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Some Statistical Investigations on the Nature and Dynamics of Electricity Prices

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Some Statistical Investigations on the Nature and Dynamics of Electricity Prices*

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Abstract

It is widely accepted that in liberalized electricity markets log-returns display fat-tailed densities. Besides qualitative assessments, so far precise characterizations of the shape of the distribution have been seldom provided. In this work, we characterize the conditional and unconditional probability density functions of daily electricity log-returns, and of the underlying shocks from the NordPool market, for each of the 24 hours, through a very flexible and general family of distributions, namely the Subbotin family.

Our study contributes with novel results in the field. First, we show that price fluctuations in electricity markets are additive in nature. We do this by exploiting a scaling relationship between price level and volatility, which is in turn a new result in the electricity markets literature. Second, in line with recent studies, we uncover the existence of multiple regimes in price dynamics, and we characterize the distributional shape for each of them. Interestingly, the shocks behind electricity price dynamics are approximately Laplace if one conditions on low price levels, and closer to a Gaussian in correspondence of high initial price levels.

These results are at variance with the evidence from financial markets. The peculiar non-storable nature of electricity, and the varying strength of correlations between bidding behaviors at different load levels are suggested as possible key factors behind the specificities of electricity markets outcomes.

Keywords: Electricity Markets, Subbotin Distribution, Laplace Distribution, Additive Process, Scaling.

JEL Classifications: C16, D4, L94.

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1 Introduction

As an outcome of the liberalization policies pursued in several countries from the 80s on, the so-called day-ahead electricity market provides economists with a very challenging phenomenon. Electricity cannot be economically stored, which implies that demand and supply must be continuously balanced, so that the market price mainly reflects the demand and supply conditions prevailing at the very moment it has to be delivered to final users. Then, rather complex market systems have been set up, with the aim of reaching a reasonable trade-off between economic efficiency and system reliability. These systems are built around a market operator, whose task is to manage uniform-price, sealed-bid, bilateral auctions in order to construct aggregate demand and supply curves, and to determine equilibrium prices and quantities. The knife-edge character of such a price setting mechanism is furtherly pushed to the extreme by a very low price elasticity of demand, and by technical constraints which time by time lead to network congestion (see [18] and [17] among others). As an implication, the structure of price fluctuations in power markets is expected to be extremely interesting.

The existence of multiple periodic patterns, spikes, and time dependent volatility in one and the same process has indeed spurred efforts towards appropriate modelling of the time dependencies in electricity prices and their changes. Less attention has been paid to characterizing the shape of the distribution of electricity log-returns. While it is widely recognized that daily price fluctuations in energy markets display very fat tails (see [33], [23], [19], [1]), modelling of the probability density functions is still in its infancy. Such a lack of contributions in this area might be attributable to the strong serial dependencies in log-returns and in their volatility, which make it hard to obtain a desirable i.i.d. sample.

In this paper, we characterize the conditional and unconditional probability density functions of daily electricity log-returns, and of the underlying shocks from the NordPool market, for each of the 24 hours, through a very flexible and general family of distributions, namely the Subbotin family. Our study contributes with many novel features and results in the field. First, we show that price fluctuations in electricity markets are additive in nature. We do this by exploiting a scaling relationship between price level and volatility, which is in turn a new result in the electricity markets literature. Second, in line with recent studies ([16]), we uncover the existence of multiple regimes in price dynamics. However, for each one we also characterize the distributional nature. The shocks underlying electricity log-returns are approximately Laplacian, if one conditions on prices below some price threshold, and closer to Gaussian above that.

The plan of the paper is the following. After reviewing some background literature in Section 2, we introduce the Subbotin family of distributions (Section 3), as well as the dataset and variables used in the present work (Section 4). Section 5 is devoted to the statistical analysis of log-returns. Time dependencies are dealt with in Section 6, and the distributions of the shocks underlying electricity price dynamics are fitted in Section 7. Conclusions and discussion in Section 8 pave the way for future research.

2 Background

Virtually all analyses of electricity log-returns find evidence of fat-tailed distributions, although through different approaches.

Widespread is the so-called jump-diffusion model, stemming from the early work in [25], and from [24]. Within this approach, usually one models log-returns as a mean-reverting component, plus a Poisson number of jumps, with a given intensity, say λ. Each jump is drawn from a given distribution with first cumulant equal to θ. The distribution of log-returns is thus conceived as a mixture of distributions with Poisson mixing density. It turns out to be asymmetric if and only if $\theta \neq 0$, and leptokurtic if and only if $\lambda > 0$. Among the several applications to electricity markets, we signal [9], [19], [23], [14], [15], and [31]. All of them find $\lambda > 0$, which indirectly hints at fat tails.

In a more direct approach, one fits a power law to the tails of the distribution. The relevant parameter is the so-called tail index, which sets how quickly the tails of the distribution decay. A positive tail index is evidence of fat tails. In [1] values around 0.25 are found for Omel (Spain), 0.43-0.52 for the NordPool, 0.18-0.20 for Leipzig Power Exchange, 0.33-0.42 for ElecPool. In [6], the estimate for the NordPool log-returns right tail is 0.30. [10] find values between 0.31 and 0.38 for the PJM, and around 0.25 for the CalPX. Similar results are reported in [11] for the Australian National Energy Market.

Finally, it is worth noting that very few studies have so far dealt with the body of the distribution. Such an approach allows to give a more complete description of the distribution under analysis, including features such as its peakedness. In [10] a Cauchy-Laplace mixture is fitted to PJM and CalPX log-returns. [13] fit a Generalized Hyperbolic to NordPool log-returns. In both cases, non-Gaussian behavior is detected.

3 The Subbotin Family of Distributions

First used in economics by [5], the Subbotin probability density function reads (see [29]):

$$f(x) = \frac{1}{2ab^{1/b}\Gamma(1+\frac{1}{b})} e^{-\frac{1}{b}\left|\frac{x-\mu}{a}\right|^{b}} \qquad (1)$$

with parameters a (width), b (shape), and μ (position). $\Gamma(.)$ is the gamma function. The Subbotin reduces to a Laplace if $b = 1$, and to a Gaussian if $b = 2$.[1] As b gets smaller, the density becomes fatter-tailed and more sharply peaked.[2]

It has been proved by [32] that the Subbotin can be obtained as a mixture of normal distributions, with α-stable mixing density. A special case of this theorem holds for the Laplace, which is a geometric-stable law ([20]). Hence, a Subbotin approach is consistent with the intuition behind jump-diffusion processes. Moreover, compared to previously fitted distributions, the Subbotin has a more parsimonious specification: just 3 parameters need to be estimated (2 if the data are demeaned). Finally, the Subbotin distribution allows greater flexibility with respect to the tail behavior, whereas distributions such as the Generalized Hyperbolic only admit exponential tail decay.

4 Data and Variables

As a rule, in day-ahead wholesale electricity markets, each day 24 auctions are performed simultaneously, in order to determine prices and quantities for each of the 24 hours of the

[1] The Subbotin reduces to a degenerate if $b = 0$, and to a Continuous Uniform if $b = \infty$.

[2] The Laplace case deserves special interest, in that it has been widely detected in very diverse phenomena: from price changes in financial markets ([21]) to the growth of economic organizations (firms in Stanley et al., 1996, and in [5]; countries in [22]).

subsequent day. Due to non-storability, withdrawals and injections of power must be continuously balanced. Towards this aim, a market operator constructs aggregate demand and supply functions on the grounds of the bids collected. The electricity price for hour h - what we are interested in - is set at the intersection between the aggregate curves for hour h. Daily changes are thus due to the joint dynamics of demand and supply curves, which in turn reflect a number of technological, behavioral and institutional determinants, related to demand characteristics (e.g. very low price elasticity, periodic patterns), and to bidding strategies. The latter are enabled by the specific institutional design, and bounded by technological constraints (network transmission constraints, ramp rates, and the balance between fixed, quasi-fixed and marginal costs are some examples).[3]

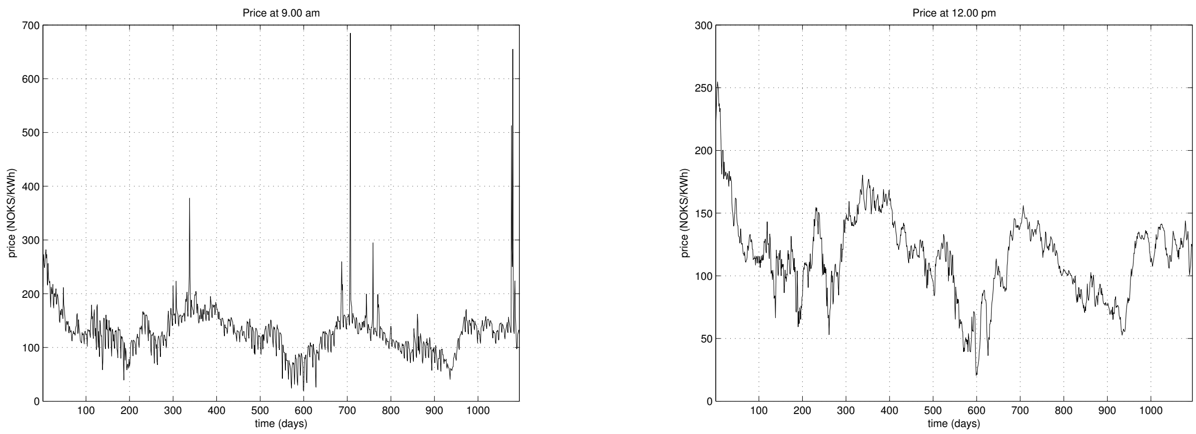

Figure 1: Time series of Price P for the sample period January 1997-December 1999. Two different hours of the day are reported.

In this study, we focus on the NordPool, the market covering Norway, Sweden, Finland and Denmark. Although the market was set up in 1992, we focus on just the 1997-1999 period (1095 observations per hour), which preliminary Kolmogorov-Smirnov tests indicate as a fairly stable one. Fig. 1 depicts the time series of prices for two hours, 9 am and 12 pm. Notice the multiple periodic patterns (seasonal in both, weekly at 9 am), and the huge spikes in the 9 am series.

All this given, daily log-return series are built for each hour. For the sake of interpretation, we keep hours separated, and analyze them as though they were outcomes of independent stochastic processes.[4] Let us denote the price at hour h of the day t as $P_{h,t}$, and let us define the logarithmic price as $p_{h,t} = \log P_{h,t}$. The normalized log-price reads:

$$p^*_{h,t} = p_{h,t} - < p_{h,t} >_t \tag{2}$$

where $< . >_t$ denotes a sample average computed along the time dimension. The daily log-return is given by:

$$r_{h,t} = p_{h,t} - p_{h,t-1} \tag{3}$$

The normalized log-return is used as well:

[3]For details on the mentioned constraints, see [7], and [26], among others.

[4]This is just a preliminary approximation. We acknowledge there are significant comovements between log-returns in different hours, if anything, because prices for all hours are set at once, and each generator can - and usually does - bid for many hours.

$$r_{h,t}^* = p_{h,t}^* - p_{h,t-1}^* \tag{4}$$

5 Statistical Properties of Log-Returns

As a first step, we explore some basic properties of the 24 $r_{h,t}$ series.

5.1 Preliminary Analysis

Log-returns for selected hours (9 am and 24 pm) are plotted in Fig. 2, top panels. Table 1 summarizes descriptive statistics for daily log-returns, hour by hour. Standard deviations are between 0.066 and 0.179, pretty high values, if compared to log-returns in foreign exchange markets, for instance.[5] The highest volatility is observed between 7 am and 10 am, i.e. the beginning of the working day. Log-returns are negatively skewed during the evening, while there is positive or null asymmetry in all other hours, especially at early morning and late afternoon. Kurtosis ranges from 8.5 (12 pm) to 40.3 (6 pm), and it is generally higher during night hours.

A look at the autocorrelograms in Fig. 2 (middle panels) reveals that different dynamic structures characterize daily log-returns in different hours. Significantly positive correlations at weekly-to-monthly lags signal a periodic pattern in hours from 6 am to 10 pm. Night-time hours (i.e. 11 pm - 5 am) show a much simpler structure: weekly and monthly periodicities are basically absent. The slow decay of weekly autocorrelations might signal an underlying long-range dependence in NordPool energy log-prices.[6]

From Fig. 2 (bottom panels) it is also clear that volatility is autocorrelated for all hours. Autocorrelograms of absolute log-returns $|r_{h,t}|$ uncover the existence of time dependencies in high order moments, with some significant weekly structure during the day.

5.2 The Distribution of Log-Returns

A priori, one would expect different autocorrelation structures to map into quite different distributional shapes. Surprisingly, this intuition happens to be wrong.

Table 1 displays Maximum Likelihood estimates of the Subbotin parameters a and b for normalized log-returns.[7] Log-return densities for all hours are well described by a Subbotin with shape parameter b between 0.572 (8 am) and 0.822 (11 pm). See also Fig. 3. Stated otherwise, NordPool log-returns have fatter-tailed and more sharply peaked densities even with respect to a Laplace law. This is even more true of night hours. Consistent with summary statistics, the highest values for the width parameter a, and relatedly the highest standard deviations, are detected for the early morning hours.

We also fitted asymmetric Subbotin laws, with no significant changes in the estimated parameters. Hence, log-returns densities can be considered symmetric up to a fair degree of confidence.

[5] As reported in [8], the standard deviation of the daily log-return of the USD-DEM exchange rate (between 1987 and 1993) equals 0.019. Similar values are found for other currencies.

[6] On this issue, see [27] and the references therein.

[7] See [4] for documentation on the estimation procedures used here.

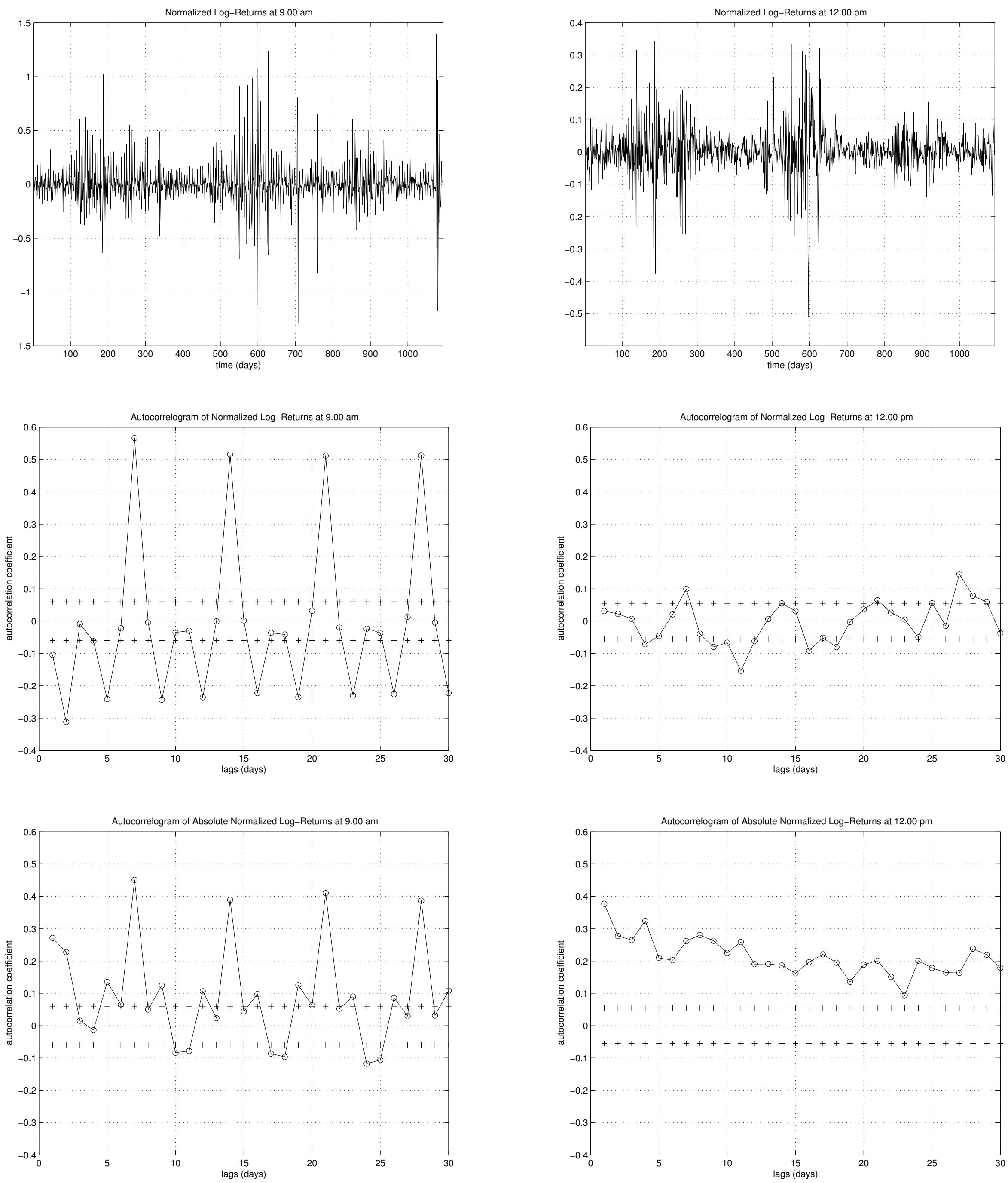

Figure 2: NordPool daily log-returns, for 9 am (left) and 12 pm (right). Top: time series plots. Middle: autocorrelograms of r^*. Bottom: autocorrelograms of $|r^*|$.

Table 1: The estimated Subbotin parameters of the normalized log-Returns r^* together with descriptive statistics in different hours.

Hour	**Subbotin Param.**		**Descriptive Statistics**							
	b Param.	a Param.	Mean	Std.	Skewness	Kurtosis	Min.	Max.	Median	Obs.
1	0.720	0.038	-0.000	0.074	-0.271	11.5	-0.488	0.581	-0.001	1461
2	0.643	0.042	-0.000	0.097	-0.253	27.6	-0.897	1.090	-0.000	1461
3	0.574	0.045	-0.000	0.121	0.044	26.2	-0.958	1.360	-0.001	1461
4	0.629	0.048	-0.000	0.112	0.711	30.8	-0.908	1.450	-0.001	1461
5	0.636	0.050	-0.000	0.115	0.686	23.6	-0.789	1.360	-0.001	1461
6	0.632	0.054	-0.000	0.124	0.549	17.5	-1.030	0.961	-0.003	1461
7	0.585	0.060	-0.000	0.147	1.160	16.1	-1.140	1.210	-0.001	1461
8	0.572	0.069	-0.000	0.165	1.240	11.7	-1.000	1.250	-0.001	1461
9	0.579	0.075	-0.000	0.179	0.882	13.4	-1.290	1.390	-0.010	1461
10	0.647	0.069	-0.000	0.147	0.890	13.3	-1.050	1.260	-0.010	1461
11	0.713	0.063	-0.000	0.121	0.685	12.7	-0.951	1.100	-0.008	1461
12	0.736	0.058	-0.000	0.108	0.770	14.2	-0.912	1.030	-0.008	1461
13	0.740	0.056	-0.000	0.104	0.788	14.5	-0.927	0.953	-0.009	1461
14	0.728	0.056	-0.000	0.107	0.911	14.6	-0.952	0.988	-0.009	1461
15	0.706	0.056	-0.000	0.109	1.040	14.1	-0.929	0.981	-0.009	1461
16	0.716	0.055	-0.000	0.106	1.170	12.8	-0.875	0.823	-0.010	1461
17	0.704	0.052	-0.000	0.105	1.290	18.6	-0.752	1.160	-0.009	1461
18	0.699	0.050	-0.000	0.108	0.360	40.3	-1.360	1.300	-0.007	1461
19	0.776	0.047	-0.000	0.086	1.070	17.2	-0.646	0.939	-0.006	1461
20	0.799	0.043	-0.000	0.076	-0.045	10.2	-0.582	0.490	-0.002	1461
21	0.814	0.042	-0.000	0.074	-0.083	15.1	-0.616	0.644	-0.001	1461
22	0.786	0.039	-0.000	0.072	-0.386	15.2	-0.659	0.550	-0.000	1461
23	0.822	0.038	-0.000	0.066	-0.483	14.4	-0.671	0.434	-0.001	1461
24	0.814	0.038	-0.000	0.066	-0.340	8.5	-0.514	0.339	-0.001	1461

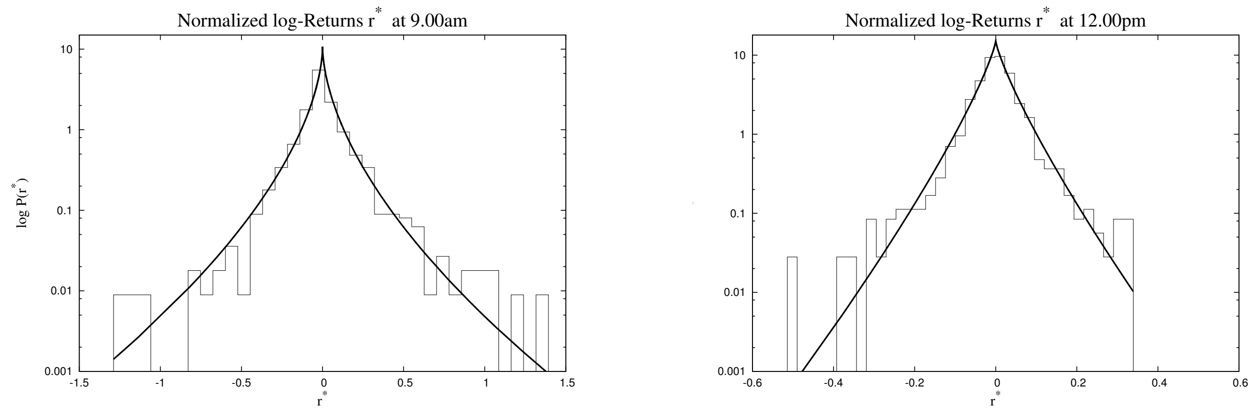

Figure 3: Binned empirical densities of the normalized log-returns r^* together with the Subbotin Maximum Likelihood fit. Notice the log scale on the y-axes. (see Tab. 1 for the values of the estimated coefficients and other descriptive statistics).

6 Time Dependencies

The persistent and systematic nature of the time dependencies in electricity log-returns and in their absolute values emerges very clearly. Our results might have been affected by such dependencies to a considerable extent. Furthermore, one might want to study the distribution of some more fundamental driving force. Filtering the time dependencies away is thus advisable.

In order to do so, we first deal with the linear autocorrelation structure, and then we tackle the issue of non-linear dependencies via the exploration of scaling relationships between price level and volatility.

6.1 Application of a Linear Filter

Linear filtering is accomplished by acknowledging that any stationary time series, $y = (y_1, ..., y_T)'$, can be represented, up to second order, by a Cholesky decomposition of the series' covariance matrix and a set of uncorrelated residuals.

The filtering procedure is based on the Cholesky factor algorithm, a model-free bootstrapping method introduced by [3].[8] It goes through the following steps:

1. Estimate the covariance matrix Σ of the vector r^*, as the Toeplitz matrix built on the autocovariance vector γ[9];

2. Calculate C as the Cholesky factor of Σ, i.e. C: $CC' = \Sigma$;

3. Extract the linearly uncorrelated, standardized residuals ϵ as follows:

[8] See also [12] and [2].

[9] A Toeplitz matrix is a matrix which has constant values along negative-sloping diagonals. In our case, given the autocovariance vector $\gamma = (\gamma_0, \gamma_1, ..., \gamma_T)$, the matrix Σ is defined as follows:

$$\left| \begin{array}{ccccc} \gamma_0 & \gamma_1 & \gamma_2 & \cdots & \gamma_T \\ \gamma_1 & \gamma_0 & \gamma_1 & \cdots & \gamma_{T-1} \\ \gamma_2 & \gamma_1 & \gamma_0 & \cdots & \gamma_{T-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \gamma_T & \gamma_{T-1} & \gamma_{T-2} & \cdots & \gamma_0 \end{array} \right|$$

$$\epsilon = C^{-1}r^* \tag{5}$$

On these grounds, one can envisage log-returns r^* as generated by applying a linear dynamical structure to the underlying ϵ shocks.

6.2 Scaling

Because the Cholesky filter is a linear one, its outcome retains the heteroskedastic properties of the original series r^*. It is thus necessary to try and construct subsamples which at least approximately meet the stationarity requirements. In order to do so, we investigate upon the existence of some relationship between the volatility of ϵ and the initial price level P.[10]

For any given hour, price levels P are binned, and standard deviations of the associated ϵ are computed. Next, all such data are pooled together, and the log-standard deviations are regressed on the logarithm of the mean price level within the corresponding bins, and on a constant.

We find that the width of the distributions of the ϵ shocks scales as a power law of the initial electricity price level:

$$\sigma(\epsilon_{h,t}|P_{h,t-1}) \propto P^{\beta}_{h,t-1} \tag{6}$$

with a scaling exponent β around -1 for most hours (see Table 3). In other words, there exists a negative relationship between the price level and the conditional volatility of shocks: when the price is high, its dynamics over the subsequent day tends to be relatively tranquil, while very noisy price changes follow low price realizations. More specifically, σ tends to behave as $\frac{1}{P}$. Fig. 4 provides graphical illustration of this relationship for two hours (8 am and 12 pm).

As a deeper implication, this results provides circumstantial evidence that the electricity price process is as additive process. The usual logarithmic transformation, and the entailed definition of log-returns, both justified for multiplicative processes (e.g. stock market prices), turn out to be inappropriate for the analysis of electricity prices.

To see this, let us assume the sequence P_t is a realization of an additive process, such that $P_t = P_{t-1} + \epsilon_t$, with ϵ_t a zero mean, finite variance shock. Let us then suppose the process is believed to be a multiplicative one, and that log of the former, additive representation is taken. This yields: $\log P_t = \log(P_{t-1} + \epsilon_t) = \log(P_{t-1}(1 + \frac{\epsilon_t}{P_{t-1}})) = \log P_{t-1} + \log(1 + \frac{\epsilon_t}{P_{t-1}}) \approx \log P_{t-1} + \frac{\epsilon_t}{P_{t-1}}$; i.e., a process with the same scaling relationship as we have detected here.

7 Distributions of Shocks

We can now study the conditional and unconditional distributions of the underlying shock sequence ϵ, whose basic properties for various hours are reported in Table 2.

[10] This approach has been chosen after having unsuccessfully applied GARCH models. An alternative way could be to fit models with long-memory in volatility.

Table 2: The estimated Subbotin parameters of the residuals ϵ together with descriptive statistics in different hours.

Hour	**Subbotin Param.**		**Descriptive Statistics**							
	b Param.	a Param.	Mean	Std.	Skewness	Kurtosis	Min.	Max.	Median	Obs.
1	1.004	0.680	-0.000	1.000	-0.398	8.9	-7.000	7.850	0.025	1461
2	0.930	0.626	0.000	1.000	-0.400	17.5	-8.570	9.040	0.030	1461
3	0.946	0.633	0.000	1.000	-0.170	17.4	-8.750	10.000	0.049	1461
4	0.944	0.635	-0.000	1.000	0.202	18.6	-7.320	10.700	0.032	1461
5	0.945	0.640	0.000	1.000	0.275	15.2	-6.780	9.580	0.023	1461
6	0.970	0.653	0.000	1.000	-0.420	12.9	-8.120	8.740	0.047	1461
7	0.991	0.666	-0.000	1.000	-0.660	13.4	-8.990	8.120	0.014	1461
8	1.055	0.713	-0.000	1.000	-0.169	5.8	-6.300	6.590	0.024	1461
9	1.026	0.690	0.000	1.000	0.050	8.3	-6.490	7.460	0.014	1461
10	1.079	0.721	0.000	1.000	0.029	7.6	-7.380	7.190	-0.005	1461
11	1.154	0.762	-0.000	1.000	-0.146	5.2	-7.700	5.090	-0.001	1461
12	1.186	0.776	0.000	1.000	-0.184	4.8	-7.700	5.330	0.004	1461
13	1.212	0.787	-0.000	1.000	-0.161	4.7	-8.180	5.120	-0.005	1461
14	1.229	0.793	0.000	1.000	-0.093	4.8	-8.260	5.280	-0.019	1461
15	1.205	0.783	-0.000	1.000	-0.038	4.4	-7.700	5.160	-0.044	1461
16	1.170	0.767	-0.000	1.000	0.004	4.1	-7.090	4.700	-0.044	1461
17	1.079	0.719	0.000	1.000	-0.048	7.2	-6.990	7.270	-0.016	1461
18	1.006	0.667	-0.000	1.000	-0.305	19.9	-11.100	8.970	-0.008	1461
19	1.110	0.738	-0.000	1.000	-0.085	7.0	-7.320	7.310	-0.020	1461
20	1.191	0.772	0.000	1.000	-0.508	6.2	-8.370	5.000	0.026	1461
21	1.118	0.733	-0.000	1.000	-0.611	10.6	-9.660	6.230	-0.020	1461
22	1.116	0.729	-0.000	1.000	-0.835	12.8	-10.300	5.560	-0.006	1461
23	1.102	0.730	0.000	1.000	-0.767	9.6	-9.010	4.810	0.034	1461
24	1.095	0.736	-0.000	1.000	-0.494	4.9	-7.340	4.550	0.032	1461

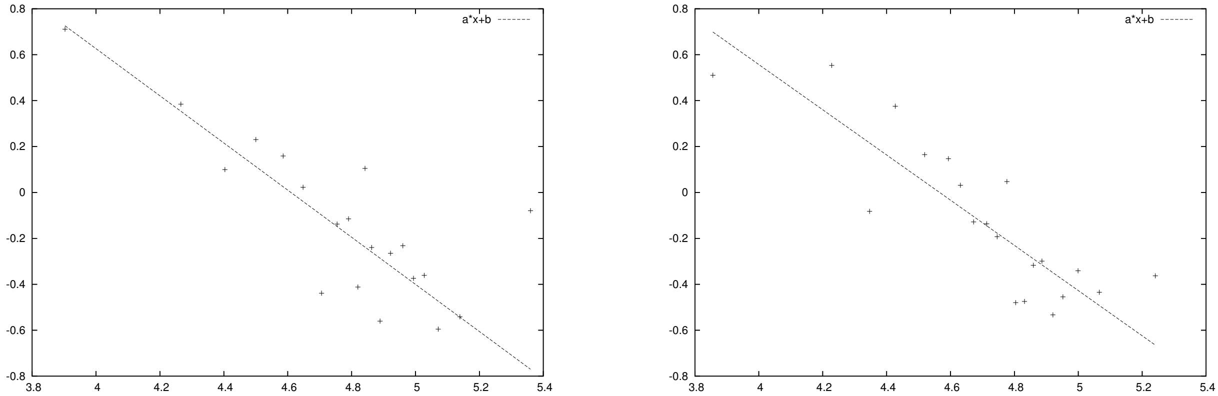

Figure 4: The volatility of the ϵ shocks, σ, scales as a power law of the initial price level P. Two hours are shown: 8 am (left), 12 pm (right). See Tab. 3 for the values of the estimated coefficients in all the 24 hours.

7.1 Unconditional Distributions

First, we fit the Subbotin to the 24 series of the ϵ shocks.[11] As data are standardized by definition, only 2 parameters need to be estimated. Results in Table 2 make clear that the shocks underlying the electricity log-returns are well described by Laplace laws: regardless of the hour considered, the shape parameter b is quite close to 1. Fitted Subbotin distributions are superimposed to the empirical ones in Fig. 5. However, some deviation from the Laplace shape exists: night hours display slightly lower b than peak-load hours. Despite the apparently systematic nature of such deviations, perhaps they are too small to be significant. Again, estimating asymmetric Subbotin models does not yield significant differences with respect to the patterns detected here.

It is worth noting that the filtered shocks ϵ are less leptokurtic than the log-returns r. At first sight, this can look surprising. Indeed, if one thinks of the moving average representation of r, the latter can be conceived as a weighted sum of the ϵ shocks. If the central limit theorem held, the shape of the distribution of r ought to be closer to Gaussian. However, one of the assumptions behind the CLT, i.e. homoskedasticity of the ϵ shocks, is not satisfied here. Our conjecture is that non-linear time dependencies contribute to increasing the fat-tailedness of electricity log-returns.

7.2 Conditional Distributions

Based on the detected scaling relationship, we can now group data in rather homoskedastic classes. We choose 6 equipopulate groups of 162 observations each, dubbed from I to VI in ascending price order.[12] An interesting point here is that, according to the detected scaling relationship, higher (lower) volatilities correspond to lower (higher) price levels. But, to the extent that price levels are in a one-to-one correspondence with demand levels, they proxy for

[11] For a correct interpretation of the findings to be described, it is worth noting that these shocks display an autocorrelated volatility structure.

[12] Here we report minima and maxima prices (in NOKs/kwh) for each of the 6 groups, for 9 am. Group I: 70.39-98.31. Group II: 98.45-117.22. Group III: 117.23-128.82. Group IV: 128.87-138.75. Group V: 138.81-153.13. Group VI: 153.13-190.18. Because groups are equipopulated, group boundaries for other hours are generally different. Bins corresponding to prices below the minimum of group I and above the maximum of group VI have been selected out. The main reason is that log-returns associated to very high prices display a spiky behavior. Thus, they can hardly be thought of as homegeneous to log-returns in other groups.

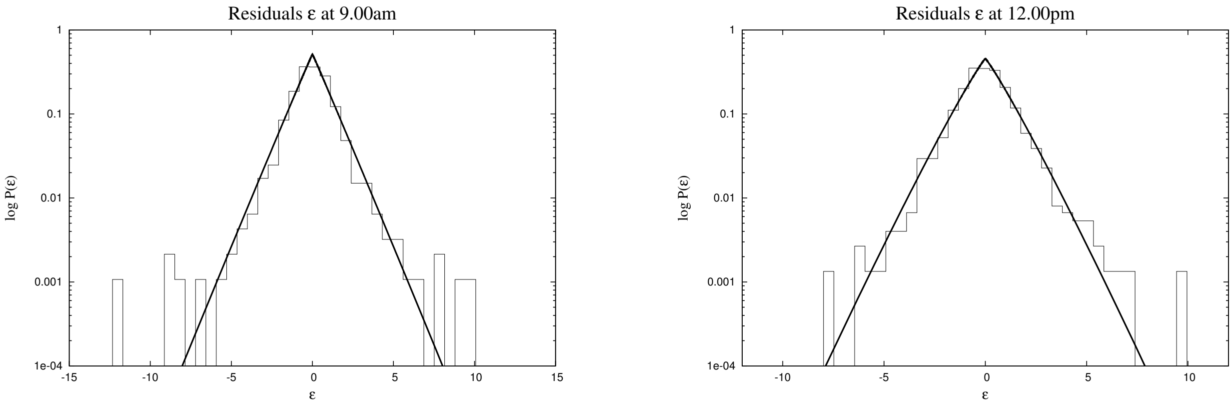

Figure 5: Binned empirical densities of the residuals ϵ together with the Subbotin Maximum Likelihood fit. Notice the log scale on the y-axes. (see Tab. 2 for the values of the estimated coefficients and other descriptive statistics).

system capacity utilization rates, a variable which is commonly recognized as an extremely important one in shaping generators' bidding behavior.

A Subbotin distribution is fitted to each subsample, yielding the results in Table 3. In order to easen the interpretation of results, in Fig. 6 we plot kernel estimates of the relationship between estimated Subbotin parameters (a and b) versus mean prices for the corresponding bins, for all hours pooled together.

What we find is that, up to approximately a price equal to 110 NOKs/MWh, estimated b's cluster around 1, only to increase up to a region around 1.5. Such an increase is not gradual: rather, it looks like a regime change. The a parameters decay with respect to increasing price levels, which is consistent with the scaling law in (6). A similar interpretation can be given to the 3-dimensional representation reported in Fig. 7. In it, each combination of the a and b parameters (horizontal axes) is associated to a price level (vertical axis). Fig. 7 shows that low prices are associated to high values of a (i.e. high volatility) and low values of b (namely, very fat tails). The opposite holds for high price levels. We can thus conclude that, following a low price realization, the shocks underlying electricity price dynamics display high volatility and fatter tails, than after a high price.

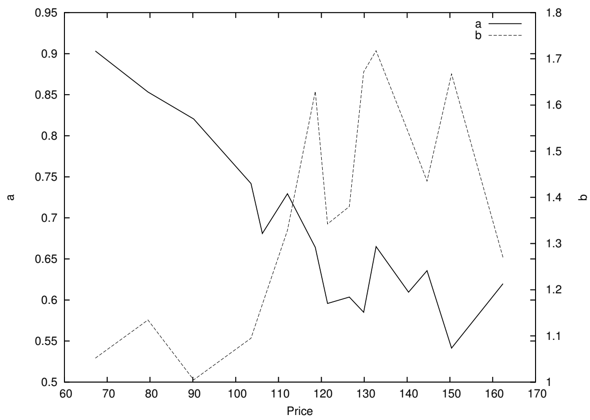

Figure 6: Kernel estimate of the relationship between price level and Subbotin parameters (a and b): 2-dimensional plot.

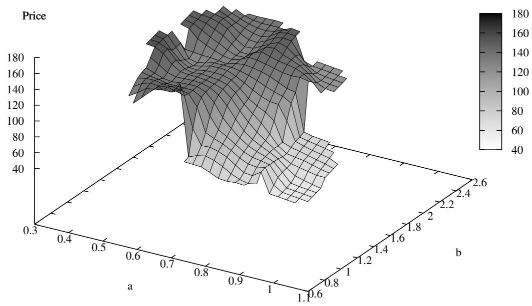

Figure 7: Kernel estimate of the relationship between price level and Subbotin parameters (a and b): 3-dimensional plot.

Table 3: Scaling parameters and the estimated Subbotin parameters of the residuals ϵ in different hours, for six groups.

Hour	**scaling**		b						a					
	α	β	I	II	III	IV	V	VI	I	II	III	IV	V	VI
1	4.700	-1.042 ± 0.153	0.940	1.002	1.217	1.184	2.061	2.035	0.859	0.885	0.645	0.564	0.609	0.662
2	5.197	-1.173 ± 0.151	1.153	0.772	1.376	1.543	1.717	1.924	0.897	0.750	0.641	0.517	0.562	0.504
3	3.999	-0.914 ± 0.160	1.107	0.922	1.215	1.831	1.770	0.999	0.925	0.761	0.568	0.533	0.558	0.425
4	4.840	-1.107 ± 0.106	1.143	0.915	1.002	1.565	1.902	2.181	1.063	0.804	0.523	0.509	0.520	0.506
5	4.712	-1.078 ± 0.098	1.083	1.078	0.955	1.996	2.159	2.019	1.069	0.836	0.520	0.583	0.562	0.493
6	5.095	-1.152 ± 0.113	0.930	1.136	0.932	2.028	2.248	2.005	1.000	0.836	0.539	0.607	0.536	0.477
7	4.938	-1.091 ± 0.157	1.247	1.012	1.075	1.374	2.225	1.532	0.972	0.784	0.737	0.578	0.582	0.458
8	3.822	-0.829 ± 0.149	0.874	1.244	1.026	1.118	1.409	1.342	0.837	0.762	0.567	0.623	0.642	0.510
9	3.663	-0.803 ± 0.188	1.096	1.548	1.253	1.533	1.124	1.011	0.851	0.706	0.545	0.629	0.620	0.538
10	3.195	-0.699 ± 0.197	1.171	1.438	1.415	1.092	1.215	1.047	0.822	0.716	0.607	0.626	0.650	0.598
11	3.172	-0.682 ± 0.169	1.117	1.364	1.271	1.797	1.854	1.269	0.834	0.776	0.617	0.781	0.776	0.730
12	3.903	-0.837 ± 0.190	1.003	1.156	1.329	1.460	1.645	1.189	0.814	0.707	0.641	0.752	0.693	0.668
13	4.436	-0.949 ± 0.160	1.061	1.238	1.219	2.433	1.258	1.637	0.867	0.786	0.638	0.778	0.616	0.699
14	4.676	-1.001 ± 0.141	1.203	1.191	1.342	1.636	1.409	1.291	0.914	0.813	0.653	0.741	0.658	0.622
15	4.745	-1.018 ± 0.116	1.051	1.198	1.161	1.605	1.544	1.374	0.886	0.802	0.619	0.712	0.658	0.653
16	4.203	-0.910 ± 0.170	1.047	1.170	1.183	1.719	1.538	1.730	0.841	0.781	0.655	0.717	0.672	0.714
17	3.388	-0.741 ± 0.179	1.162	1.112	1.606	1.525	1.173	1.201	0.862	0.736	0.695	0.594	0.623	0.639
18	2.795	-0.633 ± 0.210	1.276	1.397	1.660	1.867	1.095	0.999	0.799	0.767	0.629	0.567	0.597	0.546
19	3.923	-0.850 ± 0.170	1.145	1.137	1.971	1.551	1.137	1.206	0.838	0.791	0.830	0.571	0.622	0.612
20	4.461	-0.967 ± 0.158	1.067	0.964	1.541	1.356	1.771	1.020	0.823	0.749	0.791	0.591	0.650	0.534
21	4.887	-1.061 ± 0.150	0.895	0.993	1.284	1.232	1.568	1.465	0.741	0.783	0.718	0.545	0.577	0.615
22	4.761	-1.036 ± 0.184	1.036	0.969	1.634	1.145	1.376	1.235	0.811	0.755	0.729	0.542	0.553	0.598
23	4.835	-1.050 ± 0.183	1.348	0.957	1.384	1.412	1.249	1.238	0.939	0.786	0.760	0.591	0.513	0.584
24	4.495	-0.984 ± 0.136	0.852	1.098	1.480	1.326	1.198	2.018	0.807	0.865	0.743	0.652	0.534	0.672

8 Discussion and Conclusions

Let us briefly review our findings:

1. The distribution of normalized log-returns r^* is well approximated by a Subbotin, with shape parameter b between 0.5 and 0.8.

2. Once the linear autocorrelation structure is filtered out, the underlying shocks ϵ are distributed according to a Laplace law (Subbotin with $b = 1$), regardless of the hour considered.

3. The volatility of shocks ϵ scales as a power law of the initial price level P, with scaling exponent around -1. This is typical of an additive process;

4. Conditional on the initial price level, the estimated b is around 1 (Laplace) for prices below a certain price threshold, between 1 and 2 above it.

Finding 1 means that electricity log-returns are extremely fat-tailed. Technically, this might be reflecting that r^* can be seen as a linear combination of lagged heteroskedastic shocks, more specifically Laplacian shocks (Finding 2).

The volatility of the ϵ shocks is related to the price level according to a scaling law (Finding 3). Day-ahead price dynamics is thus generally more noisy after a low price than following a high one. Notice that scaling relationships are commonly found in the economic growth and industry dynamics literatures, while no such regularities are detected in financial markets. In a way, this signals that electricity markets have little to share with markets such as the foreign exchange markets, or stock exchanges. The additive nature of the electricity price process, as compared to the multiplicative nature of, say, stock marhet prices, makes this distinction even clearer.

As suggested by Finding 4, conditional distributions display heterogeneous shapes, depending on the initial price level. Besides being more volatile, price dynamics following a low price level is also characterized by relatively extreme fluctuations. The detected heterogeneity in distributional shapes across price levels also means there is no evidence of universality, which defines a further specificity of electricity markets.

In sum, the sequence of price levels P is an additive process. More specifically, it is the outcome of the time aggregation of Laplacian shocks ϵ, whose volatility is autocorrelated at weekly frequencies. The ϵ sequence is in turn resulting from superposition of shocks whose distributions depend on the price level, each one corresponding to a different regime.

These findings pose a major research question as to what features of electricity markets are behind the detected specificities. The peculiar nature of electricity as a non-storable commodity, technical constraints in the generation and transmission of electicity, as well as the set of behavioral rules enabled by the day-ahead market design - all of them might be crucial in this respect.

Theoretical results already available in the literature can give us some guidance towards a preliminary interpretation of at least some of our results. For instance, as regards the observed heterogeneity in distributional shapes, it is worth noting that, to the extent that price levels proxy for the system capacity utilization rates, distributional shapes are not independent of how far is the system from full capacity load.

It has been shown by [30] that the (deterministic) low-demand Nash equilibrium strategy for generators is to bid at marginal cost. The intuition is that, as long as demand is far below system capacity, bidding above marginal cost engenders the risk of not being called into operation. On the contrary, when demand is high, the uniform price converges to the highest admissible price - say, the price cap imposed by regulators. This is because, when demand approaches system capacity, all generators, even the less efficient ones, are very likely to be called into operation. Demand uncertainty makes this dichotomy only a bit milder. Hence, market design implies that at low demand levels competition is relatively fierce. Consistently, empirical evidence reported in [34] and [35] shows that mark-ups set by generating companies are increasing in the marginal cost level of their plants. Bids submitted by owners of base-load plants - the market clearing plants at low demand levels - tend to be closer to the actual marginal cost, a parameter out of their control. While base-load bidders behavior is significantly constrained, higher degrees of freedom are allowed to generators when demand is high. Differences in distributional shapes in correspondence of different price levels might reflect this.

Before getting too deep into theoretical speculations, we acknowledge that further research need to be devoted towards more precise assessments of the regularities uncovered in this paper. However, in light of the theoretical intuitions just recalled, the preliminary evidence presented in this paper looks quite promising towards a deeper understanding of the peculiar nature and dynamics of electricity markets.

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