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A Multivariate Perspective for Modelling and Forecasting Inflation's Conditional Mean and Variance

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A Multivariate Perspective for Modelling and Forecasting Inflation's Conditional Mean and Variance

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Abstract

We test the importance of multivariate information for modelling and forecasting inflation's conditional mean and variance. In the literature, the existence of inflation's conditional heteroskedasticity has been debated for years, as it seemed to appear only in some datasets and for some lag lengths. This phenomenon might be due to the fact that inflation depends on a linear combination of economy-wide dynamic common factors, some of which are conditionally heteroskedastic and some are not. Modelling the conditional heteroskedasticity of the common factors can thus improve the forecasts of inflation's conditional mean and variance. Moreover, it allows to detect and predict conditional correlations between inflation and other macroeconomic variables, correlations that might be exploited when planning monetary policies. A new model, the Dynamic Factor GARCH (DF-GARCH), is used here to exploit the relations between inflation and the other macroeconomic variables for inflation forecasting purposes. The DF-GARCH is a dynamic factor model with the additional assumption of conditionally heteroskedastic dynamic factors. When comparing the Dynamic Factor GARCH with univariate models and with the traditional dynamic factor models, the DF-GARCH is able to provide better forecasts both of inflation and of its conditional variance.

Keywords: Inflation, Factor Models, GARCH.
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1 Introduction

The use of conditionally heteroskedastic models for inflation has originally been suggested by Engle (1982 and 1983). He shows a quite clear evidence of conditional heteroskedasticity in the UK and US inflation series. There is a stream of literature that indeed considers the GARCH model by Bollerslev [1986] for forecasting inflation and inflation uncertainty. Since the hypothesis by Friedman [1977] and Ball [1992] that higher variability of inflation would lead to decreased output, ceteris paribus, and that higher rates of inflation are generally associated with higher variability of inflation, there has been a whole stream of literature that uses different kinds of GARCH models to test this relation, e.g. Engle [1983], Grier and Perry [1998], Fountas et al. [2000], Kontonikas [2004]. If this hypothesis is true, then higher rates of inflation would also be associated with low levels of output, which implies a positively sloped Phillips curve. When dealing with more recent data, inflation is proved to be less conditionally heteroskedastic. However, we still share the original idea that a model which is able to forecast levels and conditional variance may prove to be very useful for monetary policy purposes. Moreover, given the long lags necessary for monetary policies to be effective, it is of crucial importance to have forecasts not only of inflation but also of inflation uncertainty. Central banks have to evaluate carefully the risks that the economy faces for price stability, which in turn is often considered as the avoidance of excess inflation but also of deflation. This definition implies the necessity of knowing not only inflation levels but also its confidence intervals. Policy makers have then to act as risk managers (see Kilian and Manganelli [2007]). Although inflation does not show appreciable ARCH effects, some of the dynamic factors that govern the economy instead do, as we show in this paper. The observed inflation series can then be seen as a linear combination of GARCH processes and conditionally homoskedastic processes, thus resulting in a Weak GARCH process (see Drost and Nijman [1993] for the definition of Weak GARCH and Nijman and Sentana [1996] on the contemporaneous aggregation of GARCH processes).

Here we apply the DF-GARCH by Alessi et al. [2006], which explicitly takes into account the conditional heteroskedasticity of the dynamic common factors driving the economy. The DF-GARCH is a multivariate dynamic factor model as the well known models by Stock and Watson [2002] and Forni et al. [2005] that in addition models the vector of dynamic factors as a Multivariate GARCH, therefore allowing for estimation and forecast not only of the conditional variance but also of the conditional covariances of the whole dataset. Given the typical dimension of macroeconomic datasets used in central banks, there are some limitations if we want to estimate directly a Multivariate GARCH on all the series. In the BEKK formulation by Engle and Kroner [1995] too many parameters are required, while on the contrary if we use the DCC formulation by Engle [2002] we have too few parameters for too many series, thus constraining too much the dynamics of conditional covariances. Factor models provide us a useful technique of dimension reduction that makes feasible the estimation of Multivariate GARCH for large datasets. According to our intuition, modelling not only the conditional mean, but also the conditional variance and covariance of our data, may improve the forecast of the levels of inflation. This result was first found by Engle [1982] when forecasting UK inflation's mean and variance. The importance of modelling conditional variance when forecasting levels was again recently established by Stock and Watson [2006], who find the stochastic volatility model to be one of the best univariate models for predicting inflation. About the use of factor models, it is well known that when dealing with aggregate macroeconomic variables few driving shocks are enough to explain the bulk of the variance. Factor models thus seem to provide a good representation of macroeconomic datasets. They provide us with better forecasts and are able to take into account all the relevant information included in the large dataset used by central banks. On the performance of factor models see for example D'Agostino and Giannone [2006].

In section 2 we present the different multivariate models used. In section 3 we describe the data and the forecasting procedure. In section 4 we determine the number of dynamic and static factors. In section 5 we test for the conditional heteroskedasticity of the dynamic factors. In section 6 we report the results of the forecasts of the inflation series. In section 7 we analyze the performance of the DF-GARCH in forecasting and insample estimation of the conditional variance of inflation and of conditional covariances. Finally, in section 8 we outline the possible further developments of this work.

2 Competing models for inflation forecasting

We rapidly review the different models that we compare in the empirical application. The benchmark for multivariate models is the factor model in the static representation

$$\mathbf{x}_t = \mathbf{\Lambda}\mathbf{F}_t + \boldsymbol{\xi}_t \ , \tag{1}$$

We know that the static factors can be estimated by means of static (see Stock and Watson [2002]) or generalized principal components (see Forni et al. [2005]). Although in theory using generalized principal components should be better than using just static principal components in terms of forecasting performance, the evidence is mixed (e.g. see D'Agostino and Giannone [2006] and Boivin and Ng [2005]). We therefore decide to use only the procedure by Stock and Watson, not to add to an already complex model a step that seems to give no big improvement. Once we retrieve an estimate of the static factors $\hat{\mathbf{F}}_t$ as static principal components $\mathbf{S}\mathbf{x}_t$, the h-step ahead forecast of the common part is given by

$$\hat{\boldsymbol{\chi}}_{t+h|t} = \hat{\mathbf{\Gamma}}_h^x \mathbf{S}(\mathbf{S}'\hat{\mathbf{\Gamma}}_0^x \mathbf{S})^{-1}\hat{\mathbf{F}}_t \ .$$

Notice that the main limit of this forecasting method is due to the fact that no dynamic model is estimated for the static factors, therefore we can rely only on their insample estimation. For this reason we now consider a class of dynamic factor models in state-space form where the evolution of the static factors is explicitly modelled.

The departure point for the models we consider in this paper is a state-space form made of equation (1) and an equation that specifies the time evolution of the static factors

$$\mathbf{F}_t = \mathbf{A}\mathbf{F}_{t-1} + \mathbf{H}\mathbf{u}_t \ , \tag{2}$$

where the dynamic factors $\mathbf{u}_t$ are conditionally distributed as

$$\mathbf{u}_t | \, \mathcal{I}_{t-1} \sim \mathrm{N}\left(0, \mathbf{Q}_t\right) \ .$$

We consider two cases that differ only in the way in which the conditional covariances of the dynamic factors ($\mathbf{Q}_t$) are modelled.

1. $\mathbf{Q}_t$ does not depend on time, therefore the dynamic factors are conditionally homoskedastic. This model was already proposed in Giannone et al. [2004].

2. $\mathbf{Q}_t$ follows a Multivariate GARCH with BEKK representation. This is the DF-GARCH. Its estimation, together with results from Monte Carlo simulations and an application to financial asset returns, is explained in detail in Alessi et al. [2006].

The parameters $\mathbf{A}$ and $\mathbf{H}$ are estimated as in Giannone et al. [2004]. By means of these estimates we have an estimate for the dynamic factors $\hat{\mathbf{u}}_t$ just by inverting (2). Once we have the dynamic factors, $\mathbf{Q}_t$, if required, is estimated by usual Maximum Likelihood. In both cases we can use the Kalman filter approach in its classical or modified version (see appendix A), to reestimate the static factors. Thus we have two other dynamic factor models to be compared with the usual one by Stock and Watson. In both models, at the end of the Kalman filtering estimation we have an estimate of the static factors $\tilde{\mathbf{F}}_t$ with which we can compute the h-step ahead forecast of the common part as

$$\begin{aligned} \hat{\boldsymbol{\chi}}_{t+h|t} &= \hat{\boldsymbol{\Lambda}}\tilde{\mathbf{F}}_{t+h|t} \ , \\ \tilde{\mathbf{F}}_{t+h|h} &= \hat{\mathbf{A}}\tilde{\mathbf{F}}_{t+h-1|t} \ . \end{aligned} \tag{3}$$

Notice that when modelling $\mathbf{Q}_t$ as a Multivariate GARCH we obtain not only an estimate of the static factors, but also an estimate of conditional variance and covariances of the series. Given that the idiosyncratic part of factor decomposition is often considered as a simple measurement error, we consider the forecast of the common part as a forecast of the series of interest, i.e. $\hat{\mathbf{x}}_{t+h|t} = \tilde{\boldsymbol{\chi}}_{t+h|t}$.

We compare all results with a univariate AR(p)-GARCH(1,1) and a simple AR(p) for the inflation series, where the autoregressive order p is computed according to the Akaike Information Criterion. In the next sections we compare the performance of all models in forecasting the levels of inflation. We also try to evaluate the performance of the conditionally heteroskedastic models (DF-GARCH and AR-GARCH) in forecasting the conditional variance of inflation. However, the lack of a real measure of inflation conditional variance allows us to make only qualitative evaluations. For the DF-GARCH, given an estimate of the conditional covariance of the dynamic factors $\tilde{\mathbf{Q}}_t$, the predicted conditional covariance of $\mathbf{x}_t$ is defined as

$$\tilde{\boldsymbol{\Gamma}}^x_{t+h|t} = \tilde{\boldsymbol{\Gamma}}^\chi_{t+h|t} = \hat{\boldsymbol{\Lambda}}\hat{\mathbf{H}}\tilde{\mathbf{Q}}_{t+h|t}\hat{\mathbf{H}}'\hat{\boldsymbol{\Lambda}}' \ .$$

Notice that, also for the conditional covariance, we do not consider here any contribute derived from the idiosyncratic component. In principle, we could model each series of this component as a univariate GARCH, as in Alessi et al. [2006].

3 Data and forecasting procedure

We start with a panel of 158 US macro time series (from the Global Insight Database) with monthly observations from December 1986 to November 2006.[1] We remove the series with too many zero entries: these are the series for which no appreciable monthly growth is present. We remove these series because, if too many zeroes are present, the estimation of covariances may be heavily affected by such entries which we believe are lacking any useful information.

[1]This dataset is the standard one used in factor models literature, see e.g. Stock and Watson [2002], Giannone et al. [2004], and D'Agostino and Giannone [2006]. In appendix B there is a list of the variables used.

The remaining series are 130. We consider four price indexes: PCE (all items), PCE core (i.e. excluded oil and food), CPI (all items), CPI core (i.e. excluded oil and food). We transform data to obtain stationarity, and, in particular, for every price index p_t we define the monthly inflation rate as

$$\pi_t = 1200 \log \left(\frac{p_t}{p_{t-1}} \right) .$$

The insample length is always denoted as T and the number of series with n.
We compute the h-steps-ahead forecast with $h = 1, \dots, 12$ by using a rolling forecasting scheme, therefore keeping a constant insample length $T = 156$. The first insample runs from December 1986 to November 1999. We repeat the forecast for about 6 years (i.e. 73 times), therefore the first out-of-sample observations that we forecast are for December 1999, while the last are for November 2006. The Root Mean Squared Error (RMSE) is defined as usual as

$$\text{RMSE}_h = \sqrt{\frac{1}{73} \sum_{k=1}^{73} (\pi_{T+k+h-1} - \hat{\pi}_{T+k+h-1|T})^2} \qquad h = 1, \dots, 12 ,$$

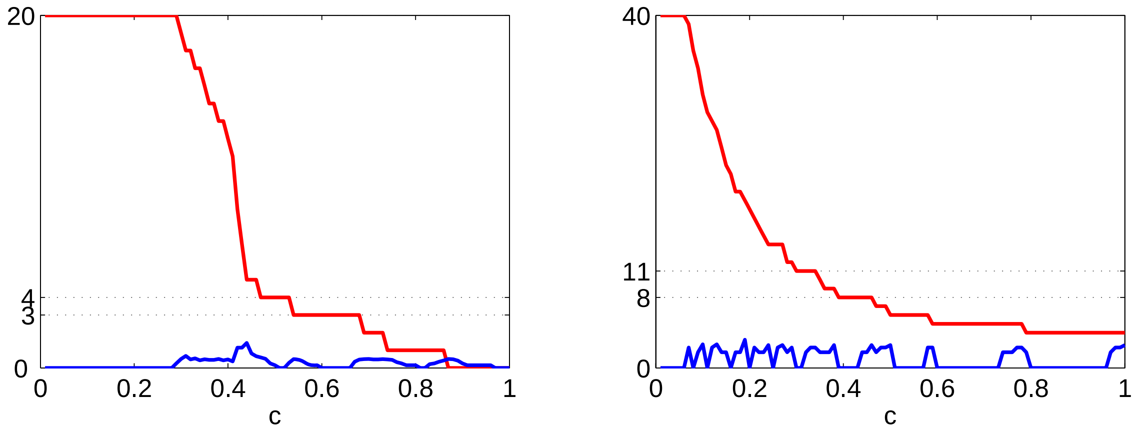
where π_{t+h} is the true value of inflation while $\hat{\pi}_{t+h|t}$ is the h-steps-ahead forecast given all the available information at time t. For the DF-GARCH we have that $\hat{\pi}_{t+h|t} = \tilde{\chi}^{\pi}_{t+h|t}$, where by the superscript $^{\pi}$ we indicate the series of the common component corresponding to inflation. Remember that the forecast is based only on the estimated common component of the inflation series. Each time we repeat the forecast, we reestimate all the parameters of all the models and the factor decomposition, except for the number of static and dynamic factors that is estimated once and forever at the beginning of the forecasting exercise.

Finally, notice that when estimating the factor model we need to standardize data. Therefore, in order to have the h-steps-ahead forecast of inflation, we need to destandardize the forecast obtained from the factor model. We do this by using the estimated insample mean and standard deviation. We consider all values greater than five standard deviations as outliers and we replace them with the insample mean of the series.

4 The number of factors

To determine the number of dynamic factors we apply, as usual, the criterion by Hallin and Liška [2007]. The criterion looks for the largest dynamic eigenvalue that is bounded as n goes to infinity. It makes use of a penalty function that depends on a constant c. For each value of c the criterion is computed together with its variance S_c for different subsamples and the optimal number of dynamic factors $\hat{q}^T_{c,n}$ is given. In figure 1.(a) we look for the first zero variance interval of c corresponding to a stable value of $\hat{q}^T_{c,n} < q_{max}$: that is the number of dynamic factors. By inspection of the graph we can say that we have either 3 or 4 dynamic factors. To determine the number of static factors, we first use a simple heuristic procedure i.e. we compute the explained variance by the chosen number of dynamic factors and we take the number of static factors that explains at least the same amount of variance.[2] The variance

[2] The same procedure is used in D'Agostino and Giannone [2006].



(a) The number of dynamic factors. (b) The number of static factors.

Figure 1: Determining the number of factors.

No. of factors	1	2	3	4	5	6	7	8	9	10	11	12
EV(dynamic)	0.27	0.45	0.56	0.64	0.70	0.75	0.79	0.82	0.84	0.87	0.89	0.90
EV(static)	0.16	0.30	0.38	0.44	0.49	0.53	0.56	0.58	0.61	0.63	0.65	0.67

Table 1: Percentage of cumulated explained variances.

explained by the i-th dynamic (static) factor is defined as:

$$EV_i(\text{dynamic}) = \frac{\int_{-\pi}^{\pi} \lambda_i(\theta)\, \mathrm{d}\theta}{\sum_{j=1}^{n} \int_{-\pi}^{\pi} \lambda_j(\theta)\, \mathrm{d}\theta},$$

$$EV_i(\text{static}) = \frac{\mu_i}{\sum_{j=1}^{n} \mu_j},$$

where $\lambda_i(\theta)$ is the i-th largest eigenvalue of $\hat{\mathbf{\Sigma}}^x(\theta)$ and μ_i is the i-th largest eigenvalue of $\hat{\mathbf{\Gamma}}^x_0$. In table 1 we report the values for the largest 12 dynamic and static factors. According to this criterion, when we choose 3 or 4 dynamic factors, we need from 7 to 10 static factors i.e. each dynamic factor is loaded with 1 or 2 lags.

In practice, the number of dynamic factors is a quantity that must be fixed by some criterion, since it represents the number of economic shocks that hit the economy. The dynamic factor model requires that the q largest dynamic eigenvalues diverge but that the $(q+1)$-th does not. If the model is well identified so it must be q. As for the number of static factors, we can apply a refinement of the criterion by Bai and Ng [2002] that is just an adaptation of the procedure used by Hallin and Liška [2007] to the case of static factors (see Alessi et al. [2007]). Figure 1.(b) shows the plot for the modified PC_2 criterion; we see indeed that $r = 8$ or $r = 11$. Therefore, given that by assumption we need that $r = q(s+1)$, we choose for each dynamic factor a number of lags $s = 1$ for $q = 3$ giving $r = 6$ or $s = 2$ for $q = 4$ giving $r = 12$.

Inflation series	**Variance of χ_t**
PCE core	0.68
PCE	0.82
CPI core	0.72
CPI	0.94

Table 2: Variance of the common part of inflation series for all the insample estimations.

In the following, we report the results for cases $q = 4$ and $r = 12$.
Once the number of factors is determined, we estimate the common part for each of the n series. In table 2 we report the variance of the common part of the four considered series of inflation, averaged over the 72 times we have repeated the insample estimation of the factor model.

5 Testing for conditional heteroskedasticity

As expected, when testing for ARCH effects on the standardized residuals of an AR(p) model of inflation, we find little evidence of conditional heteroskedasticity. This phenomenon becomes evident when using more observations than the insample length that we choose (i.e only for $T \sim 200$) and at long horizons (more than one year lags). Indeed, Engle [1983] finds evidence of ARCH effects at one and two-years lags but not at shorter horizons. We want to provide a model that is able to forecast not only the levels of inflation but also its conditional uncertainty. In particular, we want to provide also more reliable measures of inflation uncertainty than a GARCH model. Given the low degree of conditional heteroskedasticity in recent inflation series, we do not expect that a model which considers conditional heteroskedasticity improves levels forecasts with respect to its homoskedastic counterpart. We would be satisfied with a model that forecasts levels with the same accuracy of a homoskedastic one, but that is also able to provide reliable forecasts of conditional variances that can be used as proxies of inflation uncertainty. In addition, our model is able to provide also conditional correlations forecasts that are very useful in the context of a monetary policy rule with more than one target variable.
The lack of conditional heteroskedasticity in inflation is not in contrast with the hypotheses of our model, as we ask for conditional heteroskedasticity of the dynamic factors $\mathbf{u}_t$ and not for the whole series $\mathbf{x}_t$. If $\mathbf{u}_t$ is a multivariate GARCH process, then $\mathbf{F}_t$, which are contemporaneous linear combinations of $\mathbf{u}_t$, are Weak GARCH processes and so are the $\mathbf{x}_t$. Therefore, the hypothesis of conditionally heteroskedastic dynamic factors is perfectly consistent with the observed weak conditional heteroskedasticity of the inflation series considered. Once we have an estimate of $\mathbf{u}_t$, we can test for GARCH effects through the ARCH test by Engle. Results for the first and the last insample used are in tables 3 and 4. It is evident that at least two of the four dynamic factors have ARCH effects, confirming our initial hypothesis. For this reason, we have to estimate the multivariate GARCH on the dynamic factors and not on the whole dataset. Indeed, although this might be feasible when using the DCC formulation (at least in terms of time required and number of parameters), the lack of conditional heteroskedasticity at the aggregate level prevents us from doing it. On the other hand, the conditional heteroskedasticity of dynamic factors suggests to apply the Multivariate GARCH (as a BEKK or a DCC) on them and not directly on the observable series.

ARCH order	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
u_{1t}	2.84*	3.21	3.91	4.51	4.69	5.25	6.41	7.93	8.35	12.71
u_{2t}	0.04	0.29	4.14	4.08	4.21	4.34	15.29†	15.74†	15.58*	15.77
u_{3t}	10.38†	13.81†	13.68†	13.96†	14.35†	14.32†	14.31†	16.52†	16.56*	16.38*
u_{4t}	8.15†	11.61†	12.41†	12.40†	12.16†	15.67†	16.83†	18.33†	10.13	9.95

Table 3: ARCH-test on $\hat{\mathbf{u}}_t$ for conditional heteroskedasticity († significant at 95%, * significant at 90%). Insample observations from 1986:M12 to 1999:M11.

ARCH order	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
u_{1t}	11.54†	11.79†	13.31†	15.32†	15.45†	15.54†	15.36†	16.20†	17.42†	17.36*
u_{2t}	1.32	10.34†	10.48†	12.68†	12.96†	12.83†	12.93*	13.57*	14.37	16.51*
u_{3t}	2.23	2.52	2.55	2.85	2.28	2.42	2.30	2.28	2.12	2.93
u_{4t}	0.01	7.44†	9.17 †	9.22*	12.20†	13.21†	13.20*	13.21	13.03	12.94

Table 4: ARCH-test on $\hat{\mathbf{u}}_t$ for conditional heteroskedasticity († significant at 5%, * significant at 10%). Insample observations from 1992:M12 to 2005:M11.

To clarify the distinction between GARCH and Weak GARCH, let us consider a return series without any conditional mean specification $y_t = \sqrt{h_t}\epsilon_t$, with $\epsilon_t \simeq (0,1)$. We know that the usual definition of a GARCH process implies that the coefficients of the GARCH are such that satisfy the definition of the conditional moments of the returns

$$\mathrm{E}[y_t|y_{t-1}\ldots] = 0 \quad \text{and} \quad \mathrm{E}[y_t^2|y_{t-1}\ldots] = h_t\ .$$

While in Weak GARCH the coefficients are such that only a condition on the best linear predictors of the returns and their variance is satisfied

$$\mathrm{Proj}[y_t|y_{t-1}\ldots] = 0 \quad \text{and} \quad \mathrm{Proj}[y_t^2|y_{t-1}\ldots] = h_t\ .$$

Thus a GARCH is also a Weak GARCH, but not the viceversa. Results about Weak GARCH processes as outcomes of contemporaneous and time aggregation of GARCH processes are in Nijman and Sentana [1996] and Drost and Nijman [1993] respectively.

Finally, although we know that dynamic factors are not identified unless we impose economic restrictions, it is quite tempting to try to interpret the results obtained in this section. Figure 2 shows, for the last insample used in the rolling scheme, the correlation between the dynamic factors and the observed series grouped into industrial production growth rates, price growth rates (i.e. inflation), asset returns, interest rates, and employment indexes. It is worth noticing that the first factor is highly anticorrelated with industrial production growth rates and correlated with inflation. Also the second factor is highly correlated with inflation series, while the fourth seems to be linked with asset returns. Nothing can be said about the third factor. According to the ARCH test for the last insample, the first, second and third factors display conditional heteroskedasticity. These factors (especially the second and fourth) turn out to be the ones correlated with price series, which are typically conditional heteroskedastic series.

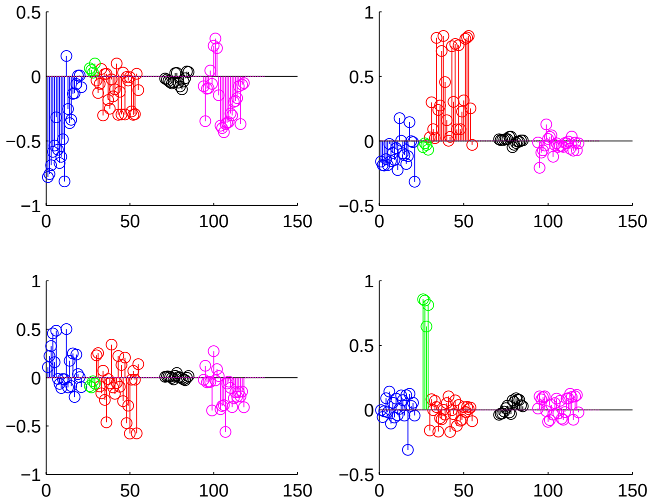


Figure 2: Correlation between dynamic factors and the observed series. Industrial growth rates: blue. Inflation: red. Asset returns: green. Interest rates: black. Employment indexes: magenta.

6 Forecasts of conditional mean

We report here the results of the forecasting exercise with 4 dynamic factors each loaded with 2 lags, i.e. 12 static factors. In table 5 we show the RMSE for the four different inflation variables and for different forecasting horizons ($h = 1, 3, 6, 9, 12$ months ahead). We report the RMSE for the univariate cases AR and AR-GARCH, for the Stock and Watson [2002] model, the DF-GARCH and its homoskedastic counterpart. The last two models outperform the others at least at horizons larger than one, which are the ones of major interest for policy makers, given that monetary policies typically take some months to become effective. Therefore a simple AR(1) specification of the dynamics of the static factors is enough to achieve better forecast performances with respect to the classical dynamic factor model. This result was already used by Giannone et al. [2004]. However, as expected, the conditionally heteroskedastic models do not improve significantly the performance with respect to their homoskedastic counterparts. Although, in principle, in the estimation of the DF-GARCH model a different specification of the conditional variance of the dynamic factors should have an influence also on the level forecast, this effect is too small to give any improvement in the forecasts, given the weak conditional heteroskedasticity of inflation series. This is anyway good news for us because, if we prove to have a good estimator of the conditional variance of inflation, we can have a model that in terms of level forecasts outperforms all the others and, in addition, it is able to give a reliable measure of inflation uncertainty that is theoretically consistent with the conditional mean estimation. We evaluate the performance of the DF-GARCH by means of three different tests of predictive accuracy. In table 6 we show the results of the usual Diebold and Mariano [1995] test of equal predictive accuracy. Given the predictions of two competing models (say a and b) we compute, for each horizon h, the difference between the squared errors obtained with the two models: $d_T^{(h)} = \mathrm{E}[(\hat{\mathbf{x}}^a_{T+h+k-1|t} - \mathbf{x}_{T+h+k-1})^2 - (\hat{\mathbf{x}}^b_{T+h+k-1|t} - \mathbf{x}_{T+h+k-1})^2]$. We test for $d_T^{(h)} = 0$. If the computed statistic is significantly larger than zero, model b has a better forecast performance than model a, and viceversa. According to this test, the model by Stock

PCE core	AR	AR-GARCH	SW	χ_t homoskedastic	DF-GARCH
h=1	1.0250	1.0386	1.0273	0.9852	**0.9851**
h=3	1.0190	1.0387	1.0406	**0.9351**	0.9352
h=6	1.0375	1.0409	1.0561	**0.9363**	**0.9363**
h=9	1.0106	1.0111	1.0008	**0.8971**	**0.8971**
h=12	1.0340	1.0298	0.9921	**0.8934**	**0.8934**
PCE	**AR**	**AR-GARCH**	**SW**	**χ_t homoskedastic**	**DF-GARCH**
h=1	2.2909	2.2807	**2.1256**	2.1511	2.1509
h=3	2.3624	2.3589	**2.2943**	2.2962	2.2962
h=6	2.3978	2.3670	2.3906	**2.2438**	**2.2438**
h=9	2.3533	2.3487	2.3189	**2.2407**	**2.2407**
h=12	2.5389	2.5274	2.5011	**2.4617**	2.4618
CPI core	**AR**	**AR-GARCH**	**SW**	**χ_t homoskedastic**	**DF-GARCH**
h=1	1.0698	1.0660	**1.0411**	1.0431	1.0429
h=3	1.0427	1.0421	1.0541	0.9580	**0.9579**
h=6	1.0632	1.0609	1.0242	0.9495	**0.9494**
h=9	1.1054	1.1096	1.1603	**0.9881**	**0.9881**
h=12	1.1713	1.1734	1.2492	**1.0167**	**1.0167**
CPI	**AR**	**AR-GARCH**	**SW**	**χ_t homoskedastic**	**DF-GARCH**
h=1	2.7641	2.7888	**2.3713**	2.4792	2.4794
h=3	2.9463	2.9594	2.7136	2.6492	**2.6491**
h=6	2.8866	2.8627	2.7048	2.6083	**2.6082**
h=9	2.8100	2.7941	2.7487	**2.5794**	**2.5794**
h=12	3.2159	3.2094	3.1352	**3.0126**	3.0127

Table 5: RMSE for CPI core and CPI relative to the out-of-sample period 1987:M1-2005:M12.

and Watson delivers forecasts which are not significantly better than those of the univariate GARCH, while improvements are obtained with the DF-GARCH especially for long horizons and for the core variables. We do not report results for testing between the DF-GARCH and its homoskedastic counterpart, because, given the similarity in RMSEs, it is obvious that according to this test the two models are equally informative. Notice that, although some of the models we are comparing may be considered as nested, this test is already useful to make a first distinction between them. When the null hypothesis of equal predictive accuracy is rejected with high significance levels, then, no matter if the models are nested, we already have an indication of which one is better. The problem with nested models arises when we cannot reject the null hypothesis of equal predictive accuracy. In this case we consider also the test for nested models by Clark and West [2007] which adds just a correction term to $d_T^{(h)}$ and once again tests for equal predictive accuracy. We may consider the univariate GARCH as nested in the DF-GARCH. Results are in table 7 and confirm the improvement in forecasts made by the DF-GARCH. We do not show the results of the test between the DF-GARCH and its homoskedastic counterpart, since it is already obvious from the RMSE that the two

a =AR-GARCH b =SW	PCE core	PCE	CPI	CPI core
h=1	0.2917	1.7384*	4.0018†	0.4482
h=3	-0.0519	0.7713	2.6380†	-0.2178
h=6	-0.1955	-0.2464	1.7225*	0.7469
h=9	0.1366	0.3508	0.5284	-0.5731
h=12	0.4672	0.2985	0.6820	-0.7793
a =AR-GARCH b =DF-GARCH	**PCE core**	**PCE**	**CPI**	**CPI core**
h=1	1.2862	1.6543*	3.1010†	0.3900
h=3	2.3088†	0.8019	3.6413†	1.6258
h=6	1.7602*	1.6900*	2.6971†	2.3772†
h=9	2.2557†	1.7509*	2.3859†	1.9752†
h=12	2.3079†	1.2065	2.4801†	2.5553†
a =SW b =DF-GARCH	**PCE core**	**PCE**	**CPI**	**CPI core**
h=1	1.5634	-0.6049	-2.2434†	-0.0813
h=3	2.0749†	-0.0244	0.8720	2.3486†
h=6	1.7472*	1.7775*	1.2507	1.9028*
h=9	1.6994*	1.1171	1.7711*	2.7254†
h=12	1.8361*	0.6211	1.6279	3.4923†

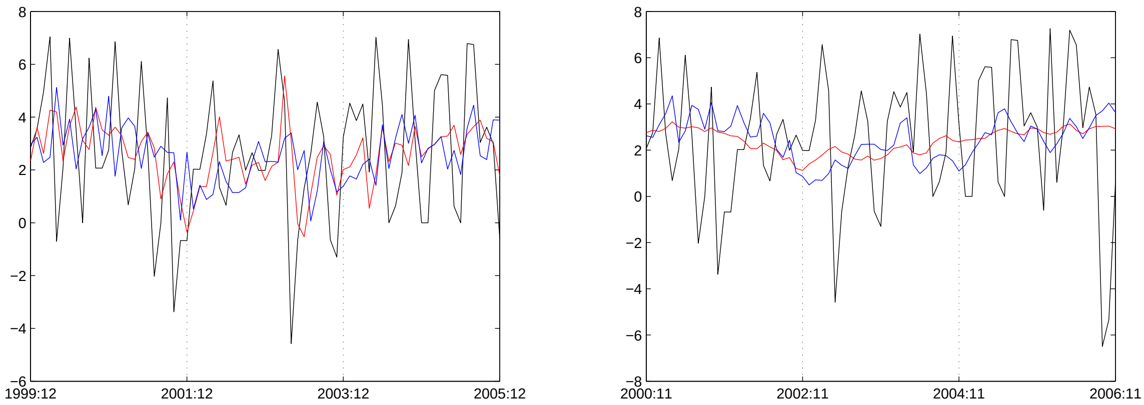
Table 6: Values of the Diebold and Mariano statistics distributed as a standard Normal († significant at 95%, * significant at 90%). Model *b* is better than model *a* when we have significant positive values.

models have equal predictive accuracy.

Finally we used the Mincer and Zarnowitz [1969] regressions to compare the predictive power of the different models. We simply regress the real values of inflation on a constant and the forecast and we compute the multiple correlation coefficient R^2. Results are in table 8 and confirm the previous results with the DF-GARCH and its homoskedastic counterpart having the largest predictive power for horizons greater than 1 period and for core inflation variables. Hence, summing up, we have a multivariate model (the DF-GARCH) that outperforms the univariate models and the classical dynamic factor model, but that gives almost identical forecasts to the ones given by its homoskedastic counterpart. In order to consider the DF-GARCH a better model with respect to the others considered here, we must evaluate its performance in forecasting and estimating the conditional variance of inflation. The small values of R^2 are explained by figure 3 where we show the real inflation and the forecasts made with DF-GARCH with horizons 1 and 12 months for CPI. Predictions are slightly lagging and have lower variance than the real series, confirming the difficulty in forecasting inflation levels. However, although the variance of the univariate forecasts is higher, this does not improve the performance of GARCH, on the contrary sometimes this model misses completely the

a =AR-GARCH b =DF-GARCH	PCE core	PCE	CPI	CPI core
h=1	3.2869†	3.2891†	4.3859†	2.0263†
h=3	3.7389†	2.1075†	4.7057†	3.0699†
h=6	2.5872†	2.6981†	3.4964†	4.0254†
h=9	3.6867†	2.8715†	3.2062†	3.3972†
h=12	3.4734†	1.9306*	3.2125†	4.0327†

Table 7: Values of the Clark and West statistics distributed as a Student-t with 72 degrees of freedom († significant at 95%, * significant at 90%). The DF-GARCH is better than GARCH when we have significant positive values.



(a) $h = 1$.

(b) $h = 12$.

Figure 3: Inflation forecast for CPI. Observed series: black. DF-GARCH forecast: red. AR(p)-GARCH(1,1) forecast: blue.

variations in the real data. The DF-GARCH prediction is smoother as it is always the case when using the common part of factor models as an index to forecast a real variable. We do not try here to add a forecast for the idiosyncratic part to improve the performance, but we believe that what our model is not capturing is really due to idiosyncratic effects. If these are interpreted as measurement errors, as it is often done in the literature, then a model that does not consider this part is a good model from a structural point of view. This is precisely one of the reasons why factor models were introduced: to get rid of measurement errors and having a more realistic index of economic activity.

7 Forecasts and estimation of conditional variance

Hereafter, we concentrate only on the two models that give forecasts and estimates also of the conditional variance: the univariate GARCH and the DF-GARCH. However, the conditional variance of inflation is not observable, thus we do not have a real benchmark for it. It is then impossible to compare the two models by using RMSE as we do for levels. Still we can qualitatively compare them, or at least, if we believe in GARCH predictions, we can check if our DF-GARCH is equally good. Indeed, given the wide use of GARCH made in the literature

PCE core	AR	AR-GARCH	SW	χ_t homoskedastic	DF-GARCH
h=1	0.0000	0.0016	**0.0121**	0.0101	0.0100
h=3	0.0012	0.0074	**0.0085**	0.0001	0.0001
h=6	0.0011	0.0017	0.0002	0.0153	**0.0154**
h=9	0.0009	0.0005	0.0009	0.0455	**0.0456**
h=12	0.0033	0.0020	0.0001	0.0446	**0.0447**

PCE	AR	AR-GARCH	SW	χ_t homoskedastic	DF-GARCH
h=1	0.0082	0.0095	**0.0825**	0.0624	0.0625
h=3	0.0043	0.0028	**0.0046**	0.0000	0.0000
h=6	0.0000	0.0013	0.0033	**0.0227**	**0.0227**
h=9	0.0055	0.0076	0.0000	**0.0249**	**0.0249**
h=12	0.0081	0.0059	**0.0106**	0.0042	0.0043

CPI core	AR	AR-GARCH	SW	χ_t homoskedastic	DF-GARCH
h=1	0.0490	0.0556	**0.0560**	0.0376	0.0379
h=3	0.0481	0.0502	0.0481	0.1263	**0.1265**
h=6	0.0309	0.0336	0.0597	0.1200	**0.1201**
h=9	0.0139	0.0142	0.0053	0.0775	**0.0776**
h=12	0.0004	0.0001	0.0038	**0.0519**	**0.0519**

CPI	AR	AR-GARCH	SW	χ_t homoskedastic	DF-GARCH
h=1	0.0023	0.0001	**0.1608**	0.0983	0.0983
h=3	0.0034	0.0057	0.0058	**0.0219**	**0.0219**
h=6	0.0014	0.0003	0.0043	0.0525	**0.0526**
h=9	0.0000	0.0011	0.0020	0.0430	**0.0431**
h=12	0.0265	0.0221	**0.0362**	0.0042	0.0043

Table 8: R^2 for the Mincer and Zarnowitz regressions.

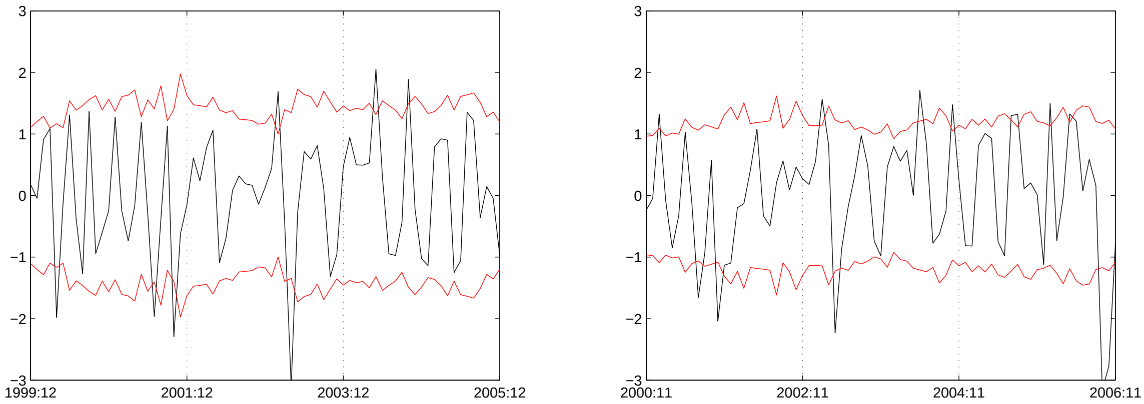
for forecasting the conditional variance of inflation, we can consider it as a benchmark model at least for historical reasons.

The general model for a time series, based on the first two conditional moments, has a specification for the conditional mean and conditional variance. Let us define the conditional mean out-of-sample forecast or insample estimate of the inflation series as μ_t and the conditional variance as h_t. The model for inflation dynamics is

$$\pi_t = \mu_t + \nu_t \ ,$$

$$\nu_t = \varepsilon_t \sqrt{h_t} \quad \text{and} \quad \varepsilon_t \sim \mathrm{N}(0,1) \ ,$$

where by h_t we mean the conditional variance of π_t. Therefore, for the DF-GARCH we have: $\mu_{t+h|t} = \tilde{\chi}^{\pi}_{t+h|t}$ and $h_{t+h|t} = \tilde{\Gamma}^{\pi}_{t+h|t}$. Here, for a given sample of the rolling scheme, $t = T+1, \ldots, T+12$ when considering out-of-sample forecasts, while $t = 1, \ldots, T$ for in-sample estimates. In figure 4 we show, for DF-GARCH, the innovations for the forecasted



(a) $h = 1$.

(b) $h = 12$.

Figure 4: Confidence intervals for CPI forecasted with DF-GARCH. Estimated residuals $\nu_{t+h|t}$: black. Forecasted 90% confidence interval $\pm 1.65\sqrt{h_{t+h|t}}$: red.

inflation series, i.e. $\nu_{t+h|t} = (\pi_{t+h} - \mu_{t+h|t})$ and the 90% forecasted confidence intervals, under the assumption of conditional normality. Namely, for all the 5 years of monthly forecasts, we plot $\pm 1.65\sqrt{h_{t+h|t}}$. If the assumption of conditional normality holds for our model, then the residuals should be contained in the predicted confidence interval 90% of the times. The simplest method for determining the adequacy of a Value-at-Risk measure is to test the hypothesis that the proportion of violations is equal to the expected one. Kupiec [1995] develops the likelihood ratio statistic

$$LR = 2\log\left[\left(1 - \frac{\tau}{T}\right)^{T-\tau}\left(\frac{\tau}{T}\right)^{\tau}\right] - 2\log\left[(1-p)^{T-\tau}p^{\tau}\right] \simeq \chi_1^2 ,$$

under the null hypothesis that the observed exception frequency, τ/T , equals to the expected one, p , where τ is the number of days over a period T that a violation has occurred. Results for the 5-th and 95-th percentiles are in table 9. Both GARCH and DF-GARCH perform well, although the DF-GARCH tends to overpenalize the 95-th percentile of noncore variables, and underpenalize the 5-th percentile of core variables.[3]
In figure 5 we plot the confidence intervals estimated for the last insample of the rolling scheme. The performance of our model seems qualitatively good when looking at CPI. Notice that for PCE we have a big outlier (probably due to some error in data cleaning) that is well detected by the DF-GARCH, although this penalizes the rest of the performance. The confidence intervals for the core variables (not shown here) are quite flat and this is imputable again to the lack of conditional heteroskedasticity.

We do not have an observable proxy of inflation's conditional variance, however we can build an indicator as in Engle [1983]. We fit an AR model, with 3, 6 or 12 lags, for the first five years of insample data (i.e. 60 observations). We then compute the standard error of the regression which we interpret as the standard deviation of the estimation. Then we drop the first observation and we add a new one at the end of the sample. We reestimate the AR, again with 60 observations. In this way, we obtain a series of variance estimates obtained under the

[3] A more sophisticated version is the test proposed by Christoffersen [1998], where it is also possible to examine whether the violations are randomly distributed through time.

	Prob $\{\nu_{t+h|t} < -1.65\sqrt{h_{t+h|t}}\}$		Prob $\{\nu_{t+h|t} > 1.65\sqrt{h_{t+h|t}}\}$	
CPI core	**DF-GARCH**	**GARCH**	**DF-GARCH**	**GARCH**
h=1	$2.81^{\dagger}$	$0.13^{\dagger}$	$0.13^{\dagger}$	$0.03^{\dagger}$
h=3	$0.84^{\dagger}$	$0.09^{\dagger}$	$0.09^{\dagger}$	$0.56^{\dagger}$
h=6	$2.44^{\dagger}$	$0.05^{\dagger}$	$0.11^{\dagger}$	$0.05^{\dagger}$
h=9	6.67	$0.58^{\dagger}$	$0.58^{\dagger}$	$0.58^{\dagger}$
h=12	$2.01^{\dagger}$	$0.00^{\dagger}$	$2.01^{\dagger}$	$0.47^{\dagger}$
CPI	**DF-GARCH**	**GARCH**	**DF-GARCH**	**GARCH**
h=1	$0.03^{\dagger}$	$0.47^{\dagger}$	$0.13^{\dagger}$	$0.03^{\dagger}$
h=3	$0.56^{\dagger}$	$0.56^{\dagger}$	4.40	$0.56^{\dagger}$
h=6	$0.11^{\dagger}$	$0.11^{\dagger}$	4.83	$0.11^{\dagger}$
h=9	$1.98^{\dagger}$	$1.98^{\dagger}$	$1.98^{\dagger}$	$0.02^{\dagger}$
h=12	$1.04^{\dagger}$	$0.00^{\dagger}$	5.79	$1.04^{\dagger}$

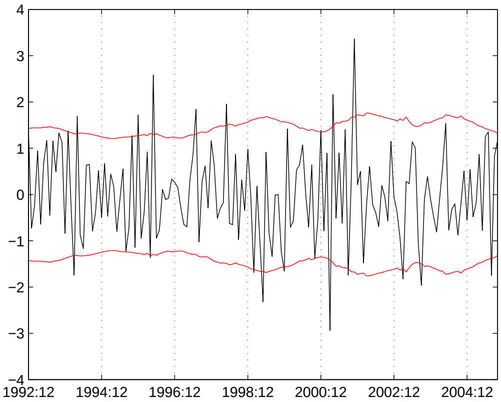
Table 9: Values of the *LR* Kupiec statistic for CPI and CPI core distributed as a χ^2 with one degree of freedom (†: we accept the null hypothesis of a correct model specification, i.e. p-value > 0.05).

	Relative RMSE			
AR lags	**PCE core**	**PCE**	**CPI core**	**CPI**
3	2.00	0.83	2.56	0.72
6	2.00	0.97	0.46	0.86
12	0.69	1.71	0.22	1.64

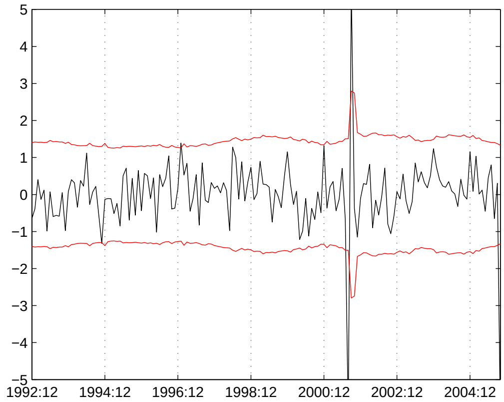
Table 10: RMSE for insample volatility estimates, relative to univariate GARCH, when considering as a benchmark the proxy suggested by Engle. Values smaller than one indicate a better performance of DF-GARCH.

assumption that the model and its variance are constant for the preceding five years. As noted by Engle: "[...]the statistical properties of this procedure are not clear as the assumptions are continually changing, but the interpretation is quite straightforward". We can compute the RMSE between this proxy and the estimates obtained with the GARCH or the DF-GARCH. Results are displayed in table 10. For noncore variables the DF-GARCH performs better than GARCH when using a low number of lags in the conditional mean specification. The viceversa holds for core variables. If we assume that core variables are more persistent than noncore variables then our results are good, otherwise the proxy suggested by Engle seems to depend too much on the chosen conditional mean model. We can only say that DF-GARCH is performing at least as well as the univariate GARCH.

Summing up, the intervals predicted by the DF-GARCH contain the majority of observations and for noncore variable follow quite well the fluctuations of the series. Moreover, if we consider a proxy of inflation's conditional variance as in Engle [1983], the DF-GARCH has a performance comparable with a univariate GARCH. We thus have a model that forecasts



(a) CPI.



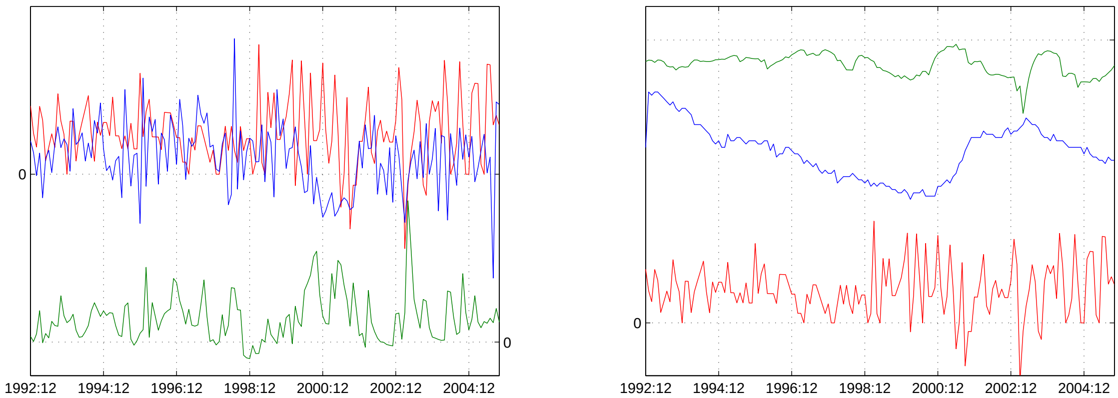
(b) PCE.

Figure 5: Insample estimated confidence intervals for CPI and PCE. Estimated residuals ν_t: black. Estimated 90% confidence interval $\pm 1.65\sqrt{h_t}$: red.

inflation levels better than the univariate and classical factor models and equally well when compared to factor models estimated in state space form. Moreover, our model forecasts conditional variances at least as well as univariate models as GARCH. But our model provides also forecasts and estimates of conditional covariances. In figure 6 we show estimates of conditional covariances for the last insample of the rolling scheme. We consider economically interesting couples of series, in particular we show the conditional covariances between CPI and total industrial production growth rate (Δy_t) or unemployment rate (υ_t). As expected the first covariance is positive while the second is negative. This is in line with the unconditional covariances and with the conventional economic literature. For our data we have: $\text{cov}(\Delta y_t, \pi_t) = 0.14$ and $\text{cov}(\upsilon_t, \pi_t) = -0.06$. The performance of the DF-GARCH is remarkable in estimating the right sign of conditional covariances and in following the peaks and troughs in the comovements between the variables.
Reliable estimates of the conditional covariances when dealing with monetary rules as the Taylor rule are essential if central bankers want to act as risk managers with two targets: one for inflation and one for economic growth. A multi-target rule is often said to be inconsistent although in practice it is sometimes implicitly used by the Federal Reserve. Our model could be useful in deepening our knowledge of the mechanisms that relate the nominal and real sectors of the economy and may thus help in throwing light upon the theorized inconsistency and possibly making monetary rules more efficient.

8 Further research

In this paper we test the importance of multivariate information for modelling and forecasting inflation's conditional mean and variance. In particular, we apply a conditionally heteroskedastic factor model, originally proposed in Alessi et al. [2006], to inflation forecasting. The model imposes a conditional mean structure to the static factors, and provides also a forecast of conditional variance and conditional covariances for all the macroeconomic variables present in our dataset. Results both for inflation levels and its conditional variance are encouraging. There are many possible ways to extend this work, either in the economic field of monetary policies and in the econometric field of factor models.



(a) $\mathrm{cov}(\Delta y_t, \pi_t | \mathcal{I}_t - 1)$.

(b) $\mathrm{cov}(u_t, \pi_t | \mathcal{I}_t - 1)$.

Figure 6: Insample estimated conditional covariances. CPI: red. Industrial production growth rate or unemployment: blue. Estimated conditional covariance: green (scale on the right hand side).

The availability of conditional covariance forecasts is an additional information that can give insights on the structural process of inflation and its interpretation in terms of relations with other macroeconomic variables (e.g. measures of economic activity), which could be useful for example in monetary policy issues. This road naturally leads to models of monetary rules in which not only the target variables enter, but also their conditional variances (as measures of uncertainty) and the conditional covariances between them. Concerning the estimation part, we could also try to improve our procedure by employing the Quasi Maximum Likelihood estimator proposed by Doz et al. [2006] when estimating the parameters of the state space form of the DF-GARCH. Finally, from the results of insample estimation part we could also test the Friedman [1977] hypothesis as Engle [1983] did.

In Alessi et al. [2006] we apply the DF-GARCH to asset returns, which have high conditional heteroskedasticity but probably not enough dynamics to fully justify the dynamic approach. In this paper we apply the same method to inflation, which has enough dynamics in the levels but less conditional heteroskedasticity than asset returns. The latter fact is especially clear for core indicators which are indeed computed without taking into account the most fluctuating price indexes (i.e. oil and food). Moreover recent data do not fluctuate much due to the high stability in Western economies in the last twenty years (the phenomenon known as Great Moderation). The results on inflation volatility by Engle are obtained from data of the 1970s which are definitely more conditionally heteroskedastic. An ideal field of application of the DF-GARCH are disaggregated price indexes which are more dynamic and conditionally heteroskedastic than asset returns and aggregated inflation. Applying our method to these series may be a good way to compute an aggregate inflation index with its confidence bands, and may also be useful in shedding light into price dynamics. Indeed, one of the first applications of dynamic factor models in economic literature is related to the aggregation of heterogeneous microeconomic series (see Forni and Lippi [1997]). The issue of aggregation of economic time series in a factor model context is also considered in Zaffaroni [2004] from a general perspective and in Altissimo et al. [2007] when considering precisely the aggregation of sectoral price indexes in order to study the dynamics of the aggregated inflation indicator.

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A The modified Kalman filter

We explain here in detail the estimation of the state-space model

$$\begin{aligned} \mathbf{x}_t &= \hat{\mathbf{\Lambda}}\mathbf{F}_t + \boldsymbol{\xi}_t \quad && \text{measurement equation ,} \\ \mathbf{F}_t &= \hat{\mathbf{A}}\mathbf{F}_{t-1} + \hat{\mathbf{H}}\mathbf{u}_t && \text{transition equation ,} \end{aligned}$$

where

$$\begin{aligned} \boldsymbol{\xi}_{t|t-1} &\sim \mathrm{N}\left(0, \hat{\mathbf{R}}_t\right) \quad \hat{\mathbf{R}}_t \text{ diagonal ,} \\ \mathbf{u}_{t|t-1} &\sim \mathrm{N}\left(0, \mathbf{Q}_t\right) \ , \\ \mathbf{Q}_t &= \hat{\mathbf{C}}_0{}'\hat{\mathbf{C}}_0 + \hat{\mathbf{C}}_1{}'\mathbf{u}_{t-1}\mathbf{u}'_{t-1}\hat{\mathbf{C}}_1 + \hat{\mathbf{C}}_2{}'\mathbf{Q}_{t-1}\hat{\mathbf{C}}_2 \ . \end{aligned}$$

The multivariate GARCH representation considered here is a full BEKK, but the following procedure can be easily modified to allow for a DCC representation.

Initialization

Initial values are built as:

$$\begin{cases} \mathbf{F}_{1|1} & = \hat{\mathbf{F}}_1 \\ \mathbf{P}_{1|1} & \text{sufficiently large} \\ \mathbf{u}_{1|1} & = \hat{\mathbf{u}}_1 \\ \mathbf{Q}_{1|1} & = \hat{\mathbf{Q}}_1 \\ (\mathbf{u}_1\mathbf{u}'_1)_{|1} & = \mathbf{u}_{1|1}\mathbf{u}'_{1|1} + \mathbf{Q}_{1|1} \ , \end{cases}$$

where the variables with the hat have been obtained during the estimation step presented in section 3, $\hat{\mathbf{Q}}_1$ has been obtained by the multivariate GARCH model, and the state initial covariance matrix $\mathbf{P}_{1|1}$ must represent the high uncertainty about the initial value of the state vector.

Prediction

The steps described in this and the following section must be repeated together for time $t = 2 \dots T$. First we predict the unobserved state vector

$$\mathbf{F}_{t|t-1} = \hat{\mathbf{A}}\mathbf{F}_{t-1|t-1} \ ,$$

and its conditional covariance matrix

$$\mathbf{P}_{t|t-1} = \hat{\mathbf{A}}\mathbf{P}_{t-1|t-1}\hat{\mathbf{A}}' + \hat{\mathbf{H}}(\mathbf{u}_t\mathbf{u}'_t)_{|t-1}\hat{\mathbf{H}}' \ ,$$

where

$$\begin{cases} (\mathbf{u}_t\mathbf{u}'_t)_{|t-1} & = \mathbf{Q}_{t|t-1} \\ \mathbf{Q}_{t|t-1} & = \hat{\mathbf{C}}_0{}'\hat{\mathbf{C}}_0 + \hat{\mathbf{C}}_1{}'(\mathbf{u}_{t-1}\mathbf{u}'_{t-1})_{|t-1}\hat{\mathbf{C}}_1 + \hat{\mathbf{C}}_2{}'\mathbf{Q}_{t-1|t-1}\hat{\mathbf{C}}_2 \ . \end{cases} \tag{A-1}$$

The conditional covariance matrix for the state vector is obtained by using the GARCH estimated parameters $\hat{\mathbf{C}}_0$, $\hat{\mathbf{C}}_1$ and $\hat{\mathbf{C}}_2$; they are applied on the updated conditional covariance of the transition error $\left(\mathbf{u}_{t-1}\mathbf{u}'_{t-1}\right)$, which in turn has been obtained by the Kalman update, as we see in the next step.
The prediction error is given by

$$\boldsymbol{\eta}_{t|t-1} = \tilde{\mathbf{x}}_t - \tilde{\mathbf{x}}_{t|t-1} = \tilde{\mathbf{x}}_t - \hat{\mathbf{\Lambda}}\mathbf{F}_{t|t-1} \ ,$$

whose conditional covariance is built by using the predicted conditional covariance of the static factors and the known conditional covariance of the measurement errors, as obtained previously by univariate modelling of the idiosyncratic parts:

$$\mathbf{Y}_{t|t-1} = \hat{\mathbf{\Lambda}}\mathbf{P}_{t|t-1}\hat{\mathbf{\Lambda}}' + \hat{\mathbf{R}}_t \,.$$

Update

We compute the Kalman gain

$$\mathbf{K}_t = \mathbf{P}_{t|t-1}\hat{\mathbf{\Lambda}}'\mathbf{Y}_{t|t-1}^{-1} \,,$$

and we build more accurate inferences, exploiting information up to time t,

$$\mathbf{F}_{t|t} = \mathbf{F}_{t|t-1} + \mathbf{K}_t\boldsymbol{\eta}_{t|t-1} \,,$$

$$\mathbf{P}_{t|t} = \mathbf{P}_{t|t-1} - \mathbf{K}_t\hat{\mathbf{\Lambda}}\mathbf{P}_{t|t-1} \,.$$

By inverting the transition equation and recalling the paper by Giannone et al. [2004], we get

$$\mathbf{u}_{t|t} = \mathbf{\Phi}^{-1/2}\mathbf{M}'\left(\mathbf{I}_r - \hat{\mathbf{A}}L\right)\mathbf{F}_{t|t} \,, \tag{A-2}$$

and then

$$(\mathbf{u}_t\mathbf{u}_t')_{|t} = \mathbf{u}_{t|t}\,\mathbf{u}_{t|t}' \,. \tag{A-3}$$

Equation (A-3), when put in the context of the following prediction step (A-1), is not precise. As noted by Harvey et al. [1992], a correction term should be added on the right hand side in order to take the into account the conditional variance of the dynamic factor. However, the same authors show that, when applied to the factor model by Diebold and Nerlove [1989], the effect of this correction may be empirically negligible. The differences between their estimation procedure and ours, including the update passage described in (A-2), let us prefer avoiding the estimation of the correction term.

Smoothing

Smoothing would be especially useful when extending our procedure to a higher number of lags in the GARCH structure of dynamic factors' conditional covariances. In any case, the smoothing procedure is recommended for getting a more precise estimate of the common and idiosyncratic components of the dataset. Following de Jong [1989] and Durbin and Koopman [2001], the following fixed interval smoother can be applied for $t = T, T-1, \ldots, 2$ in order to find more precise insample values of the static factors and of dynamic factors' conditional covariances. First we compute

$$\mathbf{r}_{t-1} = \mathbf{L}_t'\mathbf{r}_t + \hat{\mathbf{\Lambda}}'\mathbf{Y}_{t|t-1}^{-1}\boldsymbol{\eta}_{t|t-1} \,,$$

$$\mathbf{F}_{t|T} = \mathbf{F}_{t|t-1} + \mathbf{P}_{t|t-1}\mathbf{r}_{t-1} \,,$$

where $\mathbf{L}_t = \hat{\mathbf{A}}\left(\mathbf{I}_r - \mathbf{K}_t\hat{\mathbf{\Lambda}}\right)$, $\mathbf{r}_T = 0$. At each step, we also find the smoothed state variance matrix

$$\mathbf{P}_{t|T} = \mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1}\mathbf{\Theta}_{t-1}\mathbf{P}_{t|t-1} \,,$$

where $\mathbf{\Theta}_t$ has been obtained by

$$\mathbf{\Theta}_{t-1} = \hat{\mathbf{\Lambda}}'\mathbf{Y}_t^{-1}\hat{\mathbf{\Lambda}} + \mathbf{L}_t'\mathbf{\Theta}_t\mathbf{L}_t \,,$$

with initial value $\mathbf{\Theta}_T = 0$. At the end of each step, we get smoothed values for the dynamic factors and their conditional covariances $\mathbf{Q}_t$

$$\mathbf{u}_{t|T} = \mathbf{Q}_{t|t-1}\hat{\mathbf{H}}'\mathbf{r}_t \ ,$$

$$\mathbf{Q}_{t|T} = \mathbf{Q}_{t|t-1} - \mathbf{Q}_{t|t-1}\hat{\mathbf{H}}'\mathbf{\Theta}_t\hat{\mathbf{H}}\mathbf{Q}_{t|t-1} \ .$$

B The US database

Data from the McGraw-Hill DRI database used in the paper.
Transformation codes (in parenthesis):
1 = No transformation
2 = Monthly growth rate
3 = Logarithm
4 = Monthly difference

1. (2) INDUSTRIAL PRODUCTION INDEX - TOTAL INDEX UNITS 2002=100, SA
2. (2) INDUSTRIAL PRODUCTION INDEX - PRODUCTS, TOTAL UNITS 2002=100, SA
3. (2) INDUSTRIAL PRODUCTION INDEX - FINAL PRODUCTS UNITS 2002=100, SA
4. (2) INDUSTRIAL PRODUCTION INDEX - CONSUMER GOODS UNITS 2002=100, SA
5. (2) INDUSTRIAL PRODUCTION INDEX - DURABLE CONSUMER GOODS UNITS 2002=100, SA
6. (2) INDUSTRIAL PRODUCTION INDEX - NONDURABLE CONSUMER GOODS UNITS 2002=100, SA
7. (2) INDUSTRIAL PRODUCTION INDEX - BUSINESS EQUIPMENT UNITS 2002=100, SA
8. (2) INDUSTRIAL PRODUCTION INDEX - MATERIALS UNITS 2002=100, SA
9. (2) INDUSTRIAL PRODUCTION INDEX - DURABLE GOODS MATERIALS UNITS 2002=100, SA
10. (2) INDUSTRIAL PRODUCTION INDEX - NONDURABLE GOODS MATERIALS UNITS 2002=100, SA
11. (2) INDUSTRIAL PRODUCTION INDEX - MANUFACTURING UNITS 2002=100, SA
12. (2) INDUSTRIAL PRODUCTION INDEX - RESIDENTIAL UTILITIES UNITS 2002=100, SA
13. (2) INDUSTRIAL PRODUCTION INDEX - BASIC METALS UNITS 2002=100, SA
14. (2) INDUSTRIAL PRODUCTION INDEX - FOODS AND TOBACCO UNITS 2002=100, SA
15. (2) INDUSTRIAL PRODUCTION INDEX - CLOTHING UNITS 2002=100, SA
16. (2) INDUSTRIAL PRODUCTION INDEX - CHEMICAL PRODUCTS UNITS 2002=100, SA
17. (2) INDUSTRIAL PRODUCTION INDEX - DEFENSE AND SPACE EQUIPMENT UNITS 2002=100, SA
18. (2) INDUSTRIAL PRODUCTION INDEX - ENERGY MATERIALS UNITS 2002=100, SA
19. (1) PURCHASING MANAGERS' INDEX - SA
20. (1) NAPM PRODUCTION INDEX - PERCENT
21. (2) DISPOSABLE PERSONAL INCOME - BILLIONS OF CHAINED 2000 DOLLARS , SA
22. (2) REAL PERSONAL CONSUMPTION EXPENDITURES - DURABLE GOODS QUANTITY INDEX 2000=100, SA
23. (2) REAL PERSONAL CONSUMPTION EXPENDITURES - NONDURABLE GOODS QUANTITY INDEX 2000=100, SA
24. (2) REAL PERSONAL CONSUMPTION EXPENDITURES - TOTAL QUANTITY INDEX 2000=100, SA
25. (2) REAL PERSONAL CONSUMPTION EXPENDITURES - SERVICES QUANTITY INDEX 2000=100, SA
26. (2) S & P'S COMMON STOCK PRICE INDEX - COMPOSITE 1941-43=10
27. (2) S & P'S COMMON STOCK PRICE INDEX - INDUSTRIALS 1941-43=10

28. (2) S & P'S COMPOSITE COMMON STOCK - PRICE-EARNINGS RATIO, NSA

29. (2) COMMON STOCK PRICES - DOW JONES INDUSTRIAL AVERAGE

30. (1) NAPM COMMODITY PRICES INDEX - PERCENT

31. (2) PRODUCER PRICE INDEX - CRUDE MATERIALS 1982=100, SA

32. (2) PRODUCER PRICE INDEX - FINISHED CONSUMER FOODS 1982=100, SA

33. (2) PERSONAL CONSUMPTION EXPENDITURES - DURABLE GOODS PRICE INDEX 2000=100, SA

34. (2) PERSONAL CONSUMPTION EXPENDITURES - NONDURABLE GOODS PRICE INDEX 2000=100, SA

35. (2) PERSONAL CONSUMPTION EXPENDITURES - SERVICES GOODS PRICE INDEX 2000=100, SA

36. (2) PERSONAL CONSUMPTION EXPENDITURES - TOTAL LESS FOOD AND ENERGY PRICE INDEX 2000=100, SA

37. (2) PERSONAL CONSUMPTION EXPENDITURES - TOTAL GOODS PRICE INDEX 2000=100, SA

38. (2) CONSUMER PRICE INDEX-U - ALL ITEMS 82-84=100, SA INDEX BASE: 1982-84 = 1.000

39. (2) CONSUMER PRICE INDEX-U - FUEL OIL, COAL AND BOTTLED GAS 82-84=100, SA

40. (2) CONSUMER PRICE INDEX-U - FRUITS & VEGETABLES 82-84=100, SA

41. (2) CONSUMER PRICE INDEX-U - FOOTWEAR 82-84=100, SA

42. (2) CONSUMER PRICE INDEX-U - USED CARS 82-84=100, SA

43. (2) CONSUMER PRICE INDEX-U - ENERGY 82-84=100, SA

44. (2) CONSUMER PRICE INDEX-U - FOOD 82-84=100, SA

45. (2) CONSUMER PRICE INDEX-U - TRANSPORTATION 82-84=100, SA

46. (2) CONSUMER PRICE INDEX-U - MEDICAL CARE 82-84=100, SA

47. (2) CONSUMER PRICE INDEX-U - ENERGY COMMODITIES 82-84=100, SA

48. (2) CONSUMER PRICE INDEX-U - SHELTER 82-84=100, SA

49. (2) CONSUMER PRICE INDEX-U - SERVICES 82-84=100, SA

50. (2) CONSUMER PRICE INDEX-U - ALL ITEMS LESS ENERGY 82-84=100, SA

51. (2) CONSUMER PRICE INDEX-U - ALL ITEMS LESS FOOD 82-84=100, SA

52. (2) CONSUMER PRICE INDEX-U - ALL ITEMS LESS SHELTER 82-84=100, SA

53. (2) CONSUMER PRICE INDEX-U - ALL ITEMS LESS MEDICAL CARE 82-84=100, SA

54. (2) CONSUMER PRICE INDEX-U - ALL ITEMS LESS FOOD AND ENERGY 82-84=100, SA

55. (2) SPOT MARKET PRICE INDEX - ALL COMMODITIES 1967=100, NSA

56. (3) HOUSING STARTS - NONFARM 1947-58 TOTAL FARM & NONFARM 1959-2006 THOUS.,SA

57. (3) HOUSING STARTS - NORTHEAST THOUS., SA

58. (3) HOUSING STARTS - MIDWEST THOUS., SA

59. (3) HOUSING STARTS - SOUTH THOUS., SA

60. (3) HOUSING STARTS - WEST THOUS., SA

61. (3) HOUSING AUTHORIZED - TOTAL NEW PRIVATE HOUSING UNITS THOUS.,SA

62. (1) NAPM VENDOR DELIVERIES INDEX - PERCENT

63. (1) NAPM NEW ORDERS INDEX - PERCENT

64. (1) NAPM INVENTORIES INDEX - PERCENT

65. (2) NEW ORDERS (NET) - CONSUMER GOODS AND MATERIALS, 1996 DOLLARS BILLIONS OF 1982 DOLLARS, SA

66. (2) NEW ORDERS - NONDEFENSE CAPITAL GOODS, 1996 DOLLARS BILLIONS OF 1996 DOLLARS, SA

67. (1) FOREIGN EXCHANGE RATE - CANADA CANADIAN $ PER U.S.$

68. (1) FOREIGN EXCHANGE RATE - JAPAN YEN PER U.S.$

69. (1) FOREIGN EXCHANGE RATE - SWITZERLAND SWISS FRANC PER U.S.$

70. (1) FOREIGN EXCHANGE RATE - UNITED KINGDOM CENTS PER POUND

71. (1) INTEREST RATE - FEDERAL FUNDS (EFFECTIVE) PERCENT PER ANNUM, NSA

72. (1) INTEREST RATE - U.S.TREASURY BILLS,SEC MKT,3-MO. PERCENT PER ANNUM, NSA

73. (1) INTEREST RATE - U.S.TREASURY BILLS,SEC MKT,6-MO. PERCENT PER ANNUM, NSA

74. (1) INTEREST RATE - U.S.TREASURY CONST MATURITIES,1-YR. PERCENT PER ANNUM, NSA

75. (1) INTEREST RATE - U.S.TREASURY CONST MATURITIES,5-YR. PERCENT PER ANNUM, NSA

76. (1) INTEREST RATE - U.S.TREASURY CONST MATURITIES,10-YR. PERCENT PER ANNUM, NSA

77. (1) BOND YIELD - MOODY'S AAA CORPORATE PERCENT PER ANNUM

78. (1) BOND YIELD - MOODY'S BAA CORPORATE PERCENT PER ANNUM

79. (1) SPREAD FYGM3-FYFF

80. (1) SPREAD FYGM6-FYFF

81. (1) SPREAD FYGT1-FYFF

82. (1) SPREAD FYGT5-FYFF

83. (1) SPREAD FYGT10-FYFF

84. (1) SPREAD FYAAAC-FYFF

85. (1) SPREAD FYBAAC-FYFF

86. (2) MONEY STOCK - M1 (CURR,TRAV.CKS,DEM DEP,OTHER CK'ABLE DEP) BILLIONS OF DOLLARS, SA

87. (2) MONEY STOCK - M2 (M1+O'NITE RPS,EURO$,G/P& B/D MMMFS& SAV& SM TIME DEP BILLIONS OF DOLLARS, SA

88. (2) MONETARY BASE, ADJ FOR RESERVE REQUIREMENT CHANGES(MIL$,SA)

89. (2) DEPOSITORY INST RESERVES:TOTAL,ADJ FOR RESERVE REQ CHGS(MIL$,SA)

90. (2) DEPOSITORY INST RESERVES:NONBORROWED,ADJ RES REQ CHGS(MIL$,SA)

91. (2) COMMERCIAL AND INDUSTRIAL LOANS - LARGE WEEKLY REPORTING BANKS, BILLIONS OF CURRENT DOLLARS, SA

92. (2) CONSUMER CREDIT OUTSTANDING - NONREVOLVING(G19), BILLIONS OF CURRENT DOLLARS, SA

93. (2) COMMERCIAL AND INDUSTRIAL LOANS OUTSTANDING - BILLIONS OF 2000 DOLLARS, SA

94. (1) INDEX OF HELP-WANTED ADVERTISING IN NEWSPAPERS 1967=100, SA

95. (2) EMPLOYMENT - RATIO HELP-WANTED ADS NO. UNEMPLOYED CIVILIAN LABOR FORCE

96. (2) CIVILIAN LABOR FORCE - EMPLOYED, TOTAL THOUS., SA

97. (2) CIVILIAN LABOR FORCE - EMPLOYED, NONAGRIC.INDUSTRIES THOUS., SA

98. (1) UNEMPLOYMENT RATE - ALL WORKERS, 16 YEARS & OVER PERCENT, SA

99. (2) UNEMPLOYMENT BY DURATION - PERSONS UNEMPLOYED LESS THAN 5 WKS THOUS., SA

100. (2) UNEMPLOYMENT BY DURATION - PERSONS UNEMPLOYED 5 TO 14 WKS THOUS., SA

101. (2) UNEMPLOYMENT BY DURATION - PERSONS UNEMPLOYED 15 WKS + THOUS.,SA

102. (2) UNEMPLOYMENT BY DURATION - PERSONS UNEMPLOYED 15 TO 26 WKS THOUS., SA

103. (4) NAPM EMPLOYMENT INDEX - PERCENT

104. (2) EMPLOYEES ON NONAGRICULTURAL PAYROLLS - THOUS., SA

105. (2) EMPLOYEES ON NONAGRICULTURAL PAYROLLS - TOTAL PRIVATE UNITS, THOUS., SA

106. (2) EMPLOYEES ON NONAGRICULTURAL PAYROLLS - GOOD-PRODUCING UNITS, THOUS., SA

107. (2) EMPLOYEES ON NONAGRICULTURAL PAYROLLS - CONSTRUCTION UNITS, THOUS., SA

108. (2) EMPLOYEES ON NONAGRICULTURAL PAYROLLS - MANUFACTURING UNITS, THOUS., SA

109. (2) EMPLOYEES ON NONAGRICULTURAL PAYROLLS - DURABLE GOODS UNITS, THOUS., SA

110. (2) EMPLOYEES ON NONAGRICULTURAL PAYROLLS - NONDURABLE GOODS UNITS, THOUS., SA

111. (2) EMPLOYEES ON NONAGRICULTURAL PAYROLLS - SERVICE PROVIDING UNITS, THOUS., SA

112. (2) EMPLOYEES ON NONAGRICULTURAL PAYROLLS - TRADE TRANSPORTATION AND UTILITIES UNITS, THOUS., SA

113. (2) EMPLOYEES ON NONAGRICULTURAL PAYROLLS - WHOLESALE TRADE UNITS - THOUS., SA

114. (2) EMPLOYEES ON NONAGRICULTURAL PAYROLLS - RETAIL TRADE UNITS - THOUS., SA

115. (2) EMPLOYEES ON NONAGRICULTURAL PAYROLLS - FINANCIAL ACTIVITIES UNITS - THOUS., SA

116. (2) EMPLOYEES ON NONAGRICULTURAL PAYROLLS - PROFESSIONAL AND BUSINESS SERVICES UNITS - THOUS., SA

117. (2) EMPLOYEES ON NONAGRICULTURAL PAYROLLS - EDUCATIONAL AND HEALTH SERVICES UNITS - THOUS., SA

118. (2) EMPLOYEES ON NONAGRICULTURAL PAYROLLS - GOVERNMENT UNITS: THOUS., SA

119. (1) UNIVERSITY OF MICHIGAN INDEX OF CONSUMER EXPECTATIONS

120. (2) CAPACITY UTILIZATION - MANUFACTURING (SIC) PERCENT OF CAPACITY, SA

121. (2) CAPACITY UTILIZATION - FINISHED PROCESSING (CAPACITY) PERCENT OF CAPACITY, SA

122. (2) CAPACITY UTILIZATION - NONMETALLIC MINERAL PRODUCT NAICS=327 PERCENT OF CAPACITY, SA

123. (2) CAPACITY UTILIZATION - FABRICATED METAL PRODUCT NAICS=332 PERCENT OF CAPACITY, SA

124. (2) CAPACITY UTILIZATION - MOTOR VEHICLES AND PARTS NAICS=3361-3 PERCENT OF CAPACITY, SA

125. (2) CAPACITY UTILIZATION - AEROSPACE AND MISCELLANEOUS TRANSPORTATION EQ. PERCENT OF CAPACITY, SA

126. (2) CAPACITY UTILIZATION - PAPER NAICS=322 PERCENT OF CAPACITY, SA

127. (2) CAPACITY UTILIZATION - PETROLEUM AND COAL PRODUCTS NAICS=324 PERCENT OF CAPACITY, SA

128. (2) CAPACITY UTILIZATION - CHEMICAL NAICS=325 PERCENT OF CAPACITY, SA

129. (2) CAPACITY UTILIZATION - PLASTICS AND RUBBER PRODUCTS NAICS=326 PERCENT OF CAPACITY, SA

130. (2) CAPACITY UTILIZATION - PRIMARY & SEMIFINISHED PROCESSING (CAPACITY) PERCENT OF CAPACITY, SA