INSTITUTE
OF ECONOMICS

Scuola Superiore
Sant'Anna

LEM | Laboratory of Economics and Management

Institute of Economics
Scuola Superiore Sant'Anna

Piazza Martiri della Libertà, 33 - 56127 Pisa, Italy
ph. +39 050 88.33.43
institute.economics@sssup.it

LEM
Working Paper Series

The footprint of evolutionary processes of learning and selection upon the statistical properties of industrial dynamics

Giovanni Dosi °
Marcelo C. Pereira §
Maria Enrica Virgillito °

° Institute of Economics, Scuola Superiore Sant'Anna, Pisa, Italy
§ University of Campinas, Brazil

2015/04 **March 2015**

ISSN (online) 2284-0400

The footprint of evolutionary processes of learning and selection upon the statistical properties of industrial dynamics

Giovanni Dosi*[1], Marcelo C. Pereira†[2] and Maria Enrica Virgillito‡[1]

[1]*Sant'Anna School of Advanced Studies*
[2]*University of Campinas*

March 2015

Abstract

Evolutionary theories of economic change identify the processes of *idiosyncratic learning* by individual firms and of *market selection* as the two main drivers of the dynamics of industries. Are such processes able to robustly account for the statistical regularities which industrial structures and dynamics display? In this work we address this question by means of a simple agent-based model formalizing the mechanisms of learning and selection. The interplay between these two engines shapes the dynamics of entry-exit and market shares and, collectively, the productivity and the size distributions and their patterns of growth. As such, and despite its simplicity, the model is able to robustly reproduce an ensemble of empirical stylised facts, including ample heterogeneity in productivity distributions, persistent market turbulence and fat-tailed distribution of growth rates.

Keywords

Firms Growth Rate, Productivity, Fat Tail Distributions, Learning Processes, Market Selection.

JEL Classification

C63-L11-L6

Acknowledgements

We thank Marco Grazzi, Luigi Marengo, Andrea Roventini, the participants to the Workshop "Explaining Economic Change", Rome, November 2014 and to the seminar at SKEMA, University of Nice Sophia-Antipolis for helpful comments and discussions.
We gratefully acknowledge the support to one of us (M.C. Pereira) by CNPq, Conselho Nacional de Desenvolvimento Científico e Tecnológico – Brasil, project 400548/2014-3.

* gdosi<at>sssup.it
† marcelocpereira<at>uol.com.br
‡ m.virgillito<at>sssup.it (corresponding author)

1 Introduction

Evolutionary theories of economic change identify the processes of *idiosyncratic learning* by individual firms and of *market selection* as the two main drivers of the dynamics of industries. The interplay between these two engines shapes the dynamics of entry-exit and market shares and, collectively, the productivity and the size distributions and their patterns of growth. Learning (what in the empirical literature is sometimes broadly called the *within effect*) stands for various processes of idiosyncratic innovation, imitation, and changes in techniques of production. Selection (what is usually denominated the *between effect*) is the outcome of processes of market interaction where more competitive firms gain market share at the expense of less competitive ones, some firms die, and others enter. The ensuing industrial dynamics presents some remarkable and quite robust statistical properties – "stylized facts" – which tend to hold across industries and countries, levels of aggregation and time periods (for a critical survey, see Dosi et al., 1995).

In particular, the stylized facts include:

- persistent heterogeneity in productivity and all other performance variables;
- persistent market turbulence, due to change in market shares and entry-exit phenomena;
- skewed size distributions;
- fat-tailed distribution of growth rates.

Different theoretical perspectives address the interpretation of one or more of such empirical regularities. One stream of analysis, which could go under the heading of *equilibrium evolution*, try to interpret size dynamics in term of *passive* (Jovanovic (1982)) or *active* learning (Ericson and Pakes (1995)).
Another stream – from the pioneering work by Ijiri and Simon (1977) all the way to Bottazzi and Secchi (2006a) – studies the result of both mechanisms in terms of the ensuing exploitation of new business opportunities, captured by the joint operation of learning and selection.
A third stream, including several contributions by Metcalfe (see among others Metcalfe (1998)), focuses on the processes of competition/selection often represented by means of a replicator dynamics where market shares vary as a function of the relative competitiveness or "fitness".
Finally, many evolutionary models unpack the two drivers of evolution distinguishing between some idiosyncratic processes of change in the techniques of production, on the one hand, and the dynamics of differential growth driven by differential profitabilities and the ensuing rates of investment, on the other hand (such as in Nelson and Winter (1982)). Yet, in other evolutionary models selection is represented by means of an explicit replicator dynamics, (such in Silverberg et al. (1988) and in Dosi et al. (1995)).
In the following contribution we shall present a simple agent-based model, formalizing the mechanisms of learning and selection, able to robustly reproduce an ensemble of empirical stylised facts, including ample heterogeneity in productivity distributions, persistent market turbulence and fat-tailed distribution of growth rates. In particular, Section 2 briefly summarizes the empirical stylized facts and Section 3 discusses the main theoretical models aimed at explaining some of them. Section 4 presents the model and Section 5 analyses the simulation results.

2 Empirical stylised facts: productivity, size and growth

During the last few decades, enabled by the availability of longitudinal micro data, an increasing number of studies have identified a rich ensemble of stylised facts related to *productivity*, *size* and firm *growth rate* distributions. Let us consider some of them, germane to the model which follows.

2.1 Productivity distribution and growth

As extensively discussed in Doms and Bartelsman (2000), Syverson (2011), Dosi (2007) and Foster et al. (2008), among many others, *productivity dispersion*, at all levels of disaggregation, is a striking and very robust phenomenon. Moreover, such heterogeneity across firms is *persistent over time*, (cf. Bartelsman and Dhrymes (1998), Dosi and Grazzi (2006) and Bottazzi et al. (2008)), with autocorrelation coefficients in the range $0.8-1$. An illustration is provided in Figure 1 for one 2-digit Italian sector and two 3-digit thereof: notice the wide support of the distribution that goes from 1 to 5 in log terms. The distribution and its support are quite stable over time and so is the "pecking order" across firms, as suggested by the high autocorrelation coefficients: see Figure 2. These findings, robust to the use of parametric and non-parametric analysis, empirically discard any idea of firms' revealed production process as the outcome of an exercise of optimization over a commonly shared production possibility set – which, under common relative prices, ought to yield quite similar input/output combinations. Rather *asymmetries* are impressive, which tells of a history of both firm-specific learning patterns and of co-existence in the market of low and high productivity firms, with no trace of convergence (see Dosi et al. (2012)). Even more so, asymmetries are pronounced in emerging economies (with some reduction along the process of development): see on China, Yu et al. (2015). In that, the survival of dramatically less efficient firms hint at a structurally imperfect mechanism of market selection, which demand a theoretical interpretation.

A less explored phenomenon related to the dynamic of productivity is the *double exponential nature* of its growth rate distributions. Extensive evidence is provided in Bottazzi et al. (2005) and Dosi et al. (2012): see Figure 3 for an illustration. The double exponential nature of growth rate in productivity does not only reveals an underlining multiplicative process that determines efficiency changes, but also suggests processes of idiosyncratic learning characterized by discrete, relatively frequent "big" events (see also below, on the size growth rates).

2.2 Size distribution

Firm size distributions are skewed. And this is an extremely robust property which, again, holds across sectors, countries and levels of aggregation. But, how skewed is "skewed"?

The *power-law nature* (Pareto or Zipf law, according to the slope of the straight line, in a log-log plot)[1] of the firm size distribution, has been investigated by many authors[2] since the pioneering work by Simon and Bonini (1958). This is not the place to discuss the possible generating mechanism of such distribution (an insightful discussion is in Brock (1999)). Here, just notice that, being the Pareto a scale free distribution, in principle it should be scale invariant, or equivalently, it should be detectable irrespectively of the considered level of aggregation. However

[1] See Newman, 2005 for a succinct overview.

[2] See Stanley et al., 1995 and Axtell, 2001 for US manufacturing data.

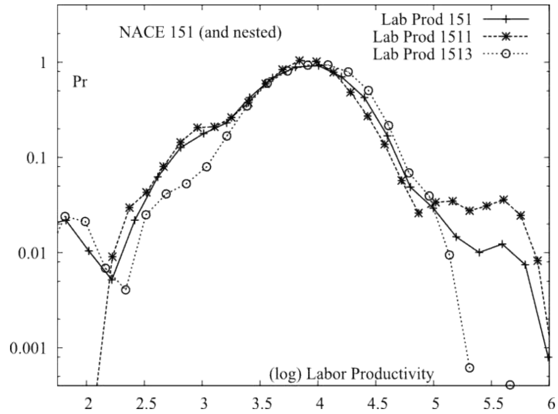


Figure 1: Empirical distribution of labour productivity. Source: Dosi et al. (2013).

SECTOR	ISIC Code	Labor Prod.		Π_l Growth rates	
		AR(1)	Std.Dev.	AR(1)	Std.Dev
Production & processing of meat	151	1.0021	0.0016	-0.3446	0.0915
Knitted & crocheted articles	177	1.0056	0.0023	-0.2877	0.1005
Wearing apparel & acc.	182	1.0035	0.0012	-0.3090	0.0871
Footware	193	1.0029	0.0019	-0.3903	0.0793
Articles of paper and paperboard	212	1.0053	0.0008	-0.3027	0.0603
Printing and services related to printing	222	0.9962	0.0011	-0.4753	0.1103
Plastic products	252	1.0030	0.0010	-0.3150	0.0557
Articles of concrete, plaster & cement	266	0.9985	0.0016	-0.4572	0.0979
Metal products	281	1.0034	0.0012	-0.4125	0.0715
Treatment, coating of metal & mech. engin.	285	1.0051	0.0013	-0.1846	0.0679
Special purpose machinery	295	1.0011	0.0011	-0.3040	0.0495
Furniture	361	0.9994	0.0001	-0.4472	0.0808

Figure 2: AR(1) coefficients for Labour Productivity in levels and first differences, Italy, Istat Micro.1 Dataset. Source: Dosi and Grazzi (2006).

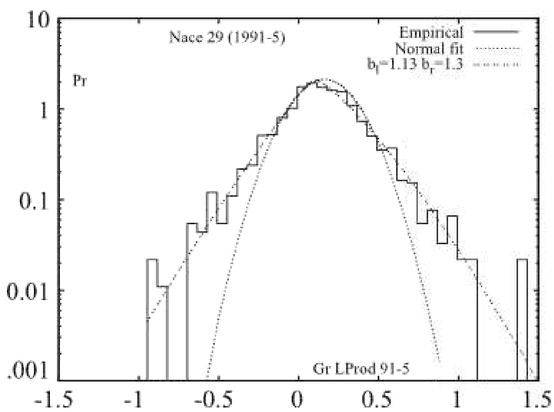


Figure 3: Tent shaped productivity growth rate. Italy, Istat Micro.3 Dataset. Source: Dosi et al. (2012).

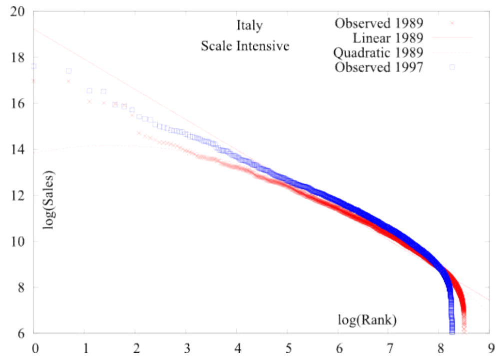


Figure 4: Skewness in the size distribution. Elaboration on Fortune 500. Source: Dosi et al. (2008).

Bottazzi et al. (2007) and Dosi et al. (2008) find that the size distributions fairly differ across sectors in terms of shape, fatness of the tails and even modality. Plausibly, technological factors, the different degree of cumulativeness in the process of innovation, and the predominance of process vs. product innovation might strongly affect sector-specific size distributions (Marsili, 2005). Also, it might well be that the empirical findings on the Zipf (Pareto) law distribution are a mere effect of aggregation as already discussed in Dosi et al. (1995). Hence, what should be retained here is the skewness of the distribution (see Figure 4).

2.3 Market turbulence

Underneath the foregoing invariances, however, there is a remarkable turbulence involving changes in market shares, entry and exit (cf. the discussion in Baldwin and Rafiquzzaman (1995) and Doms and Bartelsman (2000)). A good deal of such turbulence is due to "churning" of going firms, with $20\% - 40\%$ of entrants die in the first two years and only $40\% - 50\%$ survive beyond the seventh year in a given cohort.

I	Sales	Value Added
mean	0.130	0.161
stdev	0.048	0.045
min	0.0544	0.082
max	0.559	0.601

Table 1: Turbulence Index. Italy, Istat Micro.3 Dataset. Source: Grazzi et al. (2013)

An index that synthesizes the market turbulence is shown in Table 1 presenting some synthetic statistics on Italian micro data, both in terms of sales and value added market shares. The I index reads as the sum of the absolute value of market share variations

$$I = \sum_i |s_i(t) - s_i(t-1)| \quad O \leq I \leq 2 \tag{1}$$

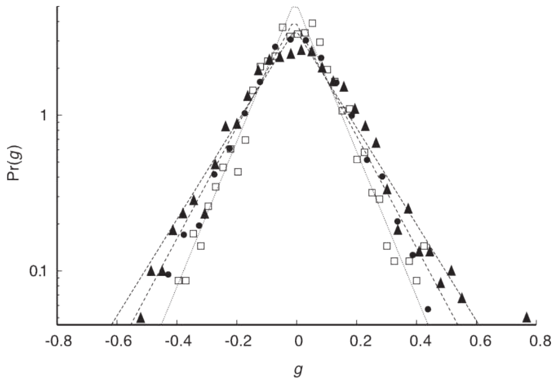


Figure 5: Tent shaped size growth rate, Italy, Istat Micro.1 Dataset. Source: Bottazzi and Secchi (2006a).

2.4 Fat-tailed distributions of growth rates

A huge empirical literature testifies the emergence of the *Laplace distribution* in growth rates. A typical empirical finding is illustrated in 5. This applies across different levels of sectoral disaggregation, across countries, over different historical periods for which there are available data. This stylised fact is robust to different measures of growth, e.g., in terms of sales, value added or employment (for more details see Bottazzi et al. (2002), Bottazzi and Secchi (2006a), Bottazzi et al. (2008) and Dosi (2007)).

Firms grow and decline by relatively lumpy jumps which cannot be accounted by the cumulation of small, "atom-less", independent shocks. Rather "big" episodes of expansion and contraction are relatively frequent. More technically, this is revealed by fat-tailed distributions (in log terms) of growth rates. What determines such property?

In general, such fat-tailed distributions are powerful evidence of some underlying correlation mechanism. Intuitively, new plants arrive or disappear in their entirety, and, somewhat similarly, novel technological and competitive opportunities tend to arrive in "packages" of different "sizes" (i.e., economic importance). This is what Bottazzi (2014) calls the *bosonic nature* of firm growth, in analogy with the correlating property of a family of elementary particles – indeed the bosons.

2.5 Scaling growth-size relationship

The multiplicative process of firm growth is (roughly) uncorrelated with size, at least for not too small firms. This is what goes under the heading of the *Gibrat law*.

However, the *variance* of growth rates falls with size. How the level and growth rate of size are related to each other? Since Stanley et al. (1996), an extensive literature of empirical papers found a negative correlation between the variance of growth rates and size (see Sutton (2002), Lee et al. (1998)). An illustration of the phenomenon is provided in Figure 6. The underlying idea is that firms can be described as a collection of independent units, each of them characterised by an independent growth process. The bigger the firm, the higher the number of its components ("lines of business"). Hence, under an assumption of independent growth processes for each unit, the variance of the growth rate decreases proportionally to the inverse square root of size. Bottazzi and Secchi (2006b) envisage the cause of this negative correlation in product differentiation. They argue that bigger firms tend to operate in more sub-markets. As a direct

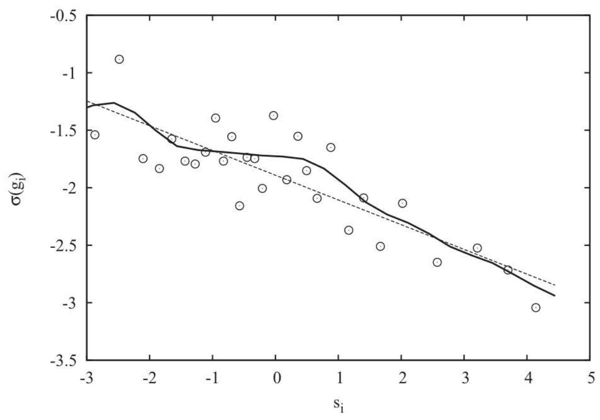


Figure 6: Variance-size relation. Elaboration on PHID, Pharmaceutical Industry Dataset. Source: Bottazzi and Secchi (2006b).

consequence, the variance of the growth rate reduces as the size of firms increases.

To summarize, the challenge for the theory is to account for: [i] persistent heterogeneity in productivities, [ii] skewness in size distributions, [iii] market turbulence, [iv] fat-tailed distribution of growth rates, [v] negative variance-size correlations. How do incumbent theories fare in this respect?

3 Theoretical interpretations

Let us start from that family of models which we call in Dosi et al. (1995) of " evolutionary equilibria", including Jovanovic (1982), Ericson and Pakes (1995), Hopenhayn (1992a), Hopenhayn (1992b) and Pakes and Ericson (1998), where there is the explicit attempt to link heterogeneity among firms, their growth and possibly survival. The evolutionary aspect lies in the idiosyncratic productivity enhancing process that leads more productive firms to expand their own capacity, or equivalently, their market shares and less productive firms to shrink their weight in the market up their death. Implicitly, there is some process of selection that reward the more productive and penalize the less efficient at work. However, competitive selection never appears because rational agents "learn" their equilibrium size. These models, at a first glance distinguishable in passive (as Jovanovic (1982)) and in active (as Ericson and Pakes (1995)) learning models, address issues as growth/death rates conditional on age, the dependence or not of the current size on the initial one, and the entry-exit rate in equilibrium. The "rational" attributes stem from the fact that they are characterized by profit-seeking maximizing agents over an infinite time horizon. In each period they decide the equilibrium size and whether to stay or not in the market according to their technological rational expectations. Thus, selection is never at work because it is, so to speak, "anticipated in the heads of the agents" and, as a consequence, every observed variable is an equilibrium one: hard to believe indeed! Moreover, this class of models seems unable to offer any guidance on the interpretation of productivity and size distributions (except the rationalization of whatever observation as an equilibrium one) and of the ubiquitous fat-tailedness of growth rates.
The abandonment of strong rationality and equilibrium assumptions bears abundant fruits. Sim-

ple characterization of firm growth as a multiplicative process, such as in the seminal Ijiri and Simon (1977) work, yields insight into the explanation of observed size distributions. Ijiri and Simon (1967) decompose the total growth rate as the sum of an idiosyncratic and an industry time-variant component. The growth rate at the industry level is assumed to be constant, as the initial size of the firms. Finally, idiosyncratic shocks are modelled as an $AR(1)$ process capturing [i] the independence of growth rates from size (Gibrat Law); [ii] one-period autocorrelation of growth rates (or single period Markow process); and [iii] a reversion to the mean behaviour. The growth rates are described as the sum of the independent micro-shocks. The size trajectories turn out to be random walks with drift yielding lognormal distributions.

However, the interactiveness intrinsic to the dynamics of firm growth is still missing. And this is what different families of evolutionary and Simonesque models are meant to capture.

In such a perspective, the micro patterns of the industrial dynamics are the outcome of twin processes of *learning* and *selection* among *boundedly rational, interacting agents*. The aggregate regularities emerge as the result of the continuum coupling of *change* and *coordination*. Models like the ones proposed in Nelson and Winter (1982), Silverberg and Lehnert (1993), Dosi et al. (1995), Dosi et al. (2000a), Bottazzi et al. (2001) and Winter et al. (2003) are somewhat different and complementary examples of the evolutionary-modelling approach. Evolutionary models that address the pattern of industry evolution, with particular reference to the learning and selection process, can be subdivided into three different categories: [i] the group of models that focus on the selection process (see Metcalfe (1998) for an extensive discussion); [ii] models that mainly investigate the pattern of firm growth as a process of cumulation of learning opportunities folding together the two different processes (from Ijiri and Simon (1977) to Bottazzi and Secchi (2006a)); and [iii] models that unpack and treat separately the two processes (Silverberg et al. (1988) and Dosi et al. (1995)).

Of particular interest for our purpose here, are those Simonesque models which parsimoniously but powerfully account for correlating mechanisms both on learning and competition, such as Bottazzi and Secchi (2006a), nesting such correlations into firm specific increasing returns in (correlated) business opportunities. Building upon the "island" model by Ijiri and Simon (1977), they introduce the hypothesis of exploitation of business opportunity via a Polya Urn scheme, wherein in each period "success breeds success". It is a two-step model, where in the first step an assignment procedure of the *fixed number* of business opportunities M is realised. In the second step, these business opportunities act as source of growth.[3] The dynamics of firms growth rate is still of *Gibrat type* but with the significant difference that the number of opportunities M are not assigned with a constant probability $1/N$, being N the number of firms, but proportionally to the number of opportunities that in *each period* the firm already has access to, which is correlated to the opportunities of all other firms. At each time step a micro-shock of type $i \in \{1, ..., N\}$ is extracted from an urn. Once it is extracted, the ball is replaced and, additionally, a new ball of the same colour is introduced. This implies that, once one type i has been extracted, the probability of that type being re-extracted increases. This procedure is repeated M times, the number of the total business opportunities. Indeed this cumulative process is at the core of the emergence of fat-tailed distributions. Bottazzi and Secchi (2006a) demonstrated that when N and the ratio M/N increase, the limit distribution of this scheme is Laplace distributed and this occurs independently from the distribution function of the shocks. This explanation, which

[3] A further development along similar lines with a birth process of new types of opportunities (i.e., balls of different colors) is in Marengo and Zeppini (2014).

we share indeed, of the "tent shape" relies on the idea that a big chunk of the micro-shocks M are concentrated in few firms. However, a significant drawback of this representation is that the assignment procedure occurs once each period: dynamic increasing returns displace in *space* (that is, cross-sectionally), as a cumulation of many shocks in few firms, but never in *time*. It turns out to be rather difficult imagine that firms update their expertise every period – say, a year – when the urn is open.
Nevertheless, our conjecture, is that synchronous cumulative processes are only one of the drivers of the apparent correlations underlying the "tents" in growth rates. Indeed, we suggest that a rather large ensemble of evolutionary processes, characterized by different forms of *idiosyncratic* (i.e., firm-specific) *learning* and *competitive interactions* yields the observed distributions of growth rates. This is the hypothesis we are going to explore in this work. Also, we shall provide an explicit account of learning and selection dynamics that is able to robustly explain the stylised facts flagged earlier.

4 The model

The model is an evolutionary agent-based model microfounded upon simple behavioural rules and heuristics. The absence of any rational expectations (technological or otherwise) is intentionally pursued. Most of individual (and more so organizational) decisions and behaviours take place under conditions of environmental complexity, radical uncertainty and massively imperfect information. The microeconomics of our model fully acknowledges all that and is inscribed within the perspective of "bounded rationality", written large (let us just mention the classic March and Simon (1958), Nelson and Winter (1982), and Gigerenzer and Selten (2002)).
The three processes that takes place in the simulated industry are learning, selection and entry-exit.

4.1 Idiosyncratic learning processes

We build upon a simplified version of Dosi et al. (1995) whereby learning is represented by some multiplicative stochastic process upon firms productivities – or more generically "levels of competitiveness" – a_i of the form

$$a_i(t) = a_i(t-1)(1+\theta_i(t)) \tag{2}$$

where the $\theta_i(t)$ are realizations of a sequence of random variables $\{\Theta_i\}_{i=1}^{N}(t)$, and $N(t)$ is the number of firms. Such dynamics is meant to capture the idiosyncratic accumulation of capabilities within each firm (see Teece et al. (1994) and Dosi et al. (2000b)). This entails various types of process innovation – yielding higher production efficiency and plausibly lower output prices – and new and improved products. This latter dynamics is not explicitly formalized in our utterly simple model but one may think of new products as higher value added ones, thus implicitly proxied by higher value of a_i.
The process is a multiplicative random walk with drift: the multiplicative nature is well in tune with the evidence on productivity dynamics under the further assumption that if a firm draws a negative θ_i, it will stick to its previous technique (negative shocks are quite unreasonable!).
We experiment with different learning processes, whereby $\theta_i(t)$ is drawn from a set of possible alternative distributions, namely Normal, Lognormal, Poisson, Laplace and Beta. Consider the

foregoing model as our *Baseline Regime*. Further, we experiment, as one already did in Dosi et al. (1995), with two other more extreme regimes, named *Schumpeter Mark I* and *Schumpeter Mark II*, somewhat dramatizing the role attributed to entrants vs. incumbents by the "young Schumpeter" (Schumpeter, 1912) and the "old Schumpeter" (Schumpeter, 1947).

Under *Schumpeter Mark I*

$$\theta_i(t) = 0 \tag{3}$$

for all incumbents. Conversely, under *Schumpeter Mark II*

$$\theta_i(t) = \pi_i(t) \left(\frac{a_i(t-1)}{\sum_i a_i(t-1)s_i(t-1)} \right)^{\gamma} \tag{4}$$

where $\pi_i(t)$ is the same draw process as under the *Baseline Regime*. At one extreme, in the first case, incumbents do not learn after birth. Advances are only carried by new entrants. At the opposite extreme, in the latter case, incumbents do not only learn, but do it in a cumulative way so that a draw by any firm is scaled by its extant relative competitiveness. This captures what Paul David, quoting Robert Merton, called the "Matthew effect". [4]

4.2 Market selection and birth-death processes

Competitive interactions are captured by a "quasi replicator dynamics"

$$\Delta s_i(t, t-1) = A s_i(t-1) \left(\frac{a_i(t)}{\bar{a}(t)} - 1 \right) \tag{5}$$

where

$$\bar{a}(t) = \sum_i a_i(t) s_i(t-1) \tag{6}$$

where $s_i(t)$ is the market share of firm i which changes as a function of the ratio of the firm's productivity (or "competitiveness") to the weighted average of the industry. It is a "quasi-replicator" since a genuine replicator lives on the unit simplex. The "quasi" one may well yield negative shares, under certain parametrizations, in which case the firm is declared dead and market shares are accordingly recomputed. Being A an elasticity parameter that captures the intensity of the selection exerted by the market, in terms of market share dynamics and, indirectly, of mortality of low competitiveness firms. Below, we shall study the effect of different degrees of *market selectiveness* upon industry structures and dynamics. In that, the *competitive process as such* induces ex post correlation in growth rates: the growth of the share of any one firm induces the fall of the total share of its competitors.

Finally, we assume that entry of new firms occurs proportionally to the number of incumbents present in the market

$$E(t) = \omega(t) N(t-1) \tag{7}$$

where $E(t)$ is the number of entrants at time t, $N(t-1)$ is the number of incumbents in the previous period and $\omega(t)$ is a random variable uniformly distributed on a finite support (which in the following, for simplicity,we assume drawn from a uniform distribution). The idea that the number of entrants is proportional to the number of incumbents is strongly corroborated

[4] *"For unto every one that hath shall be given, and he shall have abundance: but from him that hath not shall be taken even that which he hath"* (Matthew 25:29, King James Version). For more details see Dosi and Sylos Labini (2007).

empirically: (see e.g., Geroski (1991) and Geroski (1995). However, the evidence supports also a rough proportionality between entry and exit: thus, in the simplest version of the model, we assume a constant number of firms with the number of dying firms offset by a equal number of entrants.

The productivity of entrants follows a process similar to Eq. 2 but applied to the *average productivity* of the industry, whose stochastic component $\theta_j(t)$ is again a random extraction from alternative distributions (Normal, Lognormal, Poisson, Laplace and Beta)

$$a_j(t) = (1 + \theta_j(t)) \sum_i a_i(t) s_i(t-1) \tag{8}$$

Of course, here $\theta_j(t)$ can get negative values. Indeed, the location of the mass of the distribution – over negative or positive shocks – captures barriers to learning by the entrant or, conversely, the advantage of "newness".

4.3 Timeline of events

- There are N initial incumbent firms. They have at time 0 equal productivities and equal market shares.
- At the beginning of each period, except under Mark I Regime, firms learn according to the process specific to each regime.
- Firms acquire or lose market share, according to the replicator.
- Firms exit the market according to the rule: $s_i(t) < s_{min}$.
- The number, the competitiveness and the size of entrants are determined and market shares of incumbents are adjusted accordingly.

	Value
Number of firms (N)	150
Initial productivity	1
Initial market share ($1/N$)	0.006667
s_{min}	0.001
A	1
γ	1
$Beta(\beta_1, \beta_2)$	$(1, 5)$
$Normal(\mu, \sigma)$	$(0.05, 0.8)$
$Lognormal(\mu_1, \sigma_1)$	$(-3.5, 1)$
$Laplace(\alpha_1, \alpha_2)$	$(0.01, 0.015)$
Number of time steps	200
Number of MC runs	50

Table 2: Parameters and simulation setup.

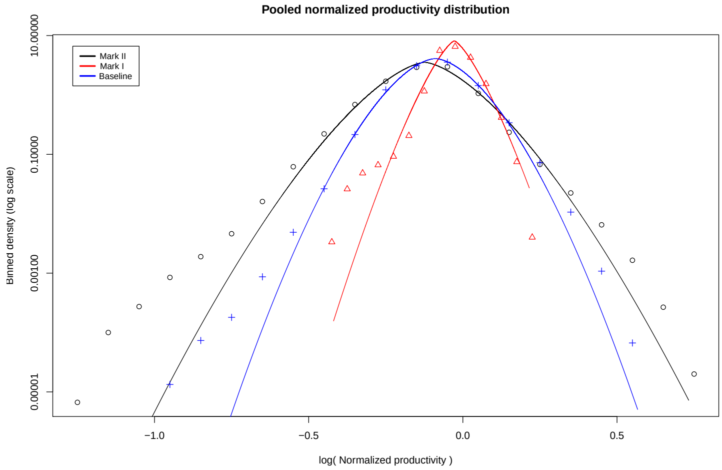


Figure 7: Log normalized productivity distribution across regimes.

5 Model properties and simulation results

Let us consider the model properties together with the simulation results which we shall present according to the empirical "stylised facts" the model is able to replicate. The model is parametrized according to Table 2 and the benchmark distribution for the extraction of the micro-shocks is Beta distribution if not otherwise specified.

5.1 Productivity distribution

The productivity distributions obtained from the three regimes are presented in Figure 7.
The normalized productivity distributions are calculated as

$$n_i(t) = \log a_i(t) - \log \sum_i a_i(t) s_i(t-1) \tag{9}$$

As already mentioned, $n_i(t)$ turns out to be rather disperse on its support with a persistency of such asymmetries over time. It is interesting to notice how the support of the distribution (in logs) spans between $(-1, 0.5)$ in the Baseline Regime, shrinks in the Mark I Regime $(-0.5, 0.2)$, and expands in the Schumpeter Mark II $(-1.4, 0.8)$. If innovation is endogenous the support is wider and even more so if it is cumulative as under the Schumpeter Mark II engine.
The autocorrelation structure of productivities is reported in Table 3. The] parameters are in tune with empirical evidence.

Autocorrelation	$AR(1)$
Baseline Regime	0.970
Schumpeter Mark I Regime	0.986
Schumpeter Mark II Regime	0.987

Table 3: Autocorrelation coefficients across regimes.

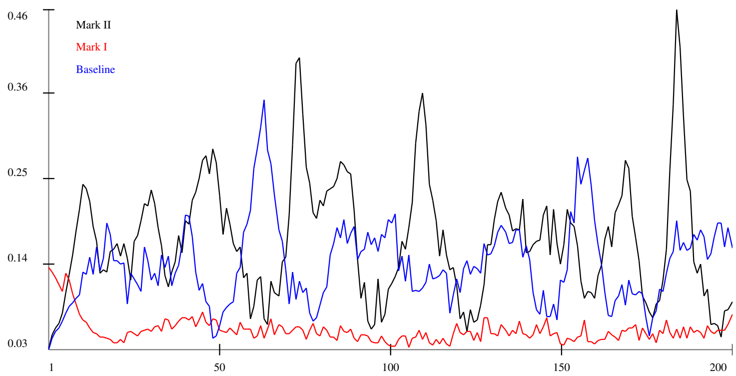


Figure 8: Market turbulence index.

5.2 Market turbulence and concentration

The dynamic of entry and exit is a key aspect of our model: in each scenario a churning market is a major aspect of competition whereby new firms enter and others die. The dynamics are illustrated in Table 4 which lists a series of descriptive statistics about the number of entrants, the average age, the average productivity growth, the average size growth, the entry-exit turbulence, and the market concentration.
Interestingly, and somewhat counter-intuitively, firms' life expectation is the highest under the non-endogenous learning (Schumpeter Mark I) regime (21 periods) and the lowest in the cumulative regime (4 periods). This is a robust sign of the effects of "creative destruction" when innovation is endogenous.

As expected, and in agreement with Table 4, the three regimes show increasing degrees of turbulence, from Mark I to Mark II (see Figure 8). The average reported values are strikingly close to the empirical data shown in Table 8. Finally, the market concentration, measured by the Herfindahl-Hirschman Index[5] in typical simulation runs, is presented in Figure 9.

The Mark I Regime concentration is relatively low and stable over time; conversely, in the Baseline and in the Mark II regimes, phases of high and low concentration alternate, signalling the presence of endogenous cycles which are not related to "market power" but, rather, to the path-dependent dynamics of innovative success.

5.3 Size distribution

Regarding firm size distribution, proxied by the market shares s_i, Figure 10 shows the rank-size plot (in logs) fitted against a Lognormal distribution. If the size distribution follows a power law, being r_i the rank of firm i, then

$$s_i r_i^\tau = C \tag{10}$$

linearising we have

$$\log r_i = \alpha + \tau \log s_i \tag{11}$$

so τ is the slope parameter: under the Zipf law (which is a restriction of the Pareto scaling) τ is equal to one. Recall from Section 2 above that the empirical size distributions, at relatively

[5] $H(t) = \sum_i s_i^2(t) \quad 0 < H \leq 1$

Baseline Regime	Average	Min	Max	S.E.
Number of entrants	12.14	0	15.64	2.974
	(0.792)	(0.000)	(1.198)	(0.479)
Average age	8.376	1	11.02	1.067
	(0.286)	(0.000)	(0.383)	(0.171)
Average productivity growth	0.046	0.045	0.051	0.001
	(0.0005)	(0.0004)	(0.0008)	(0.0002)
Average shares growth	-0.091	-0.138	-0.001	0.019
	(0.0065)	(0.000)	(0,0096)	(0,0039)
Turbulence Index	0.137	0.032	0.170	0.017
	(0,006)	(0.0002)	(0.070)	(0,01)
Herfindahl-Hirschman Index	0.175	0.007	0.229	0.052
	(0,014)	(0.000)	(0.0175)	(0.0086)
Schumpeter Mark I Regime	Average	Min	Max	S.E.
Number of entrants	3.799	0	13	1.334
	(0.243)	(0.000)	(0.355)	(0.1629)
Average age	20.66	1	24.23	4.746
	(0.239)	(0.000)	(0.353)	(0.178)
Average productivity growth	0.002	0	0.372	0.026
	(0.000)	(0.000)	(0.000)	(0.000)
Average shares growth	-0.027	-0.115	-0.013	0.012
	(0.0005)	(0.000)	(0.0008)	(0.0004)
Turbulence Index	0.058	0.042	0.137	0.014
	(0.001)	(0.000)	(0.001)	(0.0007)
Herfindahl-Hirschman Index	0.029	0.007	0.040	0.008
	(0.001)	(0.000)	(0.002)	(0.001)
Schumpeter Mark II Regime	Average	Min	Max	S.E.
Number of entrants	17.16	0	22.74	4.111
	(1.249)	(0.000)	(1.849)	(0.750)
Average age	5.306	1	9.571	0.8396
	(0.2305)	(0.000)	(0.290)	(0.1319)
Average productivity growth	0.175	0.160	0.200	0.006281
	(0.0005)	(0.000)	(0.0007)	(0.002)
Average shares growth	-0.137	-0.226	-0.001	0.030
	(0.011)	(0.000)	(0.015)	(0.007)
Turbulence Index	0.174	0.032	0.252	0.026
	(0.011)	(0.0002)	(0.017)	(0.007)
Herfindahl-Hirschman Index	0.240	0.007	0.302	0.062
	(0.014)	(0.000)	(0.002)	(0.001)

Table 4: Descriptive Statistics across regimes (50 Monte Carlo runs).

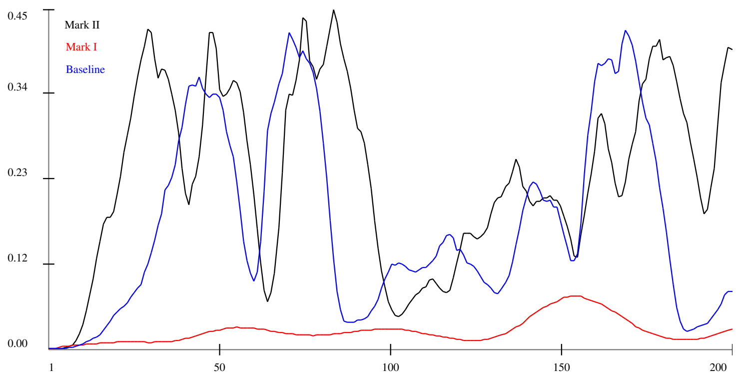


Figure 9: Herfindahl-Hirschman concentration index.

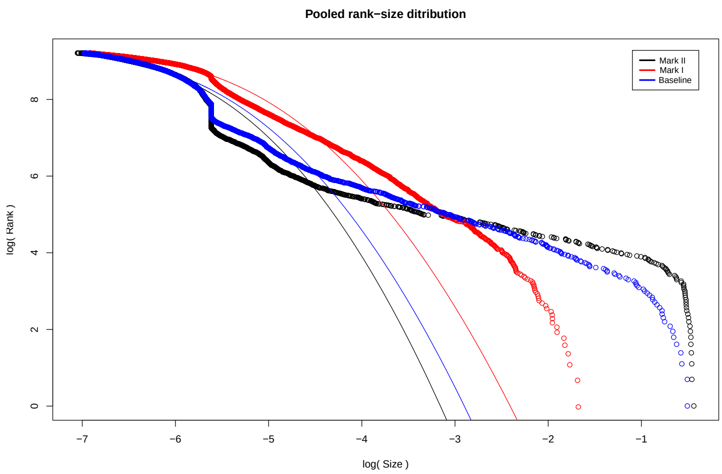


Figure 10: Log rank-size distribution across regimes.

disaggregated levels, are far from any power law. And so are generally our results, with a slope clearly different from one. However, note that beyond some cut-off point the Lognormal is no longer a good approximation of the size distribution. In fact, sizes appear to be rather skewed across all the three regimes. In case of the Schumpeter Mark I, the cut-off point is higher than in the other two cases. A bigger fraction of firms seems to be characterised by a Lognormal size distribution with respect to the Baseline and the Mark II regimes. Again, these results seem in line with the empirical evidence discussed in Section 2.2.

5.4 Firm growth

The growth rate of firm sizes is defined as

$$g_i(t) = \log s_i(t) - \log s_i(t-1) \tag{12}$$

where market shares are our proxy for size. In Figure 11 the growth rate distributions across the three regimes are presented. Notice the strong fat-tailed departure from a Normal distribution in all the three regimes.

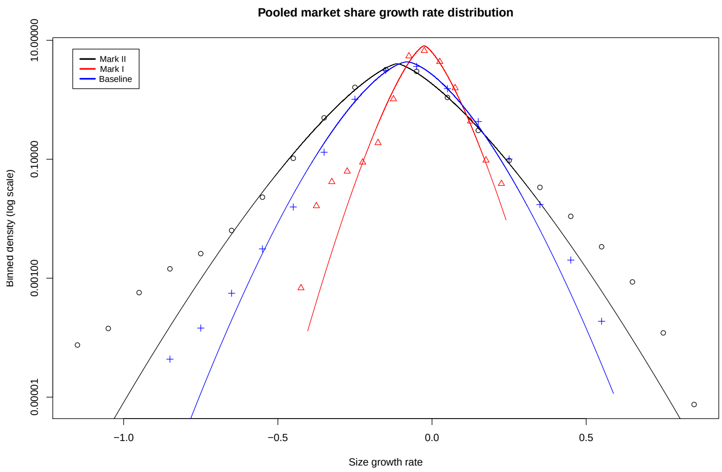


Figure 11: Growth rate distribution across regimes.

In order to test for the robustness of these results, we experimented with four alternative distributions for the innovation shocks, namely Laplace, Lognormal, Normal, and Poisson. Figure 12 shows the results for the four distributions across the three regimes. It is rather striking the departure from normality and the emergence of fat tails, independently from the shape of the micro-shocks.

To understand how "fat" the tails are, we estimated the parameters of a symmetric Subbotin distribution, defined by the parameters m, a and b. m is a location measure, a is the scale parameter and b informs the fatness of the tails. The Subbotin distribution

$$f_S(x) = \frac{1}{2ab^{1/b}\Gamma(1/b+1)} e^{-\frac{1}{b}\left|\frac{x-\mu}{a}\right|^b} \tag{13}$$

according to the value of the parameter b, can yield [i] a Normal distribution, if $b = 2$, or [ii] a Laplace distribution, if $b = 1$. The estimation of the Subbotin distribution parameters is presented in Table 5. Across the three regimes, the value of the b parameter is always significantly smaller than 2 (the normal case).

What is the source of the tent-shaped distribution of growth rates? Our intuition is that the replicator dynamics yields a mechanism of correlation equivalent to a Polya urn mechanism.

As demonstrated by Schreiber (2001) and discussed in Pemantle (2007), *the stochastic replicator is a generalized Polya urn scheme*. As discussed in the latter, suppose that, at each time $t \geq 0$, there is a population of firms $N(t)$ whose only attribute is the relative productivity $m_i(t) = a_i(t)/\bar{a}(t)$, $i \in \{1, ..., N\}$. These firms can be represented by an urn with balls of colors m, being $m_i(t)$ the relative "fitness" of firm i and the market share $s_i(t)$ its outcome. At each time step t a ball (firm) of color (relative productivity) m is extracted from the urn and then returned to the urn plus (with some probability) a new ball of the same color of color m. Being $m_i(t)$ a measure of the fitness of firm i in the population, its representation $s_i(t)$ (market share) change by an amount proportional to its fitness against the others $N(t) - 1$ balls. Repeating this mechanism allows for the growth of firm i to be proportional to its own success against all the other firms, weighted by their own representation in the population. Clearly

$$\Delta s_i(t, t-1) = a_i(t)s_i(t-1) / \sum_i a_i(t)s_i(t-1) \tag{14}$$

	Baseline Regime b	Schumpeter Mark I Regime b	Schumpeter Mark II Regime b
Beta shocks	1.539	1.397	1.367
	(0.006)	(0.003)	(0.006)
Gaussian shocks	1.637	1.426	1.402
	(0.005)	(0.003)	(0.007)
Laplace shocks	1.489	1.360	1.284
	(0.007)	(0.003)	(0.007)
Lognormal shocks	1.467	1.329	1.327
	(0.006)	(0.003)	(0.006)
Poisson shocks	1.327	0.900	1.231
	(0.007)	(0.001)	(0.007)

Table 5: Estimation of the b parameters across regimes under different innovation shocks

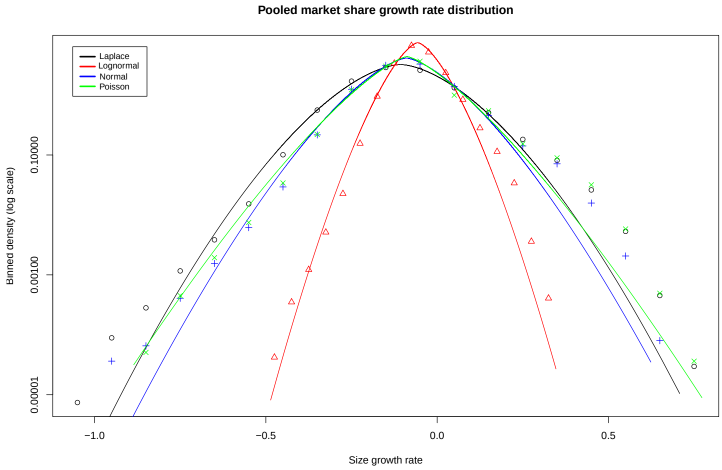
The finite number of opportunities in the Bottazzi and Secchi (2006a) model reads, in our case, as the finite dimension of the market, given that it lies in the unit simplex: $\sum_i s_i(t) = 1, \forall t \geq 0$. We explored such intuition throughout our simulations under the three alternative regimes. Indeed, the model is able to robustly reproduce the fat-tailed distribution of growth rates under the three learning regimes, and, in the *Schumpeter Mark II* case, a strict Laplace distribution emerges. In our replicator process, the correlation at the origin of the fat tails occurs both in "space" (that is, within each period) as in Bottazzi and Secchi (2006a) and over time.

5.5 Scaling growth-variance relationship

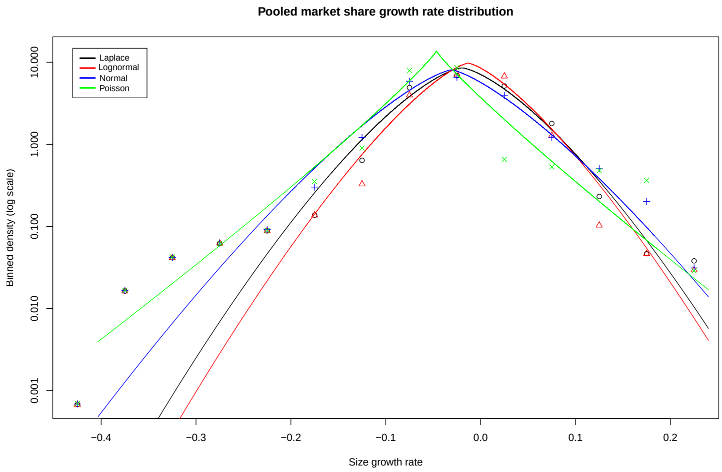
Figure 13 shows the negative correlation between the variance of growth rate and size, together with an *OLS* fit for the data

$$\sigma(g_i) = \alpha + \beta s_i \tag{15}$$

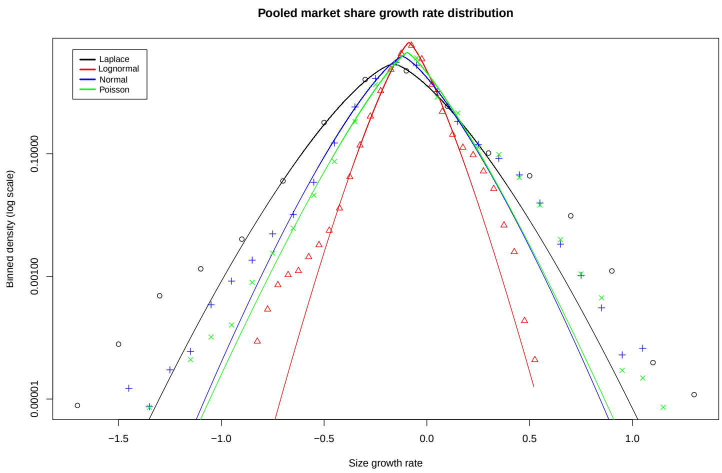
The slope coefficient β is presented in Table 6 (here the independent variable is market share rather than sheer size, but the two, in finite markets, easily map). Across the three regimes, the negative linear correlation is always present. What is the source of this persistent relationship? One interpretation of variance scaling has been extensively discussed in Bottazzi and Secchi (2006b), as already mentioned above. In our model there is no possibility of a process of differentiation, but nonetheless the negative scaling holds. Why? Recalling equation 5, it is straightforward how the size growth rate depends on the firm relative productivity $m_i = a_i/\bar{a}$. The higher the size (share) of firm i, other things being equal, the nearer the firm is to the market weighted average, since it disproportionally contributes the average itself. It is as if size as such "bends" the selection landscape. In order to visualize this, think of the extreme of a monopolist: in this case, its own competitiveness is also the market average and, of course, the replicator dynamics impact upon growth variance is nil. At the opposite extreme, smaller firms might be by far more or less competitive than the mean but, because of their size, they have a negligible effect upon the "selection" process. Hence, their growth rates are more prone to fluctuate. This induce the emergence of a *core-periphery* structure where bigger firms tend to have a lower variation in their growth rate while the smaller ones fuel the turbulence of market



(a) *Baseline Regime.*



(b) *Schumpeter Mark I Regime.*



(c) *Schumpeter Mark II Regime.*

Figure 12: Firm growth rates under different distributions of innovation shocks.

Scaling variance-growth	β
Baseline Regime	-0.2562
	(0.006)
Schumpeter Mark I Regime	-0.190
	(0.01)
Schumpeter Mark II Regime	-0.4121
	(0.008)

Table 6: Estimation of the slope coefficient of scaling variance-growth correlation across regimes.

shares. Needless to say, this interpretation of the variance scaling relation does *not* contradict the diversification hypothesis. It just adds to it a scale dependence of the very yardstick of competitive selection.

5.6 Cumulativeness and selection

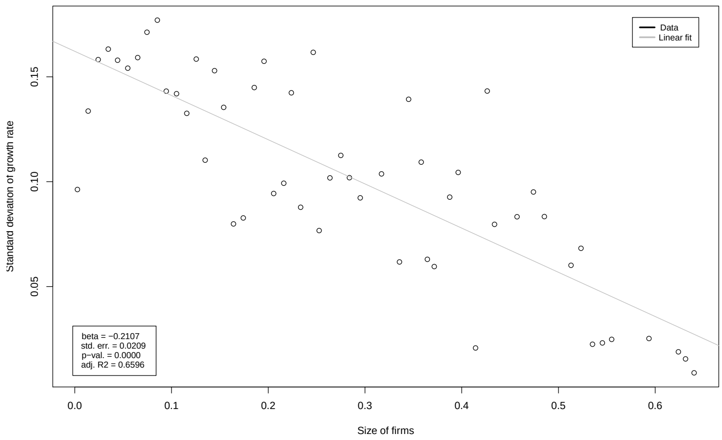
We have argued above that the very competition process robustly induces fat-tailed distributed growth rates, even in absence of learning. But of course, cumulative learning is also plausibly enhancing such fat-tailedness, and so does the increase in the "fierceness" of competition. Let us explore this conjecture in the case of the Schumpeter Mark II regime. To do this, we study the effects on the distribution of firms growth rate of the γ parameter, which captures the degree of cumulativeness in the learning process, and the A parameter, which embodies the selectivity of the market mechanism.

We start by analysing the effect of cumulativeness. As expected, the increase in the γ parameter, as shown in Figure 14, induces a more tent-shaped distribution of the growth rates, up to the point of becoming "super Laplacian", with the estimate of b – the tail estimate of the distribution – falling below one. Notice that the estimates under such regime are those which look closer to the empirical ones, circumstantial evidence that Matthews effects are quite widespread. It is worth underlining that the causality here goes from the cumulative learning process to the selection mechanism and, then, to the dynamics of firm growth, *in absence of any contemporaneous correlation among the productivity shocks themselves.*

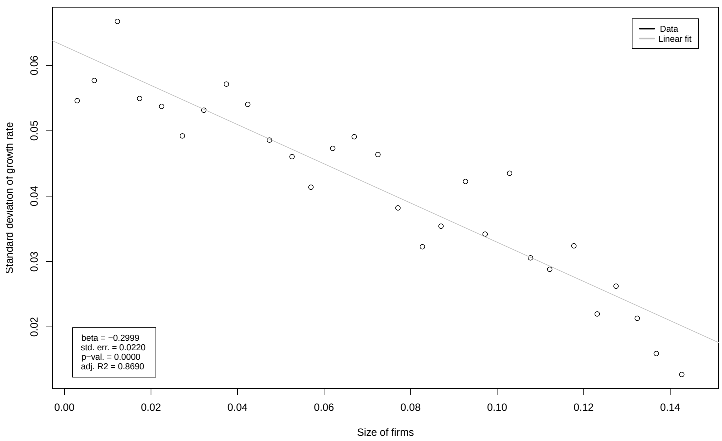
Regarding the effect of parameter A, Figure 15 illustrates the asymmetric effects of different degrees of market selectiveness. On the one side, A influences the support of the tail of surviving, negative-growth firms and, on the other, the growth of successful ones, that is, the conditions of the coexistence of both high growth firms: the "gazzellas" at the upper-right tail and the "laggards" at the lower-left tail. This hints that the selection parameter is positively correlated to the asymmetry of the distribution.

6 Conclusions

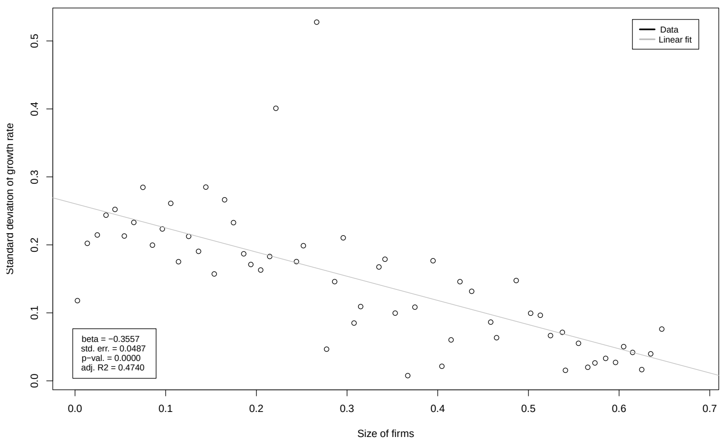
There is an ensemble of "stylised facts" on industrial dynamics which ubiquitously emerges across industries, levels of aggregation, times and countries. They include: wide and persistent asymmetries in degrees of relative efficiency, however measured; skewed size distributions; negative scaling relations between firm sizes and variance in firm growth rates; and, last but not least, fat-tailed distributions of growth rates themselves.



(a) *Baseline Regime.*



(b) *Schumpeter Mark I Regime.*



(c) *Schumpeter Mark II Regime.*

Figure 13: Scaling growth-variance relationship across regimes.

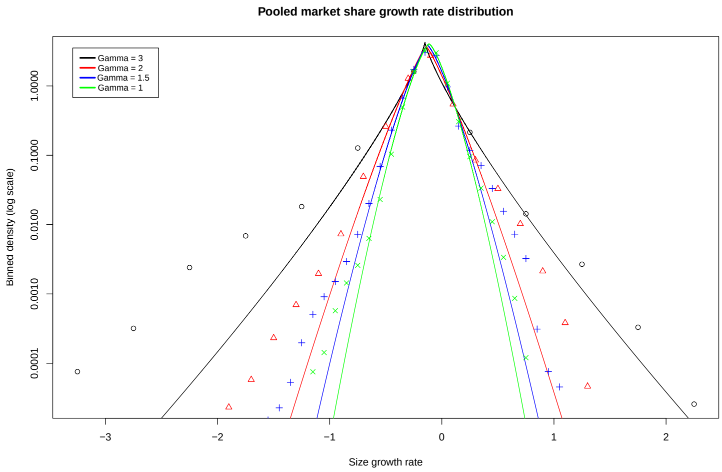


Figure 14: Schumpeter Mark II Regime. Firm growth rate distributions under different degrees of cumulativeness.

	Schumpeter Mark II
Cumulativeness	b
$\gamma = 1.5$	1.293
	(0.008)
$\gamma = 2$	1.176
	(0.011)
$\gamma = 3$	0.811
	(0.014)
	Schumpeter Mark II
Selection	b
$A = 0.2$	1.423
	(0.01)
$A = 0.5$	1.355
	(0.007)
$A = 1.5$	1.419
	(0.007)
$A = 2$	1.458
	(0.004)

Table 7: The effect of cumulativeness and selection pressure.

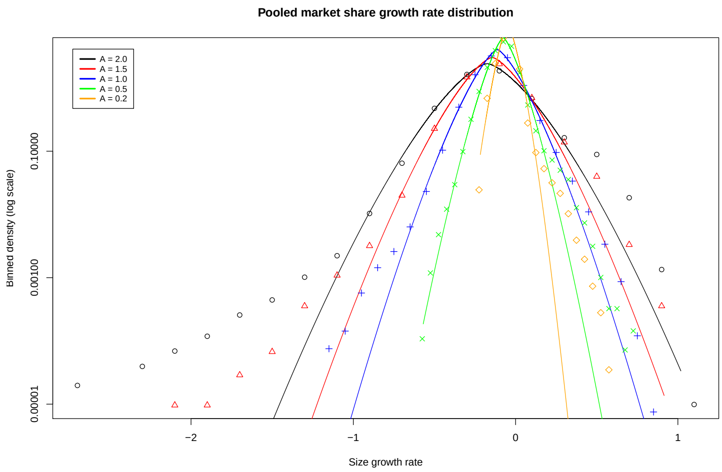


Figure 15: Firm growth rates under different selection pressure.

We show that all these regularities can be accounted for by a very simple agent-based evolutionary model whereby the dynamics is driven by some learning process by incumbents and entrants (or at least by entrants alone) together with some process of competitive selection. They correlate the growth (and survival) fates across firms, even in absence of correlations in the original "technological shocks" themselves. We also show that this interpretation is robust to different learning and competition regimes, including the extreme archetypes wherein incumbents do not learn at all or, other, with incumbents building success upon success.

References

Axtell, R. L. (2001). "Zipf Distribution of U.S. Firm Sizes". *Science* 293.5536, 1818–1820.

Baldwin, J. and M. Rafiquzzaman (1995). "Selection versus evolutionary adaptation: Learning and post-entry performance". *International Journal of Industrial Organization* 13.4, 501–522.

Bartelsman, E. J. and P. J. Dhrymes (1998). "Productivity Dynamics: U.S. Manufacturing Plants, 1972–1986". *Journal of Productivity Analysis* 9.1, 5–34.

Bottazzi, G. (2014). "On the Bosonic Nature of Business Opportunities". *15 ISS Conference, Jena, Germany.*

Bottazzi, G. and A. Secchi (2006a). "Explaining the distribution of firm growth rates". *RAND Journal of Economics, RAND Corporation* 37.2, 235–256.

Bottazzi, G. and A. Secchi (2006b). "Gibrat's law and diversification". *Industrial and Corporate Change* 15.5, 847–875.

Bottazzi, G., G. Dosi, and G. Rocchetti (2001). "Modes of knowledge accumulation, entry regimes and patterns of industrial evolution". *Industrial and Corporate Change* 10.3, 609–638.

Bottazzi, G., E. Cefis, and G. Dosi (2002). "Corporate growth and industrial structures: some evidence from the Italian manufacturing industry". *Industrial and Corporate Change* 11.4, 705–723.

Bottazzi, G., M. Grazzi, and A. Secchi (2005). “Input output scaling relations in Italian manufacturing firms”. *Physica A: Statistical Mechanics and its Applications* 355.1, 95 –102.

Bottazzi, G., E. Cefis, G. Dosi, and A. Secchi (2007). “Invariances and diversities in the patterns of industrial evolution: Some evidence from Italian manufacturing industries”. *Small Business Economics* 29.1-2, 137–159.

Bottazzi, G., A. Secchi, and F. Tamagni (2008). “Productivity, profitability and financial performance”. *Industrial and Corporate Change* 17.4, 711–751.

Brock, W. (1999). “Scaling in economics: a reader’s guide”. *Industrial and Corporate change* 8.3, 409–446.

Doms, M. and E. J. Bartelsman (2000). “Understanding Productivity: Lessons from Longitudinal Microdata”. *Journal of Economic Literature* 38.3, 569–594.

Dosi, G. (2007). “Statistical Regularities in the Evolution of industries. A guide trough some evidence and challenges for the theory”. *Perspectives on innovation.* Malerba, F. and Brusoni, S. Editors (2007).

Dosi, G. and M. Grazzi (2006). “Technologies as problem-solving procedures and technologies as input–output relations: some perspectives on the theory of production”. *Industrial and Corporate Change* 15.1, 173–202.

Dosi, G. and M. Sylos Labini (2007). “Technological paradigms and trajectories”. *Elgar Companion to Neo-Schumpeterian Economics.* Hanusch, H. and Pyka, A. Editors (2007), 333.

Dosi, G., O. Marsili, L. Orsenigo, and R. Salvatore (1995). “Learning, market selection and the evolution of industrial structures”. *Small Business Economics* 7.6, 411–436.

Dosi, G., S. Winter, and Y. Kaniovski (2000a). “Modeling industrial dynamics with innovative entrants”. *Structural Change and Economic Dynamics* 11, 3.

Dosi, G., R. Nelson, and S. Winter (2000b). *The nature and dynamics of organizational capabilities.* Oxford University Press.

Dosi, G., A. Gambardella, M. Grazzi, and L. Orsenigo (2008). “Technological revolutions and the evolution of industrial structures: assessing the impact of new technologies upon the size and boundaries of firms”. *Capitalism and Society* 3.1, 6.

Dosi, G., M. Grazzi, C. Tomasi, and A. Zeli (2012). “Turbulence underneath the big calm? The micro-evidence behind Italian productivity dynamics”. *Small Business Economics* 39.4, 1043–1067.

Dosi, G., M. Grazzi, L. Marengo, and S. Settepanella (2013). “Production theory: accounting for firm heterogeneity and technical change”. 2013/22.

Ericson, R. and A. Pakes (1995). “Markov-Perfect Industry Dynamics: A Framework for Empirical Work”. *Review of Economic Studies* 62.1, 53–82.

Foster, L., J. Haltiwanger, and C. Syverson (2008). “Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?” *American Economic Review* 98.1, 394–425.

Geroski, P. (1991). *Market dynamic and entry.* Wiley-Blackwell.

Geroski, P.A. (1995). "What do we know about entry?" *International Journal of Industrial Organization* 13.4. The Post-Entry Performance of Firms, 421 –440.

Gigerenzer, G. and R. Selten (2002). *Bounded rationality: The adaptive toolbox.* Mit Press.

Grazzi, M., R. Sanzo, A. Secchi, and A. Zeli (2013). "The building process of a new integrated system of business micro-data 1989–2004". *Journal of Economic and Social Measurement* 4, 291–324.

Hanusch, H. and A. Pyka (2007). *Elgar Companion to Neo-Schumpeterian Economics.* Edward Elgar.

Hopenhayn, H. (1992a). "Entry, Exit, and firm Dynamics in Long Run Equilibrium". *Econometrica* 60.5, pp. 1127–1150.

Hopenhayn, H. (1992b). "Exit, selection, and the value of firms". *Journal of Economic Dynamics and Control* 16.3, 621–653.

Ijiri, Y. and H. Simon (1967). "A model of business firm growth". *Econometrica: Journal of the Econometric Society*, 348–355.

Ijiri, Y. and H. Simon (1977). *Skew Distributions and the Sizes of Business Firms.* North-Holland.

Jovanovic, B. (1982). "Selection and the Evolution of Industry". *Econometrica* 50.3, 649–70.

Lee, Y. et al. (1998). "Universal features in the growth dynamics of complex organizations". *Physical Review Letters* 81.15, 3275.

Malerba, F. and S. Brusoni (2007). *Perspectives on innovation.* Cambridge University Press.

March, J. and H. Simon (1958). *Organizations.* Wiley.

Marengo, L. and P. Zeppini (2014). "The arrival of the new". *LEM Working Papers Series.*

Marsili, O. (2005). "Technology and the size distribution of firms: evidence from Dutch manufacturing". *Review of Industrial Organization* 27.4, 303–328.

Metcalfe, J. S. (1998). *Evolutionary Economics and Creative Destruction.* Routledge.

Nelson, R. R. and S. G. Winter (1982). *An Evolutionary Theory of Economic Change.* Cambridge, MA: Belknap Press of Harvard University Press.

Newman, M. (2005). "Power laws, Pareto distributions and Zipf's law". *Contemporary physics* 46.5, 323–351.

Pakes, A. and R. Ericson (1998). "Empirical implications of alternative models of firm dynamics". *Journal of Economic Theory* 79.1, 1–45.

Pemantle, R. (2007). "A survey of random processes with reinforcement". *Probab. Surveys*, 1–79.

Schreiber, S. J. (2001). "Urn models, replicator processes, and random genetic drift". *SIAM Journal on Applied Mathematics* 61.6, 2148–2167.

Schumpeter, J. (1912). *Theorie der wirtschaftlichen Entwicklung.* Duncker & Humblot.

Schumpeter, J. (1947). *Capitalism, Socialism, and Democracy.* New York and London: Harper & Brothers Publishers.

Silverberg, G. and D. Lehnert (1993). “Long waves and ’evolutionary chaos’ in a simple Schumpeterian model of embodied technical change”. *Structural Change and Economic Dynamics* 4.1, 9–37.

Silverberg, G., G. Dosi, and L. Orsenigo (1988). “Innovation, Diversity and Diffusion: A Self-organisation Model”. *Economic Journal* 98.393, 1032–54.

Simon, H. A. and C. P. Bonini (1958). “The Size Distribution of Business Firms”. *The American Economic Review* 48.4, 607–617.

Stanley, M. et al. (1995). “Zipf plots and the size distribution of firms”. *Economics letters* 49.4, 453–457.

Stanley, M. et al. (1996). “Scaling behaviour in the growth of companies”. *Nature* 379.6568, 804–806.

Sutton, J. (2002). “The variance of firm growth rates: the ’scaling’ puzzle”. *Physica A: Statistical Mechanics and its Applications* 312.3–4, 577 –590.

Syverson, C. (2011). “What Determines Productivity?” *Journal of Economic Literature* 49.2, 326–65.

Teece, D. J., R. Rumelt, G. Dosi, and S. Winter (1994). “Understanding corporate coherence: Theory and evidence”. *Journal of Economic Behavior & Organization* 23.1, 1 –30.

Winter, S., Y. Kaniovski, and G. Dosi (2003). “A baseline model of industry evolution”. *Journal of Evolutionary Economics* 13.4, 355–383.

Yu, X., G. Dosi, J. Lei, and A. Nuvolari (2015). “Institutional change and productivity growth in China’s manufacturing: the microeconomics of knowledge accumulation and creative “restructuring””. *Industrial and Corporate Change* forthcoming.