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Volatility-price relationships in power exchanges: A demand-supply analysis*

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Abstract

The evidence of volatility-price dependence observed in previous works (Karakatsani and Bunn 2004; Bottazzi, Sapio and Secchi 2005; Simonsen 2005) suggests that there is more to volatility than simply spikes. Volatility is found to be positively correlated with the lagged price level in settings where market power is likely to be particularly strong (UK on-peak sessions, the CalPX). Negative correlation is instead observed in markets considered to be fairly competitive, such as the NordPool. Prompted by these observations, this paper aims to understand whether volatility-price patterns can be mapped into different degrees of market competition, as the evidence seems to suggest.

Price fluctuations are modelled as the outcome of dynamics in both sides of the market - demand and supply, which in turn respond to shocks to the underlying preference and technology fundamentals. Negative volatility-price dependence arises if the market dynamics is accounted for by common shocks which affect valuations uniformly. Positive dependence is related to the impact of asymmetric shocks. The paper shows that under certain conditions, these volatility-price patterns can be used to identify the exercise of market power. Identification is however ruled out if all shocks affect valuations uniformly.

JEL Classifications: C16, D4, L94.

Keywords: Electricity, Market, Volatility, Supply Curve, Demand Curve, Fundamentals, Shocks.

1 Introduction

Among the goals of the power industry restructuring, attaining a high degree of efficiency is viewed as a primary one. Widespread demand for lower energy prices urged regulators to introduce market-based trading in key segments of the industry, such as electricity production (Joskow 1996, Newbery 2002). Whether markets have been successful in yielding lower power prices is still a debated issue. Much better established is the evidence that liberalized power exchanges can be extremely volatile. Simonsen (2005) reports a 16% value for the annualized volatility of NordPool daily returns - a value much higher than for other energy markets, such as coal, natural gas, and petroleum (Schwartz and Smith 2000). Sharp and short-lived spikes are commonly observed in market sessions characterized by a very tight balance between

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demand and the available generating capacity. Besides, power exchange volatility has been shown to possess a rich temporal structure, which has been analyzed through reduced-form models, such as GARCH, regime-switching, and jump-diffusion models (see Weron 2007 for a survey). Further statistical studies have shed light on patterns of correlation between the lagged electricity price level and measures of volatility. Karakatsani and Bunn (2004) fitted a power law on the relationship between the lagged price level and the residual volatility of the price model, and observed various patterns across hours in the UK market. Bottazzi, Sapio and Secchi (2005) and Simonsen (2005) observed negative correlation for the NordPool market; Sapio (2005, 2008) and Bottazzi and Sapio (2008) described positive correlation between volatility and the price level in the APX and CalPX, as well as regime breaks in NordPool and Powernext. These patterns seem to be somewhat related to different degrees of market competition. Positive correlation in the UK is observed on-peak, when the demand-capacity ratio is very high, and in a market, such as the CalPX, which is widely recognized as probably the most striking example of how market power can lead to the collapse of a trade institution (Joskow 2001, Wolak 2003). Negative correlation is instead found in the NordPool, which heavy reliance on hydropower suggests it may be fairly competitive. If such association finds a theoretical foundation, the volatility-price patterns can be seen as "footprints" left by power producers who exercise market power.

Analyzing the mechanisms which give rise to volatility-price patterns is appealing for this reason, but not only. Making sense of volatility patterns can improve our understanding of possible trade-offs between two key goals of energy policy: ensuring low and stable prices. Such knowledge can be precious to discipline policy-making, by clarifying the menu of the attainable policy results. Moreover, accurate modelling of the time structure of volatility enables more precise market forecasts (Weron 2006). As such, it can foster the design of efficient tools for risk management, a key step on the way to the long-term achievement of liquid and robust power exchanges (Powell 1993).

The paper seeks to explain the emergence of volatility-price patterns in power exchanges, and to understand whether volatility patterns can be mapped into different degrees of competition, and hence used to identify anti-competitive behaviors.. The analysis is carried out by means of a structural approach, based on direct modelling of the demand and supply curves. The basic insight behind this approach is that price fluctuations are due to dynamics in both sides of the market - demand and supply. These are in turn responsive to changes in technology and preference fundamentals, such as the demand responsiveness to price signals and the structure of production costs.

Among the advantages of using a structural approach, it allows to map the evidence into specific features of the market curves. The properties of demand and supply curves convey relevant information as to the degree of market competition and transparency. For instance, a low value of the price-elasticity of demand is a source of market power. Further, if the supply curve lies above the marginal cost curve and is steeper, this is due to market power abuse by multi-plant power generating companies (Ausubel and Cramton 1996). In addition to this, and importantly, the design of any policy measure to mitigate volatility needs to outline batteries of incentives which directly touch on consumption habits and investment plans. Knowledge of the main sources of volatility is critical, in that the required policy actions may differ dramatically whether volatility is due to demand or supply factors.

Results from this paper reveal that negative volatility-price dependence takes place if the bulk of market dynamics is due to *uniform shocks*, i.e. common shocks whose impact on valuations is uniform across agents. The reason is that, if all valuations on either side of the market change by the same amount, the shock weighs proportionally more on low valuations, which are the marginal ones when the price is low. Positive price dependence is a sign that *asymmetric shocks* are the main volatility drivers. These shocks typically affect the degree of heterogeneity among commodity valuations. Under certain conditions, their impact is amplified by the price level. As the main implications for competition policy, the identification of market power by means of volatility-price patterns requires the occurrence of asymmetric shocks. Indeed, if all shocks affect valuations uniformly, volatility patterns do not vary across market regimes. Since markets with only uniform shocks are hard to imagine, the association between volatility-price patterns and competition suggested by the evidence might have some foundation.

This paper fills a gap in the existing theoretical literature on the volatility of power exchanges. Some previous works place a similar emphasis on the role of market structure in explaining volatility. Mount (2000) posits a perfectly inelastic demand and a constant-elasticity supply function. As shown, the market-clearing price is more volatile, the lower the supply elasticity. However, agents behaviors are not explicitly modelled. While the analysis in the present paper eschews modelling behaviors too, it is general enough as to include several optimizing models as special cases. Mount (1998) proposes a Cournot model with displaced quadratic cost functions, and demonstrates that volatility is higher in an oligopolistic setting than under perfect competition, in that market power makes the supply curve more inelastic. Yet, no account is given for the volatility-price patterns observed more recently. Barlow (2002) and De Sanctis and Mari (2007) assume a stochastically varying demand function, and aim to account for the emergences of spikes. Kanamura's (2007) paper on the natural gas market is another valuable attempt within the structural approach, but its goal is to explain the evidence of inverse leverage effects.

The paper is structured as follows. Section 2 reviews the main facts on the volatility of power exchanges, with a major focus on volatility-price patterns. Section 3 describes the role of fundamentals in driving market dynamics, and offers some definitions and taxonomies. Section 4 breaks down volatility into components associated to single fundamental drivers, gives a flavour of the possible patterns, and illustrates some examples. In Section 5, the linkage between price dependence patterns and the properties of market curves is investigated, while Section 6 proposes some market power implications. Discussion and conclusions are in Section 7.

2 Evidence on the volatility of power exchanges

The volatility of electricity markets is commonly associated with the existence of significant rigidities on both the demand and the supply sides of the market (cf. Alvarado and Rajaraman 2000, Stoft 2006). First, there exists wide evidence of short-run demand inelasticity: Considine (1999), Halseth (1999) and Earle (2000) report estimated values between 0.05 and 0.5. This is partly related to electricity being a necessary consumption good and production input in several economic activities, and partly to the fact that the short-run dynamics of the retail price is not affected by wholesale price movements, due to lack of metering. Second, power supply is subjected to capacity constraints, ramp rates, fixed and quasi-fixed costs. These technical constraints require a certain amount of base-load power to be supplied inelastically.[1]

The analysis of volatility in power exchanges has benefited from up-to-date statistical and econometric techniques (Bunn 2004). A very common approach in the relevant literature deals

[1] In their study on the Spanish market Omel, Guerci et al. (2005) have documented that a fraction of power suppliers (*passive sellers*) offer their capacity at a zero ask price.

with various versions of the GARCH model, which entails the estimation of autoregressive and moving average components of the conditional volatility of electricity prices. A cursory review of research in this field includes Bellini (2002), Worthington et al. (2002), Knittel and Roberts (2005), Cavallo, Sapio and Termini (2005), and Guerci et al. (2006). These works, among the many, shed light on a few interesting empirical properties of power exchange volatility, such as the existence of clustering patterns, persistence, mean-reversion, and the so-called *inverse leverage effect* - i.e. stochastic innovations have an asymmetric impact on the volatility level, with positive shocks causing larger increases in volatility (Knittel and Roberts 2005; Hadsell, Marathe and Shawky 2004). Some refinements of the existing econometric techniques have been called upon in order to cope with the observed discontinuities in the temporal behavior of electricity prices. The jump-diffusion process (Merton 1976) has received widespread applications in studies on the power market: see the papers by Johnson and Barz (1999), Escribano et al. (2002), Villaplana (2004) and Knittel and Roberts (2005). As an alternative way to capture the sudden shifts observed in electricity prices, the regime-switching model (Hamilton 1989) posits the existence of multiple price regimes and aims to estimate the associated switching probabilities. The jump-diffusion and the regime-switching approaches have been merged in the works by Ethier and Mount (1998), Huisman and Mahieu (2003), Weron, Bierbrauer and Trueck (2004), Geman and Roncoroni (2006), and De Sanctis and Mari (2007), whose estimates reveal the dual nature of volatility - large probability to persist in any given state, along with extremely sizeable jumps due to transitions between states.

While these works provide satisfactory accounts of the time behavior of the volatility level, there are good reasons to expect that volatility is not homogeneous across price levels. As an interesting clue to this, spikes are more likely when the load and prices are high (Mount 2000). Closer to a demand-supply focus, demand is more elastic when prices soar: energy users are induced to adopt energy saving habits, or to arbitrage between market segments, e.g. by rebalancing their portfolios. Hence, demand responsiveness may not be uniform. Sources of supply rigidity seem to apply differentially across price levels, too. At times of high load, incentives exist for suppliers to withhold capacity in order to exploit the emerging profit opportunities, making the supply responsiveness greater when price is high (see Harvey and Hogan 2001). Alternatively, power suppliers may choose to over-produce as a way to cause network congestion and therefore higher prices. Also, fixed and quasi-fixed costs have a stronger impact on average costs when the quantity traded is small, and so are prices. It is however worth recalling circumstances in which the supply elasticity may actually display an inverse relation with the price. First, in most power markets the supply stack has a rapidly increasing slope, which implies a convex supply curve, as commonly observed. Second, a supplier with market power may be completely indifferent about having marginal units dispatched, as there is no penalty to the supplier of setting offers for marginal units at very high levels (Ausubel and Cramton 1996; Mount, Ning and Oh 2000). This may give rise to a convex supply function too. Finally, in systems based on a large share of hydropower production, wet winters may cause the filling fraction of water reservoirs to reach maxima during the following summer and fall. In order to prevent the reservoirs from flooding, hydroelectric plants are ready to produce at whatever low prices the market may determine (Simonsen 2005).

An empirical approach which seems better suited to understanding the varying impact of rigidities on volatility was adopted by Karakatsani and Bunn (2004), Bottazzi, Sapio and Secchi (2005), and Simonsen (2005), who studied the relationship between measures of volatility and the lagged price level. The former dealt with the UK power market. The latter two papers, focused on the NordPool market, have been subsequently extended by Sapio (2005), Sapio (2008), and Bottazzi and Sapio (2008) to other markets, such as the CalPX, the Dutch

APX, and the French Powernext.

Karakatsani and Bunn (2004) estimated a price model via GLS, including some of the main fundamental variables of the electricity market as regressors, i.e. the demand level and curvature, and the demand-capacity margin. The volatility of the model residuals ϵ was modelled as a power law of the lagged price level:

$$\sqrt{V[\epsilon_t]} = \chi p_{t-1}^{\rho} \tag{1}$$

Estimates of the power exponent for peak hours were positive, whereas estimation on off-peak hours yielded negative values. These patterns, as the authors suggested, could be due to the fact that producers employ different bidding strategies and use different degrees of "arbitrariness" in their behaviors under different relative scarcity.

Simonsen (2005) computed daily logarithmic returns of NordPool wholesale day-ahead prices, sorted them in ascending price order, and after smoothing them through a median filter, fitted a stretched exponential plus a constant:

$$\sqrt{V[\Delta \log p_t]} = k_0 + e^{k_1 p_{t-1}^{k_2}} \tag{2}$$

where k_0, k_1 and k_2 are costant parameters, $V[.]$ the variance operator, and Δ the first difference operator. The resulting estimates showed a negative price level dependence of volatility for the lowest prices, and a much milder dependence for higher prices. Interestingly, the gap between volatilities corresponding to the lowest and the highest price levels amounts to about an order of magnitude.

Bottazzi, Sapio and Secchi (2005), Sapio (2005), Sapio (2008) and Bottazzi and Sapio (2008) modelled the standard deviation of returns as a power function of the lagged price level, after removing time dependencies from returns. Taking natural logarithms, this reads

$$\sqrt{V[\Delta \log p_t]} = \chi p_{t-1}^{\rho} \tag{3}$$

where χ and ρ are constant coefficients, $\Delta \log p_t$ is a sequence of normalized log-returns. The parameter ρ tunes the type of volatility-price pattern. Estimation of the power law scaling model is performed using a binning procedure.[2] Results allow to detect two kinds of volatility-price dependence patterns: (i) monotonically decreasing, with a break beyond a price threshold (NordPool, Powernext), (ii) monotonically increasing (CalPX, APX). Fig. 1 depicts examples of the patterns observed.

These patterns seem to be associated to different degrees of market competition. Positive correlation is observed in cases when the market is likely to have been strongly affected by market power abuse (on-peak sessions in the UK; the CalPX). Negative correlation is instead found in the NordPool, which could be considered fairly competitive thanks to its high share of hydropower, which makes it harder for producers to implement capacity withholding strategies. The research question thus arises as to whether increasing voaltility patterns can be univocally associated to imperfect competition, and decreasing patterns to perfect (or nearly perfect) markets.

[2]For any given time series, data were grouped into equipopulate bins. Next, sample standard deviations of log-returns in each bin were computed, and the logarithm of the sample standard deviations was regressed on a constant and on the logarithm of the median price level within the corresponding bins.

3 Fundamentals, valuations, and shocks

A common way of assessing market performance - vis-à-vis social welfare - is by comparing the quoted price to the underlying market fundamentals. This lies at the heart of e.g. the market efficiency hypothesis in the analysis of financial markets: the price is (informationally) efficient if it includes all information regarding the economic value of the issuer company (see Leroy 1989 for a survey). In commodity markets, the definition of fundamentals may change, depending on the physical characteristics of the commodity. Restructuring the power industry has often been welcomed as a move towards greater transparency, which is but a way to shape the wide-spread claim that market trading will drive prices down to the cost-based value of electrical power. The reference fundamental here is productive efficiency, and in a transparent market the level and variability of power prices closely track the underlying dynamics of input prices. However, demand factors have a relevant role to play: power markets are structurally constrained to clear at all times, as storage is not economically viable. Hence, fluctuations in demand are not smoothed out.

A wider notion of *fundamentals* of an electricity market is thus the set of variables which define the structure of the consumption and production decision problems, resulting in the market curves. Fundamentals refer to both sides of the market, and accordingly can be categorized in *demand fundamentals* and *supply fundamentals*. Demand fundamentals have to do with consumer choice rooted in preferences and bounded by budget and time constraints. Supply fundamentals are instead linked to the technological parameters behind the production choices. Fundamentals are ultimately tied to the valuations of the commodity by users and producers. As a simple way to visualize fundamentals, one can see them as the parameters of demand and supply curves. The location of market curves is indeed affected by the average value of electricity to producers (marginal costs) and consumers (reservation prices). Slopes take up the heterogeneity of valuations across agents. For instance, steep supply stacks denote a wide source diversification among plants.

While the dynamics of a transparent power exchange tracks the evolution of fuel costs, more generally electricity prices reflect the fluctuations in both demand and supply fundamental drivers. Shocks to agent valuations modify the shape of the market curves, and the power price changes to ensure market clearing. The volatility of fundamental drivers is transmitted to power prices, more or less intensely depending on how sensitive is price to fundamentals. Relatedly, it will prove useful to operate a distinction between shocks to fundamentals. Some shocks affect energy valuations uniformly across sellers and purchasers. These types of common shocks can be termed *uniform shocks*. Other shocks, *asymmetric shocks*, only hit individuals or groups of agents, thereby modifying the degree of heterogeneity among valuations; or they are common shocks yielding heterogeneous individual impacts.

Uniform and asymmetric shocks may lead to different volatility outcomes. Shocks that affect valuations uniformly have the same impact on demand and/or supply, regardless of the price level. For example, suppose temperature increases, leading to greater use of air conditioning. If all energy users adjust their electricity consumption levels in the same extent, the demand curve shifts, with no impact on its slope. Shocks that engender asymmetric response among companies or among users will instead change the curve slopes. Think of a positive shock to the marginal cost of a rather inefficient plant: all other cost levels being unaffected, the slope of the supply curve increases. It is worth stressing that in such a case, shocks are amplified by the price level. This insight may be extremely relevant in understanding volatility-price dependence patterns.

In line with the above, let us provide some definitions. A *demand fundamental* δ is a non-price argument of the demand function D. A *supply fundamental* σ is a non-price argument of the supply function S. Demand and supply functions are assumed continuous and differentiable.[3] Formally, demand equals

$$D = D(p, \delta, \sigma) \tag{4}$$

with the standard assumption $\partial D/\partial p \leq 0$, while supply reads

$$S = S(p, \delta, \sigma) \tag{5}$$

with $\partial S/\partial p \geq 0$ as usual. Derivatives with respect to δ and σ are left unrestricted.

Whenever a common shock affects agents uniformly, its impact on either supply or demand is price-invariant:

$$\frac{\partial^2 D}{\partial\delta\partial p} = \frac{\partial^2 D}{\partial p\partial\delta} = 0 \qquad \frac{\partial^2 S}{\partial\sigma\partial p} = \frac{\partial^2 S}{\partial p\partial\sigma} = 0$$

An *asymmetric shock response* yields a price-dependent change in demand or supply:

$$\frac{\partial^2 D}{\partial\delta\partial p} = \frac{\partial^2 D}{\partial p\partial\delta} \neq 0 \qquad \frac{\partial^2 S}{\partial\sigma\partial p} = \frac{\partial^2 S}{\partial p\partial\sigma} \neq 0$$

Unlike in the standard analysis of market curves, demand and supply fundamentals are treated as arguments of the demand and supply functions. More specifically, they are assumed to be random variables, driven by geometric random walks:

$$\dot{\delta} = \delta g_\delta \qquad \dot{\delta} = \delta g_\delta$$

where $g_\delta \approx iid(0, v_\delta)$, $g_\sigma \approx iid(0, v_\sigma)$, with g_δ and g_σ mutually independent.[4] This assumption is useful for the sake of simplicity, as well as for its empirical relevance concerning the dynamics of fuel prices (see Serletis and Herbert 1999, Schwartz and Smith 2000, Asche et al. 2003, Postali and Picchetti 2006).

4 Volatility break-down

It is tempting to try and reduce the volatility of power markets to a function of the volatilities of fuel prices, and furtherly try and disentangle the respective contributions. Yet, electricity prices vary over time in response to changes in other fundamentals too, e.g. demand participation, the level of water reservoirs, and so forth. Thus, more generally one would like to break down the volatility of power markets into individual components, imputable to each one of the most relevant power fundamentals. Benini et al. (2002) have noted how volatility is due to uncertainty in fundamental drivers. A simple framework for doing so is the following.

[3] The author is aware that results obtained using continuous market curves are generally not achievable by taking the continuous limit of discrete curves. Von der Fehr and Harbord (1993) were among the first to show this, using an auction-theoretic approach better suited to deal with the discrete curves observed in reality. However, continuous models, such as the Supply Function Equilibrium model, have offered rather accurate predictions of electricity market outcomes (Baldick, Grant and Kahn 2004). Hence the choice of a continuous approach - besides its analytical convenience.

[4] Time subscripts have been omitted for the sake of notational parsimony. Assuming zero means does not affect the results on volatility.

Consider the demand and supply functions introduced in the previous section (Eq. 4 and Eq. 5 respectively). Because power is not economically storable, supply and demand need to perfectly match at all times. Given a uniform price auction format, the market-clearing condition is that the price p^* ensures $D(p^*, \delta, \sigma) = S(p^*, \delta, \sigma)$. As an outcome, the market-clearing price is a function of all fundamentals:

$$p^* = p^*(\delta, \sigma) \tag{6}$$

Demand and supply evaluated at equilibrium read:

$$D^* = D^*(\delta, \sigma) \tag{7}$$

and

$$S^* = S^*(\delta, \sigma) \tag{8}$$

Having obtained a formulation for the market-clearing price, its rate of change can now be computed, by taking the time derivative $\dot{p^*}$ and dividing it by p^*.[5] Because p^* is a function of fundamentals, the price rate of change is a weighted sum of the rates of change in fundamentals:

$$\frac{\dot{p^*}}{p^*} = \epsilon_{p\delta} g_\delta + \epsilon_{p\sigma} g_\sigma \tag{9}$$

The weight $\epsilon_{px} \equiv \frac{\partial p}{\partial x}\frac{x}{p}$ is the elasticity of the market-clearing price with respect to the generic fundamental x. Now, the variance of the price rate of change can be computed:

$$V\left[\frac{\dot{p}}{p}\right] = \epsilon_{p\delta}^2 v_\delta + \epsilon_{p\sigma}^2 v_\sigma \tag{10}$$

where $V[.]$ is the conditional variance operator. This is a weighted sum of fundamental variances, with weights equal to the squared ϵ_{px} or, as we shall call them from now on, the *variance contribution* of x.[6]

How much a fundamental contributes to the overall variance depends on (i) the variance of the rate of change of that fundamental, and (ii) how sensitive the price is to fluctuations in that fundamental. The proposed volatility break-down sheds light on a distinction between *volatility sources* (fluctuations in fuel prices, water reservoirs, demand participation and so on) and *volatility transmission* (tuned by the price reactiveness to various fundamentals). The former are most likely exogenous with respect to the power price, and as such, unlikely to be affected by the values of power prices in previous market sessions. For instance, it is not very likely that the volatility of the world-wide brent market be influenced by the outcomes of a local power exchange. Having posited random walk dynamics in fundamentals, the variances of fluctuations in fundamentals are constant, and volatility-price patterns emerge only if variance contributions depend on the price level. Intuitively, one can expect the overall pattern to behave approximately like the variance contribution of the most volatile fundamental. If all fundamentals behave in the same way - all increasing or all decreasing with price - this will likely be mirrored in the variance of $\dot{p}/p$. If instead different variance contributions follow

[5]The reader may feel that a discrete-time formulation would be better suited to the periodic nature of power auctions. While we take this criticism, working in continuous time simplifies the analysis.

[6]Notice that the above formulation does not include any covariance term: this holds because of orthogonality between the g_δ and g_σ sequences, as assumed in Section 3.

different patterns, then either one of the patterns prevails, or one could observe some non-monotonic price dependence. All of this is summarized in

Proposition 1.

(i) If variance contributions are all decreasing (increasing) with price, then the price growth variance is decreasing (increasing) with price.

(ii) Let $\partial\epsilon^2_{p\delta}/\partial p < 0$ and $\partial\epsilon^2_{p\sigma}/\partial p > 0$. The price growth variance is decreasing with price if and only if

$$\frac{\partial\epsilon^2_{p\delta}}{\partial p} < -\frac{v_\sigma}{v_\delta}\frac{\partial\epsilon^2_{p\sigma}}{\partial p} < 0 \tag{11}$$

Proof. See Appendix.

The above framework can readily be generalized to allow for multiple fundamentals on both the demand and the supply sides. Accordingly, fundamentals would be denoted by vectors $\underline{\delta}$ and $\underline{\sigma}$, whose components are assumed to be random variables driven by random walks with mutually orthogonal innovations. Demand and supply are then multivariate functions of price and the respective fundamental vectors. Under such additional assumptions, the results summarized above hold qualitatively.

Let us now illustrate the proposed volatility breakdown with some examples.

4.1 A stochastic demand-supply model

Mount's (2000) analysis sought to understand how uncertainty about the electricity load is amplified by the structure of offers to sell power. To this aim, he posited a perfectly inelastic demand function

$$D = a \tag{12}$$

and the following supply function

$$S = c + p^h \tag{13}$$

where $h > 1$ makes it convex. In terms of the taxonomy outlined in Section 3, shocks to the a and c variables are uniform, whereas shocks to h are asymmetric. The market-clearing price reads

$$p = (a - c)^{1/h} \tag{14}$$

In Mount's original model, fluctuations in the market-clearing price were obtained by assuming stochastic dynamics in demand, given the supply parameter. Hereby this model is slightly generalized by allowing for (mutually orthogonal) random walk fluctuations in supply parameters, too. According to Eq. 10, the variance of price rates of change reads

$$V\left[\frac{\dot{p}}{p}\right] = \left(\frac{a}{hp^h}\right)^2 V\left[\frac{\dot{a}}{a}\right] + \left(\frac{c}{hp^h}\right)^2 V\left[\frac{\dot{c}}{c}\right] + (\log p)^2 V\left[\frac{\dot{h}}{h}\right] \tag{15}$$

i.e. it amounts to a weighted sum of fundamental volatilities. The existence of volatility patterns can be detected, as it will be analyzed later, by studying the relationship between volatility contributions and the price level p.

Whether and how volatility contributions are related to price, it is quite immediate in this case. Volatility contributions from parameters a and c are decreasing in p, whereas the contribution of h is increasing in p whenever $p > 1$.

4.2 Symmetric Affine SFE

In the Supply Function Equilibrium (SFE) model, introduced by Klemperer and Meyer (1989), suppliers maximize profits, given the market-wide demand function, by choosing their individual supply curves - i.e. a continuum of price-output pairs. Green and Newbery (1992) and Green (1996) pioneered the use of the SFE model in the analysis of electricity markets.[7]

It its simplest rendition, the SFE model depicts n symmetric oligopolists, who face an affine demand function

$$D = a - bp \tag{16}$$

and produce their output under a quadratic cost function technology, i.e. $c(S) = cS + 0.5hS^2$. Marginal costs are thus affine functions of output. If we assume a and b fluctuate randomly, shocks to a have to be seen as uniform shocks, as they do not affect the slope of demand. Movements in b are instead due to asymmetric shocks.

Solving the associated profit maximization problem, it follows that individual supply functions are themselves affine, and the aggregate supply reads[8]

$$S = n\beta(p - \alpha) \tag{17}$$

where the supply parameters, α and β, are the solutions to the mentioned profit maximization problem, whose optimum values are $\alpha = c$

$$\beta = \frac{n - 2 + \sqrt{(n-2)^2 + 4bh(n-1)}}{2h(n-1)} \tag{18}$$

As the impact of n and b is tuned by p, movements in these parameters can be seen as asymmetric shocks. The market clearing price reads

$$p^* = \frac{a + n\alpha\beta}{b + n\beta} \tag{19}$$

According to Eq. 10, the variance of price returns reads

$$V\left[\frac{\dot{p}}{p}\right] = \left(\frac{a}{(b+n\beta)p}\right)^2 V\left[\frac{\dot{a}}{a}\right] + \epsilon_{pb}^2 V\left[\frac{\dot{b}}{b}\right] + \left(\frac{n\beta c}{(b+n\beta)p}\right)^2 V\left[\frac{\dot{c}}{c}\right] + \epsilon_{ph}^2 V\left[\frac{\dot{h}}{h}\right] \tag{20}$$

with

$$\epsilon_{pb}^2 = \frac{b}{(b+n\beta)p}\left[\left(1 + \frac{n}{z}\right)p - \frac{nc}{z}\right]^2$$

[7] See also Baldick, Grant and Kahn (2004) and references therein.

[8] A crucial assumption here is that, in each period, producers solve the maximization problem after having observed the actual realizations of the stochastic processes driving fundamentals.

$$\epsilon_{ph}^2 = \frac{nh}{b+n\beta}\frac{\partial\beta}{\partial h}\left(1-\frac{c}{p}\right)$$

and

$$z \equiv \sqrt{(n-2)^2 + 4bh(n-1)}$$

These rather cumbersome expressions for the variance contributions all depend on p: consistent with Proposition 2, the price-dependence patterns can be negative (a and c) as well as positive (b and h).

5 Volatility contributions and the reactiveness of demand and supply

In this section, the width of price fluctuations is traced back to movements in market curves. This is done by studying the relationship between variance contributions and the price level, as shaped by the properties of demand and supply. The two following preliminary results, proved in the Appendix, will be useful.

Lemma 1. The slope of the price function with respect to the generic fundamental x can be expressed in terms of the supply and demand slopes, as follows[9]

$$\frac{\partial p^*}{\partial x} = \frac{\frac{\partial D}{\partial x} - \frac{\partial S}{\partial x}}{\frac{\partial S}{\partial p} - \frac{\partial D}{\partial p}} \tag{21}$$

As this Lemma clarifies, the market-clearing price is more sensitive to (i.e. reflects more) a given fundamental if (i) market curves have very different reactions to fluctuations in the fundamental under focus, and (ii) market curves have very similar values of their price-elasticities.[10] While the latter effect is in line with the extant literature, the intuition behind the effect associated with the numerator of Eq. 21 deserves more description. If demand and supply respond in opposite ways to a given random shock - say, demand plummets as supply soars or vice versa - then equilibrium-preserving price adjustments need to be distributed on a rather wide support. Conversely, when demand and supply move in the same direction, any demand movement is compensated by - or compensates - a change in supply. As a result, the price is more likely stable.

Lemma 2. The variance contribution of the generic fundamental x depends on price p as follows:

[9] Or, equivalently,

$$\frac{\partial p^*}{\partial x} = \frac{\frac{\partial S}{\partial x} - \frac{\partial D}{\partial x}}{\frac{\partial D}{\partial p} - \frac{\partial S}{\partial p}}$$

[10] Note that, assuming demand is downward-sloping and supply is upward-sloping, the difference $\frac{\partial S}{\partial p} - \frac{\partial D}{\partial p}$ is always positive, and is null only when both demand and supply are perfectly inelastic.

$$\frac{\partial \epsilon_{px}^2}{\partial p}\frac{p}{\epsilon_{px}^2} = -2\left(1 - \frac{x}{\epsilon_{px}}\Gamma_x\right) \tag{22}$$

where

$$\Gamma_x = \frac{(\frac{\partial^2 D}{\partial x \partial p} - \frac{\partial^2 S}{\partial x \partial p})(\frac{\partial S}{\partial p} - \frac{\partial D}{\partial p}) - (\frac{\partial D}{\partial x} - \frac{\partial S}{\partial x})(\frac{\partial^2 D}{\partial p^2} - \frac{\partial^2 S}{\partial p^2})}{(\frac{\partial S}{\partial p} - \frac{\partial D}{\partial p})^2} \tag{23}$$

The above Lemma indicates that price-dependence patterns in variance contributions are related to

- the response of demand and supply to price signals;
- the response of demand and supply to fundamentals, for a given price level;
- non-linearities in demand and supply functions;
- the uniform or asymmetric nature of shocks.

The Γ_x term subsumes all of these effects in a single key indicator. The magnitude and sign of Γ_x are crucial to assess the kind of volatility-price pattern at work. Whether and how variance contributions depend on p is related to whether market curves are linear or not, and even more importantly, to the "type" of shocks.

The following *price-independence condition* is obtained by setting Eq. 22 equal to zero, and shows how the above mentioned properties of demand and supply have to combine for variance contributions to be unrelated to the price level:

$$\Phi_x \equiv \frac{\frac{\partial^2 D}{\partial x \partial p} - \frac{\partial^2 S}{\partial x \partial p}}{\frac{\partial D}{\partial x} - \frac{\partial S}{\partial x}} - \frac{1}{p} - \frac{\frac{\partial^2 S}{\partial p^2} - \frac{\partial^2 D}{\partial p^2}}{\frac{\partial S}{\partial p} - \frac{\partial D}{\partial p}} = 0 \tag{24}$$

If the left-hand side is greater, the ensuing price-dependence is positive; it is negative otherwise. Given these intermediate steps, one can now study the conditions behind price dependence in volatility contributions. Analyzing the above price-independence condition leads to the following

Proposition 2. Let $\frac{\partial^2 S}{\partial p^2} - \frac{\partial^2 D}{\partial p^2} < -\frac{1}{p}\left(\frac{\partial S}{\partial p} - \frac{\partial D}{\partial p}\right)$.
(i) If shocks are uniform, then variance contributions are decreasing in price.
(ii) Let $\frac{\partial^2 D}{\partial x \partial p}\frac{\partial D}{\partial x} > 0$, $\frac{\partial^2 S}{\partial x \partial p}\frac{\partial S}{\partial x} > 0$, and $\frac{\partial D}{\partial x}\frac{\partial S}{\partial x} < 0$. If the variance contribution of a fundamental is increasing in price, then its fluctuations are due to asymmetric shocks.

Proof. See Appendix.

This proposition states that a sufficient condition for a negative price dependence pattern is that x is hit by uniform shocks only. The reason is that, if all valuations by agents on either side of the market change by the same amount, the underlying shock has a proportionally greater impact on low valuations, which are the marginal ones when prices are low. Moreover, Proposition 2 establishes a necessary condition for positive price dependence: whenever an increasing volatility-price pattern is observed, then we know that asymmetric shocks are at work, but the asymmetric nature of a shock per se does not allow predictions on the emerging variance pattern.

A direct consequence of Proposition 2 is the upcoming

Corollary 1. If demand and supply functions are linear or affine, the variance contribution of a fundamental hit by a uniform shock goes like the inverse of p^2.

Proof. See Appendix.

This result follows from two key premises. First, uniform shocks have a greater proportional impact on low valuations. Second, the price-elasticity of affine demand and supply curve is not costant: affine market curves are very inelastic when $p \approx 0$, and very elastic when $p >> 0$. Hence the market is very unstable when price is low, as predicted by the corollary.

6 Volatility and market power

Given the proposed understanding of volatility-price dependence, hereby some implications for market power analysis are drawn. The seeming association between volatility-price patterns in markets and sessions which might be characterized by strong market power raises the question of whether volatility-price patterns can be mapped into different degrees of competition. The conjecture inspired by the existing evidence is that a competitive market will yield a negative volatility-price dependence, whereas market power will imply a positive correlation. This conjecture is true if one can rule out the remaining cases (i.e. market power associated with a decreasing pattern; competition associated with an increasing pattern). In a sense, this outlines an identification problem.

The unprecedented levels of volatility, witnessed by liberalized electricity markets, have frequently been interpreted as a negative side effect of market power exploitation by power suppliers. Electricity pools are particularly prone to this, due to low demand responsiveness to price signals, as well as by the large minimum efficient scale of power plants (see Wolak and Patrick 1997, Wolfram 1999, Borenstein et al. 2002, Green 2004, Stoft 2006). The occurrence of sharp and short-lived spikes in power exchanges is perhaps the most striking consequence of anti-competitive behaviors. More subtle, yet empirically relevant, are the volatility-price patterns which may convey useful information on market power, too.

An early insight on the determimants of volatility was provided by von der Fehr and Harbord (1993), who shed light on how the market can oscillate between low-demand and high-demand equilibria. Whenever the load-capacity ratio is expected to grow beyond a certain threshold, generating companies respond by playing the high-demand equilibrium, and the highest admissible price results. Otherwise, price offers are kept close to marginal costs. Transitions between equilibria give rise to price variance. On these grounds, Barlow (2002) and De Sanctis and Mari (2007) have made sense of how suppliers can induce price jumps. As suggested by these works, the exercise of market power is associated with a demand-supply interdependency. This is because in order to fully reap the benefits of a dominant position, price-making suppliers have to take duly account of the properties of demand. Other oligopolistic models embody a demand-supply interdependency. In the Cournot model, the Lerner index is inversely related to the price-elasticity of demand. In the affine SFE, the supply slopes chosen by generating companies increase with the demand slope parameter, as suppliers are better off restricting their output when demand is less responsive to price.

Based on these insights, a first way to formalize market power is by letting supply be responsive to demand fundamentals δ,

$$\frac{\partial S_{ic}}{\partial \delta} \neq 0 \tag{25}$$

where the subscript ic denotes variables in an imperfectly competitive market.[11] Further to linking the anti-competitive conduct and volatility in electricity pools, oligopolistic behavior can imply an inelastic supply, which exacerbates volatility. Coupled with the typically low responsiveness of electricity demand, this makes the market particularly noisy. The intuition is that a multi-plant supplier may be completely indifferent about having marginal units dispatched, as there is no penalty to the supplier of setting offers for marginal units at very high levels (Ausubel and Cramton 1996, Mount, Ning and Oh 2000). Incentives for this to happen are however weaker when demand is relatively low, as multi-plant generators are most likely to obtain sales from only few plants, and thus enjoy less leeway. The supply curve under imperfect competition is therefore expected to lay below the perfectly competitive one, and the gap between the two is supposed to increase with the price level. Formally,

$$S_{ic}(p) < S_c(p) \qquad \frac{\partial^2 S_{ic}}{\partial p^2} < 0 \qquad \frac{\partial^2 S_{ic}}{\partial p^2} < \frac{\partial^2 S_c}{\partial p^2}$$

These *market power conditions* are useful for a thorough assessment of the link between market power and volatility.

In assessing the volatility implications of market power, the distinction between uniform and asymmetric shocks proves extremely valuable. Even more so if one wishes to use volatility-price patterns as a mean to identify instances of anti-competitive behaviors. The main question here is whether - and to what extent - given patterns can be univocally associated with market power.

Identification is possible only to the extent that volatility-price patterns can be mapped into market regimes: e.g. if perfect competition is associated with a decreasing volatility-price pattern, and market power with an increasing pattern. As it has been shown, under certain conditions different types of shocks map into different volatility patterns: Proposition 2 suggests that uniform shocks are responsible for negative volatility-price dependence, whereas positive dependence hints at the influence of asymmetric shocks. More precisely, uniform shocks alone can never give rise to increasing patterns, and can only imply decreasing ones.[12] Therefore, in a hypothetical market with only uniform shocks, volatility-price patterns would not allow to identify market power. Identification requires that at least part of the overall market variance be accounted for by asymmetric shocks. Identification is ruled out if all shocks affect valuations uniformly, because the resulting volatility-price pattern would be decreasing under any market regime. This leads to the following

Proposition 3. Identification of market power exercise by means of volatility-price patterns is not possible if all shocks are uniform.

Proof. See Appendix.

[11] Conversely, if all producers are price-takers, they do not use demand information to make their choices, and therefore

$$\frac{\partial S_c}{\partial \delta} = 0$$

[12] At least at the conditions outlined in Proposition 2.

The above proposition illustrates only a necessary condition, in that asymmetric shocks may as well give rise to downward-sloping volatility-price relationships (see Proposition 2). Yet, it is worth looking more closely into the necessary and sufficient conditions for identification.

Toward this aim, note that market power can yield impacts on both the level and composition of volatility. Thanks to market power, electricity producers manage to sell at higher price than under perfect competition. But by Proposition 2, the higher prices play down the variance contribution due to uniform shocks, and scale up the influence of asymmetric shocks. Market power modifies the structure of volatility, making the market more vulnerable to movements involving e.g. the price-elasticity of demand or the supply stack profile.

Further effects are at work. As indicated by market power conditions, the strategies of multi-plant generators - who can at times ask extremely large prices on their latest units - reduce the elasticity of supply vis-a-vis the competitive regime, more so when price is high. This makes it more likely for anti-competitive practices to map into an increasing volatility-price pattern: at low price levels, different market regimes would perform the same, but at higher levels, the strategies of suppliers endowed with market power would imply very inelastic supply curves, which jointly with inelastic demand would enhance volatility. Finally, imperfect competition involves some interdependency between market curves. Here, the guess is that positive volatility-price dependence is the case if, at high prices, producer choice is more sensitive to demand shocks. All of these considerations lead to

Proposition 4. Volatility-price patterns allow to identify market power if, for all demand fundamentals,

$$\Phi_{\delta,c}\Phi_{\delta,ic} < 0 \tag{26}$$

and for all supply fundamentals,

$$\Phi_{\sigma,c}\Phi_{\sigma,ic} < 0 \tag{27}$$

where

$$\Phi_{\delta,c} = \frac{\frac{\partial^2 D_c}{\partial\delta\partial p}}{\frac{\partial D_c}{\partial\delta}} - \frac{1}{p_c} - \frac{\frac{\partial^2 S_c}{\partial p^2} - \frac{\partial^2 D_c}{\partial p^2}}{\frac{\partial S_c}{\partial p} - \frac{\partial D_c}{\partial p}} \qquad \Phi_{\delta,ic} = \frac{\frac{\partial^2 D_{ic}}{\partial\delta\partial p} - \frac{\partial^2 S_{ic}}{\partial\delta\partial p}}{\frac{\partial D_{ic}}{\partial\delta} - \frac{\partial S_{ic}}{\partial\delta}} - \frac{1}{p_{ic}} - \frac{\frac{\partial^2 S_{ic}}{\partial p^2} - \frac{\partial^2 D_{ic}}{\partial p^2}}{\frac{\partial S_{ic}}{\partial p} - \frac{\partial D_{ic}}{\partial p}}$$

$$\Phi_{\sigma,c} = \frac{\frac{\partial^2 S_c}{\partial\sigma\partial p}}{\frac{\partial S_c}{\partial\sigma}} - \frac{1}{p_c} - \frac{\frac{\partial^2 S_c}{\partial p^2} - \frac{\partial^2 D_c}{\partial p^2}}{\frac{\partial S_c}{\partial p} - \frac{\partial D_c}{\partial p}} \qquad \Phi_{\sigma,ic} = \frac{\frac{\partial^2 S_{ic}}{\partial\sigma\partial p}}{\frac{\partial S_{ic}}{\partial\sigma}} - \frac{1}{p_{ic}} - \frac{\frac{\partial^2 S_{ic}}{\partial p^2} - \frac{\partial^2 D_{ic}}{\partial p^2}}{\frac{\partial S_{ic}}{\partial p} - \frac{\partial D_{ic}}{\partial p}}$$

Proof. See Appendix.

The conjecture inspired by the empirical evidence is true if, along with the conditions established by Proposition 4, we have $\Phi_{\sigma,c} < 0$, $\Phi_{\delta,c} < 0$, as well as $\Phi_{\sigma,ic} > 0$, $\Phi_{\delta,ic} > 0$. If so, one can conclude that NordPool, Powernext and the UK market (off-peak) have been fairly competitive, whereas the outcomes of the CalPX, APX and UK market (on-peak) have been affected by anti-competitive behaviors.

7 Concluding remarks

This paper has dealt with the determinants and the market power content of volatility-price dependence patterns in power exchanges, as detected by a number of empirical papers (Karakatsani and Bunn 2004, Bottazzi, Sapio and Secchi 2005, Simonsen 2005). A structural approach has been followed, based on direct modelling of demand and supply curves. The shape and location of market curves change in response to random shocks to individual valuations of the electricity commodity. The price fluctuates in such a way as to preserve market clearing, giving rise to volatility.

Common shocks affecting valuations uniformly determine shifts in demand and/or supply, which magnitudes are independent of the lagged price level. Because their proportional impact on low valuations is higher, volatility is negatively associated with price. Conversely, asymmetric shocks modify the slope of the market curves - thus, under certain conditions, their impact can as well be magnified by high prices, and generate positive correlation between volatility and the price level.

The observed volatility-price patterns can be used to identify market power under certain conditions. Volatility patterns are useful to detect anti-competitive behaviors to the extent that one can univocally map them into market regimes - e.g. increasing patterns with market power, decreasing patterns under perfect competition. Because uniform shocks imply negative volatility-price correlation regardless of the market regime, a necessary condition for identification is that at least some shocks hit valuations asymmetrically. If this has been the case, one can conclude that NordPool, Powernext and the UK market (off-peak) have been fairly competitive, whereas the CalPX, APX and UK market (on-peak) have been affected by anti-competitive behaviors. Further work needs to be done in order to validate these claims empirically.

The analysis performed in this paper suggests two main avenues for future research. *First*, the results on the relevance of asymmetric shocks can provide a novel viewpoint on the issue of fuel diversification. The advantages of energy source diversification as a mean to mitigate volatility and increase the security of supply has been discussed at length in Stirling (1994), Costello (2005), Li (2005), Roques et al. (2006), and Hanser and Graves (2007) among others. Bunn and Oliveira (2007) have analyzed the emergence of diversification and specialization patterns within an agent-based platform. In the proposed framework, diversification can be understood if one considers the prices of different fuels as supply fundamentals. Let us set aside the benefits associated to negative correlation between fuel price innovations, which are ruled out by the orthogonality assumptions of this paper. If multiple fundamentals represent different fuel prices, asymmetric shocks can be seen as shocks affecting only one or some of the fuels. If the fuel mix is diversified enough, then most of the shocks hitting supply fundamentals are likely to be asymmetric. But as from Proposition 3, asymmetric shocks provide a necessary condition for identification of market power. Hence, diversification enables detection of anti-competitive behaviors. Less diversified markets are less likely to be hit by asymmetric shocks. Markets relying on, say, just one power source, are very prone to the impact of common uniform shocks. The NordPool is an example of this, as it relies on hydropower for most of the time. All hydropower plants are going to be affected by shocks to water resources in approximately the same fashion. The bulk of volatility is attributable to uniform shocks which, by Proposition 2, give rise to decreasing volatility-price patterns, regardless of the market regime. The NordPool market might as well be very inefficient, yet volatility-price patterns would not allow detection of this.

A *second* remark, related to the issue of fuel price volatility, hints at a potentially fruitful area of research. The value of the fuel price volatility is sensitive to the balance between (i) the width of the time window used to compute the power price volatility, (ii) the type of fuel contracts included in the generating companies portfolios. With a significant share of long-term fixed rate contracts, the volatility of fuel prices may be very low, unless the time window is wide enough, as to allow changes in the fuel portfolios composition. This issue resembles the problem of macroeconomic price rigidity, as represented by staggering models (see Blanchard and Fischer 1989). Understanding how fuel portfolios are updated over time is another key step towards a thorough assessment of volatility in power exchanges.

8 Appendix

Proof of Proposition 1.

The statement in (i) is immediate. The necessary and sufficient condition in (ii) is proved by taking the derivative of $V[\dot{p}/p]$ - defined in Eq. 10 - with respect to p:

$$\frac{\partial V[\dot{p}/p]}{\partial p} = \frac{\partial \epsilon_{p\delta}^2}{\partial p} v_\delta + \frac{\partial \epsilon_{p\sigma}^2}{\partial p} v_\sigma \tag{28}$$

Suppose we want to check $V[\dot{p}/p] < 0$. All we need is to use the above equality and perform a few simple algebraic steps to isolate $\partial\epsilon_{p\sigma}^2/\partial p$. Eq. 11 results.

Proof of Lemma 1.

Given the equilibrium demand and supply of power, $D^* = D^*(\delta, \sigma)$ and $S^* = S^*(\delta, \sigma)$, consider their partial derivatives with respect to e.g. the supply fundamental σ. By the chain rule, these read

$$\frac{\partial S^*(\delta,\sigma)}{\partial \sigma} = \frac{\partial S(p,\sigma)}{\partial p}\frac{\partial p^*(\delta,\sigma)}{\partial \sigma} + \frac{\partial S(p,\sigma)}{\partial \sigma} \tag{29}$$

$$\frac{\partial D^*(\delta,\sigma)}{\partial \sigma} = \frac{\partial D(p,\delta)}{\partial p}\frac{\partial p^*(\delta,\sigma)}{\partial \sigma} + \frac{\partial D(p,\delta)}{\partial \sigma} \tag{30}$$

Market-clearing requires $D^* = S^*$. Hence, demand and supply at equilibrium must have the same derivative with respect to σ: $\frac{\partial S^*(\delta,\sigma)}{\partial \sigma} = \frac{\partial D^*(\delta,\sigma)}{\partial \sigma}$. Imposing this equality in the above system implies:

$$\frac{\partial S}{\partial p}\frac{\partial p^*}{\partial \sigma} + \frac{\partial S}{\partial \sigma} = \frac{\partial D}{\partial p}\frac{\partial p^*}{\partial \sigma} + \frac{\partial D}{\partial \sigma} \tag{31}$$

This can be solved for $\partial p^*/\partial\sigma$, yielding Eq. 21.

Proof of Lemma 2.

By definition, $\epsilon_{px} \equiv \frac{\partial p^*}{\partial x}\frac{x}{p}$. Up to a constant x, the relationship between ϵ_{px} and the price level can be understood by studying the sign of the derivative of $\frac{\partial p^*/\partial x}{p}$ with respect to p. In doing so, it is useful to take account of Eq. 21, which states how $\partial p^*/\partial x$ varies with demand and supply slopes. Algebra shows that

$$\frac{\partial \epsilon_{px}^2}{\partial p} = 2\epsilon_{px}\frac{x}{p^2}\left[\frac{\partial^2 p^*}{\partial x \partial p}p^* - \frac{\partial p^*}{\partial x}\right] \tag{32}$$

Using the definition of ϵ_{px} we get

$$\frac{\partial \epsilon_{px}^2}{\partial p} = -2\frac{\epsilon_{px}^2}{p} + 2\epsilon_{px}\frac{x}{p}\frac{\partial^2 p^*}{\partial x \partial p} \tag{33}$$

The latter cross derivative can be computed by using Eq. 21. The result is the expression in Eq. 23, which we call Γ_x. Finally, divide both sides of Eq. 34 by ϵ_{px}^2/p to obtain Eq. 22.

Proof of Proposition 2.

Note first that, by setting $\frac{\partial^2 S}{\partial p^2} - \frac{\partial^2 D}{\partial p^2} < -\frac{1}{p}\left(\frac{\partial S}{\partial p} - \frac{\partial D}{\partial p}\right)$, the last two addenda of Phi_x give a negative sum. Part (i) of the Proposition then holds because cross-derivatives $\partial^2 D/\partial x \partial p$ and $\partial^2 S/\partial x \partial p$ are both zero for uniform fundamentals; hence, $\Phi_x < 0$. An an implication, price dependence can only be negative.

Let us now deal with part (ii). Because of the premise, the condition for a positive price dependence holds to the extent that the last two addenda of Φ_x are not too large. If $\frac{\partial^2 D}{\partial x \partial p}\frac{\partial D}{\partial x} > 0$, $\frac{\partial^2 S}{\partial x \partial p}\frac{\partial S}{\partial x} > 0$, and $\frac{\partial D}{\partial x}\frac{\partial S}{\partial x} < 0$, then the left-hand side of Φ_x is always positive. Holding the denominator fixed, the value at the left-hand side is greater, the larger are the cross-derivatives in absolute value. But cross-derivatives are not null only for asymmetric shocks.

Proof of Corollary 1.

Whenever market curves are linear or affine in price, second derivatives with respect to p are null, i.e. $\frac{\partial^2 D}{\partial p^2} = \frac{\partial^2 S}{\partial p^2} = 0$. Plugging the definitions of uniform and relative fundamentals in Eq. 23, it is easy to see that $\Gamma_x = 0$. As a result,

$$\frac{\partial \epsilon_{px}^2}{\partial p}\frac{p}{\epsilon_{px}^2} = -2$$

Therefore, $\epsilon_{px}^2 \sim 1/p^2$. This holds for both demand and supply fundamentals.

Proof of Proposition 3. Suppose that δ and σ are hit by uniform shocks. By Proposition 2, a decreasing pattern of volatility results. This holds whether market power conditions hold or not. Hence, if all shocks are uniform, volatility-price patterns are qualitatively invariant across market regimes, and cannot be used to identify market power.

Proof of Proposition 4. Eq. 26 holds if $\Gamma_{\delta,c}$ and $\Gamma_{\delta,ic}$ have different signs, and similarly for Eq. 27. But by Eq. 24, this implies that perfect and imperfect competition yield different volatility-price patterns. Hence, identification is possible. The expressions for $\Gamma_{\delta,c}$, $\Gamma_{\delta,ic}$, $\Gamma_{\sigma,c}$ and $\Gamma_{\sigma,ic}$ are obtained through substitution of market power conditions and the condition in footnote 11 into Eq. 24. In doing so, consider that, if $\partial S/\partial \delta = 0$, then $\partial^2 S/\partial \delta \partial p = 0$ too.

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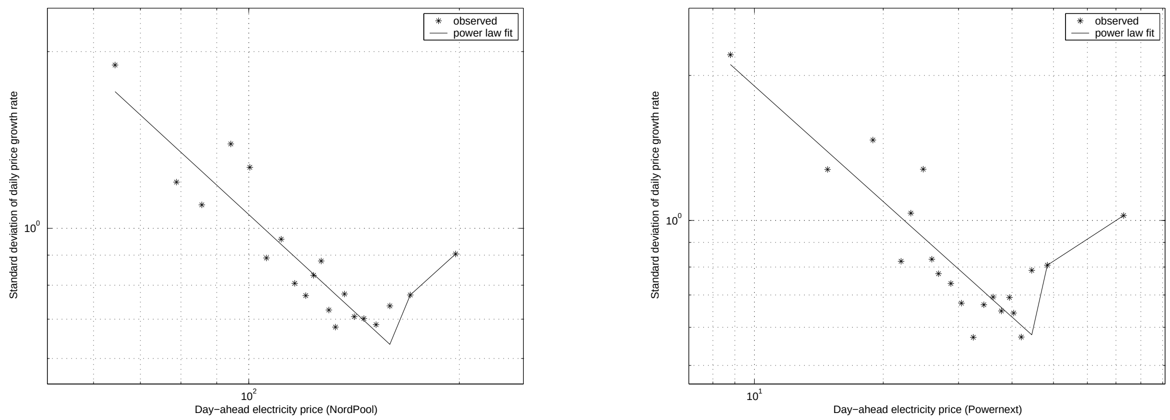

Figure 1: Linear fit of the relationship between log of the conditional standard deviation of normalized log-returns, $\log \sigma_{t-1}(r_t)$, and lagged log-price level $\log(P_{t-1})$. Clockwise: NordPool (3 p.m.), Powernext (10 a.m.), APX (8 p.m.). Similar patterns are observed for other hours within each market. Source: Sapio (2008).