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Financial Fragility and Growth Dynamics of Italian Business Firms

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Abstract

This work explores a number of properties investigated in the empirical literature on firm size and growth dynamics: (*i*) the distribution and the autoregressive structure of firm size; (*ii*) the existence of size-growth scaling relationships; (*iii*) the distribution and the autoregressive structure of scaling-free growth rates. The major novelty concerns our exploiting of a credit rating index to condition all the analyses upon firms' financial fragility and access to credit. We find that the distributions of both firm size and firm growth rates are fatter tailed among less solvable firms than in the rest of the sample, both at the bottom and at the top extreme of the distributions. As a result, we conclude that not only small and/or slowly growing firms might suffer from difficulties in raising external financing, but also big and fast growing ones might be exposed to financial constraints.

JEL codes: L11, C14, D21, G30

Keywords: firm size, firm growth, financial constraints

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1 Introduction

The size of the firm, in terms of sales, market shares, productive capacity, and its growth records can be considered as important measures of the present and past economic success of a firm, for a number of reasons. First, inasmuch as, without any revenue, no economic activity results sustainable. In addition, due to the minimal amount of sunk costs which are implicitly present in any economic activity, the unit cost of production is likely to be larger for smaller firms, at least below a certain threshold level, and, therefore, it is natural to expect that a company does its best to try to seize out the largest share of the market. At the same time, economic theory teaches us that the ability of a company to implement rent-seeking or competition distorting strategies and, ultimately, to charge higher prices to its customer is likely to increase with its market power. More in general, firm size and firm growth could be, indeed, considered two of the key ingredients that make a company a viable and profitable economic activity. This is the reason why these issues have a long standing tradition in economics, and the number of contributions devoted to their analysis is huge, even if the empirical support of these conjectures appears to be, often, questionable (see Bottazzi *et al.* (2006a). From the empirical side, starting with the seminal investigations conducted by Hart and Prais (1956) for UK and by Simon and Bonini (1958) for US, a series of works focused on studying the statistical properties of firm size distribution and, mainly within a linear and Gaussian framework inspired by the work of Gibrat (1931), explored the relationships between firm size and its logarithmic growth rates, and the autoregressive structure of growth processes (among the many see, for instance, Evans (1987), Hall (1987), Dunne *et al.* (1988) and the critical surveys in Sutton (1997) and Lotti *et al.* (2003)). Then, in the recent years, a new stream of research has focused on the existence of scaling laws between size and growth, and on the empirical properties of the distribution of 'scaling-free' growth rates, within a richer statistical framework than in the past (see Stanley *et al.* (1996), Amaral *et al.* (1997), Bottazzi and Secchi (2006a)).

A first contribution of the present work will be to repeat a number of statistical exercises already performed in the literature using an extensive source of data on more than 40000 Italian firms developed by *Centrale dei Bilanci* (CEBI), the Italian member of the European Committee of Central of Balance Sheet Data Offices: to the best of our knowledge, it has not yet been employed for the study of size-growth dynamics, and covers a relatively recent time window 1996-2003 not yet explored by previous studies on Italian data.[1]

In addition, we will also move a step forward. Indeed, the major novelty introduced by the present analysis concerns an attempt to link size-growth dynamics with a direct measure of firms' financial conditions and, indirectly, with the existence of financial constraints. We do that exploiting the presence in our dataset of a financial rating index which CEBI itself has developed since its foundation in the early 80's for the purpose of credit risk analysis on behalf of the Italian Central Bank and of the merchant banks who are amongst its major shareholders. The index ranks, once per year, all the companies included in the database in terms of their expected ability to pay back their debts or, alternatively, to default. We will call this rating 'default risk' and use it to run comparative analyses of size-growth dynamics conditioning upon firms' overall financial conditions. In spite of the fact taht we are aware that the choice of black-boxes the assessment of firms' financial stability into a single index could represent a questionable approach, we do believe that this way of proceeding also presents important

[1]This database has been made available to us by Unicredit Banca d'Impresa Research Office under the mandatory condition of censorship of any individual information.

advantages. First, it allows for succinct and straightforward classification of firms, and avoid us to enter into much more complicated and, in turn, much more cumbersome descriptions of the financial situation of a firm as it would have been the case if we had tried to capture one by one several different aspects like the relation between debt and cash flow, the structure of the former or the ability of self-financing. Second, and more importantly, rating indexes are exactly the kind of information which merchant banks and other financial institutions look at when they are asked to yield new loans. It is in this respect that there is room to interpret the present work also as an investigation on how, and to what extent, firms' economic performance relates with the costs payed by the firm over their debt and, indirectly, with their ability to access external financing, at least in first approximation. Despite the fact that these are obviously crucial dimensions affecting firms' investment decisions and, thereby, capable to drive size and growth dynamics, the empirical investigation of relationship between size, growth and financial structure represents a relatively under-explored topic in the literature. More precisely, there certainly are works attempting to analyse how financial factors, such as liquidity constrains, leverage or the degree of exposure to international capital markets, impact on firm size and growth (see, among the others, Fazzari *et al.* (1988), Holtz-Eakin *et al.* (1994), Gilchrist and Himmelberg (1995) and Bond *et al.* (2003)), but they have been primarily interested in identifying the relevant correlations among the many variables considered, usually including different proxies of firms' financial structure among the explanatory variables in panel data econometric models, sometimes quite sophisticated. Instead, liquidity or credit constraints type of arguments have only recently been put at the forefront as the key explanation of the observed statistical properties of the firm size distribution and, in particular, of its evolution over time, and even fewer attempts have been made in the direction of exploring how they interact with the properties of growth rates distributions.[2] Lastly, such kind of arguments, though interesting in their own, could be of particular relevance for the Italian case, as they seem particularly well suited to explain the predominance of small-medium sized firms characterizing the productive system of the country.

The structure of the work is as follows. In Section 2 we present a short description of the data and go through the choices implemented in order to obtain a homogeneous and uniform sample. In Section 3 we provide a non parametric statistical description of firm size distribution, focusing on its shape and its stationarity over time. Then, Section 4 analyzes the inter-temporal dynamics of firm size, focusing on the autoregressive structure of the process. In Section 5 we explore the scaling relationship existing between firm size and firm growth rates and, finally, in Section 6 we characterize the probability distribution and the autoregressive profile of firm growth rates. Section 7 summarizes the results and concludes.

2 Data Description and Sample Selection

The data we employ come from the Centrale dei Bilanci (CEBI) database. It contains a rich set of balance sheet and asset structure variables for a large sample of Italian business firms operating in all economic sectors from 1996 to 2003. They are all *limited liability* firms whose accounting books, by the rule of Italian legislation, must be made publicly available at the Chambers of Commerce. CEBI collects the data and performs preliminary cleaning. In

[2] A noticeable exception is Fagiolo and Luzzi (2006) who directly measure liquidity constraints using firms' cash flow and analyse the properties of both size and growth distribution on a different database on Italian firms. Cabral and Mata (2003) and Bertinelli *et al.* (2006) present (indirect) empirical evidence on the first point. Cooley and Quadrini (2001) represent a reference point from a theoretical point of view.

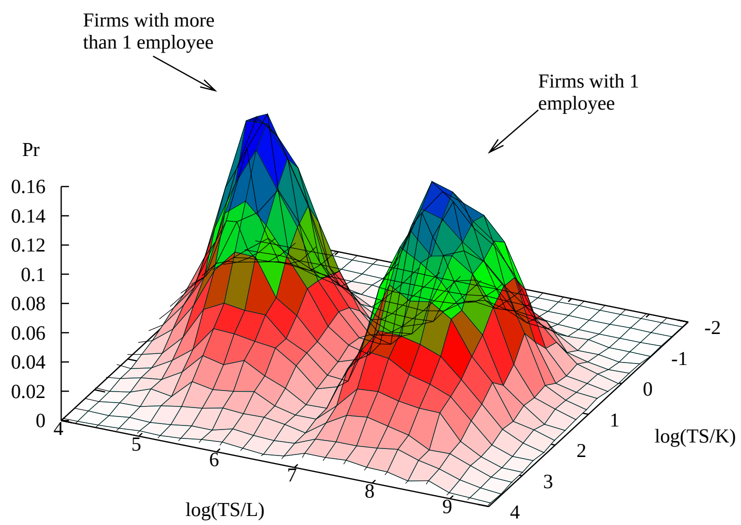

Figure 1: Bivariate empirical density in 2002 of output per worker (TS/L) and output per unit of capital (TS/K) - Manufacturing.

particular, only balance sheets complying with the IV EEC directive are considered.

The original data were filtered according to three criteria. *First*, we only considered the time window $1998-2003$, as the database covers a substantially lower number of firms in the previous years and we wanted to work with comparable sample sizes in all the years under analysis. *Second*, we removed all the firms reporting only one employee, on the basis of different reasons. The main one was that we wanted to focus the study on 'true firms', that is on business entities presenting at least a minimum level of organizational structure, but this is generally not the case for firms with only one employee. Moreover, these latter obviously capture all the phenomena connected with self-employment, which we also prefer to ignore here. After all, firms with only one employee and firms with more than one employee fall into two categories which are, in all probability, representative of two different worlds: netting out the effect of the first category should keep us safe from the generation of statistical properties merely resulting from the aggregation of intrinsically diverse phenomena. We present an example of how severe this problem might be in Figure 1, where the bivariate empirical densities of Total Sales per worker (TS/L) and per unit of capital (TS/K) are reported for firms operating in Manufacturing: it is apparent that the two groups of firms present completely different structures.[3] *Third*, motivated by a similar attempt of working with 'true firms', we further restricted the sample to firms declaring, in each year, Total Sales greater than one million of euros. At the end of the day, we are left with around $15000-20000$ firms active in Manufacturing and $10000-15000$ operating in the Services, depending on the year.[4]

On the top of these considerations, we selected three proxies of firm size, that is Total Sales

[3]Similar exercises were replicated with some of the other variables present in the database and with different years, with the same result, also in the Service industry.

[4]The data are organized according to the Ateco classification, released by the Italian Statistical Office and substantially corresponds to the European NACE 1.1 taxonomy. Firms with code ranging from 15 to 36 belong to Manufacturing, while Services include firms with code in the range 50-74.

Class	Rating	Definition	Number of firms 1998	2000	2002
Low	1	high reliability	1114	1396	1531
	2	reliability	1293	1602	1664
	3	ample solvency	1483	1698	1671
Mid	4	solvency	4170	4549	4310
	5	vulnerability	2360	2621	2405
	6	high vulnerability	1969	2016	2083
	7	risk	2249	2691	2311
High	8	high risk	350	433	457
	9	extremely high risk	93	121	130
Total			15081	17127	16562

Table 1: Population of firms, total and by rating classes in 1998, 2000 and 2002 - Manufacturing.

(TS), Value Added (VA) and Tangible Assets (K), corresponding to different ways of looking at the issue.[5] Working with three different measures allows us to achieve the twin purposes of enriching the analysis and checking the robustness of the results with respect to changes in the way size is measured.

The list of variables used is completed by the financial rating index mentioned in the introduction. It is built by CEBI drawing from the information reported in both the balance sheets themselves and in external sources, according to a multivariate discriminant analysis whose details are proprietary ownership of CEBI itself and were not disclosed to us. The rating assigns to each firm, once per year, a score ranging from 1 to 9, in increasing order of financial fragility: 1 applies to highly solvable and less risky firms, while 9 identifies firms suffering from an high risk of default. The number of firms belonging to each category remains quite stable over time, as shown in Table 1. In order to simplify the subsequent analysis, we grouped the firms in 3 classes, named, for a matter of brevity, in terms of the associated 'proximity to default risk': Low Risk firms are those characterized by good rating (1-3), Mid Risk firms include ratings from 4 to 7, while High Risk firms are those in severe financial difficulties (rated 8-9). The choice of this particular grouping was made with the purpose of building three classes containing firms with similar financial profiles, so that the identification of class-specific patterns in size and growth properties can be more easily mapped in terms of financial conditions and access to credit.[6]

[5]For reference, *Gross* Tangible Assets ("Immobilizzazioni Materiali Lorde") have been preferred over *Net* Tangible Assets ("Immobilizzazioni Materiali Nette") because this choice should avoid incurring into reporting distortions related to balance sheet policies aiming at lowering taxable income.

[6]We explicitly took into account the lower discriminatory power of the class 7 which emerged during our discussions at Unicredit, and we decided to cautiously include it into our Mid Risk class. We performed robustness checks, and the results we will present in the following were never affected by including class 7 into our High Risk category.

3 Firm size distribution

There exists a long standing tradition that has focused on the highly asymmetric nature of firm size distribution, an empirical regularity suggesting the co-existence of productive units characterized by extremely heterogeneous size (among the many contributions in this field, see the classical studies by Hart and Prais (1956), Simon and Bonini (1958), Steindl (1965), Quandt (1966), Ijiri and Simon (1977), and the recent works by Stanley *et al.* (1996), and Cabral and Mata (2003). For a critical survey, see Kleiber and Kotz (2003)). These works tend to conclude that the firm size distribution is well approximated, at least in the upper tail, by skewed and fat-tailed distributions, such as Zipf's or, more generally, Pareto-types of laws. However, some recent investigations have suggested that the precise shape does not seem to be invariant with respect to the size proxy used in the empirical analysis. For instance, Bottazzi *et al.* (2006b) show that the size distribution of Italian manufacturing firms displays a peculiar bimodality when firm size is proxied in terms of number of employees. Moreover, in accordance with a conjecture raised previously in Dosi *et al.* (1995), it has been shown that the very shape of the aggregate size distribution (i.e. considering the manufacturing industry as a whole) emerges as a "spurious" result due to the aggregation of intrinsically diverse sectors wherein highly non homogeneous activities are performed. Indeed, when one focuses the study at a finer level of sectoral aggregation (looking, for instance, at the 2 or 3 digits level), the robust conclusion is the emergence of an extreme heterogeneity in the shapes assumed by the size distributions observed in the various sectors (cfr. the classical study by Hymer and Pashigian (1962), and the recent works by Bottazzi and Secchi (2003) and Bottazzi *et al.* (2006b)). Many sectors display their own peculiarities, for instance in terms of multimodality and degree of asymmetry, with some skewed to the right (i.e. with more weight more towards the biggest firms) and others skewed to the left (i.e. with more weight towards the smallest sizes). The only common property concerns the widths spanned by the supports, which are remarkably spread, whatever the sector considered. This result, confirming what is observed also at the aggregate level, suggests that the presence of widespread heterogeneity in firm size is a robust property of industrial structures, still holding even among firms performing more homogeneous productive activities, and casts severe doubts on the empirical foundation of the notion of 'optimal size' of the firm.

In this section we investigate these issues by means of kernel estimates of firm size probability densities, proceeding along two directions. First we briefly focus on the aggregate level, asking whether the properties emerging within Manufacturing, investigated more frequently in the aforementioned literature, apply also to Services. We then move to disaggregated exercises, but instead of following the literature along the way of a finer sectoral decomposition, we run comparative analysis at a different level, that is controlling for the different degree of financial fragility, as identified through our risk classes.

As a leading example of what we have found, Figure 2 reports the densities estimated in different years using (the log of) Value Added as a size proxy, comparing aggregate Manufacturing with aggregate Services.[7] To help the visual investigation of features such as skewness and fat-tails, it's worth recalling that, on a log scale, a Log-Normal distribution appears as the parabola labeled as Gaussian Fit in the plots. Three main features deserve to be highlighted. First, in agreement with the findings of many previous contributions (see Hart and

[7] If not else specified, we use Epanenchnikov kernel and set the bandwidth according to the 'optimal routine' described in Section 3.4 of Silverman (1986). All the estimates in this work were performed using *gbutils* and *subbotools*, two packages for parametric and non-parametric analysis of panel data. They are distributed under the General Public License, and freely available at *www.sssup.it/bottazzi/software*.

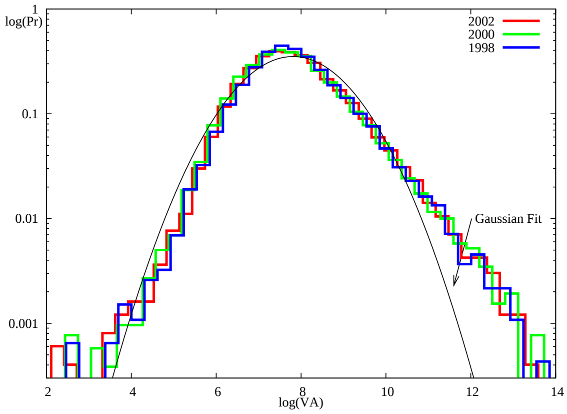

Figure 2: Empirical density of firm size in different years for the Manufacturing (**left**) and the Service (**right**) industry. Size is proxied with Value Added (VA). A Gaussian fit is also reported

Prais (1956), Simon and Bonini (1958), Quandt (1966) and Ijiri and Simon (1977)) the size distributions estimated at the aggregate level span fairly wide supports; second, and contrary to what is suggested in Stanley *et al.* (1996), the upper tail is stably fatter than it would be in the case of a Log-Normal distribution; third, we observe a considerable degree of stationarity: over time the overall shape, the support spanned and the central location do not change much. This last property, possibly influenced by the relatively short time window considered, is confirmed by the figures in Table 2 (cfr. line *Total*), where the mean and the variation coefficient (V.C.) are reported for the same years: both the statistics remain almost identical in the Manufacturing industry, but drop among Service firms in such a way that does not have a strong impact on the overall shape of the densities.

Motivated by considerations of space, we do not report the kernel densities estimated using Total Sales and Tangible Assets, Table 2 helps appreciating how the same set of properties was also observed with respect to the other two measures of size used in this work.[8] On the one hand, the same statistics are both rather stable over time in the Manufacturing, at least once one neglects the upward trend in the mean of Total Sales due, to a large extent, to a nominal effect. On the other, firms in the Service sector display a downward trend in the average size, more pronounced when looking at Assets. The numbers relative to the different risk classes suggest a possible explanation for such a peculiar behavior observed in the Services. For instance, the reduction in the average value of Total Sales corresponds to a remarkable increase in the average size of High Risk firms and, at the same time, to a clear reduction in the average size of Low Risk firms: there is a 'reallocation effect' at work, which shows up almost identical for all the variables.

This introduces the novel question we want to pose in this section, that is whether controlling for firms' financial conditions could provide useful insights about how robust the properties of firm size distribution are with respect to this level of disaggregation not previously explored in the literature. Given that considerable stationarity over time was observed also at this level of analysis, Figure 3, 4 and 5 report, on a log scale, the kernel density of firm size estimated only for 2002 using Total Sales (TS), Value Added (VA) and Tangible Assets (K), respectively, and disaggregating the firms by risk classes. The plots clearly say that all of the aggregate

[8]Results are of course available upon request.

	Rating	Value Added					
		Mean			V.C.		
		1998	2000	2002	1998	2000	2002
MANUF.	Low Risk	7978	7634	7120	3.51	3.57	3.15
	Mid Risk	5445	5255	5632	3.71	3.79	3.64
	High Risk	2712	5214	2618	5	5.82	10.26
	Total	5998	5904	5955	3.72	3.83	3.59
SERV.	Low Risk	6133	4411	4030	11.76	7.95	5.27
	Mid Risk	9427	6865	7257	23.4	19.93	19.05
	High Risk	1883	-2481	3310	8.22	-3.98	10.58
	Total	8289	5776	6149	22.88	20.25	18.51
		Total Sales					
		Mean			**V.C.**		
	Rating	1998	2000	2002	1998	2000	2002
MANUF.	Low Risk	25713	26708	23751	4.11	4.9	3.01
	Mid Risk	23624	24464	27692	3.87	4.45	4.72
	High Risk	15569	23608	48331	2.88	4.3	13.78
	Total	23877	25049	27309	3.94	4.6	6.29
SERV.	Low Risk	31523	30192	22200	7.75	13.65	4.33
	Mid Risk	33567	31634	34240	10.84	9.09	7.93
	High Risk	16479	21750	22836	2.12	3.89	7.23
	Total	32282	30784	30256	10.21	10.32	7.62
		Assets					
		Mean			**V.C.**		
	Rating	1998	2000	2002	1998	2000	2002
MANUF.	Low Risk	13543	13861	13529	5.45	6.23	5.69
	Mid Risk	10545	10972	12458	5.58	5.58	5.61
	High Risk	6063	7718	17075	5.33	4.63	8.91
	Total	11156	11653	12942	5.59	5.86	5.92
SERV.	Low Risk	8679	7081	4552	17.4	18.7	7.41
	Mid Risk	35372	21434	26386	35.4	34.7	35.4
	High Risk	3770	6378	13369	7.13	10.2	15.01
	Total	27423	16902	19531	38.64	36.67	39.11

Table 2: Mean and variation coefficient (VC) of firm size in 1998, 2000 and 2002. Size is proxied with Total Sales, Value Added and Tangible Assets. Figures are in thousands of Euro.

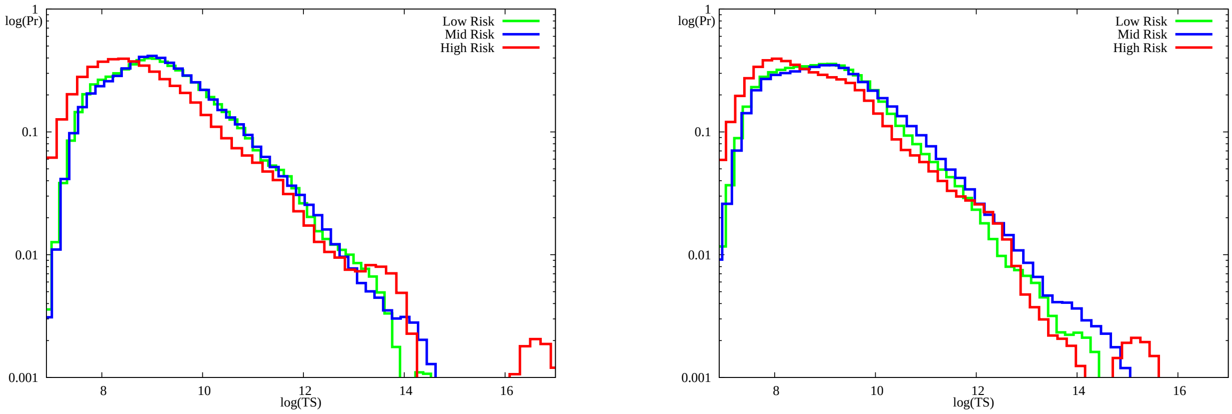

Figure 3: Empirical density of firm size in 2002 for the Manufacturing (**left**) and Service (**right**) industry. Size is proxied with Total Sales (TS).

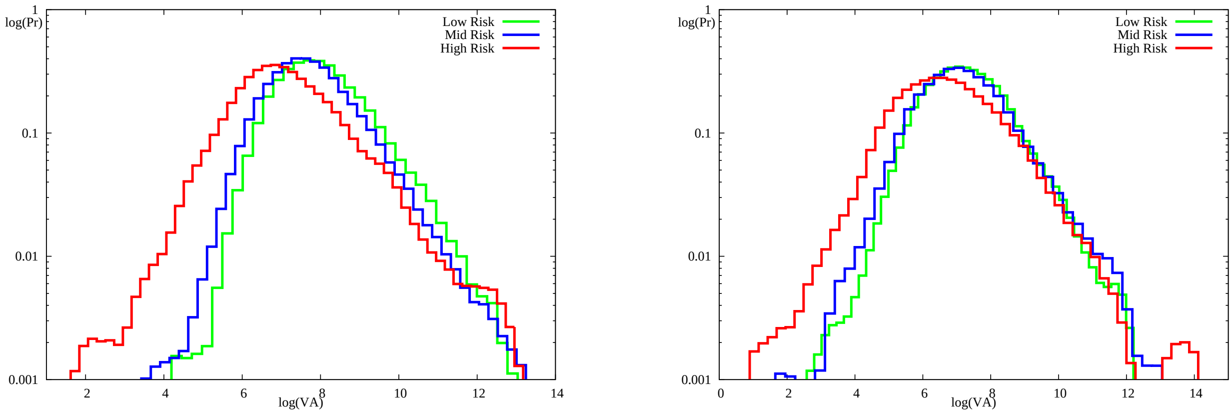

Figure 4: Empirical density of firm size in 2002 for the Manufacturing (**left**) and Service (**right**) industry. Size is proxied with Value Added (VA).

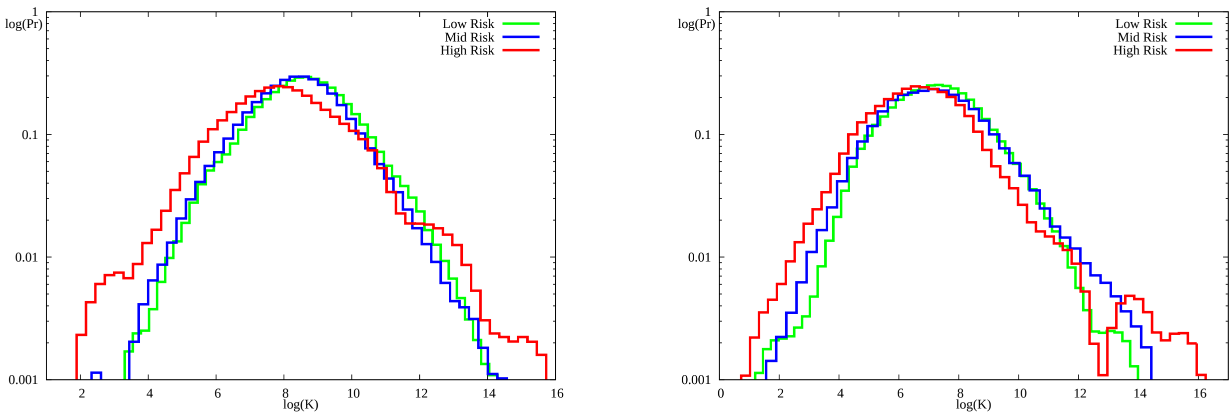

Figure 5: Empirical density of firm size in 2002 for the Manufacturing (**left**) and Service (**right**) industry. Size is proxied with Tangible Assets (K).

results are invariant with respect to financial conditions. The densities, indeed, are characterized by a right-skewed and fat-tailed shape, and clearly span quite large supports, irrespective of the sector, the rating group and the size proxy chosen.[9]

However, it should be noted that novel results also emerge from a careful comparison across the risk classes. Indeed, High Risk firms present a lower tail which is systematically fatter than in the other two classes, a fact that suggests that small sizes are, on average, more concentrated among badly rated firms. This feature, confirmed also by the variation coefficients reported in Table 2, is more pronounced in the Manufacturing industry than in the Services, and when Value Added and Tangible Assets are used as size proxies. The behavior of High Risk firms becomes even more peculiar when one observes that, particularly for Total Sales and Tangible Assets, their size distributions are fatter in the upper tail, too, so that also the biggest firms in the sample are, on average, more concentrated among very bad ratings.

On the interpretative side, finding an explanation is not easy. It's indeed equally convincing to argue that, on the one hand, big firms are those characterized by higher capability of self-financing their activities (and hence should be well rated), but also that, on the other hand, big firms have become big exactly because they have heavily resorted to external resources as a way to finance their growth (and hence should be badly rated). Similarly, it is not clear a priori whether one should expect to observe small firms to be well or bad rated. Here age also plays an important role, as it is likely that small firms, if young, are indebted due to initial start up costs, but they also have higher potential for growing in the future than equally small, but old, firms. Essentially, whether better ratings, and, hence, sounder financial conditions and easier access to credit, should be associated with big or small size seems to be an empirical question. What we observe is that, overall, a non trivial relationship emerges between size and financial conditions suggesting that, insofar as the rating index succeeds in capturing the way banks grant credit, two main patterns emerge. On the one hand, the smallest firms are those most likely subject to the most severe difficulties in raising external funds, while, on the other hand, among the biggest firms many are badly rated, as if their current position is associated with past high indebtment.

4 Autoregressive profile in firm sizes

We showed that the distribution of sizes is stationary, but we will still need to explore the profile of size levels of each firm over time. A critical aspect concerns, in particular, how persistent the relative positioning of small and big firms are. The literature in the field has traditionally looked at these issues by means of simple autoregressive stochastic processes, mainly due to the fact that the prominent interest, since the early studies, was centered around testing the Law of Proportionate Effects (Gibrat (1931)). Basically, it postulates that growth is independent of size and, therefore, that the inter-temporal dynamics of size are well described by a geometric Brownian motion, driven by small and uncorrelated growth shocks. The list of contributions on this topic is huge, with mixed results. On the one hand, a unit root was found in the autoregressive structure of size for samples of medium-large firms, independently from the size proxy used and from the sectoral level of aggregation considered (among the many examples, cfr. Hart and Prais (1956), Simon and Bonini (1958), Hymer and Pashigian (1962), Mansfield (1962), Bottazzi and Secchi (2003)). On the other hand,

[9]More precisely, in the case of Value Added and Tangible Assets we observe fairly similar shapes, while the densities estimated for Total Sales are more skewed, but this profile is plausibly influenced by the $1,000,000$ € threshold imposed to the data from the beginning.

systematic violations of the law were often observed when the analysis focused on the growth trajectories of young, and typically small, firms (cfr. Hall (1987), Audretsch *et al.* (1999), Heshmati (2001), Evans (1987), Dunne and Hughes (1994) and Lotti *et al.* (2003)).[10] In this section, in line with most of the studies, we also explore the autoregressive structure of the firm size time series, both at the aggregate sectoral level and distinguishing by different rating classes.

Specifically, we look at the size of each firm i in logarithms and eliminate nominal trends by working with deviations from the annual sectoral (Manufacturing or Services) average, that is

$$s_i^{\mathrm{X}}(t) = \log(S_i^{\mathrm{X}}(t)) - \frac{1}{N}\sum_{i=1}^{N}\log(S_i^{\mathrm{X}}(t)) \qquad \mathrm{x} \in \{\mathrm{TS,VA,K}\} \quad . \tag{1}$$

Then, we fit the AR(1) model

$$s_i^{\mathrm{X}}(t) = \beta\, s_i^{\mathrm{X}}(t-1) + \epsilon_i(t) \tag{2}$$

stacking the observations belonging to different years. This means we are implicitly assuming different firms as different realizations of the same stochastic process, and seems the best choice one can make in order to exploit the extremely large longitudinal dimension of the panel to reduce the possible biases in the estimates of β which could arise due to the relatively short time dimension. In addition, two main econometric issues have to be addressed, namely serial correlation and heteroskedasticity in the error terms $\epsilon_i(t)$. To cope with the first source of problems we adopt here the approach described in Chesher (1979). Accordingly, we assume that the stochastic variable $\epsilon_i(t)$ is characterized by an AR(1) structure

$$\epsilon_i(t) = \rho\, \epsilon_i(t-1) + u_i(t) \quad , \tag{3}$$

where $u_i(t)$ are $i.i.d.$ disturbances, and we rewrite (2) as

$$s_i^{\mathrm{X}}(t) = \gamma_1\, s_i^{\mathrm{X}}(t-1) + \gamma_2\, s_i^{\mathrm{X}}(t-2) + u_i(t) \quad . \tag{4}$$

Therefore, $\gamma_1 = \beta + \rho$, $\gamma_2 = -\rho\beta$ and the parameters β and ρ are identified through[11]

$$\beta = \frac{1}{2}\left[\gamma_1 + \sqrt{\gamma_1^2 + 4\gamma_2}\right] \qquad \rho = \frac{1}{2}\left[\gamma_1 - \sqrt{\gamma_1^2 + 4\gamma_2}\right] \tag{5}$$

with corresponding errors easily obtained propagating $(\sigma^2_{\gamma_1}, \sigma^2_{\gamma_2}, \sigma_{\gamma_1\gamma_2})$ to β and ρ via the Taylor's expansion of (5). Then, to correct for possible heteroskedasticity in the estimates of the variance and covariance matrices of the coefficients γ $(\sigma^2_{\gamma_1}, \sigma^2_{\gamma_2}, \sigma_{\gamma_1\gamma_2})$, we use a standard jackknife estimator (cfr. MacKinnon and White, 1985). Finally, since non-robust techniques, such as Ordinary Least Squares, can suffer from undesired sensitivity to outlying points we estimate the γ parameters using Least Absolute Deviation regression (Huber, 1981), obtained by minimizing the mean absolute deviation of residuals rather than their mean square deviation.

The results for the two aggregate sectors are reported in Table 3 (cfr. line *Total*, columns *Levels*). In the Manufacturing industry we find $\beta = 1.0021$ with a standard error of 0.0006 for Total Sales, $\beta = 0.9933$ with a standard error of 0.0008 for Value Added and $\beta = 0.9973$ with

[10]See also the critical surveys in Sutton (1997), Lotti *et al.* (2003) and Dosi (2006).

[11]The identification problem here is solved "[...] by appealing to the literature on the stochastic theory of the firm where it is argued that β is close to unity even if the law of proportionate effects is not in operation." (Chesher, 1979), p. 407).

Variable	Rating	AR(1) - Autoregressive Analysis Manufacturing Levels	 Growth	 Service Levels	 Growth
Total Sales	Low Risk	1.0052 0.0010	0.002 0.021	1.0029 0.0014	0.082 0.033
	Mid Risk	1.0011 0.0008	0.029 0.017	1.0044 0.0010	0.082 0.024
	High Risk	0.9903 0.0055	0.032 0.061	0.9966 0.0062	-0.039 0.109
	Total	1.0021 0.0006	0.022 0.014	1.0040 0.0008	0.071 0.020
Value Added	Low Risk	1.0088 0.0012	-0.028 0.032	1.0084 0.0017	-0.010 0.043
	Mid Risk	0.9862 0.0009	-0.131 0.024	0.9963 0.0012	-0.093 0.020
	High Risk	0.9722 0.0099	0.049 0.093	0.9916 0.0097	0.065 0.069
	Total	0.9933 0.0008	-0.093 0.019	1.0003 0.0010	-0.060 0.017
Assets	Low Risk	1.0024 0.0009	-0.089 0.052	1.0003 0.0016	-0.001 0.071
	Mid Risk	0.9957 0.0006	-0.042 0.027	0.9953 0.0010	0.015 0.022
	High Risk	0.9898 0.0043	0.127 0.078	0.9883 0.0050	0.165 0.225
	Total	0.9973 0.0005	-0.048 0.024	0.9961 0.0008	0.010 0.028

Table 3: Estimates of the size AR(1) coefficients β in (3) and in (10) together with their robust standard errors.

a standard error of 0.0005 for Tangible Assets. Similarly, for the Services sectors we obtain $\beta = 1.0040$ with a standard error of 0.0008 for Total Sales, $\beta = 1.0003$ with a standard error of 0.00010 for Value Added and $\beta = 0.9961$ with a standard error of 0.0008 for Tangible Assets. In short, irrespectively of the size proxy used and of the industry considered, even if statistically different from 1 in some instances, the autoregressive coefficient is for all practical purposes equal to this value since the observed deviations are not likely to generate any measurable modification of the process over the relatively short time window under analysis. Therefore, one can conclude that the process of firm growth, if measured as simple log difference of size, seems reasonably approximated by a geometric Brownian motion: in accordance with Gibrat's Law, the relative positioning of small and big firms is very likely to reinforce over time, and quite sticky dynamics characterize the evolution of both market shares (VA and TS) and investment (K).

The same conclusions broadly apply also when one controls for firms' financial conditions, as captured by their annual rating index. Indeed, when repeating the estimation within each of the three risk classes, we observed (cfr. Table 3) a quite impressive homogeneity across the different groups: the estimates of β_j are all very close to 1, with a lower bound of $0.9722(\pm 0.0093)$ found using Value Added for High Risk firms operating in the Manufacturing. Consistently with what observed at the aggregate level, the firm growth process is everywhere well described by a geometric Brownian motion, with no significant variation neither with respect to the particular size proxy chosen nor with respect to the industry considered.

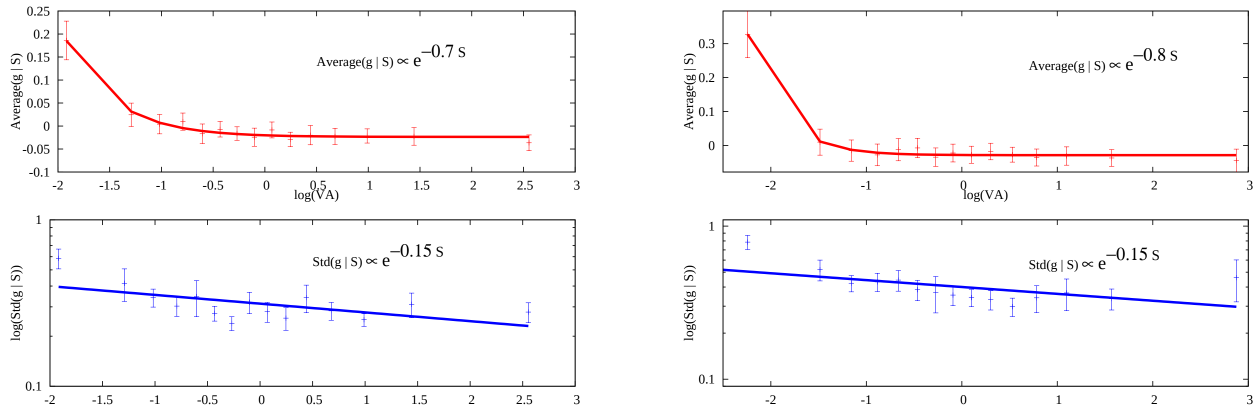

Figure 6: Scaling relation of the conditional average growth rate $E[g|s]$ and of the conditional growth rates standard deviation $\sigma[g|s]$ with respect to firm's size s, computed using equipopulated bins and data from 2002 for the Manufacturing (**left**) and Service (**right**) industry. Size is proxied with Value Added (VA).

5 Size-growth relationships

A second assumption needed for the Gibrat's Law to hold concerns independence between firm size and firm growth. Traditionally, two kinds of tests have been performed, asking (i) whether logarithmic growth rates correlate with size; and (ii) whether the variability of growth scales with size. The existing evidence on the first point speaks firmly against the validity of the Gibrat's hypothesis. Rather, since the seminal work of Hymer and Pashigian (1962), growth is found to decrease with size, and the result is robust to different levels of sectoral aggregation and to the use of different size proxies (see Kumar (1985), Hall (1987), Evans (1987), Dunne *et al.* (1988), Dunne and Hughes (1994), among others). Here, in addition to checking the existence of such a negative relationship, we also try to gather information about its shape.

Let us start at the aggregate level. To get an initial idea, we look at the graphs of the empirical distribution of growth rates conditional on size. Specifically, we again consider size in deviation from annual average

$$s_i^{\mathrm{X}}(t) = \log(S_i^{\mathrm{X}}(t)) - \frac{1}{N}\sum_{i=1}^{N}\log(S_i^{\mathrm{X}}(t)) \qquad \mathrm{x} \in \{\mathrm{TS,VA,K}\} \tag{6}$$

and then, after dividing the observations in 15 equipopulated size classes, we plot the empirical relation between average size and average growth computed within each size class, together with its 2 standard deviation confidence band. By way of example, the top panels of Figure 6 show the results obtained using Value Added as size proxy. Visual inspection of these two plots clearly points out that a negative and possibly non linear relationship emerges both in the Manufacturing and in the Service industry : small firms experience higher growth rates than larger firms. To check the statistical significance and the shape of the observed relationship we revert to non linear regression techniques. Guided by the exponential shape which seems emerging from the graphs, we fit the model

$$\Psi_m(h^X(t)|s^X(t)) = \eta_1 + e^{\eta_m\,(s^X(t)+\eta_2)} + u(t) \qquad \mathrm{x} \in \{\mathrm{TS,VA,K}\} \tag{7}$$

where $h^X(t)$ is the growth rate computed as the log difference of size, $s^X(t) - s^X(t-1)$, Ψ_m is the mean of this latter within each size class, and $u(t)$ is an i.i.d. error term. The estimates

Variable	Rating	Scaling Relations: Manufacturing η_m	Manufacturing η_{std}	Service η_m	Service η_{std}
Total Sales	Low Risk	-3.7 24%	-0.13 9%	-3.5 30%	0.13 10%
	Mid Risk	-4.8 8%	0.009 92%	-3.1 16%	-0.009 116%
	High Risk	-3.2 20%	-0.02 161%	-1.4 34%	0.02 210%
	Total	-5.3 6%	-0.03 24%	-3.4 11%	0.03 10%
Value Added	Low Risk	-0.9 9%	-0.13 9%	-1.0 10%	-0.19 7%
	Mid Risk	-0.7 3%	-0.13 6%	-0.8 4%	-0.13 5%
	High Risk	-0.7 20%	-0.02 181%	-0.6 14%	-0.12 26%
	Total	-0.7 2%	-0.14 4%	-0.8 3%	-0.14 4%
Assets	Low Risk	-0.5 42%	-0.09 12%	-0.4 40%	0.07 14%
	Mid Risk	-0.7 5%	-0.03 21%	-0.2 50%	0.01 100%
	High Risk	-0.2 101%	-0.17 17%	-0.01 10%	0.04 75%
	Total	-0.7 4%	-0.04 12%	-0.2 34%	0.03 17%

Table 4: Estimates of the η_m coefficient in (7) and of η_{std} coefficient in (8), together with robust standard errors in percentage values.

of η_m (see Table 4, lines 'Total'), robustly confirm that the relationship between average size and average growth is significant and well approximated by a negative exponential function, not only for Value Added, but also for the other two size proxies.

Concerning the relationship between firms' size and the variability of growth rates, the existing evidence is less conclusive. Indeed, both early investigations (cfr. for instance Hymer and Pashigian (1962)) and recent works (Stanley *et al.* (1996)) found that the standard deviation of growth is significantly higher among small firms, but there are also other studies which did not find the same evidence, in particular on Italian data (see Bottazzi *et al.* (2006b). Moreover, when the relationship is actually present, there are contrasting results about its precise shape: Amaral *et al.* (1997) find that the standard deviation of growth scales with size according to a Power Law, while a linear model seems appropriate according to other works (cfr. Bottazzi and Secchi (2003)).

We begin, as before, with a simple graphical inspection of the data using Value Added. After clustering firms into 15 size classes we plot (cfr. the bottom panels of Figure 6) the empirical relation between the average size and the (log of) standard deviation of growth rates computed within each size class. Once again, a negative relation, and possibly exponential, seems to emerge in both of the sectors. Then, in order to check and quantify this visual impression, we run the non-linear regression

$$\Psi_{std}(\hat{u}^X(t)|s^X(t)) = e^{\eta_{std}\,(s^X(t))} + g(t) \qquad \mathrm{x} \in \{\mathrm{TS,VA,K}\} \tag{8}$$

where $\hat{u}^X(t)$, being the residuals from (7), represents firm growth rates net of the scaling found between average growth and average size, Ψ_{std} is the standard deviation of growth computed in each size class, and $g(t)$ is an i.i.d. error term. Table 4 (cfr. line *Total*) shows that, with the only exception of the Service industry when Total Sales and Tangible Assets are considered, the coefficient η_{std}, though rather small, is statistically significant for all the size proxies and always negative: variability of growth decreases exponentially with size.

But now, are the aggregate results robust with respect to firms' financial conditions? To answer this question we simply run regressions (7) and (8) within each single risk class, and the answer is positive (see Table 4). Concerning the relation between average size and average growth, the estimated coefficients η_m are almost always negative and significant, and sensibly bigger for Total Sales. The same happens with the estimates of η_{std}: apart from few exceptions, showing up mainly within Services, they robustly support the existence of a negative exponential scaling relationship between size and the standard deviation of growth rates.

6 Empirical properties of firm growth rates

The results presented in the previous section suggest that there are substantive reasons to go beyond the simple linear stochastic framework underlying traditional 'Gibrat's Law based' investigations. This is actually what it is also suggested by a strand of research appeared in the recent years (cfr. Stanley *et al.* (1996), Amaral *et al.* (1997), Bottazzi and Secchi (2006a)) which studies the growth dynamics driving the evolution of firm size within a richer and more complex statistical framework. The basic idea is to approach size-growth dynamics looking at the growth rates distribution, asking whether its shape is consistent with the Gaussian or other thin-tailed distributions implied by Gibrat's types of models where the shocks driving size dynamics are small and uncorrelated, or, rather, present fat tails, whose existence, in turn, hints at the working of some powerful correlating mechanisms across different firms. Accordingly, particular attention has been devoted to isolate the properties of 'true' growth rates, that is cleaned from the kind of scaling relationships with size we have investigated in the previous section. Much in the spirit of these recent advances, we will measure growth in terms of $\hat{g}(t)$, the scaling-free residuals from equation (8), and we pursue three exercises. First, we apply non parametric techniques to estimate growth rates densities, investigating the extent of stationarity over time and whether any peculiar feature emerges. Second, turning to a parametric approach, we fit the shape of the distribution, directly addressing the question of fat-tailness. Third, we explore the autoregressive structure of the firm growth process. Once again, our main point of departure from existing works will concern the possibility to check the robustness of results with respect to firms financial conditions, running separate analyses for each risk class.

Growth rates distribution

We start looking at the kernel densities at the aggregate level. Figure 7 reports, by way of example, the estimates obtained at the aggregate level in terms of Value Added, for 3 different years (1998, 2000 and 2002). Two are the features which emerge. First, the overall shape does not change from one year to the other, suggesting strong stationarity of the salient properties over time. Second, the plots display tails clearly fatter than those implied by a Gaussian distribution and peculiar tent-shaped behavior quite in accordance with the Laplacian (symmetric exponential) specification which has been almost invariably found in the literature, at

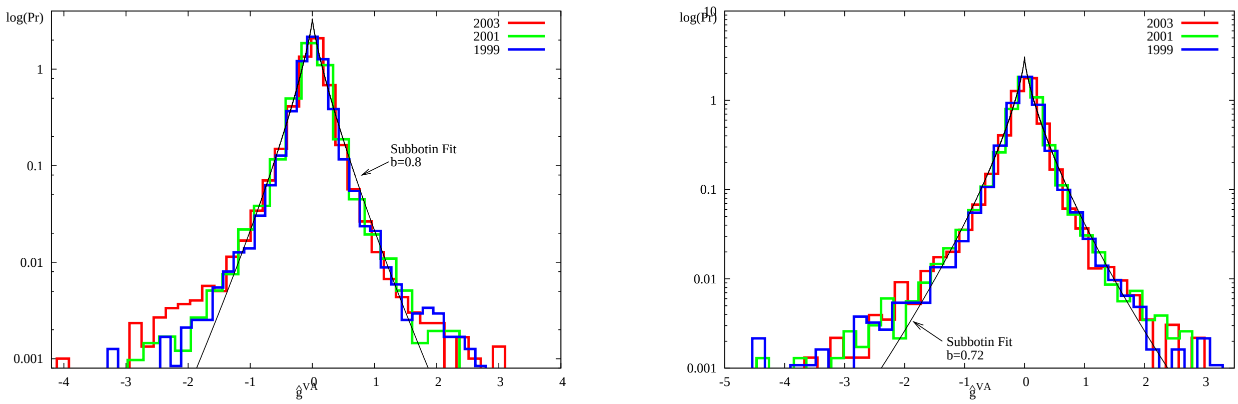

Figure 7: Empirical growth rates densities in different years for the Manufacturing (**left**) and Service (**right**) industry. Size is proxied with Value Added (VA).

different level of sectoral aggregation and across different countries.[12].

In order to statistically characterize this regularity we follow the approach detailed in Bottazzi and Secchi (2006b). Basically, we estimate via standard maximum likelihood techniques the parameters of a flexible family of probability densities, known as the Subbotin distribution (Subbotin, 1923), which encompasses different and more commonly used probability functions consistent with different tail behaviors. Formally, its functional form reads

$$f(x) = \frac{1}{2ab^{1/b}\Gamma(1/b+1)} \, e^{-\frac{1}{b}\left|\frac{x-\mu}{a}\right|^{b}} \tag{9}$$

where $\Gamma(x)$ is the Gamma function computed in x, μ is a *positioning* parameter capturing the central location of the distribution, a is the *scale* and captures the width of the support, and b is a *shape* parameter, the key one in order to discriminate among different tail behaviors. To fix the ideas consider that the Subbotin density reduces to a Gaussian one when $b = 2$ and, as a general rule, the lower is b, the fatter are the tails. Therefore, leptokurtic behavior (i.e. tails fatter than the Gaussian) are present for $b < 2$, while platykurtic shapes (i.e. tails thinner than a Gaussian) emerge for $b > 2$. Finally, the value $b = 1$ corresponds to the tent-shaped Laplacian behavior repeatedly found to characterize growth rates in previous works.

The estimates for a and b obtained at the aggregate sectoral level are reported in the bottom part (cfr. line *Total*) of Table 5.[13] The impression conveyed by the simple visual inspection of the plots is clearly confirmed, and not only for Value Added. Indeed, the estimates of b are consistent with tails even fatter than a Laplace distribution: they are always significantly smaller than 1 for all the size proxies considered, both in the Manufacturing and in the Service industry. Note that, at the same time, the shape and the width of the distribution varies with the proxy used. For instance, in the Manufacturing industry, the Subbotin fit give $b = 0.90 \pm 13‰$ and $a = 0.13 \pm 1.4‰$ when Total Sales is used as size proxy, $b = 0.82 \pm 12‰$ and $a = 0.18 \pm 1.9‰$ for Value Added, and $b = 0.52 \pm 8.5‰$ and $a = 0.08 \pm 1.3‰$ for Tangible Assets. Hence, the density of growth rates measured in terms of Tangible Assets spans a narrower support and, more interestingly, is noticeably fatter tailed, suggesting that

[12]See the works by Stanley *et al.* (1996) and Bottazzi and Secchi (2003) on US data, Bottazzi and Secchi (2006a) and Fagiolo and Luzzi (2006) on two different Italian data-banks, Bottazzi *et al.* (2005) on French firms and the study of Bottazzi and Secchi (2005) about the worldwide pharmaceutical industry

[13]Notice that the parameter μ is set to 0 by the normalization in (1).

		Subbotin Fit			
		Manufacturing		Service	
		b	**a**	**b**	**a**
Total Sales	Low Risk	0.97 26‰	0.11 2.1‰	0.82 25‰	0.11 2.6‰
	Mid Risk	0.94 16‰	0.14 1.7‰	0.76 15‰	0.13 2.0 ‰
	High Risk	0.76 59‰	0.25 1.5%	0.78 61‰	0.25 15 ‰
	Total	0.90 13‰	0.13 1.4‰	0.75 12‰	0.13 1.6‰
Value Added	Low Risk	0.96 26‰	0.16 3.1‰	0.81 26‰	0.18 4.5‰
	Mid Risk	0.82 14‰	0.18 2.3‰	0.76 16‰	0.21 3.4‰
	High Risk	0.68 58‰	0.33 23 ‰	0.79 72‰	0.35 25 ‰
	Total	0.82 12‰	0.18 1.9‰	0.77 13‰	0.21 2.8‰
Assets	Low Risk	0.60 19 ‰	0.09 2.6‰	0.49 19‰	0.11 4.0‰
	Mid Risk	0.50 10 ‰	0.08 1.5‰	0.54 13‰	0.12 2.7‰
	High Risk	0.47 48 ‰	0.10 10 ‰	0.52 55‰	0.22 22 ‰
	Total	0.52 8.5‰	0.08 1.3‰	0.51 10‰	0.12 2.3‰

Table 5: Estimates of the shape parameter b and of the scale parameter a of the Subbotin density in (9) together with robust standard errors in percentage values.

more lumpy dynamics are at work when size is proxied with this particular variable, a result which is in all probability due to the indivisibilities inherently related with the process of accumulation (or dismission) of productive capital.[14]

The picture does not change substantially when one repeats the same exercises at the risk class level, controlling for firms' different financial conditions. This is shown in Figure 8, Figure 9 and Figure 10 where, given the stationarity observed over time, we report the kernel densities of the growth rates using only the observations for the year 2002. The most important features outlined at the aggregate level do survive: the peculiar fat-tailed and tent-shaped behavior, together with the fatter tails characterizing the distribution of Tangible Assets, are still there, invariant with respect to the risk class disaggregation. Nevertheless, and in close analogy with what noted analyzing firm size distribution, peculiar features characterize the densities estimated for the High Risk firms: they span noticeably wider supports than those estimated for the other two classes, and present sensibly fatter tails, both upper and lower, whatever the proxy used and in both the aggregate sectors.

The estimates of the Subbotin parameters, reported in the same Table 5, confirm these points and add further insights. Concerning Manufacturing, two different effects are present. First, the value of the shape parameter b, still smaller than 1, decreases systematically as the financial conditions of the firms worsen, irrespectively of the size proxy: using Total Sales it goes from $b = 0.97 \pm 26‰$ in the Low Risk class to $b = 0.94 \pm 16‰$ in the Mid Risk and $b = 0.76 \pm 59‰$ in the High Risk class. The same pattern is also observed with Value Added,

[14]Note that this kind of interpretation is consistent with the findings in Bottazzi *et al.* (2006b), where a similar result emerged when using the number of employees as a proxy for size, another measure which is obviously characterized by indivisibility and can be considered quasi-fixed given a plant technique.

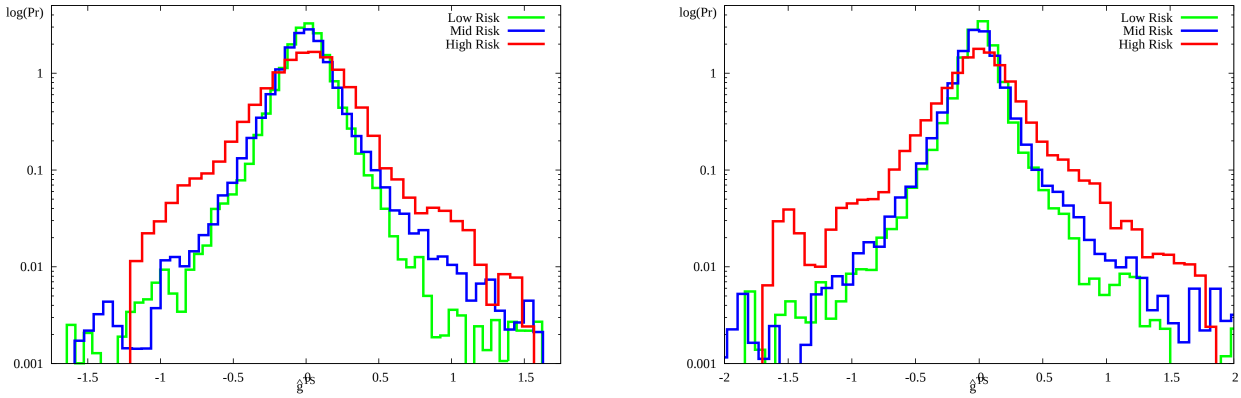

Figure 8: Empirical growth rates densities in 2002 for the Manufacturing (**left**) and Service (**right**) industry. Size is proxied with Total Sales (TS).

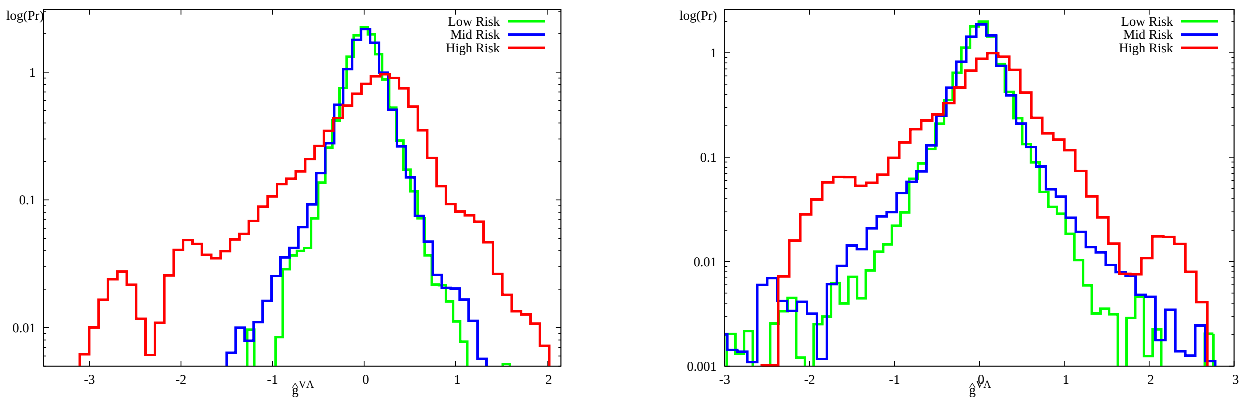

Figure 9: Empirical growth rates densities in 2002 for the Manufacturing (**left**) and Service (**right**) industry. Size is proxied with Value Added (VA).

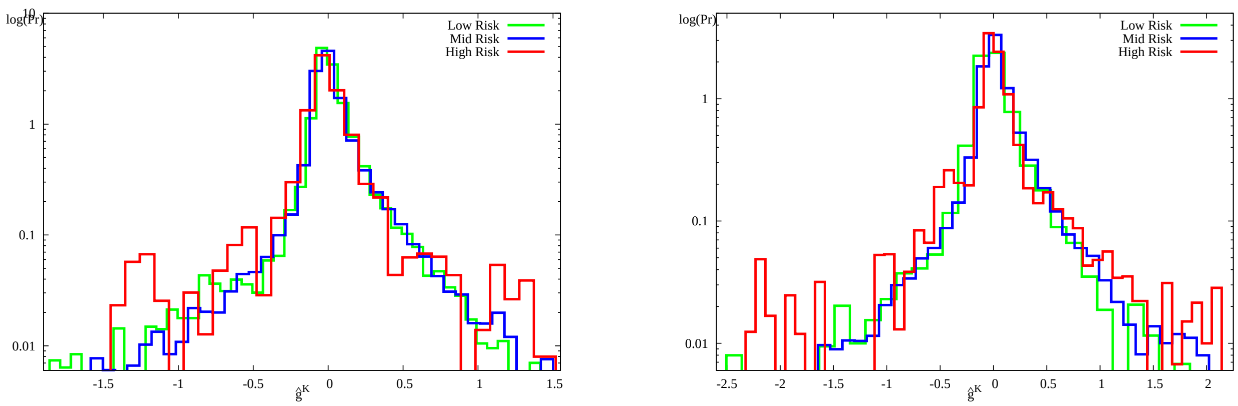

Figure 10: Empirical growth rates densities in 2002 for the Manufacturing (**left**) and Service (**right**) industry. Size is proxied with Tangible Assets (K).

where one obtains $b = 0.96 \pm 26‰$, $b = 0.82 \pm 14‰$, $b = 0.68 \pm 58‰$ respectively for the three classes, and also with Tangible Assets, although in a milder form, as the estimated values are $b = 0.60 \pm 19‰$, $b = 0.50 \pm 10‰$ and $b = 0.47 \pm 48‰$. Second, we observe that, in general, the Subbotin parameter a, and, hence, the width of the support spanned, increases with the firm's risk rating. Notice that only this second effect is observed also among firms operating in the Services, whereas the estimated b's do not change significantly across risk classes.

Summing up, we found strong evidence supporting the idea that empirical growth rates densities are stationary over time and display remarkable fat tails, even heavier than the Laplacian behavior found in previous studies. The results, robust to sectoral disaggregation and to the use of different size proxies, substantially survive once we control for firms' financial conditions and suggest that the fundamental sources of growth should be attributed to highly correlated, relatively frequent and relatively 'big' shocks, which are much likely to originate either from the very process of competition across firms or from some intrinsic lumpiness associated with the discreteness of events like entering/leaving a new market, building a new plant, introducing a new product and so on and so fort. Moreover, when we group firms according to their financial records, at least as long as the latter are captured by the financial risk rating, we find that the growth rates of less solvable firms display fatter tails and higher dispersion, especially for what concerns those firms active in Manufacturing. These represent two clear signals that a greater degree of turbulence characterizes the growth process of this group of firms. What is interesting to notice is that such turbulence does not necessarily associate with bad growth records, as one might expect, but also with extremely good performance: indeed not only the lower tail, but also the upper is fatter within High Risk firms than in the other groups.

Autoregressive profile in firm growth rates

We conclude our account of growth rates' statistical properties by means of a simple exploration of their autoregressive structure. The exercise aims primarily to understand whether self-reinforcing mechanisms, ultimately driven by competitive advantages or disadvantages are at work. This would be our interpretation in case strong autocorrelations will emerge. Similar investigation conducted in the past found mixed results about both the sign and magnitude of such autocorrelations. Bottazzi and Secchi (2005) report positive but small coefficients, and similar results are discussed in Kumar (1985), while no autocorrelation was found in Hall (1987), Contini and Revelli (1992), Boeri and Cramer (1992). All of these studies measure growth through simple log-differencing of size. Instead, we use $\hat{g}^{\mathrm{X}}(t)$, the 'scaling-free' residuals from regression (8), and we estimate the AR(1) model

$$\hat{g}_i^{\mathrm{X}}(t) = \beta\, \hat{g}_i^{\mathrm{X}}(t-1) + \epsilon_i(t) \qquad \mathrm{x} \in \{\mathrm{TS,VA,K}\} \quad . \tag{10}$$

Similarly to what we did above dealing with the autoregressive structure of size, we run a stacked regression, so that the large cross-sectional dimension of the panel is exploited to counterbalance the relatively short time series dimension, and we control for heteroskedasticity via a jackknife estimator of the standard errors. The results, reported in Table 3 above (cfr. columns *Growth*), suggest that, independently from the financial conditions and the sector of activity, the growth process of Italian business firms does not display any persistence. Indeed, both in Manufacturing and in Services, most of the estimates of the autoregressive coefficient β either are not statistically different from 0 or, when they are, as in some instances observed using Total Sales and Value Added, they are always rather small.

7 Conclusions and further research

In the course of this work we have gone through a number of statistical properties of the size and growth processes characterizing a large sample of Italian business firms, and we test whether controlling for their financial conditions significantly affect the conclusions achieved by past and recent research in the field of industrial dynamics.

In brief, we found that, no matter whether we looked at size in terms of Sales, Value Added or we focused on measures of potential capacity (Assets), the firm size distribution displays a clear rightly-skewed and leptokurtic shape, and the inter-temporal dynamics of size are well approximated by a Geometric Brownian motion. Further, we uncovered a non linear and negative dependence between average size and both average growth and the variance of growth. Finally, we documented that firms growth rates, once scaling relationships with size are filtered out, display fat-tailed and tent-shaped densities, and they don't exhibit any persistence over time. All of these results are robust at different level of aggregation, namely between aggregate sector of activity (Manufacturing vs Services) and across firms characterized by different financial ratings and, hence, by different degrees of financial fragility.

This latter dimension of analysis, however, has proved to be able to convey interesting and novel evidence. Indeed, controlling for firms' financial conditions allowed us to uncover that (*i*) both very small and very big sizes are much more concentrated within High Risk and less solvable firms, and that (*ii*) the same firms also experience much more turbulent growth records, with both extremely bad and extremely good growth episodes much more concentrated within this class than among other firms. These results, at least to the extent that the financial rating index can be considered a measure of the firms' ability to access external financing, allow us to conjecture about the existence of credit constraint type of mechanisms affecting firm size and growth dynamics. On the one hand, the result about size distribution of High Risk firms suggests that (*i*) small firms are those more likely subject to difficulties in raising external financing, and that (*ii*) it is very likely that the 'dominant' position of many big firms has been achieved through heavy indebtment, or at least relies on it. On the other hand, the peculiarity observed in the upper tail of High Risk firms' growth rates signals either a weakness on the part of many fast growing firms, who sustain their growth through heavy indebtment, or a possible malfunctioning of the (Italian) credit markets, which are not prompt enough to sustain many fast growing firms.

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