Laboratory of Economics and Management
Sant'Anna School of Advanced Studies

Piazza Martiri della Libertà, 33 - I-56127 PISA (Italy)
Tel. +39-050-883-341 Fax +39-050-883-344
Email: lem@sssup.it Web Page: http://lem.sssup.it

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Do Liquidity Constraints Matter in Explaining Firm Size and Growth? Some Evidence from the Italian Manufacturing Industry

Giorgio Fagiolo [*]

and

Alessandra Luzzi [*]

[*] *Sant'Anna School of Advanced Studies*

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Do Liquidity Constraints Matter in Explaining Firm Size and Growth? Some Evidence from the Italian Manufacturing Industry*

Giorgio Fagiolo[†] Alessandra Luzzi[‡]

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Abstract

The paper investigates whether liquidity constraints affect firm size and growth dynamics using a large longitudinal sample of Italian manufacturing firms. We run standard panel-data Gibrat regressions, suitably expanded to take into account liquidity constraints (proxied by cash flow scaled by firm sales). Moreover, we characterize the statistical properties of firms size, growth, age, and (scaled) cash flow distributions. Pooled data show that: (i) liquidity constraints engender a negative, statistically significant, effect on growth once one controls for size; (ii) smaller firms grow more (and experience more volatile growth patterns) after controlling for liquidity constraints; (iii) the stronger liquidity constraints, the more size negatively affects firm growth. We find that pooled size distributions depart from log-normality and growth rates are well approximated by fat-tailed, tent-shaped (Laplace) densities. We also study the evolution of growth-size distributions over time. Our exercises suggest that the strong negative impact of liquidity constraints on firm growth which was present in the pooled sample becomes ambiguous when one disaggregates across years. Finally, firms who were young and strongly liquidity-constrained at the beginning of the sample period grew persistently more than those who were old and weakly liquidity-constrained.

Keywords: Firm Size, Liquidity Constraints, Firm Growth, Investment, Gibrat Law.

JEL Classification: L11, G30, D21

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[†]Corresponding author. Address: Sant'Anna School of Advanced Studies, Laboratory of Economics and Management, Piazza Martiri della Libertà 33, I-56127 Pisa, Italy. Email: `giorgio.fagiolo@sssup.it`. Phone: +39-050-883341. Fax: +39-050-883344.

[‡]Sant'Anna School of Advanced Studies, Pisa, Italy. Email: `luzzi@sssup.it`

1 Introduction

This paper investigates whether liquidity constraints faced by business firms affect the dynamics of firm size and growth. Since the seminal work of Gibrat (1931), the "Law of Proportionate Effects" (LPE) has become the empirical benchmark for the study of the evolution of firm size over time. In one of its most widely accepted interpretations, the LPE states that the growth rate of any firm is independent of its size at the beginning of the period examined[1]. In turn, the underlying "random walk" description of firm (logs of) size dynamics entailed by the LPE implies, under quite general assumptions, a skewed lognormal limit distribution for firm size[2].

In the last decades, a rather large body of empirical literature has been trying to test the LPE "null hypothesis" and its further implications upon industrial organization[3]. Yet, the evidence provided by these contributions is rather mixed, if not contradicting. For example, panel data analyses suggest that LPE should only hold for large manufacturing firms. Moreover, there seem to be many indications supporting the idea that, when only surviving firms are considered, average growth rates – as well as their variance – are decreasing with both size and age. However, despite their statistically significance, estimated growth-size correlations appear to be rather weak, especially once sample selection biases are taken into account.

More recently, the robustness of the existing evidence in favor of (or rejecting) a LPE-type of dynamics has been questioned by (at least) two streams of research[4]. First, as noticed in Bottazzi and Secchi (2004), investigations of firm growth and size dynamics are typically carried out using aggregated data (over different sectors). This might lead to the emergence of statistical regularities – as the LPE – which could only be the result of aggregation of persistently heterogeneous firm dynamics.

Second, and more important to our discussion here, the traditional approach has stressed the investigation of growth-size relationships without extensively addressing a de-

[1] The LPE, also known as Gibrat's Law (GL), can be stated in terms of expected values as well. As Sutton (1997) puts it: the "expected value of the increment firm's size in each period is proportional to the current size of the firm". See also Mansfield (1962).

[2] Notice that the LPE might also form the core of a simple, stochastic, model of firm dynamics. See e.g. Simon and Bonini (1958), Ijiri and Simon (1977) and Geroski (2000). For a more general discussion cf. Steindl (1965), Sutton (1997, 1998), Cabral (1995), and Mitzenmacher (2002).

[3] An exhaustive survey of the LPE literature is of course beyond the scope of this paper. The interested reader may refer to Geroski (2000) and Lotti, Santarelli, and Vivarelli (2003) (and references therein) for quite complete overviews.

[4] The LPE can be violated not only because a statistically significant correlation between average growth and size is present, but also because: (i) higher moments of the growth rates distribution (e.g. variance) show some size-dependence (Bottazzi, Cefis, and Dosi, 2002); (ii) the (limit) size distribution departs from log-normality (Bottazzi, Dosi, Lippi, Pammolli, and Riccaboni, 2001); (iii) growth rate distributions are not normally distributed (Bottazzi and Secchi, 2003).

tailed analysis of other determinants of firm growth (Becchetti and Trovato, 2002). More specifically, the majority of contributions has focused on panel data regressions including as explanatory variables only size- and age-related measures, as well as non-linear effects, time and industry dummies, etc. . Very little attention has been paid to other determinants of firm growth and size dynamics, such as financial factors.

In the last years, however, a growing number of contributions has provided robust empirical evidence showing that financial factors (e.g. liquidity constraints, availability of external finance, access to foreign markets, etc.) can have a significant impact on firms investment decisions[5]. For example, liquidity constraints – measured by scaled measures of cash flow – have been shown to negatively affect firm's investment (Bond, Elston, Mairesse, and Mulkay, 2003) and to increase the likelihood of failure (Holtz-Eakin, Joulfaian, and Harvey, 1994). Moreover, small and young firms seem to invest more, but their investment is highly sensible to liquidity constraints (Fazzari, Hubbard, and Petersen, 1988a; Gilchrist and Himmelberg, 1995).

If financial factors significantly impact on firms' investment decisions, then they are likely to affect firm size and growth dynamics as well. For instance, highly liquidity-constrained firms might face difficulties in financing their investments and thus suffer from lower growth rates in the future (Fazzari, Hubbard, and Petersen, 1988a; Devereaux and Schiantarelli, 1989). At the same time, size and age may affect the ability of the firm to weaken its liquidity constraints and to gain access to external financing[6]. Notice that while the causal relationship going from liquidity constraints to size – through investment and growth – should typically occur at a higher frequency, one expects size and age to affect liquidity constraints over a longer time-scale. Larger and older firms might indeed face difficulties in financing their investment with internal sources – e.g. because of low cash flow – but, at the same time, easily access to external financing because e.g. they belong to well-established socio-economic networks built over the years.

In this paper, we explore whether the emergence of any LPE-type of dynamics – or violations thereof – might be influenced by taking directly into account financial factors (i.e. liquidity constraints) in studying the patterns of firm size and growth. Following Elston (2002), we argue that controlling for liquidity constraints may help in discriminating between "financial-related" and "sheer" size-effects. While the first type of size-effect should account for higher growth rates due to better access to external capital and/or

[5]Space constraints prevent us to discuss here this rather large literature. See, among others, Fazzari and Athey (1987); Hoshi, Kayshap, and Scharfstein (1991); Hall (1992); Bond and Meghir (1994); Schiantarelli (1996); Fazzari, Hubbard, and Petersen (1996); Kaplan and Zingales (1997); Hu and Schiantarelli (1997); Hubbard (1998); Mairesse, Hall, and Mulkay (1999).

[6]Harhoff, Stahl, and Woywode (1998) find that limited-liability firms experience significantly higher growth than unlimited-liability ones. Moreover, Lang, Ofek, and Stulz (1996) provide evidence in favor of a negative correlation between leverage and future firms' growth.

higher cash flow, the latter might explain higher growth rates in terms of economies of scale and scope only. Since existing contributions investigating the LPE did not introduce any controls for financial constraints[7], their estimates of the impact of firm size on future firm growth might have been the result of a composition of "sheer" and "financial-related" effects.

We employ a database containing observations on 14277 (surviving) Italian manufacturing firms of different sizes from 1995 to 2000. The sample was originated from the AIDA database, covering 90% of all Italian firms with sales larger than 1M Euros. We firstly perform standard (pooled) Gibrat's type regressions to assess the impact of liquidity constraints on employees growth rates. We employ cash flow scaled by firm sales (SCF) as a proxy of liquidity constraints and we control for size and age, as well as their lagged values and fixed time and sectoral effects[8]. We show that liquidity constraints engender a negative, statistically significant, effect on growth once one controls for sheer size. Moreover, smaller firms grow more, even after controlling for liquidity constraints.

However, both the goodness-of-fit and the magnitude of estimated coefficients appear to be very weak[9]. Therefore, as suggested in Bottazzi, Cefis, and Dosi (2002), we move towards a more detailed exploration of the statistical properties of the joint distribution of firms size and growth, conditioned on SCF and age.

First, we explore the properties of the joint, pooled, distribution of size, growth, and SCF. We find that size distributions depart from log-normality, while growth rates are well approximated by fat-tailed, tent-shaped (Laplace) densities. Moreover, firms facing stronger liquidity constraints grow less and experience more volatile growth patterns. The stronger liquidity constraints, the larger the absolute value of the observed size-growth correlation. Growth rate distributions seem however to be quite robust to SCF, once one controls for mean-variance time-shifts in their distributions.

Second, we investigate the evolution over time of the distributions of size and growth, conditioning on liquidity constraints and/or age. Our exercises suggest that the absolute value of the size-growth correlation has substantially decreased through time for any level of SCF. In addition, liquidity constraints do not seem to engender a strongly negative impact on firm growth in any given year. Thus, the negative impact of liquidity constraints on firm growth, which was quite strong in our pooled sample, becomes ambiguous when one disaggregates over time.

[7] With the exceptions of Becchetti and Trovato (2002) and Elston (2002). Cf. also Carpenter and Petersen (2002).

[8] See Section 2 for a discussion of econometric and data-related issues involved in the exercises presented in the paper.

[9] Significant but very small growth-size correlations are typically the case in the majority of empirical studies which find a violation of the LPE, cf. Lotti, Santarelli, and Vivarelli (2003).

We also find that firms who were young and strongly liquidity-constrained at the beginning of the sample period experienced in the following years higher average growth rates than those who were old and weakly liquidity-constrained. Furthermore, we show that SCF, size, and growth rates are more variable among younger firms than among older ones. Therefore, one is able to detect a shift to the right in size distributions for young and strongly liquidity-constrained firms.

The paper is organized as follows. In Section 2 we describe the dataset that we employ in our empirical analyses and we discuss some important measurement and data-related issues. Section 3 presents the results of standard regression analyses. The properties of (pooled) distributions are explored in Section 4. In Section 5 we study the evolution over time of size and growth distributions, while the effects of age and size on firm growth dynamics is briefly examined in Section 6. Finally, Section 7 concludes.

2 Data

Our empirical analysis is based on firm-level observations from the AIDA database, developed by Italian Chambers of Commerce and further elaborated by Bureau Van Dijk[10]. The database contains longitudinal data from 1992 to 2000 about size, age, and financial variables obtained by the balance sheets of 90% of all Italian firms (i.e. "lines of business") whose sales have exceeded 1M Euros for at least one year in the observed period[11]. We begin by studying firms belonging to the manufacturing sector as a whole[12].

In order to keep statistical consistency, we analyze data for the period 1995-2000. Furthermore, we focus on unconsolidated budgets so as to avoid as much as possible effects on growth and size due to mergers and acquisitions. Indeed, if one instead considers consolidated budgets, mergers and acquisitions of lines of business belonging to any parent firm may show up in the consolidated budget of the parent firm[13].

Our balanced sample consists of $N = 14277$ observations (per year). We use annual data on employees (EMP) as our main proxy for firm size. Alternative measures of firm size such as sales (SAL) and value-added (VA) – computed as after tax net operating profits minus total cost of capital – are also considered in order to check the robustness of our results.

[10] See `http://www.bvdep.it/aida.htm` for additional details.

[11] A company enters the database the year its sales exceed 1M Euros. Data for previous years are then recovered. Notice that no lower bounds for employees are in principle present.

[12] According to the ATECO 2 classification, we study all firms whose principal activity ranges from code 15 to code 37. For the manufacturing sector, the ATECO 2 classification matches the ISIC one with some minor exceptions.

[13] An alternative strategy allowing to wash-away mergers and acquisitions effects is to build "superfirms" (Bottazzi, Dosi, Lippi, Pammolli, and Riccaboni, 2001; Bottazzi, Cefis, and Dosi, 2002).

In line with existing literature, we employ cash flow, scaled by some measure of firm size, as our proxy for firms' liquidity constraints. More formally, we define the *scaled cash flow* variable (or "cash flow ratio"):

$$SCF_{i,t} = \frac{CF_{i,t}}{SAL_{i,t}}, \tag{1}$$

where $i = 1, ..., N$ are firms' labels, $t = 1996, ..., 2000$ and $CF_{i,t}$ is calculated as net firm revenues plus total depreciation (credits depreciation included)[14]. We also employ firm age (AGE) at the beginning of 1995 (the year of firm's birth is directly available in the database). Growth rates for each size variable $X = EMP$, SAL, VA are computed as:

$$X\_GR_{i,t} = \Delta \log(X_{i,t}) = \log(X_{i,t}) - \log(X_{i,t-1}). \tag{2}$$

As explained above, our goal is to study the extent to which liquidity constraints might affect firm size-growth dynamics. To do so, we focus on single equation models, as well as on statistical properties of joint distributions. Thus, we do not address here the investigation of structural models of firms' investment behavior and growth. Nonetheless, several data-related issues require a more detailed discussion.

First, cash flow ratios are used as a proxy of liquidity constraints. The rationale is that a low cash flow ratio (i.e. a small SCF) may imply, especially for small firms, strong liquidity constraints (Fazzari, Hubbard, and Petersen, 1988b). In fact, firms holding a large cash flow ratio are likely to finance internally their investments. Furthermore, in presence of imperfect capital markets, a high cash flow ratio might also function as a "screening device" to gain a better access to external financing. In presence of credit rationing, larger cash flow ratios might then be used to get additional external funding, especially when firms have some convenience to "go external" for tax reasons[15]. Indeed, high cash flow firms can always choose the right mix between internal and external financing if they have this option (i.e. if the "signalling" effect is present)[16].

Second, as noticed by Becchetti and Trovato (2002) and Elston (2002), cash flow can be

[14] Cf. e.g. Audretsch and Elston (2002), Kaplan and Zingales (1997) and Fazzari, Hubbard, and Petersen (1996). All subsequent results do not dramatically change if alternative specifications for the cash-flow ratio – e.g. $\log(CF)/\log(SAL)$ or $\log(CF/SAL)$ – are taken into account. We also repeated all subsequent regression exercises and distribution analyses employing alternative scaling variables (e.g. total assets, capital stock, etc.), but we did not observe any remarkable departure from our basic findings.

[15] See however Galeotti, Schiantarelli, and Jaramillo (1994), Goyal, Lehn, and Racic (2002) and Albuquerque and Hopenhayn (2004) for alternative approaches employing debt-related variables as measures of liquidity constraints. See also Section 7.

[16] Furthermore, cash flow is a relatively "fast" variable which can better account for the short-term impact of liquidity constraints on investments and growth. Since we expect the feedback from size-age to liquidity constraints to be slower (especially for smaller, young, firms) and we only have data about 6 years, scaled cash flow seems to have an additional justification from a "dynamic" perspective.

highly correlated with other size measures such as sales or value-added. To minimize the consequences of this problem, we scale CF with SAL and we always use employees (EMP) as our size measure whenever liquidity constraints enter the picture. In addition, we check the robustness of all exercises involving size-related measures only by using alternative size measures such as sales (SAL) and value-added (VA).

Third, we only observe "surviving" firms in our sample. This can generate a survival bias, as high-growth (small) firms may be over-represented in our data (Lotti, Santarelli, and Vivarelli, 2003). Unfortunately, we do not have any direct empirical evidence which might allow us to distinguish between missing values and entry-exit events. Hence, we performed a preliminary descriptive analysis on size-growth distributions of firms that were excluded from our database because some missing values did appear in their records. We did not find any statistically significant distribution difference between "included" firms and "excluded" ones. Therefore, we argue that survival biases should not dramatically affect the results that follow.

Fourth, it must be noticed that firms in our database are defined in terms of "lines of business". This type of unit of analysis allows us to appreciate the effects of liquidity constraints on growth in a consistent way (given that we choose to focus on unconsolidated budgets so as to avoid mergers and acquisitions effects). In fact, if one had plant-level data, cash flow ratios would have not been the right measure for liquidity constraints, as each plant has typically access to credit in proportion to cash flow ratios of the parent firm. Moreover, decisions affecting growth are not undertaken at the plant level, but more likely at the parent-firm level.

Fifth, firms in our database can possibly belong to groups consisting of a "controller" and many "controlled" firms. In all these cases (3.4% of all observations), the cash flow ratio of a controlled firm might not be a good proxy for its actual liquidity constraints. In order to avoid this problem, we considered information about parental affiliation. More specifically, we defined the dummy variable D_{subs} (to be employed in our regressions) which is equal to one if the share of a firm's ownership held by shareholders is greater than 50% at the end of each year. On the contrary, as far as distribution analyses are concerned, we simply dropped such observations from our sample (this procedure did not change our results in any substantial way).

Table 1 reports summary statistics for firm size (EMP, VA, SAL) and their growth rates (EMP_GR, VA_GR, SAL_GR), liquidity constraints (SCF) and age (AGE). All size distributions (as well as scaled cash flow) are extremely skewed to the right as expected, while growth rates appear to be almost symmetric and quite concentrated around their average values.

Correlation matrices for all key variables are reported in Tables 2 and 3. As expected,

CF is highly (positively) correlated with VA both simultaneously and at one-year lags, while smaller but still relevant correlations emerge with SAL and, in particular, with EMP. On the contrary, CF/SAL is weakly correlated with all size measures. This seems to be an additional justification for employing SCF as our proxy for liquidity constraints and EMP as size-measure in our exercises.

Both average values and coefficients of variation present weak time-trends, suggesting some non-stationarity of size and SCF distributions. However, once one defines size and SCF variables in terms of their 'normalized' values with respect to year-averages:

$$\widetilde{X}_{i,t} = \frac{X_{i,t}}{N^{-1}\sum_{j=1}^{N} X_{j,t}}, \tag{3}$$

all distributions become stationary over time. Indeed, as Fig. 1 shows for the log of standardized EMP variable ($\widetilde{EMP}$), all first moments exhibit almost no variation over time.

Similar results hold for all other size measures and for SCF. Thus, all our pooled distribution analyses will be performed in terms of $\widetilde{X}_{i,t}$ variables[17]. As far as regression analyses are concerned, we will begin by employing non-standardized values and we will introduce year dummies to control for non stationarities. We will also check the robustness of our results by using pooled distributions of standardized values $\widetilde{X}_{i,t}$. In this case, growth rates are accordingly computed as $\widetilde{X}\_GR_{i,t} = \Delta \log(\widetilde{X}_{i,t})$. Summary statistics and correlation structure for pooled, standardized, distributions are reported in Tables 4 and 5.

3 Evidence from Panel-Data Regressions

In this section we present the results of regression analyses conducted on our pooled samples of observations. We begin by a standard LPE estimation exercise where firm growth is regressed against logs of firm size at the beginning of the period, firm age at the beginning of the sample period, as well as non-linear size-age effects (e.g. size and age squared) and lagged values of both firm growth and size (Evans, 1987; Hall, 1987). We then introduce financial constraints by adding scaled cash flow ($SCF_{i,t}$) to the regression (and lagged values thereof).

We employ non-standardized, pooled, values while controlling for time fixed effects (D_{time}). We also control for industry effects (D_{ind}), defined according to 14 Ateco macro-classes, and for a "subsidiary" dummy (D_{subs}) accounting for companies who are controlled by more than 50%. This allows us to check whether affiliation to a large corporate relaxes

[17]For a more detailed discussion on this standardization procedure, see Kalecki (1945), Hart and Prais (1956), and Bottazzi, Dosi, Lippi, Pammolli, and Riccaboni (2001).

financial constraints.

Lagged values of both size and growth (as well as $SCF_{i,t-k}$, $k > 2$, and interactive terms such as $AGE * EMP$) never appear to be significant in our regressions. We then start from the "saturated" model:

$$\begin{aligned} EMP\_GR_{i,t} &= \alpha_1 \log(EMP_{i,t-1}) + \alpha_2 SCF_{i,t-1} + \alpha_3 \log(AGE_i) + \\ &\quad + \alpha_4 \log^2(EMP_{i,t-1}) + \alpha_5 \log^2(AGE_i) + \alpha_6 SCF_{i,t-2} + \\ &\quad + \alpha_7 D_{time} + \alpha_8 D_{ind} + \alpha_9 D_{subs} + \varepsilon_{i,t}, \end{aligned} \tag{4}$$

where $\varepsilon_{i,t}$ is a white-noise term.

We rely on Likelihood Ratio Tests (LRTs) to drop in each step one (or more) covariate(s) and eventually get to our preferred model. As shown in Table 6, this selection procedure allows us to discard both $\log^2(AGE_i)$ and $\log(AGE_i)$ at 5% significance level. We also employ an alternative model selection procedure based on maximization of (pseudo) R^2s (see Table 7).

We begin by standard growth-size regressions and we compare models obtained by adding covariates. Among all possible specifications, we show results of regressions that reach the highest R^2 values (which is always very low due to the large number of observations). This combined procedure suggests that, by omitting $\log(CF_{i,t-2})$ from the "saturated" model, one gets the highest R^2 levels. However, if one also drops $\log^2(AGE_i)$ and $\log(AGE_i)$ – as suggested by LRTs – the goodness of fit does not decrease very much. Therefore, the two criteria taken together indicate the following "preferred" model:

$$\begin{aligned} EMP\_GR_{i,t} &= \beta_1 \log(EMP_{i,t-1}) + \beta_2 SCF_{i,t-1} + \beta_3 \log^2(EMP_{i,t-1}) \\ &\quad + \beta_4 D_{time} + \beta_5 D_{ind} + \beta_6 D_{subs} + \varepsilon_{i,t}. \end{aligned} \tag{5}$$

Estimation of (5) shows that size effects are significant and negative (Lotti, Santarelli, and Vivarelli, 2003). Our data therefore confirm a rejection of the LPE: smaller firms grow more, even when one controls for financial constraints[18]. In addition, firms with higher cash flow ratios enjoy higher growth rates, once one controls for sheer size. Notice that a positive and significant non-linear size effect is present. Introducing $\log^2(EMP_{i,t-1})$ in the regression also implies a higher magnitude for the growth-size correlation. Finally, the "subsidiary" dummy (as well as all other dummies) is significant in all model specifications, suggesting that, given the same degree of liquidity constraints, firms who belong to a large

[18] Our results also suggest that younger firms grow more, as shown by a significant negative effect of $\log(AGE_i)$ in Table 7. Anyway, this effect does not seem to be pivotal in our model, as indicated by the LR test procedure.

company grow more than those who are "isolated", possibly because of an easier access to external financing.

The foregoing results seem to be robust *vis-à-vis* a number of different checks. *First*, although lagged values of SCF should be in principle important to explain current growth, our data suggest that $SCF_{i,t-k}$, $k \geq 2$, is hardly significant in the saturated model (see also last column in Table 7) and that any statistical significance is only due to the large number of observations we have in our dataset[19]. *Second*, if we only consider "financial-related" size effects and we exclude sheer size effects – i.e. if we discard $\log(EMP_{i,t-1})$ – in our regressions, scaled cash flow remains significant and negative. This seems to be a strong indication in favor of the robustness of this measure as a proxy for firm financial constraints. *Third*, both significance levels and signs of estimates do not change if one considers year-standardized variables and omits fixed time effects in the regressions. *Fourth*, if one also controls for $EMP\_GR_{i,t-1}$ in the preferred model, all estimates remain significant and their signs do not change. The lagged growth term appears to be significant but in general its magnitude turns out to be quite small. *Fifth*, although firms heterogeneity was taken into account only through industry-specific effects, our main results do not change if one estimates a first-difference version of our preferred model (without industry dummies) to eliminate firms fixed effects. Indeed, due to the low time-series variability in our sample, random effects turn out to be almost irrelevant.

Our findings are in line with previous ones obtained by Becchetti and Trovato (2002), who show that small surviving Italian firms experience higher growth rates, but the latter are negatively affected by the availability of external finance. Carpenter and Petersen (2002) reach similar conclusions as far as small firms in the U.S. are concerned. On the contrary, Audretsch and Elston (2002) show that medium sized German firms are more liquidity constrained (in their investment behavior) than either the smallest or the largest ones[20].

Regression results seem to support the idea that firm growth is negatively affected by both size and liquidity constraints. However, the goodness-of-fit of our preferred model is not very encouraging. In presence of such low R^2s (and quite small estimated coefficients), one might also be tempted to conclude that, albeit significant, the effect of size and scaled CF on average growth rates is irrelevant (at least as policy issues are concerned)[21]. In order to further explore whether our data really exhibit departures from the LPE, we turn

[19] Although we decided to drop lagged SCF values here, an in-depth investigation of their effects – as well as of those of cumulated SCF values – is certainly one of the main points in our agenda.

[20] Elston (2002) finds that cash flow (not scaled) – after controlling for size and age – positively affects growth of German Neuer-Markt firms. We find similar results in our sample, too. However, this outcome may be biased by the high and positive correlation between size (employees) and cash flow. See also Section 7.

[21] For a discussion on the interpretation of LPE empirical results see Sutton (1997) and Geroski (2000).

now to a more detailed statistical analysis of pooled size and growth distributions. We shall investigate in particular the properties of the joint growth-size distribution conditional on scaled cash flow observations.

4 Statistical Properties of Pooled Size and Growth Distributions

If the benchmark model of stochastic firm growth underlying the LPE holds true, then for any measure of firms size S_t, the dynamics of S_t reads:

$$S_t = S_{t-1} R_t, \tag{6}$$

where R_t is a random variable. If $r_t = \log R_t$ are i.i.d. random variables with finite mean and variance, then S_t is well approximated, for sufficiently large t, by a log-normal distribution. Moreover, growth rates $g_t = \Delta \log(S_t)$ should be normally-distributed.

We then begin by checking whether our pooled (year-standardized) firm size distributions depart from log-normal ones. As size-rank plots suggest, the distributions of $\widetilde{EMP}$, $\widetilde{SAL}$, and $\widetilde{VA}$ can hardly be approximated by a log-normal (cf. Fig. 2 for the evidence about employees), as the mass of these distributions seems to be shifted to the left. As Fig. 2 shows, a similar finding holds for $\widetilde{SCF}$ as well[22].

Furthermore, pooled standardized growth rates appear to follow a tent-shaped distribution, with tails fatter than those of a Gaussian one (cf. Fig. 3). Growth rate distributions are indeed well described by a Laplace (symmetric exponential) functional form:

$$h(x; a, b) = \frac{1}{2a} e^{-\frac{|x-b|}{a}}, \tag{7}$$

where $a > 0$. Tent-shaped distributions have been recently found to robustly characterize growth rates both at an aggregated level (Stanley *et al.*, 1996; Amaral *et al.*, 1997) and across different industrial sectors and countries (Bottazzi and Secchi, 2003, 2004).

The foregoing two pieces of evidence (i.e. departures from log-normality of size distributions and fatter tailed, tent-shaped, growth rate distributions) suggest that the underlying growth-size dynamics is not well described by a simple Gibrat-type process with independent increments. This conclusion is further reinforced by looking at how growth average and standard deviation vary with average (within bins) firm sizes[23]. As already found in

[22] These results are also confirmed by non-parametric kernel estimates (performed using a normal kernel with a 0.2 bandwidth) of size and $\widetilde{SCF}$ densities. Results are available from the Authors upon request.

[23] In this and all subsequent plots, we do not depict confidence intervals as they typically lie very close

regression exercises, small firms seem to grow more on average, but their growth patterns appear to be more volatile than those of larger firms. This holds true for all measures of size (cf. Fig. 4 for the employees distribution). In fact, one typically observes a rapidly declining pattern for both average and variance of growth rates as size increases, which however stabilizes for larger firm size values[24].

But how do liquidity constraints affect the joint size-growth distribution? To explore this issue, we investigated how average and standard deviation of size ($\log \widetilde{EMP}$) and growth ($\widetilde{EMP\_GR}$) pooled distributions (as well as their correlation) change with respect to (within-bins) averages of $\widetilde{SCF}$[25].

Our exercises point out that – in line with regressions results – firms facing *stronger* liquidity constraints are associated with *lower* average growth and *smaller* average size (cf. Fig. 5). We also find that smaller cash flows imply more volatile growth patterns, while no clear implications can be drawn as far as within-bins size variability is concerned (cf. Fig. 6).

As a consequence, one observes statistically-detectable differences in both size and growth distributions associated to strongly vs. weakly liquidity-constrained firms. As Fig. 7 (left) shows for the 1st and the 10th decile of $\widetilde{SCF}$, less liquidity constrained firms exhibit size distributions which are substantially shifted to the right. Once one controls for any mean-variance shift and compares standardized size distributions (i.e. with zero-mean and unitary variance), any statistically detectable difference disappears.

Accordingly, growth rate distributions maintain their characteristic tent-shaped pattern but the estimated Laplace coefficient (i.e. the estimate for a in eq. 7) decreases with cash flow: see Fig. 7 (right). Notice that a lower Laplace coefficient (i.e. a steeper Laplace fit) might be interpreted as evidence in favor of fatter-tailed growth distributions, *as long as the variance of the distribution remains constant*[26]. Again, a comparison of standardized growth distributions (i.e. with zero-mean and unitary variance) associated to high vs. low

to the statistics values.

[24] A similar result is obtained if one splits the pooled sample of firms into "small" (e.g. those belonging to the 1st quartile) vs. "large" ones (e.g. those belonging to the 4th quartile) and separately estimates our preferred regression model on each sub-sample. Indeed, the LPE seems to hold for large firms but fails for small ones. This conclusions is quite robust to alternative ways of splitting the pooled sample into large vs. small firms.

[25] More formally: given the triple distribution ($\widetilde{EMP\_GR}_t, \log \widetilde{EMP}_{t-1}, \widetilde{SCF}_{t-1}$), we computed statistics of ($\widetilde{EMP\_GR}_t, \log \widetilde{EMP}_{t-1}$) for each 5%-percentile of $\widetilde{SCF}_{t-1}$. The qualitative implications we present in this Section are not dramatically altered if one employs non-standardized values. Notice also that, in line with our analysis in Section 3, we do not consider further lags for our SCF variable. An interesting extension would be to study the moments of the multivariate distribution ($\widetilde{EMP\_GR}_t$, $\log \widetilde{EMP}_{t-1}$, $\widetilde{SCF}_{t-1}, ..., \widetilde{SCF}_{t-d}$), where $d \geq 2$.

[26] If $b = 0$ and X is distributed as a Laplace(a), then $var(X)$=$2a^2$. Thus, smaller estimates for a imply fatter tailed distributions only if one controls for their variance.

cash flow levels no longer reveals statistically detectable differences: estimated Laplace coefficients (cf. Fig. 8) are nearly constant with respect to cash flow. The same pattern also emerges when one plots estimated Laplace coefficients against log of size (not shown).

Our data suggest that liquidity constraints do not dramatically affect the fatness of growth-rates tails. Less liquidity-constrained firms do not enjoy high-probability (absolute values of) large growth shocks, as compared to more liquidity-constrained ones, after one controls for existing mean-variance trends.

Finally, growth-size correlation[27] is always negative for all cash flow levels: cf. Fig. 9. Nevertheless, the stronger liquidity constraints, the stronger the effect of size on growth. Note also that within-bins correlation magnitudes are always significantly larger than the non-conditioned one (i.e. correlation computed across all SCF values).

5 Liquidity Constraints and the Evolution of Size-Growth Distributions over Time

In the last Section, we characterized some statistical properties of pooled growth and size distributions conditioning on cash flow. We employed pooled data since we observed that, once one controls for year-averages, size and growth distributions are almost stationary over time. Yet, size and growth distributions do exhibit some trends in their moments, as Table 1 shows. In this Section, we start exploring in more detail the nature of the observed shifts in non-standardized growth, size, and SCF distributions. Next, in Section 6, we shall investigate the effect of age on growth-size dynamics. We shall compare, in particular, the performance of firms who were more liquidity-constrained and younger at the beginning of our sample period with that of firms who held larger SCF and were older.

Let us begin with unconditional size, growth, and scaled cash flow (non-standardized) distributions. Kernel estimates and statistical tests all confirm that no dramatic shifts over time are present in our data. All size year distributions (i.e. EMP_t, SAL_t, and VA_t, t = 1995, ..., 2000) exhibit some shift to the right due to an increasing mean (due e.g. to overall economic growth) and become slightly more concentrated and less skewed to the right (see Fig. 10). Cash flow ratios seem to be even more stationary (cf. bottom-right panel). Any observed shift seem however to be entirely due to changes in mean and variance over the years and disappear when one standardizes the variables.

Similar findings are obtained for growth rate distributions. Moments of EMP_GR_t, SAL_GR_t, and VA_GR_t (t = 1996, ..., 2000) are almost stable across years (if any,

[27] Growth-size correlations computed here are not partial ones. Hence, they cannot be compared to regression results which include other explanatory variables.

average growth is U-shaped and growth rate standard deviations seem to decline over time). Tent-shaped distributions emerge in all years and estimated Laplace coefficients are nearly stable. The unique exception concerns employee growth rate distributions, which exhibit tails becoming fatter with time. However, when one controls for mean and variance, estimated Laplace coefficients for EMP_GR_t become almost stationary over time, cf. Fig. 11.

Consider now the evolution over time of size-growth distributions conditional on one-year lagged cash flow ratios, e.g. $(EMP\_GR_t,\ EMP_{t-1}|SCF_{t-1})$[28]. We are interested in asking whether the evidence about the relationship between liquidity constraints and moments of size-growth distributions (e.g. average and standard deviation) that we obtained using pooled data (see Section 4) is robust to time-disaggregation.

Our results provide a mixed answer to this question. On the one hand, average size tends to increase in all years with cash flow (Fig. 12, top-left, for the evidence about 1996 and 2000). On the other hand, average growth rates appear to be increasing with SCF only in 1996: as one approaches the end of our sample period, average growth rates seem to be constant with respect to liquidity constraints (Fig. 12, bottom-left). Nevertheless, growth rates appear to be more volatile the smaller SCF (Fig. 12, bottom-right) in every year taken in consideration. No clear indication however emerges as to whether strongly liquidity-constrained firms enjoy less volatile size distributions (Fig. 12, top-right).

Thus, the negative impact of liquidity constraints on firm growth, which was quite strong in our pooled sample, becomes more ambiguous when one disaggregates across years. We argue that an explanation for this result can be rooted in the way firms perceive liquidity constraints over the business cycle. If we do not wash away growth trends in the whole manufacturing sector, our data may still embed the effects of firms expectations about the impact of sheer size and liquidity on their future investments and growth. Since these expectations typically depend on the business cycle, one might well observe in our data different correlation patterns between financial constraints and growth across years.

Yet, notice that many other properties which we have found in our pooled sample robustly hold also for non-standardized data across time. For example, both size and growth distributions (conditional on SCF) are quite stable over time. As Fig. 13 shows, size distributions for highly and weakly liquidity-constrained firms only shift to the right as we move from 1995 to 2000, while estimated Laplace coefficients for growth rate distributions do not exhibit any detectable time-difference when we compare high and low cash flow firms (cf. Fig. 14) after having controlled for their variance. In addition, the correlation between EMP_GR_t and EMP_{t-1} remains always negative for each t and each cash flow

[28] We employ SCF_{t-1} only because the correlation with size and growth variables typically become almost irrelevant for SCF_{t-k}, $k > 1$.

bin. The magnitude of the size-growth correlation turns out to be larger the stronger liquidity constraints are and appears to have substantially decreased in the sample period (see Fig. 15).

6 Size, Age, Liquidity Constraints, and Firm Growth

The evidence discussed at the end of the last Section indicates that our panel of firms has somewhat shifted over time towards a Gibrat-like growth-size dynamics. The decrease of growth-size correlation across years is probably due to the overall growth experienced by the whole manufacturing sector in the sampled period (on average, all firms grew by 4.3287% from 1995 to 2000). On the one hand, evidence on pooled data suggests that small, younger, and weakly liquidity-constrained firms should have benefited from higher growth. On the other hand, smaller firms are typically younger but more liquidity-constrained than larger ones. In addition, we observed that the negative correlation between liquidity-constraints and growth may be weakened by time-disaggregation.

Thus, an interesting issue concerns whether across-years performances of firms who were young but strongly liquidity-constrained at the beginning of the period could be larger than that experienced by firms who were older but held large cash flows. More generally, we are interested here in assessing whether firm age might allow us to better understand how liquidity constraints affect size-growth patterns. In order to answer these questions, we start by analyzing how age affects cash flow ratios, size, and growth distributions over time.

To begin with, firm age in our dataset is log-normally distributed, as confirmed by both density estimates - see Fig.16 - and Kolmogorov-Smirnov tests. Controlling for age only, we find that younger firms grow more, are smaller and more liquidity-constrained as expected (Fazzari and Athey, 1987; Cabral and Mata, 2003). Furthermore, cash flow ratios, size, and growth rates are more variable among younger firms than among older ones[29].

Suppose now to simultaneously control for age and liquidity constraints. More specifically, let us define "young" (resp. "old") those firms who belong to the first (resp. tenth) decile of the $\log AGE$ distribution in 1995[30]. Accordingly, let us call "strongly liquidity-constrained" (SLC) and, respectively, "weakly liquidity-constrained" (WLC) firms who belong to the first and, respectively, tenth decile of the SCF distribution in 1995. Consider now the sub-sample of "young" and SLC firms (YSLC) - and, accordingly, the sub-sample

[29] More precisely, binned average (resp. standard deviation) of log of employees and log of scaled cash flow are increasing (resp. decreasing) with respect to (within-bins) averages of log of age. Conversely, binned growth-rates average and standard deviation are both decreasing with (within-bins) averages of log of age.

[30] Incidentally, "young" firms have less than 12 years, while "old" ones have more than 42 years.

of "old" and WLC firms (OWLC) - and let us study size and growth distributions of YSLC and OWLC firms over time.

As Fig. 17 shows, YSLC firms grew persistently *more* not only than OWLC ones, but also than firms who were just "young" in 1995. Furthermore, YSLC firms who were "small" in 1995 (i.e. who belonged, in addition, to the first decile of the 1995 employees distribution) experienced from 1996 to 2000 better-than-average growth rates and outperformed YSLC firms who were "large" in 1995 (i.e. who also belonged to the tenth decile of the 1995 employees distribution). This means that, by focusing on young, cash-constrained (and possibly small) firms, we are eliciting a sub-sample of dynamic firms who, despite (or even thanks to) low levels of cash flow ratios, are able to enjoy high growth rates in the subsequent periods[31].

These pieces of evidence indicate that some weak catching-up process has been occurring during 1995-2000: see Fig. 18. Indeed, YSLC firm size distributions shift to the right, while OWLC ones are almost unchanged.

Shifts in size distributions from 1995 to 2000 are however better detectable if one controls for age only (as the number of observations increases). Fig. 19 shows that the distribution of "old" firms in 1995 was to the right of "young" ones (and slightly more concentrated). Between 1995 and 2000, "young" firms grew more than "old" ones: size distributions in 2000 are closer than in 1995 (Figs. 20 and 21). To the contrary, "old" firms enjoyed very weak growth, as their 1995 and 2000 size distributions almost coincide (cf. Fig. 22). Similar results can be obtained by exploring how cash flow distributions change over time for "young" and "old" firms.

Finally, log of size distributions all depart from normality but we do not find any statistically detectable change in their shape. Contrary to Cabral and Mata (2003), who report (for Portuguese firms) evidence about shifts towards less skewed size distributions over time, Italian ones (conditional on age and/or cash flow) are always well approximated by highly skewed densities across the entire period of observation.

7 Conclusions

In this paper, we have analyzed the relationships between liquidity constraints and firm growth dynamics for Italian manufacturing firms. Our main goal was to assess whether any detectable departure from the "Law of Proportionate Effects" might be better explained by taking into account the link between financial factors and growth.

[31] Since we are only observing surviving firms, further analyses are required to take care of any selection effects present in our data.

Gibrat-type regression exercises on pooled data show that liquidity constraints (as proxied by cash flow scaled by firm sales) engender a negative, statistically significant, effect on growth once one controls for sheer size. Moreover, smaller firms grow more, even when one controls for liquidity constraints.

This evidence against the LPE is further reinforced by a statistical analysis of pooled distributions. We find that size distributions depart from log-normality, while growth rates are well approximated by fat-tailed, tent-shaped (Laplace) densities. Moreover, firms facing stronger liquidity constraints grow less and experience more volatile growth patterns. The negative impact of size on growth seems to increase in magnitude as liquidity constraints become more severe. Growth rate distributions seem however to be quite robust to cash flow ratios, once one controls for mean-variance shifts.

We also studied the evolution of size, growth, and scaled cash flow distributions over time. Our exercises suggest that the magnitude of the size-growth correlation has substantially decreased through time for any level of cash flow. Moreover, the negative impact of liquidity constraints on firm growth – which we found to be quite strong in our pooled sample – becomes more ambiguous when one disaggregates across years.

We also find that firms who were young and strongly liquidity-constrained at the beginning of the sample period grew persistently more than those who were just young, and than those who were old and weakly liquidity-constrained. Those firms turn out to be typically small and quite dynamic entities, which are capable of experiencing high performances despite they were highly cash-constrained at the beginning of the sample period.

Shifts to the right in size distributions for young and strongly liquidity-constrained firms can also be detected. However, in contrast to existing literature (Cabral and Mata, 2003), size distributions remain quite skewed in the entire period.

Many interesting issues remain to be explored.

First, alternative proxies of liquidity-constraints could be considered in order to test the robustness of our results. For example, one might attempt to study what happens by building liquidity-constraints proxies based on firms' financial stock variables, such as leverage measures (Becchetti and Trovato, 2002). As an exploratory study, we performed regression exercises where we replaced cash-flow ratios with "gearing ratios" (G), measured by the ratio between firm's total long term debt and short term debt towards bank and net total assets. We find that the "gearing ratio" is significant and negative, probably because it accounts for a sort of "risk effect". However, if we also add to the regression our original scaled cash-flow measure, as well as a multiplicative regressor $SCF * G$, the latter term turns out to be significant and positive. This means that, given two firms with similar cash flow ratios, the one with higher gearing ratio grows more. Our data seems to support the idea that firms with higher access to external finance grow more. Moreover, a double signalling effect seems to be at work: firms with higher cash-flow ratios seem to be those with higher access to external capital.

Second, a more thorough disaggregated sectoral analysis is needed in order to ascertain the extent to which liquidity constraints affect in idiosyncratic ways the patterns of growth of different industries (Bottazzi, Cefis, and Dosi, 2002). For example, an interesting exercise might involve mapping technological specificities into different properties of growth-size-SCF dynamics.

Finally, one could try to jointly estimate investment and growth equations (and include as independent variables size, age, and proxies for liquidity constraints) in order to better understand the causal relations going from financial factors to growth and back. Along these lines, building upon recent contributions by Klette and Griliches (2000), Hall (2002), and Bougheas, Holger, and Strobl (2003), one might study the effect of financial constraints on R&D investment and growth.

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$Variable$	$Year$	$Mean$	$StDev$	$VarCoeff$	$Median$	$Kurtosis$	$Skewness$
	1995	96.01	1026.27	10.69	35.00	6897.89	77.36
	1996	98.60	705.22	7.15	38.00	6459.91	70.60
EMP	1997	100.95	683.15	6.77	40.00	5707.06	65.36
	1998	101.30	650.69	6.42	40.00	5280.79	62.37
	1999	100.14	572.44	5.72	40.00	4439.05	55.80
	2000	101.58	400.63	3.94	41.00	806.09	23.67
	1995	20145.52	194548.99	9.66	6669.00	8153.74	81.48
	1996	20224.12	160180.08	7.92	6800.00	6155.25	67.89
SAL	1997	22420.34	196892.42	8.78	7123.00	5611.88	64.68
	1998	22856.39	186945.17	8.18	7477.00	5721.81	65.10
	1999	23719.82	193310.99	8.15	7516.00	5579.45	63.93
	2000	25707.81	155260.23	6.04	8245.00	1107.74	28.96
	1995	5055.55	45556.89	9.01	1676.00	7162.51	75.72
	1996	4978.80	28630.23	5.75	1739.00	2393.64	40.68
VA	1997	5479.97	40985.14	7.48	1776.00	3592.83	52.64
	1998	5527.24	33573.26	6.07	1840.00	1887.94	37.58
	1999	5658.97	32799.82	5.80	1895.00	2031.77	38.27
	2000	6011.42	36595.92	6.09	1987.00	3682.48	49.43
	1995	1397.81	17971.56	12.86	323.00	5076.67	63.38
	1996	1222.26	12462.08	10.20	317.00	4182.24	52.82
CF	1997	1473.15	30412.31	20.64	316.00	8549.94	82.34
	1998	1445.30	20350.60	14.08	324.00	9696.49	89.39
	1999	1600.97	26633.23	16.64	348.00	10924.46	98.24
	2000	2140.89	45365.32	21.19	353.00	7524.51	81.42
	1995	0.07	0.33	4.57	0.05	10603.81	96.88
	1996	0.07	0.42	6.19	0.05	12628.96	109.56
CF/SAL	1997	0.06	0.13	2.05	0.05	5490.84	62.15
	1998	0.06	0.08	1.33	0.05	3606.24	18.02
	1999	0.07	0.08	1.21	0.05	4494.65	14.82
	2000	0.05	0.43	9.09	0.04	2844.92	17.77
	1996	0.12	0.30	0.41	0.09	84.90	3.15
	1997	0.03	0.24	0.14	0.02	70.32	1.20
EMP_GR	1998	0.00	0.29	0.01	0.00	56.54	−3.39
	1999	−0.06	0.48	−0.12	0.00	19.87	−1.95
	2000	0.04	0.68	0.05	0.04	13.89	−1.07
	1996	0.03	0.32	0.11	0.02	82.76	3.59
	1997	0.04	0.26	0.17	0.04	122.06	4.08
SAL_GR	1998	0.04	0.23	0.18	0.04	44.81	0.43
	1999	0.01	0.24	0.04	0.01	47.59	0.23
	2000	0.07	0.32	0.22	0.09	68.86	−4.22
	1996	0.05	0.35	7.32	0.04	38.08	1.63
	1997	0.03	0.30	9.11	0.03	63.11	1.26
VA_GR	1998	0.03	0.29	9.08	0.04	32.14	-0.67
	1999	0.03	0.31	11.64	0.03	36.93	0.02
	2000	0.03	0.39	13.25	0.05	41.41	-2.76
AGE	1995	27.93	83.83	0.33	21.00	533.58	22.81

Table 1: Summary Statistics for Non-Standardized Data. EMP = Employees. SAL = Sales. VA = Value-Added. CF = Cash Flow. GR = Growth Rate. N = 14277 Firms observed.

2000	SAL	VA	CF	CF/SAL
EMP	0.7716	0.7270	0.4071	0.0152
SAL	$*$	0.7104	0.4905	0.0140
VA	$*$	$*$	0.8171	0.0314
CF	$*$	$*$	$*$	0.0444

1999	SAL	VA	CF	CF/SAL
EMP	0.8921	0.8418	0.2626	0.0333
SAL	$*$	0.7689	0.2716	0.0171
VA	$*$	$*$	0.6983	0.1025
CF	$*$	$*$	$*$	0.1488

1998	SAL	VA	CF	CF/SAL
EMP	0.9135	0.8503	0.2603	0.0328
SAL	$*$	0.8489	0.3968	0.0126
VA	$*$	$*$	0.6691	0.0490
CF	$*$	$*$	$*$	0.1457

1997	SAL	VA	CF	CF/SAL
EMP	0.8957	0.8848	0.3426	0.0211
SAL	$*$	0.9121	0.5444	0.0126
VA	$*$	$*$	0.7065	0.0381
CF	$*$	$*$	$*$	0.0689

1996	SAL	VA	CF	CF/SAL
EMP	0.9234	0.9316	0.3990	0.0153
SAL	$*$	0.8892	0.4541	0.0004
VA	$*$	$*$	0.4557	-0.0120
CF	$*$	$*$	$*$	0.7136

1995	SAL	VA	CF	CF/SAL
EMP	0.8273	0.8455	0.7443	0.0122
SAL	$*$	0.9544	0.8524	0.0013
VA	$*$	$*$	0.8813	-0.0066
CF	$*$	$*$	$*$	0.3591

Table 2: Correlation Structure. Contemporaneous Distributions of Size Measures and Cash Flow.

2000-1999	EMP	SAL	VA	CF	CF/SAL
EMP	*	0.7340	0.8351	0.2934	0.0541
SAL	0.7670	*	0.7825	0.3645	0.0360
VA	0.6321	0.5748	*	0.8232	0.1238
CF	0.4778	0.4999	0.7727	*	0.1126
CF/SAL	0.0126	0.0109	0.0290	0.0293	*

1999-1998	EMP	SAL	VA	CF	CF/SAL
EMP	*	0.9092	0.8598	0.2747	0.0362
SAL	0.8951	*	0.7669	0.2631	0.0638
VA	0.8304	0.8273	*	0.6897	0.0729
CF	0.2574	0.3767	0.6550	*	0.0798
CF/SAL	0.0313	0.0298	0.0950	0.1350	*

1998-1997	EMP	SAL	VA	CF	CF/SAL
EMP	*	0.8939	0.8889	0.3463	0.0214
SAL	0.9120	*	0.9050	0.4993	0.0133
VA	0.8358	0.8373	*	0.6364	0.0437
CF	0.2476	0.4225	0.6441	*	0.0609
CF/SAL	0.0344	0.0217	0.0544	0.0689	*

1997-1996	EMP	SAL	VA	CF	CF/SAL
EMP	*	0.9190	0.9372	0.4610	0.0786
SAL	0.8799	*	0.8315	0.6252	0.2490
VA	0.8453	0.8230	*	0.7791	0.4807
CF	0.2855	0.3202	0.2238	*	0.8524
CF/SAL	0.0186	0.0070	0.0330	0.0597	*

1996-1995	EMP	SAL	VA	CF	CF/SAL
EMP	*	0.9057	0.9506	0.8183	0.0160
SAL	0.8062	*	0.9413	0.8267	0.0032
VA	0.7335	0.8281	*	0.7433	-0.0072
CF	0.3442	0.4511	0.4746	*	0.6858
CF/SAL	0.0113	0.0012	-0.0096	0.3661	*

Table 3: One-Year Lagged Correlations

	Obs	$Mean$	$Std.Dev.$	$Skewness$	$Kurtosis$
EMP_GR	71380	0.0394	0.2221	0.8963	26.3321
$Log(EMP)$	71382	0.0346	1.2921	−0.7437	2.7124
SAL_GR	71385	0.0387	0.3270	0.7696	65.8774
$Log(SAL)$	71385	−0.9723	1.0460	1.0193	5.5047
VA_GR	70767	0.0461	0.3192	0.1655	45.6623
$Log(VA)$	71091	−0.9778	1.1791	0.7668	5.8549
$Log(CF)$	71385	−1.1187	1.6984	0.7474	4.8691
CF/SAL	71215	0.8956	3.4897	154.0042	28210.68

Table 4: Summary Statistics for Year-Standardized Pooled Variables

$Corr$	EMP_GR	$Log(EMP)$	SAL_GR	$Log(SAL)$	VA_GR	$Log(VA)$	$Log(CF)$
$Log(EMP)$	−0.1150	1.0000	*	*	*	*	*
SAL_GR	0.2674	−0.0182	1.0000	*	*	*	*
$Log(SAL)$	0.0029	0.7604	−0.0942	1.0000	*	*	*
VA_GR	0.2498	−0.0349	0.5409	−0.0575	1.0000	*	*
$Log(VA)$	−0.0077	0.9016	−0.0465	0.8647	−0.1293	1.0000	*
$Log(CF)$	0.0263	0.6778	−0.0295	0.7543	−0.0481	0.8240	1.0000
CF/SAL	0.0276	0.1143	0.1247	0.0197	0.0430	0.1837	0.4111

Table 5: Correlation Matrix for Year-Standardized Pooled Variables

Regressors	1st step		2nd step	
Dropped	LR test	R^2	LR test	R^2
$log(EMP_{i,t-1})$	750.84** (0.0000)	0.0095	830.73** (0.0000)	0.0070
$CF_{i,t-1}/SAL_{i,t-1}$	9.21** (0.0024)	0.0226	9.52** (0.0020)	0.0214
$log(AGE_i)$	**2.19 (0.1388)**	**0.0225**	Dropped in 1st step	
$log^2(EMP_{i,t-1})$	533.76** (0.0000)	0.0132	573.90** (0.0000)	0.0115
$log^2(AGE_i)$	**0.32 (0.5689)**	**0.0226**	Dropped in 1st step	
$CF_{i,t-2}/SAL_{i,t-2}$	8.62** (0.0033)	0.0224	9.26** (0.0023)	0.0214
D_{subs}	Yes**		Yes**	
D_{ind}	Yes**		Yes**	
D_{time}	Yes**		Yes**	

Table 6: Model Selection. Results of the Likelihood Ratio Test Procedure. Each row reports the results of the LR test for the null hypothesis: "Drop from the saturated model (i.e. the model containing all regressors not previously dropped) the regressor indicated in the first column". A double (single) asterisk associated to the LR test value indicates that the regressor must NOT be dropped at 5% (10%).

Regressors	Dependent Variable: EMP_GR						
$log(EMP_{i,t-1})$	-0.0302**	-0.0306**	-0.0278**	**-0.1413****	-0.1361**	-0.1349**	-0.1175**
	(0.0009)	(0.0009)	(0.0009)	**(0.0036)**	(0.0037)	(0.0037)	(0.0043)
$CF_{i,t-1}/SAL_{i,t-1}$	-	0.0358**	0.0348**	**0.0336****	0.0329**	0.0327**	0.0186**
		(0.0040)	(0.0040)	**(0.0040)**	(0.0020)	(0.0040)	(0.0061)
$log(AGE_i)$	-	-	-0.0242**	**-**	-0.0190**	-0.0722**	-0.0276
			(0.0020)		(0.0020)	(0.0164)	(0.0186)
$log^2(EMP_{i,t-1})$	-	-	-	**0.0141****	0.0137**	0.0136**	0.0120**
				(0.0004)	(0.0004)	(0.0004)	(0.0005)
$log^2(AGE_i)$	-	-	-	**-**	-	0.0086**	0.0017
						(0.0026)	(0.0030)
$CF_{i,t-2}/SAL_{i,t-2}$	-	-	-	**-**	-	-	0.0146**
							(0.0050)
D_{subs}	Yes**	Yes**	Yes**	**Yes****	Yes**	Yes**	Yes*
D_{ind}	Yes**	Yes**	Yes**	**Yes****	Yes**	Yes**	Yes*
D_{time}	Yes**	Yes**	Yes**	**Yes****	Yes**	Yes**	Yes*
Obs	71380	71380	71380	**71380**	71380	71380	57105
R^2	0.0183	0.0195	0.0214	**0.0335**	0.0346	0.0347	0.0226
F-test	0.0000	0.0000	0.0000	**0.0000**	0.0000	0.0000	0.0000

Table 7: Model Selection. Results of the R^2 Procedure. Standard Errors in parentheses. In boldface our preferred model. A double (single) asterisk indicates significance at 5% (10%).

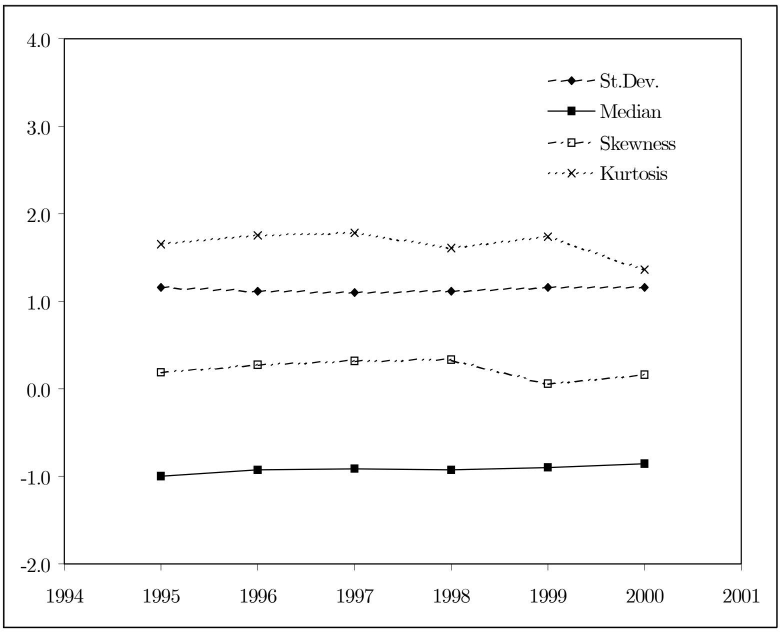


Figure 1: Moments of $\log(EMP_{i,t}/\overline{EMP}_{i,t})$ distribution against time.

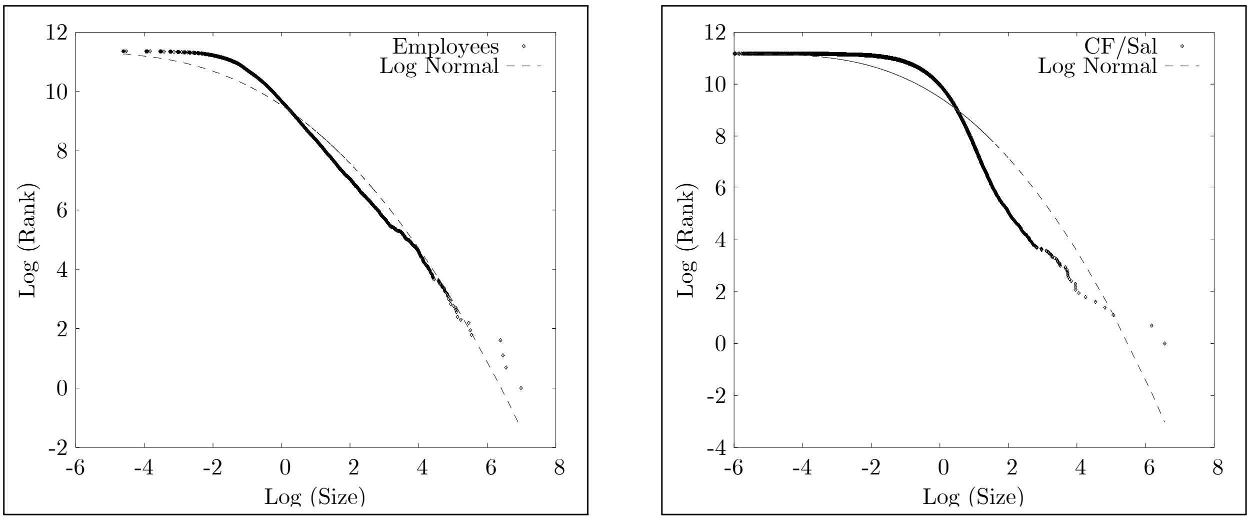


Figure 2: Pooled (Year-Standardized) Employees (Left) and CF/Sales (Right) Distributions. Log Rank vs. Log Size Plots.

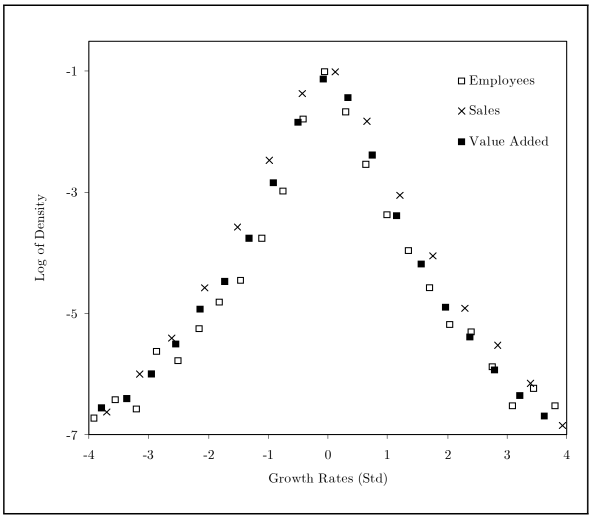


Figure 3: Pooled (Year-Standardized) Firm Growth Rates. Binned Empirical Densities vs. Laplace Fit.

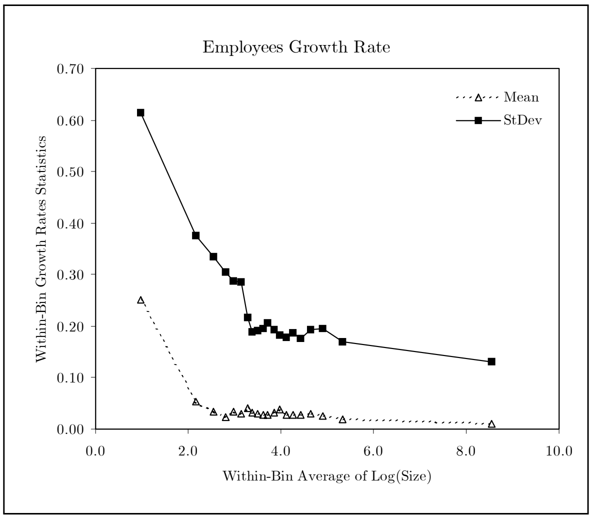
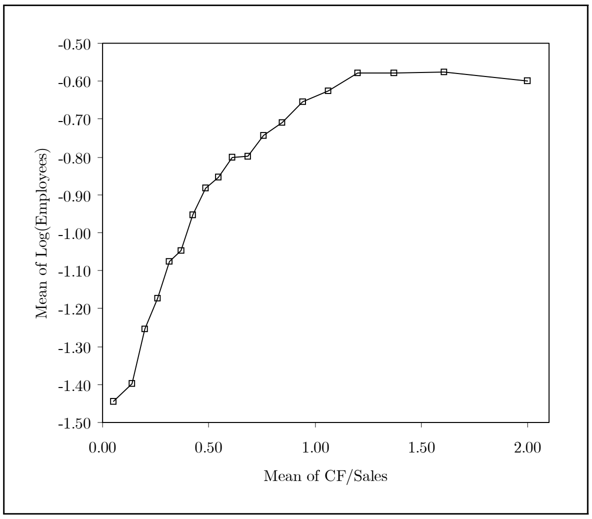


Figure 4: Mean and Standard Deviation of Pooled (Year-Std) Employees Growth Rates as a Function of (Within-Bins) Average log(Size). Bins computed as 5-percentiles.



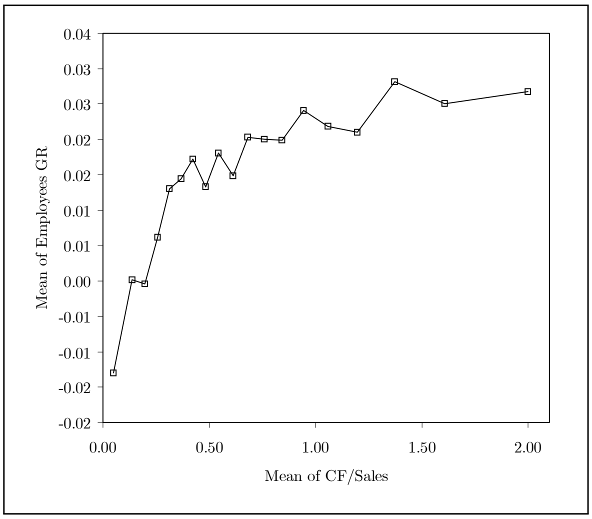


Figure 5: Mean of Pooled (Year-Std) Size (Left) and Growth Rates (Right) Distributions as a Function of (Within-Bins) Average CF/Sales. Bins computed as 5-percentiles.

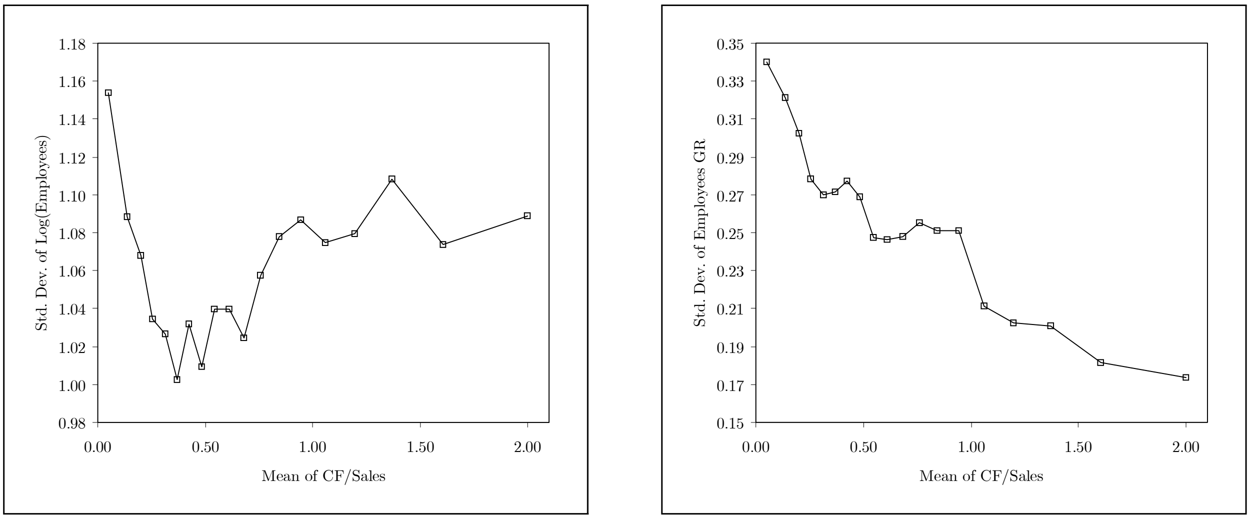


Figure 6: Standard Deviation of Pooled (Year-Std) Size (Left) and Growth Rates (Right) Distributions as a Function of (Within-Bins) Average CF/Sales. Bins computed as 5-percentiles.

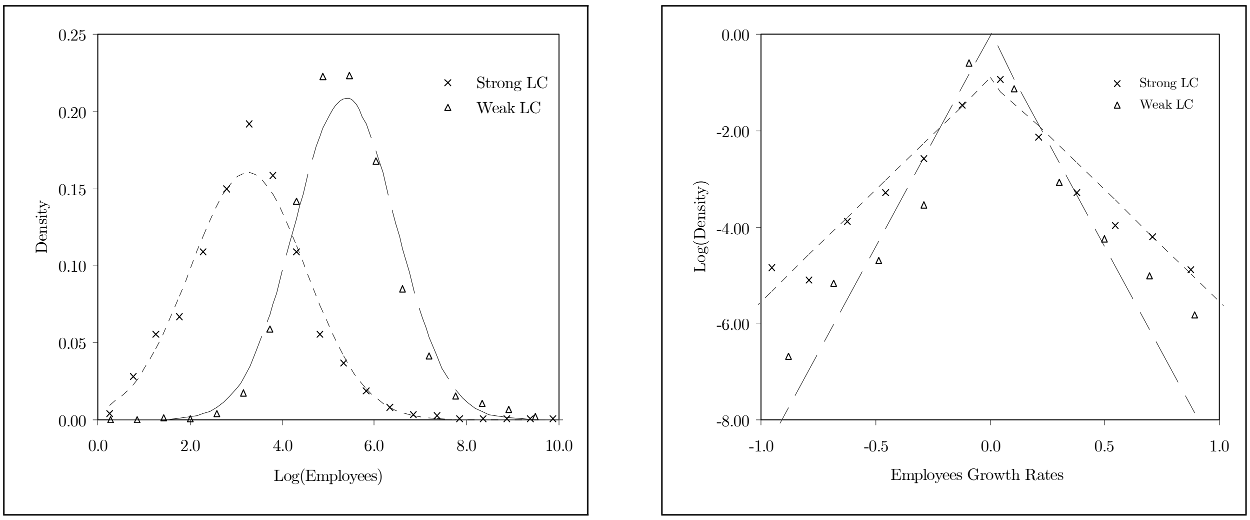


Figure 7: Pooled Log of Employees (Left) and Employees Growth Rate (Right) distributions conditional to CF/Sales. Strong Liquidity Constraints (LC) means CF/Sales in the 1st Decile. Weak Liquidity Constraints (LC) means CF/Sales in the 10th Decile.

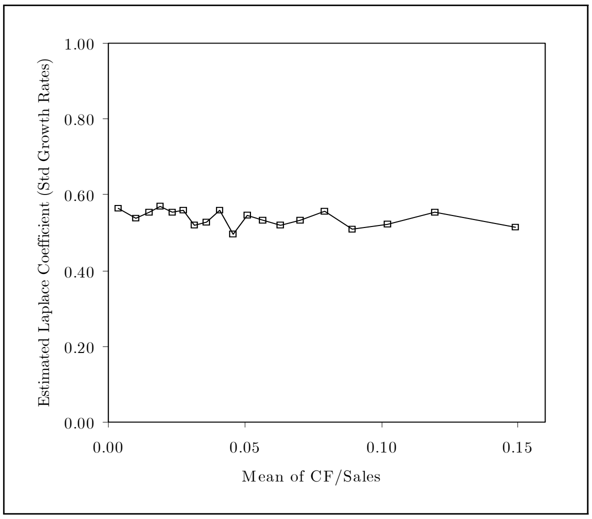


Figure 8: Estimated Laplace Coefficients for Laplace Fit of Employees Growth Rates (Zero-Mean, Unitary Variance) Conditional on (Within-Bins) Averages of CF/Sales. Bins computed as 5-percentiles.

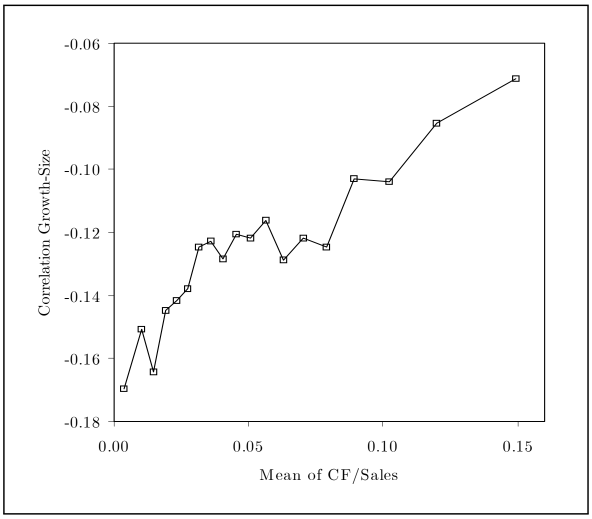


Figure 9: Correlation between Log(Employees) and Employees Growth Rates Conditional on (Within-Bins) Averages of CF/Sales. Bins computed as 5-percentiles.

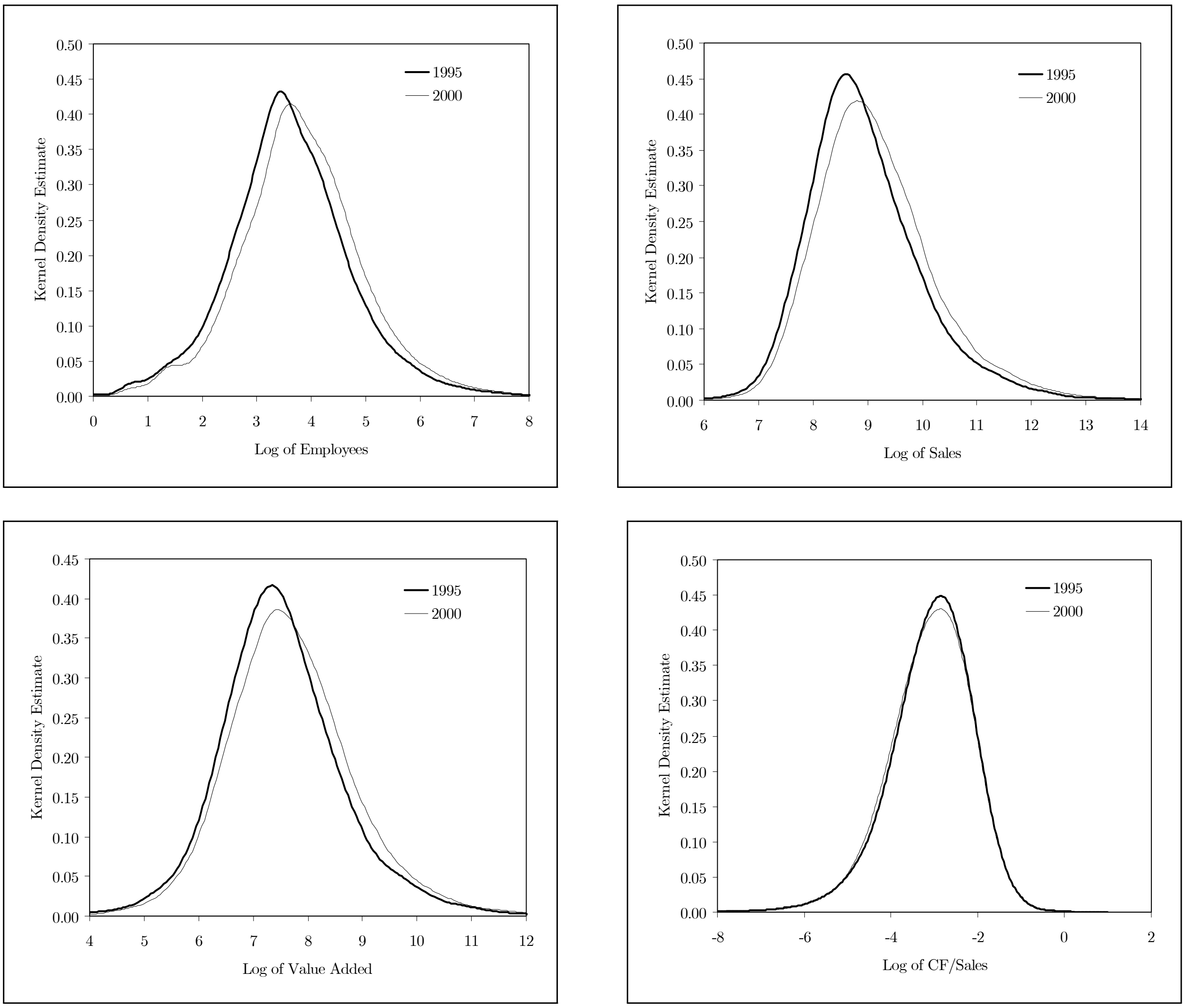


Figure 10: Non Parametric Kernel Density Estimates of Non-Standardized Size and CF/Sales Distributions. Top-Left: Employees. Top-Right: Sales. Bottom-Left: Value Added. Bottom-Right: CF/Sales (Log Scale). Kernel Density Estimates are performed employing a Normal Kernel and a 0.2 Bandwidth.

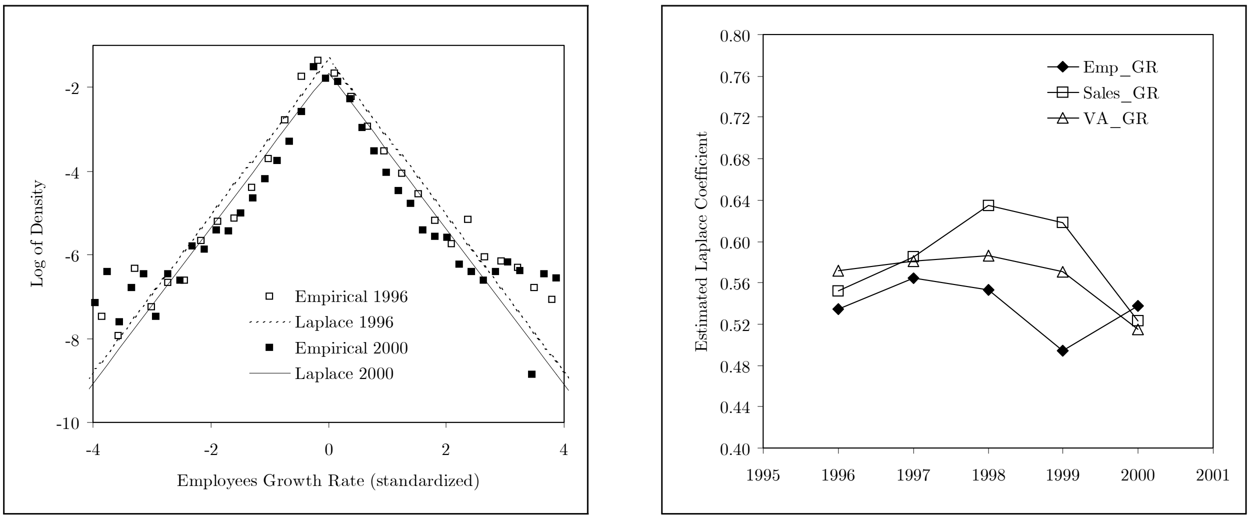


Figure 11: Shifts of Growth-Rates Distributions over Time. Left: Empirical Distributions and Laplace Fit for 1996 and 2000 Standardized Employees GR Distributions (Zero Mean, Unitary Variance). Right: Estimated Laplace Coefficient for 1996 and 2000 Standardized Employees, Sales, and Value-Added GR Distributions (Zero Mean, Unitary Variance).

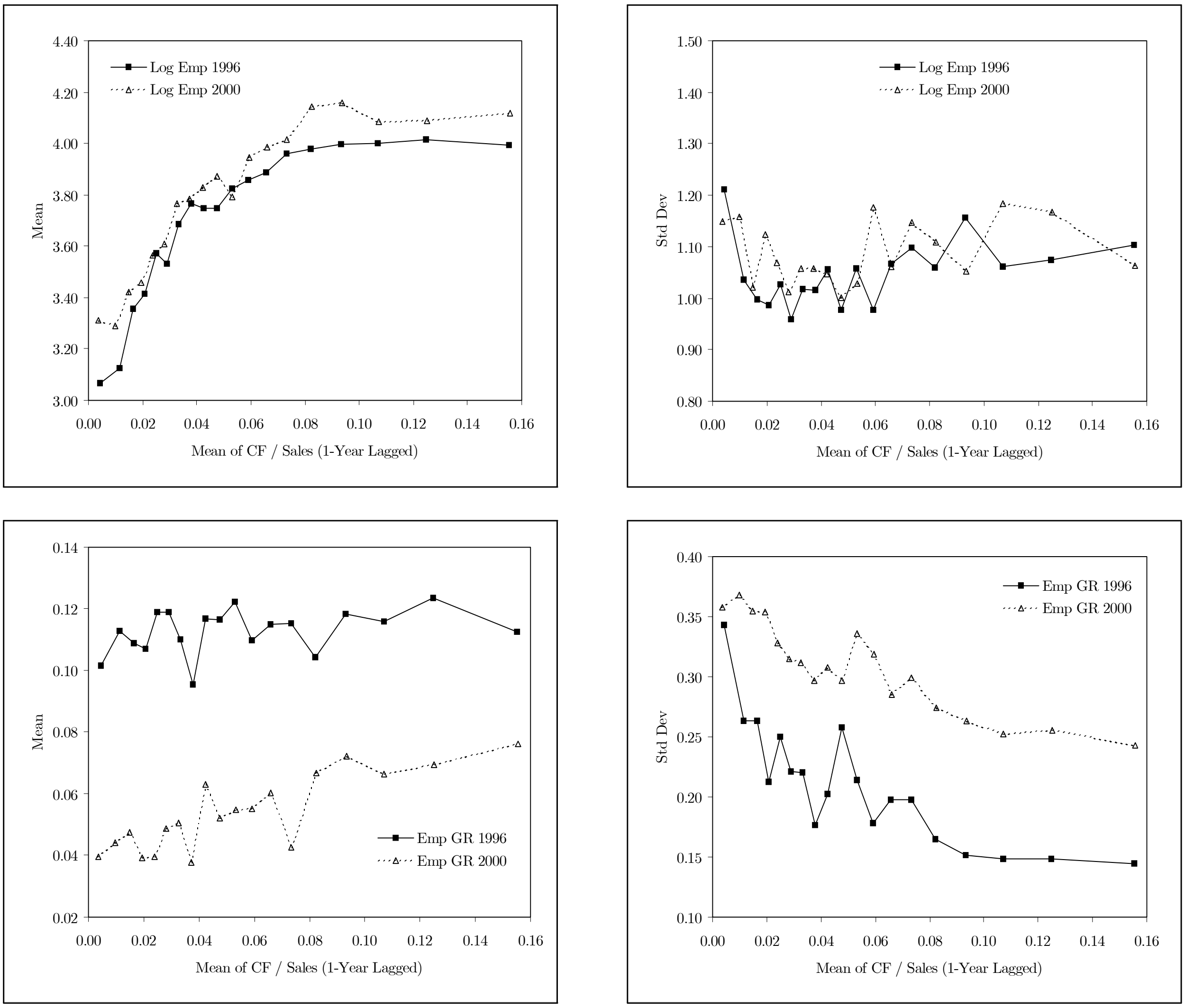


Figure 12: Average and Standard Deviation of Employee Size and Growth Year Distributions conditioned on (Within-Bins) Averages of (1-year Lagged) CF/Sales. Top-Left: Average of Log of Employees. Top-Right: St.Dev. of Log of Employees. Bottom-Left: Average of Employees GR. Bottom-Right: St.Dev. of Employees GR. Bins computed as 5-percentiles.

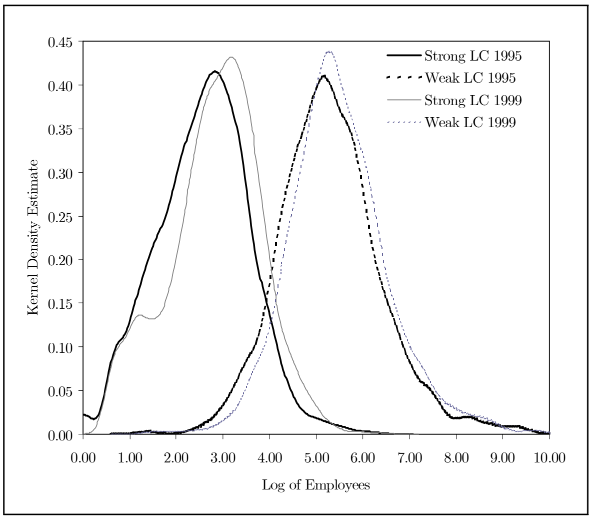


Figure 13: Non Parametric Kernel Density Estimation of Log of Employees Year Distributions conditional on CF/Sales. Strong Liquidity Constraints (LC) means CF/Sales in the 1st Decile. Weak Liquidity Constraints (LC) means CF/Sales in the 10th Decile. Kernel Density Estimates are performed employing a Normal Kernel and a 0.2 Bandwidth.

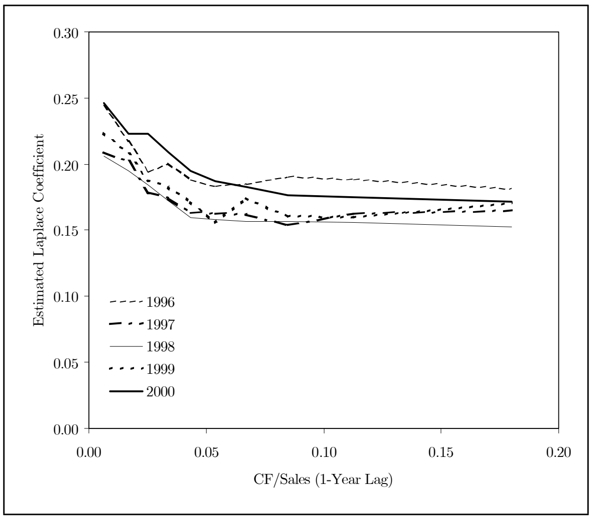


Figure 14: Estimated Laplace Coefficients for Laplace Fit of Employees Growth Rates Distributions conditional on CF/Sales.

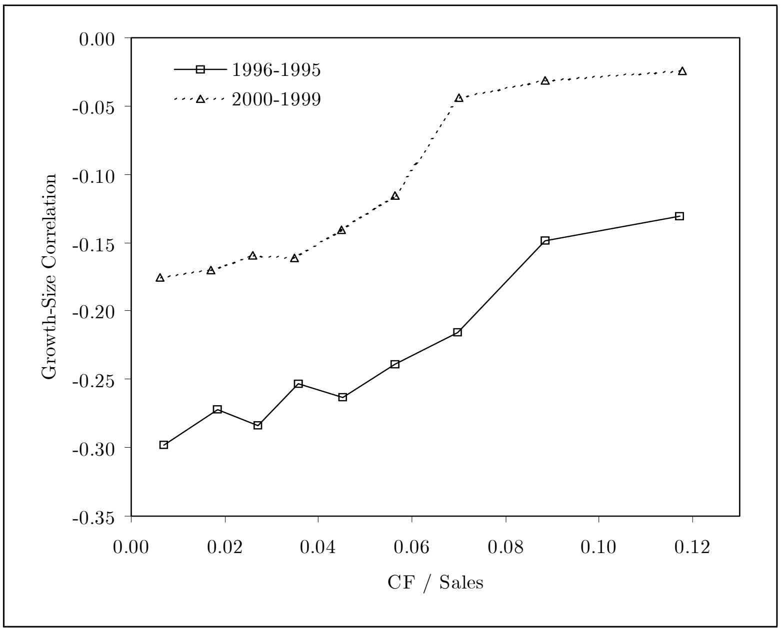


Figure 15: Correlation between Employees Growth Rate at time t and Size Distributions at time $t-1$, conditional on CF/Sales at time $t-1$. Bins computed as 10-percentiles.

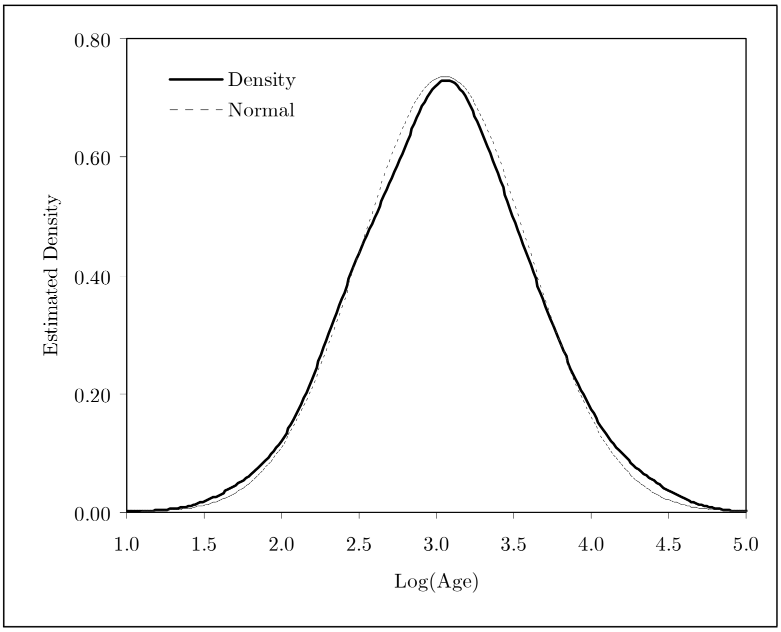


Figure 16: Non Parametric Kernel Density Estimation of Log of Age Distribution. Normal fit shown as dotted line. Kernel Density Estimates is performed employing a Normal Kernel and a 0.2 Bandwidth.

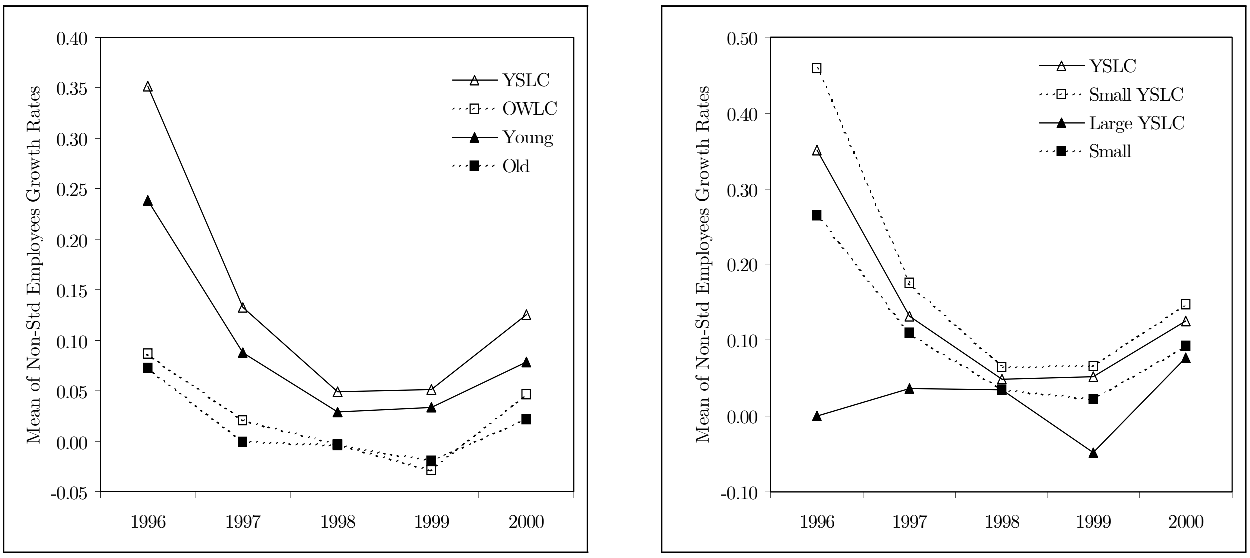


Figure 17: Average of Growth Rates Distributions Conditioned on Age, CF/Sales, and Size in 1995. YSLC: Young Firms with Small CF/Sales (1st Decile). OWLC: Old Firms with Large CF/Sales (10th Decile). Small (Large) Firms: Firms within the 1st (10th) Decile of 1995 Employees Distribution.

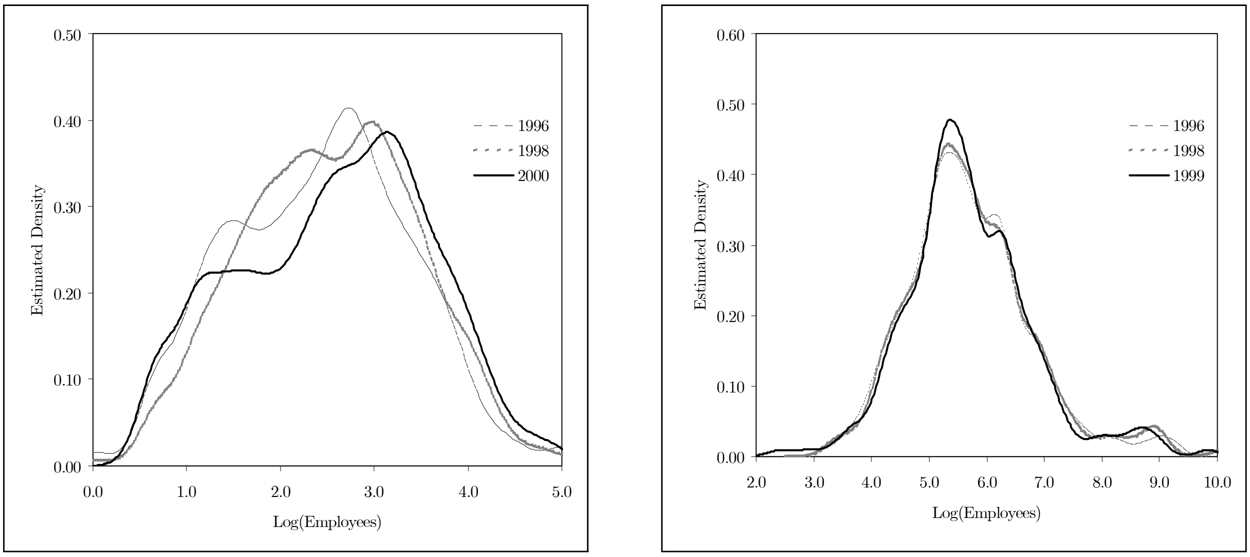


Figure 18: Non Parametric Kernel Density Estimation of Log of Employees Distributions over Time. Left: YSLC Firms. Right: OWLC Firms. YSLC: Young Firms (1st Decile) with Small CF/Sales (1st Decile). OWLC: Old Firms (10th Decile) with Large CF/Sales (10th Decile). Normal Kernel. Bandwith=0.2

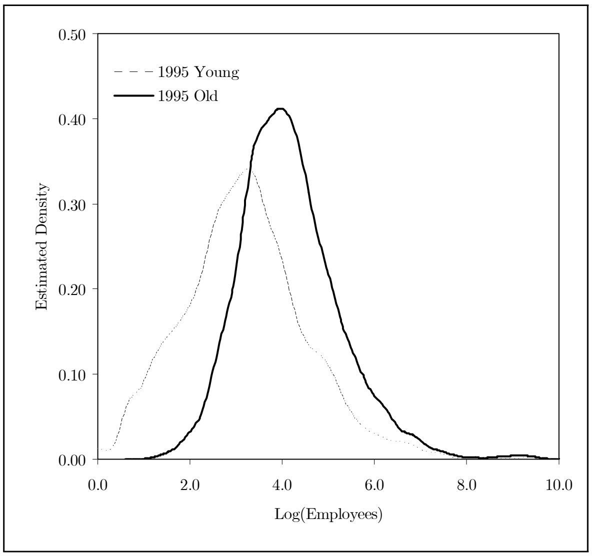


Figure 19: Non Parametric Kernel Density Estimation of Log of Employees Distribution. Young Firms vs. Old Firms in 1995. Young Firms: Firms in the 1st Decile. Old Firms: Firms in the 10th Decile. Normal Kernel. Bandwith=0.2

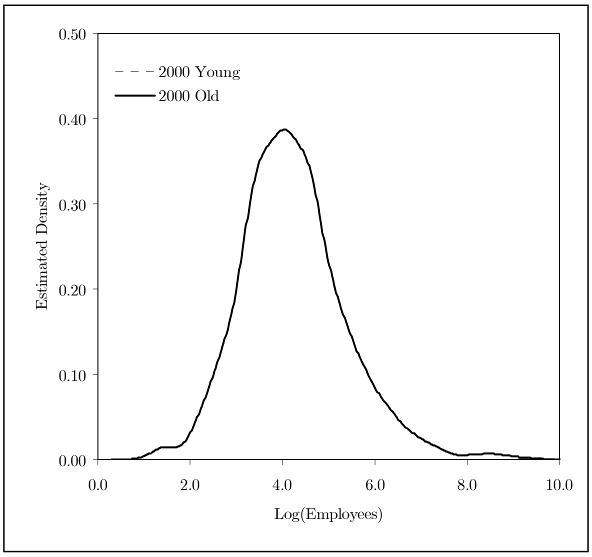


Figure 20: Non Parametric Kernel Density Estimation of Log of Employees Distribution. Young Firms vs. Old Firms in 2000. Young Firms: Firms in the 1st Decile. Old Firms: Firms in the 10th Decile. Normal Kernel. Bandwith=0.2

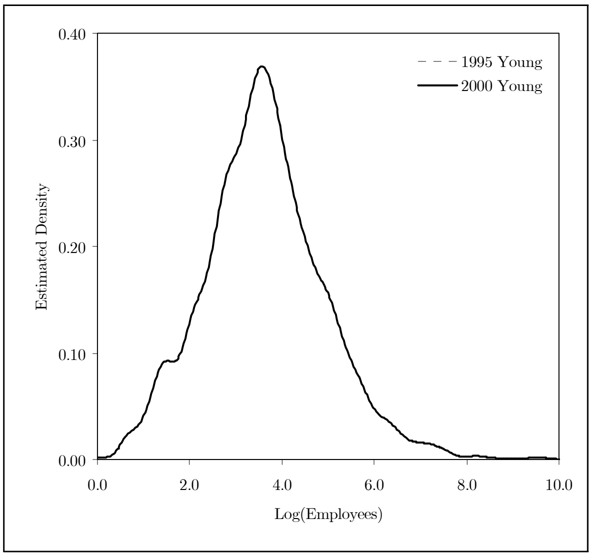


Figure 21: Non Parametric Kernel Density Estimation of Log of Employees Distribution. Young Firms in 1995 and 2000. Young Firms: Firms in the 1st Decile. Old Firms: Firms in the 10th Decile. Normal Kernel. Bandwith=0.2

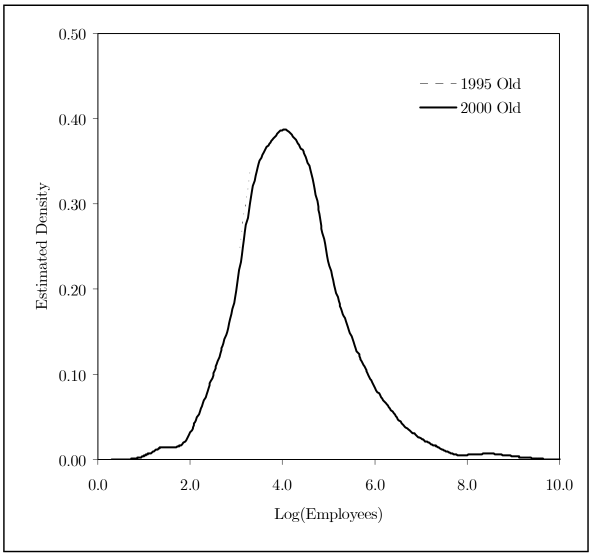


Figure 22: Non Parametric Kernel Density Estimation of Log of Employees Distribution. Old Firms in 1995 and 2000. Young Firms: Firms in the 1st Decile. Old Firms: Firms in the 10th Decile. Normal Kernel. Bandwith=0.2