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International parity relationships between Germany and US: a multivariate time series analysis for the post Bretton-Woods period

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International parity relationships between Germany and US: a multivariate time series analysis for the post Bretton-Woods period*

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Abstract

This paper investigates the effects of replacing the consumer price index (CPI) with the wholesale price index (WPI) in the cointegrating international parity relationships found by Juselius and MacDonald (2000).

Our empirical analysis outstandingly produced results similar to the ones obtained by Juselius and MacDonald, suggesting that the cointegration relationships in the international parity conditions hold both for CPI and WPI.

JEL Classifications E31, E43, F31, F32

Keywords: VAR model, cointegration, purchasing power parity, uncovered interest rate parity.

1 Introduction

Recently, basic issues in international monetary economics concerning the validity of parity conditions are receiving a growing interest also in econometrics.

Seemingly simple questions about the determinants of exchange rates between countries such as Europe and US, do not still find adequate responses

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rigorously grounded on empirical data. Is the exchange rate determined by the level of prices as the Purchasing Power Parity (*ppp*) suggests? Is the exchange rate determined by the spread between the interest rates in the two countries as the Uncovered Interest rate Parity (*uip*) claims? How prices react to changes in exchange rates and interest rates?

Answering to these issues becomes problematic when economic theory assumes that *ppp* and *uip* are stationary relations while they are empirically nonstationary both in the short and medium-long run such as a span of 20-25 years (Rogoff (1996). Rogoff refers to this problem as "The Puchasing Power Parity Puzzle" and talks about "the embarrassment of not being able to reject the random walk model" for the *ppp* while other authors doubt about the usefulness of *ppp* and *uip*[1].

This paper is aimed to show that the *ppp* and *uip* relations are indeed extremely interesting when they are jointly modelled and we should not be embarrassed when we deal with "random walk" parity conditions. Indeed, just because *ppp* and *uip* behave in a nonstationary way, we may investigate the cointegration relations between the two parities i.e. the stationary long run relations between pseudo random walks (the *ppp* and the *uip*) that share common trends.

This paper is based on a recent paper by Juselius and MacDonald (2000) and two "Journal of Econometrics" articles by Juselius (1995) and Johansen and Juselius (1992). The basic feature in all these articles is that the joint modelling of international parity conditions, namely *ppp* and *uip*, produces stationary relations showing an important interaction between the goods (via the *ppp*) and the capital markets (via the *uip*)[2].

Since "there is no "right" *ppp* measure" (Rogoff 1996)[3], we replaced the consumer price index (CPI) considered by Juselius and MacDonald with the wholesale price index (WPI) to check whether the international parity relationships still cointegrate. To our surprise we outstandingly produced similar results to those by Juselius and MacDonald, suggesting that the cointegration relationships in the international parity conditions hold also if we use different measures of prices and *ppp*. What is striking in our results is that even if there is no direct cointegration relation between CPI and WPI both in Germany and the US, the cointegration relation found between *ppp* and *uip* still holds notwithstanding of how *ppp* is measured.

The paper is organised as follows. In section 2 we define the international parity conditions. In section 3 we discuss the choice of the variables, the data set and we provide some preliminary visual analysis of the variables and the parity conditions. In section 4 we explain the statistical model we use to test the

[1] See Colombo and Lossani (2000).

[2] See Juselius 1995.

[3] Rogoff (1996) put forward the idea to use even the McDonald's "Big Mac" index to produce a PPP measure with the data provided by The Economist. We found this idea very entertaining, but unfortunately the actual dataset would not allow us to use VAR (that needs long time series) and cointegration techniques in a reliable way. The Big Mac index data set by The Economist contains only the data of the last 10 years on a semester basis so that VAR modelling and cointegration are precluded.

parities. In section 5 we test parity conditions using a model with a minimal number of variables, which exclude the short interest rates. In section 6 we extend the model including also the short term interest rates. Using the moving average (MA) representation, the weakly exogenous variables and the long run impacts of shocks are also discussed. Section 7 concludes and summarises the main results.

2 International parities conditions

2.1 The absolute *ppp*

The absolute *ppp*, is defined as:

$$p_t - p_t^* - s_t = 0 \tag{1}$$

where p_t is the log of the domestic price level (in our case the German wholesale price level index), p_t^* is the log of the foreign price level (in our case the wholesale price level index in the US), s_t denotes the log of the spot exchange rate (home currency price of a unit of foreign currency). The *ppp* states that, once converted to a common currency, the price levels in the two countries should be equal.

If the *ppp* holds empirically, we would expect that:

$$p_t - p_t^* - s_t \sim I(0)$$

where $I(0)$ stands for zero order integrated process.

The empirical analysis confirms two main aspects:

- The *ppp* is a relation valid only in the very long run (temporal horizon of more than 50 years). On a shorter temporal horizon we observe persistent deviation from *ppp* (Rogoff 1996).

The nature of the empirical support for *ppp* is very dependent on the sample period. If a relatively long span of data is used such as a century, there is mounting evidence that *ppp* is valid, although the adjustment speed of *ppp* is too slow to be consistent with a traditional version of *ppp* (Rogoff 1996) and for the recent floating experience there is little evidence that *ppp* behaves like a $I(0)$ process.

Juselius and MacDonald suggest that there are a number of possible reasons why the *ppp* has a so little empirical support in the short and medium run. One reason could lie in the rather weak correspondence between the measured prices series used by researchers - usually the CPI - and the true theoretical prices; other variables not mentioned by theory, such as institutional factors, might also be relevant. Another reason, which is a objection to traditional *ppp* is that there may be important real determinants (such as productivity shocks, differences in technology and preferences), which are responsible for introducing a stochastic trend into real exchange rates.

2.2 The *uip*

The condition of *uip*, is defined as:

$$E_t \Delta_l s_{t+l} - i_t^l + i_t^{l*} = 0 \tag{2}$$

where i_t^l denotes a long term bond yield with maturity $t + l$, E_t denotes the conditional expectations operator on the basis of time-t information set. The *uip* states that, in the capital market, the interest rate differential between the two countries is equal to the expected change in the spot exchange rates (Juselius 1995). Hence, once converted to a common currency, the interest rates in the two countries should be equal. If this were not, investors would have the incentive to move capitals from the country where the interest rate is lower to the country where the interest rate is higher till equilibrium. Thus, the *uip* is an arbitrage relation that describes an equilibrium in the capital markets (Colombo and Lossani 2000).

If the *uip* hold empirically, we would expect that:

$$E_t \Delta_l s_{t+l} - i_t^l + i_t^{l*} \sim I(0)$$

Juselius (1995) and Juselius and MacDonald (2000) maintain that empirical tests by other authors (Cumby and Obstfeld 1981) have confirmed that the *uip*, like the *ppp*, is a non stationary relation.

2.3 Combining the *ppp* with the *uip*

In this paper we also will add evidence that *ppp* and *uip* as such do not find empirical support, however, as Juselius and MacDonald (2000), our aim is to check whether a linear combination of the two parities are able to generate a stationary relation.

Before we arrive to a final equation to test that includes both parities, from the *uip* equation we have:

$$i_t^l - i_t^{l*} = E_t \Delta_l s_{t+l} \tag{3}$$

From *ppp*, differencing and taking the expected change of exchange rate, we have:

$$E_t \Delta_l s_{t+l} = E_t \Delta_l p_{t+l} - E_t \Delta_l p_{t+l}^* \tag{4}$$

Thus:

$$i_t^l - i_t^{l*} = E_t \Delta_l p_{t+l} - E_t \Delta_l p_{t+l}^* \tag{5}$$

i.e.:

$$i_t^l - i_t^{l*} - E_t \Delta_l p_{t+l} + E_t \Delta_l p_{t+l}^* = 0$$

Therefore a relation that combines the *ppp* and the *uip* may be written as:

$$i_t^l - i_t^{l*} - E_t\Delta_l p_{t+l} + E_t\Delta_l p_{t+l}^* = p_t - p_t^* - s_t \tag{6}$$

or alternatively:

$$i_t^l - i_t^{l*} = E_t(\Delta_l p_{t+l} - \Delta_l p_{t+l}^*) + ppp_t$$

that would find empirical support if:

$$i_t^l - i_t^{l*} - ppp_t - E_t(\Delta_l p_{t+l} - \Delta_l p_{t+l}^*) \sim I(0)$$

and this would be the case of either:

$$i_t^l - i_t^{l*} \sim I(0),\ ppp_t \sim I(0) \text{ and } E_t(\Delta_l p_{t+l} - \Delta_l p_{t+l}^*) \sim I(0)$$

or:

$$\begin{gathered} i_t^l - i_t^{l*} \sim I(1)\ ,\ ppp_t \sim I(1) \text{ and } E_t(\Delta_l p_{t+l} - \Delta_l p_{t+l}^*) \sim I(1) \\ \text{but } i_t^l - i_t^{l*} - ppp_t - E_t(\Delta_l p_{t+l} - \Delta_l p_{t+l}^*) \sim I(0). \end{gathered}$$

If we assume rational expectations in prices:

$$E_t(\Delta_l p_{t+l} - \Delta_l p_{t+l}^*) = (\Delta p_t - \Delta p_t^*) + v_t$$

with v_t unpredictable *i.i.d.* shock, we have that:

$$i_t^l - i_t^{l*} = (\Delta p_t - \Delta p_t^*) + ppp_t + v_t \tag{7}$$

if we relax the rational expectations hypothesis, for example if we admit *bad guys* like the chartists that do not ever conform to rational rules, we might consider that the following and more general equation might be more appropriate:

$$i_t^l - i_t^{l*} = \omega_1(\Delta p_t - \Delta p_t^*) + \omega_2 ppp_t + v_t \tag{8}$$

with ω parameters, and testing:

$$(i_t^l - i_t^{l*}) - \omega_1(\Delta p_t - \Delta p_t^*) - \omega_2 ppp_t \sim I(0) \tag{9}$$

is equivalent to test whether or not equation (8) finds empirical support.

The relation expressed in (9) is the fundamental relation that we test and would be satisfied either in the case that:

$$i_t^l - i_t^{l*} \sim I(0),\ ppp_t \sim I(0) \text{ and } (\Delta p_t - \Delta p_t^*) \sim I(0)$$

or:

$$i_t^l - i_t^{l*} \sim I(1)\ ,\ ppp_t \sim I(1) \text{ and } (\Delta p_t - \Delta p_t^*) \sim I(1).$$

Before starting the tests, a rationale choice of the variables, the sample period and the data set will be discussed in the next section.

3 Choice of the variables, data set and a visual analysis

3.1 Choice of the variables and data set

The variables that enter in equation (8) are:
- p_t, the home price index
- p_t^*, the foreign price index
- i_t, the home interest rate
- i^*, the foreign interest rate
- s_t, the spot exchange rate

Our analysis focuses only on two "big" countries, namely Germany and the US and is referred to the recent float period after the end of the Bretton Woods system (1975-1998)[4].

The choice of the countries and the sample period may be justified in the following way:

- It is always worth not to mix different regimes. A economic relation might have economic meaning in one period and be nonsense for anotherone in which a different regime prevails. Therefore it is worth to divide the sample in regime periods, and conduct a different analysis for the post war era and for the post Bretton woods period.

- The two countries, Germany and US, are to be considered two "big" countries during the last thirty years. I.e., in the last 25-30 years, a change in one of the two countries will probably affect the other one. Conversely, if we refer to the immediate post war period, we would expect that Germany follows the changes in the US economy, i.e. we would expect to consider Germany a small country and the US a big one.

Our analysis faces also other issues concerning which category of prices and interest rates should be analyzed. Should we consider the CPI or the Big Mac index? Generally the CPI is chosen, but there is no right answer to such a question; we chose the WPI. Moreover, if there is no a right *ppp* measure, there is no a right measure for the *uip* too! Shall we consider the long or the short interest rate? Generally the long interest rate is chosen, we will consider both.

Therefore our database database consists of the following variables:
- p_t, the German, or "home" wholesale price index
- p_t^*, the US, or "foreign", wholesale price index
- i_t^l, the German long bond yield (10 years)
- $i_t^{l^*}$, the US long bond yield (10 years)
- s_t, the spot exchange rate, USdollar/DeutcheMark
- i_t^s, the German three month Treasury bill rate
- $i_t^{s^*}$, the US three month Treasury bill rate

This database was provided by Prof. Juselius and was extracted from the International Monetary Fund CD-rom 1998. Data sources such as Datastream

[4] Actually the end of the Bretton Woods system, the monetary regime based on convertibility indirectly linked to gold, is generally dated between 1971 and 1973.

also contain the same and updated values. All the data are monthly, not seasonally adjusted. The starting date of our sample is July 1975, because short term interest rate for Germany are available only from that date. We transformed prices and the exchange rate with their natural log, the yearly interest rates were taken in percentage (i.e. divided by 100) and divided by 12 to obtain the monthly rates.

3.2 Visualizing data

The visual inspection of the data in any econometric analysis is a critical first step in any econometric analysis (Enders 1995). The graphs of the time series of all the variables relevant for the paper are shown in levels and differences.

3.2.1 Prices and Inflation rates

In this subsection we want to show that prices seem to be $I(2)$, inflation rates $I(1)$ but $(\Delta p_t - \Delta p_t^*) \sim I(1)$.

Prices and its differences, i.e. inflation rates, show a rapid increase in the 70s and a more stable pattern in the 80s and 90s both in Germany (see Fig. 1, LGEWPI time series, we call LGEWPI, the log of German wholesale price index)[5]

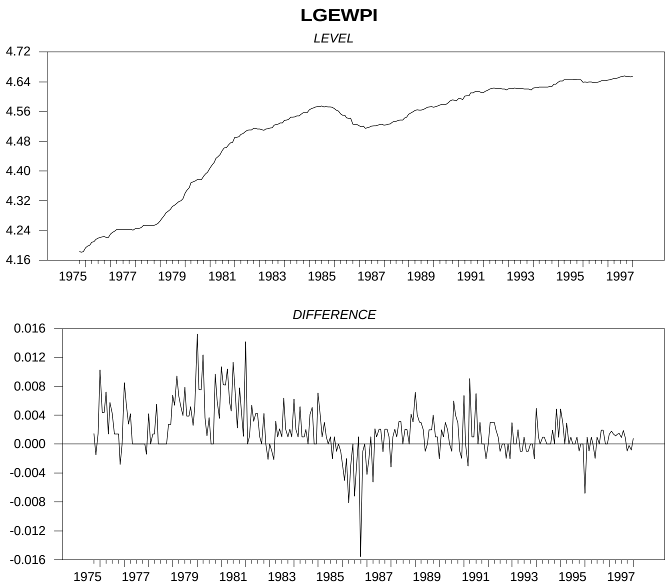

Fig.1: The log of WPI index in Germany.

and in the US (see Fig.2 LUSWPI time series, we call LUSWPI, the log of the US wholesale price index).

[5] All the graphs and the cointegration analysis in this paper has been produced by CATS for RATS software, Hansen and Juselius (1994).

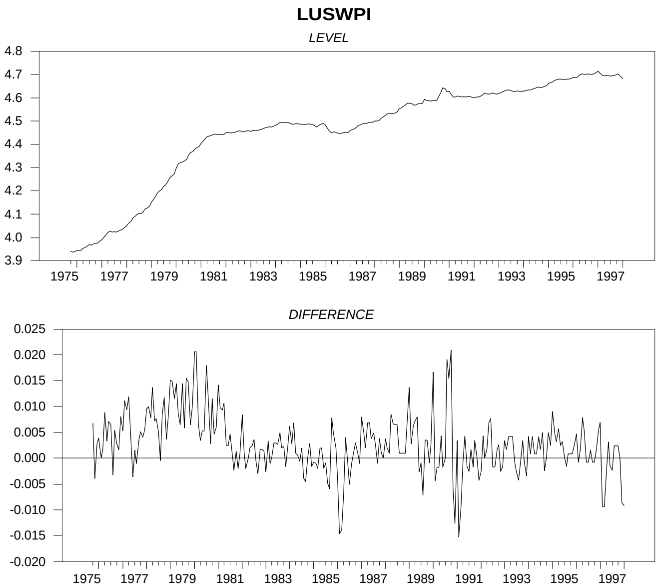

Fig.2: The log of WPI index in the US.

We also noticed that the wholesale price index is much more volatile than the consumer price index shown in the next two figures for Germany and US (see Fig. 3, LGECPI time series, we call LGECPI, the log of the consumer price index in Germany; in Fig. 4 see LUSCPI where we call LUSCPI the log of the US consumer price index).

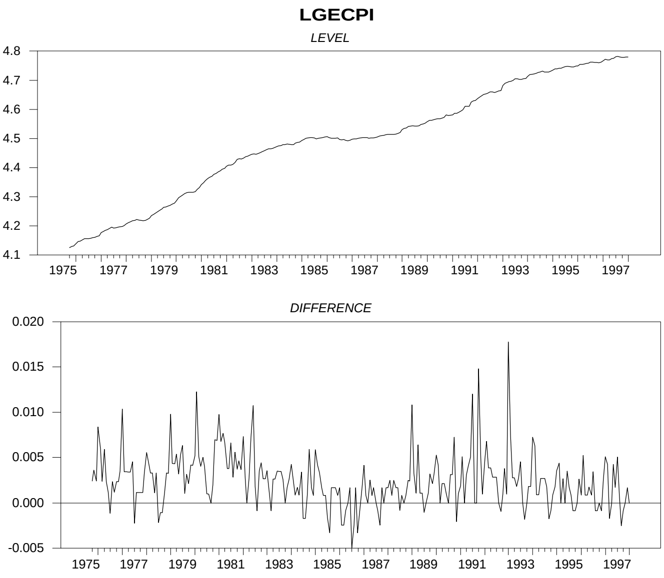

Fig.3: The log CPI index in Germany.

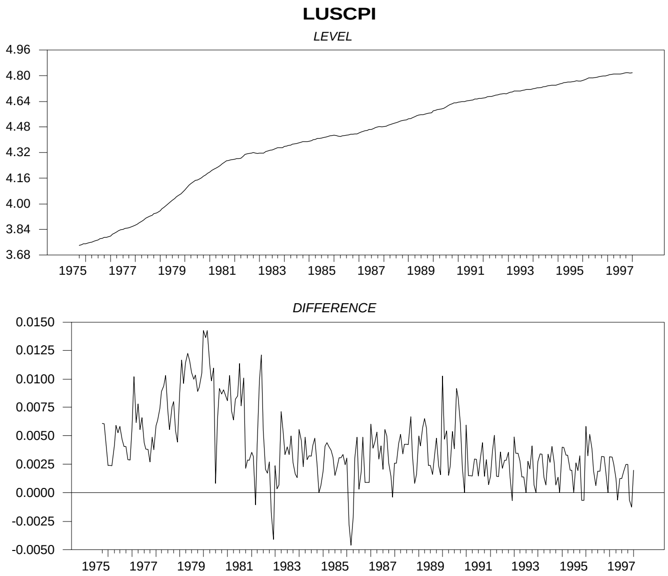

Fig. 4: The log of CPI index in the US.

The common feature in Fig. 1-4 is that prices show a decidedly positive trend or drift throughout the whole period, while their first differences i.e. inflation rates, have a positive mean. These are obvious results. But from these simple graphs we may find other interesting characteristics:

- prices seem to be highly autocorrelated and inflation too. Moreover it seems also that both mean and variance vary over time! Both graphs, either of prices or inflations, seem decidedly different from the one obtained from a white noise I(0) process distributed as a uniform distribution with zero mean and unit variance (see Fig. 5).

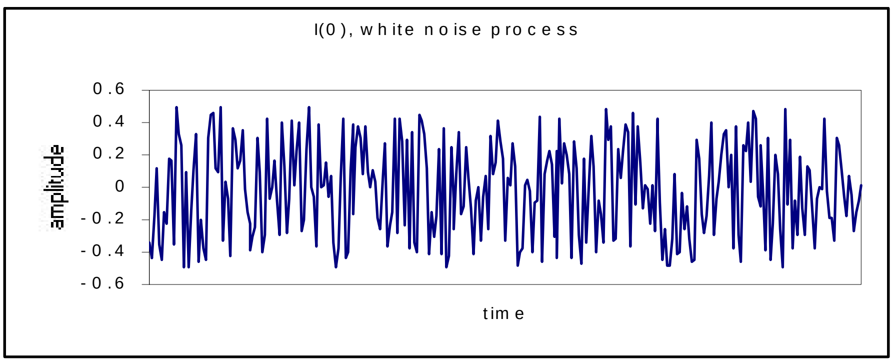

Fig 5: Graph of a white noise process.

From these first graphs we might have some idea about the type of process of both prices and inflation. Inflation seem far to be a pure I(1) process as the one in Fig. 6,

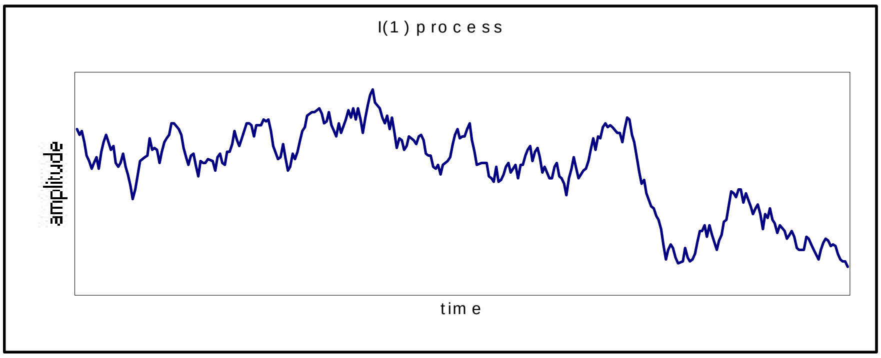

Fig. 6: Example of a I(1) process generated by white noise.

but still is quite different from the I(0) process in Fig. 5. Is inflation a I(0) or a I(1) process? The time series reported in Fig. 7 show something interesting: the time series is artificial and is composed by 75% by a I(0) component (the one in Fig. 5) and by 25% by a I(1) (the one in Fig. 6), the autocorrelation seems present and both mean and variance seem to vary with time! This is a similar pattern of inflation rates. Therefore we have the suspect that inflation rates contain some I(1) structure hidden by noise.

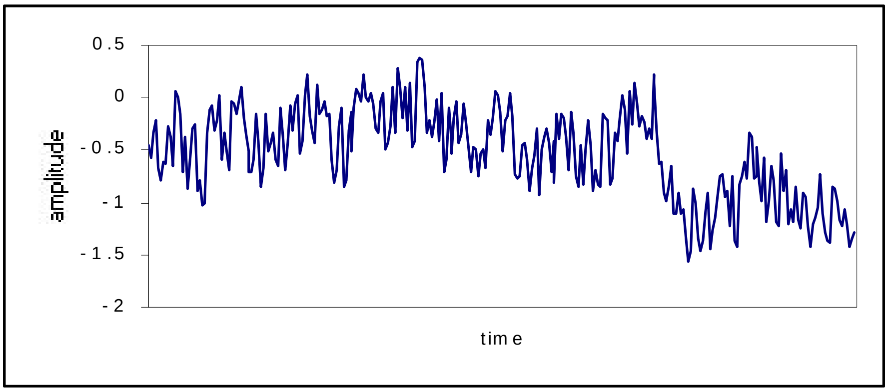

Fig. 7: Mixed process composed by 75% by a I(0) and 25% by a I(1) process.

If inflation rates contain a I(1) component, thus prices contain a I(2) component. Fig. 8 shows an artificial process generated by the summation of a I(1) (by 75%) with a I(2) process (by 25%). We observe a smooth trending behaviour similar[6] to the one observed in prices, either CPI or WPI. Concluding, we can state that prices might contain a I(2) component that should be taken into account using a truly I(2) procedure, or alternatively inflation rates should be analysed in a I(1) framework.

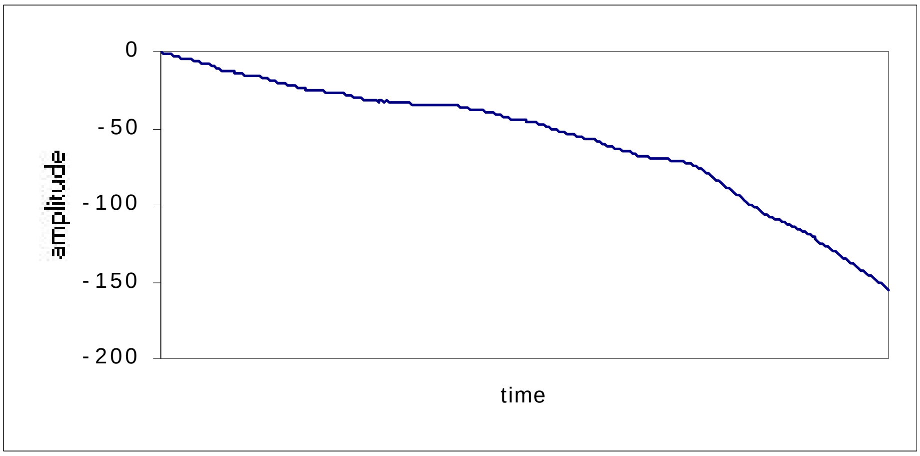

Fig. 8: Mixed process composed by 75% by a I(1) and by 25% by a I(2) process.

If we wish to compare the relation between WPI and CPI in Germany (see Fig. 9, LGEPWC time series, we call LGEPWC the spread between the log of the wholesales and consumer price indexes in Germany) and in the US (see Fig. 10, LUSPWC time series, we call LUSPWC the spread between the log of the wholesales and consumer price indexes in the US) we observe a similar pattern to Fig. 8, characterised by a smooth trending behaviour.

[6]Except for the case that here the trend is negative. Mixed processes as well as pure I(2) processes might also generate growing time series or changes in trends but the peculiar feature of these processes is that the time series is rather smooth with a strong autocorrelation.

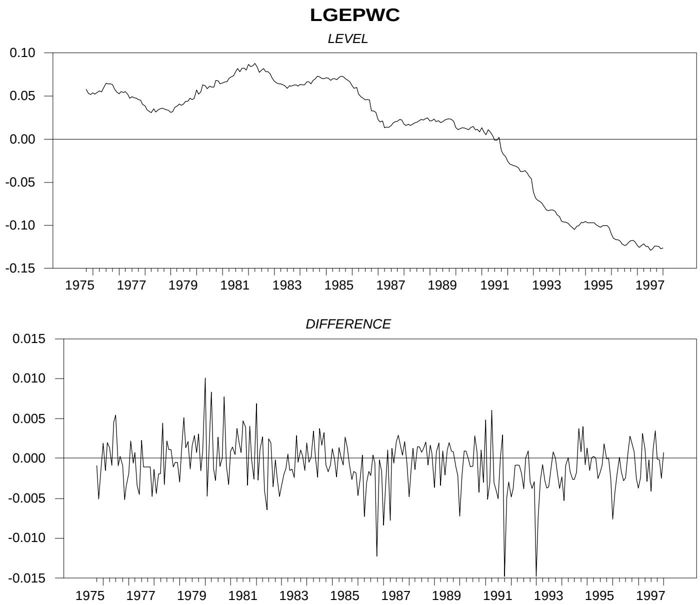

Fig. 9: The spread between price indexes in Germany.

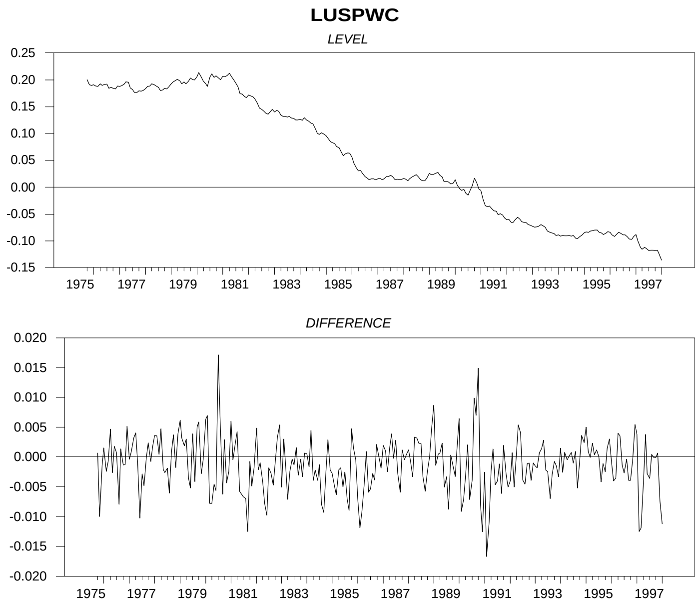

Fig. 10: The spread of price indexes in the US.

Moreover, if we look at the relation $(p_t - p_t^*)$ (see Fig. 11, GEUSPR time series, we call GEUSPR the spread of the log of the wholesales price index between Germany and US), we observe a trending behaviour similar to Fig. 8, characterised by a strong autocorrelation and a rather smooth pattern, typical of processes with components higher than I(1). We suspect that $p_t \sim I(2)$, $p_t^* \sim I(2)$ and $(p_t - p_t^*) \sim I(2)$, that is, prices alone do not cointegrate. Therefore, we conclude that $(\Delta p_t - \Delta p_t^*) \sim I(1)$ (see Fig. 11, lower panel).

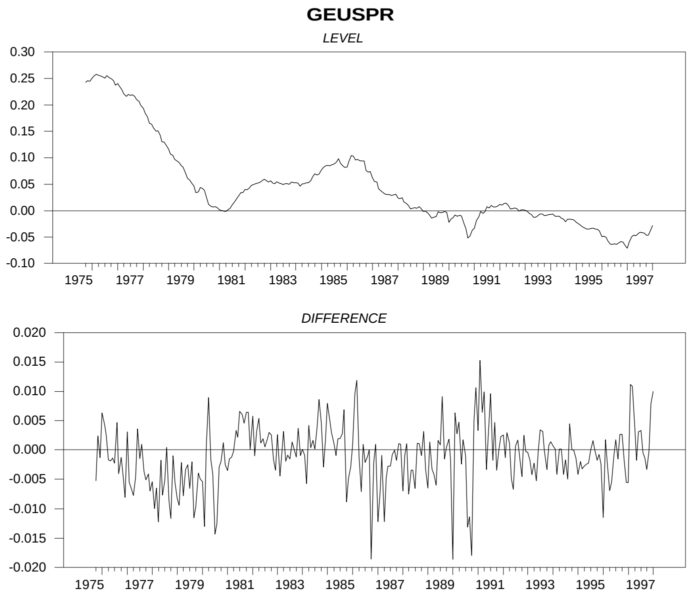

Fig. 11: The price spread between Germany and US.

3.2.2 Exchange rates and *ppp*

We have noticed that prices clearly contain structures higher than $I(1)$. Also exchange rates seem contain $I(2)$ components. Its behaviour is rather smooth, with prolonged periods of appreciation and periods of depreciation, with a trend tendency consistent with a $I(2)$ hypothesis, even if the $I(2)$ component is not as clear as in prices. However if we closely look at Fig. 12 (see LDMUSD time series, we call LDMUSD the log of exchange rate of the German Mark against the US Dollar) and at Fig. 11, we might notice that the exchange rate and the spread of prices in the long run follow a similar trend. The sharp rise of exchange rates occurred between 1980 and 1985 could be explained to different factors, such as the increase of US fiscal deficit together with a speculative bubble of world-wide dimension.

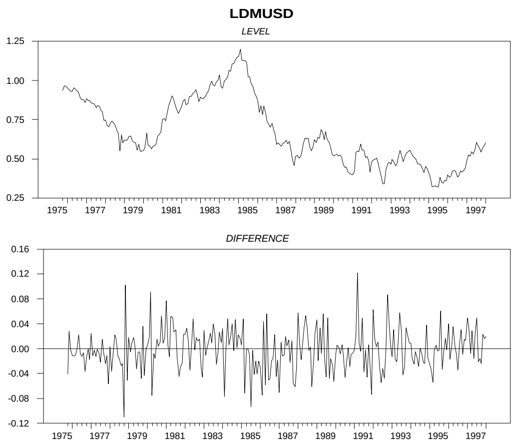

Fig 12: The log of exchange rate

In the case that spread prices (that is most probably a $I(2)$ process) share the same trend of exchange rate (that is probably a $I(2)$ process too), we might find that they cointegrate from $I(2)$ to $I(1)$, i.e. they are $CI(2,1)$. From Fig. 13 (see the PPPWGE time series, we call PPPWGE the *ppp* calculated with the wholesale prices) where we do not notice a typical trending behaviour of $I(2)$ processes, we might think that *ppp* behaves like a $I(1)$ process. As Enders (1995) pointed out referring to the *ppp*, the series seems to meander in a fashion characteristic of a random walk process, i.e. *ppp* is a $I(1)$ process.

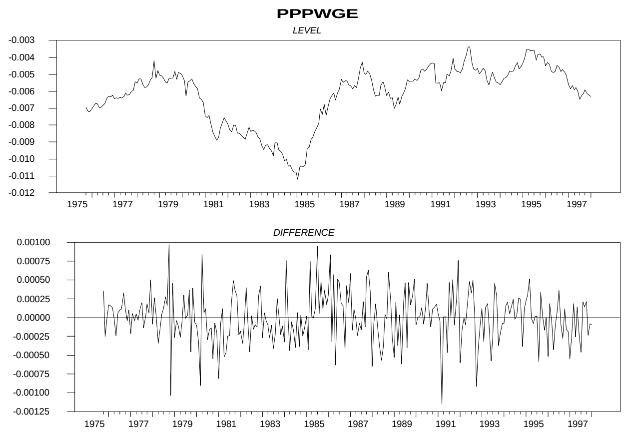

Fig 13: Purchasing Power Parity.

3.2.3 The interest rates and their spread

Let us see first the spread of interest rates. As noticed by Juselius and MacDonald, the spread between long bond interest rates follow a dynamics that is somewhat similar to the one of *ppp* (compare Fig. 14 with Fig. 13; see the

BONDSP time series, we call BONDSP the spread of the long term interest rates in the two countries). From the graph the bond spread could seem a $I(1)$ process affected by some heteroskedasticity (see lower panel).

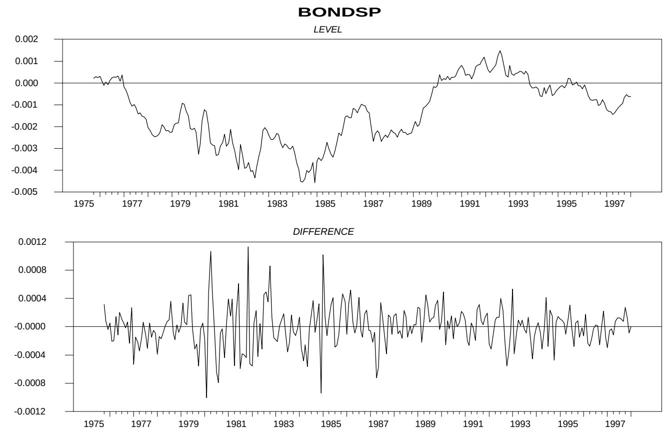

Fig. 14: The bond rate spread.

If we look at the Treasury Bill rates we observe a strong heteroskedasticity (see lower panel Fig. 15; we called BILLSP the time series of the spread between Treasury Bill rates in the two countries), and a quite irregular pattern, with no long run trending behaviour like $I(2)$ processes. Thus the short term interest rate spread might be a $I(1)$ process.

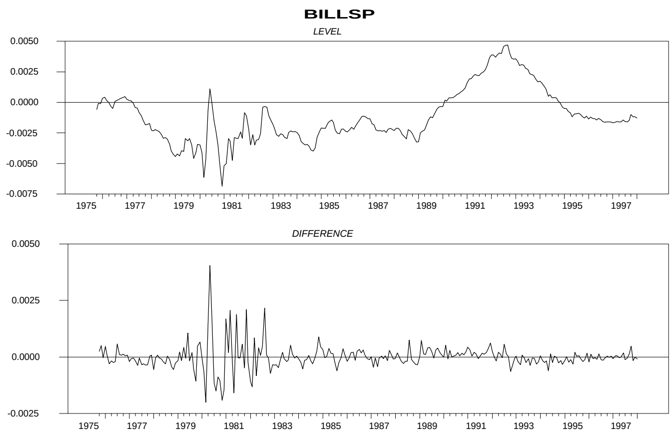

Fig. 15: The Treasury Bill rate spread.

Now, if the spread of interest rates are $I(1)$ they could be the result of the fact that the interest rates in the two countries are $I(1)$ and they do not cointegrate, or they are $I(2)$ and cointegrate.

Fig. 16 and 17 suggest that both time series are affected by ARCH structures but they do not show the typical smooth and prolonged trending behaviour of

$I(2)$ time series. Similar consideration may apply to the time series of treasury bill rates (Fig. 18 and 19), so we might think that interest rates are all I(1) processes with strong heteroskedasticity and they do not cointegrate by themselves.

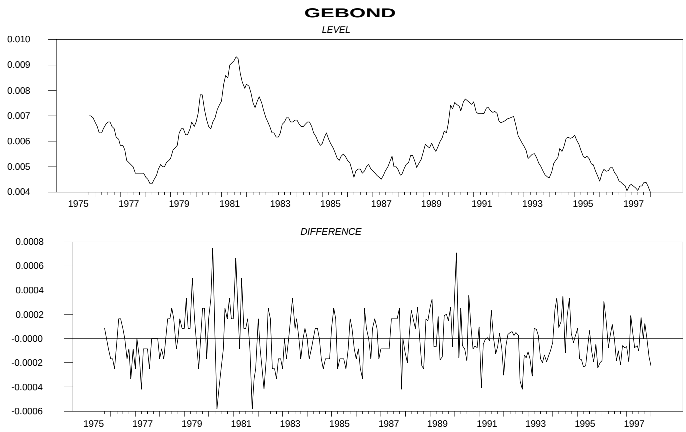

Fig. 16: The long term interest rate in Germany.

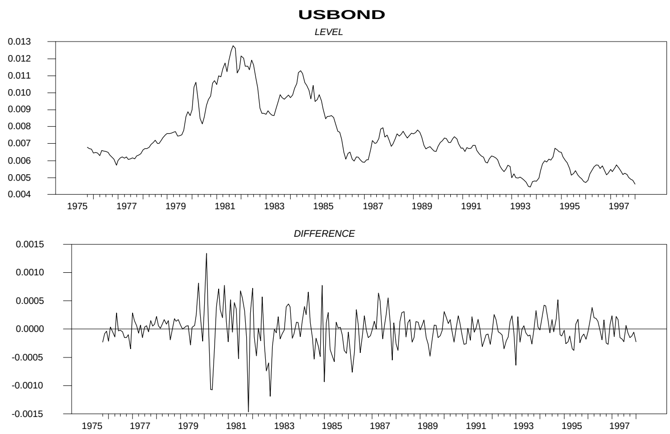

Fig. 17: the long term interest rate in the US.

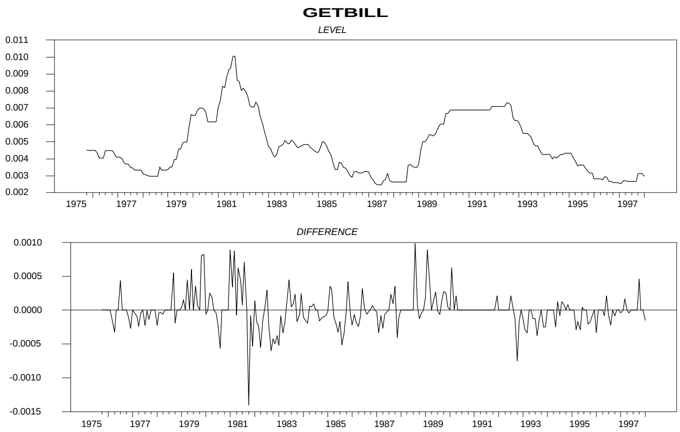

Fig. 18: the Treasury Bill rate in Germany.

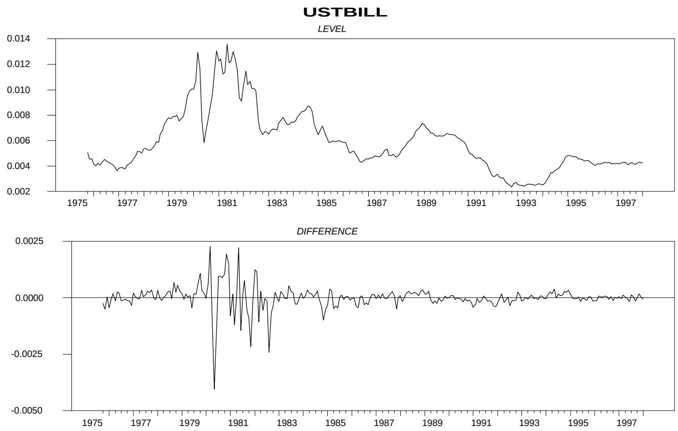

Fig 19: The US Treasury Bill rate.

3.2.4 The degree of integration of the analyzed data

Summarizing, from a simple visual inspection of the data, it appears that:

- $p_t \sim I(2)$, $p_t^* \sim I(2)$, $(p_t - p_t^*) \sim I(2)$ and $s_t \sim I(2)$
- $\Delta p_t \sim I(1)$, $\Delta p_t^* \sim I(1)$, $(\Delta p_t - \Delta p_t^*) \sim I(1)$
- $\Delta s_t \sim I(1)$
- $ppp_t \sim I(1)$
- $i^l \sim I(1)$, $i^{l^*} \sim I(1)$, $(i^l - i^{l^*})$, $I(1)$, and $i^s \sim I(1)$, $i^{s^*} \sim I(1)$, $(i^s - i^{s^*}) \sim I(1)$.

These results suggest that some variables, such as prices, are $I(2)$ and others, like inflation rates or interest rates, are $I(1)$. Luckily enough, all the economic variables $(\Delta p_t, \Delta p_t^*, i_t^l, i_t^{l*}, i_t^s, i_t^{s*}, ppp_t)$ that enter in our fundamental relation (9) should be $I(1)$ variables. Thus to test relation (9), the $I(1)$ procedure, the so called "Johansen procedure", might be sufficient[7].

4 The I(1) model[8]

The I(1) model can be formulated in two equivalent forms: the vector autoregressive model VAR and the vector moving average representation VMA. While the VAR model enables us to single out the long run relations in the data, the VMA representation is useful for the analysis of the common trends that have generated the data (Juselius 1995).

4.1 The VAR representation and the long run relations

The VAR model formulated in the correction error form is:

$$\Delta x_t = \Gamma_1 \Delta x_{t-1} + ... + \Gamma_{k-1} \Delta x_{t-k+1} + \Pi x_{t-1} + \mu + \Psi D_t + \varepsilon_t \tag{10}$$

$$\varepsilon_t \sim N_p (0, \Sigma), \; t = 1, ..., T$$

where $p = 5$ (or 7 for the extended model that includes short run interest rates) is the dimension of the VAR model, $x'_t = \left[\Delta p_t, \Delta p_t^*, i_t^l, i_t^{l*}, ppp_t\right]$ (or $x'_t = \left[\Delta p_t, \Delta p_t^*, i_t^l, i_t^{l*}, i_t^s, i_t^{s*}, ppp_t\right]$), $x'_t \sim I(1)$, k is the lag length, D_t deterministic components such as centred seasonal and intervention dummies, μ trends, Γ_1,..., Γ_{k-1}, Ψ freely varying parameters and:

$$\Pi = \alpha\beta'$$

where α and β are $p \times r$ matrices of full rank, r is the rank of the Π matrix, and $\beta' x_t$ is stationary, i.e. the stationary relations among nonstationary variables such as relation (9). The rank of the Π matrix is fundamental since it is equal to the number of stationary relations between the levels of the variables, i.e. the number of long run steady states towards which the process starts adjusting when it has been pushed away from the equilibrium (Hansen and Juselius 2000).

4.2 The VMA representation

The VMA representation is used to analyse the common trends that have generated the data, i.e. the pushing forces from equilibrium that create the non stationary property in the data.

The VMA representation is the following:

[7] The $I(1)$ procedure can be applied only to the variables that are "at most" $I(1)$. This means that not all the individual variables x_t have to be $I(1)$. They can be also $I(0)$, but not more than $I(1)$. This was the reason why it was necessary to build a model with variables that were integrated not more than $I(1)$.

[8] See Johansen (1995), Hansen and Johansen (1998) for the mathematical statistical analysis of the I(1) model and cointegration.

$$x_t = C\sum_{i=1}^{t}\varepsilon_i + C\sum_{i=1}^{t}\Psi D_i + C\mu t + C^*(L)(\varepsilon_t + \Psi D_t + \mu) \tag{11}$$

where

$$C = \beta_\perp\left(\alpha'_\perp\left(I - \sum_{1}^{k-1}\Gamma_i\right)\beta_\perp\right)^{-1}\alpha'_\perp$$

$\alpha_\perp$ and $\beta_\perp$ are $(p-r)\times(p-r)$ matrices orthogonal to α and β, while the C matrix is of reduced rank of order $(p-r)$.

The component $C\sum_{i=1}^{t}\varepsilon_i$ is really important since it represents stochastic trends of the process[9]. But how many stochastic trends are in the process? We can guess it by means of economic considerations, but we can also measure it with the rank of the C matrix. The rank is equal to the number of stochastic trends that push economic variables away from steady states. The VMA representation is of unavailable help since it shows how common trends affect all the variables of the system (see section 5.6 and 6.4).

4.3 "General to specific" and "specific to general" approach

We adopt a "general to specific" principle in statistical modelling and a "specific to general" approach in the choice of variables. By imposing restrictions on the VAR such as reduced rank restrictions, zero parameter restrictions and other parameter restrictions, the idea is to arrive to a parsimonious model with economically interpretable coefficients (Juselius and MacDonald 2000).

In the system represented by relation (10) the vector x_t is composed by five or seven variables. It had rather better to begin to analyse small models since for each added variable we have $(p+1)*k$ new parameters in the system. If the lag length is $k = 3$ (as in our case), and we have a system of five variables and we add two variables we have $(5+1)*3+(6+1)*3 = 39$ parameters more that need to be estimated. Of course when the sample is small (less than 100 observations for instance, like quarterly macroeconomic models) it is often impossible to estimate the model because the number of parameters to estimate is greater than the number of observations. In our case we have about 270 observations so we might estimate directly also system with seven variables. However is not advantageous estimate it directly. In fact, reducing at minimum the number of variables often helps in identify the cointegration relations and the identified cointegration relations remain valid in a more extended model. This property is

[9] While $C\sum_{i=1}^{t}\Psi D_i + C\mu t$ are non stationary deterministic components, $C^*(L)(\varepsilon_t)$ and $C^*(L)(\Psi D_t + \mu)$ stationary stochastic and deterministic components of the process x_t.

called invariance of the cointegration relations in extended sets. If cointegration is found within a small set of variables, the same cointegration relations are valid within any larger set of variables. The gradual expansion of the information set facilitates an analysis of the sensitivity of the results associated with the "ceteris paribus" assumption. This strategy is known as "specific to general" approach in the choice of variables (see Hendry and Juselius 2000, Juselius and MacDonald 2000). Thus we first analyse the small model ($x_t' = \left[\Delta p_t, \Delta p_t^*, i_t^l, i_t^{l*}, ppp_t\right]$) excluding short term interest rates before analysing the extended model with all the seven variables ($x_t' = \left[\Delta p_t, \Delta p_t^*, i_t^l, i_t^{l*}, i_t^s, i_t^{s*}, ppp_t\right]$).

4.4 The deterministic components

Since the asymptotic distribution of the test for cointegration depends on the assumptions made on the deterministic components, namely dummies and constant term, its choice may be crucial for inference. Without going into the details about the issues relating to the deterministic components in the cointegrated model, we need to make a sensible choice of the deterministic components in our $I(1)$ model.

4.4.1 Trends

We have first to decide whether there are trends in the data. Excluded the case of quadratic trends (since none of the variables seem to show quadratic growth), we have to decide whether there are linear trends and estimate the VAR model with an unrestricted constant that allows for trends in the variables and a linear trend restricted to the cointegration space. After we determine the cointegration rank we can test whether the trends in the cointegration rank can be set to zero with the "test for the long run exclusion".

Our tests actually showed that the trend should not to be set to zero, the p-value was in fact zero (The LR test, CHISQ = 37.98 , p-value = 0.00), so the probability that the trend is zero is zero. However, if we leave the model unrestricted the value that the trend would assume is very close to zero. Moreover what is the rationale for a trend in our set of variables? None! In fact none of the variables can follow a trend for ever. The trend detected was very small and could be typical of the period and not be justified for a longer period. There is no reason that is economically justified to expect trends in $\Delta p_t, \Delta p_t^*, i_t^l, i_t^{l*}, ppp_t$. For this reason we drop the hypothesis that there are trends both in the data and in the cointegration relations. Similarly we drop the hypothesis that there are trends in data and no trends in the cointegration relation. No drift is economically reasonable for our set of variables.

Concluding, we decided to set no trends in the data and estimate the VAR with a constant restricted to the cointegration space[10]. The only deterministic

[10] We have also run the same tests also considering trends in the data and both in the data and in the cointegration space. We noticed that there were not noticeable changes in the results probably because the trends would have been very small.

components, except the dummies allowed in our model in the data, were the intercepts in the cointegration relations.

4.4.2 Dummies

The likelihood-based inference methods on cointegration are derived upon the gaussian likelihood but the asymptotic properties of the methods depend on the *i.i.d.* assumption of the errors (Johansen 1995 p. 29). Thus the fact that the residuals are not distributed normally is not so important. Generally if we reject the normality hypothesis (which is the null hypothesis of a test for normality) we should check the skewness and the kurtosis to see whether the residuals are well-behaved. If we would not include any dummy we would get highly bad-behaved residuals especially for which regards skewness, and all the inference would result heavily distorted. To secure valid statistical inference we need to take into account for shocks that fall outside the normality confidence level. We set a dummy variable whenever the residual was larger than $|3.5\sigma_\varepsilon|$.

5 The "small" model

We needed the following dummy variables for the small model:

$$D\prime_t = [Di8111, D8610, D9008, D9102, D9103, D9601]$$

where $Dixx.yy$ is a $..., 0, 1, -1, 0, ...$ dummy measuring a transitory intervention shock in $19xx.yy$ and $Dxx.yy$ is a $..., 0, 1, 0, ...$ dummy measuring a permanent intervention shock. No shift dummy was needed and not included. We tested whether the these dummies were significant, and hence necessary and we found that all of them were significant for at least one of the variables (see Tab. 1):

$Tab.1 \quad t-values \quad of\ dummies$

	$DI8111$	$D8610$	$D9008$	$D9102$	$D9103$	$D9601$
Δp_t	-1.105	-6.047	2.625	-1.172	-0.963	-4.453
Δp_t^*	-0.816	-0.385	4.961	-4.467	-0.675	-0.550
i_t^l	-0.901	0.625	3.166	-2.895	1.706	-0.808
i_t^{l*}	-5.264	-0.671	0.909	-1.570	2.042	-0.009
ppp_t	1.493	-1.566	0.388	-0.095	-3.830	-0.921

5.1 Lag length and misspecification tests

Probably the most important requirement for unbiased results is that estimated residuals show no serial correlation. If serial correlation is found adding one lag may be sufficient to remove it. Changing the number of lags may require a change in the dummies.

Two lags and a different set of dummies were not sufficient to remove first order autocorrelation. The dummies above were based on a VAR model with three lags.

To provide an overall picture of the adequacy of the model we report some univariate and multivariate misspefication tests in Table 2. A significant test statistic is given in bold font (the $\chi^2(3)$, at 5% significance level has a critical value of 7.82).

Table 2: Misspecification tests

Multivariate tests					
Residual autocorr. $LM(1)$	$\chi^2\,(25)$	$=$	25.7	$p-val.$	0.42
Residual autocorr. $LM(4)$	$\chi^2\,(25)$	$=$	19.7	$p-val.$	0.76
Normality	$\chi^2\,(10)$	$=$	92.5	$p-val.$	0.00
Univariate tests	$\Delta^2 p_t$	$\Delta^2 p_t^*$	Δi_t^l	$\Delta i_t^{l^*}$	Δppp_t
ARCH(3)	2.05	**17.96**	5.25	**16.46**	6.73
JB(3)	**26.79**	**8.94**	5.53	**25.53**	5.90
Skewness	0.08	0.09	0.32	0.09	0.00
Ex. Kurtosis	1.71	0.85	0.44	1.74	0.65
$\hat{\sigma}_\varepsilon \times 0.01$	0.20	0.39	0.01	0.02	0.03
R^2	0.69	0.50	0.37	0.30	0.22

Looking at Table 2 it seems that there are not any problems with autocorrelations of first and fourth order since $LM(1)$ and $LM(4)$ test statistics suggest that the null hypothesis for zero autocorrelation cannot be rejected. Normality is rejected as often happens, but the rejection was mainly due to an excess of kurtosis rather than skewness. This is rather important because the properties of the cointegration estimators are more sensitive to deviation from normality due to skewness. The Jarque-Bera test statistics (distributed like a $\chi^2(3)$) suggests that the rejection from normality was mainly due to excess of kurtosis. The $ARCH(3)$ (also distributed like a $\chi^2(3)$) statistic shows that there is significant heteroskedasticity in the residual of inflation in the US and in the US bond rates. However cointegration estimates are not very sensitive to $ARCH$ structures, so we are not forced to use to a VAR model that takes into account also $ARCH$ effects. The R^2 measures the improvement in the explanatory power of the model compared to a random walk hypothesis. The model is able to explain more about changes in inflation rates than changes in interest rates and, consequently, in the purchasing parity.

To support that the model is quite well specified Fig. 20-24 are provided. Fig. 20-24 give four plots for each endogenous variable: the actual and the fitted values, the standardized residuals, a histogram of the standardized residuals with the histogram of the standardized Normal distribution as background and the correlograms for lag 1 to $T/4$. Fig. 20-24 show that the standarized residuals are quite well behaved thanks to the good selection of dummies and lags.

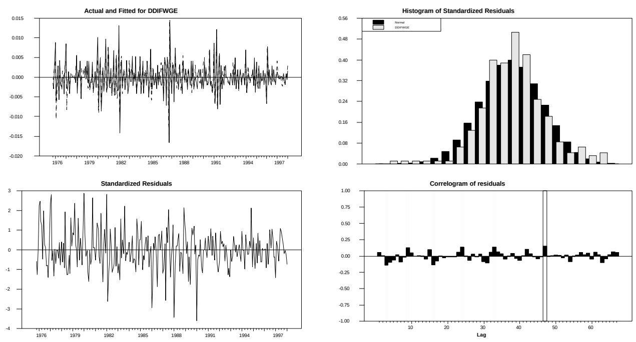

Fig. 20: estimated residuals in the German inflation.

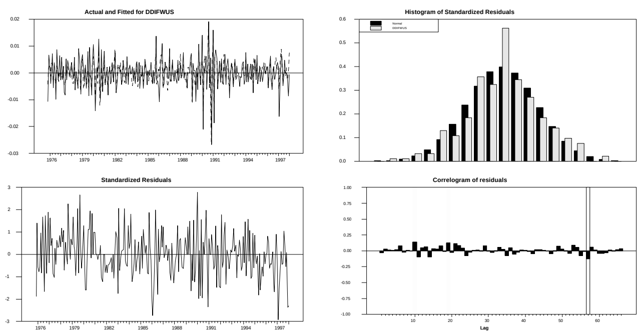

Fig. 21: estimated residuals in the US inflation.

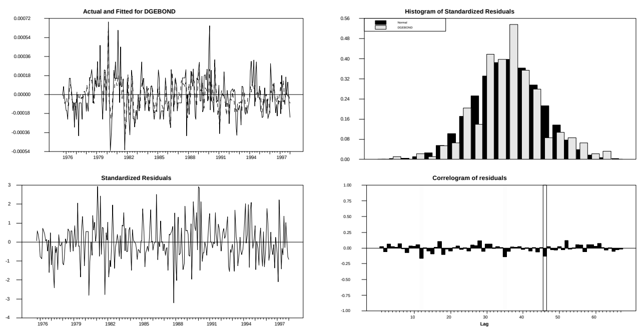

Fig. 22: estimated residuals in the German bond rate.

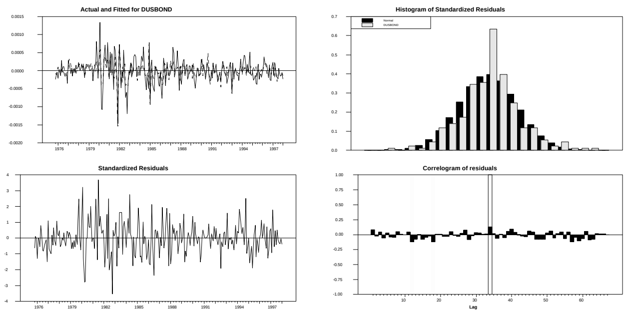

Fig. 23: estimated residuals in the US bond rate.

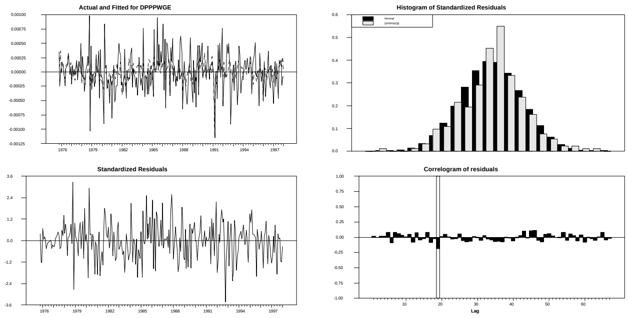

Fig. 24: estimated residuals in the PPP.

5.2 Determination of the cointegration rank

The Eigenvalues of the Π matrix are reported in Table 3. We notice that three eigenvalues are quite close to zero. How many of them are significantly different from zero? This question is fundamental since the rank of the Π matrix is equal to p less the number of zero eigenvalues.

If we could set three eigenvalues to zero, it would mean that the rank is equal to 5-3=2, i.e. there would be two linearly independent stationary relations.

To discriminate zero eigenvalues from non-zero eigenvalues, i.e. to calculate the cointegration rank, we can use the $Trace$ test and the $Lambda\ Max$ test. Table 3 shows that the null hypothesis of the $Trace$ test, r $=< 2$ against $r > 2$ cannot be rejected at 10% significance level, while the null hypothesis of the $Lambda\ Max$ test $r = 2$ against $r = 3$ is rejected by little.

Because the asymptotic distributions of these statistics can be rather bad approximations to the true small sample distributions we calculate in table 4 the five largest roots of the companion matrix of Π to help us in the choice of the cointegration rank. Either in case the model is unrestricted, or the rank of Π is set to 2 or 3, there are 3 roots that are equal or very close to one. Since the number of roots of the companion matrix of Π is complementary to the rank of the Π, since $p = 5$, $r = 2$ and $p - r$ are roots of the companion matrix set to one, $r = 2$ is our choice[11].

Table 3: Eigenvalues of the Π matrix and rank tests

Eigenvalues of the Π matrix	0.24	0.16	0.05	0.02	0.01
r	0	1	2	3	4
Trace test	**144.2**	**70.2**	21.1	6.1	1.8
Trace 90	71.7	49.9	31.9	17.8	7.5
Lambda Max test	**73.4**	**49.2**	**14.9**	4.3	1.8
Lambda Max 90	21.7	18.0	14.1	10.3	7.5

Table 4: the eigenvalues of the companion matrix

Modulus of 5 largest roots					
Unrestricted model	0.97	0.97	0.96	0.80	0.66
$r = 3$	1.00	1.00	0.93	0.81	0.66
$r = 2$	1.00	1.00	1.00	0.81	0.66

5.2.1 α, β and Π

Once fixed the cointegration rank to r=2, and normalized the first eigenvector by Δp_t and the second by Δp_t^* we obtained the estimated α, β and Π with their respective t-values (Tables 5, 6 and 7).

Table 5: beta transposed

	Δp_t	Δp_t^*	i_t^l	i_t^{l*}	ppp_t	*constant*
β_1'	1.000	−0.759	−1.100	0.627	0.845	0.008
β_2'	4.694	1.000	−2.114	−2.601	−2.077	0.006

Based on the estimated α coefficients we note that:

1) the first relation is significantly adjusting in the US inflation rate equation an to some extent to the German inflation and the US interest rate.

2) the second relation is significantly adjusting in the German inflation rate equation and to some extent to the US inflation rate

We note that the rows correspondent to Δi_t^l and Δppp_t in table 6 are not significant. This implies that the equations for Δi_t^l and Δppp_t do not contain information about the long run parameters β, i.e. i_t^l and ppp_t are weakly exogenous. we also notice that the t-value for Δi_t^{l*} is rather borderline. In the next subsection we provide a formal test of weak exogeneity.

[11] Notice that this happens only in the case when the $I(1)$ condition is satisfied, i.e. there are no $I(2)$ components in the model. In the case that for any reasonable choice of r, there remains one or more roots close to one (at least greater than 0.85), it would be a sign of the presence of $I(2)$ components violating our assumptions.

For example if we set $r = 2$ and we found 4 large roots with $p = 5$, this would have been not compatible with a $I(1)$ process.

In our case $r = 2$ and there are 3 unit roots, meaning that $x_t \sim I(1)$.

The fact that $x_t \sim I(1)$ was also confirmed by the analysis the following very small systems: $[\Delta p_t, \Delta p_t^*]$, $[i_t, i_t^*]$, $[ppp_t, i_t]$

where the rank of the Π matrix of each system was found equal to zero and the roots of the companion matrix were found equal to two.

On the contrary the system $[p_t, p_t^*]$ showed a rank $r = 2$ and 2 unit roots being inconsistent with a $I(1)$ process. These results supported and completed the visual analysis in section 3.2.

Table 6: ALPHA, T-VALUES FOR ALPHA

	$\hat{\alpha}_1$	$\hat{\alpha}_2$
$\Delta^2 p_t$	**-0.113** -2.081	**-0.071** -6.192
$\Delta^2 p_t^*$	**0.714** 7.036	**-0.062** -2.899
Δi_t^l	0.005 1.249	0.001 1.368
Δi_t^{l*}	**0.016** 2.275	0.002 1.397
Δppp_t	0.007 0.916	-0.002 -1.458

In the Π matrix, the rows give the estimates of the combined effect of the two cointegration relation. The inflation rates are both equilibrium error correcting, while the German interest rate and the ppp_t are not. Again the t-values for i_t^{l*} are borderline.

Table 7: Π matrix and t-values

	Δp_t	Δp_t^*	i_t^l	i_t^{l*}	ppp_t	*constant*
$\Delta^2 p_t$	**-0.448** -5.845	0.014 0.336	**0.275** 4.265	**0.115** 2.532	0.053 1.018	**-0.001** -3.078
$\Delta^2 p_t^*$	**0.421** 2.937	**-0.605** -7.557	**-0.654** -5.420	**0.610** 7.196	**0.734** 7.579	**0.005** 6.458
Δi_t^l	0.000 0.080	0.005 1.572	0.003 0.641	-0.006 -1.842	-0.007 -1.740	-0.000 -1.006
Δi_t^{l*}	-0.006 -0.626	**0.014** 2.567	0.013 1.579	**-0.015** -2.631	**-0.018** -2.663	-0.000 -2.012
Δppp_t	-0.018 -1.678	0.003 0.489	0.013 1.399	0.002 0.277	-0.001 -0.138	-0.000 -1.145

5.3 Long run exclusion, long run weak exogeneity, stationary tests[12]

Long run exclusion, long run weak exogeneity, stationary tests provide useful information about the choice of the variables and the properties of their time series.

The long run exclusion test investigates whether any of the variables can be excluded from the cointegration space, implying no relationship with the other variables. It is formulated as a zero row in β and the null hypothesis is that the variable does not enter in the cointegration space. In table 8 we notice a borderline value for the long bond interest rate in the US, but we preferred to keep i_t^{l*} in the cointegration space also because i_t^{l*} turns out useful for meaningful results.

The long run weak exogeneity test investigates whether one variable influence the others without being affected. It is formulated as a zero row in α and the null hypothesis is that the variable is weakly exogeous. If the null hypothesis is accepted, the variable pushes the system without being pushed. We notice that i_t^l and ppp_t turned out to be weakly exogenous and i_t^{l*} assumes again a borderline value. Considering i_t^{l*} weakly exogenous is consistent with the choice of the rank $r = 2$.

[12] See Juselius and Hansen (2000) for details and empirical examples.

The last test is the test for stationarity. It investigates whether one variable can be assumed stationary. Accepting the hypothesis implies that the variable is considered $I(0)$. The inclusion of $I(0)$ in a system to be analyzed with the I(1) procedure is legitimate, keeping into accout that for any stationary variable the rank increases by one[13]. In our system none of the variables seem stationary.

Table 8: Tests of hypothesis about some properties of x_t

	Δp_t	Δp_t^*	i_t^l	i_t^{l*}	ppp_t	$constant$	$\chi^2(\nu)$
Long run exclusion	44.59	57.25	8.65	**5.15**	9.61	13.73	$\chi^2(2)$=5.99
Long run weak exogeneity	28.46	47.91	**2.85**	6.63	**2.38**		$\chi^2(2)$=5.99
Stationarity	28.40	28.78	47.83	48.26	47.38		$\chi^2(4)$=9.49

Lastly we tested the hypothesis that i_t^l, $i_t^{l^*}$ and ppp_t were jointly weakly exogenous. The $p-value$ was 0.12, suggesting that the null hypothesis could not be rejected. The p-value increased up to 0.28 when i_t^l and ppp_t were jointly tested. Concluding, we can consider i_t^l, $i_t^{l^*}$ and ppp_t weakly exogenous variables, i.e. the driving forces of the system.

These results are quite similar to the ones by Juselius and MacDonald: inflation rates seem driven by interest rates and ppp and not viceversa.

5.4 Single Cointegration hypothesis

In table 8 it was shown that no variable in the vector x_t is stationary by itself. Looking for cointegration relations means to search for stationary linear combinations of the variables x_t. Single Cointegration tests test whether a restricted relation can be accepted leaving the other relation unrestricted. If the hypothetical relations exists empirically, this procedure maximizes the chance to find them (Juselius and MacDonald 2000).

$\mathcal{H}_1$ to $\mathcal{H}_4$ are hypothesis on pairs of variables, such as relative inflation, relative interest rates, and stationary real interest rates. We notice that $\mathcal{H}_3$, the stationary real interest for Germany, is accepted since the $p-value$ is 0.83.

Thus:

$$\Delta p_t - i_t^l + 0.004 \sim I(0)$$

$\mathcal{H}_5$ and $\mathcal{H}_6$ are tests of variants of real interest parity in which full proportionality has not been imposed. Restricting the two inflation rates to have unit coefficients and the nominal interest rates to have opposite signs ($\mathcal{H}_5$) is rejected. $\mathcal{H}_6$ that relates German interest rates with the US interest rate is accepted with a $p-value$ is 0.65.

$\mathcal{H}_7$ and $\mathcal{H}_8$ are similar to $\mathcal{H}_1$ and $\mathcal{H}_2$ leaving ppp free to vary. Both $\mathcal{H}_7$ and $\mathcal{H}_8$ are rejected.

[13]For example if we had two cointegration relation and one stationary variable, the rank of the Π matrix would be equal to three.

$\mathcal{H}_9$ and $\mathcal{H}_{10}$ are stationary real interest rates for Germany and US with ppp free to vary. Both $\mathcal{H}_9$ and $\mathcal{H}_{10}$ are accepted with rather high $p - values$ (0.66 and 0.55).

$\mathcal{H}_{11}$ simply combines $\mathcal{H}_9$ and $\mathcal{H}_{10}$ and still is strongly accepted with a $p - value$ equal to 0.48.

$\mathcal{H}_{12}$ describes an homogeneous relationship between German inflation, US inflation and German bond inflation. This relation is similar to $\mathcal{H}_3$. We notice that including the US inflation to the German real interest rate do not make results more robust.

$\mathcal{H}_{13}$ is similar to $\mathcal{H}_{12}$ but it is referred to the US. Including German inflation to the US real interest rate we cannot accept the null hypothesis.

Testing $\mathcal{H}_{14}$ is the equivalent of testing our fundamental relation in relation (9). It is accepted with a $p - value$ equal to 0.33.

Relation (9) was $(i_t^l - i_t^{l*}) - \omega_1(\Delta p_t - \Delta p_t^*) - \omega_2 ppp_t$. $\mathcal{H}_{14}$ is accepted meaning that relation (9) is empirically valid with $\omega_1 = 0.985$ and $\omega_2 = 1.273$.

Thus:

$$(i_t^l - i_t^{l*}) - 0.985(\Delta p_t - \Delta p_t^*) - 1.273 ppp_t - 0.008 \sim I(0)$$

In the next subsection we will test jointly $\mathcal{H}_{14}$ with $\mathcal{H}_3$ where $\mathcal{H}_3$ represents the stationary real interest rate in Germany.

We noticed that ω_1 and ω_2 are both values close to 1.

We therefore tested in $\mathcal{H}_{15}$ restricting ω_1 to 1. $\mathcal{H}_{15}$ was accepted with a $p - value$ equal to 0.63!

We therefore tested in $\mathcal{H}_{16}$ restricting their value to 1. $\mathcal{H}_{16}$ was accepted with a $p - value$ equal to 0.43. $\mathcal{H}_{16}$ is our preferred cointegration relation since it is perfectly economically interpretable with relation (7) where agents are assumed perfectly rational!

Thus:

$$(i_t^l - i_t^{l*}) - (\Delta p_t - \Delta p_t^*) - ppp_t - 0.008 \sim I(0)$$

Table 8: Cointegration relations

	Δp_t	Δp_t^*	i_t^l	i_t^{l*}	ppp_t	$constant$	$\chi^2(\nu)$	$p-val$
$\mathcal{H}_1$	1	-1	0	0	0	0.001	13.96	0.00
$\mathcal{H}_2$	0	0	1	-1	0	0.002	44.41	0.00
$\mathcal{H}_3$	1	0	-1	0	0	0.004	0.86	**0.83**
$\mathcal{H}_4$	0	1	0	-1	0	0.004	16.02	0.00
$\mathcal{H}_5$	1	-1	0.217	-0.217	0	0.001	13.60	0.00
$\mathcal{H}_6$	1	0.014	-1	-0.014	0	0.004	0.86	**0.65**
$\mathcal{H}_7$	1	-1	0	0	0.639	0.005	7.02	0.03
$\mathcal{H}_8$	0	0	-1	1	0.736	0.003	33.82	0.00
$\mathcal{H}_9$	1	0	-1	0	0.023	0.004	0.84	**0.66**
$\mathcal{H}_{10}$	0	1	0	-1	-1.201	-0.004	1.21	**0.55**
$\mathcal{H}_{11}$	1	-0.439	-1	0.439	0.548	0.005	0.49	**0.48**
$\mathcal{H}_{12}$	1	-0.084	-0.916	0	0	0.003	0.63	**0.73**
$\mathcal{H}_{13}$	-1.989	1	0	0.989	0	-0.006	10.67	0.00
$\mathcal{H}_{14}$	-0.985	0.985	1	-1	-1.273	-0.008	0.93	**0.33**
$\mathcal{H}_{15}$	-1	1	1	-1	-1.283	-0.008	0.93	**0.63**
$\mathcal{H}_{16}$	-1	1	1	-1	-1	-0.006	2.74	**0.43**

5.5 Fully specified cointegrating relations

We are now ready to perform a joint test of $\mathcal{H}_{14}$ (equivalent to relation (9)) with $\mathcal{H}_3$ (equivalent to stationary German real interest rate). The test statistic $\chi^2(4)$ was found equal to 2.07 with a $p-value$ of 0.72. The first vector has been normalized on the German inflation rate and the second on the German interest rate. The first vector is given by:

$$\Delta p_t - i_t^l + 0.004 \tag{12}$$

while the second representing relation (9) is:

$$(i_t^l - i_t^{l*}) - 0.985(\Delta p_t - \Delta p_t^*) - 1.273 ppp_t - 0.008 \tag{13}$$

This is the estimated fundamental relation of our paper. It combines the *ppp* and the *uip* in one relation that is strongly supported by data by a $p-value$ of 0.72.

Notice that here $\omega_1 = 0.985$ and $\omega_2 = 1.273$, while in case expectations were made fully rationally $\omega_1 = 1$ and $\omega_2 = 1$.

This evidence shows that agents behave quite close to the theoretical rational case represented by relation (7)! Therefore it was natural to jointly test $\mathcal{H}_3$ with $\mathcal{H}_{15}$ where $\omega_1$was restricted to 1. The $p-value$ increased up to 0.84!

The first vector is given by:

$$\Delta p_t - i_t^l + 0.004 \tag{14}$$

while the second vector is:

$$(i_t^l - i_t^{l*}) - (\Delta p_t - \Delta p_t^*) - 1.278ppp_t - 0.008 \tag{15}$$

Restricting also $\omega_2 = 1$, i.e. combining $\mathcal{H}_3$ with $\mathcal{H}_{16}$, the $p-value$ was still 0.69, a quite acceptable value if we consider it is perfectly consistent with the particular assumption of perfect rationality.

The first vector is given by:

$$\Delta p_t - i_t^l + 0.004 \tag{16}$$

while the second vector that represents relation (7) is:

$$(i_t^l - i_t^{l*}) - (\Delta p_t - \Delta p_t^*) - ppp_t - 0.006 \tag{17}$$

In table 9, a structural representation of the cointegration space containing all the information in (16) and (17) is finally given. The adjustment coefficients are also reported. What is noticeable is that none of the adjustment parameters referring to interest rates and *ppp* are significant, suggesting that interest rates and *ppp* are not adjusting to the two steady state relations as we would expect from weakly exogenous variables.

Table 9: A structural representation of the cointegration space

	$\hat{\beta}_1$	$\hat{\beta}_2$		$\hat{\alpha}_1$	$\hat{\alpha}_2$
Δp_t	1	-1	$\Delta^2 p_t$	$\mathbf{-0.399}$ $_{-6.0}$	0.021 $_{0.5}$
Δp_t^*	0	1	$\Delta^2 p_t^*$	-0.194 $_{-1.6}$	$\mathbf{-0.564}$ $_{-7.1}$
i_t^l	-1	1	Δi_t^l	0.008 $_{1.6}$	0.006 $_{1.9}$
i_t^{l*}	0	-1	Δi_t^{l*}	0.010 $_{1.2}$	0.015 $_{2.8}$
ppp_t^1	0	-1	Δppp_t	-0.018 $_{-1.9}$	0.002 $_{0.4}$
constant	0.004	-0.006			

[1] The ppp term has been divided by 100

5.6 Common trends

We report the VMA (common trends) representation for two different cases: (i) based on the unrestricted VAR model for $r = 2$, (ii) based on (i) but after having fully specified cointegrating relations with weak exogeneity of i_t^l, i_t^{l*} and ppp_t imposed on α.

The estimates of the C matrix in table 10 measure the total impact of permanent shocks to each of the variables on all other variables. A row of the C matrix gives an indication of which variables have been particularly important for the stochastic trend behaviour of the variable in the row.

Table 10: The estimates of the long run impact matrix C

C	$\sum \hat{\varepsilon}_{\Delta p_t}$	$\sum \hat{\varepsilon}_{\Delta p_t^*}$	$\sum \hat{\varepsilon}_{i_t^l}$	$\sum \hat{\varepsilon}_{i_t^{l*}}$	$\sum \hat{\varepsilon}_{ppp_t}$	$\sum \hat{\varepsilon}_{i_t^l}$	$\sum \hat{\varepsilon}_{i_t^{l*}}$	$\sum \hat{\varepsilon}_{ppp_t}$
Δp_t	0.015 0.771	**0.017** 2.180	**1.232** 6.878	**0.287** 2.943	0.016 0.209	**1.41** 7.54	**0.27** 2.62	−0.08 −0.97
Δp_t^*	−0.051 −1.344	**0.030** 2.016	0.398 1.144	**0.894** 4.723	**1.467** 9.669	0.42 1.57	**1.01** 6.84	**1.02** 8.98
i_t^l	0.021 1.003	0.016 1.947	**1.300** 6.941	**0.237** 2.322	−0.103 −1.264	**1.41** 7.54	**0.27** 2.62	−0.08 −0.97
i_t^{l*}	0.006 0.235	**0.024** 2.520	−0.052 −0.235	**1.137** 9.440	0.051 0.532	0.02 0.07	**1.24** 9.70	0.08 0.81
ppp_t	−0.038 −1.445	0.003 0.287	**0.556** 2.317	**-0.329** −2.524	**0.982** 9.391	0.41	**-0.23** −2.02	**0.94** 10.78

Based on both the unrestricted (left side of table 10) and restricted (right side of table 10) VAR model we note that cumulative shocks to inflation rates in Germany have no significant long run impact on any other variable. Estimated cumulative shocks to the US inflation rate assume boundary $t - values$ in the unrestricted VAR model, while cumulative shocks to long term interest rates and to *ppp* are highly significant.

The findings from the restricted VMA representation suggest that (see relation 18, a zero was set for not significant coefficients):

$$\begin{bmatrix} \Delta p_t \\ \Delta p_t^* \\ i_t^l \\ i_t^{l^*} \\ ppp_t \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & 0 \\ 0 & c_{22} & c_{23} \\ c_{11} & c_{12} & 0 \\ 0 & c_{42} & 0 \\ 0 & c_{52} & c_{53} \end{bmatrix} \begin{bmatrix} \sum \varepsilon_{i_t^l} \\ \sum \varepsilon_{i_t^{l*}} \\ \sum \varepsilon_{ppp_t} \end{bmatrix} + \begin{array}{c} \text{stationary and} \\ \text{deterministic} \\ \text{components} \end{array} \tag{18}$$

- Inflation rates are adjusting.

- German inflation rate and the long bond interest rate share the same stochastic trend.

- Shocks to the German long term interest rate speed up the German inflation and to some extent changes the *ppp* (via exchange rates as theory suggests).

- Shocks to the US long term interest rate speed up the German and US inflation, pushes the German long term interest rate implying that changes in US capital markets spread towards Europe, and has a negative effect on the *ppp* (via exchange rates as theory suggests).

- Shocks to *ppp* coming from exchange rates determine positive changes to US inflation.

6 The "extended model"

The "extended model" needed many more dummies because of the many interventions in the Treasury Bill rates that are closely linked to the monetary policy. We needed the following dummy variables for the extended model:

$D\prime_t = [D7912, Di8003, D8005, Di8006, D8011, Di8012, D8101, D8103, D8105, D8110, D8111, Di8201, D8208, D8411, D8501, D8604, D8610, D8808, D8902, D9001, D9011, D9102, D9103, D9210, D9601]$.

We tested whether these dummies were significant, and hence necessary and we found that all of them were highly significant for at least one of the variables (not shown here).

6.1 Lag length and misspecification tests

Three lags and a different set of dummies were not sufficient to remove first order autocorrelation. The dummies above were based on a VAR model with four lags.

To provide an overall picture of the adequacy of the model we report some univariate and multivariate misspefication tests in Table 11. A significant test statistic is given in bold font (the $\chi^2(4)$, at 5% significance level has a critical value of 9.48).

Table 11: Misspecification tests

MULTIVARIATE TESTS							
Residual autocorr. $LM(1)$	$\chi^2\,(49)$	=	52.7	$p-val.$	=	0.33	
Residual autocorr. $LM(4)$	$\chi^2\,(49)$	=	35.81	$p-val.$	=	0.92	
Normality	$\chi^2\,(14)$	=	144.2	$p-val.$	=	0.00	
UNIVARIATE TESTS	$\Delta^2 p_t$	$\Delta^2 p_t^*$	Δi_t^l	$\Delta i_t^{l^*}$	Δi_t^s	$\Delta i_t^{s^*}$	Δppp_t
ARCH(4)	2.7	7.2	7.54	6.98	1.01	**12.67**	4.93
JB(4)	**14.15**	0.5	3.34	**12.73**	8.17	**46.00**	**11.12**
Skewness	0.18	0.01	0.24	0.11	0.36	0.17	0.03
Ex. Kurtosis	1.16	0.11	0.28	1.06	0.74	2.49	0.65
$\hat{\sigma}_\varepsilon \times 0.01$	0.18	0.34	0.01	0.02	0.01	0.02	0.03
R^2	0.77	0.61	0.53	0.55	0.68	0.85	0.22

Looking at Table 11 it seems that there are not any problems with autocorrelations of first and fourth order since $LM(1)$ and $LM(4)$ test statistics suggest that the null hypothesis for zero autocorrelation cannot be rejected. Normality is rejected, but the rejection was mainly due to an excess of kurtosis rather than skewness. The Jarque-Bera test statistics (distributed like a $\chi^2(4)$) suggests that the rejection from normality was mainly due to excess of kurtosis (especially in the US short term interest rate). The $ARCH(4)$ (also distributed like a $\chi^2(4)$) statistic shows that there is significant heteroskedasticity in the US treasury bill rates. Comparing table 11 with table 2 in section 5, we notice that the large model which includes one more lag and several more dummies have better properties with regards to heteroskedasticity. In this case, including two new variables, it seems that ARCH structures become less relevant.

The R^2 measures the improvement in the explanatory power of the model compared to a random walk hypothesis. The larger model increased its explanation power, but this could be also effect of the many new dummies we have included in the extended model.

To support that the model is very well specified Fig. 25-31 are provided. Fig. 25-31 show that the standardised residuals are well behaved thanks to a proper choice of dummies and lags.

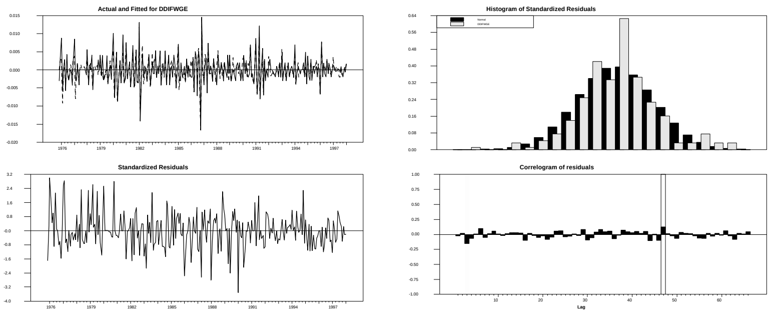

Fig. 25: The estimated residuals of German inflation.

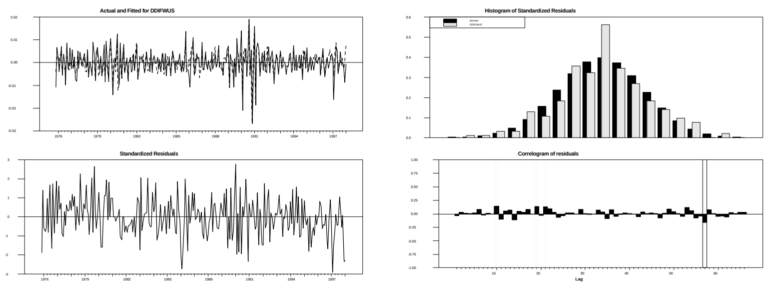

Fig. 26: The estimated residuals of US inflation.

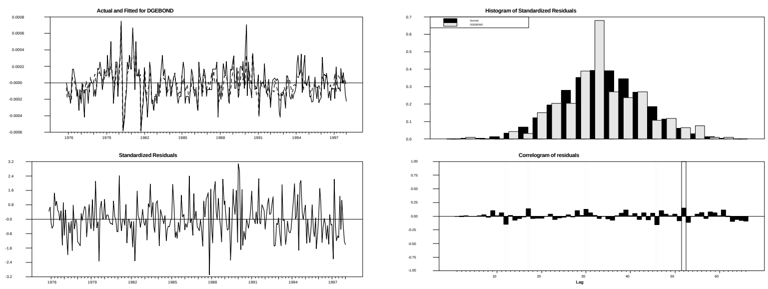

Fig. 27: The estimated residuals of the German bond rate.

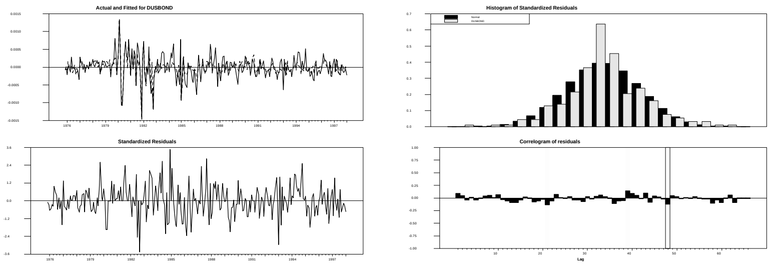

Fig. 28: The estimated residuals of the US bond rate.

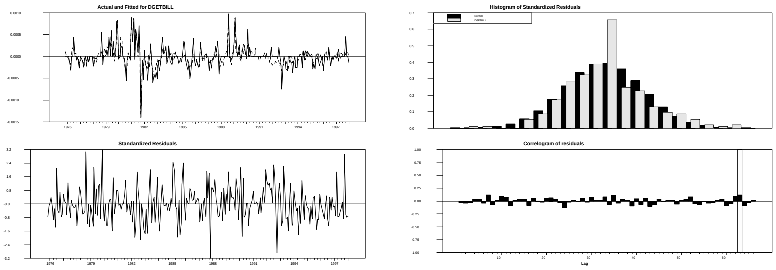

Fig. 29: The estimated residuals of the German treasury bill rate.

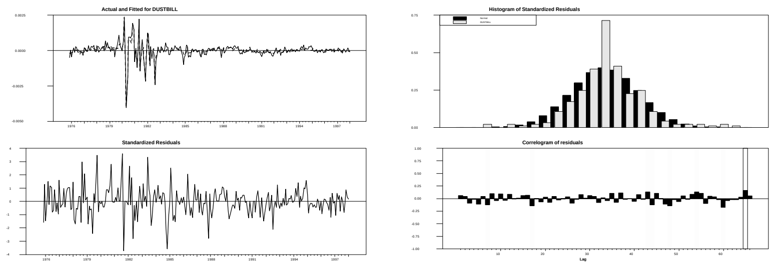

Fig. 30: The estimated residuals of the US treasury bill rates.

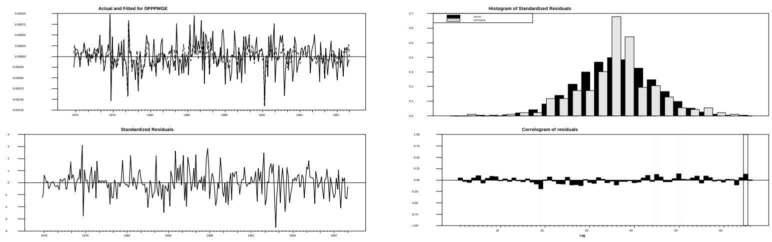

Fig. 31: The estimated residuals of the PPP.

6.2 Determination of the cointegration rank

The Eigenvalues of the Π matrix are reported in Table 12. We notice that at least three eigenvalues are quite close to zero.

Table 12 shows that the null hypothesis of the *Trace* test, r $= < 3$ against $r > 3$ cannot be rejected at 10% significance level, while the null hypothesis of the *Lambda Max* test $r = 3$ against $r = 4$ is rejected by little.

If $r = 2$, adding the two treasury bill rates, the rank is unchanged and the stochastic trends have increased to $p - r = 5$, implying that the two short term interest rates are not cointegrated with themselves nor with inflation rates, or *ppp* term.

If $r = 3$, $p - r = 4$; including short term interest rates have introduced one additional stochastic trend. This means that the short term interest rates can be jointly cointegrated or cointegrated by with the remaining variables of the system.

If $r = 4$, $p - r = 3$; the short term interest rates would be fully integrated with the long term interest rates, inflation rates and the real exchange rates.

Table 12: Eigenvalues of the Π matrix and rank tests (extended model)

Eigenvalues of the Π matrix	0.27	0.18	0.11	0.10	0.04	0.02	0.00
r	0	1	2	3	4	5	6
Trace test	**215.7**	**130.0**	**75.9**	45.7	17.6	5.8	0.6
Trace 90	126.7	97.2	71.7	49.9	31.9	17.8	7.5
Lambda Max test	**85.7**	**54.07**	**30.26**	**28.1**	11.7	5.2	0.6
Lambda Max 90	29.5	25.5	21.7	18.3	14.1	2.1	7.5

Keeping the restrictions of the cointegrating vectors in the small model and calculating the rank with $r = 3$ we obtain a fifth unit root equal to 0.85.

Table 13: the eigenvalues of the companion matrix

Modulus of 5 largest roots					
$r = 3$	1.00	1.00	1.00	1.00	0.85

It is a rather high remaining root. We suspected that short term interest rate could hide some $I(2)$ component. We therefore analysed a very small model consisting of only the two short term interest rates. The results are interesting. The rank of this small system should be between 0 and 1 (see table 14). But what is more interesting is that the roots of the companion matrix are slightly higher than one. This might indicate some slight explosive nature caused by nonlinearity or I(2) structure, but with two independent I(2) processes we normally find more than two unit roots in the companion matrix (see note in section 5.2). The fifth rather high root might be due to the structure (high kurtosis, nonlinear structure like ARCH) of short term interest bonds. Thus, our final choice for the extended model is $r = 3$.

Table 14: Eigenvalues of the Π matrix and rank tests for short term interest rates

Eigenvalues of the Π matrix	0.05	0.2
r	0	1
Trace test	17.7	5.2
Trace 90	17.8	7.5
Lambda Max test	12.5	5.2
Lambda Max 90	10.3	7.5
Modulus of 2 largest roots		
r unrestricted	1.0047	1.0047

6.3 Structural hypothesis test

An advantage of the principle of the "specific to general" approach is that we can keep the two cointrating relations found in the previous section unaltered. Hence the additional impact of the two new variables, the short term interest rates, should be described by a third cointegrating relation.

To obtain information about the new cointegrating relation we first estimate the partially restricted long run structure keeping two cointegration relation unchanged ($\mathcal{H}_3$ and $\mathcal{H}_{16}$) but leaving unrestricted the third one ($\mathcal{H}_{17}$). The hypothesis was accepted with a $p-value$ of 0.19, and the third cointegrating relation suggested that it could contain information about the spread between long and short interest rates in the two countries (table 15).

Table 15: The third unrestricted cointegrating relation

	Δp_t	Δp_t^*	i_t^l	i_t^{l*}	i_t^s	$i_t^{s^*}$	ppp_t	$constant$	$\chi^2(\nu)$	$p-val$
$\mathcal{H}_{17}$	0.617	-0.626	0.617	-0.894	-0.614	1.000	0.268	0.002	11.2	0.19

This led to test the following restricted hypothesis $\mathcal{H}_{18}$ that was accepted with a $p-value$ of 0.32:

Table 16: The third restricted cointegrating relation

	Δp_t	Δp_t^*	i_t^l	i_t^{l*}	i_t^s	$i_t^{s^*}$	ppp_t	$constant$	$\chi^2(\nu)$	$p-val$
$\mathcal{H}_{18}$	0.317	-0.317	0.317	-1	-0.317	1	0	0.001	12.6	0.32

The third vector can be written as:

$$(\Delta p_t - \Delta p_t^*) + \left(i_t^l - i_t^s\right) - 3.15\left(i_t^{l*} - i_t^{s^*}\right) \sim I(0) \qquad (19)$$

that relates the spread between prices with the spread of between interest rates.

In table 17 a structural representation of the cointegration space is finally given. The adjustment coefficients are also reported. It is noticeable that none of the adjustment parameters referring to *ppp* are significant, suggesting that *ppp* is weakly exogenous variables. Some of the adjustment parameters are significant for the interest rates, but their absolute values are very close to zero.

Table 17: A structural representation of the cointegration space (extended model)

	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$		$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$
Δp_t	1	-1	0.317	$\Delta^2 p_t$	**-0.53**	**0.20**	**0.50**
					-7.7	3.2	3.1
Δp_t^*	0	1	-0.317	$\Delta^2 p_t^*$	-0.20	**-0.33**	0.32
					-1.5	-2.8	1.1
i_t^l	-1	1	0.317	Δi_t^l	0.00	**0.01**	**0.03**
					1.1	2.3	2.8
i_t^{l*}	0	-1	-1	Δi_t^{l*}	0.00	**0.02**	**0.05**
					1.2	3.8	2.8
i_t^s	0	0	-0.317	i_t^s	0.00	0.01	**0.05**
					0.9	0.7	4.1
i_t^{s*}	0	0	1	i_t^{s*}	**0.02**	0.00	0.01
					3.2	1.3	-0.5
ppp_t[1]	0	-1	0	Δppp_t	-0.01	0.00	0.00
					-1.5	0.2	-0.2
constant	0.004	-0.005	0.001				

[1] The ppp term has been divided by 100

6.4 Common trends

As was shown in sections 5.3 and 5.6, there is a close relationship between long run weak exogeneity and common trends. In section 5.6 it was shown that the weakly exogenous variables were the ones that generated the common trends that affected all the variables in the system.

The long run weak exogeneity test is formulated as a zero row in α and the null hypothesis is that the variable is weakly exogenous. If the null hypothesis is accepted, the variable pushes the system. From table 17 we have some idea about which variables are not weakly exogenous, but it is more difficult to choose the one between interest rates that has to be excluded to be a common trend. In fact we set $r = 3$, so we expect that $p - r = 7 - 3 = 4$ common trends.

We tested which variable was weakly exogenous setting a zero row in α for each variables (table 18). Table 18 shows that the short term interest rate in the US is very unlikely weakly exogenous.

Table 18: tests for weak exogeneity

	i_t^l	i_t^{l*}	i_t^s	$i_t^{s^*}$	ppp_t
$\chi^2(3)$	4.68	7.63	1.36	15.35	3.68
$p - value$	0.18	0.05	0.71	0.00	0.30

Lastly, in table 19 we report the VMA (common trends) representation based on the fully specified cointegrating relations with weak exogeneity of i_t^l, i_t^{l*}, i_t^s and ppp_t imposed on α.

The estimates of the C matrix in table 10 measure the total impact of permanent shocks to each of the variables on all other variables. A row of the C matrix gives an indication of which variables have been particularly important for the stochastic trend behaviour of the variable in the row.

Table 19: The estimates of the long run impact matrix C

C	$\sum \hat{\varepsilon}_{i_t^l}$	$\sum \hat{\varepsilon}_{i_t^{l*}}$	$\sum \hat{\varepsilon}_{i_t^s}$	$\sum \hat{\varepsilon}_{ppp_t}$
Δp_t	**1.37** 3.82	0.39 0.90	0.08 0.23	−0.03 −0.18
Δp_t^*	0.24 0.48	1.12 1.82	0.25 0.49	**0.97** 3.71
i_t^l	**1.37** 3.82	0.39 0.90	0.08 0.23	−0.03 −0.18
i_t^{l*}	0.14 −0.35	**1.20** 2.75	0.17 0.43	0.08 0.41
i_t^s	0.65 1.54	0.44 0.86	**1.28** 3.03	−0.14 −0.65
i_t^{s*}	**-1.16** −2.28	**1.63** 2.63	0.93 1.82	0.42 1.58
ppp_t	0.38 0.87	−0.08 −0.15	0.08 0.19	**0.94** 10.78

$$\begin{bmatrix} \Delta p_t \\ \Delta p_t^* \\ i_t^l \\ i_t^{l*} \\ i_t^s \\ i_t^{s*} \\ ppp_t \end{bmatrix} = \begin{bmatrix} c_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{24} \\ c_{11} & 0 & 0 & 0 \\ 0 & c_{42} & 0 & 0 \\ 0 & 0 & c_{53} & 0 \\ c_{61} & c_{62} & 0 & 0 \\ 0 & 0 & 0 & c_{74} \end{bmatrix} \begin{bmatrix} \sum \varepsilon_{i_t^l} \\ \sum \varepsilon_{i_t^{l*}} \\ \sum \varepsilon_{i_t^s} \\ \sum \varepsilon_{ppp_t} \end{bmatrix} + \begin{array}{c} \text{stationary and} \\ \text{deterministic} \\ \text{components} \end{array} \quad (20)$$

The findings from the restricted VMA representation suggest that (see relation 20):

- German inflation rate and the long bond interest rate share the same stochastic trend. Real long term interest rates are constant in Germany.

- Shocks to the German long term interest rate speed up the German inflation and to some extent changes the *ppp* (via exchange rates as theory suggests).

- Shocks to the US long term interest pushes the US short interest rate as if the FED adjusts responds to the capital markets rather than anticipating them.

- Shocks to *ppp* coming from exchange rates have significant effects on the US inflation proably because the US are not only a big exporter but also a big importer.

6.5 The role of short-term interest rate

To gain a further perspective on the role of the short relative to the long term interest rate we report in Table 8 a comparative analysis of the combined effect measured by $\hat{\Pi}_r = \hat{\alpha}_r \hat{\beta}'$, where the subscript r stands for the restricted estimates as reported in table 9 and 17. It seems that short term interest rates were significantly important for the inflation rates in Germany, but not for the US. However if we have a close look table 19, it seems that short term interest rate do not have permanent effects on prices. Conversely, US inflation adjusts (table 20) and are affected in the long (table 19) run by shocks in ppp_t. All the other variables, either because the $t - values$ are too small or the absolute value of the impact is very close to zero, seem not to be strongly affected by any other variables (table 19) but in the long run these small effects have a tendency to cumulate (table 20).

Table 20: The combined long run effect $\hat{\Pi}_r = \hat{\alpha}_r \hat{\beta}'_r$

	Δp_t	Δp_t^*	i_t^l	i_t^{l*}	ppp_t		
$\Delta^2 p_t$	**−0.42**	−0.02	**0.42**	−0.02	−0.02		
	−5.7	0.5	5.7	−0.5	−0.5		
$\Delta^2 p_t^*$	**0.37**	**−0.56**	**−0.37**	**0.56**	**0.56**		
	2.7	−7.2	−2.7	7.2	7.2		
Δi_t^l	0.00	0.00	-0.00	−0.00	-0.00		
	0.4	1.9	−0.4	−1.9	−1.9		
Δi_t^{l*}	−0.00	**0.01**	0.00	**−0.01**	**−0.01**		
	−0.6	2.8	0.6	−2.8	−2.8		
ppp_t	−0.02	0.00	0.02	−0.00	−0.00		
	−1.9	0.3	1.9	−0.4	−0.4		
	Δp_t	Δp_t^*	i_t^l	i_t^{l*}	ppp_t	i_t^s	i_t^{s*}
$\Delta^2 p_t$	**−0.57**	0.04	**0.88**	**−0.69**	**−0.20**	**−0.16**	**0.49**
	−7.4	0.8	6.5	−3.3	−3.2	−3.1	3.1
$\Delta^2 p_t^*$	0.23	**−0.43**	−0.03	0.00	**0.32**	−0.10	0.32
	1.6	−5.3	−0.1	0.0	2.8	−1.0	1.0
Δi_t^l	0.01	−0.00	0.01	**−0.05**	**−0.01**	**−0.01**	**0.03**
	1.0	−0.0	1.5	−2.9	−2.3	−2.8	2.8
Δi_t^{l*}	−0.01	**0.01**	**0.04**	**−0.08**	**−0.03**	**−0.02**	**0.05**
	−1.0	2.1	2.7	−3.3	−3.8	−2.8	2.8
ppp_t	−0.02	0.04	0.02	0.00	0.00	0.00	−0.01
	−1.7	0.5	0.8	0.1	−0.2	0.2	−0.2
Δi_t^s	0.01	**0.00**	−0.02	−0.06	−0.01	0.00	−0.01
	1.5	2.5	−1.3	0.0	−1.3	0.5	−0.5
Δi_t^{s*}	−0.01	0.01	0.02	0.00	−0.0	−0.00	−0.01
	−1.7	0.6	0.8	0.1	−0.2	0.2	−0.2

7 Conclusions

Two building blocks of international monetary economics are the *ppp* and *uip* conditions. They are normally assumed stationary, i.e. $I(0)$. Recently it was discovered that *ppp* and *uip* are not stationary at all, but they do behave like most of the economic time series. Basically they move like random walk, that is they are $I(1)$. This fact has been represented just an enigma for many economists.

Juselius (1995) and Juselius and MacDonald (2000), exploiting the $I(1)$ property of the *ppp* and *uip*, put forward the idea that *ppp* and *uip* were linked together producing a stationary relationship. Just because *ppp* and *uip* were $I(1)$, they could produce a stationary relationship like $uip - ppp \sim I(0)$.

This paper provided evidence that the cointegrating international parity relationships discovered by Juselius and MacDonald hold also in the case we used a different price index measure. This result is quite interesting since the wholesale price index we used, was not directly cointegrated (section 3.2.1) with the CPI index used by Juselius and MacDonald.

We found that the fundamental relation that combines the *ppp* with the *uip* $(i_t^l - i_t^{l*}) - \omega_1(\Delta p_t - \Delta p_t^*) - \omega_2 ppp_t \sim I(0)$, consistent with a world in which rational and chartists agents (section 2.3) coexist, was accepted with a very high $p - value$; but the more interesting result was that the estimated coefficients ω_1 and ω_2 were very close to one ($\mathcal{H}_{14}$ section 5.4), i.e. very close to a world dominated by rational agents (section 2.3). We therefore tested whether a particular restriction to this model could be accepted: we tested the model imposing ω_1 and ω_2 equal to one. This theoretical case, in which fully rational agents were allowed, was strongly accepted ($\mathcal{H}_{16}$)!

We found also another interesting cointegration relation: real interest rates are stationary in Germany ($\mathcal{H}_3$)! Jointly tested $\mathcal{H}_{16}$ and $\mathcal{H}_3$ were accepted with very high p-values.

These results were also confirmed in the extended model which included short term interest rates ($\mathcal{H}_{18}$, section 6.3): the same hypothesis were accepted and another interesting cointegrating relation between the spread of interest rates was found (section 6.3 equation (19)).

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