Laboratory of Economics and Management
Sant'Anna School of Advanced Studies

Via Carducci, 40 - I-56127 PISA (Italy)
Tel. +39-050-883-341 Fax +39-050-883-344
Email: lem@sssup.it Web Page: http://lem.sssup.it

LEM
Working Paper Series

Heterogenous Agents, Complementaries, and Diffusion. Do Increasing Returns Imply Convergence to International Technological Monopolies?

Andrea Bassanini[*]
and
Giovanni Dosi [†]

[*] *OECD, Paris, France*
[†] *St.Anna School of Advanced Studies, Pisa, Italy*

1999/04 **December 1999**

ISSN (online) 2284-0400

Heterogeneous Agents, Complementarities, and Diffusion

Do Increasing Returns Imply Convergence to International Technological Monopolies?[*]

Andrea P. Bassanini[†]

Giovanni Dosi[‡]

1 Introduction

This work concerns some generic properties of the international diffusion of technologies and products in markets which are interdependent but display, to varying degrees, location specific forms of dynamic increasing returns and externalities.

[*]The views expressed here do not reflect those of the OECD Secretariat or its Member Countries. We are indebted to Yuri Kaniovski for very helpful suggestions. We thank also Brian Arthur, Robin Cowan, Klaus Desmet, Judith Gebauer, Michael Horvath, Andrea Prat, Aldo Rustichini, Valter Sorana, and participants to the 3rd Workshop on Economics with Heterogeneous Interacting Agents, Ancona, Italy, May 1998, and to the Conference on Economic Models of Evolutionary Dynamics and Interacting Agents, Trieste, Italy, September 1998, for their comments. Financial support from International Institute for Applied Systems Analysis (IIASA), Banca Nazionale del Lavoro (BNL), Italian Research Council (CNR), and Italian Ministry of University and Scientific Research (MURST) is gratefully acknowledged. All errors are ours.

[†]OECD, Paris, France. E-mail: andrea.bassanini@oecd.org.

[‡]Sant'Anna School for Advanced Studies, Pisa, Italy. E-mail: g.dosi@sssup.it.

Building on a related paper [Bassanini and Dosi (1998)], in the following we study the ways the spatial dimension of positive feedback in adoption affects the long-run properties of the diffusion processes, especially with regards to the possible emergence of either region-specific or global monopolies (dominant designs) of particular technologies[1]. Two stylized facts are at the origin of this work[2]:

a) A stable pattern of market sharing between competing technologies with no overwhelming dominant position has been rarely observed in markets with increasing returns (or positive feedbacks) of some kind [see Tushman and Murmann (1998)]. For example, even in the case of operating systems, which is often quoted as a case of market sharing, Apple MacIntosh has never held a market share larger than 20% (a partial exception being the submarket of personal computer for educational institutions). This fact has also triggered suspicion of market inefficiencies: Single dominant designs or technological paradigms may prevail even when the survival of more than one technology may be socially optimal [Dosi (1982), Katz and Shapiro (1986), David (1992)]. Think for example to the competition between Java-based architectures and ActiveX architectures for web-based applets: Given that with any of the two paradigms the standard tasks that can be performed are different, the general impression of experts is that society would benefit from the survival of both.

b) International diffusion may sometimes lead to different standards in different countries or conversely to the diffusion of the same standard in every country. For example, while in all the English-speaking world the QWERTY keyboard represents the standard, in the French-speaking world a slightly different version (the AZERTY keyboard) is by far the more adopted one. On the contrary in the VCR market VHS is the worldwide technological leader while the original competitor, Beta, has disappeared.

To explain these two stylized facts we analyze properties of a fairly general and nowadays rather standard class of models of competing technologies, originally suggested by Arthur (1983) and Arthur et al. (1983) and subsequently made popular by Arthur (1989) [and further explored by Cowan

[1]Here the terms monopolies and market sharing are referred to competing technologies without any reference to who is producing what. For instance, a technological monopoly occurs when all users adopt the same technology, even when it is produced by many different suppliers.

[2]For more thorough discussions of the evidence on patterns of innovation and diffusion, see, among others, Dosi (1988, 1992) and Freeman (1992, 1994).

(1991), Dosi et al. (1994), Dosi and Kaniovski (1994) and Kaniovski and Young (1995) among others]. This class of models will be presented in details in section 2.

Despite mixed results of some pioneering work on the dynamics of markets with network effects [e.g. Katz and Shapiro (1986)], unbounded increasing returns are commonly called for as an explanation of the emergence of technological monopolies. In a related paper [Bassanini and Dosi (1998)] we show that the emergence of technological monopolies depends on the nature of increasing returns with respect to the degree of heterogeneity of the population: Given a sufficiently high heterogeneity amongst economic agents, limit market sharing may occur even in the presence of unbounded increasing returns. Furthermore we suggest that the observation of the widespread emergence of monopolies is intimately related to properties of different rates of convergence (to monopoly and to market sharing respectively) more than to properties of limit states as such. It is shown that a market can approach a monopoly with a higher speed than it approaches any feasible limit market shares where both technologies coexist. When convergence is too slow the external environment is likely to change before any sufficiently small neighborhood of the limit can be attained. The empirical implication is that among markets with high turnover of basic technologies and increasing returns to adoption, a prevalence of stable monopolies over stable market-sharing should be observed.

Clearly this result also challenges the ideas that internationally unbounded increasing returns are necessary and sufficient to yield global dominance of particular technologies, and by the same token, the emergence of overwhelmingly dominant locational clusters in production.

In order to analyze the international diffusion dynamics and account for the foregoing stylized facts it is useful to start with distinguishing three interrelated processes.

At a first level, diffusion is driven by microdecisions concerning the adoption or not of a new technology. This is what Katz and Shapiro (1994) in their review of the literature on systems competition and dynamics of adoption under increasing returns call *technology adoption decision.* It basically to the decision of a potential user to place a demand in a particular market (or, relatedly, of a producer to start investing in a new technology). Indeed a good deal of literature on determinants of diffusion patterns, and, relatedly, the observed "retardation factors", pertain to this domain of analysis [see Dosi (1992)]. Moreover, relevant questions in this case are the conditions for an actual market of positive size and the conditions allowing penetration of a new (more advanced) technology into the market of an already established one [Rohlfs (1974), Oren and Smith (1981), Farrell and Saloner (1985,1986), Silverberg et al. (1988), Katz and Shapiro (1992)]. For example purchasing or not a fax or substituting a compact disc player for an analogical record player are technology adoption decisions.

Second, following again the terminology of Katz and Shapiro (1994), call *product selection* the choice between different technological solutions which perform (approximately) the same function and are therefore close substitutes. Relevant questions here are whether the market enhances variety or standardization, whether the emerging market structure is normatively desirable and what is the role of history in the selection of market structure [Arthur (1983,1989), Katz and Shapiro (1985,1986), David (1985), Church and Gandal (1993), Dosi et al. (1994), Brock and Durlauf (1995,1998), An and Kiefer (1995), Bryniolfsson and Kemerer (1996), Durlauf (1997)]. Choosing between VHS or Beta in the VCR market or between Word or Wordperfect in the wordprocessors market are typical examples of product selection decisions. However it is easy to extend this domain to include the choice between alternative technological systems (e.g. in energy generation, solar vs. nuclear energy and within the latter, PWR vs. gas-cooled reactors, etc...).

A third level concerns the spatial dimensions (either literally geographical or institution-related) which influence the above decision processes, or, more than that, straightforwardly represent a distinct domain of decision for micro agents. So, for example, an expanding literature on "national systems of innovation" [Lundvall (1992), Nelson (1993) and Freeman (1995), among others] emphasizes the long-term impact of the diverse architectures of national institutional systems as drivers of technological learning. Partly overlapping analyses of incentives and constraints to the location of investments by capital-mobile firm across heterogeneous environments focus upon the interaction between firm-specific capabilities and location-specific advantages in MNCs investments [Cantwell (1989)]. Call the latter *location selection decisions*, which in the model presented below will refer to the choice between different locations where to make an investment in a specific technology, or, more metaphorically, the choice of spatially-bound agents across different technologies, production processes and outputs. This has been for a long time a field rich of qualitative investigation by economic geography and regional economics, recently discovered, at the cost of a lot of institutional simplification by economic theory [e.g. Arthur (1990), Krugman (1991a, 1991b), Rauch (1993), Krugman and Venables (1996), Venables (1996)].

In the following we bring together the three domains of analysis and explore some formal properties of the dynamics of competition among alternative products and technologies diffusing across heterogeneous environments. We consider conditions of convergence to different market structures in a world where there are many regional markets and many different technologies. Agents choose both technology and location. Regional markets are institutional entities which cannot disappear (although their relative size can grow or shrink). Rather a diffusion pattern is realized for every region. This leads to a natural question which underlies the second stylized fact recalled above: What drives convergence to the same or different market structures in different interrelated regional markets?

Intuitively convergence to the same standard is an outcome of the relative weight and strength of international spillovers as compared to nationwide (or regional) increasing returns. For example, in the case of typewriter keyboards, geographical areas with the same language tend to be reflected in spillover clusters due to free "migration" of typists, similar training institutions, etc.... On the other hand, historically, gaining leadership in the European market, with the consequent bias in the related home video market, was crucial to VHS to resolve in its favor the battle for leadership in the Japanese market as well [Cusumano et al. (1992)]. However very little modeling effort has been made so far to formally explore this intuitive explanation. Below we will indeed consider a generalization of one-market models of competing technologies in order to establish conditions of convergence to the same or different technological monopolies in different but interrelated markets.

If one does not disaggregate a system made of many regional markets, the emergence of stable but different dominant designs would look like technological market sharing. How does one reconcile "stylized facts" and theoretical statements on the emergence of different technological monopolies in different interdependent regional markets with "stylized facts" and theoretical statements on overwhelming emergence of dominant designs?

Of course it is trivially true that, with mutually independent markets, different trajectories could emerge in different markets as if they were different realizations of the same experiment. In this paper we show that results on speed of convergence stated in Bassanini and Dosi (1998) can be extended also to the case when markets are interdependent: Even though at high level of aggregation a system of different local monopolies looks like a stable market sharing, we show that it has the same rate-of-convergence properties of a "univariate" system converging to a monopoly.

The remainder of the paper is divided as follows. Section 2 provides one motivating example, formally defines the standard class of models of competing technologies we refer to and summarizes results on rate of convergence to a stable market structure in univariate models. Section 3 establishes our main results on convergence to a stable market structure in multi-markets models. Section 4 briefly summarizes the results.

2 Unbounded Returns and Dominant Designs: A Sample Selection Bias

The class of competing technology dynamics models that we consider takes as the only basic assumption the fact that adopters enter the market in a sequence which is assumed to be exogenous. More than one agent can enter the market in each period [see e.g. Katz and Shapiro (1986) and Dosi and Kaniovski (1994)], but in order to simplify the treatment we abstract from this complication. The simple theoretical tale that underlies these models can be summarized as follows:

Every period a new agent enters the market and chooses the technology which is best suited to its requirements, given its preferences, information structure and the available technologies. Preferences can be heterogeneous and a distribution of preferences in the population is given. Information and preferences determine a vector of payoff functions (whose dimension is equal to the number of available technologies) for every type of agent. Because of positive (negative) feedbacks, these functions depend on the number of previous adoptions. When an agent enters the market it compares the values of these functions (given its preferences, the available information, and previous adoptions) and chooses the technology which yields the maximum perceived payoff. Which "type" of agent enters the market at any given time is a stochastic event whose probability depends on the distribution of types (i.e. of preferences) in the population. Because of positive (negative) feedbacks, the probability of adoption of a particular technology is an increasing (decreasing) function of the number of previous adoptions of that technology.

More formally we can write a general reduced form of payoff functions of the following type:

$$\Pi^i_j(\vec{n}(t)) = h_i(a^i_j, \vec{n}(t)),$$

where $j \in D$, D is the set of possible technologies, $i \in S$, S is the set of possible types, $\vec{n}(t)$ is vector denoting number of adoptions for each technology at time t ($n^j(t)$ is the number of adoptions of technology j at time t), $\vec{a}_i$ represents the network-independent components of agent i's preferences (a_i^j identifies a baseline payoff for agents of type i from technology j), and $h_i(.)$ is an increasing function (that can differ across agents) capturing increasing returns to adoption. If, at time t, an agent of type i comes to the market, it compares the payoff functions choosing A if and only if[3]:

$$\Pi_A^i(n^A) = \arg\max_{j \in D} \left\{ \Pi_j^i(\vec{n}) \right\}. \tag{1}$$

Strategic behaviors (including sponsoring activities from the suppliers of technologies) and expectations can be considered as already implicitly included in the foregoing formalization.

In the remainder of this paper we assume that the order of agents entering the market is random, hence $i(t)$ can be considered as an iid sequence of random variables whose distribution depends on the distribution of the population of potential adopters. With this assumption, the dynamics of the foregoing model can be seen in terms of generalized urn schemes:

Consider the simplest case where two technologies, say A and B, compete for a market. Let us denote A's market share with $X(t)$. Given the relationships between (a) total number of adoptions of both technologies $n(t) = t - 1 + n^A(0) + n^B(0)$, (b) the current market share $X(t)$ of A, and (c) number of adoptions of one specific technology, $n^i(t)$, $i = A, B$, that is, $n^A(t) = n(t)X(t)$, the dynamics of $X(t)$ is given by the recursive identity

$$X(t+1) = X(t) + \frac{\xi^t(X(t)) - X(t)}{t + n^A(0) + n^B(0)}.$$

Here $\xi^t(x), t \geq 1$ are random variables independent in t such that

$$\xi^t(x) = \begin{cases} 1 & \text{with probability} \quad f(t,x) \\ 0 & \text{with probability} \quad 1 - f(t,x) \end{cases},$$

and $\xi^t(\cdot)$ is a function of market shares dependent on the feedbacks in adoption. $f(t,x)$ equals the probability that (1) is true when $X(t) = x$ and is

[3]We assume that, if there is a tie, agents choose technology A. Qualitatively, breaking the tie in a different way would not make any difference.

sometimes called *urn function*. Designating $\xi^t(x) - E(\xi^t(x)) = \xi^t(x) - f(t,x)$ by $\zeta^t(x)$ we have

$$X(t+1) = X(t) + \frac{[f(t, X(t)) - X(t)] + \zeta^t(X(t))}{t + n^A(0) + n^B(0)}. \tag{2}$$

Provided that there exist a limit urn function $f(\cdot)$ (defined as that function $f(.)$ such that $f(t,.)$ tends to it as t tends to ∞) and the following condition is satisfied

$$\sum_{t\geq 1} t^{-1} \sup_{x\in[0,1]\cap R(0,1)} \mid f(t,x) - f(x) \mid < \infty, \tag{3}$$

asymptotic patterns of this process can be studied by analyzing the properties of the function

$$g(x) = \lim_{t\to\infty} f(t,x) - x.$$

Particularly, treating $g(x)$ in the same way of the right hand side of an ordinary differential equation, it is possible to show that the process (2) converges with positive probability to every stable zero[4]. The foregoing formal representation is employed for every result of the present paper.

In some cases, eq. (1) can be expressed directly in terms of shares rather than total numbers[5]; in this case $f(.,.)$ is independent of t and (3) is trivially verified. In this respect note that, from an interpretative point of view, *total numbers* and *shares* are likely to capture quite distinct economic and technological phenomena. For example, network externalities are often well-captured by the shares dynamics, while more idiosyncratic, cumulative and type-specific processes of learning are more naturally represented as functions of varying numbers of adopters.

As we noticed in the introduction, there seems to be a general consensus that the widespread emergence of dominant designs should be explained on the basis of the presence of unbounded increasing returns to adoption[6]. Unbounded increasing returns to adoption are neither necessary nor sufficient to lead to the emergence of technological monopolies. Let us start with an example drawn from Bassanini and Dosi (1998).

[4]A convenient review of analytical results on generalized urn schemes can be found in Dosi et al. (1994). The reader is referred to that for the results that are not proved in this paper.

[5]This is particularly relevant in the frequent case when product selection sequentially follows technology adoption [see Bassanini and Dosi (1998)].

[6]See Bassanini and Dosi (1998) for references on this debate.

Example 1 *Consider payoff functions of this type:*

$$\Pi_j(n^j) = a_j + r_j n^j,$$

where r_j, a_j, $j = A, B$, *are bounded random variables which admit density. Such a function allows agents to be heterogeneous also in terms of the degree of increasing returns which they experience. By applying (1), dividing payoff functions by total number of adoptions, and rearranging we have that* A *is chosen if and only if:*

$$X(t) \geq \frac{r_B}{r_A + r_B} + \frac{a_B - a_A}{(t + n^A(0) + n^B(0))(r_A + r_B)}. \tag{4}$$

Denoting the random variables on the right hand side with $\varsigma(t)$, *from (4) we have that the adoption process can be seen as a generalized urn scheme with urn function* $f(t, x) = F_{\varsigma(t)}(x)$, *where* $F_{\varsigma(t)}(\cdot)$ *is the distribution function of* $\varsigma(t)$. *Now suppose that* r_A *and* r_B *are highly correlated and both have bimodal distributions very concentrated around the two modes, in such a way that the distribution of* r_A/r_B *is also bimodal and very concentrated around the two modes too. Furthermore suppose that the two modes are far away from each other. To fix the ideas say that for a percentage* α *of the population* r_A/r_B *is uniformly distributed on the interval* $\left[\frac{1}{1-b}, \frac{1}{1-a}\right]$, *while for a percentage* $1 - \alpha$ *of the population* r_A/r_B *is uniformly distributed on the interval* $\left[\frac{1}{1-d}, \frac{1}{1-c}\right]$, *with obviously* $0 < a < b < c < d$. *First, let us consider the case of* $a_j = 0$, $j = A, B$. F_ς *is by construction independent of* t, *implying the following urn function:*

$$f(x) = F_{\varsigma(t)}(x) = \begin{cases} 0 & \text{if } \ y \leq a \\ \alpha\frac{1}{b-a}(y - a) & \text{if } \ a < y \leq b \\ \alpha & \text{if } \ b < y \leq c \\ \alpha + (1 - \alpha)\left[\frac{1}{d-c}(y - c)\right] & \text{if } \ c < y \leq d \\ 1 & \text{if } \ y > d \end{cases}$$

If $b < \alpha < c$,*then there are three stable fixed point of* $f(x)$ *and, as said above, it can be shown that there is a set of initial conditions (that imply giving both technologies a chance to be chosen "at the beginning of history") for which market sharing is asymptotically attainable with positive probability. If* $a_j \neq 0$ *but have bounded support and admit density, then condition (3) applies and the same argument holds: In fact, relying on the fact that* r_j, a_j *are bounded it is easy to show that* $\sup_{x \in [0,1] \cap R(0,1)} \mid f(t, x) - f(x) \mid < K/t$, *where $K > 0$ is a constant. The essential ingredient of this example is that the distribution of r_A/r_B is bimodal and very concentrated around the two modes. The argument has nothing to do with the particular (and extreme) distributional form assumed here: Following the same constructive procedure adopted here it is easy to build examples with any other distributional form. The only requirement is that the two modes are sufficiently distant. In other words the only requirement is a sufficient degree of heterogeneity in the population to counterbalance the pro-standardization effects of increasing returns to adoption.*

The argument so far suggests that, the distribution of the fine characteristics and preferences of the population of agents might determine the very nature of the attainable asymptotic states themselves. Short of empirically convincing restrictions on the distribution of agents' (usually unobservable) characteristics, Bassanini and Dosi (1998) propose instead an interpretation of the general occurrence of technological monopolies (*cum* increasing returns of some kind) grounded on the relative speed of convergence to the underlying (but unobservable) limit states. When convergence is too slow the external environment is likely to change before any sufficiently small neighborhood of the limit can be attained. Theorems 1 and 2 recalled below show that convergence to technological monopolies tends to be (in probabilistic terms) much faster than to a limit where both technologies coexist. The empirical implication is that in markets with high turnover of basic technologies, when market structure dynamics shows a relatively stable pattern, a prevalence of technological monopolies over stable market sharing is likely to be observed. Therefore the emergence of dominant designs, at a careful look, appears to come out of a sample selection bias: Including cases where market share turbulence never seem to settle down would provide a more mixed picture.

Theorem 1 (Bassanini and Dosi (1998, theorem 2)) *Let $\epsilon > 0$ and $c < 1$ be such that*

$$f(t,x) \leq cx \ \ for \ \ x \in (0,\epsilon) \quad (f(t,x) \geq 1 - c(1-x) \ \ for \ \ x \in (1-\epsilon, 1)).$$

Then for any $\delta \in (0, 1-c)$ and $\tau > 0$

$$\lim_{t\to\infty} \mathcal{P}\{t^{1-c-\delta} X(t) < \tau | X(t) \to 0\} = 1$$
$$\left(\lim_{t\to\infty} \mathcal{P}\{t^{1-c-\delta}[1 - X(t)] < \tau | X(t) \to 1\} = 1\right),$$

where $X(\cdot)$ stands for the random process given by (2).

Theorem 2 (Bassanini and Dosi (1998), theorem 5) *Let* $\theta \in (0,1)$ *be a stable root and* $f(\cdot)$ *be differentiable at* θ *with* $\frac{d}{dx}f(\theta) < 1/2$. *Then for every* $\delta, \tau > 0$

$$\lim_{t\to\infty} \mathcal{P}\{t^{1/2+\delta}|X(t)-\theta| < \tau | X(t) \to \theta\} = 0.$$

Theorems 1 and 2 show that convergence to 0 and 1 can be much faster (almost of order $1/t$ as $t \to \infty$) than to an interior limit (which can be almost of order $1/\sqrt{t}$ only)[7]. Furthermore, if we use L^2 convergence, instead of weak convergence, we can dispose of the assumption of differentiability of the urn function and obtain similar results[8].

Let us now extend the analysis to the international area where, as mentioned earlier, we do observe emergence of different standards and dominant designs in different countries. Moreover, note that at high level of aggregation a system of different local monopolies looks like a stable market sharing. However in the next section we shall show that the foregoing results can be extended also to the case of many interdependent markets. We prove that a system of locally dominant designs has the same rate-of-convergence properties of a "univariate" system converging to a single dominant designs. Furthermore we provide a characterization of different convergence patterns. Not contrary to the intuition, it is the balance between local and global feedbacks which determines whether the system can converge to the same or different monopolies in every market.

3 International Diffusion and the Emergence of Technological Monopolies

Suppose that two technologies, say A and B, compete for a complex market which consists of m interacting parts (which can be thought of as economic regions or even countries) of infinite capacity. At any time $t = 1, 2, \ldots$ a new agent enters one of the markets and has to adopt one unit of one technology. The type of agent and the region are randomly determined on the basis of the distribution of agents in the population. There can be positive (negative)

[7]Note that, provided that the number of asymptotic steady states is finite, theorem 2 and 5 do not provide only a statement on conditional convergence but also on absolute convergence.

[8]see Bassanini and Dosi (1998), lemma 1.

feedbacks not only inside every single region but also across them. Thus the probability of adoption is a function of the vector of previous adoptions in all regions. For the i-th region we consider the following indicators: $x_i(1-x_i)$ – the current market share of $A(B)$; γ_i – the frequency of additions to the region (i.e. this is the ratio of the current number of units of the technologies in the region to the total current number of units of the technologies in the market). We assume also that initially, at $t=0$, there are $n_i^A(n_i^B)$ units of the technology $A(B)$ in the i-th region and $n_i^A, n_i^B \geq 1$. Finally n stands for the initial number of units of the technologies on the market, i.e. $n = n_1^A + n_1^B + n_2^A + n_2^B + \ldots + n_m^A + n_m^B$.

Similarly to the setup above we can say, with no loss of generality, that at time instants $t = 1, 2 \ldots$. a unit of technology (either A or B) will be adopted with probability $f_i(t, \vec{X}(t), \vec{\gamma}(t))$ in the i-th region. Also it will be A with probability $f_i^A(t, \vec{X}(t), \vec{\gamma}(t))$ and B with probability $f_i^B(t, \vec{X}(t), \vec{\gamma}(t))$. Here $\vec{f}(.,.,.)$ is a vector function which maps $\mathbf{N} \times R(\vec{0}, \vec{1}) \times R(S_m)$ in S_m and $\vec{f}(.,.,.) = \vec{f}^A(.,.,.) + \vec{f}^B(.,.,.)$. By $R(\vec{0}, \vec{1})$ we designate the Cartesian product of m copies of $R(0,1)$, S_m is defined by the following relation

$$S_m = \{\vec{x} \in R^m : \ x_i \geq 0, \ \sum_{i=1}^{m} x_i = 1\}. \tag{5}$$

Here $R(0,1)$ and $R(S_m)$ stand for the sets of rational numbers from $[0,1]$ and, correspondingly, vectors with rational coordinates from S_m. By $\vec{X}(t)$ we designate the m-dimensional vector whose i-th coordinate $X_i(t)$ represents the share of technology A in the i-th region at time t. Also $\vec{\gamma}(t)$ stands for the m-dimensional vector whose i-th coordinate $\gamma_i(t)$ is the frequency of total adoptions to the i-th urn by the time t. The dynamics of $\vec{X}(\cdot)$ and $\vec{\gamma}(\cdot)$ can be described as a generalized scheme of multiple urns with balls of two colors, in analogy with to the one-dimensional case[9].

Consider the process of adoptions of technology A at time t in market i, which is obviously a stochastic variable. We can define the following "conditional adoption" stochastic variable:

$$\xi_i^t(\vec{x}, \vec{\gamma}) = E[\xi_{i,A}^t(\vec{x}, \vec{\gamma}) | \xi_{i,A}^t(\vec{x}, \vec{\gamma}) + \xi_{i,B}^t(\vec{x}, \vec{\gamma}) = 1]. \tag{6}$$

[9]See also Dosi and Kaniovski (1994).

$\xi^t_{i,j}(\vec{x},\vec{\gamma})$, $j = A, B$, stands for adoption of technology j at time t in market i. Then

$$\xi^t_i(\vec{x},\vec{\gamma}) = \begin{cases} 1 & \text{with probability} \quad g_i(t,\vec{x},\vec{\gamma}) \\ 0 & \text{with probability} \quad 1 - g_i(t,\vec{x},\vec{\gamma}) \end{cases} \tag{7}$$

where $g_i(t,\vec{x},\vec{\gamma}) = f^A_i(\vec{x},\vec{\gamma})[f_i(\vec{x},\vec{\gamma})]^{-1}$[10].

Hereafter we assume that the functions $g_i(t,.,.) = g_i(.,.)$, that is the conditional probability of adoption, is independent of t. Let us ignore the time instants t when $X_i(\cdot)$ does not undergo any changes. Then we obtain a new process $Y_i(\cdot)$ which has the same convergence properties as $X_i(\cdot)$ providing that technologies are adopted in the i-th region infinitely many times with probability one. We will implicitly assume the latter condition throughout this section[11]. For particular cases $Y_i(\cdot)$ turns out to be a conventional urn process, or anyhow can be studied by means of some associated univariate generalized urn process.

To implement this idea, introduce $\tau_i(k)$ – the moment of the k-th addition of a technology to the i-th region, i.e.

$$\begin{aligned} \tau_i(1) &= \min\left\{t : \xi^t_{i,1}(\vec{X}(t),\vec{\gamma}(t)) + \xi^t_{i,2}(\vec{X}(t),\vec{\gamma}(t)) = 1\right\}, \\ \tau_i(k) &= \min\left\{\begin{array}{c} t : t > \tau_i(k-1), \\ \xi^t_{i,1}(\vec{X}(t),\vec{\gamma}(t)) + \xi^t_{i,2}(\vec{X}(t),\vec{\gamma}(t)) = 1 \end{array}\right\}, k \geq 2. \end{aligned} \tag{8}$$

Designate $X_i(\tau_i(k))$ by $Y_i(k)$ and $\xi^{\tau_i(k)}_i(\vec{X}(\tau_i(k)),\vec{\gamma}(\tau_i(k)))$ by $\zeta^k_i(Y_i(k))$. Then for $Y_i(\cdot)$ we have the following recursion

$$Y_i(k+1) = Y_i(k) + \frac{1}{k+G_i}[\zeta^k_i(Y_i(k)) - Y_i(k)], \quad k \geq 1, \quad Y_i(1) = \frac{n^w_i}{G_i}. \tag{9}$$

Note the $Y_i(\cdot)$ indeed carries all information about changes of $X_i(\cdot)$. By definition $Y_i(k) = X_i(\tau_i(k))$, but between $\tau_i(k)$ and $\tau_i(k+1)$ the process $X_i(\cdot)$ preserves its value. Consequently, should we know that $Y_i(k)$ converges with probability one (or converges with positive probability to a certain value, or converges to a certain value with zero probability, i.e. does not converge) as

[10] Throughout this section we assume that $f_i(t,\vec{x},\vec{\gamma}) > 0$ for all possible $\vec{x}$ and $\vec{\gamma}$.

[11] This assumption, which in the spirit of the population models does not appear to be too strong, is needed in order to obtain asymptotic results.

$k \to \infty$, the same would be true for $X_i(t)$ as $t \to \infty$. (We will systematically use this observation below without further explicit mention).

The next theorem provides sufficient conditions for convergence of $Y_i(\cdot)$ (and consequently $X_i(\cdot)$) to 0 and 1 with positive (zero) probability.

Theorem 3 *Let $g_i(.) : R(0,1) \to [0,1]$ be a function such that for all possible $\vec{x} \in R(\vec{0},\vec{1})$ and $\vec{\gamma} \in R(S_m)$*

$$g_i(\vec{x},\vec{\gamma}) \leq g_i(x_i). \tag{10}$$

Designate by $Z_i(\cdot)$ a conventional urn process with $g_i(\cdot)$ as the urn-function and n_i^A, n_i^B as the initial numbers of balls. Then $\mathcal{P}\{Y_i(k) \to 0\} > 0$ ($\mathcal{P}\{Y_i(k) \to 1\} = 0$) if $\mathcal{P}\{Z_i(k) \to 0\} > 0$ ($\mathcal{P}\{Z_i(k) \to 1\} = 0$). Also, when

$$g_i(\vec{x},\vec{\gamma}) \geq g_i(x_i), \tag{11}$$

the statement reads: if $\mathcal{P}\{Z_i(k) \to 1\} > 0$ ($\mathcal{P}\{Z_i(k) \to 0\} = 0$), then $\mathcal{P}\{Y_i(k) \to 1\} > 0$ ($\mathcal{P}\{Y_i(k) \to 0\} = 0$).

The theorem is proved in the appendix.

The next statement gives slightly more sophisticated conditions of convergence and nonconvergence to 0 and 1 of the process (9) and, consequently, $X_i(\cdot)$.

Theorem 4 *Set $U_i(\vec{x}) = \sup_{\vec{\gamma}} \{g_i(\vec{x},\vec{\gamma})\}$, $\quad L_i(\vec{x}) = \inf_{\vec{\gamma}} \{g_i(\vec{x},\vec{\gamma})\}$. If there is $\epsilon > 0$ such that $L_i(\vec{x}) \geq x_i$ ($U_i(\vec{x}) \leq x_i$) for $x_i \in (0,\epsilon)$ ($x_i \in (1-\epsilon,1)$), then $\mathcal{P}\{X_i(t) \to 0\} = 0$ ($\mathcal{P}\{X_i(t) \to 1\} = 0$) for any initial combination of balls in the urn. Let $g_i(\vec{x},\vec{\gamma}) < 1$ ($g_i(\vec{x},\vec{\gamma}) > 0$) for all $\vec{x} \in R(\vec{0},\vec{1})$ and $\vec{\gamma} \in R(S_m)$. Also let $\epsilon > 0$, $c < 1$ and a function $f_i(\cdot)$ be such that $U_i(\vec{x}) \leq f_i(x_i) \leq cx_i$ ($L_i(\vec{x}) \geq f_i(x_i) \geq 1 - c(1-x_i)$) for $x_i \in (0,\epsilon)$ ($x_i \in (1-\epsilon,1)$). Then $\mathcal{P}\{X_i(t) \to 0\} > 0$ ($\mathcal{P}\{X_i(t) \to 1\} > 0$) for any initial numbers of balls in the i-th urn.*

The theorem is proved in the appendix.

Theorem 4 gives sufficient conditions of convergence with positive probability to 0 and 1. The assumptions of theorem, however, are compatible with the case of independent urns (independent markets) as a particular case. More generally they represent conditions of weak feedbacks across markets (across regions) as compared to the extent of intramarket (intraregion) spillovers: Local feedbacks in that case are so important that a single market may converge to one technology, say A, even though all the related markets converge to the other one. In a sense, for the function $g_i(\vec{x}, \vec{\gamma})$, the most important argument is x_i.

Conversely let us now consider the case of strong positive cross-regional feedbacks. Strong positive feedbacks can be characterized in terms of the urn function as follows: We can say that spillovers are strong if $\exists \vec{\delta}, \vec{\chi}, \vec{\delta} > \vec{0}, \vec{\chi} < 1$, such that $\vec{\delta} < g(\vec{x}, \vec{\gamma}) < \vec{\chi}$ when $x_j = 0$ and $x_k = 1$ for some $k \neq i$. In other words strong positive spatial feedbacks are such that adoptions of one type in other regions can be a partial substitute for adoptions of that type in the same region: even if a region has always chosen technology $A(B)$ in the past, technology $B(A)$ can still be chosen in the future, if it has been frequently chosen in at least another region.

Let us define a set $A \subset [0,1]^m$ as *reachable* if there exists t such that $\mathcal{P}\{\vec{X}(t) \in A \Big| \vec{X}(0) \text{ given}\} > 0$. We can derive the following result:

Theorem 5 *If $g(\vec{0}, .) = \vec{0}$ and $\forall i = 1, \dots, m$ $\exists \eta_i > 0$ and $c_i \in [0, 1)$, such that $0 < g_i(\vec{x}, .) \leq c_i x_i$, $\forall \vec{x} \in \prod_{i=1}^m [0, \eta_i]$ and $\prod_{i=1}^m [0, \eta_i]$ is reachable, then $\mathcal{P}\{X_i(t) \to 0\} > 0$. If $g(\vec{1}, .) = \vec{1}$ and $\forall i = 1, \dots, m$ $\exists \eta_i > 0$ and $c_i \in (0, 1)$, such that $1 > g_i(\vec{x}, .) \geq 1 - c_i(1 - x_i)$, $\forall \vec{x} \in \prod_{i=1}^m [1 - \eta_i, 1]$ and $\prod_{i=1}^m [1 - \eta_i, 1]$ is reachable, then $\mathcal{P}\{X_i(t) \to 1\} > 0$. Also if $g_i(\vec{x}, .) \geq \alpha > 0$ in a neighborhood of every $\vec{y}$ such that $y_i = 0, y_k = 1$ for some $k \neq i$, then $\mathcal{P}\{X_k(t) \to 1, X_i(t) \to 0\} = 0$ and if $g_i(\vec{x}, .) \leq \beta < 1$ in a neighborhood of every $\vec{y}$ such that $y_i = 1, y_k = 0$ for some $k \neq i$, then $\mathcal{P}\{X_i(t) \to 1, X_k(t) \to 0\} = 0$.*

The theorem is proved in the appendix.

If $\vec{0} < g(\vec{x}, \vec{\gamma}) < \vec{1}$ when $\vec{x} \in [R(0,1)]^m$ then any neighborhood of $\vec{x} = \vec{0}$ and $\vec{x} = \vec{1}$ are reachable and therefore complete worldwide monopoly of technology A or B may emerge with positive probability. Moreover theorem 5 tells us that, if feedbacks are strong, asymptotically either one technology emerges everywhere, or the other does, or market sharing is the only other possible outcome.

Subject to these conditions of weak or strong spillovers we can provide a generalization of theorem 1 to study the rate of convergence to technological monopolies.

Theorem 6 *Let $\epsilon, \eta_i > 0$ and $c, c_i < 1$ be such that either the assumptions of theorem 4 or those of theorem 5 hold. (In the second case, denote* $\max_i\{c_i\}$

with c*). Then for any* $\delta \in (0, 1-c)$ *and* $\tau > 0$

$$\lim_{n\to\infty} \mathcal{P}\{n^{1-c-\delta} Y_i(n) < \tau | Y_i(t) \to 0\} = 1$$
$$\left(\lim_{n\to\infty} \mathcal{P}\{n^{1-c-\delta}[1 - Y_i(n)] < \tau | Y_i(t) \to 1\} = 1\right),$$

where $Y_i(\cdot)$ *stands for the random process given by* (9)*.*

The theorem is proved in the appendix.

Again, as in section 2, an important observation, which theorems 6 provides, is that convergence to 0 and 1 can be much faster (almost of order $1/t$ as $t \to \infty$) than to an interior limit (which can be almost of order $1/\sqrt{t}$ only). An intuitive explanation, that holds rigorously only when $g_i(\vec{x}, \vec{\gamma}) = g_i(x_i)$, is that the variance of $\zeta_i^k(x)$, which characterizes the level of random disturbances in the process (9), is $g_i(x)[1 - g_i(x)]$. This value vanishes at 0 and 1 but it does not vanish at θ, being equal to $\theta(1-\theta)$.

4 Conclusions

This paper has been motivated by two 'stylized facts' concerning the dynamics of diffusion of different technologies competing for the same market niche and a stylized fact concerning the international location of production.

a) A stable pattern of market sharing with no overwhelming dominant position is rarely observed in markets with network esternalities. Unbounded increasing returns to adoption are often called for an explanation of this fact. However the argument is generally based on an incorrect interpretation of the popular Brian Arthur (1989) model. We recalled a simple counterexample, drawn from Bassanini and Dosi (1998), to show that unbounded increasing returns are neither necessary nor sufficient to lead to technological monopolies even in a stable external environment.

b) International diffusion may lead sometimes to different standards in different countries or to the diffusion of the same standard in every country, even without intervention of any regulatory agency. Intuitively when convergence to the same standard is not an accident of history, it is an outcome of the relative weight of international spillovers as compared to nationwide esternalities.

The crucial question we tried to address in this paper is: can a model that account for the former fact accomodate also the latter? By establishing some mathematical properties of generalized urn schemes, we build on a class of competing technology dynamics models to develop an explanation for the former "fact" and to provide sufficient conditions for convergence to the same or to different technological monopolies in different countries. Our explanation for the empirical tendency to converge to technological monopoly relies on convergence rate differentials to limit market shares. In a related paper [Bassanini and Dosi (1998)] we show that a market can approach a monopoly with a higher speed than it approaches any feasible limit market share where both technologies coexist: Convergence to market sharing is in general so slow that the environment changes before the market share trajectory becomes stable in a neighborhood of its limit. In this paper we have shown that this result hold also in a multi-market model where convergence to different local monopolies can occur, even though, at a high level of aggregation, this system may seem to converge to market-sharing. The empirical implication is again that among markets with high rate of technological change and increasing returns to adoption, a prevalence of stable monopolies over stable market sharing should be expected.

5 Appendix

Proof of theorem 3. The theorem is a straightforward consequence of the following lemma which generalizes lemma 2.2 from Hill et al. (1980):

Lemma 1 *Assume that we have a scheme of multiple urns, given by a set of the functions* $f_i^A(\cdot,\cdot)$ *and* $f_i^B(\cdot,\cdot)$, $i = 1, 2, \ldots, m$. *Let for some* i *a function* $g_i(\cdot)$ *be such that* (10) *or* (11) *holds true. Then there is a probability space, where the process* (9) *and a conventional urn process* $Z_i(\cdot)$ *can be realized and* $Y_i(k) \leq Z_i(k)$ *or* $Y_i(k) \geq Z_i(k)$ *with probability 1 for* $k \geq 1$ *depending upon whether* (10) *or* (11) *holds. The process* $Z_i(\cdot)$ *has* $g_i(\cdot)$ *as urn-functions and* n_i^A, n_i^B *as initial numbers of balls.*

Proof. Fix a probability space with $r_t, t \geq 1$, a sequence of independent random variables having the uniform distribution on $[0, 1]$. For given $\vec{x} \in R(\vec{0}, \vec{1})$ and $\vec{\gamma} \in R(S_m)$ introduce a partition of $[0, 1]$ by the points

$$t_0 = 0,\ t_1 = f_1^A(\vec{x}, \vec{\gamma}),\ t_2 = f_1(\vec{x}, \vec{\gamma}),$$

$$t_3 = f_1(\vec{x}, \vec{\gamma}) + f_2^A(\vec{x}, \vec{\gamma}),\ t_4 = f_1(\vec{x}, \vec{\gamma}) + f_2(\vec{x}, \vec{\gamma}),\ \ldots\ ,$$

$$t_{2m-1} = f_1(\vec{x},\vec{\gamma}) + f_2(\vec{x},\vec{\gamma}) + \ldots + f_{m-1}(\vec{x},\vec{\gamma}) + f_m^A(\vec{x},\vec{\gamma}),\ t_{2m} = 1.$$

Set

$$\xi^n_{i,A}(\vec{x},\vec{\gamma}) = \chi_{\{r_n\in(t_{2(i-1)},t_{2i-1})\}},\ \xi^n_{i,B}(\vec{x},\vec{\gamma}) = \chi_{\{r_n\in(t_{2i-1},t_{2i})\}},$$

where χ_A stands for the indicator of the event A. If $\tau_i(\cdot)$ are defined as above, set

$$\tilde{\zeta}^k_i(x_i) = \chi_{\{r_{\tau_i(k)}\in(t_{2(i-1)},t_{2(i-1)}+g_i(x_i)f_i(\vec{x},\vec{\gamma}))\}} \tag{12}$$

and

$$Z_i(k+1) = Z_i(k) + \frac{1}{k+G_i}[\tilde{\zeta}^k_i(Z_i(k)) - Z_i(k)],\quad k\geq 1,\quad Z_i(1) = \frac{n_i^w}{G_i}.$$

Hence $Y_i(\cdot)$ and $Z_i(\cdot)$ are given on the same probability space. If we denote with $\tilde{F}^i_k$ the σ-algebra generated by $r_t,: t\leq\tau_i(k)$, then $E[\tilde{\zeta}^k_i(Z_i(k))|\tilde{F}^i_k] = g_i(Z_i(k))$. Hence $Z_i(\cdot)$ is a conventional urn process with $g_i(\cdot)$ as urn-function.

Notice that

$$\zeta^k_i(x_i) = \chi_{\{r_{\tau_i(k)}\in(t_{2(i-1)},t_{2(i-1)}+g(\vec{x},\vec{\gamma})f_i(\vec{x},\vec{\gamma}))\}},$$

which, from (12), implies that $\zeta^t_i(x_i)\leq\tilde{\zeta}^t_i(x_i)$ or $\zeta^t_i(x_i)\geq\tilde{\zeta}^t_i(x_i)$ with probability 1 depending whether (10) or (11) holds. Now to accomplish the proof it is enough to check that

$$Y_i(t+1) = \frac{(t+G_i-1)Y_i(t) + \zeta^t_i(Y_i(t))}{t+G_i},$$

$$Z_i(t+1) = \frac{(t+G_i-1)Z_i(t) + \tilde{\zeta}^t_i(Z_i(t))}{t+G_i}.$$

The lemma is proved. ■

Proof of theorem 4. We need the following lemma:

Lemma 2 *Let $X(\cdot)$ be a conventional urn process with $f(\cdot)$ as urn-function. If there is $\epsilon > 0$ such that*

$$f(x) \geq x \text{ for } x \in (0, \epsilon) \quad (f(x) \leq x \text{ for } x \in (1-\epsilon, 1)),$$

then $\mathcal{P}\{X(t) \to 0\} = 0$ ($\mathcal{P}\{X(t) \to 1\} = 0$)for any initial numbers of balls. Also, if $f(x) < 1$ ($f(x) > 0$) for $x \in (0,1)$ and there is $\epsilon > 0$ such that

$$f(x) < x \text{ for } x \in (0, \epsilon) \quad (f(x) > x \text{ for } x \in (1-\epsilon, 1)),$$

then $\mathcal{P}\{X(t) \to 0\} > 0$ ($\mathcal{P}\{X(t) \to 1\} > 0$) for any initial numbers of balls.

Proof. Set that all conventional urn processes appearing here start from the same numbers of balls in the urn. Let $f(x) > x$ for $x \in (0, \epsilon)$. Set $g(x) = \max(f(x), x)$. Define $Y(\cdot)(Z(\cdot))$ a conventional urn process corresponding to the urn-function x ($g(x)$). Since $x \leq g(x)$, then due to lemma 2.2 from Hill et al. (1980) one has $Y(t) \leq Z(t)$, $t \geq 1$. Consequently $\mathcal{P}\{Z(t) \to 0\} \leq \mathcal{P}\{Y(t) \to 0\}$. But $Y(\cdot)$ is a Polya process, i.e. it converges a.s. to a random variable with a beta distribution. The limit, having a density with respect to the Lebesgue measure, takes every particular value from $[0,1]$ with probability 0. Hence $\mathcal{P}\{Y(t) \to 0\} = 0$ and, consequently, $\mathcal{P}\{Z(t) \to 0\} = 0$. But the urn-functions $f(\cdot)$ and $g(\cdot)$ agree in $(0, \epsilon)$, which due to lemma 4.1 from Hill et al. (1980) implies that $\mathcal{P}\{X(t) \to 0\} = 0$. Let $f(x) < x$ for $x \in (0, \epsilon)$ and $f(x) < 1$ for all $x \in (0,1)$. Set $g(x) = \max(f(x), x/2)$. Then $f(x) \leq g(x)$ and due to arguments similar to those given above $\mathcal{P}\{X(t) \to 0\} \geq \mathcal{P}\{Z(t) \to 0\}$, where $Z(\cdot)$ stands for a conventional urn process corresponding to $g(\cdot)$. Finally let us prove that $\mathcal{P}\{Z(t) \to 0\} > 0$. Put $d(x) = \min(g(x), g(\epsilon/2))$. The equation $d(x) - x = 0$ has the only root 0. Hence there is a conventional urn process corresponding to $d(\cdot)$ which converges to 0 with probability 1. Since $g(x) \in (0,1)$ and $d(x) \in (0,1)$ for all $x \in (0,1)$, this implies that $\mathcal{P}\{Z(t) \to 0\} > 0$ for any initial numbers of balls, because of lemma 4.1 of Hill et al. (1980).

Other cases can be handled by similar arguments. ■

Due to the aforementioned relationship between convergence of $X_i(\cdot)$ and $Y_i(\cdot)$, it is enough to establish the corresponding facts for $Y_i(\cdot)$.

The first statement follows by considering a conventional urn process with

$$d_i(y) = \inf_{\vec{x}:\, x_i = y} \{L_i(\vec{x})\} \quad (d_i(y) = \sup_{\vec{x}:\, x_i = y} \{U_i(\vec{x})\})$$

as the urn-function and applying lemma 2 and theorem 3.

Since convergence to 1 with positive probability can be studied by the same means, let us prove convergence with positive probability to 0 only. Let $Z_i(\cdot)$ be a conventional urn process having

$$d_i(x) = \begin{cases} f(x) & if \quad x < \epsilon/2 \\ f(\epsilon/2) & if \quad x \geq \epsilon/2 \end{cases}$$

as urn-function and starting from the same numbers of balls. Then

$$\mathcal{P}\{Z_i(t) \to 0\} = 1. \tag{13}$$

Set $l_i(t) = n_i^A(n_i^A + n_i^B + t - 1)^{-1},\ t \geq 1$. Since we assume that $g_i(\vec{x}, \vec{\gamma}) < 1$ for all possible $\vec{x}$ and $\vec{\gamma}$, the process $Y_i(\cdot)$ can move with a positive probability to the left from any point. Hence

$$\mathcal{P}\{Y_i(t) = l_i(t)\} > 0 \ \text{ for } \ t \geq 1. \tag{14}$$

For any t such that $l_i(t) < \epsilon/2$ introduce $\mu_i(t)$ as the first instant after t such that $Z_i(\cdot)$ exits from $(0, \epsilon/2)$ providing that $Z_i(t) = l_i(t)$. Due to (13):

$$\mathcal{P}\{\mu_i(t) = \infty\} \to 1 \ \text{ as } \ t \to \infty. \tag{15}$$

But due to lemma 1 $Y_i(n) \leq Z_i(n)$ for $t \leq n < \mu_i(t)$ providing that $Y_i(t) = Z_i(t) = l_i(t)$. Thus, taking into account (13) and (15), we get:

$$\mathcal{P}\{Y_i(n) \to 0 | Y_i(t) = l_i(t)\} \geq \mathcal{P}\{Y_i(n) \to 0,\ \mu_i(t) = \infty | Y_i(t) = l_i(t)\} \geq$$

$$\mathcal{P}\{Z_i(n) \to 0,\ \mu_i(t) = \infty | Z_i(t) = l_i(t)\} \to 1 \ \text{ as } \ t \to \infty.$$

Therefore to accomplish the proof it is enough to refer to (14).■

Proof of theorem 5. We need the following lemma:

Lemma 3 *Consider two multiple urn processes* $\vec{X}(.)$ *and* $\vec{Y}(.)$ *which agree in a neighborhood* N *of a point* $\theta \in (R[0,1])^m$. *Then there exists an urn process* $\vec{Z}(.)$ *with the same urn function as* $\vec{Y}(.)$ *such that* $\mathcal{P}\{\vec{Z}(t) \to \theta\} > 0$ *if* $\mathcal{P}\{\vec{X}(t) \to \theta\} > 0$. *Also consider a neighborhood* N_0 (N_1) *of* 0 (1); *if it is reachable by* $X_i(.)$, *given the initial number of balls, and if the urn functions never take the values* 0 (1) *in that neighborhood* (0 (1) *excluded*), *then* $\mathcal{P}\{\vec{Z}(t) \to \vec{0}\} > 0$ *from every initial number of balls only if* $\mathcal{P}\{\vec{X}(t) \to \vec{0}\} > 0$ $(\mathcal{P}\{\vec{Z}(t) \to \vec{1}\} > 0$ *only if* $\mathcal{P}\{\vec{X}(t) \to \vec{1}\} > 0)$.

Proof. $>$From almost sure convergence we have that $\exists t > 0$ and two vectors $\vec{n}, \vec{t} \in \mathbf{Z}_+^m$ such that

$$\mathcal{P}\left\{ \begin{array}{l} \vec{X}(s) \to \theta, X_i(t) = \frac{n_i}{t_i+G_i}, i = 1, ..., m, \\ \sum\limits_{i=1}^{m}(t_i + G_i) = t + G, \vec{X}(s) \in N, s > t \end{array} \right\} > 0,$$

which implies

$$\mathcal{P}\left\{ \begin{array}{l} \vec{X}(s) \to \theta, \vec{X}(s) \in N, s > t \\ \Big| X_i(t) = \frac{n_i}{t_i+G_i}, i = 1, ..., m \end{array} \right\} > 0. \tag{16}$$

Take $Z_i(0) = \frac{n_i}{t_i+G_i}$. Since on N $\vec{X}(.)$ and $\vec{Y}(.)$ agree, from (16) we have

$$\mathcal{P}\left\{ \vec{Z}(t) \to \theta, \vec{Z}(t) \in N, t > 0 \right\} > 0,$$

from which the first statement of the lemma follows.

Since N_0 is reachable $\exists \bar{t} > 0$ and $\exists (\vec{y}, \vec{n}, \vec{t}) \in N_0 \times \mathbf{Z}_+^{2m} : \frac{n_i}{t_i+G_i} = y_i$, $i = 1, ..., m$, $\sum\limits_{i=1}^{m}(t_i + G_i) = \bar{t} + G$ such that

$$\mathcal{P}\left\{ \vec{X}(\bar{t}) = \vec{y} \right\} > 0. \tag{17}$$

Take n_i, $t_i + G_i$ as initial conditions of the process $\vec{Z}(.)$. Again from almost sure convergence we have that $\mathcal{P}\{\vec{Z}(t) \to \vec{0}\} > 0$ from every initial number of balls implies that, $\exists \vec{z} \in N_0$ and $\exists T > 0$ such that

$$\mathcal{P}\left\{ \vec{Z}(s) \to \vec{0}, \vec{Z}(s) \in N_0, s \geq T, \vec{Z}(T) = \vec{z} \in N_0 \right\} > 0.$$

Given that the urn functions never reach 0 or 1 in N_0, $\vec{z}$ can be reached from $\vec{y}$ without leaving N_0 (through an appropriate sequence of 0 first and 1 afterwards), we have also

$$\mathcal{P}\left\{ \vec{Z}(s) \to \vec{0}, \vec{Z}(s) \in N_0, \vec{Z}(0) = \vec{y} \in N_0 \right\} > 0,$$

which implies

$$\mathcal{P}\left\{ \vec{Z}(s) \to \vec{0}, \vec{Z}(s) \in N_0 \,\Big|\, \vec{Z}(0) = \vec{y} \in N_0 \right\} > 0. \tag{18}$$

Given that on N_0 $\vec{X}(.)$ and $\vec{Y}(.)$ agree, we can choose the process $\vec{Z}(t)$ defined on the common probability space in such a way that $\vec{X}(s+\bar{t}) = \vec{Z}(s)$ for any $s < \tilde{t} = \min\left\{t : \vec{Z}(t) \notin N_0\right\}$. From (18) we have

$$\mathcal{P}\left\{\vec{X}(s) \to \vec{0}, \vec{X}(s) \in N_0 \middle| \vec{X}(\bar{t}) = \vec{y} \in N_0\right\} > 0,$$

which, taking into account (17), implies that $\exists t > 0$ such that

$$\mathcal{P}\left\{\vec{X}(s) \to \vec{0}, \vec{X}(s) \in N_0, s \geq t\right\} > 0.$$

Convergence to $\vec{1}$ can be studied with similar arguments.■

To prove the first statement of the theorem take a process $\vec{Y}(.)$ with urn function:

$$d(\vec{x},.) = \begin{cases} g(\vec{x},.) & if\ \vec{x} \in \prod_{i=1}^{m}[0,\eta_i] \\ 0 & otherwise \end{cases},$$

the apply lemma 3 and lemma 1. The second statement can be proved with similar arguments. For the third statement take a process $\vec{Y}(.)$ with urn function:

$$d(\vec{x},.) = \begin{cases} g(\vec{x},.) & if\ \|\vec{x} - \vec{y}\| < \epsilon \\ \alpha & otherwise \end{cases},$$

then apply lemma 3 and lemma 1. The last statement can be proved with similar arguments.■

Proof of theorem 6. Consider only the first case – convergence to 0. Without loss of generality we can assume that $\mathcal{P}\{Y_i(t) \to 0\} > 0$.

Let $Z_i(\cdot)$ be a conventional urn process with cx as the urn-function and the same initial numbers of balls. Then

$$EZ_i(t+1) \leq [1 - \frac{1-c}{t+G_i}]EZ_i(t), \quad t \geq 1,$$

and consequently

$$EZ_i(t) \leq Z_i(1)\prod_{j=1}^{t-1}(1 - \frac{1-c}{j+G_i}) = Z_i(1)t^{c-1}[1 + o_t(1)],$$

where $o_t(1) \to 0$ as $t \to \infty$. Hence from Markov's inequality

$$\mathcal{P}\{t^{1-c-\delta}Z_i(t) < \tau\} \to 1 \text{ as } t \to \infty \tag{19}$$

for every $\delta \in (0, 1-c)$ and $\tau > 0$.

For arbitrary $\sigma \in (0, \epsilon)$ and $v > 0$ there is N depending on these variables such that

$$\mathcal{P}\{\{Y_i(t) \to 0\}\Delta\{Y_i(n) \leq \sigma,\ n \geq N\}\} < v.$$

where $A\Delta B = (A\backslash B) \cup (B\backslash A)$. Also since $Z_i(t) \to 0$ with probability 1 as $t \to \infty$, we can choose this N so large that

$$\mathcal{P}\{\{Y_i(t) \to 0\}\Delta\{Y_i(n) \leq \sigma,\ Z_i(n) \leq \sigma,\ n \geq N\}\} < v. \tag{20}$$

To prove the theorem it is enough to show that

$$\lim_{n\to\infty} \mathcal{P}\{n^{1-c-\delta}Y_i(n) < \tau,\ Y_i(t) \to 0\} = \mathcal{P}\{Y_i(t) \to 0\},$$

or, taking into account that v in (19) can be arbitrary small, that

$$\lim_{n\to\infty} \mathcal{P}\{n^{1-c-\delta}Y_i(n) < \tau,\ Y_i(n) \leq \sigma,\ Z_i(n) \leq \sigma,\ n \geq N\} =$$
$$\mathcal{P}\{Y_i(n) \leq \sigma,\ Z_i(n) \leq \sigma,\ n \geq N\}.$$

Due to lemma 1 $Z_i(\cdot)$ majorizes $Y_i(\cdot)$ on the event $Z_i(t) \leq \sigma,\ t \geq N$, providing that these processes start from the same point. Hence,

$$\mathcal{P}\{Y_i(t) \leq \sigma,\ Z_i(t) \leq \sigma,\ t \geq N\} \geq$$
$$\limsup_{n\to\infty} \mathcal{P}\{n^{1-c-\delta}Y_i(n) < \tau,\ Y_i(t) \leq \sigma,\ Z_i(t) \leq \sigma,\ t \geq N\} \geq$$
$$\liminf_{n\to\infty} \mathcal{P}\{n^{1-c-\delta}Y_i(n) < \tau,\ Y_i(t) \leq \sigma,\ Z_i(t) \leq \sigma,\ t \geq N\} =$$
$$\liminf_{n\to\infty} E\mathcal{P}\{n^{1-c-\delta}Y_i(n) < \tau,\ Y_i(t) \leq \sigma,\ Z_i(t) \leq \sigma,\ t \geq N|Y_i(N)\} \geq$$
$$\liminf_{n\to\infty} E\mathcal{P}\{\{n^{1-c-\delta}Z_i(n) < \tau,\ Y_i(t) \leq \sigma,\ Z_i(t) \leq \sigma,\ t \geq N|$$
$$Z_i(N) = Y_i(N)\}|Y_i(N)\} =$$
$$E\mathcal{P}\{Y_i(t) \leq \sigma,\ Z_i(t) \leq \sigma,\ t \geq N|Y_i(N)\} =$$
$$\mathcal{P}\{Y_i(t) \leq \sigma,\ Z_i(t) \leq \sigma,\ t \geq N\},$$

i.e. (20) holds true.■

6 References

An, M., and N.Kiefer, Local Externalities and Societal Adoption of Technologies, *J. of Evolutionary Econ.*, **5**, 103–117, 1995.

Arthur, W.B., On Competing Technologies and Historical Small Events: The Dynamics of Choice under Increasing Returns, IIASA Working Paper WP-83-90 [Reprinted in W.B. Arthur (1994) *Increasing Returns and Path-Dependence in the Economy*, Ann Arbor: University of Michigan Press], 1983.

Arthur, W.B., Competing Technologies, Increasing Returns and Lock-In by Historical Events, *Econ. J.*, **99**, 116–131, 1989.

Arthur, W.B., "Sylicon Valley" Locational Clusters: When Do Increasing Returns Imply Monopoly?, *Math. Soc. Sci.*, **19**, 235–51, 1990.

Arthur, W.B., Y.Ermoliev and Y.Kaniovski, Generalized Urn Problem and Its Applications, *Cybernetics*, **19**, 61-71, 1983.

Bassanini, A.P. and G.Dosi, Competing Technologies, Technological Monopolies, and the Rate of Convergence to a Stable Market Structure, unpublished manuscript, 1998.

Brock, W.A. and S.N.Durlauf, Discrete Choice with Social Interactions I: Theory, NBER Working Paper #5291, 1995.

Brock, W.A. and S.N.Durlauf, A Formal Model of Theory Choice in Science, *Economic Theory*, forthcoming, 1998.

Brynjolfsson, E. and C.F.Kemerer, Network Externalities in Microcomputer Software: An Econometric Analysis of the Spreadsheet Market, *Management Science*, **42**, 1627-47, 1996.

Cantwell, J., *Technological Innovation and Multinational Corporations*, Oxford: Blackwell, 1989.

Church, J. and N.Gandal, Complementary Network Externalities and Technological Adoption, *Int. J. of Industrial Organization,* **11***,* 239-60, 1993.

Cowan, R., Tortoises and Hares: Choice among Technologies of Unknown Merit, *Econ. J.,* **101**, 801-14, 1991.

Cusumano, M.A., Y.Milonadis and R.S.Rosenbloom, Strategic Maneuvering and Mass-Market Dynamics: The Triumph of VHS over Beta, *Business History Rev.,* **66**, 51-94, 1992.

David, P., Clio and the Economics of QWERTY, *AEA Papers and Proceedings*, **75**, 332-7, 1985.

David, P., Heroes, Herds and Hystheres in Technological Theory: Thomas Edison and the Battle of Systems Reconsidered, *Industrial and Corporate*

Change, **1**, 129-80, 1992.

Dosi, G., Technological Paradigms and Technological Trajectories, *Research Policy*, **11**, 142-67, 1982.

Dosi, G., Sources, Procedures and Microeconomic Effect of Innovation, *J. Econ. Lit.*, **26**, 1120-71, 1988.

Dosi, G., Research on Innovation Diffusion: An Assessment, in A.Grübler and N.Nakicenovic (eds.), *Innovation Diffusion and Social Behavior*, Berlin: Springer, 1991.

Dosi, G., Y.Ermoliev and Y.Kaniovski, Generalized Urn Schemes and Technological Dynamics, *J. Math. Econ.*, **23**, 1-19, 1994.

Dosi, G. and Y.Kaniovski, On Badly Behaved Dynamics, *J. of Evolutionary Econ.*, **4**, 93-123, 1994.

Dosi, G., K.Pavitt and L.Soete, *The Economics of Technological Change and International Trade*, New York: NYU Press, 1990.

Durlauf, S.N., Statistical Mechanics Approaches to Socioeconomic Behavior, in S.N.Durlauf, W.B.Arthur and D.Lane (Eds.), *The Economy as an Evolving Complex System II*, New York: Addison-Wesley, 1997.

Farrell, J. and G.Saloner, Standardization, Compatibility and Innovation, *Rand J. of Econ.,* **16**, 70-83, 1985.

Farrell, J. and G.Saloner, Installed Base and Compatibility: Innovation, Product Preannouncements, and Predation, *Amer. Econ. Rev.,* **76**, 940-55, 1986.

Freeman, C., *The Economics of Industrial Innovation*, London: Pinter, 2nd ed., 1992.

Freeman, C., The Economics of Technical Change: A Critical Survey, *Cambridge J. of Econ.*, **18**, 463-514, 1994.

Freeman, C., The National Systems of Innovation in Historical Perspective, *Cambridge J. of Econ.*, 1995.

Hill, B.M., D.Lane and W.Sudderth, A Strong Law for Some Generalized Urn Processes, *Annals of Probability*, **8**, 214-26, 1980.

Kaniovski, Y. and H.P.Young, Learning Dynamics in Games with Stochastic Perturbations, *Games and Econ. Behavior,* **11**, 330-63, 1995.

Katz, M.L. and C.Shapiro, Network Externalities, Competition , and Compatibility, *Amer. Econ. Rev.*, **75**, 424–40, 1985.

Katz, M.L. and C.Shapiro, Technology Adoption in the Presence of Network Externalities, *J. Pol. Econ.*, **94**, 822–41, 1986.

Katz, M.L. and C.Shapiro, Product Introduction with Network Externalities, *J. of Industrial Econ.*, **40**, 55–84, 1992.

Katz, M.L. and C.Shapiro, Systems Competition and Network Effects, *J. of Econ. Perspectives*, **8**, 93–115, 1994.

Krugman, P., History vs. Expectations, *Quart. J. Econ.*, **106**, 651-67, 1991a.

Krugman, P., *Geography and Trade*, Cambridge, MA: MIT press, 1991b.

Krugman, P., and A.Venables, Globalization and the Inequality of nations, *Quart. J. Econ.*, **110**, 857-80, 1995.

Lundvall, B.A. (ed.), *National Innovation Systems*, London: Pinter, 1992.

Nelson, R. (ed.), *National Systems of Innovation*, Oxford: OUP, 1993.

Oren, S. and S.Smith, Critical Mass and Tariff Structure in Electronic Communications Markets, *Bell J. of Econ.*, **12**, 467–87, 1981.

Rauch, J.E., Does History Matter Only When It Matters Little? The Case of City-Industry Location, *Quart. J. Econ.*, **108**, 843–67, 1993.

Rohlfs, J., A Theory of Interdependent Demand for a Communication Service, *Bell J. of Econ.*, **5**, 16–37, 1974.

Silverberg, G., G.Dosi and L.Orsenigo, Innovation, Diversity and Diffusion: A Self-Organization Model, *Econ. J.*, **98**, 1032-54, 1988.

Tushman, M.L. and J.P.Murmann, Dominant Designs, Technology Cycles, and Organizational Outcomes, in B.Staw and L.L.Cummings (eds.), *Research in Organizational Behavior*, **20**, Greenwich, CT: JAI press, 1998.

Venables, A.J., Localization of Industry and Trade Performance, *Oxford Rev. of Econ. Policy*, **12**(3), 52-60, 1996.