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Are Output Growth-Rate Distributions Fat-Tailed? Some Evidence from OECD Countries °

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Are Output Growth-Rate Distributions Fat-Tailed? Some Evidence from OECD Countries

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Abstract

This work explores some distributional properties of aggregate output growth-rate time series. We show that, in the majority of OECD countries, output growth-rate distributions are well-approximated by symmetric exponential-power densities with tails much fatter than those of a Gaussian. Fat tails robustly emerge in output growth rates independently of: (i) the way we measure aggregate output; (ii) the family of densities employed in the estimation; (iii) the length of time lags used to compute growth rates. We also show that fat tails still characterize output growth-rate distributions even after one washes away outliers, autocorrelation and heteroscedasticity.

Keywords: Output Growth-Rate Distributions, Normality, Fat Tails, Time Series, Exponential-Power Distributions, Laplace Distributions, Output Dynamics.

JEL Classification: C1, E3.

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1 Introduction

This work investigates the statistical properties of output growth-rate time-series distributions. In each given country, we consider the time series of aggregate output growth rates and we study the shape of the resulting distribution. More precisely, we follow a parametric approach and we fit growth-rate distributions with the exponential power (EP) family of densities (Subbotin, 1923), which includes as special cases the Gaussian and the Laplace.

Our main finding is that in the U.S., and in many other OECD countries, growth-rate distributions display tails much fatter than those of a normal distribution. This implies that output growth patterns tend to be quite lumpy: large growth events, either positive or negative, seem to be more frequent than what a Gaussian model would predict[1].

We show that this result is robust to a series of alternative specifications of the analysis. First, our findings are not affected by the way we measure aggregate output (e.g., GDP or industrial production index). Second, fat tails in growth rates still emerge if one removes from the original time series outliers, autocorrelation and heteroscedasticity (if any), and then studies the shape of the ensuing distribution of residuals. Third, the existence of super-normal tails (i.e. tails fatter than Gaussian ones) is confirmed even if one employs alternative heavy-tailed density families, such as the Levy-Stable (Nolan, 2006), the Cauchy and the Student-t. However, the EP density turns out to be the family that best fits the data for the majority of countries.

We also show that growth-rate distributions do not display any significant evidence for skewness. This implies that positive and negative large growth events have almost the same likelihood, thus confirming recent results on symmetry of the magnitude of GDP fluctuations (McKay and Reis, 2006). Finally, fat tails still emerge even if one computes growth rates using longer time-lags.

Our work is motivated by two, seemingly unrelated, streams of literature. On the one hand, we refer to the rich body of contributions that in the last twenty years have been

[1]Fat-tailed distributions arise in many empirical contexts. Applications areas, apart from economics and finance, include engineering, computer science, social networks, physics, astronomy, etc.: see Adler et al. (1998) and Embrechts et al. (1997) for an introduction.

attempting to single out robust statistical properties of within-country output dynamics (cf., among others, Nelson and Plosser, 1982; Cochrane, 1988; Brock and Sayers, 1988; Rudebusch, 1993; Cochrane, 1994; Potter, 1999; Murray and Nelson, 2000). For example, as far as the U.S. are concerned, output growth-rate time series was found to be positively autocorrelated over short horizons and to have a weak negative autocorrelation over longer horizons. Moreover, still unsettled debates have focused on the questions whether U.S. GNP is characterized by a deterministic or a stochastic trend and whether output dynamics is better captured by linear or nonlinear models. Following this line of research, our study suggests that super-normal tails in the distributions of growth rates and their residuals may be considered as a candidate to become an additional stylized fact of within-country output dynamics.

On the other hand, the quest for stylized facts of aggregate output dynamics has been more recently revived by a new body of contributions investigating the properties of *cross-country* output growth-rate distributions. The main findings of these studies was indeed that GDP growth rates tend to *cross-sectionally* distribute according to densities that display tails fatter than Gaussian ones (Canning et al., 1998; Lee et al., 1998; Castaldi and Dosi, 2004)[2]. The basic exercise performed in these studies, however, has been focusing only on *cross-section* distributions, i.e. across all countries at a given year, possibly pooling all cross-section distributions together under the assumption of stationarity of moments. In this paper, on the contrary, we show that fat-tailed distributions also emerge *across time within a single country.*

Therefore, by studying the shape of within-country growth-rate distributions, we attempt to bridge earlier studies focusing on the statistical properties of within-country output dynamics to the new stream of research on cross-sectional growth-rate distributional properties.

Our analysis differs from previous, similar ones (Canning et al., 1998; Lee et al., 1998; Castaldi and Dosi, 2004) in many respects. *First*, we depart from the common practice of

[2]Interestingly, similar results were also found for cross-section firm and industry growth rates (see Stanley et al., 1996; Lee et al., 1998; Amaral et al., 1997; Bottazzi and Secchi, 2003*a*,*b*; Castaldi and Dosi, 2004; Fu et al., 2005; Sapio and Thoma, 2006). Hence, super-normal tails *cross-sectionally* emerge no matter the level of aggregation.

using annual data to build output growth-rate distributions. We instead employ monthly and quarterly data. This allows us to get longer series and better appreciate their business cycle features. *Second*, as mentioned above, we double-check the results obtained with the EP by fitting output growth-rate distributions with a number of alternative fat-tailed densities (Levy-Stable, Cauchy, Student-t). In the case of the EP, the Student-t, and the Levy-Stable, one can actually measure how far empirical growth-rate distributions are from the Normal benchmark. *Third*, we perform a detailed goodness-of-fit analysis in order to check if our data are well proxied by theoretical densities. *Finally*, we ask whether our findings are robust to controlling for the presence of outliers, autocorrelation and heteroscedasticity in output growth-rate dynamics, and we test for possible asymmetries in growth-rate distributions.

The emergence of fat tails in country-level output growth rates has several theoretical and empirical implications. First of all, it calls for theoretical models that are able to reproduce and explain this candidate new stylized fact of output dynamics. At the same time, theoretical models might employ this new evidence in their set of assumptions so as to possibly improve their performance. In fact, it has been shown that, in many cases, economic models failing to account for fat tails in their data generating process can deliver invalid implications (Ibragimov, 2005). Furthermore, gaining empirical knowledge on the shape of some important economic variables (like output growth rates) may shed some light on the properties of the processes that have generated them (with all the caveats discussed in Brock, 1999). For example, the fact that fat tails characterize not only growth-rate distributions of countries (both time-series and cross-sectionally), but also of industries and firms, hints to the existence of some common forces operating at very different aggregation levels (Lee et al., 1998). In addition, if one thinks to the growth of country output as the outcome of the aggregation of firm- and industry-level growth profiles, the emergence of fat tails in country-level growth rates seems to strongly reject the hypothesis that some form of "central limit theorem" (CLT) is at work (Castaldi and Dosi, 2004).

The paper is organized as follows. In Section 2 we describe the data and the methodology we employ in our analysis. Empirical results on growth-rate distributions for the U.S. and other OECD countries are presented in Section 3. Robustness checks are discussed in Section 4. Section 5 discusses the implications of our results in the light of the existing theoretical and empirical literature on output dynamics in macroeconomics. Finally, Section 6 concludes.

2 Data and Methodology

The main objects of our analysis are output growth rates $g(t)$, defined as:

$$g(t) = \frac{Y(t) - Y(t-1)}{Y(t-1)} \cong y(t) - y(t-1) = (1-L)y(t), \quad (1)$$

where $Y(t)$ is the output level (GDP or IP) at time t in a given country, $y(t) = log[Y(t)]$ and L is the lag operator.

We exploit two sources of (seasonally adjusted) data. As far as the U.S. are concerned, we employ the FRED database. We consider two output growth-rate series: (i) quarterly real GDP, ranging from $1947Q1$ to $2005Q3$ (GDP, 235 observations); (ii) monthly IP, ranging from $1921M1$ to $2005M10$ ($IP1921$, 1018 observations). Moreover, in order to better compare the IP growth-rate distribution with the GDP one, we also carry out an investigation on the post WWII period only, using IP observations from 1947 to 2005 ($IP1947$, 702 observations). The analyses for the other OECD countries are performed by relying on monthly IP data drawn from the "OECD Historical Indicators for Industry and Services" database ($1975M1 - 1998M12$, 287 observations)[3]. Notice that, by focusing on IP as a measure of aggregate activity, we can study a longer time span and thus improve our estimates. IP is typically a good proxy of output levels, as it tracks very closely GDP series. In fact, as Figure 1 shows, the GDP-IP cross-correlation profile mimics from time $t-6$ to time $t+6$ the GDP auto-correlation profile.

Let $T_n = \{t_1, ..., t_n\}$ be the time interval over which we observe growth rates. We

[3]We study growth-rate distributions of the following countries: Canada, Japan, Austria, Belgium, Denmark, France, Germany, Italy, the Netherlands, Spain, Sweden and the U.K.

define the within-country, time-series distribution of output growth rates as

$$G_{T_n} = \{g(t), t \in T_n\}. \tag{2}$$

We study the shape of G_{T_n} following a parametric approach. We begin by fitting growth rates with the exponential-power (EP) family of densities, also known as Subbotin distribution[4], whose functional form reads:

$$f(x; b, a, m) = \frac{1}{2ab^{\frac{1}{b}}\Gamma(1+\frac{1}{b})} e^{-\frac{1}{b}|\frac{x-m}{a}|^b}, \tag{3}$$

where $a > 0$, $b > 0$ and $\Gamma(\cdot)$ is the Gamma function. The EP distribution is thus characterized by three parameters: a *location* parameter m, a *scale* parameter a and a *shape* parameter b. The location parameter controls for the mean of the distribution, whereas the scale parameter is proportional to the standard deviation.

The shape parameter is the crucial one for our analysis: the larger b, the thinner are the tails. In fact, the EP density encompasses both the Laplace and the Gaussian distributions: if $b = 1$ the distribution reduces to a Laplace, whereas for $b = 2$ we recover a Gaussian. Values of b smaller than one indicate super-Laplace tails (see Figure 2 for an illustration). This property is the value-added of the EP density, as it allows one to precisely measure how far the empirical distribution is from the Normal benchmark and how close is instead to the Laplace one[5].

In the exercises that follow, we fit empirical distributions G_{T_N} with the EP density (3) by jointly estimating the three parameters via maximum likelihood (ML). ML estimation of EP parameters is not an easy task. For theoretical and computational issues, we refer to Agrò (1995) and Bottazzi and Secchi (2006*b*). In what follows, we perform estimation by employing the package SUBBOTOOLS[6]. Notice that, despite ML estimators are asymptotically unbiased and are always unique for $n > 100$, some upward bias may

[4]More on fitting EP distributions to economic data is in Bottazzi and Secchi (2003*a*,*b*).

[5]Notice that we are not claiming here that the EP distribution is the unique distribution that can fit growth-rate data. In Section 4.2 we shall come back to a comparison of how alternative density families perform in fitting our data.

[6]Available online at `http://cafim.sssup.it/~giulio/software/subbotools/`. See Bottazzi (2004) for details.

emerge in the estimation of the shape coefficient for small samples. However, Montecarlo studies (available from the Authors on request) show that, for sample sizes similar to those considered in this work, ML estimators of EP parameters are nearly unbiased and are characterized by a reasonably small variance. This confirms results obtained by Agrò (1995), who also shows that estimation of b is not affected by the values of (a, m).

3 Fitting the EP Density: Parameter Estimation and Goodness of Fit

In this section, we report the results of EP fits. We begin with a detailed analysis of U.S. growth-rate distributions and we then check whether the main findings of the analysis hold also for the other OECD countries under study.

3.1 U.S. Growth-Rate Distributions

Let us start by some descriptive statistics on U.S. output growth rates. Table 1 reports the first four moments of U.S. time series, together with a battery of normality tests for the null hypothesis that series come from a Gaussian distribution with unknown parameters.

Notice first that skewness levels are quite small. This justifies using a symmetric theoretical density like (3) to fit the data[7]. The relatively large figures for kurtosis suggest however that output growth-rate distributions display fat tails. Indeed, all normality tests reject the hypothesis that U.S. series are normally distributed. This is confirmed by Anscombe-Glynn's test (Anscombe and Glynn, 1983), which clearly detects that non-normality is due to excess kurtosis.

In order to better explore this evidence, we fit U.S. output growth-rate distributions with the EP density (see eq. 3). Maximum likelihood estimates, together with standard errors (in parentheses), Cramer-Rao confidence intervals, and hypothesis testing results are reported in Table 2.

[7]In Section 4.3 we will explore in details departures from symmetry and we will check whether asymmetric EP densities might perform better than the symmetric one.

Estimates indicate that, as expected, all three growth-rate time series are markedly non normal. Growth rates seem instead to distribute according to a Laplace for GDP ($\widehat{b}$ very close to one). Furthermore, they display tails even fatter than Laplace ones for IP1921 ($\widehat{b}$ smaller than one), whereas the estimated coefficient for IP1947 goes back to a value close to one[8].

These results are statistically substantiated by Cramer-Rao confidence intervals (CI), which show that $b = 1$ lies in both GDP and IP1947 CIs. Conversely, the CI for IP1921 spans entirely on the left of $b = 1$. Of course, $b = 2$ does not belong to any CIs. Since these CIs are only valid asymptotically, we also estimate exact p-values – via a standard bootstrap procedure – for two hypothesis tests: (i) H_0: b=1 vs. H_1: b$\neq$1; (ii) H_0: b=2 vs. H_1: b<2. Estimated p-values indicate that normality is strongly rejected for all three time series. For both GDP and IP1947 it is not possible to reject the Laplace hypothesis, whereas for IP1921 the coefficient is statistically smaller than one.

We turn now to a battery of goodness-of-fit tests to explore the performance of the above EP estimates. Indeed, point estimates and parameter testing suggest that U.S. growth-rate distributions are fat-tailed. But how good is the EP fit for the U.S.? A first visual assessment is contained in Figures 3–5, where we plot the binned empirical density against the ML fitted one (in semi-log scale): the EP seems to nicely describe growth-rate distributions, especially when tails turn out to be super-Laplacian.

As Table 3 suggests, the above graphical evidence is corroborated by standard goodness-of-fit (GoF) tests (see D'Agostino and Stephens, 1986, for details). In fact, no GoF test rejects the null hypothesis that data come from the fitted distributions. Moreover, both GDP and IP1947 seem to come from a Laplace distribution, whereas IP1921 appears to be well approximated by an EP with super-Laplace tails.

Similar findings are obtained if one performs generalized likelihood-ratio tests (LRTs). Table 4 reports LRTs for the null hypotheses that data come from a Laplace or a Normal distribution. Again, normality is rejected in favor of Laplace for GDP and IP1947, and in favor of a super-Laplace distribution for IP1921.

[8]This suggests that super-Laplace tails could be due to the turmoils of the Great Depression.

3.2 Do Fat Tails Emerge Also in Other OECD Countries?

The above findings for the U.S. are replicated to a large extent also in a large sample of other OECD countries. Descriptive statistics indicate that, in half of the countries under scrutiny, the distributions of IP growth rates seem to be slightly right-skewed, whereas in the other half they appear to be slightly left-skewed (see Table 5; more on that in Section 4.3). Normality tests show that almost all growth-rate distributions are markedly non normal due to excess kurtosis (cf. the Anscombe-Glynn's test). The only two exceptions are Canada and Belgium. In Canada the evidence from normality tests is mixed, while Belgium exhibits a relatively large skewness. In both cases, however, the p-values for the kurtosis tests are close to 5%, suggesting the presence of fat tails.

EP fits confirm this evidence, see Table 6. All estimated shape coefficients are significantly smaller than two (at 5%, cf. last column). The only exception is again Canada, where the null hypothesis of normality cannot be rejected (although the p-value is very close to 5%). A quick inspection of p-values for the null hypothesis $b = 1$ (see column before the last one) shows that Spain is the only clear-cut case of a growth-rate distribution with tails fatter than a Gaussian but thinner than a Laplace. Austria, France and the Netherlands seem to have tails slightly thinner than a Laplace (p-values smaller than – or close to – 5% but larger than 1%). Conversely, Japan, Belgium, Denmark, Germany, Italy, Sweden and the U.K. display Laplace tails.

Overall, the ML fit performs well in describing the data. Apart from the case of Denmark, GoF tests do not reject the hypothesis that data come from the ML fitted EP distribution (cf. Table 7, top panel). Moreover, as the bottom panel of Table 7 shows, GoF tests do not reject the Laplace null hypothesis in almost all countries.

To further check the robustness of the results in Table 6, we turn to likelihood ratio tests. Table 8 confirms that apart from Canada (which have almost normal tails) and Spain (with tails fatter than a normal but thinner than a Laplace), growth-rate distributions of all remaining countries are well approximated by a Laplace density.

4 Robustness Checks

The foregoing discussion has pointed out that within-country output growth-rate distributions are markedly non-Gaussian. The evidence in favor of Laplace (or super-Laplace) densities robustly arises in the majority of OECD countries, it does not depend on the way we measure output (GDP or IP), and it emerges also at frequencies more amenable for the study of business cycles dynamics (i.e. quarterly and monthly). As a consequence, along the time dimension, country-level growth rates display tails fatter than those characterizing the normal density. In other words, large growth events are more likely than what one should expect.

Nevertheless, this striking evidence in favor of fat tails can be biased by at least four problems. First, growth-rate series may contain outliers, some autocorrelation structure, and be possibly characterized by heteroscedasticity. This may generate spurious results due to an inappropriate pooling of time-series observations. Second, the emergence of fat tails may depend on the particular type of density employed in our fitting exercises (i.e. the EP one). Third, while data exhibit some (very mild) evidence for skewness in growth-rate distributions, we have fitted a symmetric EP. What happens when one allows for asymmetric EP densities? Finally, super-normal tails in the IP growth-rate distributions, both for the U.S. and for the other OECD countries we have analyzed, may depend on the relatively high (monthly) frequency of IP output observations. How do estimated shape coefficients behave when growth rates are computed over longer time lags? In the remainder of this section, we will discuss these issues in more detail.

4.1 Outliers, Autocorrelation, and Heteroscedasticity

A first explanation for the presence of lumpiness in growth-rate time series might refer to the presence of outliers in the raw series (Chen and Liu, 1993). Moreover, our time-series analysis relies on pooling together growth-rate observations over time. Therefore, the observations contained in G_T should come from i.i.d. random variables. If growth-rate time series exhibit (as they typically do) autocorrelation and/or heteroscedasticity, the process is no longer i.i.d. and fat tails may emerge as a spurious result due to an inappropriate pooling procedure.

To control for these notional problems, we subsequently removed from our original series outliers, autocorrelation and heteroscedasticity (whenever detected). We performed identification and correction of outliers by employing standard procedures available in TRAMO (Gómez and Maravall, 2001) on original output growth-rate series. We then fitted a battery of ARMA specifications to outliers-free series and selected the best model via the Box and Jenkins's procedure. Finally, we checked for the presence of heteroscedasticity on ARMA residuals by running Ljung-Box and Engle's ARCH tests. If heteroscedasticity was detected, we fitted the best GARCH specification to obtain an outlier-free series without autocorrelation and heteroscedasticity (i.e. "fully depurated" series).

Table 9 reports summary statistics and normality tests for U.S. series depurated from outliers only and for "fully depurated" series[9]. Normality is still rejected in all series (due to high kurtosis). Notice also that the standard deviation of "fully depurated" IP series substantially increases.

We then fit an EP density to outlier-free and "fully depurated" U.S. series. Notwithstanding outliers and "structure" have been washed away, fat tails still emerge (see Table 10) in the distributions of residuals. While all estimated shape coefficients are now larger than one, the null hypothesis of normality is strongly rejected (see last row). According to our tests, the distributions of GDP growth-rate residuals are still close to a Laplace (the p-values for H_0: $b = 1$ are slightly larger than 5% for the outlier-free series, but smaller than 5% for the "fully depurated" one). Outlier-free IP1921 growth-rate series display now a coefficient which is clearly equal to one. Although Cramer-Rao confidence intervals indicate to reject a Laplace distribution, estimated exact p-values suggest instead not to reject H_0: $b = 1$. In all other cases, coefficients are significantly larger than one (but smaller than two). GoF and likelihood-ratio tests confirm these results: see Figure 6 as an illustration of our EP fit to the outlier-free (left) to the "fully depurated" GDP growth series.

[9]Incidentally, the best model for GDP series is an AR(1) without drift, while for IP1921 and IP1947 we employed an ARMA(1,1) and a GARCH(1,1) with an additional seasonal component in order to account for residual seasonality.

Similar findings are obtained also for the other OECD countries. Table 11 reports the results for "fully-depurated" series only[10]. Although estimated shape coefficients typically increase as compared to the non-depurated ones (Cf. Table 6), in all cases (but Japan) the distributions of residuals display tails statistically thinner than the a Laplace but much fatter than a normal density. Notice that Canada's growth-rate distribution is now back to a Laplace, while Japan's one strikingly exhibits super-Laplace tails.

This evidence suggests that fat tails still characterize our series even when growth residuals are considered as *the* object of analysis, i.e. after one washes away from the growth process outliers and any structure possibly due to autocorrelation and heteroscedasticity.

4.2 Fitting Alternative Fat-Tailed Densities

Another possible weakness of the above analysis resides in the fact that it relies on a particular type of (fat-tailed) density. The use of an EP family is justified by its extreme flexibility: if the goal is to understand not only if fat tails do emerge, but also how far they are from those of a normal (or of a Laplace) distribution, the EP density turns out to be very useful. There are however other well-known examples of densities which are well-suited to fit fat- and medium-tailed distributions (for a review of theoretical underpinnings and economic applications, see Embrechts et al., 1997).

In this section we ask whether the emergence of fat tails is confirmed when alternative densities are fitted to our data. We shall firstly employ the Student-t distribution, whose density reads:

$$t(x;\lambda,\theta,\nu)=\frac{\Gamma(\frac{\nu+1}{2})}{\theta\Gamma(\frac{\nu}{2})\sqrt{\nu\pi}}\left[1+\nu^{-1}\left(\frac{x-\lambda}{\theta}\right)^2\right]^{-\frac{\nu+1}{2}}, \tag{4}$$

where λ is a location parameter, θ is a scale parameter and ν controls for the heaviness of tails (i.e., the degrees of freedom). Notice that the larger ν, the thinner are the tails. In fact, as $\nu \to \infty$ tails converge (but slowly) to those of a Gaussian.

We shall also fit our data with the Cauchy distribution:

[10] Best models are as follows. Austria*, Spain*, Sweden: AR(2); Japan: AR(4) w/ drift; France, Germany, Italy: MA(1); UK: ARMA(3,1); Canada: ARMA(3,1)+GARCH(1,1); Belgium*: MA(1)+GARCH(1,1). (*): Seasonal Component.

$$C(x;\rho,\varphi) = \frac{1}{\varphi\pi}\left[1+\left(\frac{x-\rho}{\varphi}\right)^2\right]^{-1}, \tag{5}$$

where again ρ is a location parameter and φ is a scale parameter. Since we have that:

$$t(x;\lambda,\theta,1) = C(x;\lambda,\theta), \tag{6}$$

it turns out that the Cauchy can be considered as a special case of a Student-t (with extreme heavy tails).

Finally, we employ the family of Levy-Stable (or simply 'Stable') distributions, popularized by Nolan (2006). The Levy-Stable is a 4-parameter family of distributions, i.e.:

$$S(x;\alpha,\beta,\gamma,\delta). \tag{7}$$

The only condition that a random variable must obey to be a stable one is that its shape must be preserved under addition, up to scale and shift. Two important remarks are in order. First, the stable distribution is the only possible non-trivial limit of a normalized sum of independent, identically distributed terms. This result is known as the 'generalized' central limit theorem (GCLT), because it extends the standard CLT by dropping the finite-variance assumption. Second, the density of a stable random variable cannot be generally given in closed-form. Exceptions are the Gaussian and the Cauchy distributions, which belong to the stable family. In fact, it can be shown that the parameter α works in the same way b does for the EP distribution: if $\alpha = 2$ we recover the Gaussian distribution, while if $\alpha = 1$ the stable family boils down to a Cauchy. Unfortunately, the Laplace is not a stable distribution. This prevents us to thoroughly compare stable fits with EP ones. Moreover, the parameter β controls for the skewness: the distribution is symmetric if $\beta = 0$. Finally, δ controls for location and γ for the scale.

Tables 12, 13 and 14 report the results of our fitting exercises. Given the robustness results obtained above, we go back to our original growth-rate series and we estimate density parameters via ML[11]. Furthermore, we perform GoF tests and we estimate p-

[11]To fit stable densities, we employed the package provided by John Nolan, see

values by bootstrapping the GoF test statistics under the null hypothesis that data come from the fitted distribution.

It is easy to see (Table 12) that, according to GoF tests, the Student-t distribution does not satisfactorily fit the data, especially as far as OECD IP growth rates are concerned. However, the estimates for the 'degrees of freedom' parameter $\widehat{\nu}$ are quite small, suggesting the presence of fat tails. The Student-t is on the contrary a good choice for U.S. GDP and, to some extent, for the IP1921 series. This evidence is confirmed by the results obtained by fitting a Cauchy distribution, see Table 13. Again, GoF p-values are overall poor (apart from U.S. GDP and IP1921), suggesting that the Student-t parametrization $\widehat{\nu} = 1$ is not a good one to fit our data.

Finally, Table 14 reports the results from stable fits. Notice firstly that GoF tests slightly improves, indicating that the Levy-Stable distribution does a better job as compared to the Cauchy and the Student-t. Nevertheless, the Levy-Stable seems to be out-performed by the EP[12]. Some slight evidence in favor of asymmetry is detected, cf. the estimates for β. More importantly, values of $\widehat{\alpha}$ are always between 1 and 2, strongly indicating the presence of medium-tails in all distributions. This implication is further supported by standard errors of estimates (in parentheses), which are quite small for all four parameters.

We can then confidently conclude that fat-tails robustly arise independently of the particular density employed. Yet, the EP seems to out-perform all the other three density families in describing our growth-rate data.

4.3 Skewness and Asymmetric EP Fits

Both descriptive statistics and estimates of the symmetry parameter (β) for the stable density have suggested the presence of some skewness in growth-rate distributions. In our previous analyses, conversely, we have always employed a symmetric EP (eq. 3). In what follows, we then turn to test whether our results are robust to fitting *asymmetric*

http://academic2.american.edu/~jpnolan/stable/stable.html.

[12]This casts serious doubts on whether some sort of GCLT is in place. We shall come back to this point in Section 5.

EP densities, whenever significant skewness in the data is detected.

We begin by performing D'Agostino skewness test (D'Agostino, 1970) on both U.S. and OECD data[13]. As Table 15 shows, only three series display statistically significant skewness levels: U.S. IP1921, U.S. IP1947, and Belgium. It must be noted that this evidence in favor of an overall absence of skewness is in line with recent results showing some symmetry in the magnitude of expansions and recessions of business cycles, see McKay and Reis (2006).

We fit the data of the three skewed series with an asymmetric EP distribution (Bottazzi and Secchi, 2006*b*), whose density is given by:

$$g(x; a_l, a_r, b_l, b_r, m) = \begin{cases} K^{-1} e^{-\frac{1}{b_l}\left|\frac{x-u}{a_l}\right|^{b_l}}, & x < u \\ K^{-1} e^{-\frac{1}{b_r}\left|\frac{x-u}{a_r}\right|^{b_r}}, & x \geq u \end{cases}, \tag{8}$$

where $K = a_l b_l^{1/b_l}\Gamma(1 + 1/b_l) + a_r b_r^{1/b_r}\Gamma(1 + 1/b_r)$. Notice that in the asymmetric EP density the parameters b_l and b_r allow for different tail fatness levels on the right and on the left of the mean u, respectively. Right and left scaling is instead controlled by the parameters a_l and a_r.

As Table 15 reports, estimates of b_l and b_r seem actually to differ. In the case of U.S. IP1921, both coefficients indicate super-Laplacian tails, but positive growth events seem to be more likely than negative ones. After WWII, on the contrary, IP growth becomes almost Laplacian as far as negative jumps are concerned, whereas positive growth rates seem to be super-Laplacian. A different story holds for Belgium, where negative growth large events are far more likely than positive ones.

In order to further explore the robustness of the evidence conveyed by the analysis of parameter estimates, we performed likelihood-ratio tests to check for the null hypothesis that data come from an *asymmetric* EP that has been forced to have *symmetric* parameters, the latter being equal to those obtained in our symmetric EP exercises[14]. Table

[13]The D'Agostino skewness test performs quite well in detecting departures from symmetry for given values of kurtosis, even if the distribution is not Gaussian.

[14]More precisely, we compute the likelihood of an asymmetric density (8) with a null hypothesis given by: $\widehat{b}_l = \widehat{b}_r = \widehat{b}$, $\widehat{a}_l = \widehat{a}_r = \widehat{a}$ and $\widehat{u} = \widehat{m}$, i.e. we force the shape and scale parameters of the asymmetric EP to be equal to the corresponding ML estimate of the symmetric distribution.

15 (last two columns) shows that, despite estimates of the shape coefficients differ, there is no gain whatsoever in fitting an asymmetric density to our data. This happens in all three cases where the D'Agostino skewness test detected some asymmetry in growth-rate distributions. Indeed, the improvement in the goodness of fit does not counterbalance neither the larger degrees of freedom, nor the ensuing increase in the standard deviation of estimates, both implied by an asymmetric fit.

4.4 Increasing Growth-Rate Time Lags

As mentioned above, our work departs from existing ones also because we employ monthly and quarterly data. While this choice might allow to better appreciate the business-cycle features of growth-rate distributions, it might also generate a potential problem. Indeed, lumpiness in growth events might simply depend on the fact that we have considered output data at a too high frequency. The question then becomes: What happens when one computes growth rates at different (increasing) time lags?

To explore this issue, we inspect the distribution of output growth rates where the latter are now defined as:

$$g_\tau(t) = \frac{Y(t) - Y(t-\tau)}{Y(t-\tau)} \cong y(t) - y(t-\tau) = (1 - L^\tau)y(t), \tag{9}$$

where $\tau = 1, 2, ..., 6$ when GDP (quarterly) series is employed, and $\tau = 1, 2, ..., 12$ when IP (monthly) series is under study.

In line with the results that Bottazzi and Secchi (2006*a*) report for firm growth rates, we find that the shape parameter estimated on GDP data becomes higher as τ increases (cf. for the U.S. case the left panel of Fig. 7). When we consider U.S. IP1947 series, $\widehat{b}$ first falls and then starts rising (see the right panel of Fig. 7). Therefore, as the "growth lag" increases, tails become slightly thinner: see Silva et al. (2004) for similar evidence in the contest of stock returns. Nevertheless, estimated shape coefficients remain significantly smaller than 2, especially for the IP1947 series. Interestingly, the lag-4 IP growth-rate distribution exhibits super-Laplacian tails. This means that, even if one considers longer time spans, big growth events remain more likely than what a Gaussian model would predict.

5 Implications

The foregoing evidence brings strong support to the claim that fat tails are an extremely robust stylized fact characterizing the time series of aggregate output in most industrialized economies. This has several implications, both from an empirical and from a theoretical perspective.

From an empirical perspective, the emergence of fat-tailed distributions for within-country time series of both growth rates and residuals (see section 4.1) can be interpreted as a candidate new stylized fact on within-country output dynamics, to be possibly added to the long list of its other known statistical properties. Furthermore, this finding confirms from a time-series point of view what it seems to be a general property of cross-section growth-rate distributions. As mentioned, fat tails have been indeed discovered to be the case not only for cross-sections of countries, but also for plants, firms and industries in many countries (see Stanley et al., 1996; Lee et al., 1998; Amaral et al., 1997; Bottazzi and Secchi, 2003*a*,*b*; Castaldi and Dosi, 2004; Fu et al., 2005; Sapio and Thoma, 2006, among others). In other words, the general hint coming from this stream of literature is in favor of an increasingly "non-Gaussian" economics and econometrics. A consequence of this suggestion is that we should be very careful in using econometric testing procedures that are heavily sensible to normality of residuals. On the contrary, testing procedures that are robust to non-Gaussian errors and/or tests based on heavy-tailed errors should be employed when necessary. Examples of applications here range from Gibrat-like regressions for the dependence of firm growth on size (Sutton, 1997) to cross-section country growth rates analyses (cf. e.g. Barro and Sala-i Martin, 1992).

From a theoretical perspective, our findings may pose both an opportunity and a challenge to modelers. On the one hand, fat-tailed growth-rate distributions might be embodied in economic models to replace the standard assumption of normality of growth shocks – typically employed in the literature of e.g. real business cycle (King and Rebelo, 1999). This might be an important step forward, because, as Ibragimov (2005) shows, the implications of many models in economics and finance are very sensitive to the thickness of the tails of the distributions involved in their assumptions. On the other hand, the widespread presence of fat tails in growth shocks signals that moments of output dynamics higher than the second do matter. The emergence of excess kurtosis in growth data thus suggests to go for models that are able to replicate not only the first two moments of output growth distributions, but also higher ones.

Furthermore, gaining some knowledge on the shape of the country-level output growth-rate distributions may provide some hints on its generating process. For instance, the fact that fat tails characterize the shape of growth-rate distributions, both cross-sectionally and time-series, at very different aggregation levels (e.g., firms, industries, countries), corroborates the "intriguing possibility that similar mechanisms are responsible for the observed growth dynamics of, at least, two complex organizations: firms and countries" [Lee et al., 1998, p. 3275][15].

A fascinating challenge involves the attempt to shed more light on those common mechanisms. It must be stressed, however, that such distributional findings relate to "unconditional" distributions (Brock, 1999). Therefore, making inference on the generating mechanism responsible for fat tails at different aggregation levels is not easy at all: many data generation processes can generate such distributions in the limit. Yet, since not *every* data generation process is compatible with Laplace-distributed growth shocks, our finding might place a first restriction on the set of possible models. This may hopefully help one to discriminate among different theories (e.g., business cycle ones).

For example, suppose to interpret the country-level output growth rate in a certain time period as the result of the aggregation of microeconomic (firm-level) growth shocks across all firms and industries in the same time period. The emergence of non-Gaussian distributions at the country-level strongly militates against the idea that country growth shocks are simply the result of aggregation of independent microeconomic shocks. In other words, the CLT does not seem to be at work. In addition, our evidence shows that a Levy-stable density is not in general a good proxy of country growth-rate distributions

[15] And, in fact, industries as well, see Sapio and Thoma (2006).

(see Section 4.2). This implies that not even a generalized version of the CLT (where the finiteness of the variance of shocks is dropped) seems to governs aggregation in our data. Therefore, some strong correlating mechanism linking in a similar way, at every level of aggregation, the units to be aggregated appears to be in place (more on that is in Castaldi and Dosi, 2004). This interpretation is in line with the one proposed by Lee et al. (1998) and Amaral et al. (1998), who see the widespread presence of fat tails as an indicator of the overall "complexity" of any growth process, mainly due to the strong inner inter-relatedness of the economic organizations under study.

6 Concluding Remarks

In this paper, we have investigated the statistical properties of GDP and industrial production (IP) growth-rate distributions in OECD countries by employing monthly and quarterly time-series data.

We have found that such distributions appear to be well-approximated by a symmetric exponential-power (EP) distribution, with tails much fatter than Gaussian ones. Hence, in the last century, large "growth events" have been more likely than what one should have expected. We have shown that lumpiness of growth patterns robustly emerges independently of: (i) the way we measure output (GDP or IP); (ii) the family of density employed in the ML estimation; and (iii) the length of time lags used to compute growth rates. Furthermore, we show that fat tails characterize growth rates even after one washes away outliers, autocorrelation and heteroscedasticity (if any). Finally, we did not find any strong evidence in favor of asymmetric growth-rate distributions.

Our work can be extended in at least two ways. First, our study intrinsically assumes time invariance of the underlying generating mechanism governing output dynamics. Conversely, many studies indicate some evidence towards rejecting the assumption of temporal homogeneity of per capita GDP time series over long time spans (Balke and Fomby, 1991; Gaffeo et al., 2005). Studying more carefully the robustness of fat tails in growth-rate distributions over distinct time spans could contribute to a better understanding of the (non) stationarity nature of the GDP generating processes.

Second, one might check whether applying different detrending filters on output growth-rate distributions may change the results. As pointed out by Canova (1998), different detrending methods do indeed affect business-cycle stylized facts. One could then apply to the output time series the most common filters employed in the business cycle literature (e.g. the Hodrick-Prescott and bandpass filters) and study the ensuing output growth-rate distributions. Similarly, one could employ the bandpass filter to isolate different frequency bands directly in growth-rate time series, and then investigate which frequency intervals are more conducive to fat tails and which are not.

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Table 1: U.S. output growth-rate time series: summary statistics and p-values of normality and kurtosis tests.

		Series	
	GDP	IP1921	IP1947
Statistic		Value	
Obs	234	1017	702
Mean	0.0084	0.0031	0.0028
Std. Dev.	0.0099	0.0193	0.0098
Skewness	-0.0891	0.3495	0.3295
Kurtosis	4.2816	14.3074	8.1588
Normality Test		p-Value	
Anderson-Darling	0.0019	0.0000	0.0000
Adj Jarque Bera LM	0.0000	0.0000	0.0000
Adj Jarque Bera ALM	0.0000	0.0000	0.0000
Cramer Von-Mises	0.0020	0.0002	0.0000
Lilliefors	0.0279	0.0000	0.0000
D'Agostino	0.0120	0.0000	0.0000
Shapiro-Wilk	0.0038	0.0000	0.0000
Shapiro-Francia	0.0023	0.0000	0.0000
Kurtosis Test		p-Value	
Anscombe-Glynn	0.0036	0.0000	0.0000

Table 2: U.S. output growth-rate distributions: estimated EP parameters.

	GDP	IP1921	IP1947
$\widehat{m}$	0.0082 (0.0006)	0.0031 (0.0002)	0.0030 (0.0003)
$\widehat{a}$	0.0078 (0.0006)	0.0091 (0.0004)	0.0068 (0.0003)
$\widehat{b}$	1.1771 (0.1484)	0.6215 (0.0331)	0.9940 (0.0700)
$[\widehat{b}-2\sigma(\widehat{b}),\widehat{b}+2\sigma(\widehat{b})]$	[0.8803,1.4739]	[0.5553,0.6877]	[0.8540,1.1340]
p-value for H_0: b=1 H_1:b≠1	0.1071	1.0000	0.5268
p-value for H_0: b=2 H_1:b<2	0.0001	0.0000	0.0000

Note: Standard errors of estimates $\sigma(\cdot)$ in parentheses. $\widehat{b} \pm 2\sigma(\widehat{b})$ are Cramer-Rao confidence intervals. P-values in the last two rows computed by bootstrapping the distribution of $\widehat{b}$ under H_0 and econometric sample sizes equal to those of the empirical time series. Bootstrap sample size: $M = 10000$.

Table 3: Goodness of fit tests. Test statistics and estimated exact p-values.

	H_0: b=$\widehat{b}$, H_1: $b \neq \widehat{b}$					
	GDP		IP1921		IP1947	
GoF Test	Statistic	p-value	Statistic	p-value	Statistic	p-value
KSM	0.6772	0.7354	1.1744	0.1303	0.7907	0.5540
KUI	0.8971	0.9155	0.2711	0.1928	1.2742	0.4209
CVM	0.0410	0.9166	0.2090	0.2519	0.1100	0.5301
AD2	0.2934	0.9427	0.9905	0.3637	0.7471	0.5186
	H_0: $b = 1$, H_1: $b \neq 1$					
	GDP		IP1921		IP1947	
GoF Test	Statistic	p-value	Statistic	p-value	Statistic	p-value
KSM	0.6069	0.8450	1.6195	0.0042	0.7847	0.5659
KUI	0.9998	0.8083	2.1735	0.0028	1.2643	0.4391
CVM	0.0409	0.9134	0.5025	0.0397	0.1083	0.5421
AD2	0.3697	0.8724	5.6816	0.0015	0.7351	0.5261

Note: KSM=Kolmogorow-Smirnov (D) test; KUI=Kuiper (V) test; CVM: Cramer-VonMises (W^2) test; AD2: Anderson-Darling quadratic (A^2) test. Test statistics adjusted for small-sample bias according to D'Agostino and Stephens (1986, Table 4.2, p.105). Exact p-values estimated by bootstrapping the distribution of the test statistics under the null hypothesis H_0, with econometric sample sizes equal to those of the empirical time series and $(a, m) = (\widehat{a}, \widehat{m})$. Bootstrap sample size: $M = 10000$.

Table 4: Likelihood ratio tests.

	GDP	IP1921	IP1947
LL($\widehat{b}$)	755.844	2822.742	2314.933
LL($b = 1$)	755.099	2801.059	2314.928
LL($b = 2$)	747.978	2570.602	2249.369
H_0: $b = 1, (a, m) = (a_1^*, b_1^*)$; H_1: $b \neq 1$			
Statistics	1.490	43.365	0.010
p-value	0.685	0.000	1.000
H_0: $b = 2, (a, m) = (a_2^*, b_2^*)$; H_1: $b < 2$			
Statistics	15.731	504.279	131.128
p-value	0.001	0.000	0.000

Note: LL($\widehat{b}$)= Log likelihood associated to ML estimates $(\widehat{b}, \widehat{a}, \widehat{m})$. LL($b = 1$): Log likelihood associated to (a_1^*, m_1^*), i.e. ML estimates of (a, m) subject to $b = 1$. LL($b = 2$): Log likelihood associated to (a_2^*, m_2^*), i.e. ML estimates of (a, m) subject to $b = 2$. Test statistics: $-2 \cdot \Delta LLT(b = b_0) = -2[LL(b = b_0) - LL(\widehat{b})]$, for $b_0 = 1, 2$. P-values are computed using the fact that $-2 \cdot \Delta LLT(b = b_0) \rightarrow \chi^2(3)$.

Table 5: OECD countries output growth-rate time series: summary statistics and p-values of normality and kurtosis tests.

	Countries											
	Canada	Japan	Austria	Belgium	Denmark	France	Germany	Italy	Netherlands	Spain	Sweden	UK
Statistic	Value											
Obs	286	286	286	286	286	286	286	286	286	286	286	286
Mean	0.0021	0.0027	0.0024	0.0013	0.0025	0.0013	0.0015	0.0017	0.0015	0.0017	0.0016	0.0012
Std. Dev.	0.0113	0.0404	0.0253	0.0401	0.0340	0.0130	0.0212	0.0321	0.0285	0.0401	0.0302	0.0140
Skewness	-0.2317	-0.2250	0.1707	-0.5689	0.1214	0.1525	0.0098	0.0453	-0.0350	0.2559	-0.2955	-0.1631
Kurtosis	3.5631	4.6895	5.7806	5.9446	7.2748	3.7251	9.2312	5.8380	6.5731	4.0067	37.0700	8.4090
Normality Test	p-Value											
Anderson-Darling	0.2398	0.0000	0.0012	0.0000	0.0000	0.0025	0.0000	0.0000	0.0000	0.0754	0.0000	0.0000
Adj Jarque Bera LM	0.0400	0.0000	0.0000	0.0000	0.0000	0.0300	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Adj Jarque Bera ALM	0.0400	0.0000	0.0000	0.0000	0.0000	0.0300	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Cramer Von-Mises	0.2382	0.0000	0.0026	0.0000	0.0000	0.0012	0.0000	0.0000	0.0000	0.0611	0.0000	0.0000
Lilliefors	0.3556	0.0000	0.0279	0.0000	0.0000	0.0007	0.0000	0.0021	0.0007	0.1310	0.0000	0.0013
D'Agostino	0.0523	0.0003	0.0000	0.0000	0.0000	0.0541	0.0000	0.0000	0.0000	0.0053	0.0000	0.0000
Shapiro-Wilk	0.1706	0.0000	0.0000	0.0000	0.0000	0.0108	0.0000	0.0000	0.0000	0.0096	0.0000	0.0000
Shapiro-Francia	0.1036	0.0000	0.0000	0.0000	0.0000	0.0091	0.0000	0.0000	0.0000	0.0059	0.0000	0.0000
Kurtosis Test	p-Value											
Anscombe-Glynn	0.0707	0.0002	0.0000	0.0000	0.0000	0.0309	0.0000	0.0000	0.0000	0.0071	0.0000	0.0000

Table 6: OECD countries output growth-rate distributions: estimated EP parameters.

Country	$\widehat{m}$	$\widehat{a}$	$\widehat{b}$	$[\widehat{b}-2\sigma(\widehat{b}),\widehat{b}+2\sigma(\widehat{b})]$	p-value for H_0:b=1 H_1:b≠1	p-value for H_0:b=2 H_1:b<2
Canada	0.0020 (0.0010)	0.0104 (0.0007)	1.6452 (0.2047)	[1.2358,2.0546]	0.0000	0.0516
Japan	0.0021 (0.0014)	0.0259 (0.002)	0.8491 (0.0901)	[0.6689,1.0293]	0.9301	0.0000
Austria	0.0010 (0.0014)	0.0204 (0.0014)	1.2499 (0.1446)	[0.9607,1.5391]	0.0328	0.0000
Belgium	0.0011 (0.0017)	0.0284 (0.0021)	1.0202 (0.1125)	[0.7952,1.2452]	0.4277	0.0000
Denmark	0.0000 (0.0012)	0.0215 (0.0017)	0.8063 (0.0847)	[0.6369,0.9757]	0.9756	0.0000
France	0.0010 (0.0007)	0.0106 (0.0008)	1.2623 (0.1464)	[0.9695,1.5551]	0.0256	0.0000
Germany	0.0024 (0.0008)	0.0144 (0.0011)	0.9768 (0.1067)	[0.7634,1.1902]	0.5812	0.0000
Italy	0.0010 (0.0015)	0.0237 (0.0017)	1.0778 (0.1204)	[0.8370,1.3186]	0.2627	0.0000
Netherlands	0.0019 (0.0015)	0.0223 (0.0016)	1.2133 (0.1393)	[0.9347,1.4919]	0.0521	0.0000
Spain	0.0021 (0.0029)	0.0352 (0.0024)	1.4583 (0.1755)	[1.1073,1.8093]	0.0014	0.0056
Sweden	0.0010 (0.0009)	0.0168 (0.0013)	0.8826 (0.0944)	[0.6938,1.0714]	0.8666	0.0000
UK	0.0019 (0.0006)	0.0103 (0.0008)	1.0972 (0.1230)	[0.8512,1.3432]	0.2148	0.0000

Note: Standard errors of estimates $\sigma(\widehat{\cdot})$ in parentheses. $\widehat{b}\pm 2\sigma(\widehat{b})$ are Cramer-Rao confidence intervals. P-values in the last two rows computed by bootstrapping the distribution of $\widehat{b}$ under H_0 and econometric sample sizes equal to those of the empirical time series. Bootstrap sample size: $M = 10000$.

Table 7: Goodness of fit tests for OECD countries. Test statistics and estimated exact p-values.

	GoF Test							
	KSM		KUI		CVM		AD2	
Country	Statistic	p-value	Statistic	p-value	Statistic	p-value	Statistic	p-value
	H_0: b=$\widehat{b}$, H_1: $b \neq \widehat{b}$							
Canada	0.6527	0.7766	1.0328	0.7679	0.0545	0.8378	0.3166	0.9251
Japan	0.4406	0.9864	0.8068	0.9694	0.0295	0.9717	0.3681	0.8773
Austria	0.6426	0.7987	0.9574	0.8578	0.0589	0.8099	0.5408	0.7071
Belgium	1.0041	0.2631	1.4815	0.1899	0.1231	0.4773	0.7552	0.5185
Denmark	1.7276	0.0041	2.2713	0.0009	0.3687	0.0844	1.9317	0.0997
France	1.2595	0.0804	2.2097	0.0022	0.1013	0.5552	0.5205	0.7183
Germany	0.5857	0.8764	1.0035	0.8106	0.0481	0.8815	0.4603	0.7879
Italy	0.6566	0.7784	1.3115	0.3716	0.0503	0.8642	0.3096	0.9270
Netherlands	1.3145	0.0647	2.3895	0.0004	0.1737	0.3141	1.0135	0.3453
Spain	0.5622	0.9040	1.0095	0.7963	0.0452	0.8933	0.3572	0.8853
Sweden	1.0848	0.1826	1.4733	0.1911	0.1287	0.4461	0.8782	0.4141
UK	0.8382	0.4802	1.2528	0.4470	0.0721	0.7307	0.7476	0.5155
	H_0: $b = 1$, H_1: $b \neq 1$							
Canada	0.7307	0.6551	1.4641	0.2034	0.1547	0.3760	1.5626	0.1648
Japan	0.5475	0.9165	1.0153	0.7919	0.0402	0.9238	0.7433	0.5262
Austria	0.8943	0.3974	1.2702	0.4203	0.0855	0.6540	0.8876	0.4239
Belgium	1.0107	0.2528	1.5065	0.1701	0.1261	0.4638	0.7689	0.5055
Denmark	1.7276	0.0054	2.2140	0.0021	0.3882	0.0805	2.1346	0.0811
France	1.1990	0.1098	2.2097	0.0026	0.0967	0.5923	0.6522	0.6036
Germany	0.5755	0.8915	0.9835	0.8299	0.0454	0.8912	0.4435	0.8013
Italy	0.6678	0.7556	1.3231	0.3522	0.0574	0.8231	0.3754	0.8745
Netherlands	1.3861	0.0419	2.5051	0.0000	0.2186	0.2310	1.4519	0.1885
Spain	0.7280	0.6649	1.4188	0.2477	0.0765	0.7115	1.0192	0.3521
Sweden	1.0312	0.2351	1.2939	0.3912	0.1115	0.5232	0.6802	0.5728
UK	0.9394	0.3314	1.3881	0.2745	0.0822	0.6726	0.8632	0.4390

Note: KSM=Kolmogorow-Smirnov (D) test; KUI=Kuiper (V) test; CVM: Cramer-VonMises (W^2) test; AD2: Anderson-Darling Quadratic (A^2) test. Test statistics adjusted for small-sample bias according to D'Agostino and Stephens (1986, Table 4.2, p.105). Exact p-values estimated by bootstrapping the distribution of the test statistics under the null hypothesis H_0, with econometric sample sizes equal to those of the empirical time series and $(a, m) = (\widehat{a}, \widehat{m})$. Bootstrap sample size: $M = 10000$.

Table 8: Likelihood ratio tests for OECD countries.

				$H_0: b=1$, $(a,m)=(a_1^*,b_1^*)$ H_1: $b\neq 1$		$H_0: b=2$, $(a,m)=(a_2^*,b_2^*)$ H_1: $b<2$	
Country	LL($\widehat{b}$)	LL($b=1$)	LL($b=2$)	Statistic	p-value	Statistic	p-value
Canada	878.7356	870.3870	877.4225	16.6971	0.0008	2.6262	0.4529
Japan	540.4682	539.3700	512.0700	2.1963	0.5327	56.7963	0.0000
Austria	655.4767	653.5069	645.3409	3.9396	0.2681	20.2717	0.0001
Belgium	537.1539	537.1403	514.0219	0.0274	0.9988	46.2641	0.0000
Denmark	587.7118	586.7298	561.1264	1.9640	0.5799	53.1708	0.0000
France	842.6275	841.8152	836.8383	1.6247	0.6538	11.5784	0.0090
Germany	725.8727	725.8421	695.7202	0.0612	0.9960	60.3050	0.0000
Italy	594.9010	594.6809	577.7194	0.4400	0.9319	34.3631	0.0000
Netherlands	626.4250	624.5917	612.1028	3.6666	0.2998	28.6443	0.0000
Spain	517.4279	512.9920	514.1326	8.8718	0.0310	6.5907	0.0862
Sweden	668.3535	667.2673	595.3823	2.1726	0.5374	145.9425	0.0000
UK	834.9834	834.5914	814.9782	0.7839	0.8533	40.0103	0.0000

Note: LL($\widehat{b}$)= Log likelihood associated to ML estimates $(\widehat{b},\widehat{a},\widehat{m})$. LL($b=1$): Log likelihood associated to (a_1^*,m_1^*), i.e. ML estimates of (a,m) subject to $b=1$. LL($b=2$): Log likelihood associated to (a_2^*,m_2^*), i.e. ML estimates of (a,m) subject to $b=2$. Test statistics: $-2\cdot\Delta LLT(b=b_0)=-2[LL(b=b_0)-LL(\widehat{b})]$, for $b_0=1,2$. P-values are computed using the fact that $-2\cdot\Delta LLT(b=b_0)\rightarrow\chi^2(3)$.

Table 9: Controlling for outliers, autocorrelation and heteroscedasticity. U.S. output growth-rate time series: summary statistics and p-values of normality and kurtosis tests.

	Series					
	Outliers only			"Fully Depurated" Series		
	GDP	$IP1921$	$IP1947$	$GDP^{\dagger}$	$IP1921^{\ddagger}$	$IP1947^{\ddagger}$
Statistic	Value					
Mean	0.0000	0.0019	0.0023	-0.0001	-0.0444	-0.0104
Std. Dev.	0.0091	0.0129	0.0084	0.0087	0.9957	1.0021
Skewness	-0.1636	-0.3911	-0.1015	-0.0692	-0.1858	0.1156
Kurtosis	3.7148	4.6464	4.4879	3.9990	4.6179	4.0278
Normality Test	p-Value					
Anderson-Darling	0.0035	0.0000	0.0000	0.0042	0.0000	0.0001
Adj Jarque Bera LM	0.0400	0.0000	0.0000	0.0100	0.0000	0.0000
Adj Jarque Bera ALM	0.0400	0.0000	0.0000	0.0100	0.0000	0.0000
Cramer Von-Mises	0.0027	0.0000	0.0000	0.0081	0.0000	0.0002
Lilliefors	0.0147	0.0000	0.0000	0.0493	0.0000	0.0081
D'Agostino	0.0697	0.0000	0.0000	0.0366	0.0000	0.0001
Shapiro-Wilk	0.0243	0.0000	0.0000	0.0066	0.0000	0.0001
Shapiro-Francia	0.0148	0.0000	0.0000	0.0040	0.0000	0.0001
Kurtosis Test	p-Value					
Anscombe-Glynn	0.0402	0.0000	0.0000	0.0114	0.0000	0.0001

Note: Columns 1-3 refer to original series depurated of outliers only. Columns 4-6 ("fully depurated" series) refer to original series depurated from outliers, autocorrelation and possibly heteroscedasticity. Outlier removal performed using TRAMO (Gómez and Maravall, 2001). ARMA fitting performed by selecting the best ARMA model using a Box-Jenkins selection procedure on outlier-free residuals. Best GARCH filtering applied if both Ljiung-Box and Engle's ARCH heteroscedasticity tests (on ARMA residuals) were rejected. (†) residuals from ARMA only; (‡) residuals from ARMA + GARCH.

Table 10: Controlling for outliers, autocorrelation and heteroscedasticity. U.S. output growth-rate distributions: estimated EP parameters.

	Series					
	Outliers-Free Series			"Fully Depurated" Series		
	GDP	$IP1921$	$IP1947$	$GDP^{\dagger}$	$IP1921^{\ddagger}$	$IP1947^{\ddagger}$
$\widehat{m}$	0.0000 (0.0006)	0.0027 (0.0000)	0.0026 (0.0000)	0.0000 (0.0006)	-0.0353 (0.0009)	-0.0202 (0.0013)
$\widehat{a}$	0.0073 (0.0006)	0.0094 (0.0000)	0.0067 (0.0000)	0.0071 (0.0005)	0.8280 (0.0010)	0.8480 (0.0014)
$\widehat{b}$	1.2308 (0.1568)	1.0367 (0.0019)	1.2227 (0.0034)	1.2696 (0.1628)	1.3205 (0.0026)	1.3578 (0.0039)
$[\widehat{b}-2\sigma(\widehat{b}),\widehat{b}+2\sigma(\widehat{b})]$	[0.9172,1.5444]	[1.0329,1.0405]	[1.2159,1.2295]	[0.9440,1.5952]	[1.3153,1.3257]	[1.3501,1.3655]
p-value for H_0: b=1 H_1: b≠1	0.0575	0.2719	0.0028	0.0378	0.0000	0.0000
p-value for H_0: b=2 H_1: b<2	0.0002	0.0000	0.0000	0.0005	0.0000	0.0000

Note: Columns 1-3 refer to original series depurated of outliers only. Columns 4-6 ("fully depurated" series) refer to original series depurated from outliers, autocorrelation and possibly heteroscedasticity. Outlier removal performed using TRAMO (Gómez and Maravall, 2001). ARMA fitting performed by selecting the best ARMA model using a Box-Jenkins selection procedure on outlier-free residuals. Best GARCH filtering applied if both Ljiung-Box and Engle's ARCH heteroscedasticity tests (on ARMA residuals) were rejected. (†) residuals from ARMA only; (‡) residuals from ARMA + GARCH. Standard errors of estimates $\sigma(\widehat{\cdot})$ in parentheses. $\widehat{b}\pm 2\sigma(\widehat{b})$ are Cramer-Rao confidence intervals. P-values in the last two rows computed by bootstrapping the distribution of $\widehat{b}$ under H_0 and with econometric sample sizes equal to those of the empirical time series and $(a,m)=(\widehat{a},\widehat{m})$. Bootstrap sample size: $M=10000$.

Table 11: Controlling for outliers, autocorrelation and heteroscedasticity. Output growth-rate distributions of OECD countries: estimated EP parameters.

	$\widehat{m}$	$\widehat{a}$	$\widehat{b}$	$[\widehat{b}-2\sigma(\widehat{b}),\widehat{b}+2\sigma(\widehat{b})]$	p-value for H_0:b=1 H_1:b≠1	p-value for H_0:b=2 H_1:b<2
Canada‡	0.0122 (0.0032)	0.8508 (0.0035)	1.4007 (0.0099)	[1.3809,1.4205]	0.0042	0.0022
Japan†	0.0000 (0.0000)	0.0019 (0.0000)	0.7617 (0.0047)	[0.7522,0.7712]	0.9917	0.0000
Austria†	-0.0016 (0.0001)	0.0176 (0.0001)	1.4767 (0.0106)	[1.4555,1.4979]	0.0009	0.0081
Belgium‡	0.0104 (0.0032)	0.8537 (0.0035)	1.3728 (0.0096)	[1.3536,1.3920]	0.0053	0.0010
Denmark†	0.0004 (0.0001)	0.0217 (0.0001)	1.2967 (0.0090)	[1.2787,1.3147]	0.0177	0.0004
France†	0.0019 (0.0000)	0.0106 (0.0000)	1.6051 (0.0119)	[1.5813,1.6289]	0.0000	0.0359
Germany†	0.0023 (0.0001)	0.0142 (0.0001)	1.4293 (0.0103)	[1.4088,1.4498]	0.0026	0.0035
Italy†	0.0038 (0.0001)	0.0207 (0.0001)	1.2437 (0.0086)	[1.2265,1.2609]	0.0367	0.0000
Netherlands†	0.0024 (0.0001)	0.0201 (0.0001)	1.7805 (0.0136)	[1.7533,1.8077]	0.0000	0.1622
Spain†	0.0038 (0.0001)	0.0253 (0.0001)	1.5058 (0.0110)	[1.4839,1.5277]	0.0003	0.0109
Sweden†	0.0000 (0.0001)	0.0176 (0.0001)	1.5842 (0.0117)	[1.5608,1.6076]	0.0000	0.0286
UK†	0.0021 (0.0000)	0.0105 (0.0000)	1.4821 (0.0108)	[1.4605,1.5037]	0.0006	0.0086

Note: Estimates performed on "fully depurated" series, i.e. original series subsequently depurated from outliers, autocorrelation and possibly heteroscedasticity. Outlier removal performed using TRAMO (Gómez and Maravall, 2001). ARMA fitting performed by selecting the best ARMA model using a Box-Jenkins selection procedure on outlier-free residuals. Best GARCH filtering applied if both Ljiung-Box and Engle's ARCH heteroscedasticity tests (on ARMA residuals) were rejected. (†) residuals from outliers and ARMA only; (‡) residuals from outliers, ARMA and GARCH. Standard errors of estimates $\sigma(\widehat{\cdot})$ in parentheses. $\widehat{b} \pm 2\sigma(\widehat{b})$ are Cramer-Rao confidence intervals for $\widehat{b}$. P-values in the last two columns computed by bootstrapping the distribution of $\widehat{b}$ under H_0 and with econometric sample sizes equal to those of the empirical time series and $(a,m)=(\widehat{a},\widehat{m})$. Bootstrap sample size: $M = 10000$.

Table 12: Fitting a Student-t distribution. Estimated parameters and GoF tests.

Series	$\widehat{\lambda}$	$\widehat{\theta}$	$\widehat{\nu}$	KSM	KUI	CVM	AD2
US GDP	0.0085 (0.0594)	0.0084 (0.0693)	2.2513 (1.8039)	0.8903 (0.3961)	1.7803 (0.0408)	0.2843 (0.1454)	2.2639 (0.0652)
US IP1921	0.0031 (0.0171)	0.0083 (0.0206)	1.5685 (0.1425)	0.8643 (0.4480)	1.6801 (0.0715)	0.1228 (0.4906)	0.8533 (0.4492)
US IP1947	0.0028 (0.0296)	0.0076 (0.0324)	2.4727 (0.4114)	1.7118 (0.0050)	3.0975 (0.0000)	0.8816 (0.0029)	6.2509 (0.0004)
Canada	0.0022 (0.0582)	0.0102 (0.0641)	1.5223 (0.6816)	1.4331 (0.0306)	2.6902 (0.0000)	0.6427 (0.0159)	5.9064 (0.0009)
Japan	0.0023 (0.0425)	0.0233 (0.0545)	1.6188 (0.4339)	3.8494 (0.0000)	7.4333 (0.0000)	6.3625 (0.0000)	36.7449 (0.0000)
Austria	0.0018 (0.0538)	0.0200 (0.0560)	2.8749 (1.6001)	3.0482 (0.0000)	6.0317 (0.0000)	4.1182 (0.0000)	25.3073 (0.0000)
Belgium	0.0027 (0.0462)	0.0257 (0.0499)	2.0767 (0.5859)	4.0784 (0.0000)	7.7385 (0.0000)	6.8927 (0.0000)	39.1799 (0.0000)
Denmark	0.0022 (0.0480)	0.0230 (0.0523)	2.2737 (0.7732)	3.6595 (0.0000)	7.0501 (0.0000)	5.7041 (0.0000)	33.4318 (0.0000)
France	0.0013 (0.0565)	0.0115 (0.0698)	1.8036 (0.7438)	1.6605 (0.0081)	3.0668 (0.0000)	1.0403 (0.0009)	7.9628 (0.0001)
Germany	0.0023 (0.0457)	0.0134 (0.0493)	2.1329 (0.6327)	2.0276 (0.0003)	3.7465 (0.0000)	1.4933 (0.0002)	10.9291 (0.0000)
Italy	0.0011 (0.0499)	0.0228 (0.0555)	2.3389 (0.9017)	3.5696 (0.0000)	6.9689 (0.0000)	5.6170 (0.0000)	32.9150 (0.0000)
Netherlands	0.0021 (0.0513)	0.0210 (0.0510)	2.7550 (1.0626)	3.2766 (0.0000)	6.3648 (0.0000)	4.5994 (0.0000)	27.8259 (0.0000)
Spain	0.0014 (0.0573)	0.0358 (0.0615)	1.9141 (0.2811)	4.8194 (0.0000)	9.5354 (0.0000)	10.2404 (0.0000)	54.5191 (0.0000)
Sweden	0.0019 (0.0388)	0.0163 (0.0388)	2.3133 (0.6513)	2.5150 (0.0000)	4.9236 (0.0000)	2.7009 (0.0000)	17.8114 (0.0000)
UK	0.0017 (0.0505)	0.0101 (0.0508)	2.6640 (1.0157)	0.8612 (0.4500)	1.6314 (0.0898)	0.2451 (0.1885)	2.2372 (0.0695)

Note: Standard errors of estimates and p-values of GoF tests in parentheses. GoF tests refer to the null hypothesis that data come from a Student-t distribution with parameters $(\widehat{\lambda}, \widehat{\theta}, \widehat{\nu})$. KSM=Kolmogorow-Smirnov (D) test; KUI=Kuiper (V) test; CVM: Cramer-VonMises (W^2) test; AD2: Anderson-Darling Quadratic (A^2) test. Test statistics adjusted for small-sample bias according to D'Agostino and Stephens (1986, Table 4.2, p.105). Exact p-values estimated by bootstrapping the distribution of the test statistics under the null hypothesis that data come from a Student-t distribution with parameters $(\widehat{\lambda}, \widehat{\theta}, \widehat{\nu})$, with econometric sample sizes equal to those of the empirical time series. Bootstrap sample size: $M = 10000$.

Table 13: Fitting a Cauchy distribution. Estimated parameters and GoF tests.

Series	$\widehat{\rho}$	$\widehat{\phi}$	KSM	KUI	CVM	AD2
US GDP	0.0083 (0.0005)	0.0050 (0.0004)	0.8846 (0.4159)	1.7276 (0.0545)	0.1256 (0.4713)	1.8003 (0.1215)
US IP1921	0.0031 (0.0003)	0.0065 (0.0003)	0.9394 (0.3251)	1.8606 (0.0232)	0.1494 (0.3800)	2.5632 (0.0435)
US IP1947	0.0032 (0.0003)	0.0045 (0.0002)	1.3302 (0.0600)	2.5820 (0.0000)	0.4523 (0.0507)	5.2932 (0.0016)
Canada	0.0023 (0.0006)	0.0066 (0.0005)	1.2348 (0.0938)	2.3886 (0.0004)	0.3211 (0.1115)	3.6149 (0.0134)
Japan	0.0018 (0.0014)	0.0167 (0.0013)	3.2014 (0.0000)	6.3667 (0.0000)	4.5965 (0.0000)	28.3805 (0.0000)
Austria	0.0010 (0.0012)	0.0132 (0.0010)	2.6382 (0.0000)	5.0762 (0.0000)	2.9971 (0.0000)	19.9725 (0.0000)
Belgium	0.0022 (0.0018)	0.0186 (0.0014)	3.5606 (0.0000)	6.9658 (0.0000)	5.4883 (0.0000)	32.7753 (0.0000)
Denmark	0.0016 (0.0014)	0.0157 (0.0012)	3.0038 (0.0000)	6.0214 (0.0000)	4.1113 (0.0000)	25.9262 (0.0000)
France	0.0011 (0.0006)	0.0067 (0.0005)	1.4089 (0.0355)	2.6171 (0.0000)	0.6079 (0.0211)	5.0267 (0.0025)
Germany	0.0027 (0.0009)	0.0096 (0.0007)	1.8818 (0.0009)	3.6053 (0.0000)	1.1116 (0.0009)	9.4588 (0.0000)
Italy	0.0001 (0.0014)	0.0155 (0.0012)	3.2657 (0.0000)	5.9850 (0.0000)	4.3901 (0.0000)	26.9120 (0.0000)
Netherlands	0.0033 (0.0014)	0.0147 (0.0011)	3.1575 (0.0000)	5.7933 (0.0000)	3.6824 (0.0000)	23.6593 (0.0000)
Spain	0.0031 (0.0020)	0.0220 (0.0017)	4.1418 (0.0000)	7.8756 (0.0000)	7.0009 (0.0000)	39.9212 (0.0000)
Sweden	0.0020 (0.0011)	0.0117 (0.0009)	2.2684 (0.0001)	4.4879 (0.0000)	2.0555 (0.0000)	15.0456 (0.0000)
UK	0.0022 (0.0006)	0.0070 (0.0005)	1.2959 (0.0690)	2.5191 (0.0000)	0.3446 (0.1015)	3.9675 (0.0097)

Note: Standard errors of estimates and p-values of GoF tests in parentheses. GoF tests refer to the null hypothesis that data come from a Cauchy distribution with parameters $(\widehat{\rho}, \widehat{\varphi})$. KSM=Kolmogorow-Smirnov (D) test; KUI=Kuiper (V) test; CVM: Cramer-VonMises (W^2) test; AD2: Anderson-Darling Quadratic (A^2) test. Test statistics adjusted for small-sample bias according to D'Agostino and Stephens (1986, Table 4.2, p.105). Exact p-values estimated by bootstrapping the distribution of the test statistics under the null hypothesis that data come from a Cauchy distribution with parameters $(\widehat{\rho}, \widehat{\varphi})$, with econometric sample sizes equal to those of the empirical time series. Bootstrap sample size: $M = 10000$.

Table 14: Fitting a Levy-Stable distribution. Estimated parameters and GoF tests.

Series	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\gamma}$	$\widehat{\delta}$	KSM	KUI	CVM	AD2
US GDP	1.5220 (0.0918)	-0.1790 (0.3254)	0.0055 (0.0004)	0.0084 (0.0007)	0.9343 (0.3432)	1.1846 (0.5346)	0.1075 (0.5496)	0.8347 (0.4580)
US IP1921	1.1990 (0.0467)	0.0060 (0.0724)	0.0067 (0.0003)	0.0031 (0.0004)	0.5195 (0.9494)	1.0204 (0.7628)	0.0472 (0.8856)	0.6460 (0.6849)
US IP1947	1.4840 (0.0584)	-0.0550 (0.1287)	0.0047 (0.0002)	0.0030 (0.0003)	0.7817 (0.6178)	1.1429 (0.6569)	0.0761 (0.7387)	0.6186 (0.6899)
Canada	1.7760 (0.0448)	-0.0460 (0.0469)	0.0074 (0.0003)	0.0020 (0.0007)	0.6699 (0.7374)	1.0089 (0.7962)	0.0647 (0.7552)	0.3945 (0.8458)
Japan	1.2290 (0.0899)	0.0800 (0.1463)	0.0175 (0.0013)	0.0016 (0.0019)	3.4767 (0.0010)	6.6273 (0.0004)	5.0419 (0.0008)	30.2501 (0.0582)
Austria	1.7990 (0.0763)	0.6080 (0.3572)	0.0149 (0.0008)	0.0000 (0.0016)	2.9260 (0.0010)	5.6914 (0.0007)	3.6128 (0.0004)	22.1102 (0.0123)
Belgium	1.4300 (0.0917)	-0.0750 (0.1896)	0.0204 (0.0012)	0.0004 (0.0021)	4.0941 (0.0003)	7.5255 (0.0008)	6.7418 (0.0001)	38.1219 (0.0323)
Denmark	1.4070 (0.0895)	0.1260 (0.2318)	0.0161 (0.0011)	-0.0006 (0.0019)	3.4170 (0.0007)	6.2171 (0.0002)	4.6578 (0.0004)	27.6751 (0.0359)
France	1.5520 (0.0595)	0.0520 (0.5995)	0.0079 (0.0004)	-0.0001 (0.0008)	1.9064 (0.0001)	2.4177 (0.0005)	1.1587 (0.0005)	6.0807 (0.0147)
Germany	1.5500 (0.0902)	-0.3500 (0.1971)	0.0104 (0.0006)	0.0032 (0.0011)	1.6486 (0.0115)	3.2940 (0.0010)	1.0430 (0.0003)	7.6443 (0.0153)
Italy	1.4740 (0.0890)	0.1320 (0.2277)	0.0165 (0.0010)	0.0003 (0.0019)	3.2672 (0.0001)	6.3374 (0.0010)	4.6340 (0.0005)	27.7981 (0.0315)
Netherlands	1.7830 (0.0850)	-0.3320 (0.2659)	0.0167 (0.0009)	0.0017 (0.0017)	3.3676 (0.0009)	6.3382 (0.0007)	4.6194 (0.0009)	27.4775 (0.0079)
Spain	1.7820 (0.0632)	-0.0410 (0.5152)	0.0255 (0.0013)	0.0025 (0.0026)	4.5076 (0.0008)	8.7702 (0.0005)	8.7814 (0.0009)	47.8478 (0.0060)
Sweden	1.5860 (0.0839)	-0.0680 (0.2915)	0.0127 (0.0007)	0.0014 (0.0014)	2.4971 (0.0005)	4.6093 (0.0005)	2.3793 (0.0005)	15.4577 (0.0203)
UK	1.7510 (0.0832)	-0.9500 (0.2288)	0.0079 (0.0004)	0.0030 (0.0008)	0.9512 (0.3153)	1.2665 (0.4096)	0.2025 (0.2518)	1.6222 (0.1574)

Note: Standard errors of estimates and p-values of GoF tests in parentheses. GoF tests refer to the null hypothesis that data come from a Levy-Stable distribution with parameters $(\widehat{\alpha}, \widehat{\beta}, \widehat{\gamma}, \widehat{\delta})$, under the S_0 parametrization. KSM=Kolmogorow-Smirnov (D) test; KUI=Kuiper (V) test; CVM: Cramer-VonMises (W^2) test; AD2: Anderson-Darling Quadratic (A^2) test. Test statistics adjusted for small-sample bias according to D'Agostino and Stephens (1986, Table 4.2, p.105). Exact p-values estimated by bootstrapping the distribution of the test statistics under the null hypothesis that data come from a Levy-Stable distribution with parameters $(\widehat{\alpha}, \widehat{\beta}, \widehat{\gamma}, \widehat{\delta})$, under the S_0 parametrization, with econometric sample sizes equal to those of the empirical time series. Bootstrap sample size: $M = 10000$.

Table 15: Checking for asymmetry in growth-rate distributions. D'Agostino skewness test, asymmetric EP fits and likelihood ratio tests.

	D'Agostino Skewness Test				Likelihood Ratio Test	
Series	Statistic	p-value	$\widehat{b}_l$	$\widehat{b}_r$	Statistic	p-value
US GDP	-0.3770	0.7062	-	-	-	-
US IP 1921	2.9369	0.0033	0.7441 (0.0490)	0.6639 (0.0423)	7.6122	0.1789
US IP 1947	2.3147	0.0206	1.1372 (0.1101)	0.8549 (0.0810)	6.5632	0.2552
Canada	-1.0673	0.2858	-	-	-	-
Japan	-1.0370	0.2997	-	-	-	-
Austria	0.7905	0.4292	-	-	-	-
Belgium	-2.4857	0.0129	0.7427 (0.0993)	1.3424 (0.2042)	7.4858	0.1869
Denmark	0.5642	0.5726	-	-	-	-
France	0.7073	0.4794	-	-	-	-
Germany	0.0458	0.9635	-	-	-	-
Italy	0.2110	0.8329	-	-	-	-
Netherlands	-0.1630	0.8705	-	-	-	-
Spain	1.1755	0.2398	-	-	-	-
Sweden	-1.3512	0.1766	-	-	-	-
UK	-0.7560	0.4497	-	-	-	-

Note: D'Agostino skewness test (D'Agostino, 1970) rejected only for US IP1921, US IP1947, and Belgium. Asymmetric EP fits: standard errors of estimates $\sigma(\widehat{b}_l)$ and $\sigma(\widehat{b}_r)$ in parentheses. Likelihood ratio tests (LRT) refer to the null hypothesis that data come from an asymmetric EP density (see eq. 8) wherein parameters are restricted to be homogeneous and equal to maximum-likelihood estimates computed for symmetric EP fits (see Tables 2 and 6), i.e. $\widehat{b}_l = \widehat{b}_r = \widehat{b}$, $\widehat{a}_l = \widehat{a}_r = \widehat{a}$ and $\widehat{u} = \widehat{m}$. P-values for the LRT are computed using the fact that $LRT \to \chi^2(5)$.

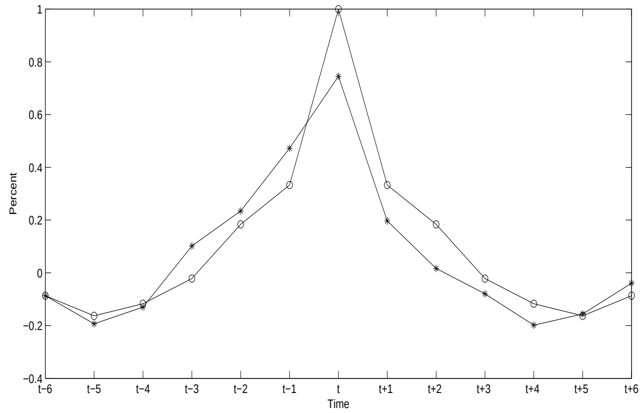

Figure 1: GDP auto-correlation vs. GDP-IP cross-correlations for U.S. FRED data. Circles: GDP auto-correlation. Asterisks: GDP-IP cross-correlations.

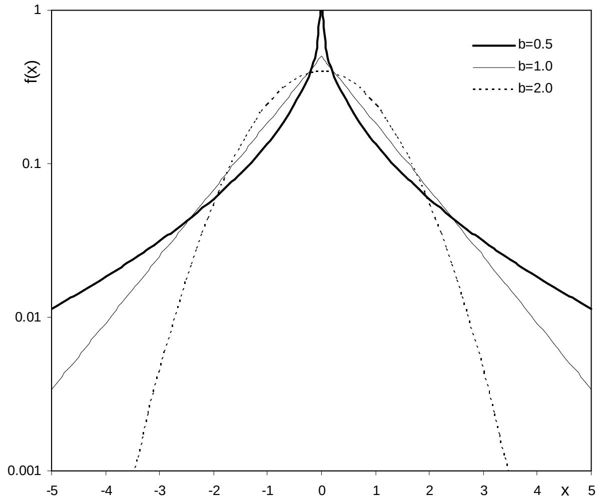

Figure 2: The exponential-power (EP) density for $m = 0$, $a = 1$ and different shape parameter values: (i) $b = 2$: Gaussian density; (ii) $b = 1$: Laplace density; (iii) $b = 0.5$: EP with super-Laplace tails. Note: Log scale on the y-axis.

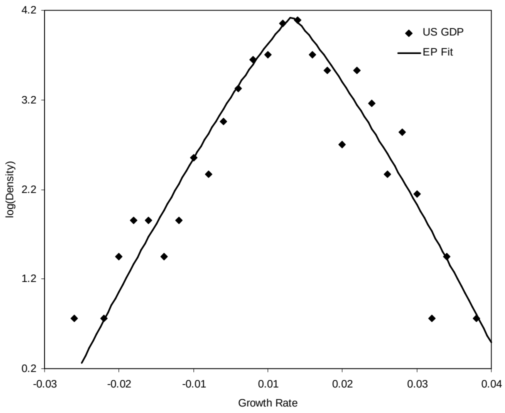

Figure 3: Binned empirical densities of U.S. GDP growth rates vs. EP fit.

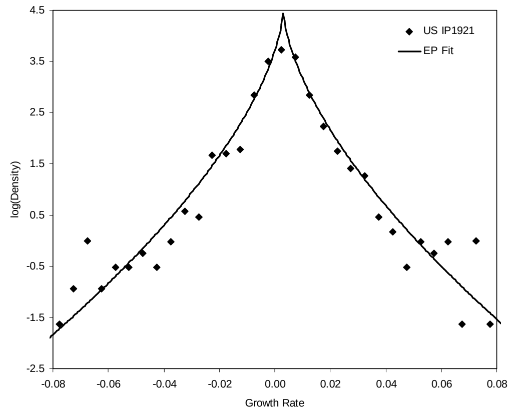

Figure 4: Binned empirical densities of U.S. IP1921 growth rates vs. EP fit.

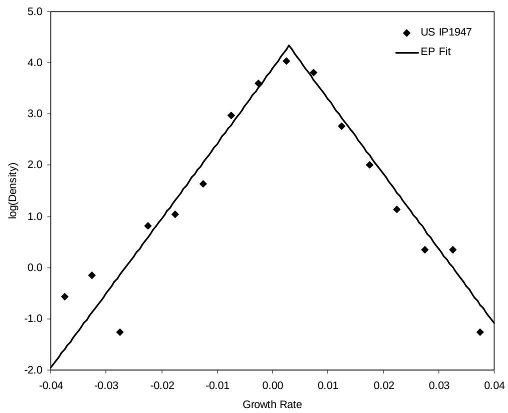

Figure 5: Binned empirical densities of U.S. IP1921 growth rates vs. EP fit.

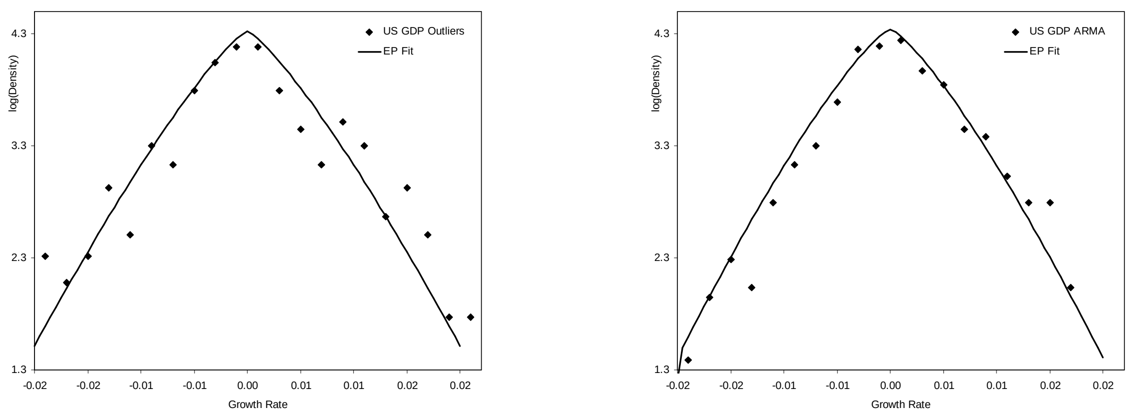

Figure 6: Controlling for outliers and autocorrelation in U.S. GDP growth rates. Binned empirical densities vs. EP fit. Left: residuals after removing outliers only. Right: residuals after removing outliers and autocorrelation (‘fully depurated series’). Outlier removal performed using TRAMO (Gómez and Maravall, 2001). Autocorrelation removal performed by fitting an ARMA model to outlier-free residuals. No evidence for heteroscedasticity detected.

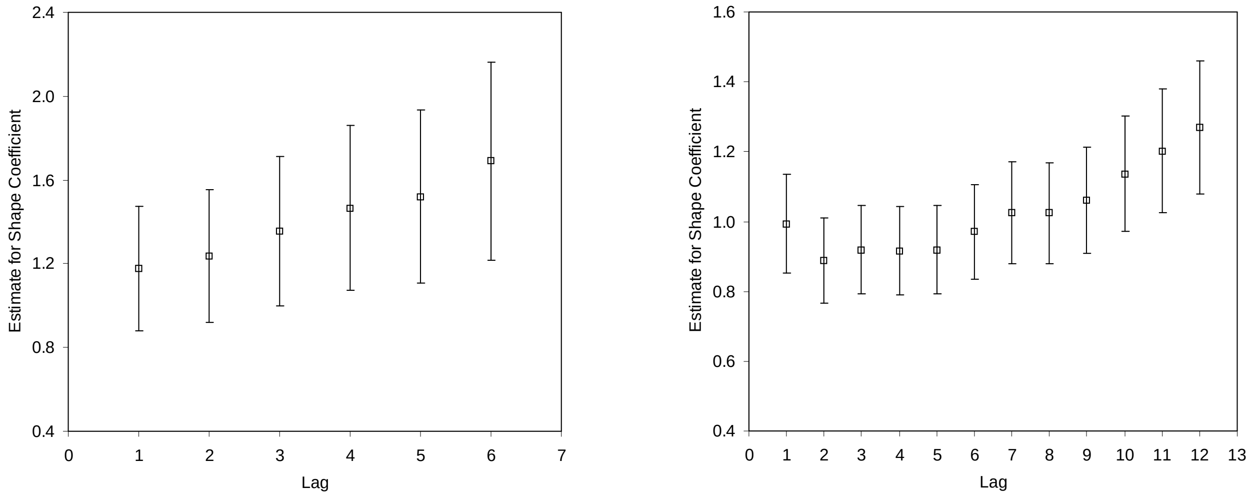

Figure 7: Increasing the time lag in the computation of growth rates. Estimates of the shape coefficient (b). Left: US GDP. Right: US IP1947. Bars represent Cramer-Rao intervals $(\widehat{b} \pm 2\sigma(\widehat{b}))$.