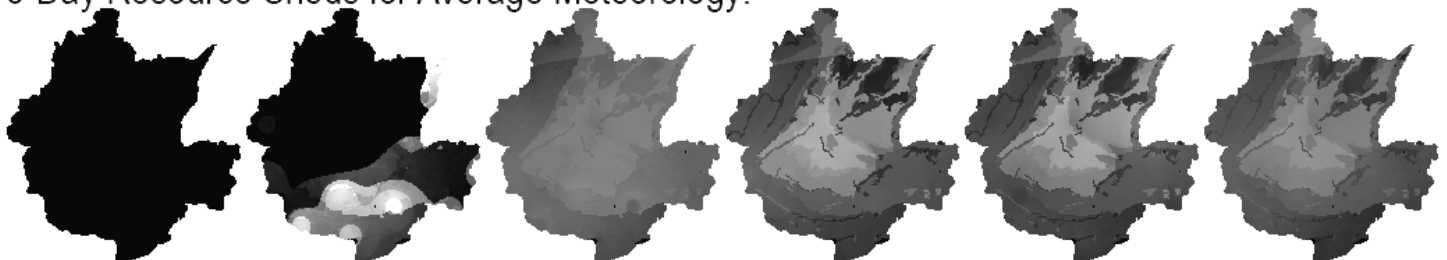
A parallel study requires average resource sheds for May 31 and August 31 for each contribution period from 1 through 31 days for linking to resource sheds in a lake that extend back into its watersheds. To calculate Maumee resource sheds over the historical record of 1950—1999 for averaging for these two dates and 31 contribution periods would require 49,600 simulation days or 5.5—11 years of computation, which is clearly impractical. Rather than calculate resource sheds for historical meteorology and then averaging them, we averaged the meteorology over all years for each day of the year and then calculated resource sheds based on this (1 year of) average meteorology. This required only 2% of the above: 992 simulation days or 40—80 days of computation. This is practical but begs the question: how well does the resource shed of average meteorology approximate the average resource shed (or how linear is the watershed model)?

To answer this question, we calculated resources sheds for selected dates for each year of record (1950—1999) for various contribution periods using about 20 machines to reduce the computation period. We averaged over the historical record and compared the resource sheds; Figure 9 shows selected average resource sheds and the resource sheds of average meteorology throughout the annual cycle. Figure 9 reveals two non-linearities in the model with respect to air temperature. The first occurs over winter, when air temperatures are close to or below freezing; the average resource shed (top Figure 9) reflects that snowmelt occurs in some years. However, average air temperature over the winter is zero and resource sheds calculated for an averaged winter date will have no snow melt (bottom Figure 9). This is because the model calculates snow

6-Day Average Resource Sheds:



6-Day Resource Sheds for Average Meteorology:



Jan 15

Mar 01

Mar 15

Jun 01

Sep 01

Oct 15

0 0.028%

Figure 9. Maumee River Six-Day Average Resource Sheds for Selected Times of the Year.

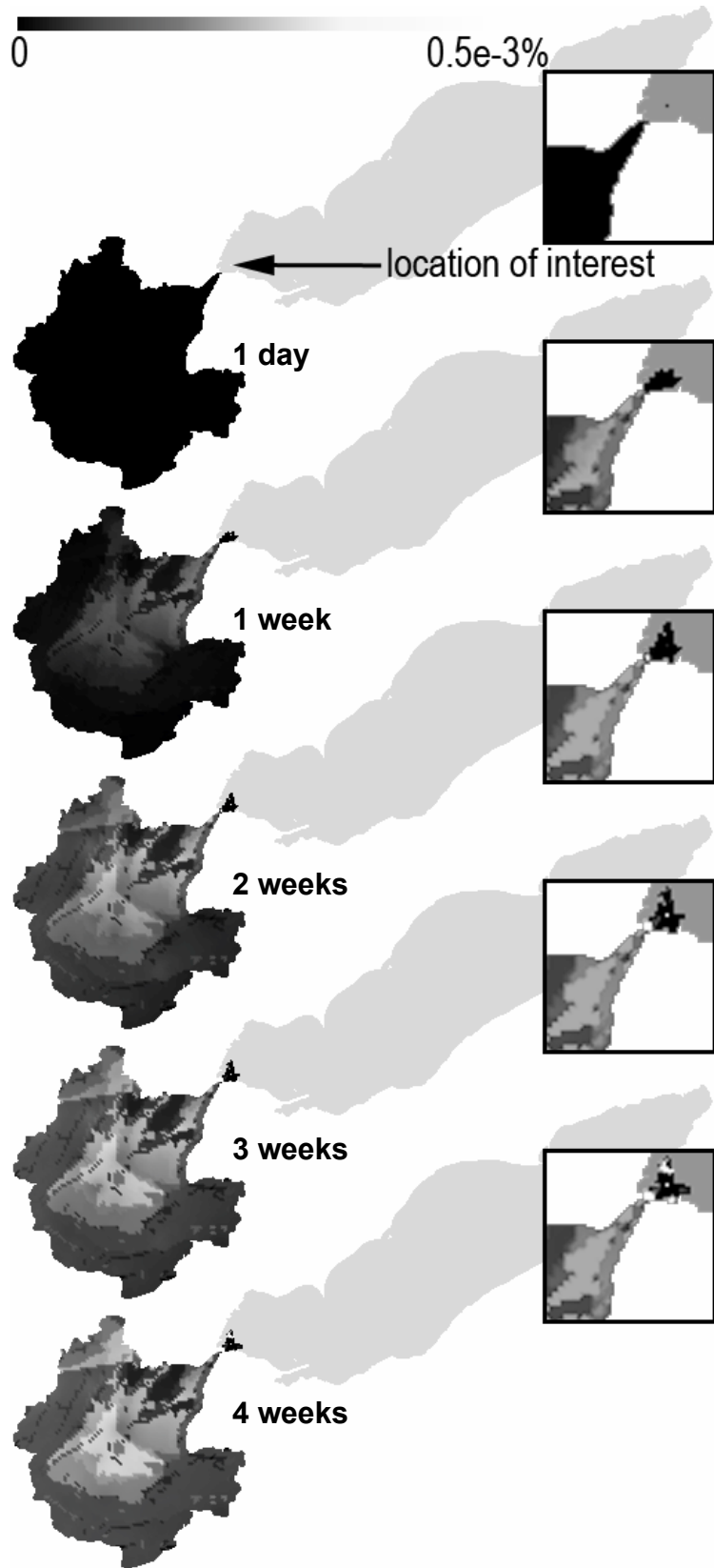


Figure 13. Maumee—Erie Linked Resource Shed for Average August 31 Conditions for Site 835