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Modeling Industrial Evolution in Geographical Space

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Abstract

In this paper we study a class of evolutionary models of industrial agglomeration with local positive feedbacks, which allow for a wide set of empirically-testable implications. Their roots rest in the Generalized Polya Urn framework. Here, however, we build on a birth-death process over a finite number of locations and a finite population of firms. The process of selection among production sites that are heterogeneous in their "intrinsic attractiveness" occurs under a regime of dynamic increasing returns depending on the number of firms already present in each location. The general model is presented together with a few examples of small economies which help to illustrate the properties of the model and characterize its asymptotic behavior. Finally, we discuss a number of empirical applications of our theoretical framework. The basic model, once taken to the data, is able to empirically disentangle the relative strength of technologically-specific agglomeration drivers (affecting differently firms belonging to different industrial sectors in each location) from site-specific geographical forces (horizontally acting upon all sectors in each location).

JEL codes: C1, L6, R1

Keywords: Industrial Location, Agglomeration, Dynamic Increasing Returns, Markov Chains, Polya Urns.

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1 Introduction

The evolution of technologies and industries clearly occurs in "spaces", both geographical spaces and more metaphorical ones wherein "distances" and boundaries are shaped by institutions, networks of interaction and associated knowledge spillovers. However, while a lot of efforts has gone into the formalization of the processes of technological and economic evolution *in general* (for some overview of the progress since the seminal Nelson and Winter (1982), cf. Dosi and Winter (2002)), it is fair to say that much less progress has been made in the formal representation of the spatial nesting of such evolutionary processes and even less so in the elaboration of models yielding empirically testable formulations.[1] This is the central concern of this work.

The basic skeleton of the class of models we present is made of a simple economy composed of a finite number of distinct locations (i.e. production sites) and populated by a finite number of firms. New firms enter the economy, select a site in which to place their activities. Conversely incumbent firms from every location face some probability of leaving them (i.e. dying). New firms are randomly selected from a notionally infinite number of potential entrants and select their production sites depending on their expected benefits (most likely including expected profits). In that, note that well in tune with evolutionary interpretations of economic change, "expectations" do not map in any precise sense into what the economic environment will eventually deliver (hence, in general "rational expectations" are deemed as just a particular case out of many possible descriptions of investment processes). We assume that the benefits "perceived" by entrants are made up of a common component, identical across the would be population, and an individual term, which captures idiosyncratic (actual or expected) returns from locating in one particular site.

Since we are interested in investigating the effect of different degrees of "agglomer-

[1] Discussions of the inroads made by evolutionary ideas in the field of economic geography are in Boschma and Frenken (2006) and Martin and Sunley (2006).

ation economies" on the ultimate distributional patters, we assume that the common term in firm preferences is composed of two elements: the intrinsic "geographic attractiveness" of a location and an "agglomeration" benefit. The latter is in general different for different locations and is assumed to be proportional to the number of firms already located there.

We describe the entry and exit process of firms and the ensuing evolution of the geographic distribution of economic activities as a finite Markov chain. This stochastic model does retain the basic evolutionary methodological prescription that sound accounts of economic phenomena - in this case evolving industrial geographies - have to be grounded into explicit *process stories* involving micro behaviors unfolding over time and bearing macro-level effects. Micro heterogeneity here fully appears even if black-boxed into the stochastic structure of the entry process, accounting for those trial-and-error behaviors and, together, those degrees of bounded rationality which are likely to underlie micro processes of exploration and adaptation. At the same time, the presence of agglomeration benefits accounts for *dynamic increasing returns* often associated with, e.g. learning-by-doing and by-using, network effects, user-producer relations and various forms of "Marshallian" externalities which characterize evolutionary dynamics in the socio-economic domain. In turn, such increasing returns are likely to be, at least partly, local, also in a strict geographical sense.

The foregoing ingredients suffice to account also for the interplay between *chance* and *necessity* involved in industrial evolution and its geographical unfolding. Indeed, the spatial distribution of economic activities is likely to depend on the intrinsic features of space itself – features that look very much like "endowments" or at least "slow" variables, like many institutional set-ups which change on a time scale much longer than the scale over which micro location decisions occur. Together, there are agglomeration forces which emerge, so to speak, along the process of agglomeration itself, with earlier locational events influencing the attractiveness of the site for future investors. In turn, such agglomeration forces might be location-specific and independent of individual sectors and technologies, or, conversely, sector-specific, applying across different locations within the same sector of activity.

The formal apparatus presented in this work is meant precisely to offer an account of the different agglomeration forces at work and together to allow the derivation of empirically testable formulations.

As compared to the incumbent literature, such a "reduced form" evolutionary model does share with New Economic Geography (NEG) (cf. Krugman (1991) and Fujita et al. (1999), among others) the interpretation of the observed spatial agglomeration patterns as phenomena of self-organization, driven by externalities and increasing returns of some kind. On the other hand, the two stream of interpretations tend to depart with respect to the micro-foundations (with NEG much more committed to rational decision-makers) and also with respect to the style of analysis whereby NEG searches whenever possible for closed form equilibrium solutions and most often builds "explanations" upon comparisons among equilibria themselves, whereas models like those presented below try to explicitly account for whatever dynamics and ask where it may lead to. Correspondingly, NEG models straightforwardly assume agglomeration phenomena as equilibrium outcomes of location decisions in monopolistically competitive markets while no such commitment is necessarily made by models closer to an evolutionary inspiration. In fact, precisely because of such an agnosticism, evolutionary-inspired models can be usefully applied also to dynamic processes such as those concerning the development of technological externalities or the diffusion of knowledge within and across geographical sites which often *do not* involve any *market* and, even less so, any equilibrium notion.

More precisely, the models in this work find their roots into the notion of *local dynamic increasing returns* explored in Arthur (1990, 1994), Dosi et al. (1994) and Dosi and Kaniovski (1994).[2] Using the formal tool of *generalized urn schemes*, these models

[2]In a similar spirit see also Brenner (2003).

begin to offer a simple spatial characterization of adaptive processes of growth accounting for the presence of positive, and possibly also negative, feedbacks over ever-growing populations of firms or customers. However, a significant drawback of generalized urn schemes rests in their limited interpretative ability over small population and short time horizons. In such a framework, the initial conditions of the system (i.e. the initial number of firms present in each location), together with the sequences of stochastic realizations, characterizes the asymptotic geographical distribution of firms. The strength of such representation is precisely its ability to account for the "power of history" to shape long-term outcomes under dynamic increasing returns of most kinds. The symmetric drawback is that such an approach hardly applies to circumstances wherein "choices" are somewhat reversible over time, while - together - one may easily account for "small" populations of agents. The representation of such alternative set-up involves repeated and reversible decisions by finite populations of agents in presence of "local" dynamic increasing returns. This is the focus of this study, formally grounded on the analytical results presented in Bottazzi and Secchi (2007).

In the following we adopt this second approach and we consider, instead of an irreversible birth dynamics, a Markov process with finite number of firms/locations and reversability of locational choices. In this framework we are able to derive the equilibrium distribution of firms across geographical locations and to obtain empirically testable models. Next we show that, by varying the relative strength of geographical attractiveness and of agglomeration positive feedbacks, the model is able to reproduce highly different degrees of spatial concentration and different temporal dynamics. In particular, when the agglomeration benefits are absent or very low, different locations attract, on average, a share of the overall population of firms that is proportional to their intrinsic attractiveness (we shall define more precisely these notions below). These shares, however, tend to fluctuate in time with a relatively high volatility. Conversely, when the strength of the agglomeration benefits increases, the system moves toward more "polarized" distributional patterns in which a small fraction of location contains almost the entire economy. At the same time, the introduction of agglomeration benefits and the ensuing polarization of spatial distribution entails a major indeterminacy (to some extent alike that shown in Arthur (1994)): locations which absorb the largest part of the economy are dynamically selected and history plays a fundamental role in it. However, the prominence of particular sites is not permanent. Rather, they represent sort of "metastable" states: over the long term, new locations do emerge displacing previous ones as leading attractive poles.

The remainder of the paper is organized as follows. Section 2 presents a stochastic model of multi-site location in which we disentangle the role played by the "intrinsic geographic attractiveness" of each site from the one due to pure agglomeration forces. Section 3 presents some small economies examples, while Section 4 studies the asymptotic behavior of our model when only entry dynamics are retained. Section 5 explores the case where all locations is characterized by the same (industry-specific) agglomeration coefficient. Finally, Section 6 discusses possible applications of the model to empirical analyses.

2 A Stochastic Process of Multi-Site Location

Assume that the economy is composed of $L \geq 2$ distinct locations, labeled by integers between 1 and L, which can be thought as "production sites" or "industrial districts" or "regions". The economy is populated by N firms. Each firm locates its productive activities in a single location. Time is discrete and at each time step $t \in \{1, 2, \dots\}$ new firms can enter the economy and incumbents can leave it. Each firm, when entering the economy, chooses to locate its production activities in the site which is expected to provide the highest benefits (which economists generally take to be the highest stream of future profits). Firms are boundedly rational and their expectations build on two terms: a common factor and an idiosyncratic component. The common factor affects the decision of any possible entrant and is meant to represent the common "perceived" advantage of locating activities in a certain site. The idiosyncratic component captures the individual preferences of that particular firm. Firms are heterogenous with respect to their revealed preferences. This heterogeneity can be due to asymmetric information or "cognitive biases", but even more plausibly, be the effect of the diverse requirements that drive the choices of different firms inside an industry.

Since for the time being we are interested only in deriving the aggregate dynamics of the system, we simply model firm heterogeneity through a random effect. Formally, we assume the following

Assumption 1. *Let $\mathcal{F}$ be the population of potential entrants and let $c_l \leq 0, l \in \{1, \ldots, L\}$ stand for the common benefits (to all firms) from locating an economic activity in l.*

When a new firm enters the economy, it is selected at random from $\mathcal{F}$ and chooses location l which satisfies

$$l = \arg\max_j \{c_j + e_j | j \in \{1, \ldots, L\}\}$$

where $(e_1, \ldots, e_L)$ represents the individual preferences of the firm.[3]

Essentially, such an assumption postulates that the entry process is defined by the probability distribution $F(\boldsymbol{e})$ of individual preferences $\boldsymbol{e} = (e_1, \ldots, e_L)$ on the population of potential entrants $\mathcal{F}$. The probability p_l that the next entrant chooses location l is indeed[4]

$$p_l = \text{Prob}\{c_l + e_l \geq c_j + e_j \forall j \neq l | \boldsymbol{c}, F(\boldsymbol{e})\} \ .$$

[3]For sake of simplicity we are neglecting here the fact that would-be entrants might have different sizes and thus also the distinction between would-be returns per unit of investment and returns per firm. In our framework it is straightforward to consider the c_l as returns to firms.

[4]Notice that this is exactly the same entry process assumed in Arthur (1990).

The dynamical process implied by this assumption is undetermined until one provides a precise definition of the distribution F. This is generally a very difficult task as it requires to model the (private and unexpressed) preferences of the whole population of possible entrants.

However, Bottazzi and Secchi (2007) show that it is possible to significantly simplify this problem without restricting too much the generality of the approach. Indeed, by introducing a minimal degree of structure in the decision process or, alternatively, by assuming a simple but plausible structure of the economy, it suffices to show that the entry decision is, in probability, only driven by the common components $\boldsymbol{c}$ of the variables (e.g. profits) which enter the decision process. In particular one may either interpret the entry decision as the outcome of a "discriminal process" (Thurstone, 1927) between different choices of location or, alternatively, one may assume that each location is composed by a large number of sub-locations characterized by the same common expected profit c_l (but allowing different firms to posses different preferences over different sub-locations). In these circumstances, it can be proved that the probability that a given location l is chosen is

$$p_l = \frac{c_l}{\sum_j c_j} \ . \tag{2.1}$$

Notice that even if the two different interpretations of the "choice" process start from highly different premises in terms of the information processing abilities of the agents and, together, of their abilities to specify their "fine-grained" preferences they do simplify our dynamical process in exactly the same way, thus adding plausibility to the assumptions underling equation (2.1).

In order to completely specify the model, at this point one has to provide the analytic expression for the "common" attractiveness of a location (that is, common to all would-be entrants). To recall, our aim is to describe the spatial distribution of economic activities under different agglomeration (or anti-agglomeration) forces. We start by assuming that the locational choice of entrant firms is affected by the actual distribution of firms that they observe when they assess their would-be location. For sake of tractability, we will try to capture this effect with a simple linear relationship, assuming the following

Assumption 2. *The common expected profit c_l from locating a new activity in location l at time t is given by*

$$c_l = a_l + b_l n_l$$

where n_l represents the number of firms present in location l at the time of choice and $a_l \geq 0$, $b_l \geq 0$.

Since this is the core relation of the family of models which we are going to discuss in the following, let us spell out at some detail its empirical grounds.

Each location $l \in \{1, \ldots, L\}$ is characterized by an "intrinsic attractiveness" parameter a_l and by an "agglomeration" parameter b_l.

The coefficient a_l captures the perceived gains that a firm would obtain by choosing to locate its activity in l, net of any agglomeration effects. In tune with the quite "agnostic" nature of our modeling skeleton, on purpose, we mean such a coefficient to capture an ensemble of phenomena, identified in the literature as *catalyzers* and "exogenous" drivers of agglomeration as distinct from the drivers which are inbuilt in the location processes themselves. Hence, they include sheer geographical aspects - e.g. a harbor or a river - and also infrastructural factors which are indeed man-made but change at time-scales plausibly slower than those characterizing the entry/exit flows addressed in our model. The intrinsic attractiveness parameter covers also the "enabling conditions" and "catalyzer" which Bresnahan et al. (2001) identify at the root of the "novel silicon valleys" (e.g. locally available skilled labor and knowledge spillovers from thereby universities which - as Adams (2002) shows - are geographically quite sticky).[5] If location decisions are in some way related to localized knowledge spillovers, a_l captures indeed their location-specific pull. Finally, suppose that the industry described by the foregoing relation is "small" as compared to the whole economy of any particular location. Then, a_l may also naturally accounts for pecuniary and non-pecuniary externalities - ranging from market availability to relationships with suppliers and customers - which are "endogenous" to the location as a whole, but exogenous to any particular ("small") sector of activity.

Conversely, the parameter b_l measures the strength of agglomeration economies in location l: it is the amount by which the advantages obtained by locating in l increases as a function of the number of firms already located there. The larger is the value of b_l the higher is the incentive for firms to locate as the number of firms that have already settled there increases. In a way, this is "agglomeration in action", with relative advantages of particular locations straightforwardly stemming from the very history of location decisions. Again, multiple (possibly complementary) dynamics are captured by positive b_l. Local network externalities are an obvious example, but equally important processes include the development of "social networks" (Sorenson, 2005), "horizontal" and "vertical" development of knowledge clusters (Maskell and Malmberg, 2007), "face-to-face" coordination and learning dynamics (Storper and Venables, 2004) and locally nested processes of "corporate filiation" along the life cycles of industries (Klepper, 2001). Needless to say, the dynamic-increasing-returns story which our modeling skeleton is meant to capture is consistent with the well known "Silicon Valley" example but also with the dynamics of e.g. Emilia Romagna districts in Italy (Brusco, 1982) or the german production clusters in Baden-Württemberg (Herrigel, 1996).

Finally, notice that in our baseline formulation we assume linear increasing returns to the number of location events at any one site. As a first approximation, the assump-

[5] On the localized dimension of knowledge spillovers see also Jaffe et al. (1993), among others.

tion seems to us as the most unbiased benchmark. However, the model, appropriately modified, can easily account for non linearities in agglomeration economies and also “anti-agglomeration” factors above certain thresholds (due e.g. to congestion phenomena or increasing rents).

Concerning the exit of incumbent firms from the economy, we also take the simplest possible approach and consider the

Assumption 3. *All firms are randomly chosen, with equal probability, to exit the economy.*

Moreover, in the following we assume that entry rates are positive, constant and equal to exit rates. The idea behind this assumption comes from the observation that the share of firms belonging to a given sector which enter and leave a given location in a relatively short period of time (e.g. a year) is typically much larger than the net growth of industry size, so that the time-scale at which spatial reallocations occur is generally quite short.[6] Broadly in line with this piece of evidence, we keep constant the number of locations L and the number of firms N present in the industry.

Analysis of the model

In our model at each time step, a firm leaves the economy according to Assumption 3 and, after such an exit, a single firm is allowed to enter the economy according to Assumption 1. Notice that the “entrant” may well “choose” (or in any case happen to pop up at) a location different from the one where “death” occurred. Thus, the process is designed to capture both the genuine formation of new firms and the reversibility of locational decisions of incumbents which might close a production unit in one site just to open up another one elsewhere. Let us summarize assumptions and results discussed above in the following

[6] For a detailed comparative cross-country overview cf. Bartelsman et al. (2005).

Proposition 2.1. *At the beginning of each time period t a firm is chosen at random among the N incumbents to exit the economy according to Assumption 3. Let $m \in \{1, \ldots, L\}$ be the location affected by this exit. After exit takes place, a new firm enters the economy. The probability p_l to choose location l conditional on the exit occurred in m, according to Assumption 2 and (2.1), is defined as*

$$p_l = \frac{a_l + b_l\,(n_{l,t-1} - \delta_{l,m})}{A + \boldsymbol{b} \cdot \boldsymbol{n} - b_m}, \tag{2.2}$$

where $A = \sum_{l=1}^{L} a_l$, $\boldsymbol{b} \cdot \boldsymbol{n} = \sum_{l=1}^{L} b_l\, n_l$ and the Kronecker delta $\delta_{x,y}$ is 1 if $x = y$ and 0 otherwise.

In (2.2) $n_{l,t-1}$ is the number of firms present in location l at the previous time step $t-1$ while Kronecker delta $\delta_{l,m}$ in (2.2) implies that it is the number of firms present is location l after exit took place that affects the probability of the entering firm to be located in l. The assumption of non-negative b_l coefficients implies non-decreasing dynamic returns and, whenever $b_l > 0$, linear returns to agglomeration.

If $n_{l,t}$ is the number of firms present in location l at time t (with $\sum_{l=1}^{L} n_{l,t} = N,\ \forall t$) the occupancy vector $\boldsymbol{n}_t = (n_{1,t}, ..., n_{L,t})$ completely defines the state of the economy at this time. Due to the stochastic nature of the dynamics (as implied by Proposition 2.1), the only possible description of the evolution of the economy is in terms of probability of observing, at a given point in time, one particular occupancy vector among the many possible ones.

Let $\boldsymbol{a} = (a_1, \ldots, a_L)$ and $\boldsymbol{b} = (b_1, \ldots, b_L)$ be the L-tuples containing the parameters for intrinsic attractiveness and for the agglomeration strength of locations $\{1, \ldots, L\}$. The dynamics of the system described in Proposition 2.1 is equivalent (cfr. Bottazzi and Secchi (2007), Section 3) to a finite Markov chain with state space

$$S_{N,L} = \{\boldsymbol{n} = (n_1, \ldots, n_L) | n_l \geq 0, \sum_{l=1}^{L} n_l = N\} \quad .$$

If $p_t(\boldsymbol{n};\boldsymbol{a},\boldsymbol{b})$ is the probability that the economy is in the state $\boldsymbol{n}$ at time t, the probability that the economy is in state $\boldsymbol{n'}$ at time $t+1$ is given by

$$p_{t+1}(\boldsymbol{n}';\boldsymbol{a},\boldsymbol{b}) = \sum_{\boldsymbol{n}\in S_{N,L}} P(\boldsymbol{n'}|\boldsymbol{n};\boldsymbol{a},\boldsymbol{b})P_t(\boldsymbol{n};\boldsymbol{a},\boldsymbol{b})$$

where $P(\boldsymbol{n'}|\boldsymbol{n};\boldsymbol{a},\boldsymbol{b})$ represents the generic element of the Markov chain transition matrix. Let $\boldsymbol{\delta}_h = (0,...,0,1,0,...0)$ be the unitary L-tuple with h-th component equal to 1. Then

$$P(\boldsymbol{n'}|\boldsymbol{n};\boldsymbol{a},\boldsymbol{b}) = \begin{cases} \frac{n_m}{N}\frac{a_l+b_l\,(n_l-\delta_{l,m})}{C(\boldsymbol{n},\boldsymbol{a},\boldsymbol{b})} & \text{if } \exists l,m\in(1,\ldots,L) \text{ s.t. } \boldsymbol{n'}=\boldsymbol{n}-\boldsymbol{\delta}_m+\boldsymbol{\delta}_l \\ \\ 0 & \text{otherwise} \end{cases} \tag{2.3}$$

where

$$C(\boldsymbol{n},\boldsymbol{a},\boldsymbol{b}) = A + (1-\frac{1}{N})\boldsymbol{b}\cdot\boldsymbol{n} \quad . \tag{2.4}$$

The state space of the Markov chain that describes the evolution of the model is the set of all the L-tuples of non-negative integers whose sum of elements is equal to N. Note that when the number of locations L and/or of firms N increase, the dimension of the Markov chain becomes soon very large. For instance, for $N = 50$ and $L = 10$ the state space contains more than a billion states. On the other hand, according to Proposition 2.1, at most one firm is allowed to move at each time steps. This implies that the transition matrix of the chain contains many zeros and all transitions happen between very similar states, i.e. states that differ by the location of a single firm.

Moreover, Assumption 2 allows for a location l to have zero intrinsic attractiveness ($a_l = 0$). This kind of location is peculiar because, if at some point in time it is empty, it will never be occupied again. Indeed, according to (2.2), if $a_l = 0$ and $n_l = 0$ the probability of location l to receive the entrant firm is $p_l = 0$. One can think of this location as if it had disappeared from the economy. Since the probability that any occupied location looses a firm is always positive, one should expect that, asymptotically, all locations with zero intrinsic attractiveness become empty.[7] Consequently we assume that all the locations present attractiveness strictly greater than zero and we present a complete characterization of the "equilibrium" condition of the present model in the following

Proposition 2.2. *The finite dimensional Markov chain described in* (2.3) *admits a unique stationary distribution* $\pi(\boldsymbol{n}; \boldsymbol{a}, \boldsymbol{b})$.

On S *the Markov chain is symmetric under time reversal and satisfies the detailed balance condition. If* $\boldsymbol{n}, \boldsymbol{n} - \boldsymbol{\delta}_h + \boldsymbol{\delta}_k \in S$ *one has*

$$
\begin{aligned}
&\pi(\boldsymbol{n} - \boldsymbol{\delta}_h + \boldsymbol{\delta}_k) = T_{h \to k}(\boldsymbol{n}) \; \pi(\boldsymbol{n}) \\
&T_{h \to k}(\boldsymbol{n}) = \frac{a_k + n_k \, b_k}{a_h + (n_h - 1) \, b_h} \frac{n_h}{n_k + 1} \frac{C(\boldsymbol{n} - \boldsymbol{\delta}_h + \boldsymbol{\delta}_k, \boldsymbol{a}, \boldsymbol{b})}{C(\boldsymbol{n}, \boldsymbol{a}, \boldsymbol{b})} \quad .
\end{aligned}
\tag{2.5}
$$

If $\boldsymbol{n} \in S$ *the stationary distribution* $\pi(\boldsymbol{n})$ *reads*

$$
\pi(\boldsymbol{n}; \boldsymbol{a}, \boldsymbol{b}) = \frac{N! C(\boldsymbol{n}, \boldsymbol{a}, \boldsymbol{b})}{Z_N(\boldsymbol{a}, \boldsymbol{b})} \prod_{l=1}^{L} \frac{1}{n_l!} \vartheta_{n_l}(a_l, b_l), \tag{2.6}
$$

where

$$
\vartheta_n(a, b) = b^n \, \frac{\Gamma(a/b + n)}{\Gamma(a/b)} = \begin{cases} \prod_{h=1}^{n} [a + b(h-1)] & n > 0 \\ 1 & n = 0 \end{cases} \tag{2.7}
$$

and $Z_N(\boldsymbol{a}, \boldsymbol{b})$ *is a normalization coefficient depending on the number of firms* N *and on the* L*-tuples* $\boldsymbol{a}$ *and* $\boldsymbol{b}$.

Proof. See Bottazzi and Secchi (2007), Section 3. □

[7] For a formal proof of this statement see Bottazzi and Secchi (2007), Section 3.

3 Examples of small economies

The analysis *in general terms* of the model presented in the previous section would require a good deal of technical details which are beyond the scope of the present paper. Here in order to appreciate its main properties, we consider the behavior of our model in some simple instantiations.

Example 1: no positive agglomeration feedbacks

Consider the simplest case with two distinct locations, 1 and 2. In this case the state of the system is completely described by the number of firms belonging to one location. Let n be the number of firms located in 1. From (2.6) it is

$$\pi(n) = \left(A + (1 - \frac{1}{N})(b_1 n + b_2(N-n))\right) \binom{N}{n} b_1^n \frac{\Gamma(a_1/b_1 + n)}{\Gamma(a_1/b_1)} \, b_2^{N-n} \frac{\Gamma(a_2/b_2 + N - n)}{\Gamma(a_2/b_2)} \,. \tag{3.1}$$

If $b_1 = b_2 = 0$, the previous expression reduces to the binomial distribution of N independent trials with probability $p = a_1/(a_1 + a_2)$. This distribution has mean equal to Np and variance $Np(1-p)$. Consequently, at equilibrium, location 1 is, on average, occupied by a number of firms proportional to its relative intrinsic attractiveness (cfr. the discussion above), that is $n \sim a_1/A$.

The same property also applies to the general model with L distinct locations: when all the agglomeration parameters are set to zero, the average occupancy of each location is proportional to its intrinsic attractiveness.

`Figure 1 about here`

However, the stochastic nature of the process implies that, in general, the actual number of firms observed in one location fluctuates through time. At the same time, when the number of firms increases, due to the Central Limit Theorem, the relative amplitude of these fluctuations decreases. An example is provided in Figure 1 (left panel) for the $L = 2$ case. As can be seen, when $N = 500$, the probability to observe a deviation larger then 10% from the average value of 1/3 is extremely low.

Example 2: positive agglomeration feedbacks uniform across locations

Let us continue with the example in Figure 1, set $N = 100$ and consider different values for the agglomeration economies parameter b, keeping it equal across the two locations. As can be seen from the right panel of Figure 1, a slight increase in the value of b_1 is enough to generate a noticeable widening in the support of the distribution. Such a widening suggests a more turbulent dynamics, with larger fluctuations in the fraction of firms which occupy location 1. This phenomenon becomes stronger when the agglomeration parameter reaches a value comparable to the value of the geographic attractiveness ($b_1 \sim a_1$). In this case, the support spans the entire range $[0, 1]$ and fluctuations of any order are likely to be observed. We will briefly discuss some typical time series at the end of this Section. Here it is interesting to notice than when the parameter b further increases, the phenomenon is reversed: the set of points on which the distribution achieves relatively large values shrinks. In particular, the probability weights becomes increasingly concentrated in the two extremes, $n = 0$ and $n = 1$. The reason of this reversal is straightforward: when the agglomeration strength parameter is high, the most probable configurations are those that are associated with a highly concentrated industry. In the case of two locations and 100 firms, the occupancies displaying with the highest concentration are those near $(n_1 = 100, n_2 = 0)$ and $(n_1 = 0, n_2 = 100)$. As can be seen for the right panel of Figure 1, when $b = 4$ they are, by a large extent, the most probable ones.

`Figure 2 about here`

The behavior described above is not restricted to the two locations case but has a general character. For instance, the same behavior is observed when three distinct locations are considered. This case is illustrated in Figure 2, where the probability of each fractional occupancy (f_1, f_2, f_3) is shown, where $f_i = n_i/N$. Of course $f_1 + f_2 + f_3 = 1$, so that these vectors all belong to the 2-dimensional unit simplex and can be represented using barycentric coordinates. In this coordinate system the triplet (f_1, f_2, f_3) is represented by a point inside the triangles of Figure 2 whose distance from vertex i is equal to $f_j^2 + f_h^2 + f_j f_h$, where j and h stand for the other two vertices. A point inside the triangles of Figure 2 represents a possible distribution of the N firms across the three locations. The number of firms for a given locations decreases with its distance from the point.

Example 3: uniform agglomeration feedbacks with diverse "intrinsic attractiveness" of locations

Set the geographic attractiveness of location 1, $a_1 = 2$, while the attractiveness of the other two locations is, $a_2 = a_3 = 1$. Consider the case of a homogeneous b. As can be seen in Figure 2(a), when the value of b is low, the distribution is concentrated around the center of the triangle. That is, the three locations contain roughly comparable shares of firms. Location 1 having the highest value of a, results the more attractive one, so that the probability mass is shifted toward its vertex. In this case, fluctuations are relatively modest. When b is increased, as in Figure 2(b), the picture changes: the probability is spread on a larger support. When $b = 1$, Figure 2(c), the distribution is uniform. This happens because the agglomeration strength parameter has, in each location, a value equal to the geographic attractiveness of that location.

For any $L > 0$, if $b_i = a_i$, $\forall i$, the generic expression (2.6) reduces to

$$\pi(\boldsymbol{n}) \sim \left(A + (1 - \frac{1}{N})\boldsymbol{a} \cdot \boldsymbol{n} \right) , \tag{3.2}$$

that is, it becomes proportional to an hyperplane. In terms of the fractional occupancy vector $\boldsymbol{f} = (f_1, \ldots, f_L)$, the distribution (3.2) is defined over the $L-1$-simplex and is sloped in such a way that its highest point (that is the point with greatest probability) is located in the vertex of the simplex associated with the most attractive location. In these circumstances when one moves away from this location the probability falls linearly: hence the distribution displays rather heavy "tails". Such "decay" of the probability gets slower as the degrees of locational attractiveness become more similar. In particular, when all the parameter a's are equal, the distribution becomes uniform.

Example 4: agglomeration feedbacks with different intensities

`Figure 3 about here`

In the foregoing examples we analyzed cases with identical b values only. In other terms, we assumed that the strength of agglomeration effects is equal in all locations. If one considers different values of b the picture changes. Consider the case with two locations. Assume $a_1 = 1$, $a_2 = 2$ and set $b_2 = 0$. In Figure 3 the probability distribution of the fraction of firms in location 1 is shown for different values of the parameter b_1. The left panel reports the distributions for $N = 100$, and the right panel for $N = 50$. As can be seen, in both cases, a small increase in the value of b_1 is enough to generate a big shift of the distribution to the right. This shift implies a larger average population for location 1. If the value of b_1 is further increased, the shape of the distribution starts to change, so that the probability of finding the large majority of firms in location 1 tends toward 1. Notice that when the number of firms is lower, the impact of the parameter b is somewhat reduced. This is not surprising, as the "effective" strength of the agglomeration coefficient depends on the number of firms composing the industry. Roughly speaking, the relative attractive strength is proportional to the total number of firms times the dynamic externality ($N\,b$). An analogous example for the case of three locations is reported in Figure 4.

When local positive returns to localization are absent ($b = 0$) the distribution is around the center of the simplex. Since the value of a_1 is lower, the probability weight is nearer to the $1 - 3$ line. A slight increase in the agglomeration strength of location 1 ($b_1 = .1$), is enough to move the weight toward the vertex with the same label (panel b). The shape of the distribution does not change and the effect is similar to the one obtained with an increase of the parameter a_1. If also the agglomeration strength of location 2 is increased (panel c), the weight moves toward the $1 - 2$ line and the shape becomes more oblong. With higher values for b_1 and b_2 (panel d) the effect becomes stronger, and the probability weight is completely concentrated near the $1 - 2$ line. This implies that location 3 remains mostly empty, while the population of firms is distributed across locations 1 and 2, with a relative preference for the latter.

`Figure 4 about here`

The differences in the shape of the limit distribution for different values of the agglomeration parameters b's we observed above do also reflect different dynamical properties of the model. As we have seen before, if one considers industries shares n_l/N, the possible occupancy vectors $\mathbf{n}$ for the L-locations case map in different points inside the $(l-1)$-simplex. When the probability weight of the limit distribution is heavily clustered around an interior point, like in Figure 1(a) or Figure 4(a), the model displays a rather stable distribution of firms, with relatively minor fluctuations around the equilibrium market shares. An example of this behavior is provided in Figure 5(a). These trajectories are obtained by simulating a model with $N = 100$ firms and three locations. The geographic attractiveness of the three locations are equal to the ones considered in Figure 4, namely $a_1 = 1$, $a_2 = a_3 = 2$. The dynamics of firm shares is reported for different values of the agglomeration parameters. The case of zero agglomeration strength - panel (a) - follows the pattern described above: the share of firms located in 2 and 3 fluctuates around .4, while the share of location 1 is around .2, reflecting the lower intrinsic attractiveness of this site. If we slightly increase b_1 we recover the dynamics of panel (b): the average fraction of location 1 increases, but the shares belonging to different locations remain rather stable in time. A further increase in the value of the parameters b changes the picture. In panel (c), both locations 1 and 2 have a value of b equal to .5. This corresponds to the limit distribution of panel (d) in Figure 4. The weight of the distribution is near the $1-2$ border of the simplex. As a consequence, location 3 is persistently almost empty (see the line near the bottom border), while location 1 and 2 (nearly) share the entire population of firms. Notice, however, that the population of firms is not distributed in time-stationary shares among the two locations. On the contrary, at any time, one location typically dominates the other and attracts a larger number of firms. This cluster can last for several periods, and then abruptly disappear. When the two locations become equipopulated a reversal in the relative concentrations become more likely, with the second location becoming the most populated one (or alternatively, the location which previously attracted the largest part of firms may as well swiftly recover its dominating role). If the value of b becomes larger, the effect is reinforced: the difference in market shares is increased and is likely to persist for a longer time: see Figure 4(d).

The foregoing analysis reveals that the dynamical characters of different equilibrium distributions can be quite diverse. In fact, the equilibrium distribution represents the unconditional probability of finding the system in a given state. This probability, however, can be very different from the average fraction of firms observed over finite time windows. The proper interpretation of a distribution like the one in Figure 2(d) is that the entire population of firms will end up concentrated in one large cluster, occupying exactly one location. Nonetheless, the three locations have the same probability to become the main industry cluster. Which location is selected, is a matter of history and chance. This highly concentrated state of the industry can last for several thousand of steps, but is only a metastable state. At some point, the sequence of random allocations can lead one of the other sites to catch up, in terms of number of firms, with the most populated location and, possibly, to overtake it. At this point, in relatively few time steps, this location may become the new cluster of the industry. loosely speaking the time profile recalls what in biology are known as "punctuated equilibria" with long period of relative environmental stability intertwined by relatively sudden transitions.

Just to give an idea of the time scale of the previous dynamics, consider that the typical turbulence in entry and exit dynamics in industrial sectors is around 5%. So, with a sector of 100 firms, five time steps of the simulations can be thought as representing one year of "real" time. In the example above (see Figure 5(d)), the metastable state in which the largest part of industry is clustered in location 1 can last for several thousand of steps. That would be equivalent to several centuries of historical time. So, even if these states are only metastable, they can be indeed considered stable for all the practical purposes. Notice, however, that this relative stability is in place only for strongly "polarized" industries: if the coefficients b are zero, or very low, then one can observe significant fluctuations also on relatively short time scales (see Figure 5(a)).

4 Pure entry process and large industry limit

In the present section we study the asymptotic behavior of our model when we switch off the exit process and retain only the entry dynamics described in Assumption 2.1. This implies that the number of firms in the industry will increase linearly with time. Assuming that the process starts with no firms present in the industry, if $n_l(t)$ is the number of firms present in location l at time t, one has $\sum_l n_l(t) = t$. Let $\boldsymbol{n(t)} = (n_1(t), \ldots, n_L(t))$ be the occupancy vector at time t, the probability that the next firm chooses location l is

$$p_l(\boldsymbol{n(t)}) = \frac{a_l + b_l n_l(t)}{A + \boldsymbol{b}.\boldsymbol{n}(t)} \ , \tag{4.1}$$

with the same notation used in Proposition 2.1. The function $p_l(\boldsymbol{x}, t)$ describes the probability that the new entrant firm locates its activity in l, given the time t in which it enters the industry and the actual occupancy of all the locations $\boldsymbol{n}$.

Consider now the conditional expected occupancy of location l at time $t+1$

$$\bar{n}_l(t+1) = \mathrm{E}\left[n_l(t+1)|\boldsymbol{n(t)}\right] \ .$$

It clearly depends on the previous occupancy $n_l(t)$. More precisely, it is equal to the number of firms previously present in l plus the average number of firms which entered that location at time $t+1$. This number (between zero and one) is exactly equal to the probability in (4.1). One can thus write

$$\bar{n}_l(t+1) = n_l(t) + p_l(\boldsymbol{n(t)}) \ ,$$

which in terms of the "fractional occupancy" $\boldsymbol{x}$, where $x_l(t) = n_l/t$, reads

$$\bar{x}_l(t+1) = x_l(t) + \frac{1}{t+1}\left(p_l(\boldsymbol{n(t)}) - x_l(t)\right) \ .$$

Substituting the definition of p_l in (4.1) and the previous equation becomes

$$\bar{x}_l(t+1) - x_l(t) = \frac{1}{t+1}\frac{1}{\frac{A}{t} + \boldsymbol{b}\cdot\boldsymbol{x(t)}}\left[\frac{1}{t}\left(a_l - Ax_l(t)\right) + \sum_{j=1}^{L} x_j(t)x_l(t)(b_l - b_j)\right] \ . \quad (4.2)$$

The previous expression can be used to derive some properties of the asymptotic behavior of the system.

Case 1: positive agglomeration feedbacks with diverse "intrinsic attractiveness" of locations

First, consider the case in which at least one b is different from zero. In this case, the first term inside the square brackets vanishes, with respect to the second term, proportionally to t^{-1}. The same applies to the first term of the denominator in front of the square brackets. In this case, retaining only the leading terms in the asymptotic expansion one has

$$\bar{x}_l(t+1) - x_l(t) \sim \frac{1}{t+1} \frac{1}{\boldsymbol{b} \cdot \boldsymbol{x(t)}} \sum_{j=1}^{L} x_j(t) x_l(t)(b_l - b_j) \ . \quad (4.3)$$

Notice that the coefficients a have completely disappeared from this expression and the asymptotic behavior seems completely driven by the coefficients b. In particular, if there exists a location l which possesses an agglomeration economy coefficient greater than any other location, that is $b_l > b_j, \forall j \neq l$, then, for this location, the right hand side of (4.3) is always positive, that is $\mathrm{E}[f_l(t+1)] > f_l(t)$. This means that the expected value of the fraction of firms in l at the next time step is always *higher* than the presently realized value. This seems to suggest that, with probability one, $f_l(t) \to 1$ when $t \to \infty$.

The previous heuristic argument can be proved to be true. In Bottazzi and Secchi (2007), using formal results derived in Pemantle (1990), it is shown that for the pure entry process defined by (4.1), when the number of firms diverges, it is impossible to find finite shares of firms in two locations with different b's. In other terms, when the number of firms in the industry diverges, only two types of distributions can possibly be observed: a complete concentration in one single location, or a population of firms split across locations with the same coefficient b. Moreover, it is possible to show that only the locations with the *largest* agglomeration coefficients are populated in the limit. This finally proves our heuristic conclusion: if there exists a location whose b is larger than any other b, then, when the number of firms becomes large, the industry finds itself completely clustered in that single location. On the other hand, if there are several locations which share the highest coefficient b, a constant positive (in probability) flows of firms will be observed from the location with lower b's toward the location with higher b's. Consequently, as t increases, the industry becomes increasingly concentrated among the latter locations and, in the limit, only these locations retain a positive fraction of firms. Notice, however, that the previous analysis does not give any hint on the way in which the population of firms is distributed across these locations[8]. In fact, in tune with the original Polya model (Polya, 1931) all the shares between the most attractive locations are asymptotically attainable: history fully rules.

Case 2: no agglomeration feedbacks with diverse "intrinsic attractiveness" of locations

In order to complete our analysis, let us consider the case in which all coefficient b's are equal to zero, that is the industry lacks any agglomeration effect in any location. Following our heuristic approach and setting $\boldsymbol{b} = \mathbf{0}$ in (4.2) one has

$$\bar{x}_l(t+1) - x_l(t) = \frac{a_l - Ax_l(t)}{A\,(t+1)} \tag{4.4}$$

The right hand side of (4.4) becomes zero when

$$x_l = \frac{a_l}{\sum_{j=1}^{L} a_j}\ , \tag{4.5}$$

so that, as expected, each location contains, asymptotically, a number of firms proportional to its intrinsic geographic attractiveness. In this case, indeed, the process retains no history: the choice of each agent is identical. At each time t, the distribution of occupancies follows a multinomial laws, with probabilities given by (4.5), so that the

[8]The interested reader find in Bottazzi and Secchi (2007) a discussion of the asymptotic distribution for large t is derived in analogy with a well known Polya process.

result follows.

To sum up: we started with a model with reversible choices and a finite population of agents, we turned off the death process - thus making location decisions irreversible - and allowed the number of firms to go to infinity. By doing that in absence of any agglomeration economies, the asymptotic picture boils down a distribution of activities somewhat in tune with the conventional notion of invariant "endowment-based" comparative advantages of the different locations. Conversely under positive returns to agglomeration, the limit properties are shaped by the very location processes and their different "pulling strengths". In particular, when more than one location posses the highest agglomeration force, one recovers the path dependency property typically characterizing polya urn models under increasing returns (cfr. (Arthur, 1994) and (Dosi and Kaniovski, 1994)).

5 Industry-specific agglomeration economy

The model presented in Section 2 allowed for different agglomeration coefficients b in different locations. While this represents part of the whole agglomeration story, it is equally plausible to think of the agglomeration effect as a force acting inside a certain industry with a strength which does not depend from the specific location.

In our notation this means assuming a constant b across all locations. As showed in the previous section, this assumption is also suitable to describe cases in which the agglomeration economies are, to some extent, location-dependent but the size of the industry is large. In this case, only the site with the highest coefficient b's will contain a relevant number of firms so that, in discussing the empirical consequences of the model, one can assume all other sectors as having $a = b = 0$, that is remove them from the dynamics.

Let us consider different geographic attractiveness a_l for each different location l. The strength of the agglomeration economy is represented by an industry-specific parameter b, equal for all locations. If we assume, as in the previous sections, that all locations posses strictly positive intrinsic attractiveness a_l then we have the following

Proposition 5.1. *If $b_l = b\ \forall l \in \{1,\ldots,L\}$ with constant $b > 0$, the stationary distribution defined in* (2.6) *reduces to*

$$\pi(\boldsymbol{n};\boldsymbol{a},b) = \frac{N!\Gamma(A/b)}{\Gamma(A/b+N)} \prod_{l=1}^{L} \frac{1}{n_l!} \frac{\Gamma(a_l/b+n_l)}{\Gamma(a_l/b)} \tag{5.1}$$

where b stands for the L-tuple of constant b's.

Proof. See Bottazzi and Secchi (2007), Section 3. □

In this case locations do, in general, differ and are characterized by their specific attractiveness parameter a_l. In order to define a marginal distribution, one has to specify the parameter a of the location of interest.

Proposition 5.2. *The marginal distribution $\pi(n,a)$ of the number of firms in a location with geographic attractiveness a for the model in* (5.1) *reduces to the Polya distribution*

$$\pi(n;N,L,a,A,b) = \binom{N}{n} \frac{\Gamma(A/b)}{\Gamma(A/b+N)} \frac{\Gamma(a/b+n)}{\Gamma(a/b)} \frac{\Gamma((A-a)/b+N-n)}{\Gamma((A-a)/b)} \tag{5.2}$$

and the average occupancy of site $l \in \{1,\ldots,L\}$ with attractiveness a_l reads

$$< n_l >= N\,\frac{a_l}{A} \tag{5.3}$$

Proof. See Bottazzi and Secchi (2007). □

`Figure 6 about here`

The marginal distribution in (5.2) depends on the total number of firms N, the total number of locations L, the two global parameters $A = \sum_{j=1}^{L} a_j$ and b and the location-specific parameters a_l. Figure 6 reports the marginal distribution (5.2) for different values of the parameter b. As we observed before, an increase in the value of b induces an apparent change in the shape of the distribution and, in particular, an increase in the size of its support again hinting at more turbulent dynamics of location.

The case is indeed interesting because it highlight the relevance for the ensuing distributions of the sheer strength of agglomeration forces, even when they apply identically in all locations.

6 Empirical issues for further research

An important feature of the family of models presented above rests in its ability to be empirically estimated on the actual locations of firms, plants and employment by sector and by site. The characterization of the stationary distribution derived in equation (5.1) allows to go well beyond the exercises of indirect model validation generally found in the literature (for a discussion concerning NEG cf. Brakman and Garretsen (2006)).

As we have already began to do in Bottazzi et al. (2006) and Bottazzi et al. (2004) one may undertake at least four classes of empirical exercises.

First, one may statistically compare the whole shape of the empirical distribution of business plants with the theoretical one (see equation 5.1) in each given industrial sector. This improves upon the existing empirical literature, where only synthetic agglomeration indices are derived (cf. Devereux et al. (2004), Maurel and Sedillot (1999), Overman and Duranton (2002), Dumais et al. (2002), Ellison and Glaeser (1999), Combes and Overman (2004) for exercises in a similar spirit).

Second, one may test simpler instances of our model obtained from the general one by switching off and on geographical and technological heterogeneity, thus gaining insights on their importance in determining the observed locational profiles. For example, one may start from an utterly simple specification where all agglomeration parameters are set to zero (i.e. $b_l = 0, \forall l$) and all locations possess the same intrinsic attractiveness ($a_l = a, \forall l$). This case is a sort of "null hypothesis" benchmark whereby neither spatial specificities nor agglomeration processes play any lasting role. Nevertheless, this unrealistic specification allows to test our model against pure randomness in the vein of Ellison and Glaeser (1997, 1999) and Rysman and Greenstein (2005). Furthermore, in order to explore the relevance (or irrelevance) of geographical heterogeneity, one can consider models where locations are homogeneous and share the same geographic attractiveness $a > 0$, but agglomeration economies are now present in the form of an industry-wide agglomeration force measured by a single parameter $b > 0$. Finally, one can envisage models where one considers heterogeneous geographic attractiveness a_l for each different location l, while retaining an industry-specific agglomeration parameter b, equal for all locations. As we do in Bottazzi et al. (2004), on Italian data disaggregated by sector of activity and by location, one is able to disentangle the "pull" of each location irrespectively of the sector of activity (call it the *urbanization effect*) from *sector-specific* agglomeration (or anti-agglomeration) forces. Hence, one is able to distinguish the "horizontal" forces of agglomeration - stemming from e.g. inter-sectoral linkages and marshallian externalities - as distinct from sector-specific forms of localized increasing (or decreasing) returns, in turn, possibly associated, with the characteristics of knowledge accumulation in each line of activity.

Third, revealing evidence is likely to come from the comparison of the distributions of agglomeration parameters across different variables. So, for example, comparisons between the location patterns of firms as compared to the location pattern of employment tell how much of the purported agglomeration forces are in fact "internalized" within a few relatively big firms, or conversely, result in the proximity of several "district-like" firms.

Fourth, the increasing availability of spatially tagged time-series allows an easy intertemporal application of the foregoing model asking how agglomeration patterns have changed over the years and exploring the evolution of the "urbanization" and sector-specific forces.

7 Conclusions

In this work we have presented a family of models of evolutionary inspiration where boundedly rational heterogeneous agents decide to locate their production activities influenced by both the "intrinsic" attractiveness of individual locations and by the number of firms already operating there, entailing the possibility of local dynamic increasing returns.

Firms enter and firms die. In fact, in the current specification, such a process keeps constant the number of incumbents but relaxations are easily possible. The Markov processes define a dynamic over a finite number of states whose limit distributions can be empirically estimated. In fact, the model allows to *empirically* address the question of how relevant are agglomeration economies driven by some form of localized positive feedbacks associated with the very history of birth and death of firms in each location. Together, it allows to empirically distinguish agglomeration forces which are, so to speak, "horizontal", in the sense that they apply across sectors of activity within the same location and those which, on the contrary, are sector-specific.

Granted these achievements, one can think of several ways ahead. One such way is to make less rudimentary the representation of "space" by adding some notion of "distance" among sites with a related impact upon location decisions. A second development that comes to mind involves the explicit account of multiple sectors of activities with ensuing inter-sectoral spillovers. Third, an important extension involves the account not only of birth and death of firms but also of spatially nested growth (a sketch of a model along these lines is in Boschma and Frenken (2007), this volume). However, possibly the most important step forward involves adding a *process of learning* through which firms could change their technological capabilities over time (i.e. innovation) and a process of *selection* driving the growth and death of each firm. Doing that would largely fulfill the objective of formalizing a fully fledged evolutionary model explicitly nested in space.

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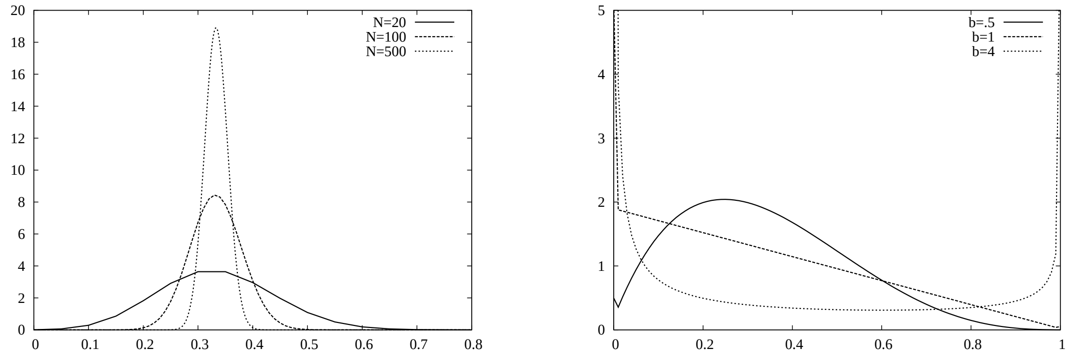

Figure 1: Two-locations model with $a_1 = 1$ and $a_2 = 2$. **Left panel:** Probability density for the number of firms in location 1 for $b = 0$ and different values of N. **Right panel:** Probability density for the number of firms in location 1 for $N = 100$ and different values of b.

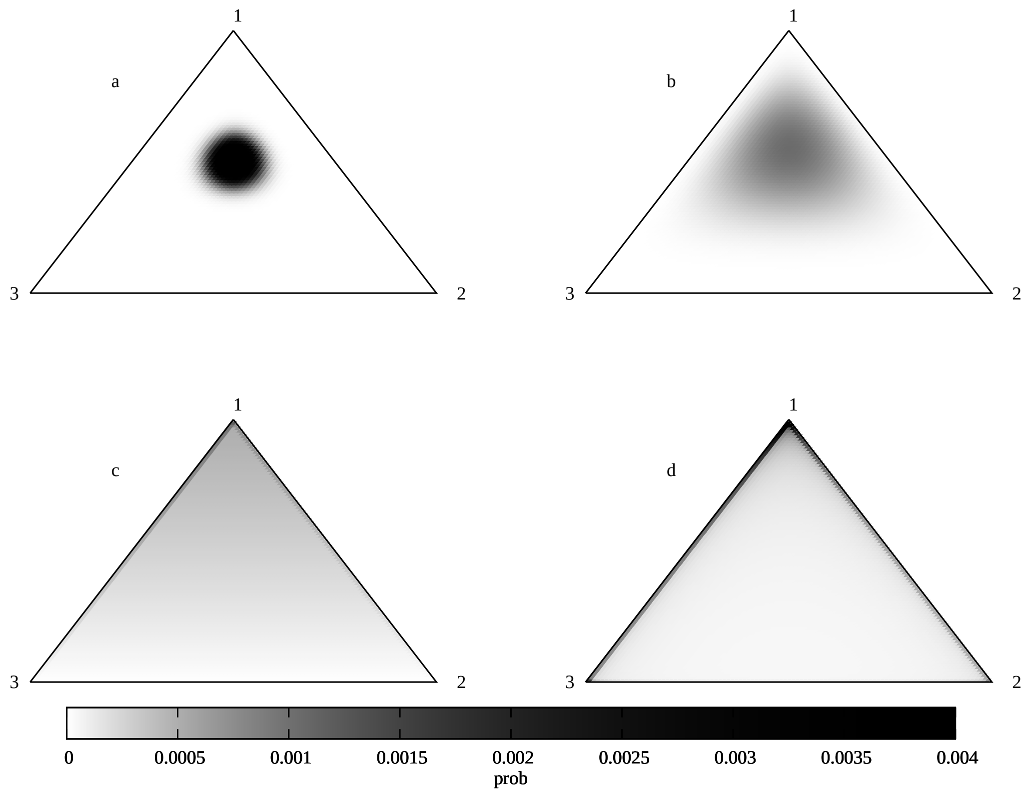

Figure 2: Model with three locations and $N = 100$ firms. All the geographic attractiveness are set to 1. The probability density of each point $(n1, n2, N - n1 - n2)$ are shown for different values of the common variable b.

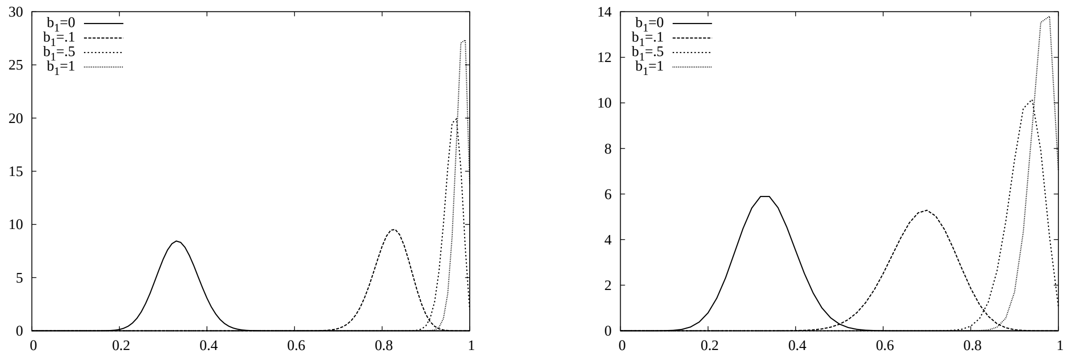

Figure 3: Probability density of the fraction of firms in location 1 for different values of b_1 with $a_1 = 1$, $a_2 = 2$ and $b_2 = 0$. The number of firms N is set equal to 100 (left panel) and 50 (right panel).

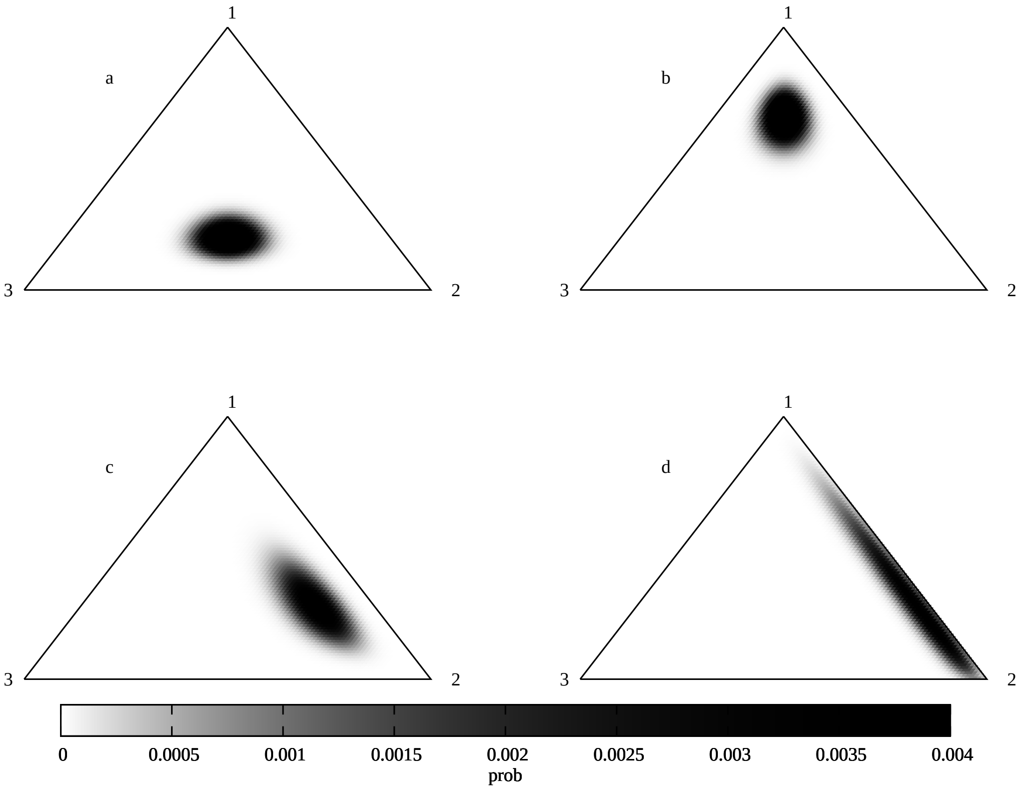

Figure 4: The model with three locations and $N = 100$ firms. The geographic attractiveness parameters are $a_1 = 1$, $a_2 = 2$ and $a_3 = 2$. Agglomeration parameters are as follows: a) $b_1 = 0, b_2 = 0, b_3 = 0$; b) $b_1 = .1, b_2 = 0, b_3 = 0$; c) $b_1 = .1, b_2 = .1, b_3 = 0$; d) $b_1 = .5, b_2 = .5, b_3 = 0$.

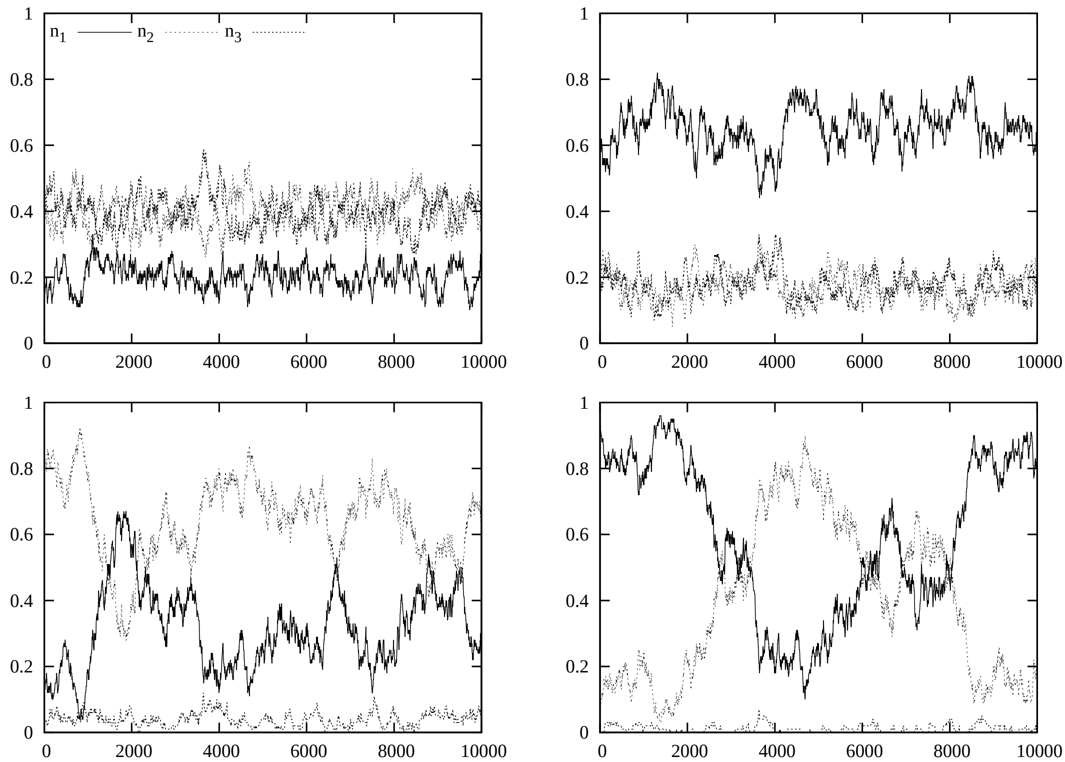

Figure 5: Temporal dynamics of the location firm shares for a model with three locations and $N = 100$ firms. The geographic attractiveness parameters are $a_1 = 1$, $a_2 = 2$ and $a_3 = 2$. Agglomeration parameters are as follows: a) $b_1 = 0, b_2 = 0, b_3 = 0$; b) $b_1 = .1, b_2 = 0, b_3 = 0$; c) $b_1 = .5, b_2 = .5, b_3 = 0$; d) $b_1 = .5, b_2 = .5, b_3 = 0$.

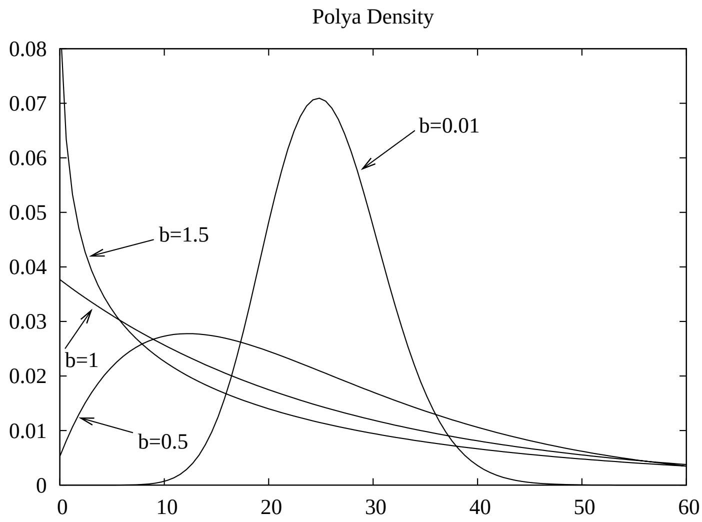

Figure 6: Polya marginal distributions (for different values of b). All distributions are computed for $N = 20000$, $L = 800$, and geographic attractiveness $a = 1$.