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An evolutionary model of firms location with technological externalities

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Abstract

In an economic geography model where both a negative pecuniary and a positive technological externality are present, we introduce an explicit dynamics of firms locational choice and we characterize its long run distribution. Our analysis shows that economic activities evenly distribute when the pecuniary externalities prevail, and agglomerate otherwise. Due to the stochastic nature of the dynamics, even when agglomeration occurs, it is only a metastable state. By giving time and firms heterogeneity a role, we are bringing the evolutionary approach inside the domain of economic geography.

Keywords: Evolutionary Economic Geography; Heterogeneity; Agglomeration; Technological externalities; Markov Chains

JEL Classification: C62, F12, R12

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1 Introduction

The present contribution intends to pursue the analysis of the effects of the evolutionary metaphor (Nelson and Winter, 1982; Freeman, 1986; Dosi, 1988) when applied inside the domain of economic geography. In principle, the validity of the assumption of evolutionary economics is all but obvious and the question whether economic interactions can be effectively thought as an evolutionary process still open. With the clear risk of oversimplifying the matter, we could say that the notion of evolution immediately entails three consequences for the economic dynamics. First, it should move from simpler to more complex structures. Second, it should progressively eliminate less efficient structures and promote the development of more efficient ones, irrespectively of the fact that this process of elimination and promotion might take place through a mechanism of adaptation by part of the economic actors or through an "adoption" by part of markets and institutions (Alchian, 1950). Third, the progressive change or renewal of the different actors and rules should proceed in a jointly integrated way.

Obviously, the central question is not whether the characteristics described above can be considered to be present in economic systems, because they certainly are. The question is whether the evolutionary accounting of their effects and causes allows for a deeper understanding and a more reliable modeling of economic interactions. In the end, one is interested to know if this accounting could help in the development of more effective policies. In principle, however, the ideas of evolutionary economic thinking can be applied to the investigation of the different domains of economics also without providing a certain and indisputable answer to the previous question. Indeed, partly following, even if not subscribing to, the Friedmanian's idea that the effectiveness of a theoretical framework should be solely judged on its ability to reproduce and explain observed phenomena, one could simply start from the "evolutionary" metaphor and see what consequences it brings to the design of economic models. As argued in Frenken and Boschma (2007), the development of an evolutionary approach to economic geography could suggest new ways of explaining the observed patterns which characterize the uneven spatial distribution of economic activities. In the spirit of the foregoing "minimalistic" research agenda, we try to complement the bottom-up theorizing suggested there with a deeper understanding of the differences that an evolutionary inspired modeling is likely to produce with respect to more traditional approaches.

To be brought to its completion our exercise requires a twofold specification. First, we need to identify a simple formal model, based on clear assumptions, which can serve as a generic analytical framework. Second, we have to consider which hypotheses are to be put forward in order to imbue this model with the spirit of the Evolutionary Economic Geography. We address the first requirement by choosing, as a starting point, the simple two-location and multi-firm model described in Krugman (1991). This model already encompasses the idea of increasing returns and of the relevance of feed-back mechanisms in shaping the aggregate economic pattern. It is well rooted in the tradition of New Economic Geography and, as such, constitutes a perfect benchmark for our comparative exercise. Concerning the second requirement, in line with the discussion in Boschma and Frenken (2006) and Boschma and Martin (2007), and partly inspired by the critical survey in Martin (1999), we assume the following three aspects as baseline characters of our evolutionary modeling. First, the interaction between economic agents should take place not only through market mechanisms, but also through localized, idiosyncratic interactions. Second, the flow of time should be present in the model and the decisions of economic agents, together with their consequences, should be put in an explicit time dimension. Third, the heterogeneity of firms behaviour should not be captured by a simple "noise" term acting as a perturbation around a deterministic equilibrium. Rather, it

should enter as an essential ingredient in the description of the model and in the determination of the final aggregate outcome (Schelling, 1978; Granovetter, 1978).

More precisely, we take as a starting point the model introduced in Forslid and Ottaviano (2003) and developed in Bottazzi and Dindo (2008). The latter extends Krugman (1991) by introducing a positive technological externality, assumed not tradable across locations, and by considering workers who are not mobile, which is equivalent to assume that firms locational decision and reallocation of capital goods take place on a much shorter time scale than the one characterizing work-force flows. Inside this simple economy, we consider a heterogeneous population of profit maximizing firms which independently choose where to locate their production. The model is characterized by a simple entry-exit process, and we consider a truly dynamic setting in which the locational decision of each firm is affected by the past decisions of others. As in Bottazzi et al. (2007), we assume that firms keep revising their decisions as new locational choices affect their profits. As we shall see this updating choice process is able to generate a self-reinforcement mechanism similar to that described in Dosi and Kaniovski (1994).

The idea that localized externalities might explain agglomeration even in absence of workers mobility, has been explored by several contributions inside the New Economic Geography literature. For instance Krugman and Venables (1996) assume a vertically structured economy with localized input-output linkages while Martin and Ottaviano (1999) consider location-specific R&D sectors which introduce different products in different locations. Baldwin and Forslid (2000) consider geographical distributions of economic activities as driven by a growth process fueled by human capital accumulation and knowledge spillovers. A drawback of these works is that, in general, they derive equilibria conditions without the complete and explicit characterization of firms profit functions. This specification becomes however necessary when one has to design the choice procedure of firms in a dynamic environment. In order to obtain explicit expression for the profit function, we take a simpler approach: we introduce technological externalities in the forms of a baseline "cost sharing" assumption, according to which fixed production costs are shared across all firms within a given location.

The cost sharing assumption makes the model in Bottazzi and Dindo (2008) particularly suitable for the present exercise because, while remaining simple and analytically tractable, it allows for a twofold dependence of firm profits on the activity of the other firms. Using the terminology of Scitovsky (1954), this dependence takes the form of both a pecuniary externality and a technological externality. In this way, firms profits depend on the interplay of an indirect interaction mediated by the market, which corresponds to a pecuniary externality, and a localized direct interaction, which corresponds to a technological externality. As it turns out, the former acts against the creation of production clusters while, by assumption, the latter promotes them.

Inside this framework, we analyze an explicit firms locational decision process. Our aim is to characterize the long run geographical distribution emerging from this process and relate it to the interplay of the two forms of externality. Since we explicitly introduce the time dimension in our analysis, we are also able to address history dependent phenomena. In particular, we are able to investigate how the initial state of the economy affects firms decisions and show that, due to firms heterogeneity, when agglomeration occurs it is characterized by a transient nature.

This chapter is organized as follows. In Section 2 we briefly describe the model and its assumptions. In Section 3 we study the static setting, and derive the geographical distribution when the model is solved by assuming instantaneous equilibrium between firms choices. Starting from the previously identified equilibria, Section 4 introduces both heterogeneity in

agents decisions and an explicit dynamics across time, discussing what kind of differences are observed with respect to the static case. Finally Section 5 summarizes our main findings and suggests some possible further developments.

2   The model

The following model is a simplification of that described in Bottazzi and Dindo (2008), where more details can be found. Consider a two locations economy. In each location there live I households.[1] Each household is a "local" worker, that is, he supplies labour to firms located where he resides, and a "global" consumer, that is, he can buy goods produced in both locations and traded in a global market. The economy has two sectors: manufacturing and agriculture. In both sectors production is localized. The agricultural good is homogeneous whereas the manufacturing good is made by differentiated products. Location $l = 1, 2$ has n_l manufacturing firms and the total number of firms is $n_1 + n_2 = N$. Without loss of generality, we assume that N is even. In order to consume manufacturing goods produced in the location where they do not reside, consumers have to pay a transportation cost $\tau \in (0, 1]$ which takes the form of an iceberg cost: for each unit of good shipped, only a fraction τ arrives at destination. This is equivalent to say that consumers pay a price p/τ for each unit of good they have to import. The higher the value of τ, the lower the cost of transporting goods. Agricultural goods are traded at no costs. Agents consumption behavior is specified by the following

Assumption 1. Each agent chooses among the agricultural good and the N different manufacturing products as to maximize the following utility function

$$\mathrm{U} = \mathrm{C}_A^{1-\mu}\mathrm{C}_M^{\mu}\,, \tag{2.1}$$

where the utility of bundle C_M is of constant elasticity of substitution (CES) type,

$$\mathrm{C}_M = \left(\sum_{i=1,N} c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{(\sigma-1)}} \quad \sigma > 1\,, \tag{2.2}$$

and each product c_i is produced by a different firm $i = 1, \ldots, N$.

   Assumption 1 implies that the N products are substitutes and that σ is the mutual elasticity of substitution (cfr. Dixit and Stiglitz, 1977). The higher σ, the more the products are substitutes and the more price differences impact consumers demand. Since, due to CES utility, agents value diversity, we have implicitly assumed that each firm produces a different product, so that N is both the number of manufacturing firms and the number of manufacturing products available for consumption.

   The agricultural sector uses only labour as input under constant returns to scale with unitary marginal costs. Due to the large number of potential producers, $2I(1 - \mu)$ at equilibrium, the agricultural market is perfectly competitive and the agricultural good is sold at its marginal cost.

   Also manufacturing firms use only labor as input and their technologies are characterized by a common, industry specific, marginal cost and a local, location specific, fixed cost. Formally one has

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[1]The generalization to different number of agents in each location will be considered in future work. Preliminary analysis shows that it doesn't significantly modify the results.

Assumption 2. The labour v_i that each firm $i = 1, \ldots, N$ needs to produce an amount y_i of output is given by

$$v_i = \beta y_i + \alpha_{l_i} \,, \tag{2.3}$$

where marginal cost β is constant across firms and across locations and the fixed costs α_{l_i} depend on the location l_i occupied by firm i.

Assumption 2 implies that we are in presence of economies of scale, that is, an increase in output causes a decrease in each firm average costs. Firm i profit is given by

$$\pi_i = p_i y_i - w_i v_i = y_i(p_i - w_i \beta) - w_i \alpha_{l_i} \,, \quad i = 1, \ldots, N \tag{2.4}$$

where w_i is firm i cost of labour.

Before looking for markets equilibria notice that, due to perfect competition and constant returns to scale in agricultural production, agricultural wages are equal to agricultural prices. Moreover, due to zero transportation costs for the agricultural goods, agricultural prices, and thus wages, must be the same in both locations. Given that consumers are not mobile and the economy is at an equilibrium, it should be also indifferent for a worker to work in the agricultural or in the manufacturing sector. As a result wages in the two sectors, and in the two locations, are equal. For this reason it is convenient to use wages to normalizes prices in the economy and set $w_i = 1$ for all i.

In order to find equilibrium prices, quantities, profits in the manufacturing sector, and the resulting geographical distribution of firms, one should in principle analyze each of the N product markets. Nevertheless the problem can be simplified by considering only one representative market for each location. In fact, location by location, firms produce using the same technology, face the same demand (due to Assumption 1 all goods are substitutes), and the same labour supply. This implies that equilibrium prices, quantities and wages are the same for all the firms in a given location. We can thus consider only two representative product markets, one for each location l, rather than the N distinct products.

We shall compute market equilibrium prices, quantities, and profits for each fixed distribution of firms, that is, fixing n_1 and n_2. First, exploiting the CES preference structure (2.2), which gives a constant elasticity of demand, and assuming that the market structure is that of monopolistic competition, we derive firms pricing behavior. Then using households budget constraints we compute their total demand for the goods produced in each location, taking into account that all goods are substitutes and transportation costs impact the prices of foreign goods. Setting supply equal to demand we are able to determine equilibrium quantities and firms profits in each location as a function of n_1 and n_2. These expressions will be used, in the next section, to asses firms geographical distribution.

Let us start from firms pricing behavior. Consistently with our assumptions, the market structure is that of monopolistic competition, that is, each firm maximizes its profits, setting its marginal revenue equal to its marginal costs, given market demand elasticity and irrespectively of other firms behavior. Using profit function (2.4) and substituting (2.3) while setting marginal profit to zero gives

$$p_l \left(1 + \frac{1}{\varepsilon} \right) = \beta \,, \tag{2.5}$$

where $\varepsilon = \partial \log c / \partial \log p$ is the demand elasticity. Given Assumption 1, it holds that

$$\varepsilon = -\sigma,$$

which together with (2.5) implies

$$p_l = \beta \frac{\sigma}{\sigma - 1} \, . \tag{2.6}$$

Since the price does not depend on the location index, local prices are equal and it it holds $p_1 = p_2$.

Denote the quantity demanded by an agent who reside in location l of a product produced in location m as d_{lm}. Each demand can be determined as a function of prices and wages using the fact that relative demands under a CES utility satisfy

$$\frac{d_{11}}{d_{12}} = \left(\frac{p_2}{p_1 \tau} \right)^{\sigma} \quad \text{and} \quad \frac{d_{22}}{d_{21}} = \left(\frac{p_1}{p_2 \tau} \right)^{\sigma} , \tag{2.7}$$

while agents budget constraints give

$$\begin{cases} \mu = n_1 d_{11} p_1 + n_2 d_{12} \dfrac{p_2}{\tau} \\ \mu = n_1 d_{21} \dfrac{p_1}{\tau} + n_2 d_{22} p_2 \, , \end{cases} \tag{2.8}$$

where, given the Cobb-Douglas formulation in (2.1), μ is the share of agents (unitary) income used to buy manufacturing goods. Solving for the demands one finds

$$d_{11} = \frac{\mu}{n_1 p_1 + n_2 p_1^{\sigma} p_2^{1-\sigma} \tau^{\sigma-1}} \qquad d_{12} = \frac{\mu \tau^{\sigma}}{n_1 p_1^{1-\sigma} p_2^{\sigma} + n_1 p_2 \tau^{\sigma-1}}$$

$$d_{22} = \frac{\mu}{n_1 p_1^{1-\sigma} p_2^{\sigma} \tau^{\sigma-1} + n_2 p_2} \qquad d_{21} = \frac{\mu \tau^{\sigma}}{n_1 p_1 \tau^{\sigma-1} + n_2 p_1^{\sigma} p_2^{1-\sigma}} \, . \tag{2.9}$$

Equating, location by location, firms supply and consumers demand, gives

$$\begin{cases} y_1 = I d_{11} + \dfrac{I d_{21}}{\tau} \\ \\ y_2 = I d_{22} + \dfrac{I d_{12}}{\tau} \, , \end{cases} \tag{2.10}$$

where τ discounts the demand of imported goods. Plugging (2.9) in (2.10) and using (2.6) one can solve for market equilibrium quantities

$$\begin{cases} y_1 = \dfrac{I \mu (\sigma - 1)}{\beta \sigma} \left(\dfrac{1}{n_1 + n_2 \tau^{\sigma-1}} + \dfrac{\tau^{\sigma-1}}{n_1 \tau^{\sigma-1} + n_2} \right) \\ \\ y_2 = \dfrac{I \mu (\sigma - 1)}{\beta \sigma} \left(\dfrac{1}{n_2 + n_1 \tau^{\sigma-1}} + \dfrac{\tau^{\sigma-1}}{n_2 \tau^{\sigma-1} + n_1} \right) \, . \end{cases} \tag{2.11}$$

Profits in each location can now be found using the latter expression together with (2.4) and (2.6). Introducing the fraction of firms in location 1, $x = n_1/N$ (so that $n_2 = (1-x)N$), and normalizing the level of wages to one, profits can be finally written as a function of x

$$\begin{cases} \pi_1(x) = \dfrac{I \mu}{N \sigma} \left(\dfrac{1}{x + (1-x)\tau^{\sigma-1}} + \dfrac{\tau^{\sigma-1}}{x \tau^{\sigma-1} + (1-x)} \right) - \alpha_1 \\ \\ \pi_2(x) = \dfrac{I \mu}{N \sigma} \left(\dfrac{1}{x \tau^{\sigma-1} + (1-x)} + \dfrac{\tau^{\sigma-1}}{x + (1-x)\tau^{\sigma-1}} \right) - \alpha_2 \, . \end{cases} \tag{2.12}$$

Each location specific profit function in (2.12) has a positive term proportional to the total demand for good produced in that location and a negative term equal to the location specific fixed costs. In turn, the total demand has a domestic component, first term in parenthesis, and an import component, second term in parenthesis. Both components depend on firms geographical distribution and transportation costs. When transportation cost is zero, $\tau = 1$, they are equal irrespectively of firms distribution. When transportation cost increases, the domestic component increases, as local consumers substitute foreign goods with local ones. For the same reason, the export component decreases. For any given positive transportation cost, when local firm concentration increases, the local component decreases as agents have more local goods to consume, all at the same price. In the same situation the export component increases, because foreign consumers have less local goods to consume and find convenient to import more. The net effects of the transportation cost on the relative profits of the two locations are appraised in the next section. However, even without knowing this effect, it is immediate to see that market forces make the average profits independent on transportation costs. Indeed one has the following

Proposition 2.1. *Consider an economy with two locations, $l = 1, 2$, and N firms, where Assumptions 1-2 are valid. Then the average firms profit $\bar{\pi}$ does not depend on transportation costs and is given by*

$$\bar{\pi} = \frac{2I\mu}{N\sigma} - x\,\alpha_1 - (1 - x)\,\alpha_2\,. \tag{2.13}$$

Proof. See Appendix. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

If now one assumes that $\alpha_1 = \alpha_2$, since location specific indexes have disappeared from any variable, only market forces are at work and our model becomes close to the one of Forslid and Ottaviano (2003). Following Bottazzi and Dindo (2008) we take a different route.

Before doing so, a last remark is necessary concerning the general equilibrium setting. In our framework labour and goods markets are at equilibrium only when total firms profits are zero, and only provided that the demand for labour in both locations is not higher than I. Concerning the former condition, notice that profits are already zero for the competitive agricultural firms but not necessarily so for manufacturing firms. Nevertheless it is possible to set zero profit for the manufacturing sector imposing a long-run equilibrium condition on the size of the economy. We shall do so in Assumption 4 below. Concerning the latter condition one has that, due to labour market segmentation (no mobility), labour demand in both locations should be lower than I. Straightforward computations show that this amounts to impose a restriction on the preferences for manufacturing goods, namely $\mu < \sigma/(2\sigma - 1)$, which we will assume to hold from now on.[2] As a result, provided that preferences for manufacturing goods are not too strong and upon imposing a long-run zero profit condition on the number of firms, prices and quantities as in (2.6) and (2.11) guarantee that both labour and good markets are at equilibrium.

2.1 Technological externalities

By retaining a dependency of the fixed cost α on the location index, we introduce a localized technological externality due to direct firms interaction, that is, not mediated by market forces

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[2]This is the same condition found in e.g. Forslid and Ottaviano (2003). See Bottazzi and Dindo (2008) for more details.

(Scitovsky, 1954).

Assumption 3. "Cost sharing" hypothesis. Firms fixed costs α_l decreases with the number of firms located in l according to

$$\alpha_l = \frac{\alpha}{2x_l}, \qquad \text{where} \quad x_l = \frac{n_l}{N}, l \in \{0, 1\}. \tag{2.14}$$

Assumption 3 represents a positive technological externality in the form of a baseline "cost sharing": the larger the number of firms in one location, the lower the fixed costs these firms bear in the production activity. Since the fixed cost payed by firms in a given location decreases proportionally with the number of firms populating that location, the total fixed cost payed remains, location by location, constant. This effect can be thought as an up front cost payed to improve access to skilled labour, the more firms in one location the smaller each firm investment in training, or as a cost for services or infrastructure use, which are evenly shared among all the active firms in one location.

An important feature of the specific form of "cost sharing" introduced in Assumption 3 is that it doesn't modify the total fixed costs payed by the industry. This has consequences on the computation of firms average profit.

Corollary 2.1. *Consider an economy with two locations, $l = 1, 2$, and N firms, where Assumptions 1-3 are valid. Total fixed costs in each location are equal to $\alpha N/2$. The average firms profit $\bar{\pi}$ does not depend neither on firms distribution x nor on transportation cost τ and is given by*

$$\bar{\pi} = \frac{2I\mu}{N\sigma} - \alpha. \tag{2.15}$$

Proof. See Appendix. $\square$

Before we start to look for geographical equilibria, that is, those spatial distributions of firms where they have no incentives to change location, notice that, without restrictions on the parameters values, there could exist economies characterized by negative profits. In this case, we would expect firms to exit the economy. On the other hand, if profits were positive we could expect firms to enter the economy. As we consider the number of firms N in the model as given, if there are not barriers to entry, it seems reasonable to set the number N to a level which implies zero profits. By force of Corollary 2.1 this can be done also without knowing the geographical equilibrium distribution. Indeed profits at a geographical equilibrium must be equal to average profits and average profits (2.15), due to Corollary 2.1, are independent on the geographical distribution x. Moreover, as we have explained at the end of the previous subsection, zero total profits are needed in order to guarantee that all markets are at an equilibrium. All together, it is enough to have the following

Assumption 4. The number of firms N is such that profits at a geographical equilibrium are zero, that is

$$N = \frac{2I\mu}{\sigma\alpha}. \tag{2.16}$$

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Even if, by construction, the previous assumption implies $\bar{\pi} = 0$, outside the geographical equilibrium profits can be both positive or negative so that their differential gives firms the incentive to relocate. Before moving to the analysis of these incentives and to the characterization of the geographical equilibria, it is useful to rewrite profits (2.12) incorporating Assumptions 3-4

$$
\begin{cases}
\pi_1(x) & = \dfrac{\alpha}{2}\left(\dfrac{1}{x+(1-x)\tau^{\sigma-1}} + \dfrac{\tau^{\sigma-1}}{x\tau^{\sigma-1}+(1-x)}\right) - \dfrac{\alpha}{2x}, \\
\pi_2(x) & = \dfrac{\alpha}{2}\left(\dfrac{1}{x\tau^{\sigma-1}+(1-x)} + \dfrac{\tau^{\sigma-1}}{x+(1-x)\tau^{\sigma-1}}\right) - \dfrac{\alpha}{2(1-x)}.
\end{cases}
\tag{2.17}
$$

Given the firms production costs α, the products elasticity of substitution σ and the transportation cost τ, the distribution of firms between the two locations, x, determines, through (2.17), the levels of profit. Notice that in (2.17), differently from (2.12), both the demand driven term and the fixed cost term are functions of the geographical distribution of firms. The first dependence is mediated by market forces (pecuniary externality) whereas the second dependence is brought in by the "cost sharing" hypothesis (technological externality).

3   Geographical equilibria

In this section we investigate the static geographical equilibria of the system, that is, those distributions of firms x such that, in the search for higher profits, each firm located in 1 has no incentives to move to 2 and *vice versa*. Geographical equilibria can be of two types: "border" equilibria and "interior" equilibria. A border equilibrium occurs when firms concentrate in one location, say 1, and profits in 1 are higher than profits in 2. As all the firms are in 1, no other firm can respond to this difference in profit opportunities. Candidates for border equilibria are $x = 1$, when all firms are in 1, and $x = 0$, when all firms are in 2. Conversely, an interior equilibrium occurs when firms distribute among the two locations, that is $x \in (0,1)$, profit levels are equal in both regions, and firms do not have any incentive to change their location. Using profits in (2.17) we will derive results for the existence and uniqueness of geographical equilibria, border and interior, for all the different parameterizations of the economy. This static[3] analysis, which owes considerably to Bottazzi and Dindo (2008) where more details can be found, is useful to understand the interplay of pecuniary and technological externalities and constitutes an useful step for the development of the evolutionary dynamic analysis of the next section.

The respective role of each externality in determining profit differentials and thus the aggregate geographical equilibrium can be judged by looking at the shape of the profit functions and by keeping in mind that, due to transportation costs, local prices are lower than foreign prices, and thus local demand impacts firms level of output more than foreign demand. Consider the pecuniary externality term alone, e.g. sets $\alpha = 0$ in (2.17). For definiteness, consider profits in 1 (results for profits in 2 follows in the same way). For small x, that is, few firms in location 1 and many firms in location 2, each firm in 1 faces a high local demand and a low foreign demand. Due to the different impact of local and foreign demand, the level of output of firms in 1 is high and profits are high too. As x increases, the local demand for these firms decreases, so that profits decrease too. As the concentration of firms

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[3]Technically our geographical equilibrium corresponds to a Nash Equilibria in Pure Strategies of the one shot game where each firm in a group of N has to chose whether to be located in 1 or in 2 and payoffs are given by profits.

in 1 increases further, for sufficiently large value of x, the demand coming from the consumers in 2, where very few firms are left, is more and more directed to 1 and the profits of firms located in 1 increase again. Profits are thus U-shaped, with $\pi_1(x)|_{\alpha=0}$ first decreasing and then increasing in x. Since a firm makes the most profits when alone in one location, we have $\pi_1(x = 0)|_{\alpha=0} > \pi_1(x = 1)|_{\alpha=0}$ so that the border distributions 0 and 1 are never an equilibrium. In fact, when all firm are located in one region is always more profitable to move to the other region. It the transportation cost is increased (decreased) the variation of profits as a function of x is more (less) pronounced but the general shape of the profit function is preserved. As a result, the overall agglomeration effect of the pecuniary externality is always "negative", in the sense that it works against concentration of production.

The above picture changes completely when one considers also the technological externality terms introduced by the "cost sharing" assumption, i.e. $\alpha > 0$ in (2.17). The panels in Fig. 1 show graphs of $\pi_1(x)$ and $\pi_2(x)$ in this case. Profits are given by the superposition of a monotonically increasing technological externality to the U-shaped market driven pecuniary externality term. With low transportation costs (high τ) the profit function is essentially determined by the "cost sharing" term and is monotonically increasing with decreasing marginal profits (upper panels of Fig. 1). In this case, when firms concentration is low, firms do not benefit from the technological externality and their profits are low too, but when concentration increases, profits increase monotonically as firms exploit the "cost sharing" opportunity. The more the firms in one location, the lower the positive contribution of an extra firm locating there, so that the marginal profit decreases. With high transportation costs (low τ) the shape of the profit function is still monotonic (bottom panels of Fig. 1), but marginal profits are first decreasing and then increasing. With low firm concentration the technological externality dominates and marginal profits are decreasing. As the concentration increases, the positive effect of the cost sharing is almost offset by the negative market interaction, which acts as a constraint on the local demand faced by firms. In this case, even if profits are still increasing the marginal profit is almost zero. As the concentration of firms increases further, profits increase more steadily because low local demand is now compensated by the foreign demand, so that the contribution of the pecuniary externality is positive too. Judging from Fig. 1, irrespectively of the transportation costs, the positive effect of technological externalities is dominating the negative effect of pecuniary externalities: firms make most profits by agglomerating on one side and border distributions are always equilibria. This is formalized in

Proposition 3.1. *Consider an economy with two locations, $l = 1, 2$ where Assumptions 1-4 are valid. Call x the fraction of firms located in 1. There always exists two, and only two, geographical equilibria given by the border distribution $x_1^* = 1$ and $x_0^* = 0$. In particular, the unique distribution where profits are equal, $x^* = 0.5$, is never an equilibrium.*

Proof. See Appendix. □

According to the previous proposition, the distribution with half of the firms located in 1 and the other half located in 2, which is the unique case where $\pi_1 = \pi_2$, is never a geographical equilibrium: even if profits are equal, incentives are such that firms move away and agglomerate. Only when all firms are located either in 1 or 2 there are no incentives to change location.

Notice that, even if transportation costs do affect the shape of each location profit function, they have no impact in characterizing the geographical equilibria of the economy. Conversely,
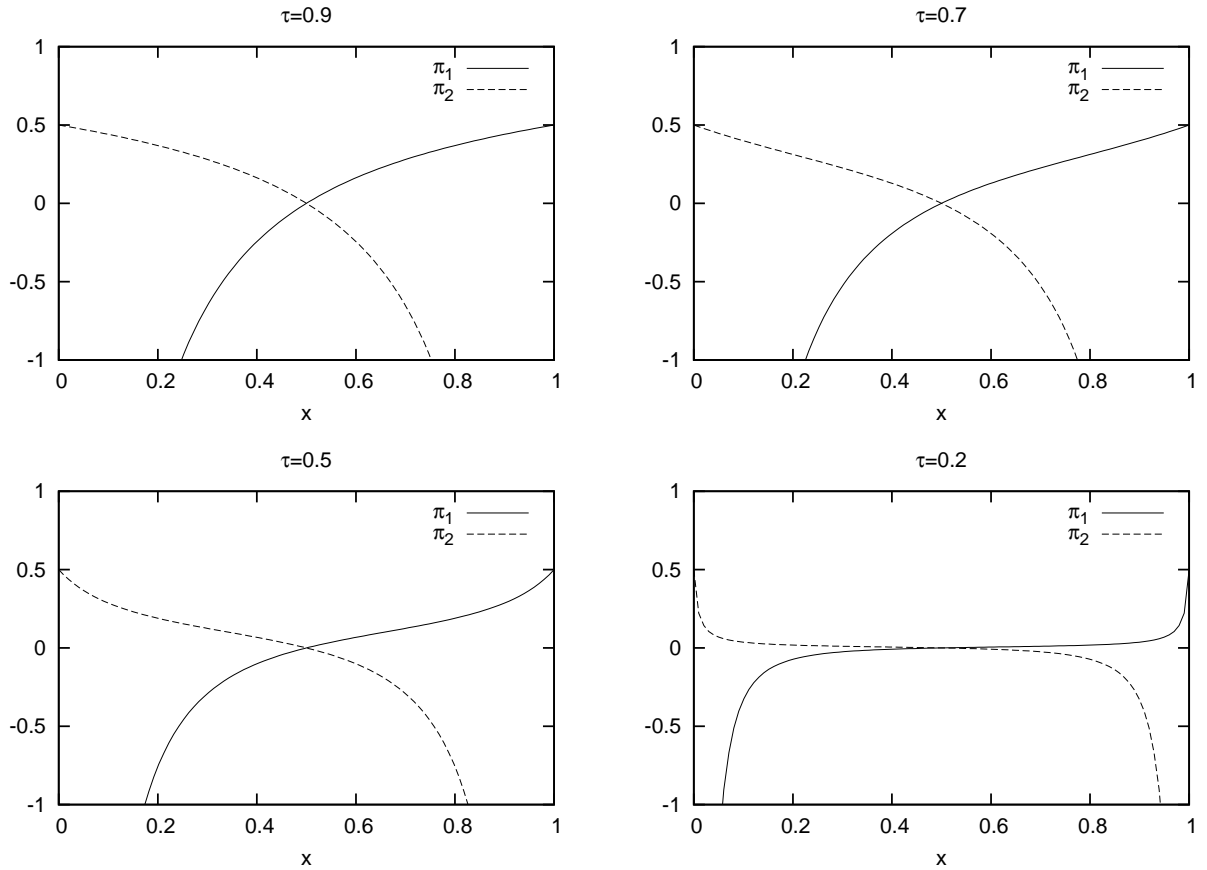
Figure 1: Location profits (2.17) as a function of firms geographical distribution for different values of the transportation cost τ. Other parameters are $\sigma = 4$ and $\alpha = 1$.

as we shall see in the following section, transportation costs will play a major role in shaping the results of the evolutionary model, even in the long run.

4 Evolutionary firm dynamics

In the previous section we have shown that, when the technological externality term is introduced, firms agglomerate in one of the two locations, irrespectively of transportation costs τ or the relevance of technological spillover as dependent on α. This abrupt behaviour would prescribe that any sector in which even a minimal level of localized non-pecuniary externalities operate should display a so-called core-periphery structure. This is clearly at odds with empirical observations. Notice that this conclusion would remain *a fortiori* valid if instead we had considered workers mobility with endogenous wage setting, thus introducing a feed-back effect which reduces (or inverts) the push of pecuniary externalities towards a symmetric geographical distribution. This effect ultimately reinforces the conclusion that in presence of technological spillovers only a core-periphery structure represents an equilibrium. We end up in the uncomfortable situation of having a single possible equilibrium, implying the impossibility of performing empirical analysis or to derive policy implication. A possible way out from this *impasse*, as we will show, is to extend the notion of geographical equilibrium to include an explicit dynamics describing firms locational decisions. The foregoing analysis is, indeed,

essentially static and thus silent on the results of firms interactions out of equilibrium. As a consequence, it is not clear what happens when the initial concentration of firms is not at an equilibrium level, in particular, whether one should expect firms to agglomerate in location 1 or in location 2.

In this section we extend our analysis by introducing heterogeneity in preferences at the single firm level and by explicitly modeling firms decisions in time, that is, by allowing for a dynamic location selection mechanism. Suppose that the individual utility of firm i derived from locating in l_i can be written as

$$\pi_i = \pi_{l_i} + e_{i,l_i} \tag{4.1}$$

where π_{l_i} are as in (2.17) and e_{i,l_i} represents an idiosyncratic profit component intended to capture firm-specific characteristics, like differences in productive efficiency leading to different fixed costs or individual preferences for one particular location, due, for instance, to existing social linkages. At every time step a firm is randomly chosen to exit the economy. At the same time a new firm enters and chooses whether to be located in 1 or in 2 by comparing the individual utilities in (4.1). As long as the distribution of $e_{i,l}$ across firms is well behaved (cfr. Bottazzi and Secchi, 2007; Bottazzi et al., 2007, for details) the resulting probability of choosing l is given by

$$\text{Prob}_l = \frac{e^{\pi_l}}{e^{\pi_1} + e^{\pi_2}}, \quad l \in \{1, 2\}. \tag{4.2}$$

The fact that the locational choice is probabilistic derives from the assumption that the new entrant possesses preferences, or faces costs, which are not fixed, but contain an individual component which is randomly extracted from a given distribution.

When the probability of choosing location l is given by (4.2), Bottazzi and Secchi (2007) show that, if the exponentials of profits are linearly changing in the number of firms, it is possible to compute the long run stationary distribution of the entry-exit process. Thus, to exploit this result we need a linearized version of the exponential profit functions. We can naturally obtain it as deviation from the middle point $x^* = 0.5$, that is, the unique point where profits are equal.

Proposition 4.1. *Consider an economy with two locations, $l = 1, 2$, where Assumptions 1-4 are valid. Denote the linearization of location l exponential profits around $x^* = 0.5$ as c_l, and the number of firms in location l as n_l. Linearized exponential profits are given by*

$$c_l = a + bn_l, \quad l = 1, 2, \tag{4.3}$$

where

$$
\begin{aligned}
a &= 1 - \frac{4\alpha\tau^{\sigma-1}}{(1 + \tau^{\sigma-1})^2}, \\
b &= \frac{4\alpha^2\sigma\tau^{\sigma-1}}{I\mu(1 + \tau^{\sigma-1})^2}.
\end{aligned} \tag{4.4}
$$

Proof. See Appendix. □

We shall call the term a in (4.4) the "intrinsic profit". This is the part of the common profit which is entirely dependent on exogenously given characteristics of the location. Conversely,

the coefficient b in (4.4) captures the marginal contribution of a firm to the profit level of the location in which it resides. We shall call it the "marginal profit".[4] In our case, this coefficient captures the total effect of pecuniary and technological externalities. Due to the leading effect of the latter it is always positive but, due to the presence of market mediated interactions, it is dependent on transportation costs. Specifically, the marginal profit is increasing with the value of τ. When transportation costs are high (low τ) each firm marginal contribution to the location profit is small, whereas when transportation costs are low (high τ) the marginal contribution is large.

Given the linearization in 4.3, the following proposition characterizes the long run geographical equilibrium distribution.

Proposition 4.2. *Consider an economy with two locations, $l = 1, 2$, where Assumptions 1-4 are valid. The economy is populated by N firms, distributed according to $\boldsymbol{n} = (n_1, n_2)$. At the beginning of each period of time a firm is randomly selected, with equal probability over the entire population, to exit the economy. Let $m \in \{1, 2\}$ be the location affected by this exit. After exit takes place, a new firm enters the economy and, conditional on the exit occurred in m, has a probability*

$$Prob_l = \frac{a + b(n_l - \delta_{l,m})}{2a + bN}$$

to chose location l, where a and b are given by (4.4). This process admits a unique stationary distribution

$$\pi(\boldsymbol{n}) = \frac{N! C(N, a, b)}{Z(N, a, b)} \prod_{l=1}^{2} \frac{1}{n_l!} \vartheta_{n_l}(a, b) \,, \tag{4.5}$$

where

$$C(N, a, b) = 2a + \left(1 - \frac{1}{N}\right) bN \,, \tag{4.6}$$

$$\vartheta_n(a, b) = \begin{cases} \prod_{h=1}^{n}[a + b(h - 1)] & n > 0 \\ 1 & n = 0 \end{cases} \tag{4.7}$$

and $Z(N, a, b)$ is a normalization factor which depends only on the total number of firms N, and the coefficients a and b.

Proof. See Propositions $3.1 - 3.4$ of Bottazzi and Secchi (2007). □

Figure 2 shows results from a simulation of the entry-exit process for two different values of the transportation cost τ. The left panel shows 50000 iterations of the process, whereas the right panel plots the corresponding long run distributions as characterized by Proposition 4.2.

With low transportation costs ($\tau = 0.95$) the long run distribution is clustered around the two extreme values, $x = 0$ and $x = 1$, confirming the prediction of the static analysis. However, the simulation of the entry-exit process (left panel of Fig. 2) shows that agglomeration is only a meta-stable state. One location can become much larger than the other for several time steps, like location 2 which, in the simulation shown, attracts almost all firms in the periods between 2000 and 3500, but at some point the cluster abruptly disappears and the other location can

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[4]In the terminology of Bottazzi and Secchi (2007) the intrinsic profit corresponds to the location "intrinsic attractiveness" whereas the marginal profit is the "social externality".
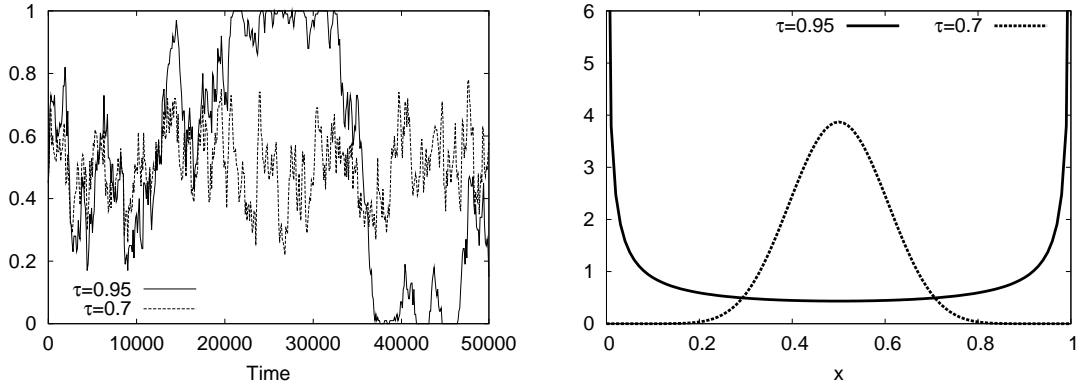
Figure 2: Entry-exit process for different values of the transportation costs. **Left panel**. 50000 simulations of the entry-exit process for different values of the transportation cost τ. **Right panel**. Long run stationary distribution of the entry-exit process simulated in the left panel. In both panels the parameters are $\sigma = 4$, $\alpha = 1$, $\mu = 0.5$ and $I = 800$.

take over. This behavior is well in accordance with the bimodal nature of the equilibrium distribution (c.f. right panel of Fig. 2). In fact, the equilibrium distribution represents the unconditional probability of finding the system in a give state. This probability can thus be very different from the frequency with which this particular state is observed over a finite time window.

Conversely, for higher transportation costs ($\tau = 0.7$), agglomeration is "almost" never observed: firms spatial distribution is now fluctuating around $x^* = 0.5$ (c.f. left panel of Fig. 2). Even if the static analysis predicts agglomeration, the equilibrium distribution of the stochastic system, reported in the right panel of Fig. 2, shows that the most likely geographical distribution has an equal number of firms per location, irrespectively of the fact that the point $x^* = 0.5$ is never a static geographical equilibrium. In general one has the following

Proposition 4.3. *Consider the entry and exit process described in Proposition 4.2. When the marginal profit is bigger than the intrinsic profit, $b > a$, the stationary distribution (4.5) is bimodal with modes in $x = 0$ and $x = 1$, when $b < a$ the stationary distribution is unimodal with mode in $x = 0.5$, and when $a = b$ the stationary distribution is uniform.*

Proof. See Appendix. □

Given our dynamic locational decision process, the previous proposition clarifies that the shape of the geographical equilibrium distribution does ultimately depend on the relative size of the marginal profit b and the intrinsic profit a. When marginal profits are bigger than intrinsic profits the distribution has mass on the borders of the $[0,1]$ interval. When marginal profits are lower than intrinsic profits the distribution has higher mass in the middle of this interval, and when they are equal every value of the geographical distribution is as likely.

Rewriting the relation $b \gtrless a$ in Proposition 4.3 using the definitions of a and b in (4.4), it is straightforward to derive the conditions for the unimodality or bimodality of (4.5) in terms of the values of τ, I, μ, and α:

$$4\alpha \left(1 + \frac{\alpha\sigma}{I\mu}\right) \gtrless \frac{(1 + \tau^{\sigma-1})^2}{\tau^{\sigma-1}}.$$
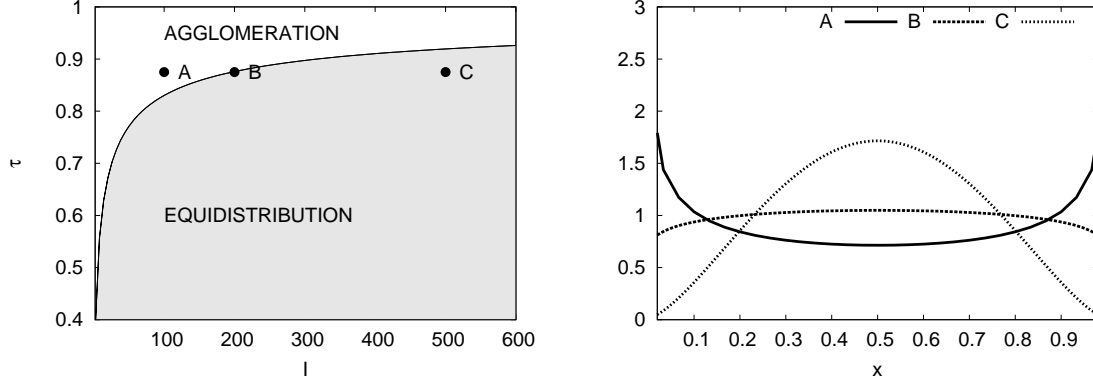
14

Figure 3: Entry-exit process for different values of the number of residents I. **Left panel**. A portion of the space (I, τ) has been divided into the "agglomeration" area (white) and the "equidistribution" area (shaded) according to Proposition 4.3. **Right panel**. Stationary geographical distributions computed at the points A, B, and C. Other parameters are $\alpha = 1$, $\mu = 0.5$ and $\sigma = 4$ whereas N is fixed by Ass. 4.

Figure 4: Entry-exit process for different values of the fixed costs α. **Left panel**. A portion of the space (α, τ) has been divided into the "agglomeration" area (white) and the "equidistribution" area (shaded) according to Proposition 4.3. **Right panel**. Stationary geographical distributions computed at the points A, B, and C. Other parameters are $I = 400$, $\mu = 0.5$ and $\sigma = 4$ whereas N is fixed by Ass. 4.
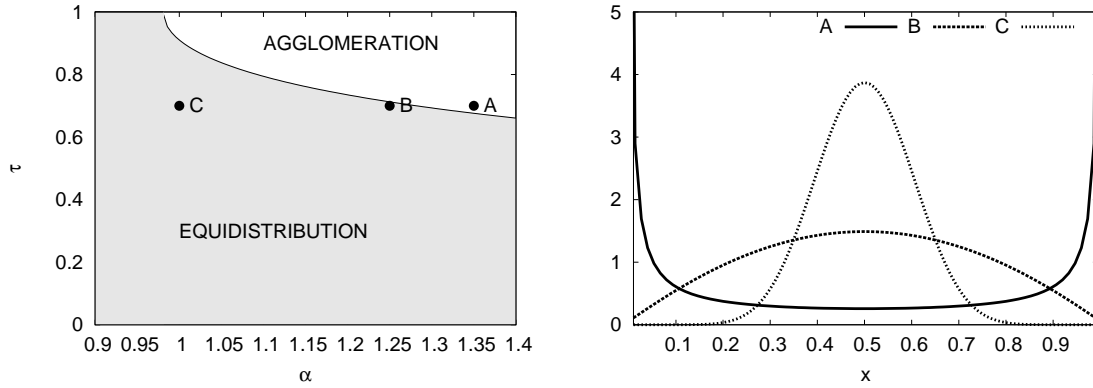
The left panels of Figs. 3 and 4 have been obtained using the latter inequality: they show which distributional shape is observed in the different regions of the plane (I, τ) and (α, τ) respectively. In the white area agglomeration is most likely (bimodal distribution), whereas in the dark area equidistribution is most likely (unimodal distribution). In the right panels the stationary distributions computed at the corresponding points A, B, and C are shown. In both figures these points have been obtained by keeping τ fixed.

The right panel of Fig. 3 shows that moving from small to large values of I while keeping τ fixed, the long run distribution changes from bimodal to unimodal. This is due to the fact that an increase of the number of residents I leads to a decrease of the marginal profit b (c.f. (4.4)). In fact, due to Assumption 4, the more the residents, the more the firms and the smaller the contribution of each firm locational decision to the profits of other firms, that is, the smaller

the marginal profit. Changing I corresponds to a sort of "size" effect: increasing the size of the economy lowers the externalities so that, due to the entry-exit process, the likelihood to observe agglomeration is lowered.

The right panel of Fig. 3 shows that, keeping τ fixed, an increase in the fixed cost parameter α leads, in general, to more agglomerated economies. It is so because an increase in α decreases the intrinsic profit a while increasing the marginal profit b. More precisely, α determines the scale of the profit differentials. Indeed, on the one hand, the difference between the maximum and the minimum profit is proportional to α and, on the other hand, because of Assumption 4, the higher α the lower N, so that a bigger profit difference is caused by a lower number of firms. As a result, increasing fixed costs decreases the profit each firm earns irrespectively of the presence of other firms, and increases the effect of each locational choice on the profits of others. Both effects go in the direction of increasing the likelihood of agglomeration. This is a sort of "scale" effect where increasing the scale of profits increases the likelihood of agglomeration.

Concerning the effect of the transportation cost on the shape of the equilibrium distribution, notice that the expression $\tau^{\sigma-1}/(1+\tau^{\sigma-1})^2$, which appears in (4.4) for both a and b, but with a different sign, is an increasing function of τ. Thus, increasing the value of τ leads to an increase of the marginal profit b and to a decrease of the intrinsic profit a. This means that low transportation costs, that is, high values of τ, favor agglomeration, while high transportation costs favor equidistribution. Indeed, when transportation costs are low, the pecuniary externality is relatively weak and the technological externality relatively strong. In terms of the entry-exit process, the choice of a firm to relocate its activity has an high impact on the level of profits. Consequently, it is likely to trigger other relocations and, eventually, a strong agglomeration is observed. Conversely, when transportation costs are high, the pecuniary and technological externalities almost offset each other. This implies that marginal profits are small and intrinsic profits dominate, so that each locational choice has a very small impact on the general level of profits. The attracting force of each location does not depend on the externality term and, given the symmetry of the two locations, equidistribution is likely to be observed.

5 Conclusion

We have analyzed a model of firms location in geographical space where firms interact both indirectly, through market interactions, and directly, through technological externalities, and where workers are not mobile. In this simple framework we have briefly discussed the general equilibrium static case, identifying the possible geographical equilibria, that is, the spatial distributions in which firms do not have any incentive to relocate their activities. We have showed that in this case the "cost sharing" assumption implies long run agglomeration, irrespectively of the number of consumers, their preferences, and transportation costs. Then we have extended the analysis including heterogeneity in firms preferences and an explicit time dynamics in their choices thus obtaining a stochastic model of firms dynamics. We have been able to characterize the long run geographical distribution of the process for different specifications of the economy. This analysis has revealed that, contrary to the static equilibrium analysis, when an explicit entry-exit dynamics is assumed to characterize the locational decision of firms, the economy can evolve towards two different long run scenarios. In the first scenario, where externalities are stronger than intrinsic location profits, which typically occurs for low transportation costs, the long run geographical distribution is bimodal with modes at

the extremal outcomes $x = 0$ and $x = 1$. Agglomeration is thus the most likely event but, as simulations show, this does not mean that once agglomeration on one side has been achieved, the situation is stable. In fact, turning points exist where the mass of firms moves from one location to the other. In the second scenario the long run geographical distribution has a unique mode. In this case, the most likely occurrence is having half of the firms located in one region and the other half in the other region. However, due to the stochastic nature of the process, fluctuations around this average level are present. This scenario is typically associated with high transportation costs, and occurs, in general, when the effect of externalities is weak with respect to the intrinsic profit levels of each location.

Summarizing, the main contribution of the foregoing analysis is to show how firms heterogeneity and an individual choice process act as breaks or constraints to firms agglomeration, even when strong incentives to locate in already populous locations exist. Moreover, having introduced an explicit time dimension, we have given history a role. Indeed the time dimension matters in two respects: first, the initial distribution of activities across two locations does influence the subsequent observed distributions and, second, when agglomeration is observed, due to stochastic fluctuations, it is only a metastable phenomenon. That is, by waiting long enough, the cluster eventually disappears just to be soon recreated, with probability $1/2$, in the other location.

Our model can be extended in several directions. First of all, the "cost sharing" assumption, while useful, is admittedly *ad hoc*. A more careful modeling is probably needed. The effort should not be restricted to the notion of technological and/or knowledge spillover, which might even be characterized by a pecuniary nature, see e.g. Antonelli (2008), but could encompass also other, possible negative, sources of not market mediated interactions, like pollution and/or congestion effects. A second extension of the model would be to generalize consumers behavior along the same lines we followed to describe firms behavior. Whereas in the present version of the model consumers are homogeneous and maximize the same CES utility function, it would be interesting to assume that consumers are heterogeneous and to explicitly model their consumption decision in time. In that case, changing the size of the economy would imply, due to varying idiosyncrasies in consumers demand, a change in the amplitude of profits fluctuations. This, in turn, would impact the likelihood of observing agglomerated outcomes, probably reducing it.

In any case, we are aware that the ultimate test bed will be to confront our model with real data. An interesting aspect of the discrete choice model we implemented is that it leads quite easily to empirical applications. An exercise in this direction has already been performed in Bottazzi et al. (2008) where the parameters characterizing the geographical equilibrium distribution have been estimated in several sectors of the Italian manufacturing industry. The present work moves in the direction of developing a theoretical framework able to provide deeper and more informative economic interpretations of these econometric exercises.

A   Appendix

Proof of Proposition 2.1 and Corollary 2.1   Average profits follow in a straightforward way by computing $\bar{\pi} = x\pi_1(x) + (1-x)\pi_2(x)$ with $\pi_1(x)$ and $\pi_2(x)$ as in (2.12) and in (2.17). Total costs in each location are equal because (2.14) implies:

$$\sum_{i=1,n_1} \alpha_i = \sum_{j=1,n_2} \alpha_j = \frac{N\alpha}{2}.$$

$\square$

Proof of Proposition 3.1 First, the unique firms distribution where profits are equal is $x = 0.5$. Indeed taking profits as in (2.17), setting $\pi_1(x)$ equal to $\pi_2(x)$ gives a first order equation in x whose unique solution is $x = 0.5$.

Geographical equilibria are Pure Strategy Nash Equilibria (PSNE) of the one stage game where each firm in a group of N (N even), has to choose to be located in $l = 1$ or $l = 2$ and profits are given by (2.17). Denote firm $i = 1, \ldots, N$ strategy as s_i. Firm i can choose to locate either in 1, $s_i = 1$, or in 2, $s_i = 0$. The strategy space has thus 2^N elements. A strategy profile will be denoted as s while s_{-i} will denote the strategy profile of the $N - 1$ firms but i. Define also

$$x(s) = \frac{\sum_{i=1,\ldots,N} s_i}{N}.$$

To complete the formalization of the game we have to specify each firm payoff for any strategy profile s. When $s_i = 1$, firm i payoff π_i is given by

$$\pi_i(1, s_{-i}) \equiv \pi_1(x(s)),$$

where $\pi_1(x)$ is as in (2.17) and $x(s)$ is defined above. When $s_i = 0$, firm i payoff is

$$\pi_i(0, s_{-i}) \equiv \pi_2(x(s)),$$

where $\pi_2(x)$ is again from (2.17). To give an example, if all firms are choosing location $l = 1$, so that $x = 1$, one has $\pi_i = \pi_1(1)$ for all $i = 1, \ldots, N$. If, instead, half of the firms are located in $l = 1$ and the other half in $l = 2$, so that $x = 0.5$, one has $\pi_i = \pi_1(0.5)$ if $s_i = 1$ and $\pi_i = \pi_2(0.5)$ otherwise. A strategy profile s^* is a PSNE if and only if

$$\pi_i(s_i^*, s_{-i}^*) \geq \pi_i(s_i, s_{-i}^*) \quad \text{for all} \quad s_i = 0, 1, \quad i = 1, \ldots, N. \tag{A.1}$$

The only candidates to be PSNE are those strategy profiles s for which $x(s) \in \{0, 1, 0.5\}$. We start by showing that every s^* such that $x(s^*) = 1$ or $x(s^*) = 0$ is a PSNE profile. From (2.17) it holds that $\pi_1(1) > \pi_2(x)$ for all $x \in [0, 1)$ and $\pi_2(0) > \pi_1(x)$ for all $x \in (0, 1]$, so that (A.1) is satisfied. Then consider a strategy profile s^* such that $x(s^*) = x^* = 0.5$. Given (A.1), it is a PSNE if and only if

$$\pi_1(0.5) \geq \pi_2\left(0.5 - \frac{1}{N}\right) \forall i = 1, \ldots, N \tag{A.2}$$

$$\pi_2(0.5) \geq \pi_1\left(0.5 + \frac{1}{N}\right) \forall i = 1, \ldots, N. \tag{A.3}$$

Since the two locations are identical and x is the share of firms choosing location 1, by symmetry it holds that $\pi_1(x) = \pi_2(1 - x)$. Using this relation we can rewrite both (A.2) and (A.3) as

$$\pi_1(0.5) \geq \pi_1\left(0.5 + \frac{1}{N}\right) \forall i = 1, \ldots, N.$$

The latter is satisfied if and only if the function $\pi_1(x)$ is not increasing at $x = 0.5$. Direct computation of $d\pi_1(x)/dx|_{x=0.5}$ (see the following proofs for the explicit expression) shows that this is the never the case, implying that the symmetric distribution is never a PSNE. $\square$

Proof of Proposition 4.2 Consider the Taylor expansion up to the first order of each term in (2.17) as a function of $z = x - 0.5$

$$\pi_l(z) = \pi_l(0.5) + z\,(-)^l \left(2\alpha \left(\frac{1 - t^{\sigma-1}}{1 + t^{\sigma-1}} \right)^2 - 2\alpha \right) + O(z^2), \quad l = 1, 2.$$

Using the expressions above to linearize $\exp \pi_l$ in (4.2), and writing them in terms of the number of firms n_l, $l = 1, 2$ we obtain the expressions of the linearized exponential payoff c_l,

$$c_1 = 1 - 2\alpha \left(\frac{(1 - t^{\sigma-1})^2}{(1 + t^{\sigma-1})^2} - 1 \right) \left(\frac{n_1}{N} - \frac{1}{2} \right)$$

$$c_2 = 1 + 2\alpha \left(\frac{(1 - t^{\sigma-1})^2}{(1 + t^{\sigma-1})^2} - 1 \right) \left(\frac{1}{2} - \frac{n_2}{N} \right).$$

This shows that a and b are given by

$$a = 1 + \alpha \left(\frac{(1 - t^{\sigma-1})^2}{(1 + t^{\sigma-1})^2} - 1 \right)$$

$$b = -\frac{2\alpha}{N} \left(\frac{(1 - t^{\sigma-1})^2}{(1 + t^{\sigma-1})^2} - 1 \right),$$

which, using Assumption 4 to eliminate N, correspond to (4.4). $\qquad\square$

Proof of Proposition 4.3 From (4.5) it follows that the distribution is symmetric around $N/2$, that is $\pi(N/2 + n) = \pi(N/2 - n)$ for every $n = 0, \dots, N/2$. Consequently it suffices to analyse the set $\{0, \dots, N/2\}$.

When $a = b$, $\theta_n(a, b)$ reduces to $n!$. As a result the probability density becomes

$$\pi(n) = \frac{N! C(N, a, b)}{Z(N, a, b)} \quad \forall n,$$

so that the distribution is uniform. The rest of the lemma will be proved by induction. First consider

$$\pi(0) \gtrless \pi(1)$$
$$\frac{N! C(N, a, b)}{Z(N, a, b)} \frac{1}{N!} \theta_N(a, b) \gtrless \frac{N! C(N, a, b)}{Z(N, a, b)} \frac{1}{(N-1)!} \theta_1(a, b)\theta_{N-1}(a, b)$$
$$a(a + b) \dots (a + b(N - 1)) \gtrless Na^2(a + b) \dots (a + b(N - 2))$$
$$b \gtrless a. \tag{A.4}$$

Second consider

$$\pi(n) \gtrless \pi(n + 1)$$
$$\frac{N! C(N,a,b)}{Z(N,a,b)} \frac{1}{n!(N-n)!} \theta_n(a, b)\theta_{N-n}(a, b) \gtrless \frac{N! C(N,a,b)}{Z(N,a,b)} \frac{1}{(n+1)!(N-n-1)!} \theta_{n+1}(a, b)\theta_{N-n-1}(a, b)$$
$$(n + 1)a(a + b) \dots (a + b(n - 1))a(a + b) \dots (a + b(N - n - 1)) \gtrless$$
$$\gtrless (n + 1)a(a + b) \dots (a + bn)a(a + b) \dots (a + b(N - n - 2))$$
$$b(N - 2n - 1) \gtrless B(N - 2n - 1)$$
$$b \gtrless a, \tag{A.5}$$

where the last step requires $n \leq N/2 - 1$, which is our case. From A.4 and A.5 it follows that when $b > a$ the maximum is in $\pi(0)$ (and by symmetry in $\pi(N)$), whereas, when $b < a$, the maximum is in $\pi(N/2)$. $\qquad\square$

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