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Framing the empirical findings on firm growth

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Abstract

This paper proposes a general framework to account for the divergent results in the empirical literature on the relation between firm sizes and growth rates, and on many results on growth autocorrelation. In particular, we provide an explanation for why traces of the LPE sometimes occur in conditional mean (i.e. OLS) autoregressions of firm size or firm growth, and in conditional median (i.e. least absolute deviation) autoregressions, but never in high or low quantile autoregressions. Based on an original empirical analysis of the population of manufacturing firms in the Netherlands between 1994 and 2004, we find that there is no peculiar role played by the median of the growth distribution, which is approximately equal to zero independent of firm size. In economic terms, this is equivalent to saying that most of the phenomena of interest for industrial dynamics can be studied without reference to the behaviour of the median firm, and many 'average' relations retrieved in the literature, starting from the negative relation between average size and average growth, are driven by the few dynamic firms in the sample rather than the many stable ones. Moreover, we observe the tent shape of the empirical firm growth rate distribution and confirm the skewness-size and the variance-size relations. The identified quantile regression patterns - autoregressive coefficients above 1 for fast decliners, and below 1 for fast growers - can be obtained by assuming negative variance-size scaling and Laplace growth rate distributions, and are robust to a mild positive relationship between skewness and size. A relationship between quantile regression patterns and previous findings is therefore uncovered.

Keywords: Firm growth; Law of Proportionate Effect; quantile regression; heterogeneity; variance-size scaling.

JEL Classifications: L11; L25; L60.

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1. Introduction

Starting from the earliest empirical tests of the Law of Proportionate Effect (LPE), to the findings of serial correlation in firms' growth rates, the industrial economics and industrial dynamics literature provides several results for firm growth. At first sight, these results seem contradictory and show that firm growth can be in some cases a completely erratic process, and in other cases a process displaying persistence (meaning that firm growth cannot be represented by a random walk process but is more structured). The main purpose of this paper is to propose a general framework, able to account for the divergent results in the empirical literature on firm growth.

The Law of Proportionate Effects (LPE), also known as Gibrat's Law after Gibrat's (1931) seminal work, predicts that firm size follows a random walk and hence that the firm growth is erratic and independent of firm size (Sutton 1997; Bottazzi et al. 2002). If the LPE is satisfied, it means that, on average, small firms do not grow faster than large firms, and that there are no firms that persistently grow in a positive or in a negative direction (i.e. there cannot be persistent outperformers and/or underperformers). The LPE originally was used to explain the highly skewed distribution of firm sizes and has become an empirical and theoretical benchmark in discussions of firm growth (see Steindl 1965; Hart and Prais 1956; Ijiri and Simon 1977; Hall 1987; Lotti et al. 2003; Bottazzi and Secchi 2005; Dosi 2007).

The vast literature testing the LPE provides mixed results (see the comprehensive reviews in Hay and Morris 1991, Sutton 1997, Caves 1998, Lotti et al. 2003 and Dosi and Nelson 2010). Some studies (Hart 1962; Hart and Prais 1956; Simon and Bonini 1958; Hymer and Pashigian 1962; Acs and Audretsch 1990) find evidence of the independence between firm growth and firm size. Other authors (Kumar 1985; Mansfield 1962; Evans 1987a, b; Fotopoulos and Louri 2004; Audretsch and Elston 2002) reject the LPE hypotheses, identifying three main problems. First, that the LPE does not hold for samples of micro and small firms, which have been found to grow faster than larger firms (Mata 1994; Reid 1995) or for new-born/infant/young or entrepreneurial firms (Audretsch, Santarelli and Vivarelli

1999; Lotti et al. 2003).[1] Second, growth processes are shown to deviate from the LPE in terms of the relationship between variance in growth rates and initial firm size (variance-size scaling). Starting with Hymer and Pashigian (1962) and Mansfield (1962) the level of variance in growth rates was found to be negatively correlated with firm size – a finding that was confirmed later by Amaral et al. (1997), Bottazzi et al. (2001) and Axtell (2001).

Third, focusing on the average or median firm may not allow the dynamics of fast growing and fast declining companies to be captured. Quantile regressions (Koenker and Bassett 1978) model the dynamics of the conditional firm size distribution as a whole, 'quantile by quantile' (see also Buchinsky 1998 and Koenker and Hallock 2001). Fotopoulos and Louri (2004) reject the LPE for all quantiles, in a sample of Greek firms for the 1992-1997 period. They find mean reversion, and especially for fast-growing firms. Even more interestingly, the work by Dahl et al. (2010) on Danish manufacturing firms in the 1994-1996 period finds explosive growth for small quantiles and mean reversion for large quantiles, while Gibrat's law is satisfied for the quantiles close to the conditional median. Coad and Hölzl (2009) find similar results for Austrian firms in the service industries. Hence, the few existing applications of quantile regression to firm dynamics, indicate that for fast growing and fast declining firms, the LPE does not work.

From a theoretical point of view, it is often claimed that the independence between the growth rate and the initial size of the firm rules out the concept of 'optimal size' based on U-shaped average cost curves (see e.g. Dosi 2007). Indeed, if the LPE holds, then firm growth is a multiplicative process independent of initial conditions, or, in other words, does not involve systematic scale effects. If the LPE does not hold, we observe a tendency towards monopoly (if firm growth follows an explosive path – i.e. the autoregressive coefficient is larger than 1) or a regression-to-the-mean, which, indirectly, supports the idea of some underlying 'optimal firm size' or 'optimal technological structure' (i.e. when the autoregressive coefficient is smaller than 1). In addition to the concept of an 'optimal firm size' there is the finding that firm size distributions are right-skewed, which signals the co-existence of many relatively small firms with a few large and very large firms, in numbers much higher than predicted by a Gaussian shape assuming a mean-reverting pattern of firm growth. However, this theoretical framework is not suited to explaining either quantile regression patterns or variance-size scaling –

[1] Gibrat's law is most often tested on samples of manufacturing firms or large scale services. However, Piergiovanni et al. (2003) and Audretsch et al. (2004) have shown that the LPE holds for the hospitality sector, an instance of a small scale service, exception for a limited number of sub-sectors.

indeed, existing theoretical explanations of variance-size scaling rest on managerial behaviour (Buldyrev et al. 1997, Sutton 2002) and corporate diversification (Bottazzi and Secchi 2006). A general, encompassing theoretical framework, therefore, is still beyond reach.

This paper performs new empirical tests of the LPE at all quantiles in the conditional size distribution, using a unique dataset of the entire population of manufacturing firms registered in the Netherlands for fiscal purposes (i.e. without any exogenous threshold on the size of the firms). This work moves a step nearer to a theoretical explanation of why the LPE breaks down, in the tails of the conditional size distribution, with a view to reconciling the different violations of the LPE with the findings that confirm it.

Our contribution is threefold. First, we find that firms in the central quantiles of firm size distributions do not grow from year to year, while the LPE breaks down in the tails (autoregressive coefficients above 1 in the left tail and below 1 in the right tail of the conditional size distribution). We hypothesize that if the methodology chosen to test the LPE does not take account of the whole conditional size distribution, this could produce misleading results. In studies that use OLS methods, the LPE is supported because the OLS results are driven by the large mass of firms that are stable (those between the 25th and the 75th quantiles are often shown not to grow). In these cases, the OLS autoregressive coefficient is 1 or very close to 1 (Hart and Prais 1956; Simon and Bonini 1958; and Bottazzi et al. 2001). When Least Absolute Deviation (LAD) estimators are used, the autoregressive coefficient is close to 1 because it is calculated on the 50th percentile (Bottazzi et al. 2007). We find evidence that, in these cases, the LPE is supported because firms do not move from their original position in the size distribution, not because firm growth is an erratic process.

Second, we prove that the pattern of quantile regression estimates identified, can be obtained by assuming variance-size scaling and Laplace distribution of conditional firm size. In other words, fast decliners (growers) appear to be driven by an explosive (mean-reverting) growth process since the variance in growth rates among small firms is wider than that among large firms. Even if we assume that the skewness of growth rates is sensitive to initial size, the equivalence between quantile regression patterns and variance-size scaling still holds (provided that the skewness is not too elastic with respect to initial firm size). The main implication of this is that quantile regression patterns are driven by the same economic mechanisms that give rise to variance-size scaling and are primarily related to managerial behaviour and corporate diversification.

Third, we provide new analytical evidence on firm growth rate distribution, in the line with the studies by Stanley et al. (1996), Bottazzi et al. (2002), Bottazzi and Secchi (2003), Dahl et al. (2010). If the LPE were to hold, we would obtain Gaussian distributions of firm growth rates. However, recent empirical evidence shows that the growth rate distributions have much fatter tails than the Gaussian distributions, and are usually asymmetric. Our contribution to this literature is based on our choice to use a dataset without exogenous thresholds on firm size. This allows us to obtain firm growth rate distributions that are symmetric, with fat tails. Our analysis shows that size does not affect the location of the distribution (which is why the LPE seems in some cases to be confirmed, e.g. Hart and Prais 1956, or Mansfield 1962), but it affects the variance as well as the magnitude and sign of the skewness.

The paper is structured as follows. Section 2 summarizes the relevant literature, Section 3 describes the data and the empirical evidence. Our empirical findings provide the background for the theoretical results developed in Section 4; proofs are provided in the Appendix. Section 5 concludes.

2. *Literature review*

Gibrat's law states that ``the probability of a given proportionate change in size during a specified period is the same for all firms in a given industry - regardless of their size at the beginning of the period" (Mansfield, 1962, pp. 1030-1031). Formally, if we denote by $S_{t,j}$ the size of firm j at time t - measured by total sales, total assets, or number of employees - Gibrat's law is true if $Prob(\frac{S_{t,j}-S_{t-1,j}}{S_{t-1,j}} \mid S_{t-1,j})$ is constant across different size classes (Sutton, 1997; Caves, 1998; Geroski, 1999; and Dosi and Nelson, 2010 provide extensive reviews of the literature).

The most common approach to testing Gibrat's law ultimately asks whether small firms grow faster than large firms on average. Following Sutton (1997), let us assume that the evolution of firm size is determined by a double-indexed stochastic process $\{S_{t,j}\}$, where $j \in J$ indexes firms, and $t = 0,1,\ldots$ indexes time. If $\varepsilon_{t,j}$ is a random variable denoting the proportionate rate of growth between period $t-1$ and t for firm j, then

$$S_{t,j} - S_{t-1,j} = \varepsilon_{t,j} S_{t-1,j} \qquad (1)$$

and

$$S_{t,j} = \left(1+\varepsilon_{t,j}\right) S_{t-1,j} = S_{0,j}\left(1+\varepsilon_{1,j}\right)\left(1+\varepsilon_{2,j}\right)\cdots\left(1+\varepsilon_{t,j}\right) \qquad (2)$$

If the variance of $\varepsilon_{t,j}$ is small, the approximation $\log\left(1+\varepsilon_{t,j}\right)=\varepsilon_{t,j}$ can be justified. Hence, taking logs and defining $s_{t,j} \equiv \log S_{t,j}$, we have that

$$s_{t,j} \simeq s_{0,j} + \sum_{t=1}^{T} \varepsilon_{t,j} \tag{3}$$

If the increments $\varepsilon_{t,j}$ are independently and normally distributed, then $s_{t,j} \equiv \ln S_{t,j}$ follows a random walk and the limiting distribution of $S_{t,j}$ is lognormal. Therefore, the growth of the firm is unrelated to its current size and depends only on the sum of the idiosyncratic shocks. In order to test Gibrat's law, previous work frequently uses the following general logarithmic specification

$$s_{t,j} = \mu + \beta s_{t-1,j} + u_{t,j} \tag{4}$$

where $s_{t,j}$ is the log-size of firm i at time t, and $u_{t,j}$ is a random variable that satisfies

$$E\left(u_{t,j} \mid S_{t-k,j},\ k>0\right)=0 \tag{5}$$

$$E\left(u_{t,j}u_{\tau,i} \mid S_{t-k,j},\ k>0\right)=\begin{cases}\sigma^2 & i=j, t=\tau \\ 0 & \text{otherwise}\end{cases} \tag{6}$$

Gibrat's law is confirmed if the hypothesis that $\beta=1$ is not rejected by the data against $\beta<1$. The assumption that the error term $u_{t,j}$ is Gaussian justifies the estimation of parameters via Ordinary Least Squares (OLS).[2] Gibrat's law is supported in early work, such as Hart and Prais (1956), Simon and Bonini (1958), and Hymer and Pashigian (1962), but later studies found that smaller firms grow faster on average (Kumar 1985; Evans 1987a; Hall 1987; Dunne et al. 1989; Dunne and Hughes 1994; Hart and Oulton 1996; Reichstein and Dahl 2004). These works control for sample selection bias due to the fact that smaller companies are characterized by higher hazard rates.

The detection of heavy tails in the distribution of firm growth rates (Stanley et al. 1996; Bottazzi and Secchi 2003) pushed a number of authors to estimate the model in Eq. 4 by means of an LAD estimator, assuming that the error term $u_{t,j}$ is drawn from a Laplace distribution (see Bottazzi et al.

[2]The test is a one-tailed test, which we also use in our empirical analysis in order to be consistent with the literature on testing for unit roots. Note also that $\beta>1$ implies explosive growth paths, i.e., firms grow faster as they get larger. This situation is conceivable for a short but not an indefinite period. Moreover, from a qualitative perspective, the implications for market structure are similar to those of $\beta=1$: concentration will increase over time, although at a faster rate.

2007; Bottazzi and Secchi 2006). Although OLS estimation minimizes the sum of the squared deviations from the mean, it is well known that the mean of a random variable is highly sensitive to extreme values. The LAD is a robust estimator because it is based on the median. Thus, in an LAD approach Gibrat's law holds if the median growth rate is independent of initial size, which occurs when $\beta = 1$.

Over and above the statistical robustness issues, one of the major economic implications of heavy tails is that fast-growing firms account for a non-negligible share of an industry population and drive the industry dynamics entirely. Therefore, large (positive and negative) growth rates cannot be dismissed as outliers. For this reason, assessing the validity of Gibrat's law on the mean or median firm, as in the OLS and LAD approaches, is far less relevant than testing it on fast-growing (in a positive or negative direction) firms. Quantile regressions, introduced by Koenker and Bassett (1978), represent an important advance in this respect because they model the dynamics of the firm size distribution as a whole, 'quantile by quantile'. It should be noted that quantile regression relaxes the restrictive assumption that the error term is identically distributed at all points in the conditional distribution - thereby satisfying the requirement of a robust estimation method. Buchinsky (1998) and Koenker and Hallock (2001) report on several applications of this methodology in Economics.

Let $Q_p(x \mid y)$ denote the *p-th* quantile of x conditional on y ($p \in (0,1)$). A quantile regression model for firm log-size reads

$$s_{t,j} = \mu_p + \beta_p s_{t-1,j} + u_{t,j}^p \qquad Q_p\left(s_{t,j} \mid s_{t-1,j}\right) = \mu_p + \beta_p s_{t-1,j} \tag{7}$$

In the model, $u_{t,j}^p$ is an i.i.d. innovation; μ_p and β_p are the constant coefficients to be estimated. More specifically, β_p is the first-order autoregressive coefficient of firm log-size; μ_p is the regression intercept. The p subscripts indicate that the model coefficients are allowed to vary across quantiles of the conditional size distribution. The coefficient β_p is interpreted as the marginal change in final size due to a marginal change in initial size, conditional on being on the *p-th* quantile of the final size distribution. The reader is referred to Koenker and Bassett (1978) for technical details on the estimator. Note that when $p = 1/2$ we are back to the LAD estimator. Gibrat's law is confirmed at the *p-th* quantile if β_p is not significantly different from 1.

One of the earliest applications of the quantile regression to test Gibrat's law is provided by Machado and Mata (2000), who analyse a sample of Portuguese manufacturing firms observed in 1983 and 1991, using a Box-Cox transformation of the quantiles. Firm size is shown to be positively related to industry growth, patents and exports, and Gibrat's Law is contradicted by the departures of the firm size distribution from lognormality.

Since 2000, several estimates of linear quantile regression models of conditional firm size or growth have been performed. Lotti et al. (2003) study 855 Italian firms born in 1987 and observed through to 1993. Gibrat's law fails during the first year of a firms' activity in 5 out of the 6 industries analysed, but β_p gets closer to 1 as the firm life-cycle evolves.[3] Fotopoulos and Louri (2004) reject Gibrat's law for all quantiles, in a sample of Greek firms in the period 1992-1997 because β_p is significantly below 1, and especially for fast-growing firms. The work by Dahl et al. (2010) on Danish manufacturing firms in 1994-1996 finds that $\beta_p > 1$ for small quantiles ($p \leq 0.4$) and $\beta_p < 1$ for large quantiles ($p \geq 0.7$), while Gibrat's law is satisfied for the quantiles close to the conditional median. Coad and Hölzl (2009) provide similar results for Austrian firms in the service industries. All these studies control for persistence in firm growth rates and industry-specific effects.

The assumption of an i.i.d. error term is questioned by evidence on serial growth rate correlation: positive according to Ijiri and Simon (1964), Singh and Whittington (1975), Chesher (1979), Bottazzi et al. (2001); and negative according to Boeri and Cramer (1992), Bottazzi et al. (2007). Coad (2007) provides a quantile regression analysis of serial growth correlation for a sample of French companies, and finds no correlation for the median firms, and negative correlation in the tails. If the error term is serially correlated, firm growth rates in one period depend on firm growth rates in the previous periods; hence Gibrat's law might be violated even if the first-order regression coefficient is equal to 1.

[3]See also Lotti et al. (2009) for more on the theoretical underpinnings of these results.

3. Empirical evidence

3.1 The dataset

The data used in this paper were collected by the Centraal Bureau voor de Statistiek (CBS) and come from the Business Register of enterprises. The Business Register database includes the entire population of firms registered for fiscal purposes in the Netherlands in the year considered. The database reports detailed information on the sector of the company at the 5-digit of the SBI (the Dutch Standard Industrial Classification), number of employees and dates of entry and exit of the firms in the market. The definition of entry and exit excludes changes in a firm's sector of activity; in these cases, the firm is regarded as continuing. The BR also supplies information on different types of exit, distinguishing between exits due to failure, mergers, acquisition, and radical restructuring. Our observation period covers the years 1994 to 2004. For each year, we selected all the manufacturing firms present in the Business Register. The population includes firms with zero employees, referred to as self-employment. In this study, the variable of interest is the size of the firm, proxied by number of employees.[4]

Table 1 presents the descriptive statistics for firm log size, where firm log size is defined as $\log(\mathrm{Size}+1)$, based on the presence of self-employment where firm size is set equal to 0. Table 1 presents the overwhelming importance of micro-firms in the dataset, which contributes to the right-skewness in the log-size distribution depicted in Figure 1. While a Gaussian distribution of log-size can derive from a random walk growth of incumbent firms (and then be a clear signal that the LPE holds), a right-skewed distribution where the upper tail resembles a Pareto or a Yule distribution can be caused by the presence of new entrants and still be consistent with the LPE (Simon and Bonini, 1958). In our case, the right-skewness of the log-size distribution, confirmed by the QQ-plot in figure 2, and the linear fit of the upper tail of the firm size distribution, shown in the log-log plot of figure 1 (right panel), could then in principle be consistent with the LPE. However, failures of the LPE often depend on the behaviour of young small firms having a higher average growth, as Lotti and Santarelli (2004) point out before analysing the evolution of the size distribution over time for cohorts of new entrants. In the next subsection we follow a similar intuition: we consider subsets of firms having a given initial size, and track their size distribution over a three-year time span.

[4] All the results shown in this section were obtained using the R software package, except for the quantile regression results that were obtained using the *qreg* and *grqreg* functions of the Stata software package.

3.2 Empirical conditional distributions

The columns in Table 2 report the quantiles of firm size for companies with 5, 10, 20 and 50 employees at time $t-1$, and companies with 5, 10, 20 and 50 employees at time $t-2$. Notice that the sample conditional median is remarkably stable: at time t, the median size of firms that at time $t-1$ were size x, is still x. In fact, the conditional median is constant even after 2 years. Further information on the shape of the conditional size distribution is provided in the last three rows in Table 2, which present scale, skewness and kurtosis statistics developed by Oja (1981) and Ruppert (1987).[5]

Scale statistics is zero for the smallest companies; for example, if we condition on $S_{t-1} = \{5, 10\}$ or $S_{t-2} = \{5\}$: the 25% and 75% firm quantiles are the same as the median. Hence, the growth rate distributions for small firms show a sharp modal peak. These distributions become smoother as we move towards the larger firms, as demonstrated by the positive scale values. Size distributions conditioned on $S_{t-1} = \{5, 10, 20\}$ and $S_{t-2} = 5$ are symmetric, but there is a positive skew when $S_{t-1} = 50$ and when $S_{t-2} = \{10, 20, 50\}$. Kurtosis is larger for smaller firms and for size distributions conditioned on S_{t-2}. Hence, smaller firms are also heavier-tailed.

Table 3 reports the gross 1-year and 2-year growth rate S_t / S_{t-1} and S_t / S_{t-2} quantiles. While the values in the 50% quantile row are all equal to 1, those in the left tail are below 1 and those in the right tail above 1. Interestingly, the left tail quantiles tend to increase with S_{t-1} and with S_{t-2}, while the right tail quantiles tend to decrease. These patterns are clearer in the most extreme quantiles. Hence, smaller companies are more likely to experience large positive or negative growth episodes.

Figure 3 depicts the empirical probability densities of firm size, conditional on size being 5, 10, 20 and 50 employees at time $t-1$, on a double logarithmic scale. Consistent with the above findings, compared to a Gaussian process, densities appear to be more sharply peaked and heavier-tailed. The densities for $S_{t-1} = 5$ and $S_{t-1} = 10$ display longer right tails; this is the effect of censoring since the number of employees cannot go below 0. The conditional size distribution of larger firms seems to be less dispersed.

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[5] Let Q_p be the *p-th* quantile. The scale is defined as $\frac{Q_{0.75} - Q_{0.25}}{Q_{0.75} + Q_{0.25}}$, the skewness as $\frac{Q_{0.75} + Q_{0.25} - 2Q_{0.50}}{Q_{0.75} - Q_{0.25}}$, and the kurtosis as $\frac{Q_{0.90} - Q_{0.10}}{Q_{0.75} - Q_{0.25}}$. See also Machado and Mata (2000).

These conditional firm size distributions provide two hints. First, we can reasonably expect that an LAD regression of current size on lagged size will yield a unitary coefficient, consistent with Gibrat's Law. This is implied by the observed stability of the conditional median. Second, Gibrat's Law may not apply to company growth at all points in the conditional size distribution: in fact, conditional extreme quantiles change across size classes. Against the background of all the above results, we analyse the sample in more depth based on the quantile regressions.

3.3 Quantile regression estimates

To prove that firm growth is unrelated to current size and depends only on the sum of idiosyncratic shocks, it is not enough that the first-order regression coefficient in Eq. 7 is equal to 1. If the error term is serially correlated, firm growth rates in one period depend on firm growth rates in the previous periods; hence Gibrat's law might be violated even if the first-order regression coefficient is equal to 1. Therefore, we extend the quantile regression model in Eq. 7 by assuming serial correlation in the error term:

$$\varepsilon_{t,j}^{p} = \rho_p \varepsilon_{t-1,j}^{p} + \upsilon_{t,j}^{p} \tag{8}$$

where ρ_p is the first-order autocorrelation coefficient of $\varepsilon_{t,j}^{p}$, and $\nu_{t,j}^{p}$ is an i.i.d. sequence. Following Chesher (1979), the quantile regression we estimate is the second order autoregressive model

$$s_{t,j} = \omega_p + \gamma_{1,p} s_{t-1,j} + \gamma_{2,p} s_{t-2,j} + \upsilon_{t,j}^{p} \qquad Q_p\left(s_{t,j} \mid s_{t-1,j}\right) = \omega_p + \gamma_{1,p} s_{t-1,j} + \gamma_{2,p} s_{t-2,j} \tag{9}$$

where the coefficients ω_p , $\gamma_{1,p}$ and $\gamma_{2,p}$ are linked to the structural coefficients μ_p , β_p and ρ_p in Eq. 7, as follows:

$$\omega_p = \mu_p\left(1-\rho_p\right) \tag{10}$$

$$\gamma_{1,p} = \beta_p + \rho_p \tag{11}$$

$$\gamma_{2,p} = -\beta_p \rho_p \tag{12}$$

The structural coefficients are retrieved from the estimated coefficients using the formulae:

$$\mu_p = \omega_p / \left(1 - \frac{1}{2}\left(\gamma_{1,p} - \sqrt{\gamma_{1,p}^2 + 4\gamma_{2,p}}\right)\right) \tag{13}$$

$$\beta_p = \frac{1}{2}\left(\gamma_{1,p} + \sqrt{\gamma_{1,p}^2 + 4\gamma_{2,p}}\right) \tag{14}$$

$$\rho_p = \frac{1}{2}\left(\gamma_{1,p} - \sqrt{\gamma_{1,p}^2 + 4\gamma_{2,p}}\right) \tag{15}$$

Gibrat's law is satisfied at the *p-th* quantile if the joint hypothesis $\beta_p = 1$, $\rho_p = 0$ cannot be rejected at the conventional significance levels.

The model is estimated on pooled data including all firms irrespective of size, and on three subsamples of firms with less than 20 employees at time $t-1$ (size class 1 - corresponding to micro firms), firms with 20 to 199 employees (size class 2 - small-medium firms) and firms with more than 199 employees (size class 3 - large firms). After pooling the data, we have 456,874 observations in the whole population sample, 395,408 micro firms, 55,243 small-medium firms, and 6,223 large firms.

Table 4 summarizes the quantile regression estimates and the corresponding structural coefficients. The whole sample and the micro firm subsample estimates of the first-order coefficients γ_1 are equal to 1 in all but the 10% quantile. In the small-medium firm subsample, γ_1 is equal to 1 for the 40% to 60% quantiles, below 1 in the right tail, and above 1 in the left tail. In the large firm subsample, γ_1 is equal to 1 in the 40% to 60% quantiles, above 1 in the 10% quantile, and negative in all the others. Second-order autoregressive coefficients γ_2 are substantially different from 0 only for the small-medium and large firm subsamples; they are negative in the left tails and positive in the right tails.

Next, we focus on the structural coefficients. The whole sample estimates are $\hat{\beta}_p = 1$ for all except the extreme quantiles, with $\hat{\beta}_{0.1}$ slightly below 1 and $\hat{\beta}_{0.9}$ slightly above 1. The pattern is similar for micro firms. For the small-medium and large firms, $\hat{\beta}_p = 1$ only for the quantiles 0.4 to 0.6 quantiles; β_p is above 1 in the left tail and below 1 in the right tail. Differences are even greater in the size class 2, especially in the right tail where $\hat{\beta}_{0.9} = 0.967$. These estimates are depicted in Figure 4, which shows 95% confidence bands. Figure 4 shows that, with the exception of the lower quantiles of large firms where departures from Gibrat’s Law are not significant, all other departures from Gibrat's Law are significant although only slightly. Most values of ρ_p are very close to 0, except for those cases where they are considerably lower than 0.

In interpreting these results, it should be remembered that the size distribution of micro firms (size class 1) is left-censored because the number of employees is bounded at 0. This might be distorting the results for micro firms. Also, the similarities in the estimates for size class 1 and the whole sample estimates are because micro firms are the most numerous in the sample. Estimates of β_p and ρ_p obtained on small-medium and large companies (size classes 2 and 3) are probably more realistic and are in line with the findings in Dahl et al. (2010) for Danish firms, and Coad and Hölzl (2009) for Austrian firms. Hence, in what follows we focus on the pattern of estimates obtained on small-medium and large firms. We try to explain why Dutch companies with large positive growth rates are mean-reverting and why the growth process in Dutch companies with large negative growth rates is explosive.

4. *Why does the LPE break down in the tails?*

We have detected slight, but statistically significant departures from the LPE for fast growing, and fast declining Dutch companies. Fast growers grow less (and fast decliners decline less) if they were large in size at the beginning of the period – in line with the findings in Coad and Holzl (2009) and Dahl et al. (2010). At the same time, we observed that both variance and skewness in the conditional size distribution fall with size. So are these empirical facts related? One useful insight is provided by Koenker (2005), who stresses that discrepancies among estimated coefficients at different quantiles - as in our case – are indicative of heterogeneity within the population. In our case, the inverse variance-size scaling is indeed an instance of heteroskedasticity in the distribution of firm growth rates. It would be reasonable to conjecture that the LPE fails at the extreme quantiles of the conditional growth rate distribution for the same reasons that give rise to variance-size scaling. The variation of the skewness, across different firm size classes, is another source of heteroskedasticity, which might affect the validity of the LPE. The pattern of quantile estimates of the LPE obtained and the variance-size and skewness-size scaling appear to be closely related.

In the following subsections, we explore these insights. Sub-section 4.1 analyses the equivalence between quantile estimates and variance-size scaling under the assumption that firm growth rates are Laplace distributed. In Sub-section 4.2 we relax the assumption of symmetry and consider Asymmetric Laplace growth rates in order to assess the impact of skewness-size scaling. Sub-section 4.3 provides theoretical interpretations of these relationships between LPE, variance and skewness.

4.1 Quantile regression coefficients and variance-size scaling

Suppose firm growth rates are distributed as Laplace random variables, as suggested in the literature (Amaral et al. 1997; Axtell 2001; Bottazzi and Secchi 2003, 2006; Fagiolo and Luzzi 2006). Laplace distributions have a probability density function (pdf)

$$f(g_{t,j}) = \frac{1}{2a} e^{-\left|\frac{g_{t,j}-m}{a}\right|} \qquad (17)$$

where m is the mean and a is the width parameter. Because Laplace distributions are symmetric, m is also the median. The variance is $V[g_{t,j}] = 2a^2$, therefore the width parameter a is proportional to the standard deviation (by a factor $\sqrt{2}$).

The p -quantile of $g_{t,j}$ is

$$Q_p(g_{t,j}) = m + a\log(2p) \qquad p \in (0,1/2]$$
$$Q_p(g_{t,j}) = m - a\log(2(1-p)) \qquad p \in [1/2,1)$$

Notice that $Q_{1/2}(g_{t,j}) = m$. However, the quantile regression focuses on conditional quantiles. Therefore, to obtain a theoretical explanation for the quantile regression results in the previous section, we assume functional dependence of the growth rate distribution parameters m and a on the lagged log-size $s_{t-1,t}$. We also assume the median growth rate to be linear in the lagged log-size,

$$m = m(s_{t-1,j}) = (\beta - 1)s_{t-1,j}$$

with $\beta \in [0,1]$, and that the width is decreasing with size according to a power law, in line with the previously cited literature:

$$a = a(s_{t-1,j}) = e^{\alpha s_{t-1,j}}$$

with $\alpha \leq 0$. The conditional growth rate quantiles can thus be written as:

$$Q_p(g_{t,j} \mid s_{t-1,j}) = (\beta - 1)s_{t-1,j} + e^{\alpha s_{t-1,j}} \log(2p) \qquad p \in (0,1/2] \tag{20}$$

$$Q_p(g_{t,j} \mid s_{t-1,j}) = (\beta - 1)s_{t-1,j} - e^{\alpha s_{t-1,j}} \log(2(1-p)) \qquad p \in [1/2,1) \tag{21}$$

These are non-linear functions of the lagged log-size. However, the quantile regression approach assumes a linear dependence. Let us Taylor-expand Eq. 20 and Eq. 21 up to the first order, around a given log-size s_p^*:

$$Q_p\left(g_{t,j} \mid s_{t-1,j}\right) \approx Q_p(g_{t,j} \mid s_p^*) + \left[\beta - 1 + \alpha e^{\alpha s_p^*} \log(2p)\right]\left(s_{t-1,j} - s_p^*\right), \qquad p \in (0,1/2] \tag{22}$$

$$Q_p\left(g_{t,j} \mid s_{t-1,j}\right) \approx Q_p\left(g_{t,j} \mid s_p^*\right) + \left[\beta - 1 - \alpha e^{\alpha s_p^*} \log(2(1-p))\right]\left(s_{t-1,j} - s_p^*\right), \quad p \in [1/2,1) \tag{23}$$

Finally, we obtain the conditional *log-size* quantiles by adding $s_{t-1,j}$ to both sides of the above formulas, which yields:

$$Q_p\left(s_{t,j} \mid s_{t-1,j}\right) \approx \mu_p + \beta_p s_{t-1,j} \tag{24}$$

where

$\beta_p \equiv \beta + \alpha e^{\alpha s_p^*} \log(2p)$ if $p \in (0,1/2]$;

$\beta_p \equiv \beta - \alpha e^{\alpha s_p^*} \log(2(1-p))$ if $p \in [1/2,1)$;

$\mu_p \equiv Q_p(g_{t,j} \mid s_p^*) - \frac{dQ_p(g_t|s_{t-1})}{ds_p^*} s_p^*$.

This is essentially a first-order quantile regression model, but now the β_p coefficient is constituted of two parts:

- $\beta\left(= \beta_{\frac{1}{2}}\right)$, which defines the median growth-size relationship, and is equal to 1 under Gibrat's Law;
- $\alpha e^{\alpha s_p^*} \log(2(1-p))$ in the right tail, or $\alpha e^{\alpha s_p^*} \log(2p)$ in the left tail, depending on the variance-size scaling coefficient α and the quantile percentage p.

More generally, we can relax the power-law scaling assumption and, using a generic function $a(s_{t-1,j})$ for the width coefficient, we can show that the quantile regression coefficient β_p then reads

$$\beta_p \equiv \beta + a'(s_p^*)\log(2p) > \beta \qquad p \in (0,1/2]$$
$$\beta_p \equiv \beta - a'(s_p^*)\log(2(1-p)) < \beta \qquad p \in [1/2,1)$$

As long as $a' < 0$, the results based on power-law scaling are confirmed. The steeper the variance-size relationship (i.e. the larger α), the greater the gap between β_p and $\beta_{\frac{1}{2}}$. Thus, we can make the following proposition.

Proposition 1. Let firm growth rate $g_{t,j}$ be Laplace distributed, conditional on initial firm log-size. If the width coefficient is decreasing in initial firm size, then

- $\beta_p > \beta_{1/2}$ in the left tail of the conditional size distribution, defined by (7) ($p \in (0,1/2)$);
- $\beta_p < \beta_{1/2}$ in the right tail ($p \in (1/2,1)$).

The pattern predicted by Proposition 1 resembles the pattern observed in our 'size class 2' and 'size class 3' subsamples in Table 4. If there is no variance-size scaling ($\alpha = 0$), all quantile regression coefficients collapse to $\beta_{\frac{1}{2}}$.

We can conclude, therefore, that the pattern of quantile regression estimates for Dutch companies with at least 20 employees - explosive growth in the left tail of the conditional size distribution, mean-reversion in the right tail - can be obtained by assuming a decreasing relationship between the variance in firm growth rates and firm size.

4.2 Quantile regression coefficients and skewness

How generalizable is this conclusion? The above analysis is grounded on the assumption of a Laplace growth distribution. Under this assumption, discrepancies among the regression coefficients at different quantiles, can only be due to changes in the width of the distribution across size classes. If firm heterogeneity is captured only by scale and location differences, we could apply a 'conditional location-scale model', as defined in Machado and Mata (2000), and abandon quantile regression. However, our results and those in Capasso et al. (2009) show that the growth rate distribution for large Dutch companies is left-skewed; it is symmetric only for medium companies. Reichstein and Jensen (2005) find that in the growth rate distribution of Danish companies the right tail is fatter than the left tail. Therefore, skewness seems to be another source of heterogeneity among firms from different size classes.

In order to understand how changes in skewness across size classes affect results, we extend the analysis to the case of an Asymmetric Laplace (AL) conditional growth distribution (Kotz et al. 2001). The probability density function of an AL random variable reads:

$$f(g) = \frac{1}{a}\frac{\kappa}{1+\kappa^2}e^{-\kappa\frac{g-m}{a}} \qquad g \geq m$$
$$f(g) = \frac{1}{a}\frac{\kappa}{1+\kappa^2}e^{\frac{1}{\kappa}\frac{g-m}{a}} \qquad g < m$$

Here a and m are respectively, scale and location parameters. Skewness is regulated by κ: specifically, $\kappa < 1$ indicates a right skew, $\kappa > 1$ indicates a left skew (i.e. it implies a more negative skewness). The symmetric Laplace distribution is obtained by setting $\kappa = 1$. Variance $V[g] = a^2(\kappa^{-2} + \kappa^2)$ is increasing in a, while the relationship with κ is U-shaped, with a minimum at $\kappa = 1$. The quantiles are given by

$$Q_p(g) = m + a\kappa \log\left(\frac{1+\kappa^2}{\kappa^2} p\right) \qquad p \le \frac{\kappa^2}{1+\kappa^2}$$

$$Q_p(g) = m - \frac{a}{\kappa} \log\left[(1+\kappa^2)(1-p)\right] \qquad p \ge \frac{\kappa^2}{1+\kappa^2}$$

We can now condition variance and quantiles on lagged log-size, s_{t-1}. We need to assume that a and κ depend on the lagged log-size. If the growth of large companies is more left skewed than the growth of small companies, as our results and those in Capasso et al. (2009) show, it is reasonable to assume that $\kappa = \kappa(s_{t-1}) \ge 1$ with $\kappa' > 0$ (i.e., larger firms are negatively skewed). In this case, the variance is increasing in κ, hence an inverse variance-size scaling can be obtained only if we assume $a = a(s_{t-1})$ with $a' < 0$. More precisely, we need $\left|\frac{a'}{a}\right| > \left|\frac{\kappa^2 - \kappa^{-2}}{\kappa^2 + \kappa^{-2}} \frac{\kappa'}{\kappa}\right|$.[6] Finally, it is useful to assume that the median also depends on lagged size ($m = m(s_{t-1})$), although we do not impose any restriction on the sign of its first derivative.

The conditional quantiles can then be rewritten as:

$$Q_p(g_t \mid s_{t-1}) = m(s_{t-1}) + a(s_{t-1})\kappa(s_{t-1}) \log\left(\frac{1+\kappa(s_{t-1})^2}{\kappa(s_{t-1})^2} p\right) \qquad p \le \frac{\kappa(s_{t-1})^2}{1+\kappa(s_{t-1})^2}$$

$$Q_p(g_t \mid s_{t-1}) = m(s_{t-1}) - \frac{a(s_{t-1})}{\kappa(s_{t-1})} \log\left[(1+\kappa(s_{t-1})^2)(1-p)\right] \qquad p \ge \frac{\kappa(s_{t-1})^2}{1+\kappa(s_{t-1})^2}$$

Taking the first-order Taylor approximation around s_p^* and adding s_{t-1} on both sides yields an expression for the conditional quantiles of the time- t log-size s_t :

$$Q_p(s_t \mid s_{t-1}) \approx \mu_p + \beta_p s_{t-1}$$

[6] In order to obtain this condition, we need to take the logarithm of conditional variance $V[g \mid s] = a(s)^2(\kappa(s)^{-2} + \kappa(s)^2)$ and find the region where its first derivative with respect to s is negative.

where

$$\beta_p \equiv 1 + \frac{dQ_p(g_t \mid s_p^*)}{ds_{t-1}}$$

is the regression parameter for the *pth* -quantile, and $\mu_p \equiv Q_p(g_t \mid s_p^*) - \frac{dQ_p(g_t|s_{t-1})}{ds_p^*} s_p^*$. The following proposition outlines the conditions behind the pattern of quantile regression estimates identified, if conditional growth rates are AL-distributed.

Proposition 2. Let the firm growth rate g_t, conditional on initial firm size, be distributed as an Asymmetric Laplace, with width and skewness decreasing with size - respectively ($a' < 0$) and ($\kappa' > 0$).[7]

If

a) the location parameter function $m(s_{t-1})$ satisfies

$$\frac{m'}{m} = \left[\left(\frac{a'}{a} + \frac{\kappa'}{\kappa}\right)\log\left(\frac{2\kappa^2}{1+\kappa^2}\right) + \frac{2}{1+\kappa^2}\frac{\kappa'}{\kappa}\right]\frac{a\kappa}{m}$$

and

b) the skewness function κ is less elastic to firm size than the width $\left(\left|\frac{\kappa'}{\kappa}\right| < \left|\frac{a'}{a}\right|\right)$

then, in the size autoregressive model specified by (7):

1. the conditional median firm size is driven by a Gibrat process: $\beta_{1/2} = 1$;
2. firms in the left tail of the conditional size distribution are driven by an explosive process: $\beta_p > 1$ for $p \in (0,1/2)$;
3. firms in the right tail of the conditional size distribution are driven by a mean-reverting process: $\beta_p < 1$ for $p \in (1/2,1)$.

See Appendix 1 for the proof.

Note that if condition (b) is satisfied, then the variance in the growth rate is decreasing in size, because the condition for inverse variance-size scaling previously indicated, i.e. $\left|\frac{a'}{a}\right| > \left|\frac{\kappa^2 - \kappa^{-2}}{\kappa^2 + \kappa^{-2}} \frac{\kappa'}{\kappa}\right|$, is also satisfied. Therefore, the message from Proposition 2 is that even if skewness decreases with size, the observed pattern of quantile regression coefficients is driven by inverse variance-size scaling - provided that skewness is less sensitive than width to initial size.

4.3 Theoretical interpretation

The picture that emerges hints at the equivalence between two pieces of evidence: lack of support for the LPE in the tails; growth rate variance is decreasing with respect to size. This equivalence is compatible with skewness decreasing with size, provided that this decline is mild. We can conjecture that the pattern of quantile regression estimates that emerges, is rooted in the same economic mechanisms that give rise to variance-size scaling in firm growth performances, which is related to managerial behaviour, risk diversification and the evolutionary processes of learning and selection. At the same time, there are reasons why we might expect that, for large firms, the right tail of the growth distribution will decline faster than the left tail, including diseconomies of growth and scale, market power, and managerial attention, all of which lead to left-skewed growth rate distributions for large firms. Our results suggest that the pattern of quantile regression estimates identified, will survive as long as these mechanisms are not too strong. We describe each of these issues in more detail.

The evidence of variance-size scaling has been rationalized in a few formal models, and is consistent with evolutionary accounts of industry dynamics. In the models proposed by Buldyrev et al. (1997) and Sutton (2002), variance-size scaling is determined by the firm's internal dynamics which is shaped by managerial behaviour. In Buldyrev et al. (1997), the firm is comprised of equally-sized units; if the proportional growth rates of these units are statistically independent, the firm's growth rate variance scales with size according to a power law with exponent $\alpha = 0.5$. At the other extreme, $\alpha = 0$ if unit growth rates are perfectly correlated. The observed scaling exponents are replicated if unit growth rates are highly, but imperfectly correlated. Sutton (2002) disputes the assumption of a

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[7] Recall that $\kappa > 1$ denotes a left skew.

positive correlation between units, and shows that correlating mechanisms are not necessary to obtain variance-size scaling. In his model, the firm consists of a number of (approximately) independent businesses, and unit growth rates are independent; the result is that variance in firm growth rates is a power law of firm size with the empirically 'correct' exponent. A third model of the scaling phenomenon is proposed by Bottazzi and Secchi (2006), and relies on the risk-spreading effects of corporate diversification (see also Bottazzi et al., 2001). After showing empirically that larger firms operate in a greater number of submarkets, the authors use a stochastic branching process as a metaphor for the cumulative nature of core competences. Larger companies manage to achieve more homogeneous growth performance because they exploit the synergies among units operating in different submarkets.

In addition to the above formal models, insights on variance-size scaling are provided in the literature on firm entry, which stresses the role of evolutionary dynamics in the processes of firm survival and firm growth. It has become a stylized fact that new entrants on average are smaller than incumbents (Geroski 1995, Caves 1998, Dosi 2007). Hence, the heterogeneity observed among small firms may be due in part to systematic differences among entrants, translating possibly into highly heterogeneous growth performance. New firms seem to be highly asymmetric in three respects: determinants of entry, and founders' experience; strategic goals; expected costs and performance. These asymmetries are likely to translate into dramatically different growth paths. The first aspect is discussed in Santarelli and Vivarelli (2007), who distinguish between 'progressive' entry determinants, such as the existence of favourable economic and technological opportunities, and 'regressive' motivations, as in the case of firm founders seeking to escape from unemployment or low-wage jobs. The latter motivations are likely to result in slower growth or annihilation by the competition. Second, not all new firms pursue growth as a strategic objective (Cabral and Mata 2003). There are cases of firms that stay small in order to avoid tax, to maintain workforce flexibility,[8] or because they are 'control averse', for example, scientist and engineer firm founders want to maintain control of their high-tech ventures (see Cressy 1995; Chittenden et al. 1996; Cressy and Olofsson 1997). At the same time, new firms based on a growth strategy will come under pressure to grow fast in order to survive and reach a minimum efficient scale. These differences in relation to strategy are likely to disappear for large firms, implying widely dispersed growth rates only for small firms. Finally, in the 'passive learning' model proposed by Jovanovic (1982), entrants have very heterogeneous expectations about their relative efficiency and, therefore, their growth performance is highly asymmetric. Firms whose cost expectations prove over-optimistic decline, and may choose to exit, while firms with realistic cost expectations will adjust their size gradually as these expectations change. Thus, large firms could be seen as the survivors in an evolutionary process through which they managed to learn, and achieve stable and homogeneous expansionary paths.

As already underlined, our theoretical results emphasize the role of skewness in conditioning the success of the LPE at the extreme quantiles. The industry dynamics literature provides some reasons for skewness-size scaling. Penrose (1959) suggests that sustaining a rapid pace of growth entails relatively high operating costs, and that fast-growers tend to be over-stretched and enjoy little slack. These diseconomies of growth – sometimes dubbed the 'Penrose effect' (Coad 2009) – may interact with diseconomies of scale, more frequent in large companies. If this occurs, large firms will prefer to avoid fast growth. Aversion to fast growth can also be due to market power: large firms controlling a significant fraction of the market may restrict their growth in order to preserve high prices (Nelson and Winter 1982; Nelson 1987). Finally, in large firms involved in diversification, managerial attention may be a binding constraint, hindering their ability to grow. All of these mechanisms could explain why the right tail in the growth rate distribution declines faster for large firms, but do not restrict the behaviour of the left tail. If diseconomies of growth and scale, market power and managerial attention are strong enough, the skew in large firms' growth rates distributions may be extremely negative, possibly violating condition (b) in Proposition 2.

Based on the above discussion, we can make two main conjectures. On the one hand, the LPE does not hold at the extreme quantiles of the conditional size distribution because learning, market selection, risk diversification and correlations among units cause growth performance among large firms to be more similar. On the other hand, the signs of the deviations from the LPE depend on the strength of the skewness-size scaling: the observed quantile regression estimate patterns require diseconomies of scale, diseconomies of growth, market power and limits on managerial attention to be not too strong.

[8] In some countries, firing regulations are less restrictive for companies with small workforces (e.g. in Italy, for companies with less than 20 employees).

5. Conclusions

This study frames the empirical evidence on firm growth by increasing our understanding of the processes that jointly drive many of the empirical findings in the literature. Assuming that firm growth rates are Laplace distributed (as in Amaral et al. 1997, Buldyrev et al. 1997, Bottazzi and Secchi 2003) with the location parameter equal to zero, and that the width of the growth distribution decreases with initial firm size (as found in this paper and several previous works), we can derive the pattern of quantile regression estimates observed by Coad and Hölzl (2009) and Dahl et al. (2010), for our dataset of Dutch firms: autoregressive coefficients above 1 for fast decliners, and below 1 for fast growers. In other words, under certain conditions variance-size scaling and the observed quantile regression patterns are equivalent. Such equivalence disappears if we assume Asymmetric Laplace growth rates with a skewness that is very steeply increasing with initial firm size.

Our analysis provides some explanations for the negative relation between firm size and mean of growth rates, and many results on size and growth autocorrelation in the literature since the 1980s. In particular, we have provided an explanation for why traces of the LPE sometimes occur in conditional mean (i.e. OLS) autoregressions of firm size or firm growth, and in conditional median (i.e. least absolute deviation) autoregressions, but never in high or low quantile autoregressions.

In this paper, a central role is played by the asymmetry in the growth rate distribution, and the changes in the shapes of the left and right tails that occur as average firm size increases. The importance we attribute to the skewness in the growth rate distribution is counterbalanced by the negligible relevance we assign to its location parameter. We assume that there is no peculiar role played by the median of the growth distribution, which is approximately equal to zero independent of firm size. In economic terms, this is tantamount to saying that most of the phenomena of interest for industrial dynamics can be studied without reference to the behaviour of the median firm, and many 'average' relations retrieved in the literature, starting from the negative relation between average size and average growth, are driven by the few dynamic firms in the sample rather than the many stable ones.

Although the main intuitions of the present study come from an analysis of the stylized facts, this work is not limited to a 'dead reckoning' based on past research, but includes an original empirical analysis of the population of manufacturing firms operating in the Netherlands between 1994 and 2004. Our analysis has enabled observation of the tent shape of the empirical firm growth rate distribution, confirmation of the skewness-size and the variance-size relations, and has allowed us to dismiss the role of location, to run the quantile autoregressions that connect us with the most recent literature, and to uncover a relationship with the previous findings.

Our study is a phenomenological work of framing what has been observed. Full understanding of the economic processes underlying the facts described requires observation of other firm-level variables. The heterogeneity of firm behaviour, the intrinsic differences between stable and dynamic firms and between small and large firms, and the importance of extreme events, implicitly raise the questions about why some firms are more dynamic than others, and why, within a set of dynamic firms, some enjoy durable success while others are not able to maintain persistent growth. Analyses of firm growth that take account of different innovative behaviours, capital structures, geographic locations would complement our study and enable investigation of the economic roots of the phenomena investigated in this paper.

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References

Acs, Z. J. and Audretsch, D. B. (1990). The determinants of small-firm growth in U.S. manufacturing. *Applied Economics*, 22(2):143–153.

Amaral, L. A., Buldyrev, S. V., Havlin, S., Leschhorn, H., Maass, P., Salinger, M. A., Stanley, H. E., and Stanley, M. H. (1997). Scaling behavior in economics: I. Empirical results for company growth. *Journal de Physique I France*, 7:621–633.

Audretsch, D. B. and Elston, J. A. (2002). Does firm size matter? Evidence on the impact of liquidity constraints on firm investment behavior in Germany. *International Journal of Industrial Organization*, 20(1):1–17.

Audretsch, D. B., Klomp, L., Santarelli, E., and Vivarelli, M. (2004). Gibrat's Law: Are services different? *Review of Industrial Organization*, 24(3):301-324.

Audretsch, D. B., Santarelli, E., and Vivarelli, M. (1999). Start-up size and industrial dynamics: Some evidence from Italian manufacturing. *International Journal of Industrial Organization*, 17(7):965–983.

Axtell, R. (2001). Zipf distribution of U.S. firm sizes. *Science*, 293:1818–1820.

Boeri, T. and Cramer, U. (1992). Employment growth, incumbents and entrants: Evidence from Germany. *International Journal of Industrial Organization*, 10(4):545–565.

Bottazzi, G., Cefis, E., and Dosi, G. (2002). Corporate growth and industrial structures: some evidence from the Italian manufacturing industry. *Industrial and Corporate Change*, 11(4):705–723.

Bottazzi, G., Cefis, E., Dosi, G., and Secchi, A. (2007). Invariances and diversities in the patterns of industrial evolution: Some evidence from Italian manufacturing industries. *Small Business Economics*, 29(1):137–159.

Bottazzi, G., Dosi, G., Lippi, M., Pammolli, F., and Riccaboni, M. (2001). Innovation and corporate growth in the evolution of the drug industry. *International Journal of Industrial Organization*, 19(7):1161-1187.

Bottazzi, G. and Secchi, A. (2003). Why are distributions of firm growth rates tent-shaped? *Economics Letters*, 80:415–420.

Bottazzi, G. and Secchi, A. (2005). Growth and diversification patterns of the worldwide pharmaceutical industry. *Review of Industrial Organization*, 26(2):195 –216.

Bottazzi, G. and Secchi, A. (2006). Explaining the distribution of firms growth rates. *RAND Journal of Economics*, 37:234–263.

Buchinsky, M. (1998). Recent advances in quantile regression models: A practical guideline for empirical research. *Journal of Human Resources*, 33(1):88–126.

Buldyrev, S. V., Amaral, L. A., Havlin, S., Leschhorn, H., Maass, P., Salinger, M. A., Stanley, H. E., and Stanley, M. H. (1997). Scaling behavior in economics: 2. Modeling of company growth. *Journal de Physique I France*, 7:635–650.

Cabral, L. M. B. and Mata, J. (2003). On the evolution of the firm size distribution: Facts and theory. *American Economic Review*, 93(4):1075–1090.

Capasso, M., Cefis, E., and Frenken, K. (2009). Do some firms persistently outperform? Utrecht School of Economics, Working Papers 09-28, Utrecht University, Utrecht, the Netherlands.

Caves, R. E. (1998). Industrial organization and new findings on the turnover and mobility of firms. *Journal of Economic Literature*, 36:1947–1982.

Chesher, A. (1979). Testing the Law of Proportionate Effect. *Journal of Industrial Economics*, 27(4):403–411.

Chittenden, F., Hall, G., and Hutchinson, P. (1996). Small firm growth, access to capital markets and financial structure: Review of issues and an empirical investigation. *Small Business Economics*, 8:59–67.

Coad, A. (2007). A closer look at serial growth rate correlation. *Review of Industrial Organization*, 31(1):69–82.

Coad, A. and Hölzl, W. (2009). On the autocorrelation of growth rates. *Journal of Industry, Competition and Trade*, 9(2):139–166.

Cressy, R. and Olofsson, C. (1997). The financial conditions for Swedish SMEs: Survey and research agenda. *Small Business Economics*, 9(2):179–194.

Cressy, R. C. (1995). Borrowing and control: A theory of entrepreneurial types. *Small Business Economics*, 7:291–300.

Dahl, M. S., Ebersberger, B., Jensen, M. B., and Reichstein, T. (2010). The devil dwells in the tails - a quantile regression approach to firm growth. *Journal of Evolutionary Economics*, 20(2):219–231.

Dosi, G. (2007). *Statistical Regularities in the Evolution of Industries. A Guide through some Evidence and Challenges for the Theory.* in Brusoni and F. Malerba (eds.), *Perspectives on Innovation,* Cambridge and New York: Cambridge University Press, 153-86.

Dosi, G., and Nelson, R.R. (2010), Technical change and industrial dynamics as evolutionary processes, in Hall B.H., and Rosenberg N. (eds.), *Handbook of the Economics of Innovation*, Vol. 1. Amsterdam/New York: Elsevier.

Dunne, P. and Hughes, A. (1994). Age, size, growth and survival: UK companies in the late 1980's. *Journal of Industrial Economics*, 42(2):115–140.

Dunne, T., Roberts, M., and Samuelson, L. (1989). The growth and failure of US manufacturing plants. *Quarterly Journal of Economics*, 104(4):671–698.

Evans, D. (1987a). The relationship between firm growth, size, and age: Estimates for 100 manufacturing industries. *Journal of Industrial Economics*, 35(4):567–581.

Evans, D. (1987b). Tests of alternative theories of firm growth. *Journal of Political Economy*, 95(4):657–674.

Fagiolo, G. and Luzzi, A. (2006). Do liquidity constraints matter in explaining firm size and growth? some evidence from the Italian manufacturing industry. *Industrial and Corporate Change*, 15(1):1–39.

Fotopoulos, G. and Louri, H. (2004). Firm growth and FDI: Are multinationals stimulating local industrial development? *Journal of Industry, Competition and Trade*, 4(3):163–189.

Geroski, P. A. (1995). What do we know about entry? *International Journal of Industrial Organization*, 13(4):421–440.

Geroski, P. A. (1999). The growth of firms in theory and in practice. *CEPR Discussion Papers* 2092.

Gibrat, R. (1931). *Les inéqualités économiques*. Librairie du Recueil Sirey, Paris.

Hall, B. (1987). The relationship between firm size and firm growth in the us manufacturing sector. *Journal of Industrial Economics*, 35(4):583–606.

Hart, P. (1962). Size and growth of firms. *Economica*, 29(1):29–39.

Hart, P. and Oulton, N. (1996). The size and growth of firms. *Economic Journal*, 106:1242–1252.

Hart, P. E. and Prais, S. J. (1956). The analysis of business concentration. *Journal of the Royal Statistical Society*, 119:150–191.

Hay, D. A. and Morris, D. J. (1991), *Industrial economics and organization, Theory and evidence*, Oxford/New York: Oxford University Press.

Hymer, S. and Pashigian, P. (1962). Turnover of firms as a measure of market behavior. *The Review of Economics and Statistics*, 44(1):82–87.

Ijiri, Y., and Simon, H. A. (1964). Business firms growth and size. *American Economic Review* 54(2): 77-89.

Ijiri, Y. and Simon, H. A. (1977). *Skew distributions and the sizes of business firms.* Amsterdam: North-Holland Pub. Co.

Jovanovic, B. (1982). Selection and the evolution of industry. *Econometrica*, 50(3):649–670.

Koenker, R. (2005). *Quantile Regression*. Cambridge: Cambridge University Press.

Koenker, R. and Hallock, K. F. (2001). Quantile regression. *Journal of Economic Perspectives*, 15(4):143–156.

Koenker, R. W. and Bassett, G. (1978). Regression quantiles. *Econometrica*, 46(1):33–50.

Kotz, S., Kozubowski, T., and Podgorski, K. (2001). *The Laplace distribution and generalizations*. Birkhäuser.

Kumar, M. (1985). Growth, acquisition activity and firm size: evidence from the united kingdom. *Journal of Industrial Economics*, 33:327–338.

Lotti, F., Santarelli, E., and Vivarelli, M. (2003). Does Gibrat's law hold among young, small firms? *Journal of Evolutionary Economics*., 13(3):213–235.

Lotti, F. and Santarelli, E. (2004). Industry Dynamics and the Distribution of Firm Sizes: A Nonparametric Approach. *Southern Economic Journal*, 70(3):443-466.

Lotti, F., Santarelli, E., and Vivarelli, M. (2009). Defending Gibrat's law as a long-run regularity. *Small Business Economics*, 32(1):31–44.

Machado, J. A. F. and Mata, J. (2000). Box-Cox quantile regression and the distribution of firm sizes. *Journal of Applied Econometrics*, 15(3):253–274.

Mansfield, E. (1962). Entry, Gibrat's law, innovation and the growth of organizations. *American Economic Review*, 52:1023–1051.

Mata, J. (1994). Firm growth during infancy. *Small Business Economics*, 6(1):27–39.

Nelson, R. R. (1987). *Understanding technical change as an evolutionary process*. Amsterdam: North-Holland.

Nelson, R. R. and Winter, S. (1982). *An Evolutionary Theory of Economic Change.* London: The Belknap Press of Harvard University.

Oja, H. (1981). On location, scale, skewness and kurtosis of univariate distributions. *Scandinavian Journal of Statistics*, 8:154–168.

Penrose, E. T. (1959). *The Theory of the Growth of the Firm*. Oxford/New York: Oxford University Press

Piergiovanni, R., Santarelli, E., Klomp, L., and Thurik, A. R. (2003). Gibrat's law and the size/firm growth relationship in Italian small scale services. *Revue d'Economie Industrielle*, 102:69-82.

Reichstein, T. and Dahl, M. (2004). Are firm growth rates random? analysing pattern and dependencies. *International Review of Applied Economics*, 18(2):225–246.

Reichstein, T. and Jensen, M. B. (2005). Firm size and firm growth rate distributions – the case of Denmark. *Industrial and Corporate Change*, 14(6):1145–1166.

Reid, G. (1995). Early life-cycle behaviour of micro-firms in scotland. *Small Business Economics*, 7(1):89–95.

Ruppert, D. (1987). What is kurtosis? *The American Statistician*, 41:1–5.

Santarelli, E. and Vivarelli, M. (2007). Entrepreneurship and the process of firms' entry, survival and growth. *Industrial and Corporate Change*, 16(3):455–488.

Simon, H. and Bonini, C. P. (1958). The size distribution of business firms. *American Economic Review*, 58(4):607–617.

Singh, A. and Whittington, G. (1975). The size and growth of firms. *Review of Economic Studies*, 42(1):15–26.

Stanley, M. H. R., Amaral, L. A. N., Buldyrev, S. V., Havlin, S., Leschhorn, H., Maass, P., Salinger, M. A., and Stanley, H. E. (1996). Scaling behaviour in the growth of companies. *Nature*, 379:804–806.

Steindl, J. (1965). *Random Processes and the Growth of Firms*. London: Griffin.

Sutton, J. (1997). Gibrat's legacy. *Journal of Economic Literature*, 35:40–59.

Sutton, J. (2002). The variance of firm growth rates: the 'scaling' puzzle. *Physica A: Statistical Mechanics and its Applications*, 312(3–4):577–590.

Appendix

Proof of Proposition 2.

Part (a). By definition, $\beta_{1/2} = 1$ requires $\frac{dQ_{1/2}(g_t|s_p^*)}{ds_{t-1}} = 0$, i.e. the median is constant across size classes. Given $\kappa > 1$, the conditional median of g_t is equal to

$$Q_{1/2}\left(g_t \mid s_{t-1}\right) = m\left(s_{t-1}\right) - a\left(s_{t-1}\right)\kappa\left(s_{t-1}\right)\log\left(\frac{2\kappa\left(s_{t-1}\right)^2}{1+\kappa\left(s_{t-1}\right)^2}\right) \tag{34}$$

Its first derivative with respect to s_{t-1} is

$$\frac{dQ_{1/2}\left(g_t \mid s_{t-1}\right)}{ds_{t-1}} = m' - \left(a'\kappa + a\kappa'\right)\log\left(\frac{2\kappa^2}{1+\kappa^2}\right) - \frac{2a}{1+\kappa^2}\kappa' \tag{35}$$

Setting this to zero yields $m' = (a'\kappa + a\kappa')\log\left(\frac{2\kappa^2}{1+\kappa^2}\right) + \frac{2a}{1+\kappa^2}\kappa'$; further algebra leads to part (a) of the proposition.

Part (b). Whether β_p is above or below 1 depends on the sign of $\frac{dQ_p(g_t|s_p^*)}{ds_{t-1}}$. Compute the first derivatives of the conditional quantiles with respect to s_{t-1} :

$$\frac{dQ_p\left(g_t \mid s_{t-1}\right)}{ds_{t-1}} = m' + \left(a'\kappa + a\kappa'\right)\log\left(\frac{1+\kappa^2}{\kappa^2}p\right) - \frac{2a}{1+\kappa^2}\kappa' \qquad p \le \frac{\kappa^2}{1+\kappa^2} \tag{36}$$

$$\frac{dQ_p\left(g_t \mid s_{t-1}\right)}{ds_{t-1}} = m' - \frac{1}{\kappa^2}\left(a'\kappa + a\kappa'\right)\log\left(\left(1+\kappa^2\right)\left(1-p\right)\right) - \frac{2a}{1+\kappa^2}\kappa' \qquad p \ge \frac{\kappa^2}{1+\kappa^2} \tag{37}$$

Plug the value of m' obtained from part (a) into Eq. 36 and Eq. 37 to yield

$$\frac{dQ_p\left(g_t \mid s_{t-1}\right)}{ds_{t-1}} = \left(a'\kappa + a\kappa'\right)\log\left(2p\right) \qquad p \le \frac{\kappa^2}{1+\kappa^2} \quad (38)$$

$$\frac{dQ_p\left(g_t \mid s_{t-1}\right)}{ds_{t-1}} = \left(a'\kappa + a\kappa'\right)\left(\log\left(2\kappa^2\right) - \frac{1+\kappa^2}{\kappa^2}\log\left(1+\kappa^2\right) - \frac{1}{\kappa^2}\log\left(1-p\right)\right) \qquad p \ge \frac{\kappa^2}{1+\kappa^2} \quad (39)$$

Let us consider three cases: (i) $p \in (0, \frac{1}{2})$; (ii) $p \in (\frac{1}{2}, \frac{\kappa^2}{1+\kappa^2})$; (iii) $p \in (\frac{\kappa^2}{1+\kappa^2}, 1)$.

It is easy to see that, under case (i), in Eq. 38 $a'\kappa + a\kappa' > 0$ implies $\frac{dQ_p(g_t|s_{t-1})}{ds_{t-1}} < 0$, i.e. $\beta_p < 1$. Case (ii) is again relevant for Eq. 38: this time, $a'\kappa + a\kappa' > 0$ implies $\frac{dQ_p(g_t|s_{t-1})}{ds_{t-1}} > 0$, i.e. $\beta_p > 1$. Finally, case (iii) is relevant for Eq. 39: in that equation, the term in brackets is positive, because $\log(1-p) < 0$ and $2\kappa^2 > (1+\kappa^2)^{\frac{1+\kappa^2}{\kappa^2}}$. So, $a'\kappa + a\kappa' > 0$ implies $\frac{dQ_p(g_t|s_{t-1})}{ds_{t-1}} > 0$ and $\beta_p > 1$. Overall, case (i) allows the conclusion that $\beta_p < 1$ when $p < 1/2$, whereas cases (ii) and (iii) show that $\beta_p > 1$ when $p > 1/2$. These conclusions prove part (b) of our proposition.

Table 1: Descriptive statistics for the yearly unconditional distributions of firm log size.

Year	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004
Sample size	58930	59771	60792	62198	61721	63019	63747	65224	64858	64532	63346
Mean	1.43	1.48	1.47	1.48	1.49	1.49	1.48	1.45	1.43	1.41	1.39
Variance	1.78	1.60	1.48	1.46	1.45	1.48	1.47	1.48	1.48	1.46	1.46
Skewness	1.25	1.35	1.41	1.42	1.45	1.42	1.44	1.44	1.48	1.48	1.49
Kurtosis	4.56	5.03	5.37	5.35	5.40	5.23	5.28	5.27	5.41	5.46	5.51
Minimum	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1 quantile	0.00	0.00	0.00	0.00	0.69	0.69	0.69	0.00	0.00	0.00	0.00
0.2 quantile	0.00	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69
0.3 quantile	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69
0.4 quantile	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69	0.69
0.5 quantile	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10
0.6 quantile	1.39	1.39	1.39	1.39	1.39	1.39	1.39	1.39	1.10	1.10	1.10
0.7 quantile	1.95	1.79	1.79	1.79	1.79	1.79	1.79	1.79	1.79	1.61	1.61
0.8 quantile	2.40	2.40	2.40	2.40	2.40	2.40	2.40	2.30	2.30	2.30	2.20
0.9 quantile	3.58	3.50	3.18	3.18	3.18	3.26	3.26	3.22	3.18	3.18	3.14
Maximum	9.47	10.49	10.56	10.50	10.50	9.94	10.46	10.46	10.46	10.46	10.20

Table 2: Quantiles of current firm size (S_t) conditional upon lagged firm size S_{t-1} = {5, 10, 20, 50} and S_{t-2} = {5, 10, 20, 50}, along with rescaled range and rescaled interquartile range, using pooled data.

	S_{t-1}				S_{t-2}			
quantiles of S_t	5	10	20	50	5	10	20	50
0.01	0	0	3	16.69	0	0	1	16.83
0.03	2	4	12	27	1	2	8	23
0.05	3	6	14	33.45	2	4	11	27
0.10	4	9	16	40	3	7	14	34.3
0.25	5	10	19	49	5	10	18	45
0.50	**5**	**10**	**20**	**50**	**5**	**10**	**20**	**50**
0.75	5	10	21	52	5	11	23	56
0.90	6	12	25	60	7	15	28	65
0.95	7	15	28	67.55	10	18	33	75
0.97	9	17	31	74.93	11	20	38	91.53
0.99	13	23	41	93.31	17	29	46.26	108
scale	0	0	0.05	0.03	0	0.048	0.122	0.109
skewness	0	0	0	0.33	0	1	0.20	0.09
kurtosis	∞	∞	4.50	6.670	∞	8	2.80	2.79

Table 3: Quantiles of the gross 1-year growth rates $S_t \ / \ S_{t-1}$ conditional upon lagged firm size S_{t-1} = {5, 10, 20, 50} and of the gross 2-year growth rates $S_t \ / \ S_{t-2}$ conditional upon S_{t-2} = {5, 10, 20, 50}, using pooled data.

	S_{t-1}				S_{t-2}			
quantiles	5	10	20	50	5	10	20	50
0.01	0	0	0.15	0.3338	0	0	0.05	0.3366
0.03	0.4	0.4	0.6	0.54	0.2	0.2	0.4	0.46
0.05	0.6	0.6	0.7	0.669	0.4	0.4	0.55	0.54
0.10	0.8	0.9	0.8	0.8	0.6	0.7	0.7	0.686
0.25	1	1	0.95	0.98	1	1	0.9	0.9
0.50	1	1	1	1	1	1	1	1
0.75	1	1	1.05	1.04	1	1.1	1.15	1.12
0.90	1.2	1.2	1.25	1.2	1.4	1.5	1.4	1.3
0.95	1.4	1.5	1.4	1.351	2	1.8	1.65	1.5
0.97	1.8	1.7	1.55	1.4986	2.2	2	1.9	1.8306
0.99	2.6	2.3	2.05	1.8662	3.4	2.9	2.313	2.16

Table 4: Quantile regression estimates: estimated and structural parameters for the whole sample and for size classes 1, 2, 3.

	Estimated parameters			Structural parameters		
quantiles of S_t	γ_1	γ_2	ω	β	ρ	μ
whole sample						
0.1	0.9542425	-5.39E-09	2.84E-08	0.954242494	5.64846E-09	2.90E-08
0.2	1	-2.54E-15	2.22E-16	1	2.55351E-15	2.22E-16
0.3	1	-2.54E-15	2.22E-16	1	2.55351E-15	2.22E-16
0.4	1	-2.54E-15	2.22E-16	1	2.55351E-15	2.22E-16
0.5	1	-2.54E-15	2.22E-16	1	2.55351E-15	2.22E-16
0.6	1	-2.54E-15	2.22E-16	1	2.55351E-15	2.22E-16
0.7	1	-2.54E-15	2.22E-16	1	2.55351E-15	2.22E-16
0.8	1	-2.54E-15	2.22E-16	1	2.55351E-15	2.05E-16
0.9	1	0.0424813	-2.33E-09	1.040815403	-0.040815403	-2.33E-09
Class 1 ($S < 20$)						
0.1	0.9622252	-2.49E-09	1.47E-08	0.962225197	2.58775E-09	1.50E-08
0.2	1	8.84E-16	-2.22E-16	1	-8.88178E-16	-2.22E-16
0.3	1	8.84E-16	-2.22E-16	1	-8.88178E-16	-2.22E-16
0.4	1	8.84E-16	-2.22E-16	1	-8.88178E-16	-2.22E-16
0.5	1	8.84E-16	-2.22E-16	1	-8.88178E-16	-2.22E-16
0.6	1	8.84E-16	-2.22E-16	1	-8.88178E-16	-2.22E-16
0.7	1	8.84E-16	-2.22E-16	1	-8.88178E-16	-2.22E-16
0.8	1	8.84E-16	-2.22E-16	1	-8.88178E-16	-2.00E-16
0.9	1	0.0556869	-8.02E-10	1.052889591	-0.052889591	-8.02E-10
Class 2 ($20 \leq S < 200$)						
0.1	0.8913632	0.111605	-0.2232448	1.002670907	-0.111307707	-2.18E-01
0.2	0.9718225	0.0357569	-0.1119261	1.007319576	-0.035497076	-1.12E-01
0.3	1.003906	0.0048254	-0.0613011	1.008689829	-0.004783829	-6.12E-02
0.4	1	-3.94E-16	-4.44E-16	1	3.88578E-16	-4.44E-16
0.5	1	-3.94E-16	-4.44E-16	1	3.88578E-16	-4.44E-16
0.6	1	-3.94E-16	-4.44E-16	1	3.88578E-16	-4.40E-16
0.7	0.9790966	0.0050933	0.0821885	0.984271291	-0.005174691	8.14E-02
0.8	0.9665415	0.0098785	0.154981	0.976656114	-0.010114614	1.55E-01
0.9	0.9575058	0.0073854	0.2830477	0.965157813	-0.007652013	2.89E-01
Class 3 ($S \geq 200$)						
0.1	0.9694636	0.0483171	-0.2766459	1.016974243	-0.047510643	-2.73E-01
0.2	0.9886211	0.0147131	-0.086611	1.003286011	-0.014664911	-8.77E-02
0.3	0.9986038	-0.0032412	0.0078472	0.99534745	0.00325635	7.85E-03
0.4	1	-8.63E-16	0	1	8.88178E-16	0.00E+00
0.5	1	-8.63E-16	0	1	8.88178E-16	0.00E+00
0.6	1	-8.63E-16	0	1	8.88178E-16	0.00E+00
0.7	1.000929	0.0001184	-0.0055223	1.001047276	-0.000118276	-5.45E-03
0.8	0.9867288	0.0060289	0.078868	0.992801414	-0.006072614	7.78E-02
0.9	0.9693917	0.0101207	0.2221831	0.979721877	-0.010330177	2.26E-01

Figure 1: Histogram of the frequencies (left) and log frequencies (right) for the empirical distribution of firm log size in year 2004.

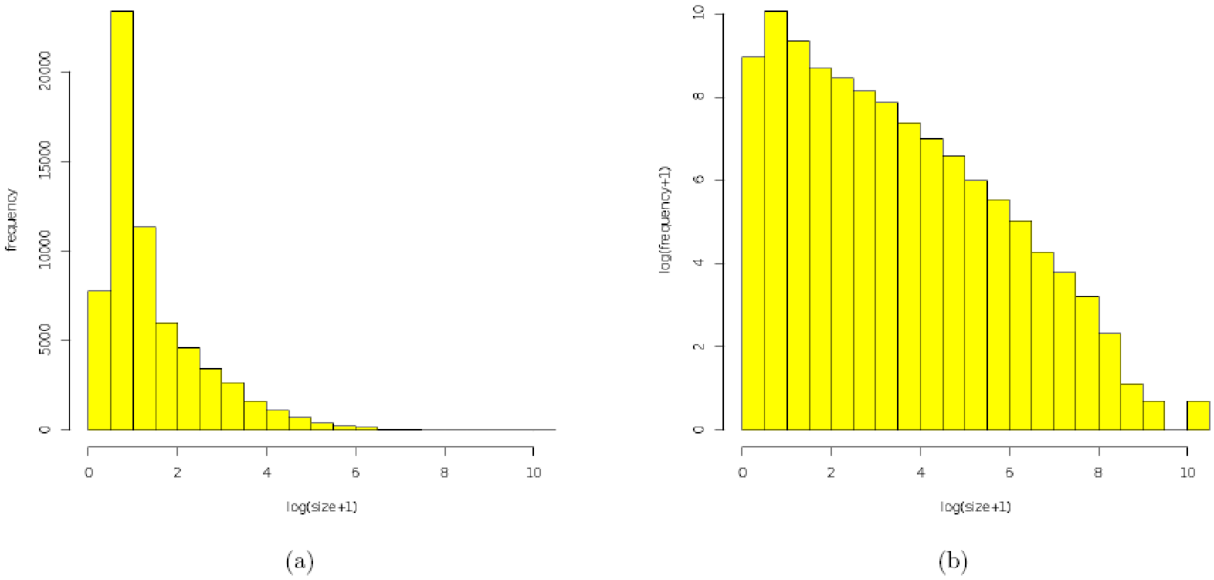

Figure 2: QQ plot of the empirical distribution of firm log size in year 2004 against a theoretical gaussian distribution.

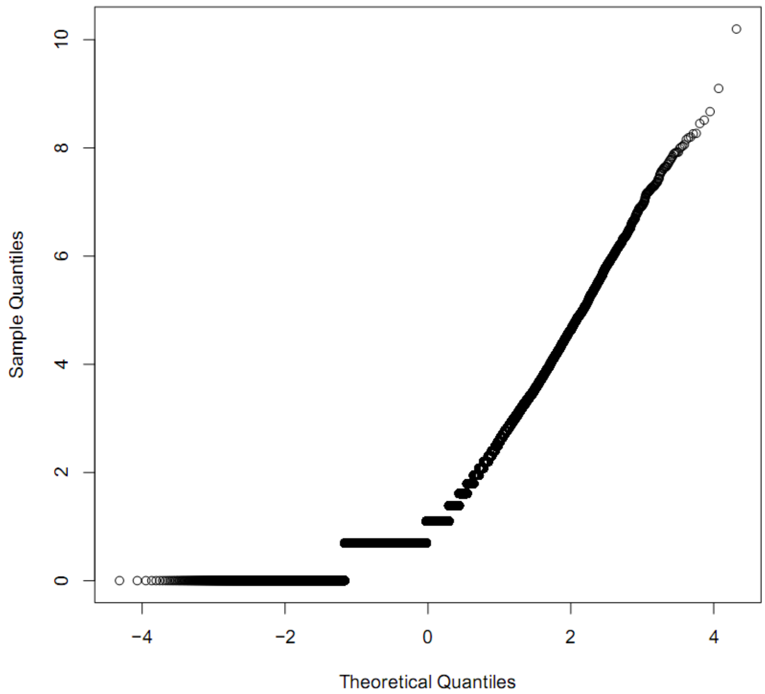

Figure 3: Empirical probability densities of firm log size S_t, conditional upon S_{t-1}= {5, 10, 20, 50} (clockwise). Notice the double logarithmic scale.

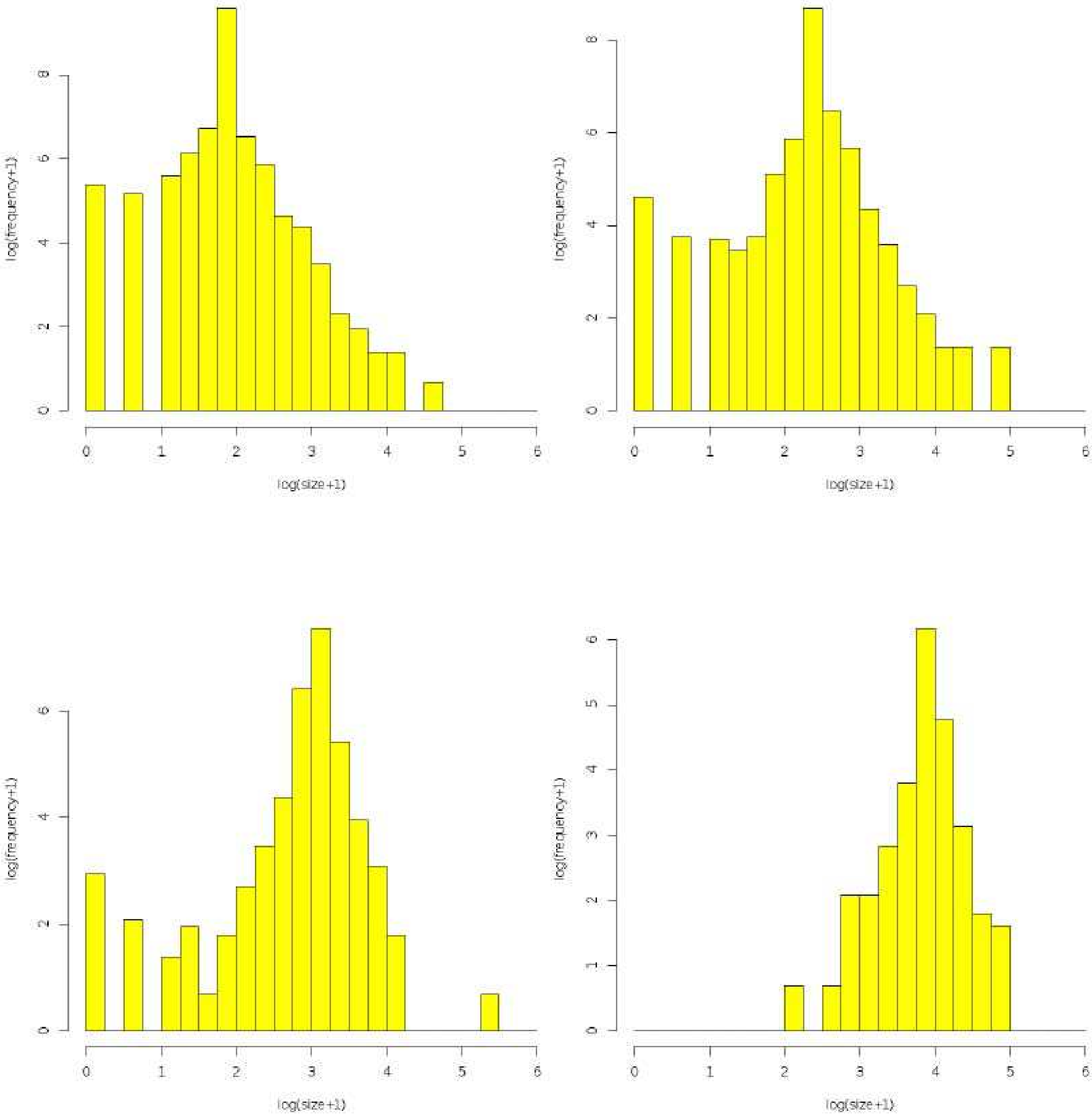

Figure 4: Quantile regression coefficients. Clockwise: whole sample, size class 1, size class 2, size class 3. Red lines indicate point estimates; shaded areas indicate the confidence intervals.