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Uncertainty, Optimal Specialization and Growth

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Uncertainty, Optimal Specialization and Growth

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Abstract

We present a novel argument demonstrating that when trade is characterized by uncertainty the comparative advantages doctrine is misleading and a positive level of diversification is growth enhancing. Applying a result developed in the mathematical biological literature, we show that, in Ricardian trade model in which capital available for investment depends on previous periods returns, incomplete specialization is optimal. We also demonstrate that, in this case, the decentralized solution is characterized by an inefficiently high level of specialization with respect to the social optimal one. Finally, we present a taxation scheme that, reconciling individual incentives and social optimum, is able to induce individual agents to adopt the optimal specialization strategy, i.e. the one that maximizes the country growth rate.

Keywords: Uncertainty, specialization, growth, government

JEL Classification: F19, O49, D80, H20

1 Introduction

The cornerstone of trade theory is the doctrine of comparative advantages. Its most classical application is the result that, in a standard two-sector Ricardian trade model, each country should fully specialize. As any economist knows, this would be optimal both from the country and the world point of view. Less known is the fact that under uncertainty this recipe does not apply any longer. In fact, the presence of uncertainty dramatically modifies almost any positive and normative results of classical trade theory. In particular in a Ricardian model of trade with uncertainty, if agents are risk averse and asset markets do not work perfectly, it can be shown that: *i*) incomplete specialization may be optimal; *ii*) countries may optimally specialize against comparative advantages.

The literature on trade under uncertainty, despite the fact that it has provided a number of interesting and unconventional results, has never attracted much attention in

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the profession, and even less in the larger audience of policy makers. But its results are relevant to both. For instance, while the orthodox view prescribes developing countries to exploit static comparative advantages and to pursue high levels of specialization, results from this literature question the use of the comparative advantage doctrine as the golden rule to be followed when deciding how much and in which sectors to specialize.

Since the fundamental paper by Brainard and Cooper (1968) two elements have characterized theoretical models belonging to this line of research. First, the assumption of risk adverse agent and thus the focus on the trade-off between risk and expected returns. Second, the static nature of the analysis. Our model differs from previous ones precisely because neither of these two elements is present. In particular, we generalize previous contributions to the trade under uncertainty literature by considering specialization decision of risk *neutral* agents. Moreover, we extend existing results by studying the dynamics of economic growth and how individual investment choices can influence it. The novel contribution of the present paper is indeed the characterization of the optimal level of specialization, i.e. the one that maximize the rate of growth - in the presence of uncertainty. We also derive the conditions under which the market selected level of specialization is dynamically inefficient with respect to the social optimal one. Finally, we show how a tax-based redistributive system is able to reconcile individual incentives and social optimum.

We consider a simple standard two-sector Ricardian trade model in which export revenues are subject to uncertainty, with one sector known having higher expected returns. Agents' decisions concern the allocation of production capacity between the two sectors. We show that the country optimal specialization level depends on whether the capital used for investments is constant at the country level, or depends on past realized returns. While with constant capital full specialization is in general optimal, this is not true when the available stock to be invested depends on past cumulated returns. Indeed, using well known results from mathematical biology, we show that in the second case (for a large range of parameters) diversification is growth enhancing. Thus the social optimum is characterized by a share of agents investing in the *wrong* sector, i.e. the one with lower expected return - which is clearly a sub-optimal strategy from the individual point of view. As a consequence, the optimal specialization level cannot be achieved through a decentralized decisional system which, on the contrary, selects a dynamically inefficient solution, i.e. full specialization. Thus, our model presents a novel instance in which an individually optimal behavior is detrimental for the group as a whole. Obviously, the optimal specialization can be reached by a Central Planner deciding for the whole stock of capital of the country. However, we show that the same outcome can be obtained in a decentralized system if an appropriate taxation system is present. We also show that whatever the initial agent's investment preferences, the optimal tax system generates incentives to press adaptive agents to move toward the optimal configuration.

The paper is organized as follows. Section 2 discusses the literature relevant for our paper. Section 3 begins with the presentation of the investment-specialization problems with constant and cumulated capital. Then we show that in the latter case the decentralized solution is characterised by *overspecialization*. Indeed the highest average growth rate is associated with incomplete specialization and can be achieved only through a centralized decision process. In section 3.3 we describe how government intervention, in the form of a redistributive tax mechanism, can induce agents to take investment decision as to obtain the social optimal country-level of specialization. Section 4 draws the conclusions and suggests potential extensions.

2 Related literature: uncertainty, specialization and growth

That trade benefits welfare and growth is one of the tenet of the economic profession. One channel through which this can happen is the increase in the international division of labour. As trade integration increases, in order to fully exploit the country's comparative advantages, a process of efficiency-enhancing reallocation of production factors takes place in each and any country. This would produce, as final effect, lower prices and higher real wages and welfare (e.g. Dornbusch *et al.*, 1977). But trade-induced specialization is also considered to favour the process of capital accumulation and growth. In the last decades several theoretical models have explored conditions under which this is true. The most important are i) the presence of demand externalities (Krugman, 1991); ii) production technologies characterized by dynamic economies of scale or learning-by-doing (Rivera-Batiz and Romer, 1991). Both of them are forces that tend to make specialization growth enhancing. Interestingly enough, they could induce changes in the country's specialization pattern that do not necessarily go in the same direction as the one indicated by the comparative advantages doctrine, and it may even happen that their presence reverts its prediction. For instance, when the rate of learning by doing in a modern sector is higher than in a traditional one, it can be optimal to specialize in the first even if the country does not enjoys comparative advantages there (Redding, 1999).

Despite the fact that trade is certainly one of the most uncertain economic activities, most of the models, both static and dynamic ones, assume a deterministic environment. Brainard and Cooper (1968) is the first paper that introduces uncertainty into a trade model. Applying the results of the static mean-variance portfolio model, they show that, in order to benefit from trade, countries should selectively protect sectors, choosing them as to have low or negative correlation among their export returns. Subsequent static models have derived no less unconventional results. Two are the most important for the present paper. The first is that static comparative advantages, unable to incorporate evaluation of risk, are not sufficient to determine the optimal pattern and level of specialization (Ruffin, 1974; Turnowsky, 1974; Helpman and Razin, 1978)[1]. The second is the acknowledgment that under uncertainty, and differently from what happens in the deterministic case, increasing specialization has also a cost, which is the reduction of the number of *active* sectors in the economy. Indeed, the more an economy is specialized the less easily negative shocks affecting (relatively large) specific sectors can be absorbed by the system[2]. This implies that, if agents are risk averse, increasing specialization may have negative effect on per-period utility.

Even less attention in the literature has been devoted to the relationship between specialization and growth under uncertainty. A notable exception is Acemoglu and Zilibotti (1997). In their model diversification occurs endogenously as a result of agents' decisions to invest either in a safe low-return asset or in a range of imperfectly correlated high-return risky projects. Due to the scarcity of capital and the indivisibility of investment projects, the beginning of the development process is characterized by limited diversification opportunities. Thus the fact that economies are lucky at the beginning of the process makes the difference in the long run. An interesting feature of the model is that the decentralized solution is inefficient due to the presence of a *pecuniary externality*. Since each new sector increases the opportunity to diversify investment, the more numerous they are the more risk averse individuals are willing to invest in risky activities. Since this externality cannot be internalized, government intervention (i.e. in the form of large public funded investments) is shown to be optimal.

Our paper is also related to the growing literature on the optimality of government intervention when the economic environment is characterized by some form of uncertainty. Indeed the existence of a Welfare State can be justified on the basis of its role of insurance supplier. This can take two forms: 1) to give incentives to increase risk-taking by investors; 2) to provide insurance when agents are characterized by some form of specificity (Sinn, 1996). For instance, Brainard (1991) shows that if human capital investment is characterized by indivisibility and specificity, government intervention can raise (per-period) welfare through the provision of a tax-financed insurance scheme. Applying a similar taxation mechanism, in our paper we demonstrate how the Welfare State is able to improve upon the decentralized solution. However, our taxation system does not aim at increasing the welfare directly, but it is a pure industrial policy instrument, inducing

[1] For an excellent survey on trade models under uncertainty see (Hoff, 1994).

[2] Bowles and Pagano (2006) argue that, since this positive role of diversification is not accounted for in the individual's profit-maximizing choices concerning occupational or sectoral location, economies guided entirely by private incentives will also tend to *overspecialize*.

agents to select the specialization level that maximizes the aggregate growth rate.

3 Optimal specialization under uncertainty

In this section we develop a simple two-sector Ricardian model of trade under uncertainty. We study investment decisions of risk neutral identical agents and the effect of the resulting country level of specialization on growth. We consider both the case in which the capital to be invested in each period is constant, and the case in which it depends on the cumulated returns of previous periods. Using insights from mathematical biology we show that the optimal specialization level differs in the two cases. In addition, we demonstrate that in the cumulative case (by far the most relevant) the decentralized solution is inefficient. We explore the robustness of the result with respect to different parameter values. Finally, we show how government intervention in the form of a redistributive taxation system can induce agents to properly diversify their investment as to produce the optimal specialization at the country level.

3.1 The model

Consider a small country open to international trade populated by N maximizing agents. There are three goods. Goods x and y are manufactured for export, while good z is imported for consumption. The latter is the numeraire good[3].

Both export sectors are subject to uncertainty in the form of stochastic foreign demand. At the beginning of the period, before the state of the world is known, each agent decides in which domestic sector to invest. Investment is characterized by some form of specificity. Thus, after uncertainty resolves, it is not possible to immediately transfer the investment made from one sector to the other[4]. While investment decisions are made before uncertainty resolves, trading decisions are taken ex-post. Following a long tradition in the trade under uncertainty literature we assume that financial and credit markets are imperfect and thus unable to provide insurance for specific investments (see for instance Newbery and Stiglitz (1984))[5].

To simplify the analysis we assume that there can exist only two states of the world, which appears with given, constant probabilities. The two states determine the rate of return on the two sectors available for export. The total return produced by exports on

[3]This assumption limits the effect of uncertainty on consumption to indirect effect through income. (Brainard, 1991).

[4]Specificity characterizes, for example, worker's investment in human capital or firms' R&D investment in frontier technologies.

[5]Saint-Paul (1992) and Obsfeld (1994) analyze the role of financial markets in spurring economic growth when uncertainty is present.

the capital invested is a linear function

$$R_{i,j} = r_{i,j}\, K$$

where $r_{i,j}$ is the rate of return from investing in the i sector when the state of the world is j and K is the amount of invested capital.

Under uncertainty, in contrast with the deterministic case, the sectoral rates of return depend on some random factors. Thus which sector enjoys the higher rate of return is determined only in probability. Without any loss of generality, we model uncertainty as the occurrence of two states of the world: state 1 occurs with probability $\pi_1 = \pi$ and state 2 with probability $\pi_2 = (1 - \pi)$. Consequently, there are two rates of return, r_l and r_u, with $r_l > r_u$, and which sector gets the highest rate of return depends on a known probability π. For sector x we will then have that:

$$r_x = \begin{cases} r_l & \text{with probability } \pi \\ r_u & \text{with probability } (1-\pi) \end{cases} \tag{1}$$

The same applies to r_y, obviously with complementary probabilities. In the following we will say that sector $i = \{x, y\}$ is "lucky" ("unlucky") if, for the realized state of the world j, $r_i = r_l$ (r_u). We introduce asymmetry in the model by assuming $\pi > 0.5$. This implies that investing in x has the same variance but higher expected returns with respect to y.

This very simple model setting can be interpreted as a (crude) formalization of the working of the law of comparative advantage in the two-sector Ricardian trade model under uncertainty. Indeed, we can interpret π as the probability of high international demand for sector x (yielding r_l) and low international demand for sector y (yielding r_u). With probability $(1-\pi)$ the opposite takes place: high demand for y and low demand for x. We would then have a probabilistic description of the pattern of comparative advantages (and thus of the trade flows) of the country. These probabilities can be also interpreted as the *ex-ante* information agents have about the country's comparative advantages. A large difference between the probability of positive sectoral demands represents a low level of uncertainty concerning the country comparative advantages. In fact the bigger the difference the easier (because closer to the deterministic case) are agents investment decision.

Define λ and $(1 - \lambda)$ as the share of capital invested in sector x and y, respectively. Note that in our model λ also represents the country's specialization level. In each period t total return reads:

$$R_t = \bar{r}\, K_t \tag{2}$$

where

$$\bar{r} = \begin{cases} \lambda r_l + (1-\lambda) r_u & \text{with probability } \pi \\ \lambda r_u + (1-\lambda) r_l & \text{with probability } (1-\pi) \end{cases} \tag{3}$$

Define $g = E(R_{t+1}/R_t)$ the expected growth rate of return on invested capital. The country problem consists in finding the optimal λ, i.e. the specialization level that maximizes g.

The dynamics of capital accumulation can be assumed to depend for varying degrees on past returns. We can express in a general form any case from total independence of available capital up to full re-investment of any return with the following expression:

$$K_{t+1} = \alpha \cdot R_t + (1-\alpha) \cdot K_0 \tag{4}$$

with $0 \leq \alpha \leq 1$. Note that for $\alpha = 0$ the capital to be invested in each period is an exogenous constant. For $\alpha > 0$, on the contrary, the available capital depends also on the returns obtained from previous investments. While the α expresses the assumptions on the cumulation process, for each of its values the expected growth rate depends on the diversification level, that is the share λ of total capital that is invested in sector x (once fixed the rates of return r_l, r_u and the probability π).

We limit our analysis to the two extreme cases of $\alpha = 0$ and $\alpha = 1$. As we will see, the optimal country specialization level, generating the highest possible expected growth rate, associated to these two cases is, in general, different.

3.1.1 Constant capital

If $\alpha = 0$ the capital invested at any period is a constant value and therefore total returns are:

$$R = \begin{cases} [\lambda r_l + (1-\lambda) r_u] \; K_0 & \text{with probability } \pi \\ [\lambda r_u + (1-\lambda) r_l] \; K_0 & \text{with probability } (1-\pi) \end{cases} \tag{5}$$

We normalize initial capital assuming $K_0 = 1$. In this case, the rate of return represents also the production level obtained at the end of any period.

Since $K_t = K_0$, the rate of growth of total returns coincides with the single-period expected total returns. Given (2) it is clear that total returns are maximized when the (expected) rate of return is maximum. In fact, the expected rate of return is the average rate of return over the two possible states of the world weighted with their probabilities:

$$E(g) = E(r) = \pi[\lambda r_l + (1-\lambda) r_u] + (1-\pi)[\lambda r_u + (1-\lambda) r_l] \tag{6}$$

Since we have assumed that $\pi > 0.5$ and $r_l > r_u$, the (constant across periods) expected rate of return is maximized for $\lambda = 1$.[6] That is, the optimal strategy for the country consists in fully specializing in the sector with the highest probability to be "lucky", i.e. in the sector in which it enjoys (in probability) a comparative advantage.

[6]Since $\frac{\partial E(r)}{\partial \lambda} = (r_l - r_u)(2\pi - 1) > 0$, the maximum is obtained for the largest λ.

3.1.2 Cumulated capital

We now consider the (more realistic) case in which available capital for investment at the beginning of each period is given by cumulated returns from previous periods, and we show how this modifies the optimal specialization rule.

If $\alpha = 1$ and the production at time t is available as capital to invest at $t+1$ equation (4) becomes:

$$K_{t+1} = R_t = \bar{r}\, K_t \tag{7}$$

where $\bar{r}$ is given in equation (3). The expected rate of growth is then:[7]

$$E(g) = [\lambda r_l + (1-\lambda) r_u]^{\pi} [\lambda r_u + (1-\lambda) r_l]^{1-\pi} \tag{8}$$

Equation (8) gives the average rate of growth associated to each possible configuration of agents' sectoral distribution. Maximizing (8) with respect to λ, we obtain that the optimal specialization level is:

$$\lambda^* = \frac{r_u}{r_u - r_l}(1-\pi) + \frac{r_l}{r_l - r_u}\pi \tag{9}$$

which implies $\lambda^* \in (0;1)$ for a large space of parameters' values. In other terms, depending on the parameters, it is well possible that incomplete specialization is optimal.

3.1.3 Individual vs. collective optimality

As we have seen the social optimal solution depends crucially on whether K_{t+1} is a function of previous returns, or not. However, we may ask whether the system-level optimal behaviour can be obtained by optimal micro-behaviors, that is whether maximizing agents can generate aggregate, system level optimal decision. To simplify our argument we assume that each period the whole stock of cumulated capital is divided evenly among the N agents irrespective of the past individual's performance[8]. Thus each agent decides upon an identical share of cumulated capital. Agents objective consists in maximising their own, current individual return[9].

[7] The average rate of growth of a variable is given by its weighted geometric mean. In a stochastic environment the weights are the probability of the events. Thus (8) is the geometric mean of growth rates realized by investing λ capital in sector x and $(1-\lambda)$ in sector y.

[8] This assumption is meant to prevent a few agents to grow big enough to identify their interest with that of the country as a whole. See Section 4 for a discussion on how the relaxation of this assumption would affect our results.

[9] The fact that each agent, even if she commands an equal share of cumulated capital, maximizes *current* individual utility and not the aggregate growth rate can be justified in different ways. For instance, it immediately follows if in each period the agent has to consume, i.e. for subsistence necessities, a share of its realized returns. The same agent's behaviour would also result from an overlapping generation model (with no bequest) in which each agent lives only one period.

In the case of constant capital it is immediate to see that the socially optimal solution (i.e. full specialization) coincides with the individual maximizing specialization decision. In fact, given the independence of available capital from past returns, agents maximize returns by specializing in the sector most likely to generate the highest return.

Different is the case of cumulated capital, i.e. $K_{t+1} = R_t$. Equation (9) gives the expression for the optimal level of specialization λ^*, i.e. the share of total investment capacity assigned to sector x. The economy's specialization level, like the distribution of production capacity among the two sectors, is determined by decisions of individual agents, each controlling an equal share of capital. In each period we have

$$k_{h,t} = \frac{K_t}{N}$$

where $k_{h,t}$ is the unit of capital decided upon by agent h at time t. Since agents utility is a linear function of the return of their own (current) investment the problem for the agent is to maximize:

$$R_{h,t} = k_{h,t}[q_h r_l + (1 - q_h)r_u] \tag{10}$$

where $R_{i,t}$ is the return on capital of agent h and q_h is the share of the capital unit that agent h invests in sector x. Thus the objective function of the agent is different from the one of the social planner. As it is immediate to verify, equation (10) is maximized for $q_h = 1$. Since agents are all identical, we obtain that a country populated by maximising agents fully specialize in sector x.

The result derived in this paragraph resembles the typical case of the "tragedy of the commons", where costs are collective and the gains private. The system may grow at maximum speed provided all agents maintain the optimal behavior λ^*. But each agent has the incentive to defect since the "greedy" behaviour would have a negligible negative effect on the system growth rate, while it would sensibly increase her utility. In our case, the social cost of defection is given by the selection of an actual $\hat{\lambda}$ which is higher then the optimal one. If all agents are identical, all of them would specialize and the system as a whole would be brought to a suboptimal growth path.

3.2 Robustness and comments of the results

Robustness In this section we check the robustness of our result with respect to the value of two crucial parameters: 1) the difference in the lucky-unlucky rates of return; and 2) the probability of the two states of the world.

Figure 1 plots the optimal level of specialization λ^* as a function of the values of these two parameters. Obviously, the higher is π (probability of x being the lucky sector) the higher is the optimal λ^* (share invested in x). However, there is large set of parameters

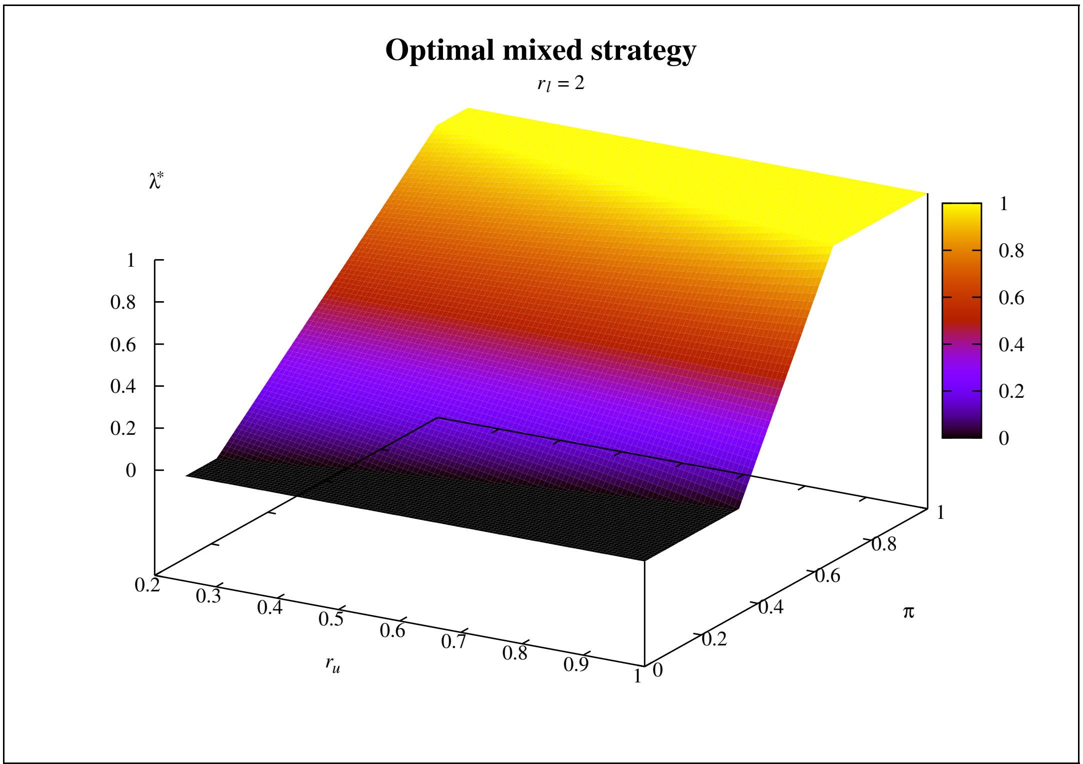

Figure 1: The optimal specialization level λ^* plotted against the probability π of sector x being the lucky sector (i.e. yielding the higher return) against varying levels of the return when the sector is unlucky r_u. The values are computed for a constant level of the return when lucky $r_l = 2$. For extreme values of π, when one of the two events is much more likely than the other, the best strategy is to specialize. Considering that diversification is a sort of insurance against unlucky, though rare, events, explains why specialization is less attractive for lower values of r_u.

combination for which incomplete specialization is optimal, i.e. $0 < \lambda^* < 1$. Only for extreme values of π, when one of the two sectors is much more likely to produce the highest rate of return, specialization appears to be the optimal choice, while more uncertain cases call for a more diversified investment strategy. Less obviously, the range of π over which some degree of diversification is optimal is larger the lower the rate of return when unlucky. This is explained by considering diversification as an insurance against bad performance. As the cost of bad luck increases, diversification becomes more attractive.

Though we have shown that diversification is better than full specialization, we may ask how much one may gain or loose from diversifying or specializing. In fact, if the difference were negligible, the whole exercise would have a mere academic interest, but not practical relevance. Figure 2 plots the difference between the average rate of return of an optimally diversified vs. a fully specialized economy. The graph shows, for any value of r_u (i.e. rate of return when unlucky) and π, the difference Δ between the average rate of return produced by adopting the best diversification λ^* as opposed to the average rate

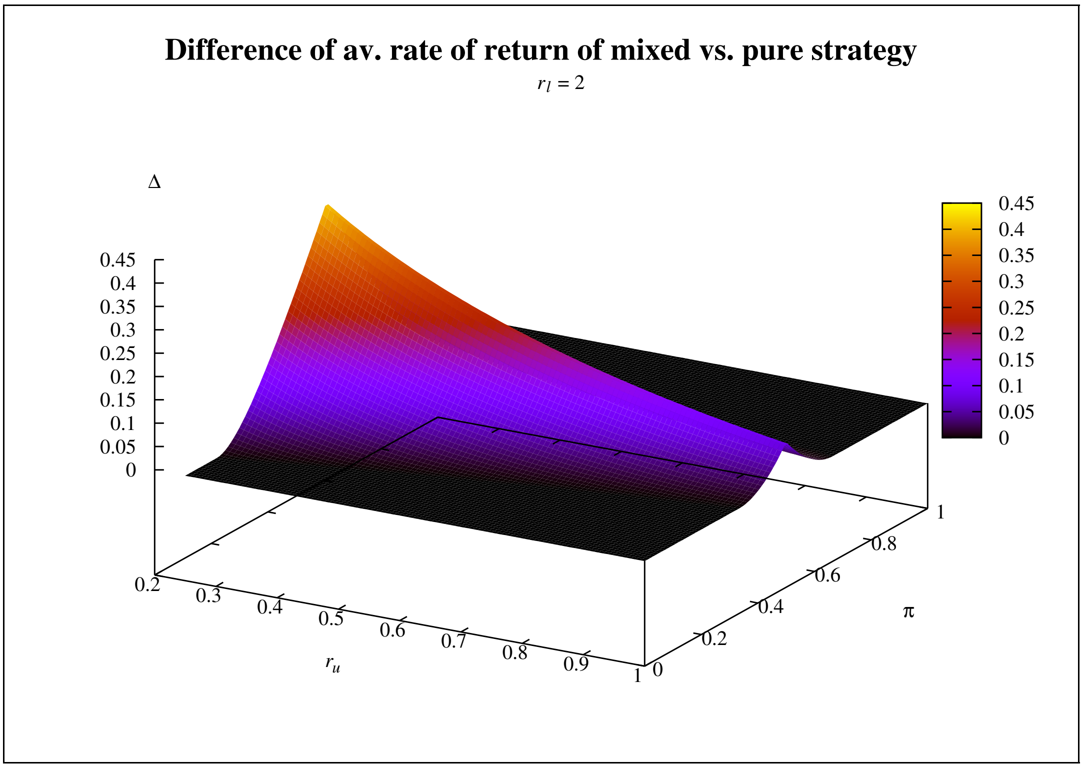

Figure 2: Gain in average rate of return by adopting an optimally diversified economy as opposed to full specialization. See figure 1 for details on the independent variables.

of return produced by full specialization. As expected the higher the variability of the environment (i.e. the closer π is to 0.5), the more growth enhancing is diversification. Moreover, for any probability distribution of the states of the world, Δ is higher the lower the unlucky return. Diversification acts as a sort of insurance policy against bad luck: the worse the damage in case of bad luck, the higher the benefit of the diversification strategy. Concluding, the difference between the rates of growths expected under the two policies of (optimal) diversification and full specialization, though obviously varying with the parameters of the model, seems relevant enough to justify a detail analysis.

Comments and interpretation of the result In section 3.1.1 we have shown that in a two-sector Ricardian model with uncertainty and optimizing risk neutral agents if the capital invested in each period is independent from previous results full specialization is optimal and there is no divergence between the market and the centralized solution. The analysis developed in section 3.1.2, on the contrary, shows that if previous periods results matter in the investment process there is a large set of parameters configuration for which incomplete specialization is optimal. This implies that optimal specialization is characterized by some agents investing in the *wrong* sector. The whole economy benefits from *some* agents 'sacrificing' themselves investing in the sector with lower expected return (acting in a 'sub-optimal' way from an individual point of view). Which is the reason for this apparently surprising result?

The intuition is the following. When all agents invest in the same sector and the sector is (ex-post) unlucky the loss is very penalizing. In an accumulation process even a single negative event can have a long run relevant effect. It is the highly path-dependent nature of the growth process that makes diversification growth enhancing. Diversification "insures" the accumulation process from abrupt stops caused by adverse realizations of random event. The 'individually' sub-rational behavior of some agents, keeping active also comparatively disadvantaged sectors, is a form of social insurance yielding higher aggregate growth in the long run[10].

More technically, our result is based on the mathematical properties of systems where the maximization of the level of a variable differs from the maximization of its growth rate. This is always the case in uncertain environments characterized by fluctuations across periods. Differently from the arithmetic mean, the geometric mean (i.e. the expected rate of growth of returns) is reduced by such fluctuations. Adopting the biological terminology we could say that in a fluctuating environment diversification has a sort of *homeostatic* property: the production of variety within each period reduces fluctuation in fitness across periods and increase the rate of growth of the population[11]. Transferring this argument to an economic context, imagine to compare two countries identical but for the value of λ, their specialization level. It is always true that, on (arithmetical) average, an optimally diversified economy has lower expected return (measured period on period) than a fully specialized economy. But, as we have shown in the previous paragraph, since incomplete specialization has lower variance with respect to full specialization, under proper limitations of the parameter space, its geometric mean (i.e. its expected growth rate) is higher.

Our results derive from the application to the trade context of a series of theoretical propositions developed in the mathematical biology literature. There, it can be accepted that phenotypes sacrifice themselves for the benefit of the genotype's (i.e. population's) growth, since natural selection at population level can ensure this. However, in an eco-

[10]Note the difference between our argument and the one in favor of diversification based on the law of large numbers. In the latter the standard result is that, in a static setting, the higher the number of (at least partially) uncorrelated stochastic sectors, the lower the variability of output and thus the higher the utility of risk averse agents. These models predict that diversification insures higher expected per-period output but they do not discuss the issue of the existence of an optimal level of diversification in a dynamic context.

[11]In the biology literature the fitness superiority of mixed strategy (i.e. incomplete specialization) over pure ones (i.e. full specialization) in fluctuating environments has been called 'coin-flipping' strategy (Cooper and Kaplan, 1982).

nomic context "natural selection" operating at country level is not likely to induce agents to sacrifice. Besides calling for a central planner forcing unwanted actions to individuals for the common benefit, we may ask whether an incentive mechanism can be devised such to obtain the same result. In the next section we show that a proper taxation mechanism is able to induce (selfish) maximising agents to adopt the (country level) optimal diversification strategy.

3.3 Government intervention

In section 3.1.3 we have demonstrated that when the capital available for investment is given by previous periods realized returns, the optimal diversification level λ^* cannot be the result of a decentralized solution mechanism. In fact, optimal diversification requires *some* agents investing in the sector yielding lower expected returns, i.e. comparatively disadvantaged. But, since this individually sub-optimal behaviour is optimal from a social point of view, we ask whether is it possible, excluding coercion, to induce agents to optimally diversify.

In this section we introduce a simple state-dependent redistributive taxation mechanism able to induce agents to make investment decisions yielding optimal diversification at the aggregate level[12]. In addition we show that this mechanism is adaptive, that is, whatever it is the assumed initial behaviour of the agents, it gives the agents the incentives to adapt their strategy in the direction of the optimal diversification.

3.3.1 The taxation system

We assume the government to be benevolent and willing to maximize social welfare, defined as total (cumulated) returns obtained by independently optimizing individuals. The government can impose industry-wide taxes under the condition that they satisfy the equilibrium budget constraint. Government cannot lend or borrow[13].

For the sake of simplicity, as before, agent's capital endowment at the beginning of each period is assumed to be an equal share of cumulated capital. Agent h, with $h = \{1, 2, ..., N\}$ invests with probability λ_h in sector x and with residual probability $(1 - \lambda_h)$ in sector y. She obtains r_l if the sector she invested into is lucky (probability π) or r_u otherwise (probability $(1 - \pi)$).

The taxation mechanism is as follows. At the end of each period, tax revenues collected taxing agents that have invested in the lucky sector are redistribute in the form of subsidies

[12] The use of state dependent taxation and transfers under uncertainty has been explored in Eaton and Grossman (1985), Brainard (1991), Sinn (1996)

[13] Otherwise full specialization would be always optimal with the government borrowing in state 1 and lending in state 2.

to the ones that have invested in the unlucky one[14]. In each period agent's income derived from a unit of capital is now described by

$$w = (1-\tau)r_i + s$$

where τ is the tax rate, s the subsidy, and i can be either lucky (l) or unlucky (u).

The government pursues a perfectly egalitarian policy in respect of luck: taxes and subsides are set as to make *ex-post* agents's incomes identical irrespective of whether the sector each agent invested into has been lucky or not. Formally:

$$(1-\tau)\, r_l = r_u + s \tag{11}$$

The government adopts a strict policy of balanced budget. That is, in each state of the world, the sum of the subsidies distributed must equal the total tax revenues collected. Defining σ as the share of lucky agents (who invested in the ex-post lucky sector), in each period it must hold that

$$\tau\, r_l\, \sigma = (1-\sigma)\, s \tag{12}$$

Substituting (11) into (12) and solving for τ we obtain:

$$\tau = (1-\sigma)\, \frac{r_l - r_u}{r_l} \tag{13}$$

Equation (13) gives the per-period tax each lucky agent pays. The total value of collected taxes is then equally distributed to the agents who invested in the unlucky sector. This tax-subsidy scheme ensures that all agents have the same (ex-post) net income.

The tax rate depends on the share agents having invested on the lucky sector, which is a random variable. Therefore, we actually need two tax rates, depending on which sector happen to be lucky. As before, we indicate with λ the share of agents that have invested in sector x. If this sector results as the lucky one, then we have that $\sigma = \lambda$ and equation (13) becomes

$$\tau_1 = (1-\lambda)\frac{r_l - r_u}{r_l} \tag{14}$$

Using (12), we obtain that the subsidy is

$$s_1 = \frac{\lambda}{1-\lambda}\, \tau_1\, r_l\, k \tag{15}$$

If, on the contrary, the lucky sector results y, then the share of lucky agents is $\sigma = (1-\lambda)$. Therefore, agents in y are taxed and the ones in x receive a subsidy given, respectively, by

$$\tau_2 = \lambda\, \frac{r_l - r_u}{r_l} \tag{16}$$

and

$$s_2 = \frac{1-\lambda}{\lambda}\, \tau_2\, r_l\, k \tag{17}$$

[14]According Sinn (1996) the role of the Welfare State is to redistribute not only from rich to poor but also from lucky to unlucky.

3.3.2 Taxing your way to optimal growth

Consider now the tax rates to be applied in order to ensure identical incomes to agents *who are optimally diversified*, that is, when $\lambda = \lambda^*$. In case x is the lucky sector, the agents that have invested in x are taxed by:

$$\tau_1^* = (1 - \lambda^*) \frac{r_l - r_u}{r_l} \tag{18}$$

On the contrary, if y is the lucky sector, the tax is imposed on investors in the y sector and its level is given by:

$$\tau_2^* = \lambda^* \frac{r_l - r_u}{r_l} \tag{19}$$

Call the taxation mechanism described by equations (18) and (19), the 'optimal taxation scheme'. If the individual subsidy is calculated as discussed in the previous paragraph, the application of the optimal taxation scheme insures that *ex-post* incomes are always identical across agents, under conditions that exactly λ^* have invested on sector x.

Let's see what happens when the optimal taxation system is applied to a population of agents with an arbitrary diversification λ. The result is that for any $\lambda \neq \lambda^*$ the application of the optimal taxation scheme generates income differences among agents, that is, the incomes of agents who invested in x or y will differ. Moreover, we can state following:

Proposition 1 *Define λ the proportion of agents that have invested in sector x. If the government applies the 'optimal taxation scheme' and collected taxes are then equally redistributed among unluckies, we have:*

$$w_{i,x} < w_{j,y} \Leftrightarrow \lambda > \lambda^* \tag{20}$$

and

$$w_{i,x} > w_{j,y} \Leftrightarrow \lambda < \lambda^* \tag{21}$$

where $w_{i,x}$ and $w_{j,y}$ are, respectively, incomes of agents who invested in sector x and y irrespective of whether their sectors result to be lucky or unlucky ($i, j = \{l, u\}$)

Proof. See Appendix A. ■

Proposition 1 states that when the proportion of agents investing in x is higher (lower) than the optimal one, their income will be lower (higher) than the ones investing in y, irrespective of the sector which ex-post turns out to provide the highest return. For example, suppose too many (in respect of optimal diversification) agents invested in x, which results to be the lucky sector ($i = l$ and $j = u$). In this case, the tax catch will be higher than that with an optimal diversification (same tax rate, but larger taxed base), and will be distributed to a smaller number of unlucky agents, who, therefore, will have a higher income than would receive in case of optimal diversification. Similar considerations can be derived for the remaining cases, by considering that tax rates are constant, while individual subsidies consists of the total tax catch divided by the number of eligible agents.

The income differences generated by the application of the optimal taxation scheme provide the incentives to the agents to modify their distribution across sectors in the direction of the optimal diversification. How this can actually happen depends on how we represent actual agent's investment decision process. In the next paragraph we implement one of the possible ways to model agents' behaviour, and then we allow agents to adaptively "learn" how to maximize their income. The result is that this process will lead the whole population of investors to adopt country-level optimal diversification.

3.3.3 Simulation setting and results

In section 3.3.1 we have presented a taxation scheme that makes agents indifferent as where to invest. Thus under this scheme any specialization pattern, including the optimal one, is feasible. Moreover, in section 3.3.2, we have shown how 'optimal taxation' applied to population with sub-optimal diversification generates income differences pressing agents toward optimal diversification. Now we implement a simulation model where agents are initialized with sub-optimal investment strategies, and are endowed with a learning system to adapt their strategy. We will see that such economic system reaches the optimal diversification level.

Agent's investment decisions are made at the beginning of each period. The decision (i.e. which sector to invest in) is taken by "flipping" a biased coin where each agent h has, at time t, $\lambda_{h,t}$ probability of choosing to invest in sector x. Then, uncertainty about the actual lucky sector resolves, the government collects taxes applying the 'optimal taxation scheme' and redistributes them as subsides to unlucky gents. At the end of the period agents revise their strategies (i.e. the $\lambda_{h,t}$) on the base of the comparison between their own result and that of their fellow investors. Agents "learn" by adjusting the probability of investing in x, according to the comparison between their own income $w_{h,t}$ and the population average income $\bar{w}_t = \sum_{h=1}^{N} w_{h,t}$. If the income (investment return net of taxes and subsidies) is above the population average, then the agent increases the next period probability to choose the same sector chosen in the current period, otherwise the probability is reduced. This system simply reinforces actions that produced positive results and punishes those generating disappointing ones. For example, $\lambda_{h,t+1} > \lambda_{h,t}$ if the agent invested in x and the (ex-post) income is above average, or if the agent invested in y and the income is below the average.

We initialized all agents to an identical initial probability $\lambda_{h,0} = 0.5$. For a sufficiently large number of agents the proportion of total capital invested in sector x can be approximated by the average probability of each agent to invest in that sector. That is:

$$\lambda_t \sim \bar{\lambda}_t = \sum_{h=1}^{N} \frac{\lambda_{h,t}}{N}$$

Thus, initially, roughly half the agent are expected to invest in each sector. In the subsequent steps of the simulation agents will revise their probabilities as indicated above, generating a dynamics of the λ, either calculated as the observed share of investors in x at each time step or as average over the $\lambda_{h,t}$'s.

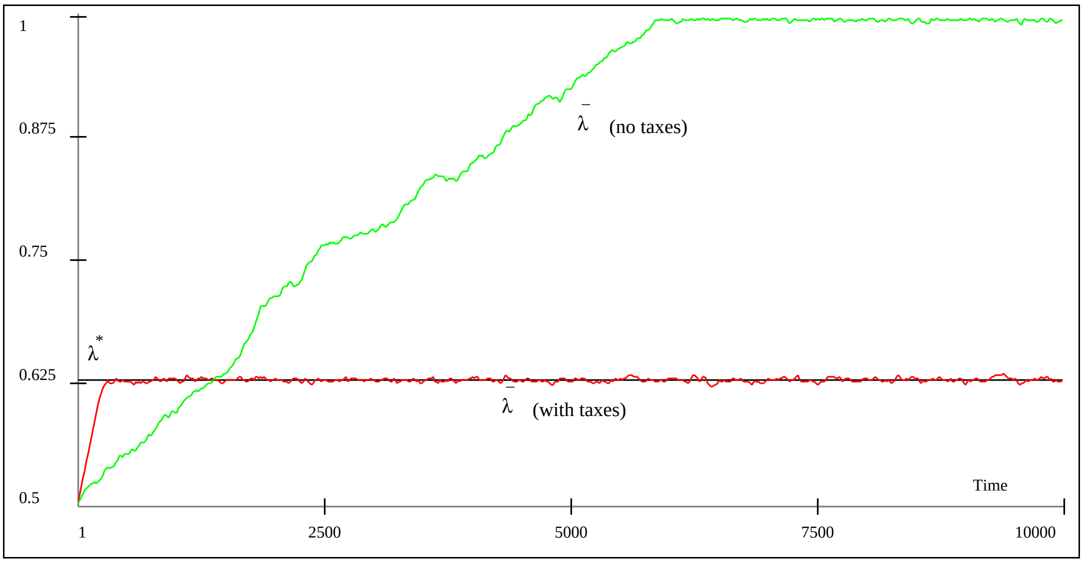

Figure 3: Values of $\bar{\lambda}_t$, for two countries, with taxes and without taxes, plotted through time. Flat line indicates λ^*. System parameters are $\pi = 0.6$, $r_l = 2.0$ and $r_u = .25$, so that the optimal specialization level is $\lambda^* = 0.628571$. Each economy is populated by 1000 agents in all simulations.

Consider now two countries that are identical but for the fact that in the first the government applies the 'optimal taxation scheme' we have described in the previous section, while in the second no such mechanism exists (i.e. tax rates and subsidies are zero). Figure 3 shows the time series for the variable λ_t for the two countries, besides the λ^* reported for comparison.

Given the initialization of the $\lambda_{h,0}$, when the simulation starts $\bar{\lambda}_t$ is 0.5 for both countries. In the one where the government applies the taxation scheme, the system very quickly moves to values of $\bar{\lambda}_t$ close to the optimal one (i.e. λ^*). On the contrary, in the country without redistribution agents modify their $\lambda_{h,t}$ according to the realization of the state of the world. That is, when x results is the lucky sector all agents will increase their $\lambda_{h,t}$, which, conversely, will be decreased when y has the highest return. Since x is lucky more frequently (60% of times, with our initialization), the agents in this country keeps on increasing their $\lambda_{h,t}$ continuously.

The simulation result clearly shows that the incentives created by the taxation scheme push the population to adopt an investing strategy yielding optimal specialization at the country level (indicated in the graph with the usual symbol λ^*). Instead, without government intervention, the egoistic behavior of agents would press them to act in an individually optimal way, which is to fully specialize in the most frequently lucky sector. But this individually optimal behaviour is globally detrimental. This is clearly shown in Figure 4, where the growth path[15] of the two economies is plotted. While the country without the scheme records the highest period-on-period *arithmetic* mean of rates of return[16] its growth rate, computed by the geometric mean, is lower than the one for the country with the taxation mechanism.

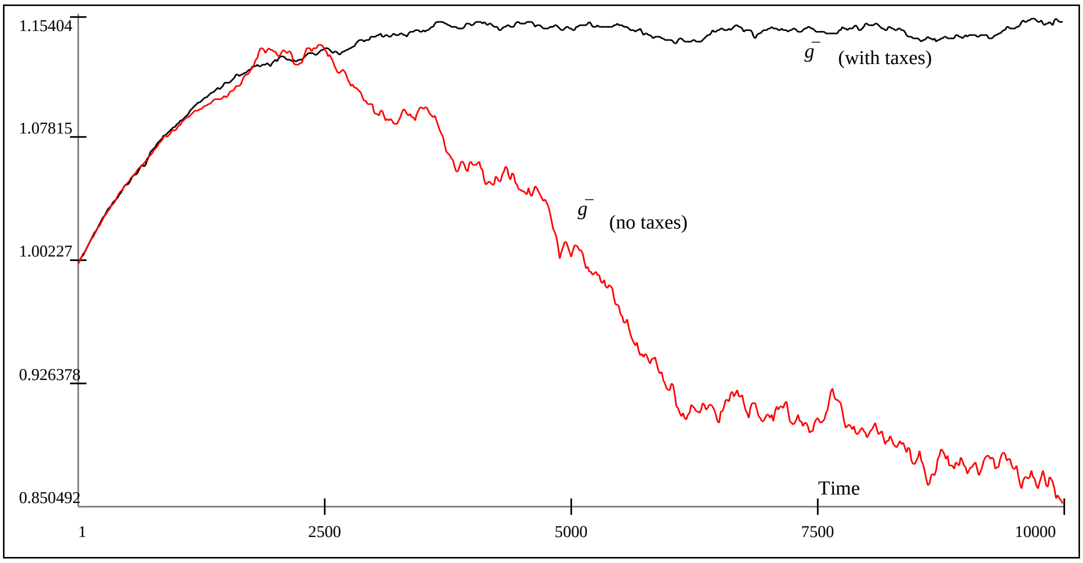

Figure 4: Moving average of overall growth rate $\bar{g}$ for two simulations with and without the taxation mechanism described in section 3.3.1 plotted through time. The use of the moving average allows to discard distant past events.

In a series of exercises not reported, we observed that the result obtained is independent from the initialization of agents' strategies. That is, any initial set of $\lambda_{h,0}$'s eventually provides optimal diversification, although, of course, the final distribution of agents' strategies will depend on the initial distribution. Therefore, for example, we may have different final populations where all agents have the same strategy $\lambda_{h,T} = \lambda^*$; or we may have a λ^* share of agents having $\lambda_{h,T} = 1$ and the rest with $\lambda_{h,T} = 0$. The taxation scheme produces the same eventual (optimal) average value, which, as any average, can

[15] Actually, we report a moving average of observed growth rate in order to avoid initial values to weight excessively into subsequent observations.

[16] At step 6000, after which (on average) agents do not modify their strategy, the country level average returns are 129.8% for the country without the scheme, compared to 117.1% for the country with the taxation scheme.

be generated by many different distributions.

This result suggests that country-level optimal diversification is compatible with heterogeneity of individual strategies. It also indicates a fruitful line of research to understand why two countries characterized by a similar level of (aggregate) diversification may show different capacity, in terms of growth performance, of adapting to 'shocks'. For example, imagine two countries that are initially characterized by the (same) optimal level of specialization. Consider both countries face a structural change, like, for example, the emergence of a new sector, changing rates of return and associated probabilities. Adapting to the shock would require larger or smaller modifications depending on which distribution is currently generating the same aggregate specialization level. Then, since the transformation of the domestic production structure always entails costs in the form of (temporary) lower growth, it is evident that the country performance depends on *how much different* is the initial distribution from the final (optimal) one.

4 Concluding remarks

In this paper we developed a simple two-sector Ricardian model of trade under uncertainty to study the effect of agents' specialization-investment decision on aggregate growth. We have derived three main theoretical results.

First, we have shown that under uncertainty, even in a two-sector Ricardian trade model, full specialization at the country level can be sub-optimal when capital available for investment in each period depends on previous periods' cumulated returns. This implies that, in a growth context, following the comparative advantages doctrine may not produce the optimal patter of specialization. Interestingly, our result is very general in that it does not depend on assumed differences in the learning by doing or technological accumulation capabilities at the sectoral level, as done for instance in Redding (1999), and it very robust to changes in the model's parameters.

Second, we have demonstrated that when capital available for investment depends on previous aggregate returns, the decentralized solution to the investment problem diverges from the social optimal one. Indeed, while the first entails full specialization, the latter is characterized by some agents investing in the comparatively dis-advantaged sector. This means an economy populated by individually profit maximizing agents tends to *overspecialize* thus recording an average growth rate that is not the maximal one. Our model thus presents a novel case in which the market solution turns out to be *dynamically* inefficient *a lá* Dosi (1988).

Third, we have demonstrated the existence of a simple tax-based redistributive system able to induce agents to properly diversify achieving optimal specialization at the country level. Furthermore, using simulations, we have shown that the application of the 'optimal' taxation scheme is growth enhancing. This is an interesting case illustrating that the Welfare State can have a positive effect on growth.

Our model formalizes a novel argument in favor of diversification of production, showing that it is a good growth policy when there is uncertainty. In order to render the basic mechanism driving our results as clear as possible we have employed the most simple and standard trade model. But our findings can be easily generalized to other frameworks. Indeed the result that diversification, reducing fluctuations across periods, increases the average growth rate applies whenever the variable under scrutiny is characterized by cumulativeness or path dependency. For the sake of simplicity we have also excluded, apart from the exploitation of comparative advantages, other factors that makes increasing specialization beneficial to growth. In fact while these have been largely studied in the literature, our model was meant to emphasizes an often disregarded aspect of the story, namely that the presence of uncertainty makes high specialization costly.

Even if our model is highly stylized, our results may have important implications for the never ending debate about the opportunity to protect and support sectors in which a country seem not to enjoy (any more) a comparative advantage. Our findings show that under uncertainty comparative advantages are *still* useful to determine in *which* sector to specialize. But the *extent* to which the comparative advantage doctrine should be followed is limited by the potentially growth-reducing effect of high specialization. The central message of our paper is that, while in a static setting efficiency considerations always impose the abandon of comparatively disadvantaged sectors, the same conclusion is not so obvious in a growth setting when uncertainty is present. Our model also suggests that the provision of an 'investment' insurance, financed through a taxation system, would have a positive growth enhancing effect. Thus a redistributive tax system, beyond social considerations, would also serve as an effective industrial policy. Since this type of 'industrial policy' is clearly much less demanding in terms of information than, for instance, the 'picking winner strategy' and easily implementable than country-wide industrialization plans our result show that there are relatively simpler and effective ways for the government to sustain growth.

This paper should be intended as a first attempt to address the complex relationship between uncertainty, specialization and growth. Taking advantage of the simplicity and flexibility of our model the next step will be the removal of some of the simplifying assumptions we have made in order to derive our results. Two extensions of the basic model seem particularly interesting. The first is to allow the realization of the random event not to be the same for all agents. In the present model, diversification of production increases the growth rate because, by introducing variety in the economy, it 'stabilizes' the growth process. Thus, we expect that the introduction of an additional source of variability (at agent's level) would increase, *ceteris paribus,* the optimal level of specialization. The second extension concerns the issue of wealth distribution. It would be interesting and important to explore in more detail how inequality may affect the optimal level of specialization. In the present paper we have already established that the way in which, at the beginning of each period, agent's capital endowment is distributed does not modify the result that the decentralized solution and the social optimal one diverge. But we expect that the distributional rule would influence the optimal level of specialization and also the dynamics of the growth process. Our preliminary results suggest that the core of our argument still holds when agents are allowed to appropriate the returns of their investments. In fact, fully specialized agents tend to increase the share of capital they control, becoming larger and larger, slow down the country growth path. Even in this case, the taxation scheme proposed above generates positive results aligning the individual interests with those of the whole population.

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A Appendix

Proof of Proposition 1 Consider the case in which the lucky sector is x. Normalizing $k_h = 1$, the income of the individual lucky agent is:

$$w_{l,x} = r_l \left(1 - \tau_1^*\right)$$

where τ_1^* is the optimal tax rate given by equation (18). Clearly the income of agents investing in x depends on the states of the world (i.e. on the fact that *ex-post* sector x is lucky or not), but it does not depend on the proportion of luckies *vs* unluckies. In other terms, whatever is the number of agents guessing correctly which is the lucky sector (i.e. the actual state of the world), this does not influence others luckies' income. On the contrary, the income of the unlucky agent *does* depend on the proportion of luckies *vs* unluckies. This is so because the individual amount of subsidy is given by the total level of taxes collected evenly divided by the number of unluckies (see derivation of equation 15).

When the lucky sector is x a share of λ agents is lucky and $(1-\lambda)$ unlucky. Individual subsidy is given by:

$$s_1 = \frac{\lambda}{1-\lambda}\,\tau_1^*\,r_l\,k$$

Income of agents in y is given by:

$$w_{u,y} = r_u + s_1 = r_u + \frac{\lambda}{1-\lambda} r_l\,\tau_1^*$$

We are now able to prove equivalence (20). We have:

$$\begin{aligned} w_{l,x} < w_{u,y} \quad &\Leftrightarrow \quad r_l(1-\tau_1^*) < r_u + \frac{\lambda}{1-\lambda} r_l\,\tau_1^* \\ &\Leftrightarrow \quad \lambda(r_u - r_l) < r_u - r_l + r_l(1-\lambda^*)\frac{r_l - r_u}{r_l} \\ &\Leftrightarrow \quad \lambda > \lambda^* \end{aligned}$$

Since the model is symmetrical, the derivation of the proof for the other case is trivially obtained following the same procedure.