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Localized technological externalities and the geographical distribution of firms *

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Abstract

Using an analytically solvable general equilibrium model, we study how the distribution of economic activities is affected by the trade-off between pecuniary externalities, as dependent on transportation costs, and localized technological externalities, as dependent on inter-regional spillovers. We model localized technological externalities as having a cost saving effect that can be interpreted as a technological advantage, like the presence of inter-firms knowledge spillovers. Under the assumption of capital mobility and labour immobility, we show that whereas decreasing transportation costs, i.e. promoting market openness, leads to sudden agglomeration, increasing inter-regional spillovers, i.e. promoting technological openness, favors a smoother transition between different levels of firms concentration and ultimately leads to a less uneven distribution of welfare.

Keywords: New Economic Geography; Agglomeration; Footloose capital models; Technological externalities; Market and technological openness.

JEL Classification: F12, F15, R12, O3.

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1 Introduction

The skewed nature of the distribution of economic activities found in both developed and developing countries, at any scale, from cities to regions, can be the result of both market mediated interactions, such as labor pooling or intermediate goods availability, and non tradable differences across geographical locations. Beside the effect of trade openness in final markets and increased mobility in factors of productions, like labour and capital, economic agglomeration is plausibly enhanced by the institutional framework, the availability of public infrastructures, higher levels of human and social capital and the local and tacit nature of technical knowledge. Indeed, the abundant presence of agglomerated production clusters away from big cities and main transport systems suggests that forces other than transportation costs, advantages due to larger local demand, or deeper factor markets, are at work. These forces are not exclusively acting in high-tech sectors, like semiconductors or ICT services, but are often pervasive of the entire economy. Analyzing the Italian manufacturing industry, Bottazzi et al. (2008) find that sectors like Food Products, Leather Products or Basic Metal Workings are highly agglomerated and their agglomeration cannot be explained by the presence of transport infrastructures or localized demand. This is not a peculiar aspect of Italian manufacturing, as similar results have been found for the US (Ellison and Glaeser, 1997), France (Maurel and Sedillot, 1999), Germany (Brenner, 2006) and the UK (Devereux et al., 2004).

If localized non-pecuniary advantages are important in describing the observed final outcome of firms locational choice, at least as much as pecuniary market-mediated interactions, it becomes relevant to investigate what aggregate effects can be observed when the institutional, cultural and social barriers which make these advantages local are, at least partially, abated. Among the non-pecuniary factors which presumably provide local advantages in production, a particular attention has been devoted to the possible presence of technological externalities (Marshall, 1920) via localized knowledge spillovers which allow co-located companies to share part of their knowledge sources, reduce innovation costs and gain competitive advantages with respect to firms located elsewhere (see for instance the review in Audretsch and Feldman, 2004). In their re-visitation of the notion of "localized learning" Malmberg and Maskell (2006) notice how the formation of international regulating and supervising authorities, the development of common commercial laws, the internationalization of the capital market and the increased mobility of ideas are likely to alter the geographical reach of knowledge spillovers. Ultimately, how does a variation in the degree of "openness" of these social and technological factors and, in particular, their interaction with the freeness of trade and the commercial "globalization", affect the geographical distribution of firms?

In the present paper we intend to address this question inside the domain of New Economic Geography (NEG). Since Krugman (1991b) this literature has mostly dealt with the effect of pecuniary externalities on the spatial distribution of economic activities. Firms agglomeration arises as the result of a market mediated circular causation: firms locate where demand is high and demand moves where there are many firms. In early models, localized technological externalities were disposed off explicitly as sources of economic agglomeration, essentially because presumed to be particularly prone to measurement problems and modeling sloppiness (Krugman, 1991a, p.53). Notwithstanding the original lack of interest, recent theoretical contributions to NEG have extended the investigation by including non-pecuniary external economies. A number of works have adopted the type of externality introduced in growth models by Grossman and Helpman (1991), postulating an R&D sector with marginal cost of innovation decreasing in the number of existing innovations. In particular Martin (1999) and Martin and Ottaviano (1999) include a Grossman-Helpman type of externality in a footloose capital model where capital is mobile and labour is not. Despite being the endogenous force behind growth, the technological externality is not causing agglomeration because its advantage can be globally exploited. In fact, patents or capital created at a lower cost in the most innovative region can migrate and exert their innovativeness also in the other region. This is not the case of Baldwin and Forslid (2000), who consider the same type of technological externality, but this time with mobile workers and immobile capital. In their model each location has its own R&D sector so that the technological externality can be a source of agglomeration. Moreover they introduce an inter-regional spillover parameter, a sort of "technological openness", which measures to what extent non-pecuniary advantages are location specific or can be shared across regions. Increasing the flow of knowledge between regions reduces the overall cost of innovation, so that location choice is less relevant and agglomerated outcomes less likely. Ultimately, the equilibria of the economy and their desirability in terms of welfare are decided by the interplay between market openness, as dependent on trade costs, and technological openness, as dependent on inter-regional spillovers. The model is however not analytically tractable, and the authors analyze stability only for a predetermined set of benchmark equilibria corresponding to full agglomerated and symmetrically non-agglomerated economies.

In the present paper we advance an analytically tractable general equilibrium model that explicitly accounts for the presence of technological externalities via localized knowledge spillovers. Following Forslid and Ottaviano (2003), we obtain analytical tractability through partial factor immobility. In particular, we impose labor immobility and assume that households are both local workers and global investors, as in Martin and Rogers (1995). In this way the mobile factor is represented by the capital, whose rent is payed to households/shareholders and consumed in the location in which they reside. We believe that this assumption better represents today increased capital mobility, specially in geographical area like the European Union, in which relative regional homogeneity leads to flows of capital which are hardly matched by flow in any other productive factor. Martin and Rogers (1995) and the more recent Dupont and Martin (2006), lacking any self-reinforcing mechanism, reproduce firms agglomeration exclusively via the so called home market effect, induced by the presence of regional differences in the exogenous endowment of factors. Conversely, in our model agglomeration can emerge with a-priori identical locations and is sustained by the endogenous effect of the technological externality. This externality is introduced through a mechanism inspired by Grossman and Helpman (1991). The enhancement of capital creation capability due to scale economies in R&D activity is modeled as a direct effect on final good producers by an increase in operating margins due to a sharing of fixed costs. Our cost sharing assumption represents a technological advantage, like the presence of inter-firms knowledge spillovers. Inside this framework, we introduce an inter-regional knowledge spillover parameter describing the degree of localization of the technological externality, much in line with Baldwin and Forslid (2000). The analytical tractability of our model allows for the explicit derivation of geographical equilibria, defined as those distribution of firms where households do not have incentives to change capital allocation.

We are aware that the empirical literature has not found an agreement on the general functioning, not to mention the specific transmission mechanism, of localized knowledge spillovers. Some issues has been raised about the actual effectiveness of the econometric models and tools adopted in their measurements, see e.g. Breschi and Lissoni (2001b,a). Despite the interest localized knowledge spillovers have attracted, the precise scale and scope of their action is still an open question (Rallet and Torre, 1999), as much as whether they uniquely act as positive externalities or, in the long run, turn out to be negative externalities due to lock-in effects (Boschma, 2005). For these reasons our formulation has all the limits of a toy-model. Nonetheless, it allows us to derive analytical result using a general equilibrium framework.

Our analysis confirms previous findings by Baldwin and Forslid (2000) about the stabilizing nature of inter-regional spillovers: the strongest the link between the two regions the larger the interval of transportation costs which lead to firms equidistribution. We also find that if the effect is strong enough, there exists a smooth equilibrium transition between agglomeration and equidistribution, with partly agglomerated economy for intermediate values of transportation costs. In this case an opening of inter-regional trade does not entail an abrupt reallocation of economic activities nor the hysteresis effect, typical of NEG model, which locks the economy in a core-periphery equilibrium also if higher trade costs are reintroduced.

Welfare analysis reveals that agglomeration can entail lower welfare level for the economic periphery, also when it represents the geographic equilibrium. The huge "welfare gap" existing between core and periphery regions, which often hinders the implementation of trade opening policies, could be reduced and eventually eliminated if market and technological integrations were pursued together. We provide conditions under which either policies are to be preferred.

The rest of the paper is organized as follows. In Section 2 we introduce the model and derive the market equilibrium. In Section 3 we find the geographical equilibria of our economy and analyze their stability by studying how changes in the distribution of capital influence capital rents in both regions. In Section 4 we complete the characterization of geographical equilibria by making explicit their dependence on the parameters ruling trade and technological openness. The welfare analysis is presented in Section 5. In Section 6 we consider the case of non a-priori symmetric regions and discuss how these asymmetries affect our results on geographical equilibria. Section 7 concludes.

2 The model

Consider an economy with two locations, $l = 1, 2$, both populated by L households[1] so that $2L$ is the total number of households. Each household is endowed with labour and capital and supply them inelastically. The economy has a modern and a traditional sector. Whereas the traditional sector supplies an homogeneous good, the modern sector supplies differentiated products. In both sectors production is localized.

Households are "local" workers and "global" consumers, that is, they are immobile and work where they reside, and they can buy goods produced in both locations. Households are also "global" investors, that is, they can supply capital to both locations. Modern goods are traded at a transportation cost which takes the form of an iceberg cost: for one unit of a differentiated good to reach the other region $\tau \in [1, +\infty)$ units must be shipped. As a result $1/\tau \in (0, 1]$ is an index of freeness of trade. Traditional goods and capital are traded at no costs.

Consumption All households have the same preferences and decide how much of the traditional good C_T and of the bundle of modern goods C_M to consume as to maximize a Cobb-Douglas utility function

$$\mathrm{U} = \mathrm{C}_T^{1-\mu}\mathrm{C}_M^{\mu}\,, \tag{2.1}$$

with $\mu \in (0, 1)$. As a result a fraction μ of each household income is spent on C_M and a fraction $(1 - \mu)$ is spent on C_T. The utility of the bundle C_M is of constant

[1] We consider the case of unequally populated regions in Section 6.

elasticity of substitution (CES) type,

$$C_M = \left(\sum_{i=1,N} c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{(\sigma-1)}} \qquad \sigma > 1 \,, \tag{2.2}$$

with c_i the consumption of good i, $i = 1, \ldots, N$. This implies that the N modern goods are substitutes, with a mutual elasticity of substitution equal to σ (cfr. Dixit and Stiglitz, 1977).

Production Each household is endowed with one unit of labour, and there is not an *a priori* distinction between workers of the modern and traditional sector. The traditional sector uses labour as unique input under constant returns to scale with unitary marginal costs. Due to the large number of potential producers, as we shall see at least $2L(1-\mu)$ at equilibrium, this market is perfectly competitive and the traditional good is sold at its marginal cost, which we take as the normalization price of the economy.

We assume a fixed number of firms, N, active in the modern sector. Both capital and labour are used in the production of modern goods. The amount of labour v_i that firm i employs to produce an amount y_i of modern output is given by the usual scale economy cost function

$$v_i = \beta y_i + \alpha_{l_i} \,, \tag{2.3}$$

where β is constant across firms and across locations, and α_{l_i} is the fixed amount of labour necessary to start production. Whereas the marginal productivity of labour is assumed to be constant and equal in the two locations, the fixed amount of labour might depend on the location l_i of firm i. We will assume that α_{l_i} is a function of firms' location, as stated below. Each firm also needs one unit of capital, available at a price r_i. This, at equilibrium, is given by the operating profits

$$r_i = p_i y_i - w_i v_i \,, \tag{2.4}$$

where p_i is the price of good i and w_i is the cost of labour of firm i.

Given the structure of preferences in (2.2) each firm produces a different product. The total number of varieties produced in each location is thus equal to the amount of capital available there.

We assume that each household is endowed with the same amount of capital $N/2L$. Assuming that households maximize their capital revenues, and since capital is moved without costs, their investment choices are symmetric so that each household invests a fraction $1/2L$ of capital in each firm.[2]

The market structure is that of monopolistic competition, that is, each firm maximizes its profits given market demand elasticity and irrespectively of other firms behavior.

Technological externality So far our assumptions closely mimic footloose capital models, such as Martin and Rogers (1995) or Dupont and Martin (2006). Departing from these works, we introduce a localized technological externality, which we model as a term of direct firms interaction not mediated by market forces, like the presence of inter-firms knowledge spillovers. More specifically we assume that the required fixed amount of labour α_l decreases with the number of firms located in a region according to

$$\alpha_l = \frac{\alpha}{\frac{n_l}{N} + \lambda \left(1 - \frac{n_l}{N}\right)}, \tag{2.5}$$

where n_l is the number of firms producing modern goods in location $l \in \{1, 2\}$. Equation (2.5) represents a positive localized externality because the presence of more local producers decreases the production cost. The parameter $\lambda \in (0, 1]$ governs the inter-regional spillover. It describes the degree to which technological externalities are de-localized and firms in one region can enjoy the cost-reducing effect of firms in the other region. Equation (2.5) is analogous to the the production function of the R&D sector as in Grossman and Helpman (1991), used also in a geographic context by Martin (1999); Martin and Ottaviano (1999); Baldwin and Forslid (2000). The latter work also consider the impact of a inter-regional spillover λ as we do here. The difference between this literature and the present work is that we do not directly model a R&D sector and a market for patents, but instead assume that each firm operates an internal R&D unit. The inter-firm knowledge spillover improves the productivity of research and development activities and generates a fixed cost reduction. This reduction increases proportionally with the number of firms, and thus R&D units, located in the same region. The advantage of our formulation is that we are able to solve the model analytically, that is, to find all its geographical equilibria, study their global stability, and perform a comparative dynamics exercise on the space of trade and technological openness parameters.

The marginal decrease of fixed costs, or increase of R&D productivity, with the number of firms located in the same region is dependent on the inter-regional

[2]This is what it is usually assumed in footloose capital models (see Baldwin et al. (2003) p.74) to avoid the complications resulting from household strategic interaction. This assumption is harmless at a geographical equilibrium, that is, at a distribution of firms where either rents in both locations are equal or all firms are in the same location. It is, however, not harmless out of equilibrium and stability results do in general depend on it.

spillover parameter λ that captures the "technological openness" of the all economy. For low values of λ the economy is split in technological separated parts and firms can exclusively exploit advantages derived by co-location in the same region. In particular when $\lambda = 0$, research and development in the two locations are completely segregated and the total fixed cost to be paid per location are constant and equal to $N\alpha$. Unless the modern sector is aggregated in one location each firm pay more than α in labour fixed costs. For positive values of λ, conversely, the economy is technologically integrated, and productivity improving positive externalities are also operating across regions. The higher the λ the higher the effect. Notably, the existence of inter-regional spillovers impacts on fixed costs in two ways. Firstly, it creates a global advantage in reducing production cost of all firms thus increasing modern sector profits. Secondly, it makes location choice less relevant and uneven outcomes less likely. In the extreme case of $\lambda = 1$ the technological externality operates across all firms and each firm pays the same fixed cost α, irrespectively of their geographical distribution.

2.1 Market equilibria

Having specified all the elements of our economy, we derive, for any given fixed distribution of firms, the equilibrium capital rents for both locations. Due to perfect competition and constant returns to scale in the traditional sector, traditional workers wages are equal to prices. Moreover, due to zero transportation costs, prices and thus wages, must be the same in both locations.

Given that workers are not mobile, at an economic equilibrium it should be indifferent to work in the traditional or modern sector. As a result wages in the two sectors are equal. For this reason it is convenient to use wages as the numeràire of the economy.

In order to find equilibrium prices, quantities, and profits of the modern sector, one should in principle analyze each of the N product markets. Nevertheless the problem can be simplified by considering only a representative market for each location. In fact, location by location, firms produce using the same technology, face the same demand (due to the CES utility all goods are substitutes), and the same labour supply. This implies that equilibrium prices, quantities and wages are the same for all the firms in a given location. We can thus consider only two representative product markets, one for each location l.

We proceed as follows. Exploiting the CES preference structure (2.2), we compute consumer demand for the goods produced in each location. We use that all goods are substitutes, that transportation costs impact the consumption of foreign goods, and that the budget constraint depends also on the capital rent. Using the monopolistic competition structure of the market, and knowing consumers demand elasticity, we derive firms pricing behavior. By setting supply equal to demand we are able to determine equilibrium quantities and capital rents as a function of the parameters of the economy and the distribution of firms across locations. The following step is to use capital rents and labour income to determine consumers demand for the traditional good. From here we can derive the traditional good required supply and the labour needed to produce it. As a result we can derive the total labour demand. Next we have to check that the labour market is at equilibrium too. Since there are two segmented labour markets, requiring that both clear amounts to posing a constraint on agents preferences and on the scale of the economy. Finally, we have to impose that capital rents are positive, or otherwise households do not have any incentive to spent their capital endowment.

Let us start from consumers demand. Denote the quantity consumed by a consumer who resides in l of a product produced in m as d_{lm} with $l, m \in \{1, 2\}$. Relative demand under CES utility satisfies

$$\frac{d_{11}}{d_{12}} = \left(\frac{p_2 \tau}{p_1}\right)^{\sigma} \quad \text{and} \quad \frac{d_{22}}{d_{21}} = \left(\frac{p_1 \tau}{p_2}\right)^{\sigma} . \tag{2.6}$$

Agents budget constraints are

$$\begin{cases} \mu I(n_1, n_2) = n_1 d_{11} p_1 + n_2 d_{12} p_2 \tau \, , \\ \mu I(n_1, n_2) = n_1 d_{21} p_1 \tau + n_2 d_{22} p_2 \, , \end{cases} \tag{2.7}$$

where $I(n_1, n_2)$ is the income of each consumer which is given by his wage, normalized to 1, plus his share of capital rent, still unknown, as depending on the geographical distribution of firms (n_1, n_2). Using the previous equations to find demands leads to

$$\begin{aligned} d_{11} &= \frac{\mu I(n_1, n_2)}{n_1 p_1 + n_2 p_1^{\sigma} p_2^{1-\sigma} \tau^{1-\sigma}} \, , \qquad & d_{12} &= \frac{\mu I(n_1, n_2) \tau^{-\sigma}}{n_1 p_1^{1-\sigma} p_2^{\sigma} + n_2 p_2 \tau^{1-\sigma}} \, , \\ d_{22} &= \frac{\mu I(n_1, n_2)}{n_1 p_1^{1-\sigma} p_2^{\sigma} \tau^{1\sigma} + n_2 p_2} \, , \qquad & d_{21} &= \frac{\mu I(n_1, n_2) \tau^{-\sigma}}{n_1 p_1 \tau^{1-\sigma} + n_2 p_1^{\sigma} p_2^{1-\sigma}} \, . \end{aligned} \tag{2.8}$$

Given the market structure of monopolistic competition, each firm, knowing consumers inverse demand, sets the output so that marginal revenues are equal to marginal costs. In location l this gives

$$p_l \left(1 + \frac{1}{\varepsilon}\right) = \beta \, , \tag{2.9}$$

where $\varepsilon = \partial \log c / \partial \log p$ is the demand elasticity, and we have used the fact that wages are normalized to one. Given (2.2), as long as the number of commodities N is large (see Dixit and Stiglitz, 1977, for the details), it holds that

$$\varepsilon = -\sigma,$$

which together with (2.9) implies

$$p_l = \beta \frac{\sigma}{\sigma - 1} \, . \tag{2.10}$$

Equating, location by location, demand and supply, we get

$$\begin{cases} y_1 &= Ld_{11} + Ld_{21}\tau \\ y_2 &= Ld_{12}\tau + Ld_{22} \, , \end{cases} \tag{2.11}$$

where, due to iceberg costs, for one unit of foreign output to be consumed τ units must be imported. Using the demand derived in (2.8) and substituting the expression for prices in (2.10) we can easily solve for market equilibrium quantities. Introducing the freeness of trade parameter $\phi = \tau^{1-\sigma}$, with $\phi \in (0, 1]$, market equilibrium quantities read

$$\begin{cases} y_1 &= \mu I(n_1, n_2) L \, \frac{\sigma - 1}{\beta\sigma} \left(\frac{1}{n_1 + n_2\phi} + \frac{\phi}{n_1\phi + n_2} \right) \, , \\ \\ y_2 &= \mu I(n_1, n_2) L \, \frac{\sigma - 1}{\beta\sigma} \left(\frac{\phi}{n_2\phi + n_1} + \frac{1}{n_2 + n_1\phi} \right) \, . \end{cases} \tag{2.12}$$

The revenue of a firm in l is given by

$$p_l y_l = \beta \frac{\sigma}{\sigma - 1} y_l \, , \tag{2.13}$$

and, using (2.4), its capital rent is

$$r_l = \frac{\beta}{\sigma - 1} y_l - \alpha_l(n_1, n_2) \, . \tag{2.14}$$

Let $x = n_1/N$ be the fraction of firms (or capital) in location 1, so that $n_2 = (1 - x)N$. Thus rents payed by each firm can be written as[3]

$$\begin{cases} r_1(x) &= \frac{\mu I(x) L}{N\sigma} \left(\frac{1}{x + (1-x)\phi} + \frac{\phi}{x\phi + (1-x)} \right) - \frac{\alpha}{x + \lambda(1-x)} \, , \\ \\ r_2(x) &= \frac{\mu I(x) L}{N\sigma} \left(\frac{1}{x\phi + (1-x)} + \frac{\phi}{x + (1-x)\phi} \right) - \frac{\alpha}{1 - x + \lambda x} \, , \end{cases} \tag{2.15}$$

Notice that this is still an implicit equation because $I(x)$ is a function of r_1 and r_2. In fact, with wages normalized to one, and remembering that each households invests a fraction $1/2L$ of capital in each firm, we have

$$I(x) = 1 + \frac{N}{2L}(x r_1(x) + (1 - x) r_2(x)) \, . \tag{2.16}$$

[3] Throughout the paper, without loss of generality, we consider x to be a real number in the interval $[0, 1]$.

Solving (2.15) for $r_1(x)$ and $r_2(x)$ we get

$$\begin{cases} r_1(x) = I(x)\dfrac{\mu L}{N\sigma}\left(\dfrac{1}{x+(1-x)\phi} + \dfrac{\phi}{x\phi+(1-x)}\right) - \dfrac{\alpha}{x+\lambda(1-x)} \\ r_2(x) = I(x)\dfrac{\mu L}{N\sigma}\left(\dfrac{1}{x\phi+(1-x)} + \dfrac{\phi}{x+(1-x)\phi}\right) - \dfrac{\alpha}{1-x+\lambda x}\ , \end{cases} \tag{2.17}$$

where

$$I(x) = 1 + R(x) = 1 + \frac{\mu - \frac{N}{2L}\sigma\alpha\left(\frac{x}{x+\lambda(1-x)} + \frac{1-x}{1-x+\lambda x}\right)}{(\sigma-\mu)}\ , \tag{2.18}$$

and $R(x)$ is the per-capita capital rent.

Equations (2.17-2.18) define an equilibrium of our economy for every geographical distribution x provided that, location by location, modern sector firms' labour demand is always smaller than L. The condition will be imposed in the following where we summarize our findings

Proposition 2.1. *Given the scale of the economy $S = N/L$, if it holds*

$$S < \tilde{S} = \frac{\mu + \sigma - 2\mu\sigma}{\alpha\sigma(1-\mu)}\ , \tag{2.19}$$

then for any geographical distribution $x \in [0,1]$ the global market for the traditional good, the N global markets for the modern goods, and the two local labour markets clear with location rents and total per capita rents given by (2.17) *and* (2.18), *respectively.*

Proof. The proof that capital rents are as in (2.17) is in the text. In order to show that under condition (2.19) both local labour markets clear, notice that under (2.5) total fixed costs in each location are non-decreasing. As a result the maximum amount of labour used by the modern sector in location $l = 1, 2$ is achieved when all firms are located in l. Consequently condition (2.19) is found by imposing that the demand for labour of the modern sector when it is totally aggregated in one region is lower than L. $\tilde{S}$ is the value of S such that the demand for labour employed in the modern sector, when fully agglomerated, is exactly L. □

Finally to make certain that households do actually invest in the modern sector we impose that the per-capita rent of the economy is positive. We have the following

Lemma 2.1. *The per-capita rent of the economy $R(x)$ is positive for any value of the transportation cost τ and of the inter-regional spillover λ provided that*

$$S = \frac{N}{L} < \bar{S} = \frac{\mu}{\alpha\sigma} \, . \tag{2.20}$$

Proof. The per-capita rent $R(x)$ in (2.18) does not depend on ϕ but does depend on λ. The given value of $\bar{S}$ has been found by equating to zero the minimum values of per-capita rents, obtained when $\lambda = 0$ and firms are not agglomerated, or $x \in (0, 1)$. □

In what follows we shall assume that the scale S of the economy is such that both constraints in (2.19) and (2.20) hold. Since it is always possible to order the values of $\tilde{S}$ and $\bar{S}$, this is equivalent to assume that $S < \min\{\tilde{S}, \bar{S}\}$.[4] Obviously the foregoing condition can only be met when $\tilde{S} > 0$, or, in terms of the preference for the modern goods, $\mu < \sigma/(2\sigma - 1)$, so that the share of income spent on modern goods is not too big.[5] Summing up, for any given elasticity of substitution, provided that preferences for modern goods are not too strong, there always exists a range of firms-to-households ratios such that markets in both locations are at equilibrium and capital rents are non-negative.

The dependence of equilibrium capital rents on the geographical distribution of firms is due to both pecuniary and technological externalities. The effect of the former goes via the sum of local and foreign demand, that is the part of capital rents in (2.17) which depends on ϕ. When concentration of local firms is low, each firm faces a high local demand and makes high profits. As the concentration of local firms increases, a higher competition lowers the profits coming from the local demand which, due to positive transportation costs, are not fully compensated by an increased foreign demand. Thus pecuniary externalities have a negative effect on agglomeration. The effect becomes stronger as transportation costs increase, that is, higher τ, or lower ϕ.

Technological externalities influence equilibrium capital rents both locally and globally. The local effect is due to the direct dependence of firm fixed costs on the geographical distribution of firms, as given by the last term of both expressions in (2.17). The higher the concentration of firms in location l, the lower the fixed costs and the higher the capital rent of firms located there. The global effect of the technological externality is due to the dependence of local rents in (2.17) on

[4]By simple computations one can show that $\bar{S} = \min\{\tilde{S}, \bar{S}\}$ when $\mu \in [0, 1/2]$ or $\mu \in (1/2, 1]$ and $\sigma < \mu^2/(2\mu - 1)$ and $\tilde{S} = \min\{\tilde{S}, \bar{S}\}$ otherwise.

[5]This is the same condition on consumers preferences found in e.g. Forslid and Ottaviano (2003). Notice that the condition is always satisfies when $\mu \leq 1/2$ and requires $\sigma < \mu/(2\mu - 1)$ when $\mu > 1/2$.

global capital rents $R(x)$ and acts as a sort of multiplier. In fact, the geographical distribution has first an impact on total fixed costs, and thus on total capital rents $R(x)$, which, in turn, have a wealth effect on consumers demand, thus affecting the capital rent each location. An increase in the concentration of firms, lowers total fixed costs payed by all firms, increases total capital rents, increases households wealth, increases total demand and, in turn, increases capital rents further in a multiplier fashion. Notice that whereas the local effect increases the rent in a given location through local agglomeration, the global effect increases capital rents in both locations, no matter where firms do actually agglomerate. As we shall see the overall effect of these two forces and its strength depend both on the inter-regional spillover λ and on the freeness of trade τ. In general, when λ is low (high) the technological externality is (not) localized and its variability with the local concentration is high (low). In the extreme case, $\lambda = 1$, both regions have equal benefits, irrespectively of the geographical distribution of firms.

3 Geographical equilibria

We assume that capital moves from one location to the other following the rent difference $\Delta(x) = r_1(x) - r_2(x)$. When $\Delta(x)$ is positive capital flows from location 2 to location 1, the other way round when $\Delta(x)$ is negative. The capital dynamics can be derived as the solution of the following dynamical system

$$\frac{dx}{dt} = \begin{cases} \max\{0, \Delta(x)\} & \text{if } x = 0 \\ F(\Delta(x)) & \text{if } 0 < x < 1 \\ \min\{0, \Delta(x)\} & \text{if } x = 1 \end{cases} \tag{3.1}$$

where F is a strictly increasing differentiable function with $F(0) = 0$. Despite different functions F correspond to different trajectories, their general properties allow to identify the interior fixed points of the dynamics in (3.1) as the solution of the equation $\Delta(x) = 0$ and to investigate their global stability looking at the sign of $\Delta'(x)$. The definition of the dynamical system at the border is due to the fact that the variable x is constrained in the interval $[0, 1]$ so that 0 and 1 are other possible fixed points, depending on the sign of $\Delta(0)$ and $\Delta(1)$ respectively. We have the following

Definition 3.1. An *interior geographical equilibrium* $\hat{x} \in (0, 1)$ is an asymptotically stable fixed point of (3.1), i.e. $\Delta(\hat{x}) = 0$ and $\Delta'(\hat{x}) < 0$. A *border geographical equilibrium* $\hat{x} \in \{0, 1\}$ is a border asymptotically stable fixed point of (3.1), that is, $x = 0$ is a border geographical equilibrium if $\lim_{x\to 0^+} \Delta(x) < 0$, $x = 1$ is a border geographical equilibrium if $\lim_{x\to 1^-} \Delta(x) > 0$.

We shall name an interior geographical equilibrium a *non-agglomerated economy* (NAG) when $\hat{x} = 1/2$, a *partially agglomerated economy* (PAG) when $\hat{x} \neq 1/2$,

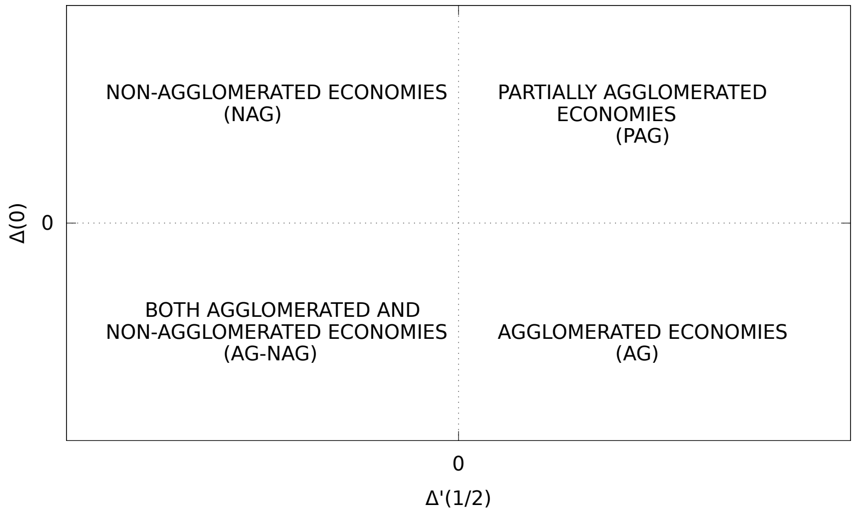

Figure 1: Existence of different type of stable geographical equilibria in the plane $(\Delta'(1/2), \Delta(0))$.

an *agglomerated economy* (AG) when at least one border is a geographical equilibrium.

Let us start from the existence of geographical equilibria corresponding to NAG and AG. Given the symmetry of the economy it always holds $\Delta(1/2) = 0$. As a result $1/2$ is locally asymptotically stable, so that NAG occurs upon having an initial condition close enough to $1/2$, whenever $\Delta'(1/2)$ is negative. The same symmetry also implies that the sign of $\Delta(0)$ is the opposite of the sign of $\Delta(1)$, so that either both or none of the two borders are geographical equilibria. In particular AG occurs whenever $\Delta(0)$ is negative.

NAG and AG are not the only possible long-run outcomes. Indeed there might be other interior fixed points leading to PAG. Given our formulation of the technological externality we are able to identify these other fixed points and study their global stability analytically. Moreover, given the functional form of $\Delta(x)$, it turns out that the knowledge of the signs of $\Delta(0)$ and $\Delta'(1/2)$ is enough also to characterize the existence and the stability of all the other geographical equilibria of our economy, as we shall show in the following proposition. These results are summarized in Fig. 1. Specific examples of the functional forms of the rent difference $\Delta(x)$ are instead given in Figure 2.

Proposition 3.1. *Assume $\tilde{S} > 0$ and $S < min\{\tilde{S}, \bar{S}\}$. Define x^-, x^+ as*

$$x^{\pm} = \frac{1}{2}\left(1 \pm \sqrt{\frac{a}{a+2b}}\right) ,$$

where

$$
\begin{aligned}
a &= (1-\phi)^2(\alpha N\sigma(1-\lambda) - \mu L(1+\lambda)^2 + 2\alpha N\mu\lambda) + 4\alpha N\phi(1-\lambda)(\sigma-\mu)\,, \\
b &= \mu\lambda(1-\phi)^2(2L-\alpha N) - 2\alpha N\phi(1-\lambda)(\sigma-\mu)\,.
\end{aligned}
$$

Consider the trajectories of the dynamical system (3.1) *given initial condition* $x(t = 0) = x_0$. *It holds that*

- *if* $\Delta(0) = \Delta'(1/2) = 0$, *all* $x \in [0,1]$ *are stable, but not asymptotically stable, fixed points of* (3.1). *The geographical equilibrium is equal to the initial condition* x_0.
- *if* $\Delta(0) \leq 0$ *and* $\Delta'(1/2) \geq 0$, *but not* $\Delta(0) = \Delta'(1/2) = 0$, *only AG occurs. The geographical equilibrium is either* 0 *or* 1 *depending on initial conditions, the former when* $x_0 < 1/2$ *the latter when* $x_0 > 1/2$.
- *if* $\Delta(0) \geq 0$ *and* $\Delta'(1/2) \leq 0$, *but not* $\Delta(0) = \Delta'(1/2) = 0$, *only NAG occurs. The geographical equilibrium is* $1/2$ *irrespectively of the initial condition.*
- *if* $\Delta(0) < 0$ *and* $\Delta'(1/2) < 0$ *both NAG and AG occur. The geographical equilibrium is* 0 *when* $x_0 < x^-$, $1/2$ *when* $x_0 \in (x^-, x^+)$, *and* 1 *when* $x_0 > x^+$.
- *if* $\Delta(0) > 0$ *and* $\Delta'(1/2) > 0$ *only PAG occur. The geographical equilibrium is* x^- *when* $x_0 \in [0, 1/2)$, *and* x^+ *when* $x_0 \in (1/2, 1]$.

Proof. Using the expressions in (2.17), after some simplifications, one finds

$$
\Delta(x) = \frac{(1-2x)((2a+4b)(x^2-x)+b)}{2(\sigma-\mu)(x+\phi(1-x))(x\phi+1-x)(x+\lambda(1-x))(x\lambda+1-x)}\,, \quad (3.2)
$$

where

$$
\begin{aligned}
a &= (1-\phi)^2(\alpha N\sigma(1-\lambda) - \mu L(1+\lambda)^2 + 2\alpha N\mu\lambda) + 4\alpha N\phi(1-\lambda)(\sigma-\mu)\,, \\
b &= \mu\lambda(1-\phi)^2(2L-\alpha N) - 2\alpha N\phi(1-\lambda)(\sigma-\mu)\,.
\end{aligned}
$$

We can restrict our analysis to the signs and derivatives of $N(x)$, the numerator of $\Delta(x)$. Indeed according to our hypothesis $\sigma > \mu$ and $\phi, \lambda \in (0, 1]$ so that the denominator of $\Delta(x)$ is always positive. This implies that both the signs and zeros of the rent difference are equal to the signs and zeros of its numerator, and that the sign of the derivative of the rent difference evaluated at its zeros is equal to the sign of the numerator rent difference derivative evaluated at the same points.

First notice that $N(x)$ is a cubic function symmetric around $x = 1/2$. It follows that the dynamics in (3.1) has at most 5 different fixed points: the two extreme values 0 and 1 and the three zeros of $N(x)$. Its parametrization in terms of a and b has been chosen so that

$$a = N'(1/2) = \Delta'(1/2) \tag{3.3}$$
$$b = N(0) = \Delta(0)\,. \tag{3.4}$$

As a result when a and b are both positive, the derivative of the rent difference computed at $x = 1/2$ is positive, the rent difference at $x = 0$ is positive and, by symmetry, negative at $x = 1$. Given that $N(x)$ is a cubic polynomial, it must cross zero in two other points in the interval $[0, 1]$, which we name x^+ and x^- and are symmetrically located around 1/2. Moreover the marginal rent difference at both x^+ and x^- must be negative, so that these points are indeed geographical equilibria corresponding to PAG. It turns out that this is the only sign combination for which the economy is in a PAG status, as can be easily checked by repeating the same reasoning for all the other sign combinations of a and b. The signs of $N(x)$, and thus also of dx/dt, shows that the system converges to x^- for initial conditions $x_0 \in (0, 1/2)$ and to x^+ for $x_0 \in (1/2, 1)$. All other results follow along the same lines, see also Fig. 2.

The asymmetric interior equilibria x^+ and x^- are the solutions of the second order equation $(2a + 4b)(x^2 - x) + b = 0$, that is,

$$x^\pm = \frac{1}{2}\left(1 \pm \sqrt{\frac{a}{a+2b}}\right)\,.$$

Notice that they are in the interval $(0, 1)$ only when a and b have the same signs. Finally notice that due to (3.3-3.4) all the conditions about the existence and basin of attraction of geographical equilibria can be given in terms of $N(0)$, and thus $\Delta(0)$, and $N'(1/2)$, and thus $\Delta'(1/2)$, rather than a and b.

□

The previous proposition implicitly states that, apart from the non generic case when both $\Delta(0)$ and $\Delta'(1/2)$ are zero, there are at most 5 different geographical equilibria, the two *border* equilibria 0 and 1 and the three *interior* equilibria 1/2, x^+, and x^-. The two *interior* equilibria x^+ and x^- exist only when the marginal rent difference at 1/2, $\Delta'(1/2)$, and the rent difference of an agglomerated economy, $\Delta(0)$ and $\Delta(1)$, have the same sign.

The existence and the basins of attraction of geographical equilibria can be established from the signs of $\Delta(0)$ and $\Delta'(1/2)$ since they completely characterize the zeros and the sign of $\Delta(x)$. When $\Delta(0)$ and $\Delta'(1/2)$ have opposite signs, the long-run outcome is either NAG, when $\Delta(0) \geq 0$, or *AG*, when $\Delta(0) < 0$. In the last case the outcome depends on the initial condition x_0. When $\Delta(0)$ and $\Delta'(1/2)$ have the same sign also the two fixed points x^+ and x^- exist. They are stable and the economy is in a PAG status, that is, partially agglomerated in either one or

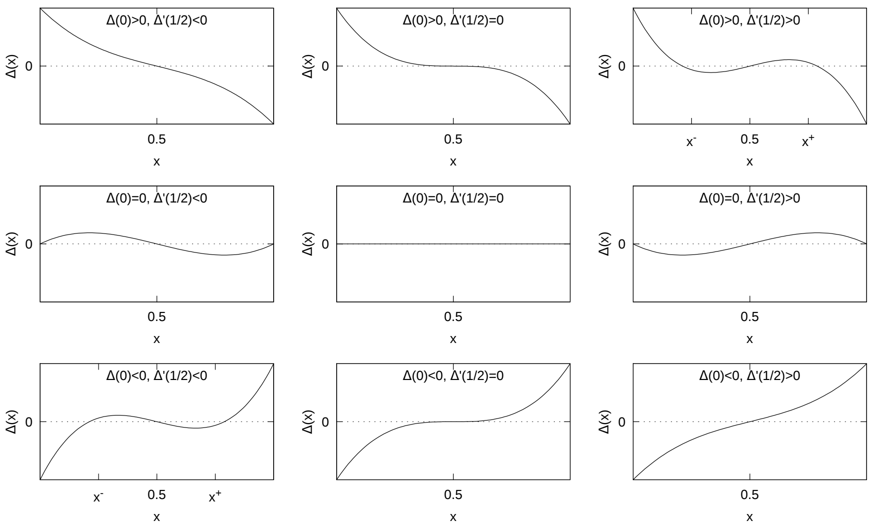

Figure 2: Capital rent differences for all the sign combinations of the coefficients $\Delta'(1/2)$ and $\Delta(0)$.

the other, when $\Delta(0)$ and $\Delta'(1/2)$ are both positive. Conversely when $\Delta(0)$ and $\Delta'(1/2)$ are both negative, they are unstable, and the economy is either AG or NAG.

Proposition 3.1 also shed lights on the possible transitions between the different type of geographical equilibria as changes in the parameters of the economy occur. We analyze the 5 different possible transitions (both coefficients equal to zero, one of the two equal to zero while the other is positive, the same when the other is negative) with the help of Fig. 2. In the special case when both $\Delta'(1/2)$ and $\Delta(0)$ are zero, rents are equal for any distribution of capital, that is, there is a continuum of interior equilibria. All these equilibria are stable but not asymptotically stable. When $\Delta(0)$ is positive and $\Delta'(1/2)$ changes sign from negative to positive the economy transits from NAG to PAG. In particular, when $\Delta'(1/2) = 0$ NAG occurs. Otherwise, when $\Delta(0)$ is negative and $\Delta'(1/2)$ changes sign, AG always occurs whereas NAG occurs for $\Delta'(1/2)$ negative and vanishes otherwise. Using the language of bifurcation theory, the two phenomena are known respectively as a sub-critical and super-critical pitchfork bifurcation. Importantly, in the former case the transition between non agglomeration and agglomeration is smooth and does not exhibit the typical hysteresis phenomenon associated with the latter. The same type of argument is valid when it is $\Delta(0)$ that changes its sign. The transition between full agglomeration and non full agglomeration exhibits hysteresis when $\Delta'(1/2) > 0$, and is smooth otherwise.

4 The effect of trade and technological openness

In this section we are concerned with the influence of the inter-regional spillover λ and of the freeness of trade ϕ on the possible geographical equilibria of our economy. We assume consumer preferences, costs structure, number of firms and households as given and subject to the two restrictions (2.19) and (2.20). For this purpose, we translate the geographical equilibria conditions given above in Proposition 3.1 in terms of the more directly interpretable policy parameters λ and ϕ. This is the content of the following

Proposition 4.1. *Consider the rent difference* $\Delta(x)$ *as in* (3.2)*, and assume that* $\tilde{S} > 0$ *and* $S < min\{\tilde{S}, \bar{S}\}$*. The functions*

$$\phi^a(\lambda) = \left(1 + \Gamma(\lambda) - \sqrt{\Gamma^2(\lambda) + 2\Gamma(\lambda)}\right)^{\frac{1}{\sigma-1}}, \tag{4.1}$$

$$\phi^b(\lambda) = \left(1 + \Theta(\lambda) - \sqrt{\Theta^2(\lambda) + 2\Theta(\lambda)}\right)^{\frac{1}{\sigma-1}}, \tag{4.2}$$

with

$$\Gamma(\lambda) = \frac{\alpha N(1-\lambda)(\sigma-\mu)}{\mu\lambda(2L - \alpha N) + \frac{1-\lambda}{2}(\mu L(1-\lambda) - N\alpha\sigma)},$$

$$\Theta(\lambda) = \frac{\alpha N(1-\lambda)(\sigma-\mu)}{\mu\lambda(2L-\alpha N)}$$

map the interval $(0,1]$ *in the interval* $(0,1]$ *such that*

$$\begin{aligned} \Delta'(1/2) \gtreqless 0 \quad &\Leftrightarrow \quad \phi \gtreqless \phi^a(\lambda) \\ \Delta(0) \gtreqless 0 \quad &\Leftrightarrow \quad \phi \lesseqgtr \phi^b(\lambda)\,. \end{aligned} \tag{4.3}$$

Moreover it holds $\phi^b(1) = \phi^a(1) = 1$, $\phi^b(0) = 0$ *and, when* $\lambda \gtreqless \tilde{\lambda} = 1 - \frac{S}{\bar{S}}$, $\phi^b(\lambda) \gtreqless \phi^a(\lambda)$*.*

Proof. As with the previous proof we characterize the signs of $\Delta(x)$ and $\Delta'(x)$ by looking at the signs of its numerator $N(x)$ and its derivative $N'(x)$. The formula for $N'(1/2)$ and $N(0)$ found in (3.3-3.4) are polynomial of second order in ϕ. The equations $\Delta'(1/2) = 0$ and $\Delta(0) = 0$ can thus be solved in terms of ϕ giving

$$\phi^a_\pm(\lambda) = \left(1 + \Gamma(\lambda) \pm \sqrt{\Gamma^2(\lambda) + 2\Gamma(\lambda)}\right)^{\frac{1}{\sigma-1}},$$

$$\phi^b_\pm(\lambda) = \left(1 + \Theta(\lambda) \pm \sqrt{\Theta^2(\lambda) + 2\Theta(\lambda)}\right)^{\frac{1}{\sigma-1}},$$

with

$$\Gamma(\lambda) = \frac{\alpha N(1-\lambda)(\sigma-\mu)}{\mu\lambda(2L-\alpha N)+\frac{1-\lambda}{2}(\mu L(1-\lambda)-N\alpha\sigma)},$$

$$\Theta(\lambda) = \frac{\alpha N(1-\lambda)(\sigma-\mu)}{\mu\lambda(2L-\alpha N)}.$$

Provided that $S = N/L < \bar{S}$ one can show that both $\Gamma(\lambda)$ and $\Theta(\lambda)$ are positive for any value of λ. This, in turn, implies that both ϕ_+^a and ϕ_+^b are larger than 1 for every λ in $[0,1]$, whereas ϕ_-^a and ϕ_-^b are two functions from $[0,1]$ to $[0,1]$ and corresponds to $\phi^a(\lambda)$ and $\phi^b(\lambda)$ given in (4.1-4.2). Taking limits to 1 and 0 it holds $\phi^b(1) = \phi^a(1) = 1$ and $\phi^b(0) = 0$. Furthermore substituting for $\tilde{\lambda}$ in $\Theta(\lambda)$ and $\Gamma(\lambda)$ it is immediate to see that $\phi^a(\tilde{\lambda}) = \phi^b(\tilde{\lambda})$ and that $\phi^b(\tilde{\lambda}) \gtreqless \phi^a(\tilde{\lambda})$ when $\lambda \gtreqless \tilde{\lambda}$. Notice at last that since $S < \bar{S}$ it is $\tilde{\lambda} \in (0,1)$. □

The dependence of geographical equilibria on ϕ and λ is the consequence of their effect on $\Delta'(1/2)$ and $\Delta(0)$ which, in turn, depends on the trade-off between negative pecuniary and positive technological externalities. Figs. 3-4 bring together Propositions 3.1 and 4.1 and show for which values of the policy parameters λ and ϕ AG, NAG, or PAG are observed. Despite in Fig. 3 the curves $\phi^a(\lambda)$ and $\phi^b(\lambda)$ are plotted for specific values of the economy parameters (L, N, μ, σ and α), their behavior and, in particular, the regions they identify are general properties of the model. Indeed it always holds that $\phi^b(0) = 0$, $\phi^b(1) = \phi^a(1) = 1$, $\phi^b(\tilde{\lambda}) \gtreqless \phi^a(\tilde{\lambda})$ when $\lambda \gtreqless \tilde{\lambda}$, and $\tilde{\lambda} \in (0,1)$. For the same reason the "bifurcation" phenomena illustrated in Fig. 4 are also general.

For high values of ϕ, $\phi > \phi^b(\lambda)$, the technological externality dominates, agglomeration on either sides is a geographical equilibrium and the long run dynamics converges either to 0 or 1 depending on initial conditions. Conversely, for low values of ϕ, $\phi < \phi^a(\lambda)$, the pecuniary externality dominates and the outcome is NAG. Irrespectively of the initial geographical distribution, capital, and thus firms, distribute equally between the two regions.

For intermediate values of the transportation cost, geographical equilibria coexist and two different scenarios are possible. For low inter-regional spillovers, $\lambda < \tilde{\lambda}$, there are values of the trade cost $\phi \in (\phi^b(\lambda), \phi^a(\lambda))$ such that the economy can be either in a NAG or AG configuration (c.f. the left panel of Fig. 4). When this is the case, the transition from NAG to AG, due to the opening-up of the economy, is abrupt. Moreover the economy shows hysteresis, that is, once the transition has occurred it is not the case that going back to a lower freeness of trade brings the economy back to its non-agglomerated state.

Things are different for higher inter-regional spillovers $\lambda > \tilde{\lambda}$, as shown in the right panel of Fig. 4. This time AG and NAG are still associated respectively with high and low values of ϕ, but the transition between the two equilibria

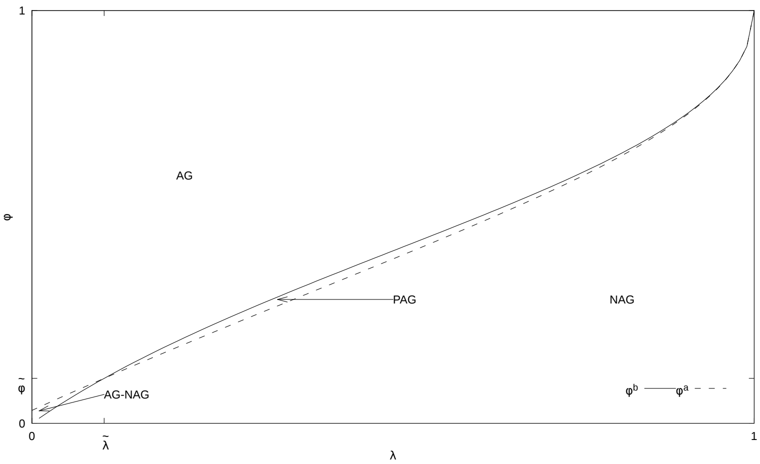

Figure 3: Existence of different type of geographical equilibria in the plane (λ, ϕ). The economy parameters are $L = 400$, $N = 150$, $\sigma = 3$, $\mu = 0.5$, $\alpha = 0.4$. The dotted and continuous curves are $\phi^a(\lambda)$ and $\phi^b(\lambda)$ respectively.

is smoother. Indeed, for intermediate values of ϕ, between $\phi^a(\lambda)$ and $\phi^b(\lambda)$, two asymmetric geographical equilibria emerge, collapsing on the border (interior symmetric) equilibria as ϕ increases (decreases). These distributions represent PAG configurations: due to local spillover, the partial concentration of modern goods production is advantageous but, due to relatively high transportation costs, a further agglomeration is not beneficial as would only increase competition in the crowded location without enough profits coming from an increased demand in the other region. Notice at last that the higher the scale of the economy, the closer S is to $\bar{S}$, which implies lower per-capita capital profits, the closer $\tilde{\lambda}$ is to 0, so that the smooth transition from NAG to AG occurs for a larger set of openness-parameters values.

5 Welfare Analysis

So far we have assumed that capital moves in order to maximize its rent, rather than households real income. This begs the question of what happens to household utilities, that is, to their welfare level. Household's utility in each location can be

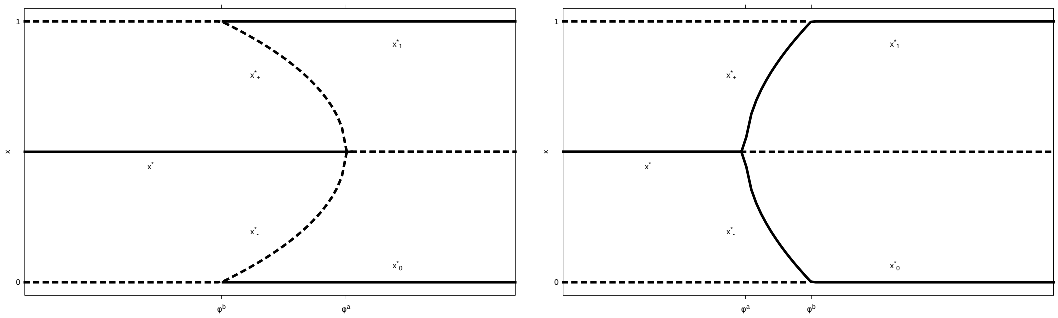

Figure 4: Internal fixed points and boundary distributions as a function of ϕ. Continuous lines denote geographical equilibria. Left panel: $\lambda < \tilde{\lambda}$. Right panel: $\lambda > \tilde{\lambda}$.

written as total income divided by the price index

$$\begin{cases} W_1(x) = \dfrac{I(x)}{P_1(x)} \\ \\ W_2(x) = \dfrac{I(x)}{P_2(x)}\,, \end{cases} \tag{5.1}$$

where, using $\rho = 1/\tau = \phi^{1/(\sigma-1)}$ as policy freeness of trade parameter[6], $I(x)$ is given in (2.16-2.18),

$$P_1(x) = \left(\frac{\beta\sigma}{\sigma-1}\left(Nx + N(1-x)\rho^{\sigma-1}\right)^{\frac{-1}{\sigma-1}}\right)^{\mu}\,, \tag{5.2}$$

and $P_2(x) = P_1(1-x)$.

The properties of $I(x)$ and $P_{1,2}(x)$ (c.f. the proof of Prop. 5.1) imply that the welfare difference

$$\Delta W(x) = I(x)\frac{P_2(x) - P_1(x)}{P_1(x)P_2(x)}$$

is negative in $x = 0$, zero only in $x = 1/2$ and positive in $x = 1$. As a result each household is better off if firms agglomerate in his/her own region. In this case local household does not pay transportation costs for modern goods and, due to the technology externality, their income benefits from the strongest possible abatement of fixed costs.

[6]In this section we use ρ instead of ϕ as the latter depends also on the preference parameter σ so that, when computing marginal changes of Welfare with respect to changes of the freeness of trade, also preferences would be (wrongly) involved.

Whereas agglomeration is clearly beneficial for the region which happens to host the modern sector, it is not clear whether it is beneficial also for the whole economy. This is an important issue in a model like ours, where regions are ex-ante identical and where workers are not mobile. Finding an answer requires to investigate what happens to the welfare of the region that specializes in the traditional sector. On the one hand, households living there have to import all the modern goods so that, due to transportation costs, they have higher real prices. On the other hand, their nominal income is the same as that of households located in the modern region, and they also profits from higher capital rents. The overall result depends on the relative strength of these two effects, which in turn are related to both trade costs and inter-regional spillovers, that is, to market and technological openness.

Welfare analysis clearly depends on which type of welfare aggregating function one considers. Since we are mainly concerned with welfare levels in the traditional region, the Max-Min formulation seems the most appropriate. We define total welfare to be equal to the minimal welfare level between the two regions:

$$W_T(x) = \min\{W_1(x), W_2(x)\}\ . \tag{5.3}$$

Given that benefits of agglomeration spill also to the traditional region, it may well be the case that agglomeration is the best outcome, also under such an egalitarian definition of total welfare as (5.3). The following proposition rules when this is the case

Proposition 5.1. *Consider total welfare as in* (5.3)*. For any given value of the inter-regional spillover λ, provided that freeness of trade ρ is such that*

$$\rho > \rho^w(\lambda) = \left(2\left(\frac{L - \frac{N\alpha}{2}}{L - \frac{N\alpha}{1+\lambda}}\right)^{\frac{\sigma-1}{\mu}} - 1\right)^{-\frac{1}{\sigma-1}},$$

AG is the global welfare maximum. Otherwise, when $\rho < \rho^w(\lambda)$, NAG is a global maximum. When $\rho = \rho^w(\lambda)$ both NAG and AG are global maxima.

Proof. The proof relies on the properties of income $I(x)$ and of price indexes $P_{1,2}(x)$. For the income function defined in (2.16-2.18) one can easily show that $I(x) = I(1-x)$, $I'(1/2) = 0$, $I'(x) \gtreqless 0$ when $x \gtreqless 1/2$, and $I''(x) > 0$. For the price index functions in (5.2) it holds $P_1(x) = P_2(1-x)$, $P_1'(x) = -P_2'(x) < 0$, and $P_1''(x) = P_2''(x) > 0$. Using these properties we can re-write the expression of the Max-Min welfare as

$$W_T(x) = \begin{cases} W_1(x) & x \leq \frac{1}{2} \\ W_2(x) & x \geq \frac{1}{2}\,. \end{cases}$$

Due to the symmetry of the economy, we can restrict our attention to the maxima of $W_2(x)$ in the interval $[1/2, 1]$. Given the behavior of $I(x)$ and $P_2(x)$ it holds both that $W_2'(1/2) < 0$ and that there exists at most one value of $x \in [1/2, 1]$ where $W_2'(x) = 0$. As a result the global maxima of the continuous and differentiable function $W_2(x)$ in the interval $[1/2, 1]$ are on its border, that is, either $x = 1/2$ or $x = 1$. In order to determine when one or the other prevail, we compare their welfare level and find

$$\frac{W_2(1)}{W_2(1/2)} \gtreqless 1 \Leftrightarrow \rho \gtreqless \rho^w(\lambda) = \left(2 \left(\frac{L - \frac{N\alpha}{2}}{L - \frac{N\alpha}{1+\lambda}} \right)^{\frac{\sigma-1}{\mu}} - 1 \right)^{-\frac{1}{\sigma-1}} ,$$

which proofs the proposition. □

AG is a welfare maximum provided that $\rho > \rho^w(\lambda)$. In this case the profits generated in the agglomerated region are so high that they offset the losses due to a high price index in the traditional region. High ρs and low λs are in fact, respectively, lowering the price index difference between AG and NAG and increasing the gains in terms of capital rents due to agglomeration. Notice that since $\rho^w(\lambda)$ is an increasing function of λ, the minimal freeness of trade ρ sufficient to make AG the welfare maximum is increasing with the strength of the inter-regional spillover λ. Conversely, when $\rho < \rho^w(\lambda)$ the welfare maximum is given by NAG. Given these differences in welfare levels, when does the geographical economic equilibrium arising from firm rents maximizing behavior lead to a total welfare maximum?

Figure 5 tries to answer this question putting together the results from Propositions 4.1 and 5.1. In the upper left area, above $\rho^w(\lambda)$, agglomerated economies are both a geographical equilibrium and the welfare maximum. In the lower right area, below $\rho^a(\lambda)$, the same is true for non-agglomerated economies. In Fig. 5 starred labels denote those geographical equilibria which are welfare maximizers. For any value of the inter-regional spillover there also exists an intermediate range of trade costs where NAG is the welfare optimum but the economic equilibria are AG or PAG. There seems to be no continuous path that links the upper-right area AG* with the lower-left area NAG* . The following lemma shows that this is a general result. By proving that the curve $\rho^w(\lambda)$ lies always above $\rho^a(\lambda)$, apart in the point $(\lambda = 1, \rho = 1)$ where they coincide, it shows that, unless $\lambda = 1$, it is never possible to move from NAG* to AG* following a path where the total welfare is always maximal.

Lemma 5.1. *Provided that $\tilde{S} > 0$ and $S < min\{\tilde{S}, \bar{S}\}$, it holds that*

$$\rho^a(\lambda) < \rho^w(\lambda) , \quad \textit{for every} \quad \lambda \in (0, 1) , \tag{5.4}$$

and $\rho^a(\lambda) = \rho^w(\lambda)$ when $\lambda = 1$.

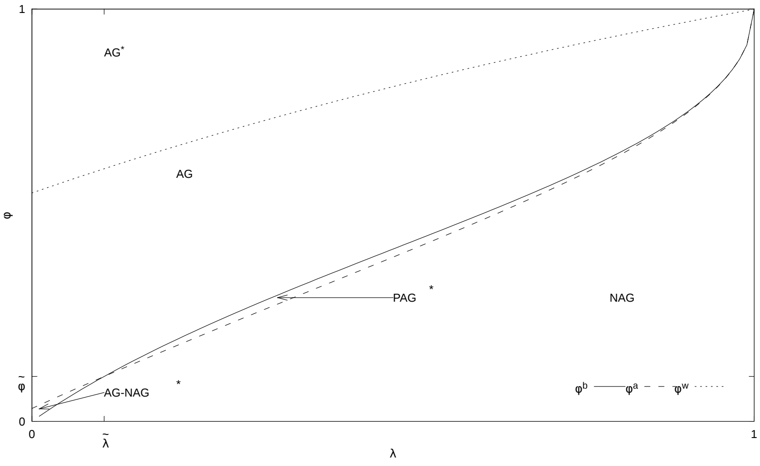

Figure 5: Geographical equilibria of Fig. 3 which are also total welfare maximizers are marked with a *. They are above the line $\phi^w(\lambda) = (\rho^w(\lambda))^{\sigma-1}$ in case of agglomerated economies and below it for non-agglomerated economies. The parameters are the same as in Fig. 3.

Proof. By evaluating $\rho^a(\lambda)$ and $\rho^w(\lambda)$ in $\lambda = 1$, one immediately sees that they are both equal to one. The rest of the statement has been proved numerically. We have defined a grid of 500 values of σ in $(1, 100]$, 100 values of μ in $(0, \sigma/(2\sigma - 1))$, which ensures that $\tilde{S} > 0$, 100 values of αS in $(0, \min\{\tilde{S}, \bar{S}\})$, and 500 values of λ in $[0, 1)$. We have checked that for every values of parameters and λ in the grid it holds that

$$\frac{d\rho^w(\lambda)}{d\lambda} < \frac{d\rho^a(\lambda)}{d\lambda} \qquad \text{when} \quad \lambda < \tilde{\lambda}$$

$$\frac{d\rho^w(\lambda)}{d\lambda} < \frac{d\rho^b(\lambda)}{d\lambda} \qquad \text{when} \quad \lambda \geq \tilde{\lambda}$$

which together with the fact that all these curves are equal to one when $\lambda = 1$, are monotonic, and $\rho^a(\lambda) \gtreqless \rho^b(\lambda)$ when $\lambda \gtreqless \tilde{\lambda}$, see Proposition 4.1, proves the result. □

5.1 Welfare enhancing policies

Having derived the total welfare for any value of the "openness" parameter, the next concern is how an economy can move toward the line $\rho = 1$, that is, the locus of the parameters space where the total welfare is the highest. Lemma 5.1 tells us that the only path that brings the economy to the highest welfare passing through welfare maxima is the one that reach the full openness state where λ and ρ are both one.

In general, under which conditions it is more beneficial to embrace policies that improve the freeness of trade and when, instead, it is better to go for inter-regional knowledge spillovers? To answer this question assume the government, by taxing households income I, can implement policies g which increase the level of freeness of trade ρ and/or the inter-regional knowledge spillovers λ.

When the policy does not entail a change in the geographical equilibrium its effect on the total welfare for $x \leq 1/2$ can be computed as[7]

$$\frac{\partial W_T(x)}{\partial g} = \frac{\partial W_1(x)}{\partial g} = \frac{1}{P_1(x)}\frac{\partial I(x)}{\partial g} + \frac{\partial W_1(x)}{\partial \lambda}\frac{\partial \lambda}{\partial g} + \frac{\partial W_1(x)}{\partial \rho}\frac{\partial \rho}{\partial g} \tag{5.5}$$

where $\partial I(x)/\partial g$ stands for the cost of the policy. Under the assumption that a given amount of money spent by the policy g has the same impact on ρ and λ, that is, $\partial\lambda/\partial g = \partial\rho/\partial g$, evaluating whether it is more welfare enhancing to increase ρ or λ amounts to compare $\frac{\partial W_1(x)}{\partial \lambda}$ with $\frac{\partial W_1(x)}{\partial \rho}$, at the different geographical equilibria.[8] This is the result of the following lemma.

Lemma 5.2. *Consider the total welfare $W_T(x)$ as in (5.3). Provided that the economy is in a AG state, it is always more beneficial to increase the freeness of trade ρ rather than the inter-regional spillover λ. Otherwise, when the economy is in a NAG state, having defined*

$$\lambda^+(\rho) = \frac{\frac{N\alpha}{L} + \sqrt{\left(\frac{N\alpha}{L}\right)^2 + 4\frac{N\alpha}{L}\frac{1+\rho}{\mu\rho^{\sigma-2}}}}{2} - 1\,, \tag{5.6}$$

it is more beneficial to increase the freeness of trade ρ when $\lambda > \lambda^+(\rho)$, to increase the inter-regional spillover λ when when $\lambda < \lambda^+(\rho)$, and indifferent when $\lambda = \lambda^+(\rho)$.

Proof. The gradient of the total welfare at the two geographical equilibria corre-

[7]Given the symmetry of the welfare function around $x = 1/2$, one can easily compute the effect of the policy also for $x \geq 1/2$.

[8]The evaluation of the policy impact can be complicated by a variation in the geographical equilibrium distribution of the economy due to its effect on λ and ρ. This occurs at the non-generic direct transition between NAG and AG and in the generic case of PAG, when the degree of agglomeration depends continuously on the values of these parameters (see e.g. the left panel of Fig. 4). Since the parameters space where this dependence occurs is small, as can be seen in Fig. 3, we skip it at this stage of the analysis.

sponding to NAG and AG is:

$$\frac{\partial W_T(x)}{\partial \lambda} = \begin{cases} \left(\frac{N\alpha\sigma}{(\sigma-\mu)L(1+\lambda)^2}\right)\left(\frac{\sigma-1}{\beta\sigma}\left(\frac{N}{2}\right)^{\frac{1}{\sigma-1}}\right)^{\mu}(1+\rho)^{\frac{\mu}{\sigma-1}}\,, & x=0.5\,, \quad \text{(NAG)} \\ 0 & x=0,1\,, \quad \text{(AG)} \end{cases}$$

$$\frac{\partial W_T(x)}{\partial \rho} = \begin{cases} \left(\frac{\sigma\mu}{\sigma-\mu}-\frac{N\alpha\sigma\mu}{(\sigma-\mu)L(1+\lambda)}\right)\left(\frac{\sigma-1}{\beta\sigma}\left(\frac{N}{2}\right)^{\frac{1}{\sigma-1}}\right)^{\mu}(1+\rho)^{\frac{\mu}{\sigma-1}-1}\rho^{\sigma-2}\,, & x=0.5\,, \quad \text{(NAG)} \\ \left(\frac{\sigma\mu}{\sigma-\mu}-\frac{N\alpha\sigma\mu}{(\sigma-\mu)2L}\right)\left(\frac{\sigma-1}{\beta\sigma}\left(N\right)^{\frac{1}{\sigma-1}}\right)^{\mu}\rho^{\mu-1}\,, & x=0,1\,, \quad \text{(AG)} \end{cases}$$

Since for AG economies $\partial W_T/\partial\lambda$ is zero the best policy is always to increase the freeness of trade. The result for NAG economies follows from the comparison of the two components of the gradient evaluated there. The expression to be evaluated gives rise to a quadratic equation in λ whose only possibly positive root is $\lambda^+(\rho)$ as in (5.6). □

When the modern sector is agglomerated, fixed production costs abatement does not depend on the inter-regional spillovers, and neither do households welfare. As a result policies that increase λ have no effects and it is preferable to improve the freeness of trade ρ. Conversely, when the modern sector is evenly spread between the two regions, Lemma 5.2 shows that it is more welfare improving to increase inter-regional knowledge spillovers when $\lambda < \lambda^+(\rho)$ and to increase freeness of trade ρ otherwise. Fig. 6 summarizes these results and plots the gradient of the welfare function on the plane (λ, ρ) for our benchmark choice of the economy parameters.

6 Exogenous regional differences

This last section before the conclusion explores the effects of regional exogenous differences on geographical equilibria found in Sections 3-4. Two forms of exogenous differences are considered. The two regions may differ for the number of households or for R&D costs. In the first case we measure the difference with the parameter δ and say that location 1 has $L(1+\delta)$ households whereas location 2 has $L(1-\delta)$ households. Without loss of generality we impose $\delta \in (0,1)$, so that region 1 has always a larger population. Similarly fixed cost differences are measured by $\epsilon \in (-1,1)$ so that fixed costs are proportional to $\alpha(1+\epsilon)$ in location 1 and $\alpha(1-\epsilon)$ in location 2. The sign of ϵ is not restricted so that any location can enjoy the lowest total fixed costs.

Repeating the market equilibrium analysis of Section 2 the expression for the

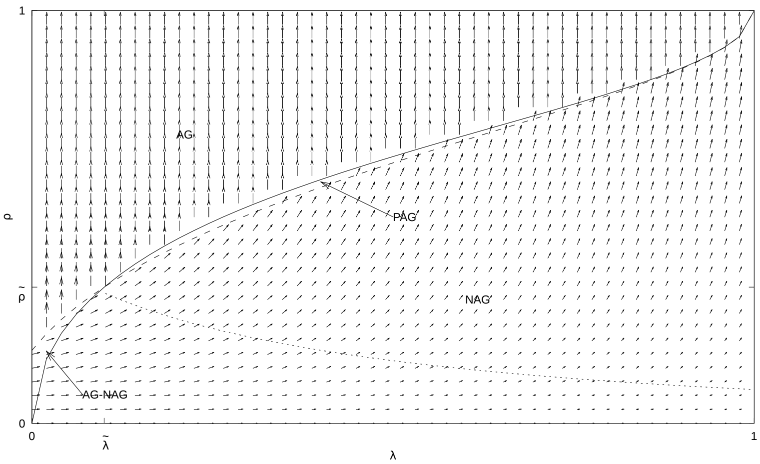

Figure 6: Gradients of the welfare function for NAG and AG economies. When the economy is in a AG state it is always better to increase freeness of trade ρ. When the economy is in a NAG state it is better to increase the freeness of trade when ρ is above the line $\lambda^{+}(\rho)$ and to increase inter-regional spillovers λ otherwise. The parameters are the same as in Figs. 3,5.

capital rent in each location becomes

$$\begin{cases} r_1(x) = I(x)\dfrac{\mu L}{N\sigma}\left(\dfrac{1+\delta}{x+(1-x)\phi} + \dfrac{(1-\delta)\phi}{x\phi+(1-x)}\right) - \dfrac{\alpha(1+\epsilon)}{x+\lambda(1-x)} \\[2ex] r_2(x) = I(x)\dfrac{\mu L}{N\sigma}\left(\dfrac{1-\delta}{x\phi+(1-x)} + \dfrac{(1+\delta)\phi}{x+(1-x)\phi}\right) - \dfrac{\alpha(1-\epsilon)}{1-x+\lambda x}\,, \end{cases} \tag{6.1}$$

where

$$I(x) = 1 + \frac{\mu}{\sigma-\mu}\left(1 - \frac{S}{2\bar{S}}\left(\frac{x(1+\epsilon)}{x+\lambda(1-x)} + \frac{(1-x)(1-\epsilon)}{1-x+\lambda x}\right)\right)$$

As a result

$$\Delta(x) = 2\alpha\left(I(x)\frac{\bar{S}}{S}\left(\frac{x_\delta - x}{x-x^2+\frac{\phi}{(1-\phi)^2}}\right) - \frac{x_\epsilon - x}{(x-x^2)(1-\lambda)+\frac{\lambda}{1-\lambda}}\right)\,, \tag{6.2}$$

where we have defined

$$x_\delta = \frac{1}{2} + \frac{\delta}{2}\frac{1+\phi}{1-\phi}\,, \quad x_\epsilon = \frac{1}{2} + \frac{\epsilon}{2}\frac{1+\lambda}{1-\lambda}\,.$$

When $\delta = 0$ and $\epsilon = 0$ the symmetric case of (2.17-2.18) is recovered. In order to keep the model tractable, we focus on the two extreme cases where the inter-regional spillover λ is either 0 or 1.

Full inter-regional spillover When $\lambda = 1$ the technological spillover operates globally and all firms "share" labour fixed costs. Agglomeration forces are weak and, in the symmetric case, the outcome is NAG for all initial conditions. Indeed since technological externalities are global and labour is not mobile, households firms equidistribute between the two. The outcome in the asymmetric case is similar in that a unique geographical equilibrium $\hat{x}$ exists and attracts all trajectories irrespectively of the initial condition. In this case however, $\hat{x}$ does not need to be equal to $1/2$, corresponding to NAG, but moves to its right or its left depending on the regional difference parameters ϵ and δ and on the freeness of trade ϕ, as characterized by the following proposition.

Proposition 6.1. *If $\lambda = 1$ there are two values $\phi^{\pm}$ of the trade openness parameter such that the geographical equilibrium is PAG if $\epsilon < 0$ and $\phi < \phi^{-}$ or if $\epsilon > 0$ and $\phi < \phi^{+}$. The PAG equilibrium belongs to the interval $(x_\delta, 1)$ in the former case, to $(0, x_\delta)$ in the latter. Otherwise the equilibrium is AG and $x = 1$ when $\epsilon > 0$ whereas $x = 0$ when $\epsilon < 0$.*

Proof. The rent difference $\Delta(x)$ in (6.2) can be written as the ratio of two polynomials. If $\lambda = 1$ the denominator is always positive, so that the study of the second order polynomial in the numerator $N(x)$ is sufficient.

When $\epsilon < 0$ it is $N(0) > 0$ and $N'(x) < 0$. The polynomial possesses a single root in $(0, 1)$ provided that $N(1) < 0$. This root is a globally attracting internal asymptotically stable fixed point. Conversely, if $N(1) \geq 0$ the system agglomerates in $x = 1$. Solving $N(1) = 0$ for ϕ leads to the identification of ϕ^{-}. The location of the PAG equilibrium in the interval $(x_\delta, 1)$ follows from noticing that $N(x_\delta) > 0$.

When $\epsilon < 0$ it is $N(1) < 0$ and $N'(x) > 0$. A similar reasoning leads to the identification of ϕ^{+} and to the statement. □

When $\epsilon = 0$, that is the two locations are equal in terms of fixed costs, the technological spillovers, acting globally, do not lead to agglomeration. However, because of its largest size, region 1 benefits from an home market effect and more firms locate there rather than in region 2. Whether the home market effect leads to an interior or a border equilibrium depends on the freeness of trade. If ϕ is large enough the equilibrium is at 1, otherwise it is interior. The same pattern occurs when $\epsilon < 0$, with the addition that the home market effect is reinforced by having lower fixed costs in region 1. Comparatively it will be observed that a larger fraction of firms settle in region 1. It can also be checked that the threshold ϕ_{-} is decreasing with ϵ. The effect of asymmetries might appear more complicated when $\epsilon > 0$ as firms located in region 1 face a trade-off between higher local demand, due to $\delta > 0$, and higher fixed costs, due to $\epsilon > 0$. However, according to proposition 6.1, the overall picture turns out to be similar, with the cost saving effect playing a leading role over the home market effect. In fact, as the freeness of trade increases, it is the region where costs are lower that gains firms till the point where all firms are located in the border configuration $x = 0$.

No inter-regional spillover When $\lambda = 0$ the technological spillover operates only locally. In the symmetric case the strong agglomeration forces induced by localized spillover induce an AG equilibrium with high freeness of trade and, possibly, a NAG equilibrium with low freeness of trade. Again regional differences shift the position of the interior equilibria but not the overall picture as shown by the following

Proposition 6.2. *Let $\lambda = 0$. If*

$$\epsilon \notin \left[\frac{(\bar{S}-S)\sigma + \delta(\sigma\bar{S} - \mu S)}{(\sigma-\mu)S}, \frac{(S-\bar{S})\sigma + \delta(\sigma\bar{S} - \mu S)}{(\sigma-\mu)S}\right]$$

only AG occurs and the economy is agglomerated in one of the two border equilibria. Otherwise, there exists a threshold value $\hat{\phi}$ so that if $\phi < \hat{\phi}$ AG and PAG coexist. When $\phi = \hat{\phi}$ the fixed point associated to the unique interior equilibrium becomes unstable through a super-critical pitchfork bifurcation and for $\phi > \hat{\phi}$ only AG occurs.

Proof. As long as $\phi > 0$, it is $\Delta(x) < 0$ in a right neighborhood of 0 and $\Delta(x) > 0$ in a left neighborhood of 1 so that the border equilibria always exist. Moreover since the denominator of $\Delta(x)$ is always positive and its numerator, $N(x)$, is a third order polynomial, there exists at most one additional interior geographical equilibrium.

Consider the extreme case $\phi = 0$. The capital rent difference reads

$$\Delta(x) = \frac{2\alpha\sigma(\bar{S}-S)}{(\sigma-\mu)S}\left(\frac{\hat{x}_0 - x}{x - x^2}\right), \quad \text{where} \quad \hat{x}_0 = \frac{1}{2} + \frac{\delta}{2} + \frac{(\sigma-\mu)S(\delta-\epsilon)}{2\sigma(\bar{S}-S)}.$$

When $\hat{x}_0 \geq 1$, the only globally stable fixed point is $x = 1$. Analogously, when $\hat{x}_0 \leq 0$, the only globally stable fixed point is $x = 1$. When $\hat{x} \in (0, 1)$, which occurs when regional asymmetries are not too big, $\hat{x}_0$ is the unique *interior* geographical equilibrium. Since the numerator $N(x)$ is a smooth function of ϕ we can conclude that in the latter case $\Delta(x)$ has a zero $\hat{x}$ close to $\hat{x}_0$ for ϕ close enough to 0. Moreover its fist order differential keeps the same sign as $N'(\hat{x}_0) < 0$. Then, since $N(x)$ is negative for $x = 0$ and positive for $x = 1$, there will be two other roots in the interval $(0, 1)$. In this case the two border equilibria and the interior equilibrium $\hat{x}$ coexist. Moreover, as ϕ increases the root of $N(x)$ in $[0, 1]$ reduces from three two one and there exists a $\hat{\phi}$ where the super-critical bifurcation occurs, so that irrespectively of the value of $\hat{x}_0$ when ϕ is close to one the unique root becomes x_ϵ and only AG occur.

The bound on the value of ϵ given in the proposition has been found by imposing $\hat{x}_0 \notin [0, 1]$. □

Technological externalities are always strong enough to make agglomerated economy a stable equilibrium. The basin of attraction of the two agglomerated equilibria and whether more equilibria exist depend however on regional differences. For large regional differences no other equilibria exist. For small regional differences, when ϕ is small enough there exist three geographical equilibria, the two border and one interior. Conversely, when ϕ is larger so that the two regions are sufficiently trade-integrated, agglomeration is the unique long-run outcome. In the extreme case when $\phi = 1$, agglomeration either in 0 or in 1 is always the unique outcome and the basins of attraction of the two border equilibria are easily determined: firms agglomerate in region 1 when the initial condition is $x_0 < (1 + \epsilon)/2$ and in 2 when $x_0 > (1 + \epsilon)/2$. In fact as trade costs are zero and the economy is fully integrated the region with the larger labour force has no exogenous advantages anymore and basins of attraction are determined by relative fixed costs.

As with symmetric locations, also in presence of asymmetries agglomeration in either locations is always an equilibrium and when the transition between agglomeration and non agglomeration occurs it does so via an abrupt change. This transition is however not the rule. Rather it follows from the absence of interregional spillovers.

7 Conclusion

We have set up an analytical model with capital mobility, workers inter-sectoral mobility and inter-regional immobility, and where agglomeration is due to technological externalities. These externalities can be interpreted as localized knowledge spillovers. Due to the analytical resolvability of our model, we have been able to compute the geographical equilibria, analyze their stability, and fully characterize their dependence on the trade-off between technological and pecuniary externalities, as regulated by transportation costs and inter-regional spillovers, and discuss their implications for total welfare. To retain analytical tractability we have considered primarily the case of symmetric regions. In the last section we have shown that our results apply also to the asymmetric case in the extreme cases of fully local and fully global technological externalities.

In general our analysis confirms previous findings by Baldwin and Forslid (2000) about the stabilizing nature of (knowledge) spillovers: the higher the spillover the larger the interval of transportation costs which lead to firms equidistribution. Moreover our analysis shows that if the spillover is high enough, there exists a smooth equilibrium transition between agglomeration and equidistribution, with partly agglomerated economy for intermediate values of transportation costs. In this case an opening of inter-regional trade does not entail an abrupt reallocation of economic activities neither the hysteresis effect, typical of NEG model, which locks the economy in a core-periphery equilibrium also if higher trade costs are reintroduced.

Welfare analysis reveals that for a relatively large part of the (λ, ϕ) parameter space, even if the agglomerated outcome represents the geographic equilibrium, it generates less welfare in the periphery region than in the core. Since, in any case, the level of welfare of the periphery in the AG equilibrium increases with trade openness, for large enough level of ϕ this solution represent the welfare optimum for both regions. However, the existence of a large "welfare gap" makes the implementation of policies based on progressive opening of the economy difficult to implement.

On the other hand, the increase of the technological openness always improves the welfare level of both locations. When the level of knowledge sharing is low, its increase represents, from the point of view of the social planner, the best policy. Beside the positive effect on welfare levels, the increase of λ has also another advantage: for an economy with strong knowledge/technological integration, the "welfare gap" between agglomerated and non-agglomerated distribution is smaller and shallower and, consequently, policy geared toward markets integration are easier and less costly to implement.

In practice an increase in technological openness can be obtained by improving global means of information sharing, developing joint education programs, unifying norms and requirements affecting economic activities and the relaxing the institutional constraints. All these policies have the effect of improving global efficiency by avoiding replicated efforts and by abating knowledge barriers adding to transaction costs. Concluding, whereas freeness of trade leads, per-se, to sudden agglomeration, knowledge-based linkages favor a smoother transition between different levels of firms concentration and ultimately lead to a less uneven distribution of welfare.

References

Audretsch, D. and M. Feldman (2004). Knowledge spillovers and the geography of innovation. In J. Henderson and J.-F. Thisse (Eds.), *Handbook of Regional and Urban Economics*, pp. 2713–2739. North-Holland (Handbooks in Economics Series).

Baldwin, R. and R. Forslid (2000). The core-periphery model and endogenous growth: Stabilizing and destabilizing integration. *Economica 67*, 307–324.

Baldwin, R., R. Forslid, P. Martin, G. Ottaviano, and R.-N. F. (2003). *Economic Geography and Public Policy*. Princeton University Press, Princeton.

Boschma, R. (2005). Proximity and innovation: A critical assessment. *Regional Studies 39*, 61–74.

Bottazzi, G., G. Dosi, G. Fagiolo, and A. Secchi (2008). Sectoral and geographical specificities in the spatial structure of economic activities. *Structural Change and Economic Dynamics 19*, 189 – 2002.

Brenner, T. (2006). Identification of local industrial clusters in Germany. *Regional Studies 40*, 991–1004.

Breschi, S. and F. Lissoni (2001a). Knowledge spillovers and local innovation systems: A critical survey. *Industrial and Corporate Change 10*(4), 975–1005.

Breschi, S. and F. Lissoni (2001b). Localised knowledge spillovers vs. innovative milieux: Knowledge "tacitness" reconsidered. *Papers in Regional Science 80*, 255–273.

Devereux, M., R. Griffith, and H. Simpson (2004). The geographic distribution of production activity in Britain. *Regional Science and Urban Economics 34*, 533–564.

Dixit, A. and J. Stiglitz (1977). Monopolistic competition and optimun product diveristy. *American Economic Review 67*(3), 297–308.

Dupont, V. and P. Martin (2006). Subsidies to poor regions and inequalities: some unpleasant arithmetic. *Journal of Economic Geography 6*, 223–240.

Ellison, G. and E. Glaeser (1997). Geographical concentration in U.S. manufacturing industries: a dartboard approach. *Journal of Political Economy 105*, 889–927.

Forslid, R. and G. Ottaviano (2003). An analytically solvable core-periphery model. *Journal of Economic Geography 3*, 229–240.

Grossman, G. and E. Helpman (1991). *Innovation and Growth in the World Economy*. MIT Press, Cambridge Mass.

Krugman, P. (1991a). *Geography and Trade*. The MIT Press, UK.

Krugman, P. (1991b). Increasing returns and economic geography. *The Journal of Political Economy 99*, 483–499.

Malmberg, A. and P. Maskell (2006). Localized learning revisited. *Growth and Change 37*(1), 1–18.

Marshall, A. (1920). *Principles of Economics*. London: Macmillan and Co.

Martin, P. (1999). Public policies, regional inequalities and growth. *Journal of Public Economics 73*, 85–105.

Martin, P. and G. Ottaviano (1999). Growing locations: Industry locations in a model of endogenous growth. *European Economic Review 43*, 281–302.

Martin, P. and C. A. Rogers (1995). Industrial location and public policy. *Journal of International Economics 39*, 335–351.

Maurel, F. and B. Sedillot (1999). A measure of the geographic concentration in french manufacturing industries. *Regional Science and Urban Economics 29*, 575–604.

Rallet, A. and A. Torre (1999). Is geographical proximity necessary in the innovation networks in the global economy? *GeoJournal 49*, 373–380.