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Defuse the Bomb: Rewiring Interbank Networks

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2015/16 June 2015

ISSN (online) 2284-0400

Defuse the Bomb: Rewiring Interbank Networks

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June 3, 2015

Abstract

In this paper we contribute to the debate on macro-prudential regulation by assessing which structure of the financial system is more resilient to exogenous shocks, and which conditions, in terms of balance sheet compositions, capital requirements and asset prices, guarantee the higher degree of stability. We use techniques drawn from the theory of complex networks to show how contagion can propagate under different scenarios when the topology of the financial system, the characteristics of the financial institutions, and the regulations on capital are let vary. First, we benchmark our results using a simple model of contagion as the one that has been popularized by Gai and Kapadia (2010). Then, we provide a richer model in which both short- and long-term interbank markets exist. By doing so, we study how liquidity shocks (de)stabilize the system under different market conditions. Our results demonstrate how connectivity, the topology of the markets and the characteristics of the financial institutions interact in determining the stability of the system.

Keywords: financial networks, systemic risk, contagion, regulation, network topology

JEL Classification: C63, G01, G21

Acknowledgements

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This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 640772 - DOLFINS .

The authors would like to thank Massimo Molinari and the conference participants at ECCS 2013, ECCS 2014, INET International Conference on Interlinkages and Systemic Risk, WEHIA 2013, Network Models in Economics and Finance, and ICCSS 2015 for the helpful discussions.

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1 Introduction

The financial crisis of the late 2000s has forced economists, both in academia and in regulatory bodies, to confront themselves with the role of the financial system's architecture. Facing the defaults of large financial institutions, regulators found themselves uncertain about the consequences that such failures would have triggered in the entire financial system since the uncertainty about the actual web of financial relationships prevented any reasonable forecast about the eventual path a crisis would follow. Given these premises, it became clear that new approaches to study financial contagion were needed (Haldane, 2009; Schweitzer et al., 2009) and an important new stream of literature has been developing, studying this very problem (e.g.: Acemoglu et al. (2013); Amini et al. (2012, 2013); Arinaminpathy et al. (2012); Battiston et al. (2012a,b); Blasques et al. (2015); Bluhm et al. (2014); Caccioli et al. (2012, 2014); Elliott et al. (2014); Gai et al. (2011); Gai and Kapadia (2010); Georg (2013); Glasserman and Young (2015); Halaj and Kok Sorensen (2014); Lenzu and Tedeschi (2012); Loepfe et al. (2013); May and Arinaminpathy (2010); Montagna and Kok (2013); Roukny et al. (2013)).

In this paper, we contribute to the debate on macro-prudential regulation by assessing which structure of the interbank market is more resilient to exogenous shocks, and which conditions, in terms of balance sheets composition, capital requirements and asset prices, guarantee the highest degree of stability. To analyze this problem, we develop two distinct models of contagion: a *benchmark model* and an *extended model*. The *benchmark model* is based on a simple framework which has been extensively studied in the literature (Amini et al., 2013; Arinaminpathy et al., 2012; Caccioli et al., 2012; Gai and Kapadia, 2010; May and Arinaminpathy, 2010; Nier et al., 2007), and which represents the most popular set up to analyze contagion in financial networks. In this case, the financial system is modeled as a static network of credit exposures between banks, encompassing any sort of interbank claims, independently on their maturity and liquidity. In this class of models, the crucial assumption is that the time scale of the defaults cascade is so quick that bank do not manage to react and modify their exposures. Instead, in the *extended model*, banks can dynamically adapt their short-term exposures in response to (negative) liquidity shocks and they do so to keep their risk-weighted capital ratio above the mandatory threshold set by the policy maker. We model a short-term interbank market which clears at each time step according to a perfect information equilibrium, in which agents take into account their liquidity needs, the liquidity shocks coming from their creditors and the ability of their debtors to repay. In this complex framework, a clear misalignment of micro-prudential and macro-prudential objectives emerges.

Our work contributes to the existing literature along two main dimensions: we study how policy regulations can influence systemic risk when banks can modify their lending behavior; and we provide a systematic overview of the different factors that can affect the stability of the interbank market. With respect to the former, other studies have analyzed what happens when banks can hoard liquidity in response to changes in market confidence, counter-party financial health, or individual financial robustness (Arinaminpathy et al., 2012; Battiston et al., 2012a; Roukny et al., 2013)); or when there are stochastic shocks to the supply of households deposits or to the returns from risky assets (Georg, 2013). However, in our work, we directly link macro-prudential policy measures to liquidity hoarding and we model changes in demand or supply of liquidity endogenously. As a consequence, our analysis can show what is the direct effect of a change in the regulations in terms of systemic risk. Furthermore, rather than focusing our attention on a specific case study (as in Montagna and Kok (2013)), we provide a more general assessment on what factors and characteristics of the interbank market determine its (in)stability, considering not only the effects of changes in the capital requirements, but also evaluating how balance sheet composition, changes in asset prices, network topology and types of shocks can influence the observed level of financial contagion.

Our findings show that the knife-edge property of diversification persists under a variety of assumptions regarding the architecture of the financial system, its connectivity, the heterogeneity of exposures and the size heterogeneity of traders. Connectivity is both a risk sharing and a risk amplification device. The probability of observing systemic crises is non-monotonic in connectivity, reaching a peak for intermediate values, while the severity of contagion episodes, when they happen, worsen as connectivity increases. This leads to robust-yet-fragile systems, in which contagion is a rare event but when it happens, it involves the entire system. We also find that systems of heterogeneous institutions are more stable to random shocks, confirming the conjecture of Haldane (2009) and the findings of previous studies such as Roukny et al. (2013) and (Georg, 2013). However, heterogeneity poses higher risks when too-big-to-fail or too-connected-to-fail banks are distressed. Our results demonstrate how the contagion risk stemming from their default is particularly high and that connectivity matters more than size as far as systemic risk is concerned.

Focusing solely on the *extended model*, the most interesting result concerns the role played by liquidity reserves and fire-sale prices. Larger cash reserves always worsen the stability of the system, since they allow banks to keep lower capital buffers. As for the fire-sale price, we find that fire-sale losses induce a more prudent behavior of creditors. Indeed, fire-sale losses make debtors more likely to be illiquid. When creditors seek to obtain the desired amount of liquidity, the illiquidity of a debtor will induce them to increase their demand of liquidity to other debtors until the desired amount is obtained. This process is thus likely to cause the closure of short-term exposures with other non-illiquid banks, removing channel of transmission of shocks. In this sense we cast new light on the challenges regulators face when designing an appropriate set of micro-incentives for macro-stability and contribute to the debate on the proper set-up of regulatory requirements (Hanson et al., 2011; Myerson, 2014).

The rest of the paper is structured as follows: Section 2 formalizes our two models of contagion; Section 3 describes the parametrizations we considered in our analysis; Section 4 summaries our findings; and Section 5 concludes discussing policy implications.

2 Models of Contagion

In this section we introduce two different simulation models of the interbank market: a benchmark static model of contagion and an extended dynamic model of contagion. In both cases, we study how stable and resilient the financial system is to an exogenous default of a bank.

We begin by introducing the *benchmark model* which is based on a simple framework that has been extensively studied in the literature (Amini et al., 2013; Arinaminpathy et al., 2012; Caccioli et al., 2012; Gai and Kapadia, 2010; May and Arinaminpathy, 2010; Nier et al., 2007), and which represents the most essential set up to analyze contagion in financial networks.

The interbank system is represented by a static network of credit exposures between N banks. More formally, a graph $G = (I, V)$ (see Figure 3), $I = \{1, \dots, n\}$ represents the set of financial institutions (nodes of the graph), and $V \subseteq I \times I$ is the set of the edges linking the banks. That is, the set of ordered couples $(i, j) \in I \times I$ indicating the presence of a loan made by bank i to bank j. Every edge

```
Initialization of the interbank network;
Exogenous default of a bank;
while at least one bank defaulted do
    for every bank i do
        if counterparty losses occured then
            update equity;
        end
        if equity < 0 then
            default bank i;
            remove bank i from the financial system;
        end
    end
end
```

Figure 1: **Simulation algorithm:** Benchmark model

```
Initialization of two interbank networks (short- and long-term);
Exogenous default of a bank;
while at least one bank defaulted do
    for every bank i do
        if counterparty losses occured then
            update equity;
        end
    end
    while short-term interbank market is not in equilibrium do
        for every bank i do
            if creditors withdraw lines of credit then
                update amount of liquidity to withdraw from the market;
                update amount of illiquid assets to sell ;
            end
            if risk-weighted capital ratio < regulatory minimum capital ratio then
                update amount of liquidity to withdraw from the market to include regulatory
                hoarding;
                update amount of illiquid assets to sell including regulatory fire-sales;
            end
        end
    end
    for every bank i do
        withdraw liquidity from the short-term interbank market;
        sell illiquid assets;
        repay short-term loans;
        if equity < 0 or bank's i became illiquid then
            default bank i;
            remove bank i from the financial system;
        end
    end
end
```

Figure 2: **Simulation algorithm:** Extended model

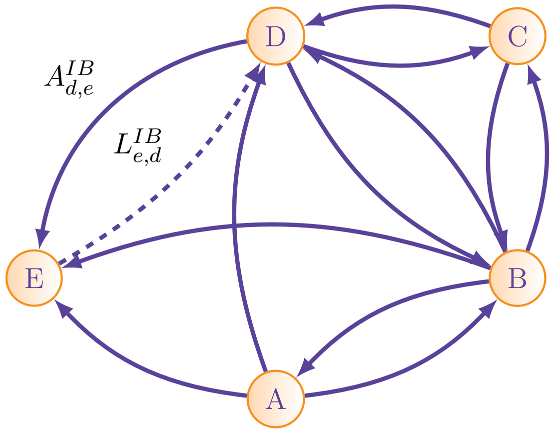


Figure 3: **Network structure - Benchmark model**: banks are connected via a network of interbank exposures. For example, the directed edge connecting bank D to bank E represents a credit of size $A^{IB}_{d,e}$ $(= L^{IB}_{e,d})$ that D has towards E.

(i, j) is weighted by the face value of the interbank claim, $A^{IB}_{i,j}$. Clearly if $(i, j) \notin I \times I$, then $A^{IB}_{i,j} = 0$. This set up allows to represent the system of interbank claims by a single weighted N-by-N matrix IB,

$$IB = \begin{bmatrix} 0 & A^{IB}_{1,2} & \dots & A^{IB}_{1,N} \\ A^{IB}_{2,1} & 0 & \dots & A^{IB}_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ A^{IB}_{N,1} & A^{IB}_{N,2} & \dots & 0 \end{bmatrix}$$

in which interbank assets are along the rows, while columns represent vectors of interbank liabilities. From this matrix, can derive the total exposure of bank i in the interbank market, $A^{IB}_i = \sum_j A^{IB}_{i,j}$ (i.e. the *out-strength* of node i), and its total interbank liabilities, $L^{IB}_i = \sum_j A^{IB}_{j,i}$ (i.e. the *in-strength* of node i).

The balance sheet structure of banks is very simple and stylized (see Table 4): interbank assets (A^{IB}_i) and liabilities (L^{IB}_i); illiquid external assets, such as mortgages (M_i); exogenously given customer deposits (D_i); and capital (E_i). In this simple model - which we use as benchmark - we only have one source of contagion: counterparty losses. Therefore, a (negative) shock can diffuse only via a direct channel of contagion. For example, once a bank is declared insolvent and goes bankrupt, its creditors suffer losses equivalent to the face value of their exposures with the defaulting bank. Furthermore, we assume zero recovery: i.e. when one's counterparty defaults, the creditor bank loses all of its interbank assets held against the defaulting bank.

The simulation of the defaults cascade is carried out using the algorithm provided in Figure 1 and it works as follow. At the beginning of each simulation run, we default a bank wiping out all its external assets. Then, we update the equity of the neighbors of the failing bank, checking whether or not the defaults cascade propagates. If no other banks fail, the process stops. Instead, if at least one bank fails, we repeat the same procedure until no more defaults occur.

However, the benchmark model is not enough to analyze how liquidity hoarding interacts with macro-prudential policies and balance sheets composition in determining the (in)stability of the financial system because long-term and short-term exposures are aggregated into a single asset category.

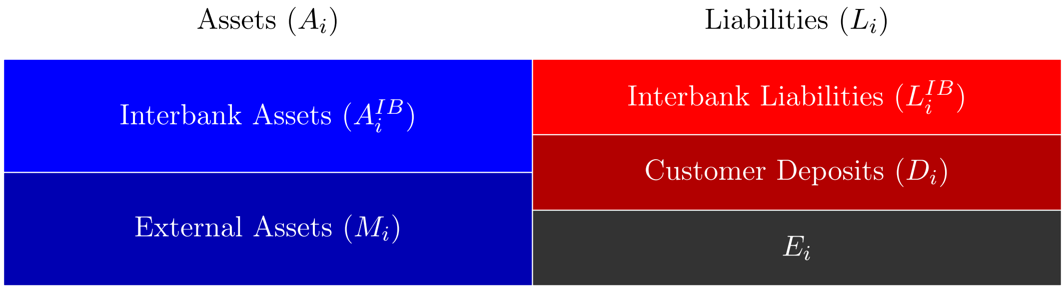
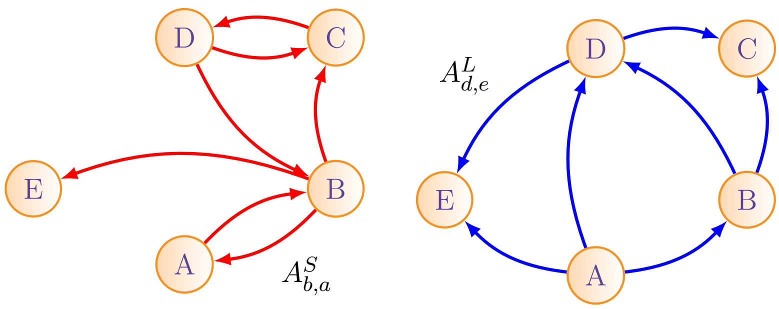


Figure 4: Balance sheet structure - Benchmark model



(a) Short-term interbank market (b) Long-term interbank market

Figure 5: **Network structure - Extended model**: two distinct network of exposures exist. Banks can be connected either via short-term credit exposures or via long-term credit exposures. For example, the directed edge connecting bank D to bank E represents a long-term credit of size $A^L_{d,e}$ $(= L^L_{e,d})$ that D has towards E. Instead, the directed edge connecting bank B to bank A represents a short-term credit of size $A^S_{b,a}$ $(= L^S_{a,b})$ that B has towards A

Moreover, interbank connectivity is static and it does not vary during the unfolding of a crises. Instead, with the introduction of a second model of contagion – that we call *extended model* – we are able to evaluate not only the effects of banks' sizes, network topology and targeted shocks on systemic risk – as done with the benchmark model – but also to explore how capital requirements, banks' characteristics and changes in asset prices affect financial contagion.

In the *extended model* the interbank market is composed of two layers: a network of *long-term* exposures and a network of *short-term* exposures (see Figures 5). While the former cannot be modified during the default cascade, the latter may undergone changes due to liquidity hoardings. Montagna and Kok (2013) is the contribution that is the closest to our modeling framework, even though also Battiston et al. (2012a); Georg (2013); Roukny et al. (2013) consider the presence of different maturities in interbank lending. In this extended modeling framework, we can explore the interplay between the network architecture of the market and the balance sheet structure of intermediaries, highlighting their complex interactions that lead to misalignments between micro- and macro-prudential policies. In particular, banks can dynamically adapt their *short-term* exposures in response to (negative) liquidity shocks and they do so to keep their risk-weighted capital ratio above the mandatory threshold set by the policy maker.

Assets (A_i)	Liabilities (L_i)
Long-term Interbank Assets (A_i^L)	Long-term Interbank Liabilities (L_i^L)
Short-term Interbank Assets (A_i^S)	Short-term Interbank Liabilities (L_i^S)
Liquid Assets/Cash (C_i)	Customer Deposits (D_i)
Illiquid Assets/Mortgages (M_i)	E_i

Figure 6: Balance sheet structure - Extended model

In this second model, three different mechanism determine how contagion spreads in the financial system: direct counterparty exposures; liquidity shocks; and changes in asset-prices. As we can see in Table 6, the balance sheet structure of the *extended model* is richer than one used in the previous section: on the assets side, we indicate with A_i^L the long-term interbank assets of bank i and with A_i^S its short-term interbank assets; while on the liability side, we denote with L_i^L the long-term interbank liabilities of bank i and with L_{ij}^S its short-term interbank liabilities.

To characterize the relative size of the different assets' categories we use parameters λ, ι and τ:

$$\lambda = \frac{C_i}{A_i} \tag{1}$$

$$\iota = \frac{A_i^L + A_i^S}{A_i} \tag{2}$$

$$\tau = \frac{A_i^S}{A_i}. \tag{3}$$

Once the size of the total assets A_i is known, the values of λ, ι and τ allow us to uniquely identify the composition of bank i's assets: parameter λ determines the level of liquidity of bank i; ι fixes the share of bank i's assets which are exposed to counterparty risk; and τ controls the share of short-term interbank assets over total assets.

In the extended model, to study the effects of macro-prudential policies on financial stability, we introduce regulations on capital. Let us denote with $RWA_i(t)$ the risk-weighted assets of bank i at time t and let us define $RWA_i(t)$ as:

$$RWA_i(t) = \gamma_C C_i(t) + \gamma_{IB}(A_i^L(t) + A_i^S(t)) + \gamma_M M_i(t), \tag{4}$$

where γ_C, γ_{IB} and γ_M represents the weights assigned to liquid non-interbank assets, interbank loans and illiquid assets respectively. Typically $\gamma_C = 0$, and this will the assumed henceforth. Therefore, the risk-weighted capital ratio $e_i(t)$ of bank i at time t can be defined as:

$$e_i(t) = \frac{E_i(t)}{RWA_i(t)}. \tag{5}$$

Then, we let the policy maker regulate the financial system by imposing a minimum risk-weighted capital ratio to the operating financial institutions (i.e. banks). In our model, we denote the mandatory minimum risk-weighted capital ratio with e^* and banks have to adjust their level of capital dynamically, while the simulation unfolds, to keep their risk-weighted capital ratio above e^*. Therefore, the following relation has to be satisfied at each time step of the simulation:

$$e_i(t) = \frac{E_i(t)}{RWA_i(t)} \geq e^*. \tag{6}$$

To adjust the composition of their balance sheet, banks can hoard liquidity and, in some cases, sell illiquid assets. In particular, as soon as a bank experiences losses which erode its equity below the regulatory threshold, i.e. $e_i(t) < e^*$, the bank will try to meet the required capital ratio by first reducing its short-term interbank exposures and then, if this adjustment is not sufficient, by selling its illiquid assets. Therefore, each bank i seeks a *regulatory adjustment* equal to:

$$RA_i(t) = \max\left(0, RWA_i(t) - \frac{E_i(t)}{e^*}\right); \tag{7}$$

and it will try to fulfill capital requirement by first resorting to *regulatory liquidity hoarding*:

$$\bar{A}_i^S(t) = \min\left(A_i^S(t), \frac{RA_i(t)}{\gamma_{IB}}\right), \tag{8}$$

which thus represents the amount of liquidity the bank has to withdraw from the market to comply with the regulations on capital. Then, if liquidity hoarding is not enough, the bank will try to sell its illiquid assets (i.e. *regulatory fire-sales*). However, the success of this strategy crucially depends on the fire-sale price of illiquid assets. Assume that the fair value of a unit of illiquid assets is 1, then, when the fire-sale price p is larger than $1 - e^*\gamma_M$, banks will have to engage in regulatory fire-sales for a quantity of

$$\bar{M}_i(t) = \min\left(M_i(t), \max\left(0, \frac{RWA_i(t) - \gamma_{IB}\bar{A}_i^S(t)}{\gamma_M - (1-p)/e^*}\right)\right). \tag{9}$$

Instead, if $p < 1 - e^*\gamma_M$, banks will never find convenient to fire-sale illiquid assets in order to meet their capital requirements. Indeed, in this case, large price discounts will induce losses which more than offset the reduction of the RWA, and $\bar{M}_i(t)$ would hence be zero.

A bank will try to adjust its RWA as far as it is possible. Once its capital ratio irremediably falls below the minimum requirement, the bank has no other choice that to keep operating with that ratio. Indeed, the only available solution would be to raise capital, which is an action that cannot be pursued in the short time scale of a contagion scenario, which is the focus of our analysis. Note also that insolvent banks, i.e. those with $E_i(t) \leq 0$, will withdraw all their funds from the short-term market, i.e. $\bar{A}_i^S(t) = A_i^S(t)$ for insolvent banks.

The dynamic adjustment of short-term exposures triggers bank runs in the interbank market. More specifically, we model runs as perfect information equilibria. Eisenberg and Noe (2001) provide the basic set up which we need to adapt to our framework where the regulatory hoarding constitute the initial demand shock.

Compared to Eisenberg and Noe (2001), we give more micro-foundation to the clearing algorithm by making rationality assumption on banks' behavior instead of assuming proportional hoarding with respect to short-term debtors. Indeed, this pure proportionality assumption may artificially lead banks to illiquidity if a debtor is illiquid. Suppose that bank i hoards a quantity $\bar{A}_i^S$ of funds, proportionally splitting this amount among its short-term creditors in quantities $\bar{A}_{i1}^S$, $\bar{A}_{i,2}^S$, ..., $\bar{A}_{in}^S$. If a debtor j is illiquid its supply of funds to i is less than $\bar{A}_{ij}^S$. If the algorithm stops here, it may be the case that also i becomes illiquid, thus reducing the fund it may supply to its hoarding creditors. A more realistic assumption is that, if i is not able to meet its demand for liquidity because of j's illiquidity, it may increase the quantities hoarded from the other debtors, up to the point in which the supply of funds from liquid debtors is enough to meet i's demand for funds. As soon as a bank j is found to be illiquid, perfect information in the short-term market makes withdrawing all the fund from it a weakly-dominating strategy for all its short-term creditors. However, no payments will be made and exposures are marked down to zero, since slow and costly default procedures will be initiated by the supervisors. It is important to note that *runs* to illiquid banks happen only when illiquidity is *revealed* by the failure to make a required payment. Indeed, in the short time scale we assume, updated balance sheet information are not made public, and the only source of information are market demand and supply of funds. To make the analysis more precise we give the following definition of illiquid bank.

Illiquid bank A bank i is illiquid if the total amount of funds it can raise is not enough to meet the demand for funds $\sum \tilde{A}_{ji}^S(t)$ of its creditors.

If we define as $\Lambda(t)$ the set of banks which become illiquid at time t, then the amount of funds bank i can raise is given by $C_i(t) + \sum_{j \notin \Lambda(t)} A_{ij}^S(t) + pM_i(t)$ which corresponds to the sum of the amount of cash available at time t, the liquidity it can recall from its creditors, and the liquidity it can obtain through fire-sales. It is clear from the definition that whether a bank is illiquid crucially depends on whether its short-term debtors are liquid or not. With this behavioral framework in mind we can define an *equilibrium in the short-term market.*

Equilibrium in the short-term market An equilibrium in the short-term market is a matrix of liquidity demands, $D^*(t) = \left(\tilde{A^S}_{ij}(t)\right)_{i,j=1}^n$, where $\tilde{A^S}_{ij}$ is the equilibrium demand for cash of i to j, and a matrix of liquidity supply, $S^*(t) = \left(\hat{A}^S_{ij}(t)\right)^n_{i,j=1}$, where $\hat{A}^S_{ij}$ is the equilibrium supply of cash of i to j, such that:

I. $\sum_{j\notin\Lambda(t)} \tilde{A}^S_{ij}(t) = \min\left(\sum_{j\notin\Lambda(t)} A^S_{ij}(t), \max\left(\bar{A}^S_i(t), \sum_j \tilde{A}^S_{ji}(t) - C_i(t)\right)\right) \quad \forall i$

II. $\sum_j \hat{A}^s_{ij}(t) = \min\left(\sum_j \tilde{A}^S_{ji}(t), C_i(t) + \sum_{j\notin\Lambda(t)} \hat{A}^S_{ji}(t) + p\tilde{M}_i(t)\right) \quad \forall i$

III. $\tilde{M}^*_i(t) = \min\left(M_i(t), \max\left(\bar{M}_i(t), \frac{1}{p}\left(\sum_j \tilde{A}^S_{ji}(t) - C_i(t) - \sum_{j\notin\Lambda(t)} \hat{A}^S_{ji}(t)\right)\right)\right)$

IV. $\tilde{A}^S_{ij}(t)/\tilde{A}^S_{ik}(t) = A^s_{ij}(t)/A^S_{ik}(t) \quad \forall i \in I, \forall j,k \notin \Lambda(t)$

V. $\hat{A}^S_{ij}(t)/\hat{A}^S_{ik}(t) = \tilde{A}^S_{ji}(t)/\tilde{A}^S_{ki}(t) \quad \forall i,j,k \in I$

VI. $i \in \Lambda(t)$ if and only if $\sum_j \hat{A}^S_{ij}(t) < \sum_j \tilde{A}^S_{ji}(t)$

VII. $\tilde{A}^S_{ij}(t) = \hat{A}^S_{ji}(t) \quad \forall i \in I, \forall j \notin \Lambda(t)$

Equilibrium demands and supply are then computed by an iterative algorithm as suggested by Eisenberg and Noe (2001).

Condition *I.* states that banks' total demand for cash cannot exceed the amount of their short-term interbank assets as that should be at least as large as to include both its regulatory demand and creditor's hoarding exceeding available cash.

Condition *II.* indicates that banks' total supply of cash does not exceed the total demand they have to meet and the liquidity they are able to raise via cash, supply of liquidity from short-term creditors and assets fire-sales.

Condition *III.* implies that equilibrium fire-sales cannot exceed the available illiquid assets and should be enough to take into account regulatory fire-sales and the liquidity demand of creditors exceeding cash reserves and funds obtained from the short-term interbank market.

Condition *IV.*, i.e. demands proportional to exposures, and condition *V.*, i.e. supplies proportional to demands, are the formalization of the behavioral assumptions we made in our micro-foundation.

Condition *VI.* is the definition of illiquid bank in equilibrium. Note that this condition, together with condition *V.*, implies that, for an illiquid bank i, it holds that $\hat{A}^S_{ij}(t) < \tilde{A}^S_{ji}(t)$ for every creditor j.

Condition *VII.* is a *market clearing condition* with illiquidity, in which the market clears with equality only for liquid debtors, while there is excess demand to illiquid debtors.

The clearing of the market conveys a crucial signal to financial institutions since it reveals which banks are unable to make the promised payments. In accordance with current financial regulation, as soon as a bank is found illiquid it does not make *any* payment and it is then subject to regulatory supervision.

Since default procedures due to illiquidity are costly and time consuming processes whose outcome is uncertain and since shorter maturity does not necessarily imply higher seniority, we assume that all the creditors, both long-term and short-term, of illiquid banks mark-to-market their exposures.

Therefore, insolvency is not the only source of contagion. Banks' defaults can be induced also by illiquidity due to bank runs. Consequently, both insolvent and illiquid banks induce losses to their direct lenders. Such losses reduce the capital buffer of the creditor which, if negative, makes it insolvent as well. Furthermore, a reduction of the capital buffer also forces banks to adjust their RWA, thus (possibly) triggering an additional round of *bank runs*. When the short-term market clears, new illiquid banks default and, together with the banks that became insolvent in the previous period, trigger another round of insolvencies. The default cascade continues as long as no additional defaults happen (see Figure 2).

We can define $\Delta(t)$ as the set of banks which are in default at time t. Then, assuming a zero recovery rate in the short run, we may update the exposures of institutions as follows:

$$A^S_{ij}(t+1) = A^S_{ij}(t) - \tilde{A}^S_{ij}(t) \qquad \text{if} \quad j \notin \Delta(t) \tag{10}$$
$$A^S_{ij}(t+1) = 0 \qquad \text{if} \quad j \in \Delta(t) \tag{11}$$

fort the short-term, and

$$A^L_{ij}(t+1) = A^L_{ij}(t) \qquad \text{if} \quad j \notin \Delta(t) \tag{12}$$
$$A^L_{ij}(t+1) = 0 \qquad \text{if} \quad j \in \Delta(t) \tag{13}$$

for the long-term. Certainly, defaulted banks continue to have liabilities to their creditors. However, as default procedures start, the timing and the amount of the final obligations become exogenous to our model. Hence, according to the update rules, we remove defaulted banks from the system by wiping-away all their interbank liabilities. Illiquid assets are updated according to:

$$M_i(t+1) = M_i(t) - \tilde{M}^S_i(t) \tag{14}$$

while cash reserves follow the rule:

$$C_i(t+1) = C_i(t) + \sum_{j \notin \Delta(t)} \hat{A}^S_{ji} + p\tilde{M}^S_i(t) - \sum_j \hat{A}^S_{ij} \qquad \text{if} \quad i \notin \Delta(t) \tag{15}$$
$$C_i(t+1) = C_i(t) + \sum_{j \notin \Delta(t)} h\tilde{a}t^S_{ji} + p\tilde{M}^S_i(t) \qquad \text{if} \quad i \in \Delta(t) \tag{16}$$

The dynamics of the capital buffer can thus be expressed as follows:

$$E_i(t+1) = E_i(t) - (1-p)\tilde{M}^S_i(t) - \sum_{j \in \Delta(t)} \left(A^S_{ij}(t) + A^L_{ij}(t)\right) \tag{17}$$

Finally, the set of defaulted banks is updated as follows:

$$\Delta(t+1) = \Delta(t) \cup \Lambda(t+1) \cup_{\{i|E_i(t+1)<0\}} i \tag{18}$$

and we look at the steady-state of the default cascade. i.e. at the state of the system at time t^* when $\Delta(t^*+1) = \Delta(t^*)$.

3 Simulations Settings

To provide a general assessment on what factors and characteristics of the interbank market determine its (in)stability we consider different parametrizations of our models where we let vary: capital requirements, banks' balance sheet composition, fire-sale prices, network topology and types of (negative) shocks hitting the system. We start by considering five different (macro) scenarios where we let vary the topology of the interbank market and the overall degree of heterogeneity of the system. Second, for each scenario, we consider different calibrations to assess how different degrees of banks' heterogeneity, market liquidity, capital regulation, and price levels affect the emergence of systemic risk. Third, we check which types of shocks make the system less stable (i.e. random vs targeted attacks). Lastly, we compare the findings of the two models of contagion.
The five (macro) scenarios we consider are[1]:

- ER1: Homogeneous banks with homogeneous exposures. In this scenario, an Erdős and Rényi (1960) network model is used to generate interbank connections and banks are assumed to have the same asset size. Interbank claims are evenly distributed among the outgoing links.

- ER2: Homogeneous banks with heterogeneous exposures. As in the previous case, all banks have the same asset size and the network is Erdős and Rényi (1960). However, we now allow banks to unevenly distribute their exposures across creditors, in such a way the link weight is power-law distributed.

- ER3: Heterogeneous banks with heterogeneous exposures. In this case, also banks are heterogeneous and their size (expressed in terms of total assets) is power-law distributed.

- FIT1: Heterogeneous banks with homogeneous exposures. We move towards a more realistic architecture for the financial system, which shows a fat-tail degree distribution. We generate a network using a fitness model (De Masi et al., 2006), with an exogenous distribution of the fitness parameter that follows a power-law and with interbank claims that are evenly distributed among the outgoing links.

- FIT2: Heterogeneous banks with heterogeneous exposures. Here, we allow also for heterogeneous exposures so that, once the network is generated using the fitness model, we draw the value of the exposures from a power-law distribution with the same exponent of the distribution of the fitness parameter.

By analyzing those five scenarios, we can show how different features that characterize the financial system can affect its stability. We start with the most simple case, where we assume that banks are homogeneous, credit is allocated with the same proportion to all neighbors, and the interbank network is generated using an Erdős and Rényi (1960) model. Then, with scenarios *ER2* and *ER3*, we add heterogeneity both in terms of banks' sizes and in terms of how credit is allocated to creditors. Lastly, in scenarios *FIT1* and *FIT2*, we change the topology of the interbank market and we use a *fitness model* to generate a network of exposures that match more closely the empirical features of real interbank networks (Montagna and Lux, 2013).

[1] Additional details are provided in Appendix C.

Furthermore, different types of shocks are considered. In the simulations, we let vary the average degree of the network, which represents the average number of creditors and debtors a randomly chosen bank has. As such, the average degree is a measure of diversification at the level of single institutions; but, from a systemic point of view, it is also a measure of connectivity, which thus represents channels through which a shock may propagate. As in previous contributions (Caccioli et al., 2012; Gai and Kapadia, 2010), we will exogenously set into default one bank and analyze the steady-state effect of contagion: i.e. the number of defaulted banks when the contagion process stops. We will focus on the *frequency of contagion*, defined as the probability that at least 10% of the intermediaries default, and on the *extent of contagion*, i.e. average fraction of defaulted banks, provided that it is larger than 10%. When heterogeneity is introduced (as in cases *ER2*, *ER3*, *FIT1* and *FIT2*) through exogenous power-law distributions, also the exponent of the distribution will be subject to analysis, since it represents the degree of heterogeneity: smaller exponents correspond to fatter tails and, consequently, to higher levels of heterogeneity.

The triggering event of contagion is the exogenous default of a bank. We assume four targets of the exogenous shock:

- *Random*, in which the exogenously defaulting bank is picked up randomly, with equal probability for each bank;
- *Too-connected-to-fail*, in which we set into default the banks with the largest number of creditors;
- *Too-exposed-to-fail*, in which the exogenous default hits the bank with the largest debt exposure (used only in the *extended model*);
- *Too-big-to-fail*, in which the exogenous default hits the bank with the largest asset size, and which is feasible only in those cases in which the asset size is heterogeneous, i.e. cases *ER3*, *FIT1* and *FIT2*

Indeed, one of the crucial policy implications of this contribution comes from our results on the role of *large* financial institutions, either in terms of connectivity or in term of asset size. When heterogeneity is introduced, the system presents potential *too-connected-to-fail* and *too-big-to-fail* banks. On the one hand, they may have detrimental effects because of their large exposures in the interbank market. On the other hand, their magnitude allows them to better absorb shocks, since their higher level of diversification or their larger capital buffers prevent them to go bankrupt after the default of a relatively small banks.

In our setting we simulate financial systems composed of 1000 banks where each different parameterization is run 500 times. Additionally, to preserve comparability with previous contributions, such as Gai and Kapadia (2010) and Caccioli et al. (2012), we keep - in the benchmark model - a fixed ratio of capital to total assets equal to 4%; while - in both models - we set the ratio of interbank assets to total capital equal to 20% (i.e. $\iota = 0.2$). Furthermore, in the extended model, we set the capital buffer $\beta = 2.5\%$, the risk weight for illiquid assets $\gamma_M = 50\%$ (which is the weight assigned by regulators to residential loans), and the risk weight for interbank claims $\gamma_{IB} = 100\%$ (which corresponds to the regulatory weight for loans to BBB- institutions, thus implying a conservative scenario).

As far as the extended model is concerned, we use logit regressions to summarize our results and to disentagle the effects that different parameters of the model have on the probability of observing

Name	Dummy	Details	Scenarios	Regressor(s)
Constant			all	x_0
Average Degree		levels & logs	all	x_1 and x_2
High Min Capital Ratio	yes	min capital = 10%	all	x_3
Low Min Capital Ratio	yes	min capital = 6%	all	x_4
Sales at loss	yes	fire-sale price = 50%	all	x_5
High liquidity	yes	$\lambda = 0.72$	all	x_6
Low liquidity	yes	$\lambda = 0.08$	all	x_7
Large short-term market	yes	$\tau = 0.18$	all	x_8
Large long-term market	yes	$\tau = 0.02$	all	x_9
High heterogeneity	yes	tail exponent = 2.5	ER2,ER3,FIT1,FIT2	x_{10}
Low heterogeneity	yes	tail exponent = 3.5	ER2,ER3,FIT1,FIT2	x_{11}
Too-connected-to-fail	yes		all	x_{12}
Too-exposed-to-fail	yes		ER2, ER3, FIT1, FIT2	x_{13}
Too-big-to-fail	yes		ER3, FIT1, FIT2	x_{14}

Table 1: List of covariates included in the logit regressions.

default cascades. The *logit model* we estimate is defined as follows:

$$Prob(contagion|threshold = h, X) = \frac{e^{X'\theta}}{1 + e^{X'\theta}} \tag{19}$$

where h is the share of intermediaries that have to fail in order for us to consider that event a "default cascade", X the regressors' matrix and θ is the vector of parameters that have to be estimated. For each regression, we use the set of covariates described in Table 1.

We also add several *interaction terms* to take into account of the fact that the effects of the regressors depend crucially on the average level of connectivity of the system. Therefore, we interact all the dummy covariates included in the model with the *average degree* regressors (both taken in levels and in logs). For example, in addition to regressor x_3 (i.e. high capital requirements), we also add covariates x_{14} and x_{15} obtained as $x_{14} = x_3 \cdot x_1$ and $x_{15} = x_3 \cdot x_2$. Our choice of regressors implies that our baseline configuration is the one where: the minimum capital ratio is set to 8%, there are no losses during fire-sales (i.e. fire-sale price = 100%), liquidity is set at its intermediate level (i.e. $\lambda = 0.4$), short-term and long-term interbank markets have the same size (i.e. $\tau = 0.1$) and the defaulting bank is chosen at random. Furthermore, if heterogeneity is present (i.e. scenarios *ER2*, *ER3*, *FIT1* and *FIT2*), the power-law exponent is set to 3.0.

To analyze the impact of the parameters we show - for each regressor - the distribution of its *marginal effects* by computing such effects for each possible combination of the other covariates. Indeed, in the case of a logit regression, we need to specify the point at which we are measuring the marginal effects of a regressor because:

$$\frac{\partial Prob(Y = 1|x)}{\partial x} = \Lambda(x'\beta)[1 - \Lambda(x'b)]\beta, \tag{20}$$

where $\Lambda(z) = \frac{e^z}{1+e^z}$. Therefore, to give a clearer idea of the impact that each covariate has on the probability of observing default cascades, we decided to combine the results obtained considering all the possible combinations available. We show the results for $h = 10\%$. Furthermore, in Table 2, we report the quality measures for the regressions performed and the number of observations (i.e. simulations) used to estimate each logit regression.

Scenario	Precision	Recall	F1-score	Observations (in millions)
ER1	0.86	0.86	0.86	3.24
ER2	0.88	0.88	0.88	14.58
ER3	0.85	0.84	0.85	19.44
FIT1	0.89	0.90	0.89	19.44
FIT2	0.90	0.91	0.90	19.44

Table 2: Quality measures[1] of logit regressions.

Main Findings	
Baseline system stability:	Non-monotonic role of diversification (i.e. system connectivity).
Extent of contagion	The financial system can be both *robust-yet-fragile* and *robust-and-not-fragile* depending on balance sheets composition.
Heterogeneity in exposures	Higher heterogeneity induces a wider contagion window
Heterogeneity in bank sizes	The financial system is more stable in the benchmark model, mixed results in the extended model.
Scale-free networks	In general, the financial system is more stable (under random attacks) than in Erdős and Rényi (1960) configurations.
Targeted attacks	Size alone matters less than connectivity in spreading contagion.
Minimum capital ratio	Higher regulatory capital ratios increase system stability.
Fire-sale price	The probability of contagion is lower when banks incur in losses while liquidating assets.
Liquidity	Higher the share of liquid assets in portfolio, lower the stability of the system.
Share of short-term exposures	Non-monotonic role of balance sheets maturity structure.

Table 3: Summary of the main findings.

4 Findings

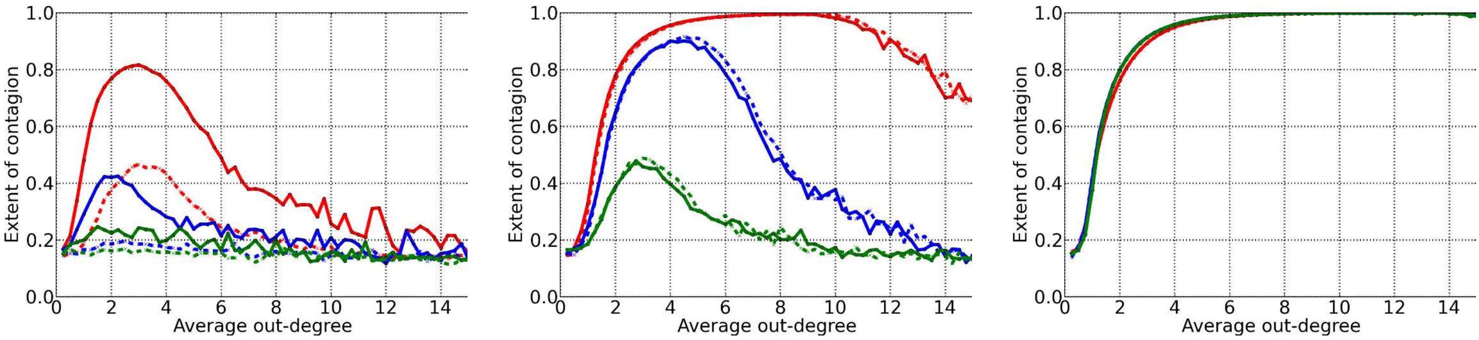
In the following section we summarize the main findings we obtain using the two models of contagion. In particular, by using the extended model, we also outline what are the effects of imposing different capital requirements on the financial system, altering the fire-sale price of illiquid assets and - more in general - considering different balance sheets compositions via tuning parameters λ and τ of the extended model [2].

Baseline system stability. The non-monotonic role diversification has on financial stability is evident in all the scenarios considered. For small average degree values, increasing connectivity increases the frequency of contagion and hence the probability of observing default cascades. Indeed, in such cases, the presence of additional transmission channels more than offset the role of diversification. However, this effect is reversed for larger values of the average degree, so that contagion becomes a rarer event due to diversification. The system has therefore two phase transitions which delimit a so-called *contagion window* (Gai and Kapadia, 2010). For example, this is what emerges by looking at estimated *marginal effects* of average degree on the probability of cascades as depicted in Figure 10a (extended model, scenario *ER1*).

Extent of contagion. However, while the frequency of contagion declines beyond a certain threshold, this is not the case - in the *benchmark model* - for the extent of contagion. Near the upper phase

[1] *Precision* is defined as the number of correctly predicted contagious events over the total number of predicted contagious events. *Recall* is defined as the number of correctly predicted contagious events over the total number of contagious events. The *F1-score* is defined as $2(\textit{Precision} \cdot \textit{Recall})/(\textit{Precision} + \textit{Recall})$.

[2] While, let's remind the reader, parameter ι is kept fixed at 0.2.



(a) $\tau = 18\%,\ \lambda = 8\%$ (b) $\tau = 18\%,\ \lambda = 40\%$ (c) $\tau = 18\%,\ \lambda = 72\%$

Figure 7: **Extended model | Extent of contagion - Threshold: 10%**
Colors identify the different levels of the minimum capital requirement: 6% red, 8% blue, and 10% green. Line styles identify fire-sale prices: solid line for fire-sale price equal to 50% of the asset value, dashed line for price equal to 100% of the asset value. The scenario considered is *FIT2* where the *too-connected-to-fail* node is defaulted.

transition the system exhibits the *robust-yet-fragile* property: contagion is a rare event, but, once it breaks up, it affects the entire network. Our experiments suggest that the same random shock may have completely different effects depending on the level of connectivity of the interbank market. Instead, in the *extended model*, the system can also be *robust-and-not-fragile*, depending on the balance sheet composition of the banks. Indeed, given the dynamic flavor of the extended model, the financial system can adjust its connectivity avoiding full collapse provided that certain conditions hold. In particular, if τ is high - i.e. banks have more short-term assets than long-term assets - the fragility of the system will depend upon the actual level of liquid assets (i.e. cash) present in the market: higher the liquidity in the interbank market, higher the level of fragility. For example, see Figure 7 where we report the extent of contagion for scenario *FIT2* when $\tau = 18\%$ and $\lambda \in \{8\%, 40\%, 72\%\}$. This happens because banks can dynamically adjust their short-term exposures, therefore reducing the probability of a systemic collapse; and because - when liquidity is low - agents are forced to recall many lines of credit to be able to adjust their risk-weighted capital ratio[1]. From a policy standpoint, this finding has two main implications. First, it means that financial markets can find a value to *preserve themselves* without external intervention. However, this can happen only when the *absolute level of capital* is not too low, as it is when liquidity is high (i.e. a lower level of capital is needed to satisfy the constraint on the risk-weighted capital ratio). Therefore, it is not sufficient to force banks to comply with regulations on capital ratios, but additional metrics are needed to distinguish *robust-yet-fragile* systems from *robust-but-not-fragile* systems.

Heterogeneity in exposures. When we compare the results *across scenarios*, it emerges that the presence of an uneven distribution of claims creates a much wider contagion window than having claims evenly distributed among debtors. For example, notice the differences between scenarios *ER1* and *ER2* in the *benchmark model* in Figures 9a and 11a. The same holds when we compare the results for the extended model (see Figure 8.

Heterogeneity in bank sizes. Instead, when we consider heterogeneity in bank sizes (i.e. we compare scenarios *ER2* and *ER3*) we observe that size heterogeneity makes the financial system more stable

[1] This can be inferred by looking back at equation 4.

in the benchmark model; while the results are mixed in the extended model where the (baseline) probability of contagion appears to pick at a lower level than in scenario *ER2.* even though at medium-to-high levels of connectivity the probability of contagion is higher in scenario *ER3.*

Scale-free networks. When we move to consider scale-free distributions for the degree and also the size of banks (scenarios *FIT1* and *FIT2*), as long as attacks to the system are random, the financial system is more stable than it is in Erdős and Rényi (1960) configurations. For example, this can be seen - in the extended model - by comparing the baseline probabilities of default cascades as depicted in Figure 8. However, the *contagion window* is larger than in *ER1* scenario and the structure remains *robust-yet-fragile* even for larger values of the average degree, no matter which degree of heterogeneity we consider. This result was expected given the extensive literature on network resilience in the case of scale-free networks (Albert et al., 2000; Cohen et al., 2000, 2001; Crucitti et al., 2004; Doyle et al., 2005; Gallos et al., 2005; Zhao et al., 2004).

When we compare the frequency of contagion between scenarios *FIT1* and *FIT2*, we reach different conclusions when comparing the results from the *benchmark model* to the ones of the *extended model.* In the benchmark model, frequency of contagion is clearly lower than in case *ER3* but also than in case *FIT1.* Additionally, heterogeneity plays a stabilizing role despite the presence of contagious links and highly connected hubs. Moreover, heterogeneity seems always to be beneficial since the frequency of contagion, for a given average degree, decreases with the power-law exponent. For exponents as small as 2.25, the maximum frequency of contagion in the benchmark model is the lowest, not even reaching 20%.

Instead, in the *extended model*, the baseline probability of contagion in *FIT2* is slightly higher than the one observed in *FIT1* (see Figure 8). Moreover, when we look at marginal effects of having high vs low heterogeneity, we discover that heterogeneity does not stabilize the system as in the benchmark model. Instead, in the extended model, it makes the system less stable and more prone to systemic risk (see Figures 22a,22b,26a,26b).

Targeted attacks. Targeted attacks alter the risk profiles of the different interbank architectures. In general, probability of contagion increases when we move from a random default to a targeted default. In the extended model, the highest probability of contagion for scenario *ER2* is found in the *too-exposed-to-fail* case (compare the marginal effects in Figures 13i and 14c). Additionally, in line with what emerges in scenario *ER1*, the *too-exposed-to-fail* attack contributes to creating a wider window of contagion since its effects are notable also for the highest level of connectivity. Therefore, no degree of *diversification* can prevent the system from being exposed to default cascades.

Whenever banks are heterogeneous also in size (as in scenarios *ER3,FIT1,FIT2*) we can test the potential for disruption of an attack to the biggest bank. Put it differently, are more *systemically important too-big-to-fail* institutions or *too-connected-to-fail/too-exposed-to-fail* institutions? Our findings suggest that size alone matters less than connectivity in spreading contagion. For example, in the extended model, if we compare Figures 17i and 21i we see that an attack to the most-connected bank has a stronger impact on the stability of the system than an attack to the biggest one. This result holds especially for low to medium levels of connectivity. Instead, when the connectivity in the system is extremely high, targeting the biggest institutions might be more dangerous (in some scenarios).

In the following, we focus our attention on the findings we obtained using solely the *extended model.* As we previously stressed, only by using the extended model we can analyze how liquidity hoarding interacts with macro-prudential policies and balance sheets composition in determining the (in)stability of the financial system. Therefore, in the following paragraphs, we describe what are the effects on systemic risk of varying minimum capital ratios, fire-sale prices, the share of liquid assets held by financial institutions, and the maturity composition of banks' balance sheets.

Minimum capital ratio. Using the *extended model*, we can also test different regulatory requirements on capital and measure how changing such requirements alters the financial stability of the interbank market. We find that the effects of having an higher regulatory capital ratio are unambiguously positive. The probability of contagion is always reduced by increasing the minimum capital ratio. This was not, however, a trivial result. Indeed, while higher capital ratios provide a larger buffer to absorb losses, they also raise the threshold that triggers runs in the short-term market because of the adjustments of the risk-weighted assets. For example, in Figures 10b and 10c, we see the marginal effects for the cases when the minimum capital ratio is set to 6% (panel (c)) and when the minimum capital ratio is set to 10% (panel (b)). To interpret the results, we have to keep in mind that - in these cases - we are looking at the marginal effects of two dummy variables where the *benchmark case* is the one where the minimum capital ratio is set to 8%. Therefore, a positive marginal effect in the case of a minimum capital ratio set equal to 6% means that - in such a case - there is an *higher* probability of contagion compared to what we would have had if we were to keep the minimum capital ratio equal to 8% (i.e. the value at the baseline). The absolute size of the marginal effects - as in the case of the average degree - dies out quickly for high levels of connectivity.

Fire-sale price. These results are probably the most counterintuitive ones and interesting, since they clearly highlight how the interplay between network architecture and behavioral rules can yield unexpected outcomes. The probability of contagion is always lower when banks incur losses as they sell their illiquid assets (i.e. the selling price of the illiquid assets is set to be equal to 50% of their book value). The reason for this fact is that banks, when facing fire-sale losses, have higher probability of becoming illiquid. All short-term creditors will thus try to obtain the desired amount of liquidity from other debtor banks. This implies that a larger fraction of credit lines are reduced and, in part, closed. The closure of short-term credit lines with short-term debtors removes possible channels through which contagion can spread, thus giving higher stability to the entire system. In general, we have observed that the beneficial effect of fire-sale losses is more evident when the fraction of short-term interbank claims is small (i.e. τ is small) and when diversification is high, i.e. in the cases in which every short-term link carries a smaller weight and thus is more likely to be removed during a run. In Figure 21d we report the marginal effects for the case when the fire-sale price is set to 50% of the original value; the baseline value is set as having a fire-sale price equal to the book value of the assets; and the scenario considered is *FIT1*.

Liquidity (i.e. λ). As far as contagion through interbank exposures is concerned, a higher share of liquid items in portfolio is detrimental for the stability of the system. Despite the fact that they provide a *cash buffer* preventing liquidity shocks to spread, they also lower the risk-weighted assets (*RWA*) and thus the initial absolute amount of capital available to absorb losses. The result is that the most stable configuration is reached when liquid assets are at their minimum, i.e. when $\lambda = 0.08$. In Figures 10e and 10f, we see that whenever we have high liquidity in the system (i.e. $\lambda = 0.72$), there is also an higher probability of observing default cascades. Furthermore, the magnitude of the (positive) marginal effects of having high liquidity are typically much larger than the (negative) marginal effects when liquidity is low. The interquantile range for the case with low levels of liquidity is approximately zero when average degree is higher than eight; while with high levels of liquidity marginal effects are still relevant up to very high levels of connectivity.

Share of short-term exposures (i.e. τ). The marginal effects of the share of short-term exposures seem to be non-linear. Let us recall that whenever $\lambda = 0.10$, the overall size of the short-term interbank market is the same as the one of the long-term interbank market. This follows from the fact that the total amount of assets allocated in the two markets is the same. Therefore, what we observe - as an example - in Figures 10g and 10h is what happens whenever one of the markets is larger than the other. In particular, the marginal effects in the two cases (i.e. $\lambda = 0.18$ in panel (g) and $\lambda = 0.02$ in panel (h)) have - for low levels of connectivity - negative marginal effects; while the opposite holds for higher levels of connectivity. In the Erdős and Rényi (1960) scenarios, the minimum value is recorded when average degree equals one. One reason that could explain this finding is that, in Erdős and Rényi (1960) models, average degree values lower than one imply that a *giant component* does not exist. Therefore, when defaults do happen, they do not percolate to the rest of the network. Once we pass that threshold, we see that the marginal effects start to become less negative, until they become strictly positive for average degree values higher than two. Consequently, when connectivity is high and market sizes are *unbalanced*, the increased number of *contagious links* makes the system more unstable since financial distress does propagate. Instead, when we move to consider scenarios where the underlying network structures are scale-free (*FIT1* and *FIT2*), the marginal effects are still negative for low levels of connectivity, but their minimum is reached at lower level of connectivity. Additionally, network unbalances where long-term assets outsize short-term assets (i.e. $\lambda = 0.02$) are more dangerous than scenarios where short-term assets outsize long-term assets. The reason being that short-term liabilities can be dynamically adjusted by the banks at running time, while long-term liabilities creates channels of contagion that cannot be cut while a default cascade unfolds.

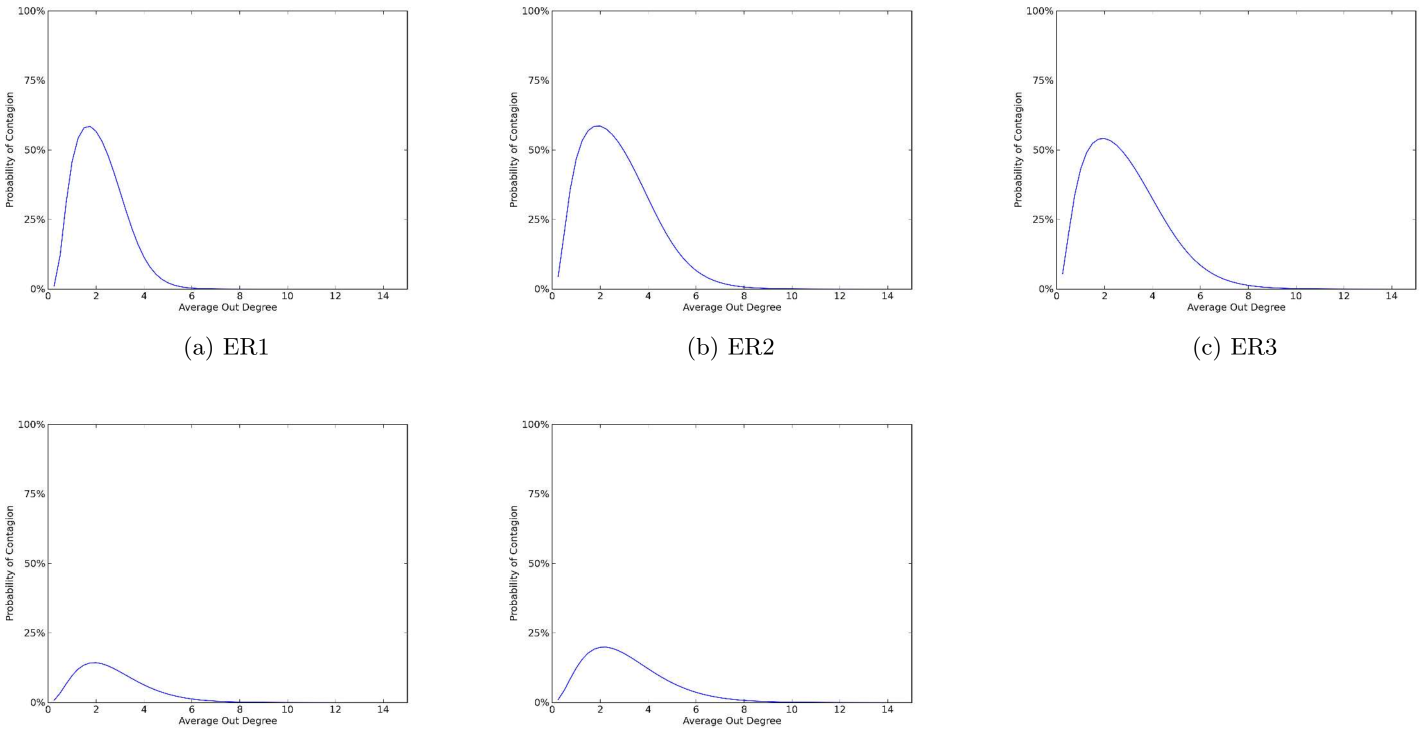
5 Conclusions

In this paper we explored the interplay between heterogeneity, network structure and balance sheet composition in the spreading of contagion.

In the first part, using an established model of contagion, we have shown that the system presents phase transitions in connectivity. Indeed, connectivity is both a driver of contagion, as it provides the channel for shocks to propagate, but it is also an hedge against contagion, via diversification. This result is consistent with what has been found also in other studies such as Gai and Kapadia (2010), Caccioli et al. (2012) and Elliott et al. (2014).

Also heterogeneity has an ambiguous role. If heterogeneity regards exclusively the link weights, the main effect is a widening of the interval of connectivity levels in which contagion is possible. This is due to the fact that diversification cannot, in this case, prevent contagious links to exist, which are a necessary condition for contagion to arise.

When size heterogeneity is introduced, also some positive effects are seen. Big banks seems to act as shock absorber, making contagion a less likely phenomenon. Heterogeneity in connectivity provides additional stabilization when the initial default is random. However, this comes with the cost of an extremely high contagion risk when the most connected or the largest institution is initially distressed (as it was also discussed in Caccioli et al. (2012) and Roukny et al. (2013)).

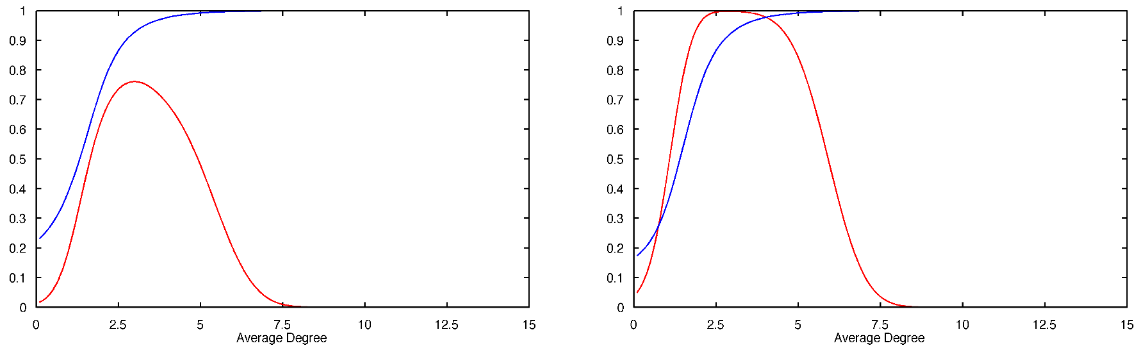


(a) ER1 (b) ER2 (c) ER3

(d) FIT1 (e) FIT2

Figure 8: **Extended model | Baseline Probabilities of Observing Default Cascades - Threshold: 10%**

Panels (a)–(e) show the estimated probability of observing default cascades where more than 10% of the total assets had been wiped out.



(a) Random Default (b) Too-Connected-To-Fail

Figure 9: **Benchmark model - [ER1] | Frequency and Extent of Contagion - Threshold: 10%**
Frequency (red) and extent (blue) of contagion as functions of the average number of interbank connections of the banks in the system.

We showed how *too-connected-to-fail* banks are more dangerous that the *too-big-to-fail* ones. In our model, despite being very correlate, the two set of institutions do not necessarily overlap. We then proved that the total amount of distressed loans matters less than the number of creditors being initially hit by the default.

In our extended model of contagion, which includes default cascades, endogenous bank runs and asset-liability management, we highlighted the complex interactions between network structure and balance sheet composition. We proved that larger capital requirements are effectively able to stabilize the system, while larger liquid reserves, despite providing a buffer in case of liquidity run, induce banks to keep a smaller amount of capital, thus making them vulnerable to contagion.

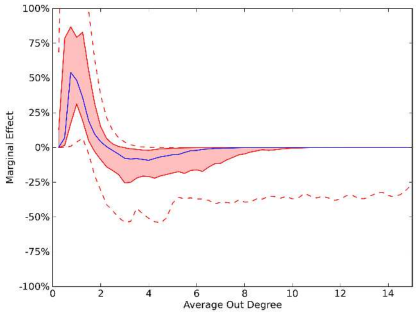
This finding is in line with what has been predicted also by other authors. For example, in Battiston et al. (2012a), it was shown that the size of default cascades would increase when the average robustness (i.e. the average equity ratios of banks) would decrease. The same result is also discussed in Nier et al. (2007) where it is found that when levels of equity falls below a given threshold, there is a sharp increase in the risk of a systemic breakdown.

The relative weight of short-term and long-term exposures also matters in this framework and an intermediate balance between the two seems optimal. Short-term exposures are indeed both a channel for liquidity shocks, but they can also be easily removed, preventing shock to propagate.

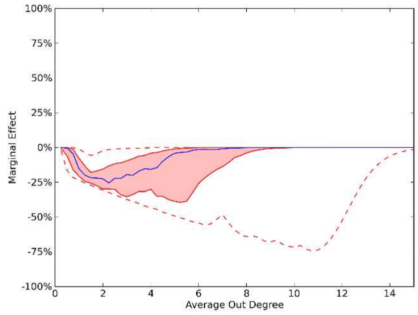
Finally, the role of fire-sales highlight the complexity of this kind of models in which several channels of contagion operate. Indeed, fire-sale losses imply higher risk of illiquidity. Hoarding banks will then seek funds from other non-illiquid banks, reducing their exposures to them and, eventually, leading to a more likely closure of the credit lines. This effectively removes channels for the propagation of contagion. In this sense, fire-sale losses induce a more prudent behavior.

This paper also provides policy suggestions for the regulation of the financial system. The role of *too-connected-to-fail* and *too-big-to-fail* institutions in financial markets is ambiguous, since they act as shock absorbers in case of random attack, but pose relevant systemic risk if distressed. Nevertheless, we proved that *too-connected-to-fail* banks should be the primary concern for a contagion-averse regulator, since their distress is more likely to trigger systemic breakdowns.

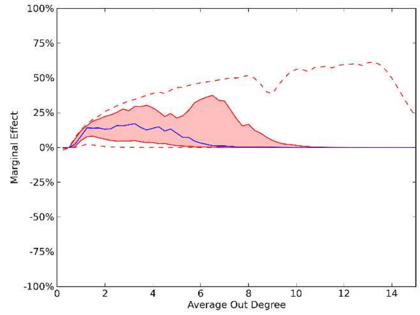
Capital requirements should also be rethought in the light of the trade-offs highlighted by our



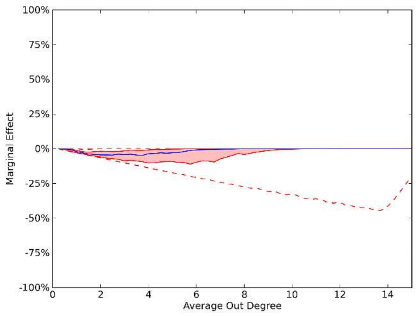
(a) Average degree



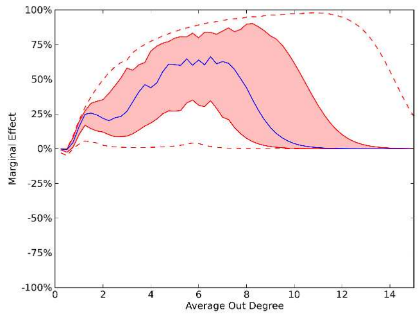
(b) High min capital ratio



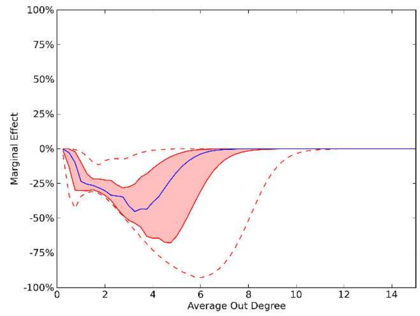
(c) Low min capital ratio



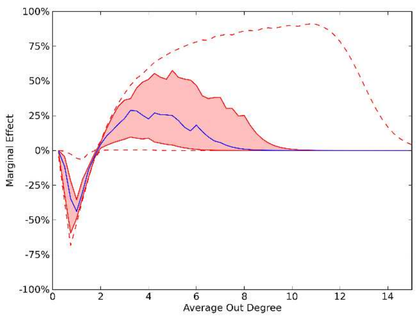
(d) Sales at Loss



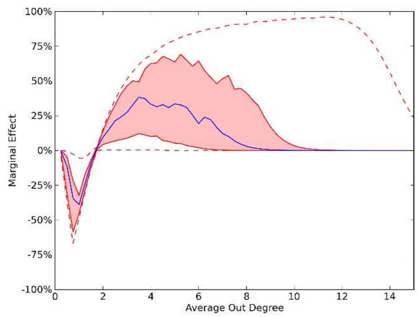
(e) High liquidity



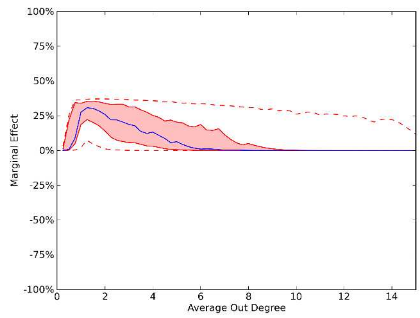
(f) Low liquidity



(g) Large short-term market

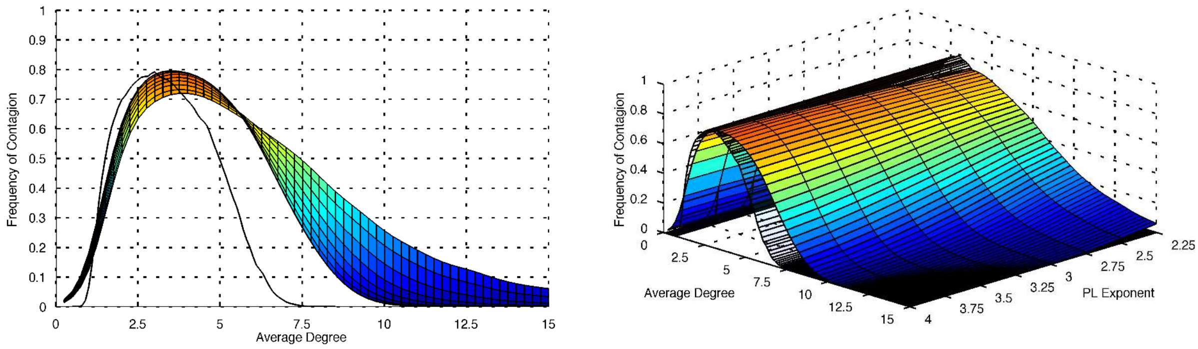


(h) Large long-term market

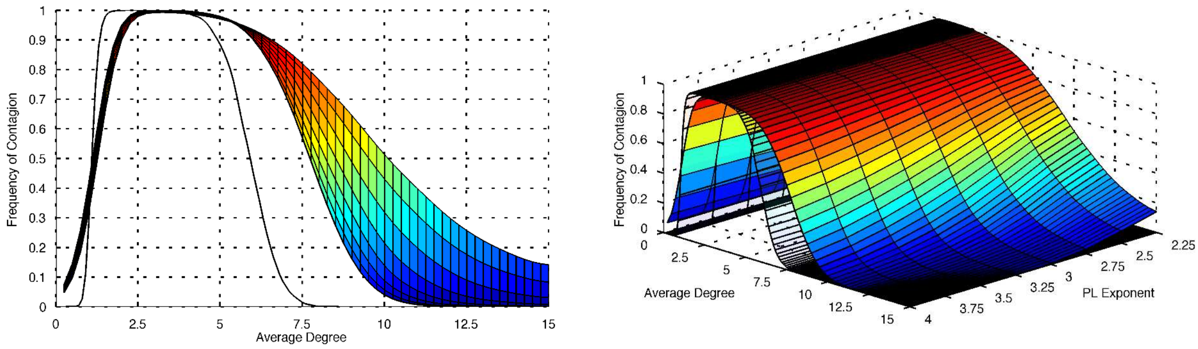


(i) Too-connected-to-fail

Figure 10: **Extended model - [ER1] | Marginal Effects - Threshold: 10%**
Panels (a)–(i) show the *marginal effects* for the different parameters: the blue line represents the median marginal effect; the shaded area marks the value of the marginal effect between the 25^{th} and the 75^{th} percentiles; while the upper and lower dashed red lines correspond, respectively, to the maximum and minimum estimated marginal effect for each specific regressor.



(a) Random Default



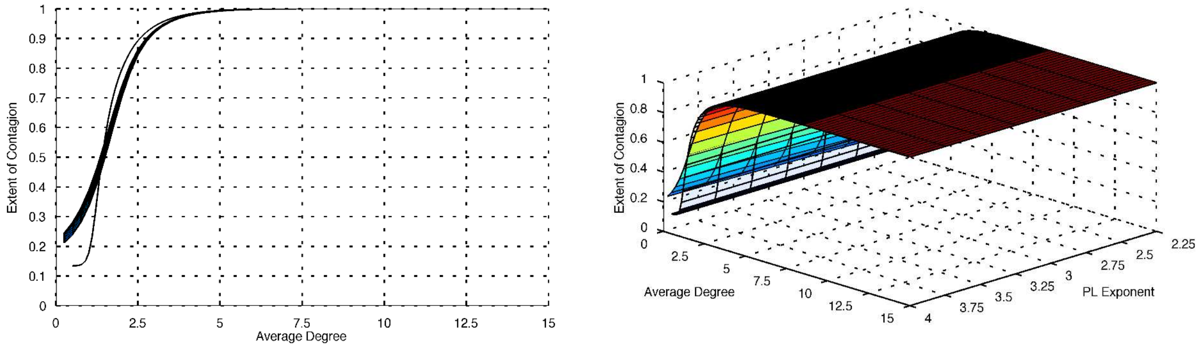
(b) Too-Connected-To-Fail

Figure 11: **Benchmark model - [ER2] | Frequency of Contagion - Threshold: 10%**
Frequency of contagion as a function of the average number of interbank connections of the banks in the system and of the exponent of the power-law distribution used to generate the interbank exposures.

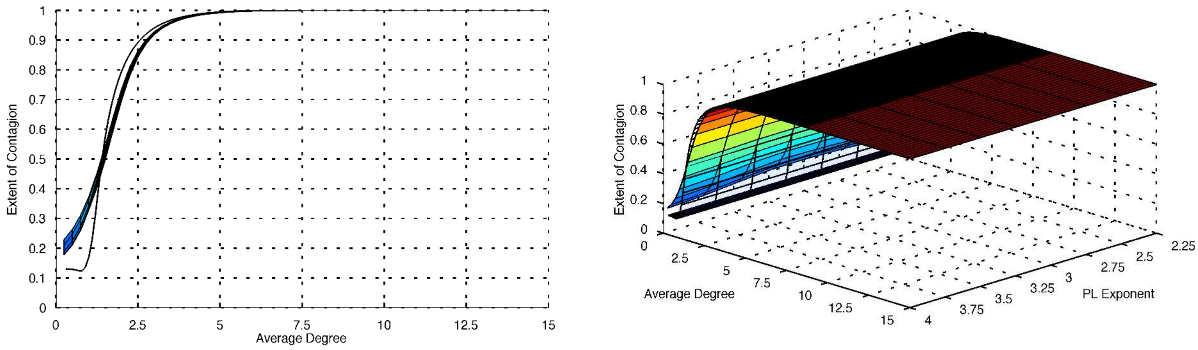
complex system approach, together with the incentives micro-prudential regulation should set. Indeed, such incentives may be strongly mis-aligned with macro-prudential objectives, if not designed in a systemic perspective. Our paper has indeed clearly highlighted how conditions that are extremely desirable from a micro-prudential point of view (e.g. larger liquid reserves and no fire-sale losses), may induce, at a macro level, systemic fragility.

Our analysis, however, is far from being complete. The model we have hereby presented can still be extended along different dimensions. From a macro-prudential standpoint, the menu of possible market regulations that can be tested shall be expanded. One addition we are currently working on is to include a *liquidity requirement* along with the capital requirements which are already part of our framework. This additional control mechanism will probably intensify the dynamics of the short-term interbank market because having mandatory liquidity requirements will increase the demand for liquidity during distressed times. Another possibility would be to introduce – alongside the other measures – *counter-cyclical capital buffers* and/or *maximum leverage ratios.*

In terms of the dynamics of the model, additional features shall be included. First, banks shall be allowed not only to alter the structure of their short-term lending but also to transform the maturity of long-term loans. Second, asset prices shall be made endogenous. In the current version of the model, asset prices are exogenously determined and they do not vary while contagion unfolds. Instead, a more realistic approach would be to let prices adjust depending on the amount of assets sold in the market at any given time and balance sheets shall be evaluated *mark-to-market.* Lastly, different types of asset

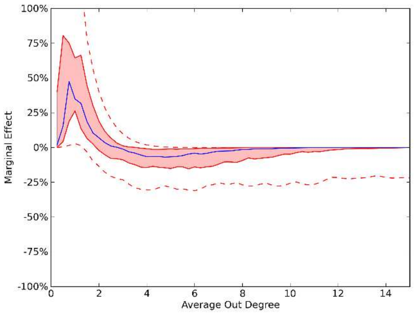


(a) Random Default

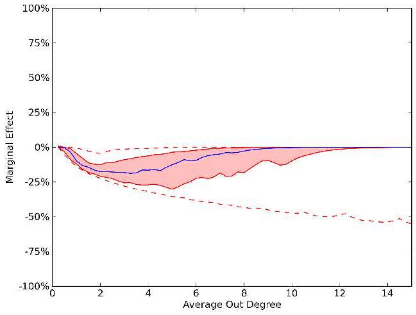


(b) Too-Connected-To-Fail

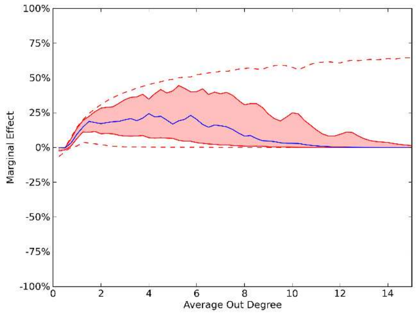
Figure 12: **Benchmark model - [ER2] | Extent of Contagion - Threshold: 10%**
Extent of contagion as a function of the average number of interbank connections of the banks in the system and of the exponent of the power-law distribution used to generate the interbank exposures.



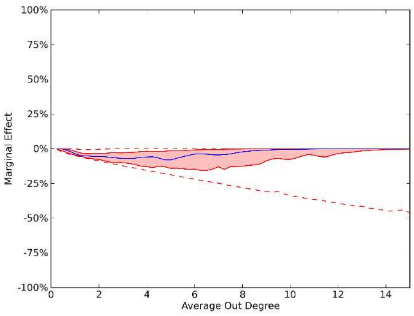
(a) Average degree



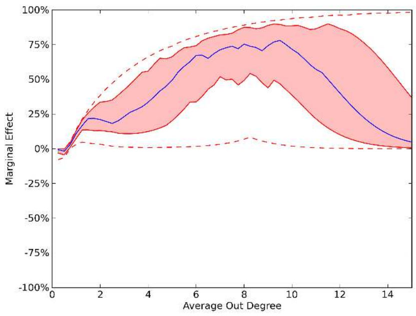
(b) High min capital ratio



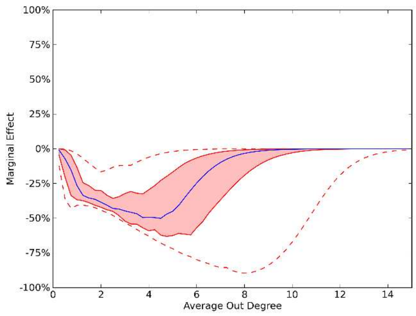
(c) Low min capital ratio



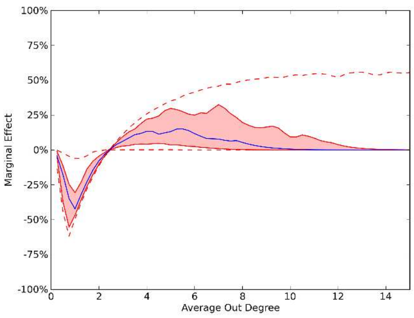
(d) Sales at Loss



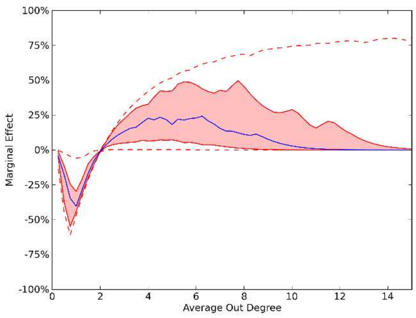
(e) High liquidity



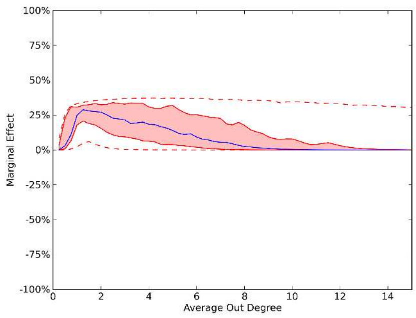
(f) Low liquidity



(g) Large short-term market

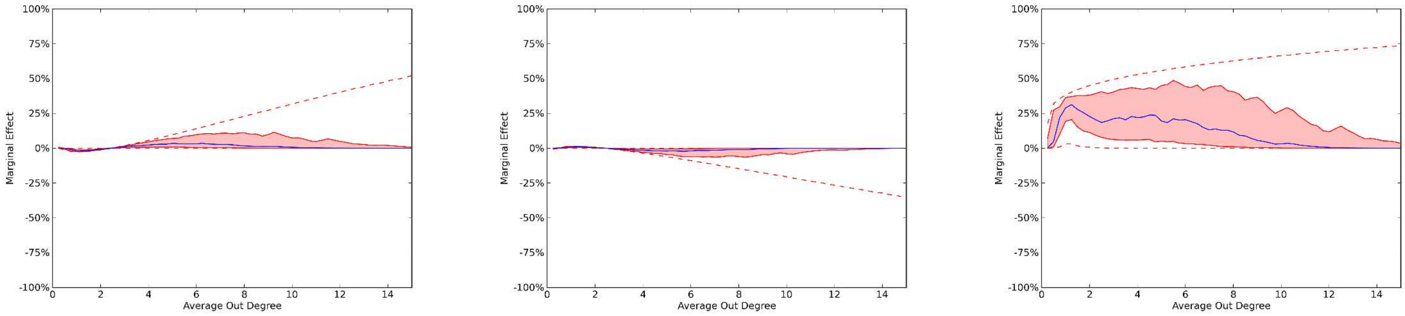


(h) Large long-term market



(i) Too-connected-to-fail

Figure 13: **Extended model - [ER2] | Marginal Effects - Threshold: 10% (A)**
Panels (a)–(i) show the *marginal effects* for the different parameters: the blue line represents the median marginal effect; the shaded area marks the value of the marginal effect between the 25^{th} and the 75^{th} percentiles; while the upper and lower dashed red lines correspond, respectively, to the maximum and minimum estimated marginal effect for each specific regressor.



(a) High heterogeneity (b) Low heterogeneity (c) Too-exposed-to-fail

Figure 14: **Extended model - [ER2] | Marginal Effects - Threshold: 10% (B)**
Panels (a)–(c) show the *marginal effects* for the different parameters: the blue line represents the median marginal effect; the shaded area marks the value of the marginal effect between the 25^{th} and the 75^{th} percentiles; while the upper and lower dashed red lines correspond, respectively, to the maximum and minimum estimated marginal effect for each specific regressor.

categories shall be introduced. This will allow us to study how the composition of banks' (overlapping) portfolios influence the stability of the financial system.

Finally, other topological features of the interbank network can be analyzed to explain the diffusion of financial contagion. Due to computation constraints, we have restricted our attention – in this first contribution – to a smaller set of network metrics. However, a larger set topological features of the interbank network can be analyzed: the distribution of node clustering; the graph-component distribution; or the community structure of the graph [1]. Correlating systemic events with such finer-grained characteristics of the interbank network will allow the policy maker to better understand how the topology of the interbank lending can be tweaked and/or controlled to improve the stability and the resilience of the financial system.

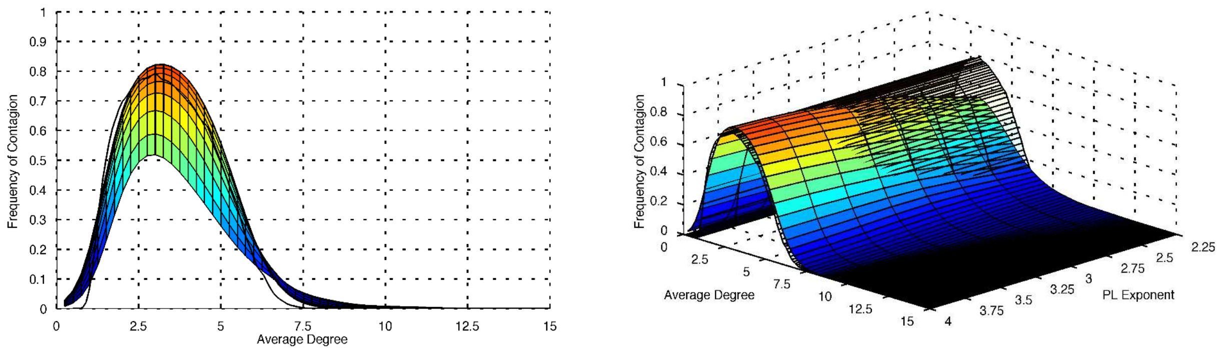
Appendix

A Network Theory: Definitions

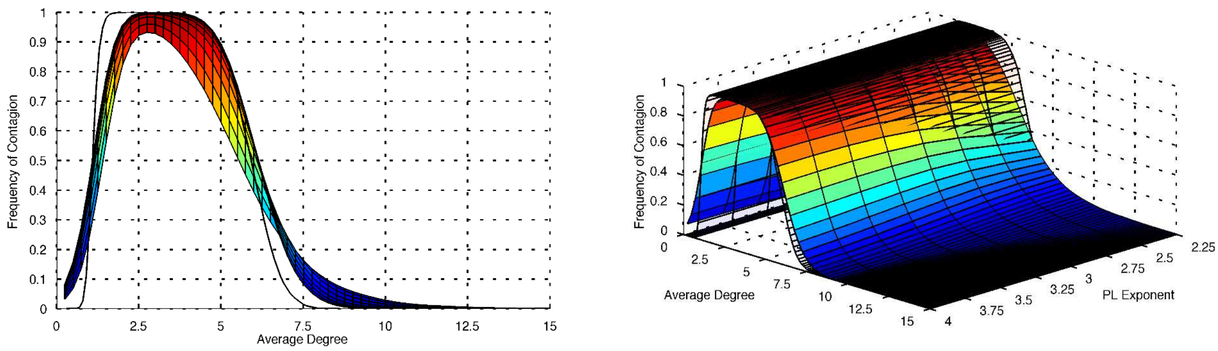
A network is simply a collection of points connected by links, which we may formalize as a set $G = (I, V)$, where I is the set of vertices (nodes), while V is the set of couples $(i, j) \in I^2$ representing the edges, which may be ordered or unordered, and we shall then speak of directed or undirected graphs respectively.

Any network can be unambiguously represented by an adjacency matrix $A(G)$, whose elements a_{ij} take the value of zero or one depending on whether $(i, j) \notin V$ or $(i, j) \in V$. If the network is undirected the adjacency matrix is symmetric. Moreover, whenever links have different weights,

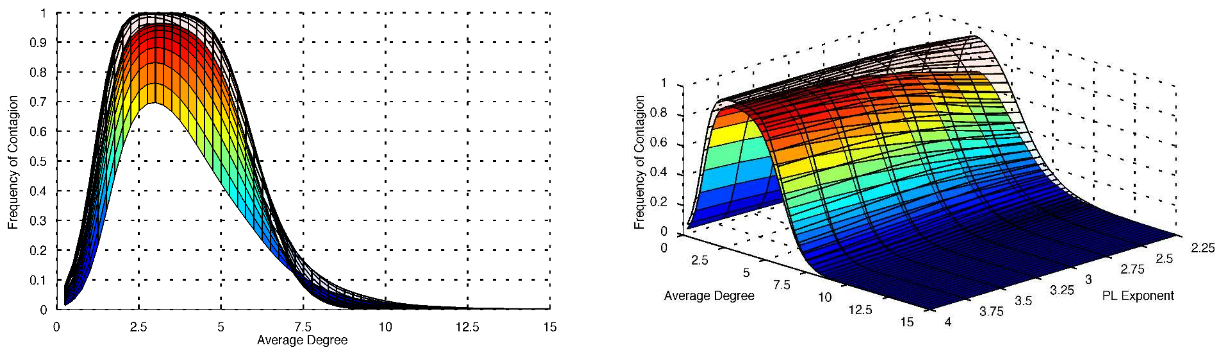
[1] A component of a graph is a minimal subset of nodes that are connected, i.e. for which any two nodes are connected by a path going through the nodes of the subset. A community structure is a partition of the nodes of the graph, induced by observed topology, where the nodes in each set of the partition are more strongly linked between each other than they are with nodes belonging to different sets of the partition. See Newman (2010) for an introduction.



(a) Random Default

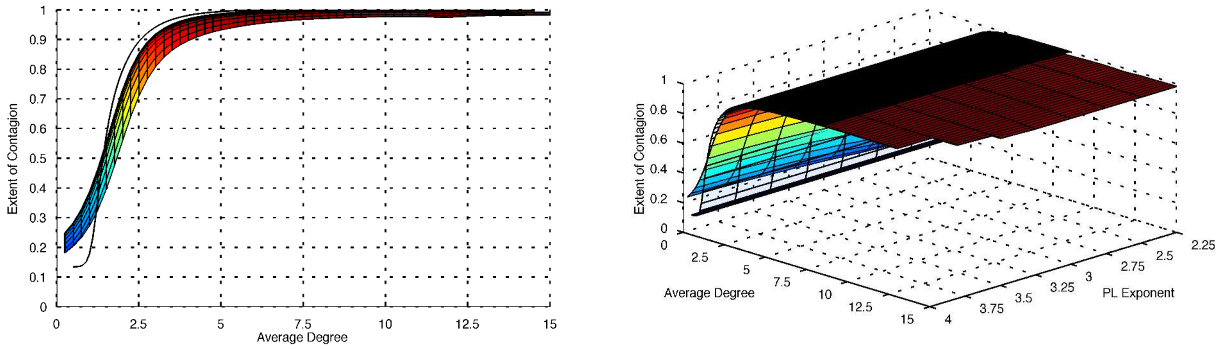


(b) Too-Connected-To-Fail

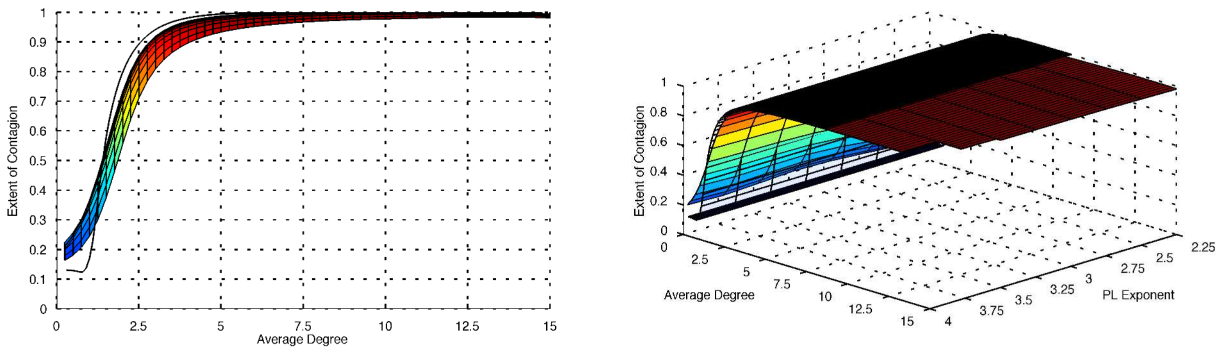


(c) Too-Big-To-Fail

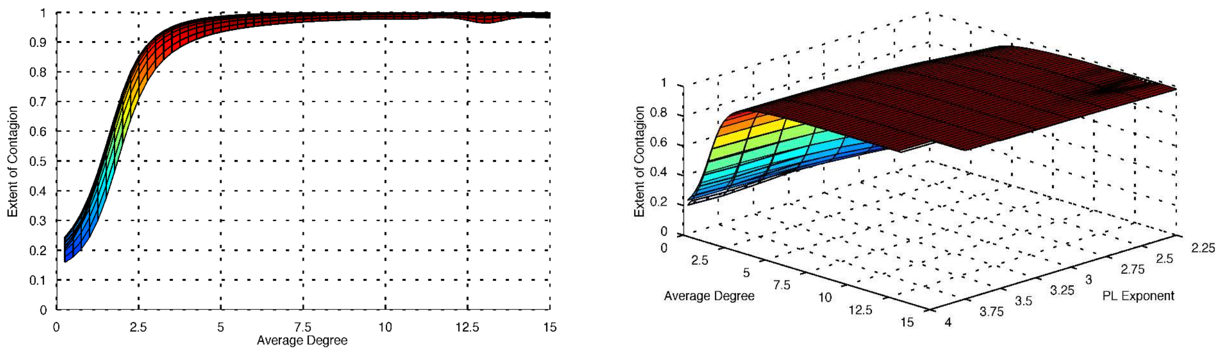
Figure 15: **Benchmark model - [ER3] | Frequency of Contagion - Threshold: 10%**
Frequency of contagion as a function of the average number of interbank connections of the banks in the system and of the exponent of the power-law distribution used to generate the interbank exposures and the banks' total size.



(a) Random Default

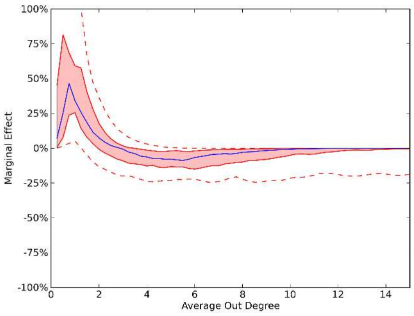


(b) Too-Connected-To-Fail

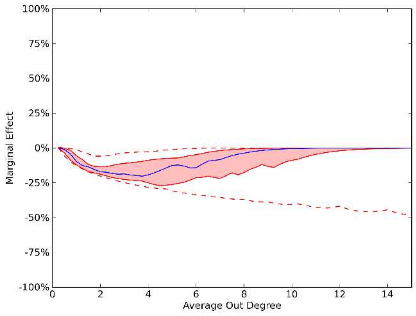


(c) Too-Big-To-Fail

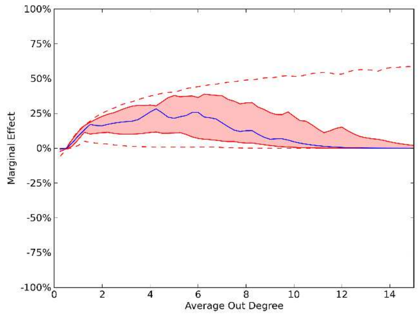
Figure 16: **Benchmark model - [ER3] | Extent of Contagion - Threshold: 10%**
Extent of contagion as a function of the average number of interbank connections of the banks in the system and of the exponent of the power-law distribution used to generate the interbank exposures and the banks' total size.



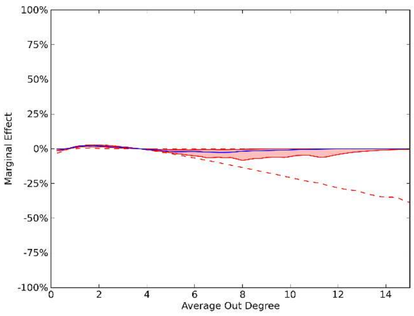
(a) Average degree



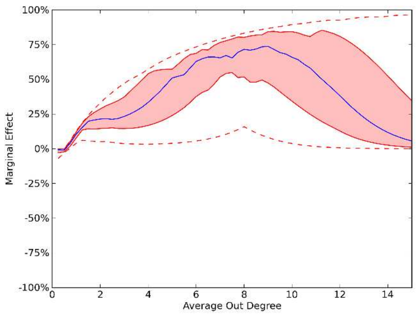
(b) High min capital ratio



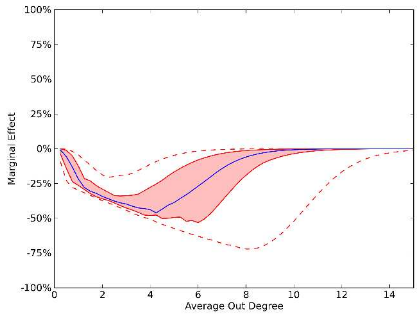
(c) Low min capital ratio



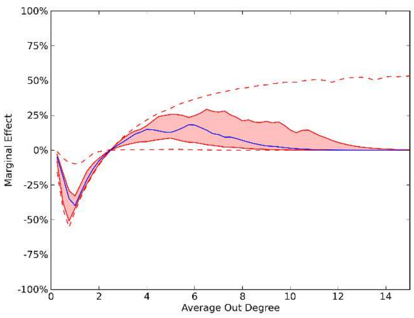
(d) Sales at Loss



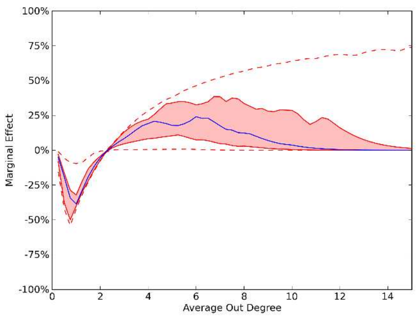
(e) High liquidity



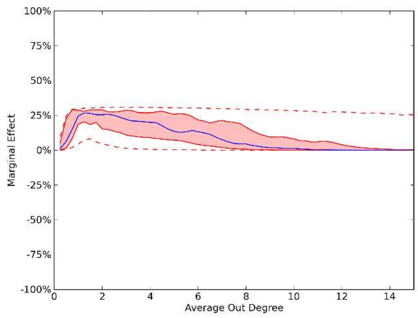
(f) Low liquidity



(g) Large short-term market

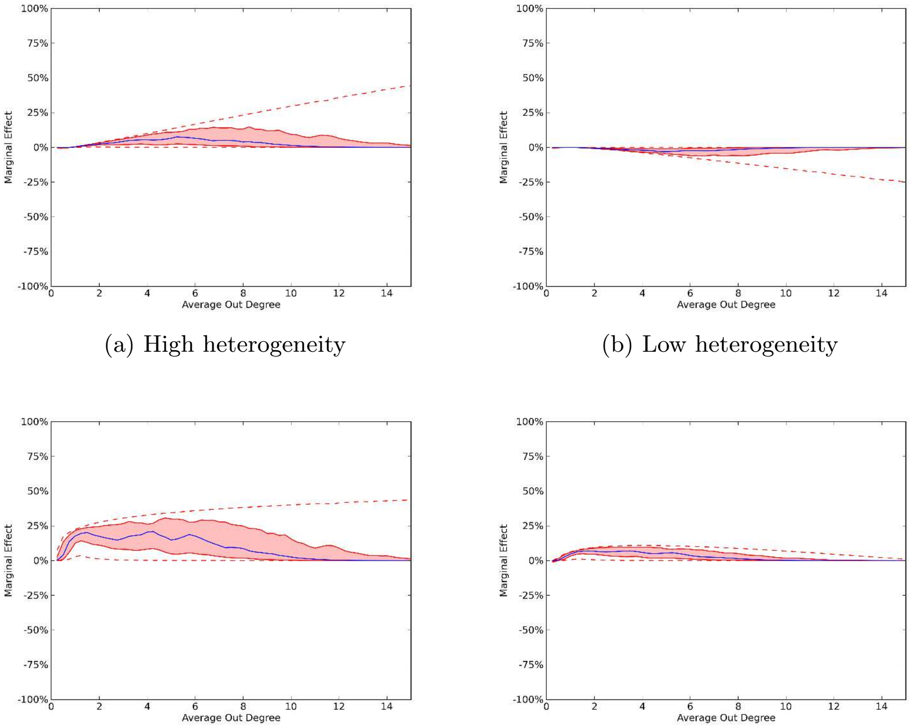


(h) Large long-term market



(i) Too-connected-to-fail

Figure 17: **Extended model - [ER3] | Marginal Effects - Threshold: 10% (A)**
Panel (a) shows the estimated probability of observing default cascades where more than 10% of the total assets had been wiped out. Panels (b)–(i) show the *marginal effects* for the different parameters: the blue line represents the median marginal effect; the shaded area marks the value of the marginal effect between the 25^{th} and the 75^{th} percentiles; while the upper and lower dashed red lines correspond, respectively, to the maximum and minimum estimated marginal effect for each specific regressor.



(a) High heterogeneity

(b) Low heterogeneity

(c) Too-exposed-to-fail

(d) Too-big-to-fail

Figure 18: **Extended model - [ER3] | Marginal Effects - Threshold: 10% (B)**
Panels (a)–(c) show the *marginal effects* for the different parameters: the blue line represents the median marginal effect; the shaded area marks the value of the marginal effect between the 25^{th} and the 75^{th} percentiles; while the upper and lower dashed red lines correspond, respectively, to the maximum and minimum estimated marginal effect for each specific regressor.

representing different intensities in the connections one may define a weighted matrix $W(G)$ whose elements w_{ij} represent the weight of the link from i to j if a link between them exist, while they are zero if no link is present between them.

A network is a natural representation of an interbank market. Banks represent the nodes of the graph, edges are given by lending relations and their weight is the value of the exposures. By taking this point of view on the financial system, one is able to analyze its properties and its architecture, in order to identify the relevant features for its stability. Indeed we are interested in the network structure of an interbank market for its consequences in the transmission of liquidity shocks and default cascades.

Network Statistics

A first step towards the understanding of the stability of financial systems passes through the analysis of their structure itself. Despite not exhaustive of the entire set of topological feature one my compute in a network, the following list provides an overview of the statistics which are both economically meaningful and relevant for financial stability. In the following definitions we consider a network of n nodes, whose adjacency matrix is A and whose weighted matrix is W.

Node Degree The in-degree k_i^{in} and out-degree k_i^{out} of a node in a directed networks are the number of incoming and outgoing links respectively:

$$k_i^{in} = \sum_{j=1}^{n} a_{ij} \quad \text{and} \quad k_i^{out} = \sum_{j=1}^{n} a_{ji}.$$

In the context of an interbank network the in-degree represent the number of creditors and the out-degree the number of debtors.

Node Strength The in-strength s_i^{in} and out-strength k_i^{out} of a node in a directed networks are the total amount of weight carried by its incoming and outgoing links respectively:

$$s_i^{in} = \sum_{j=1}^{n} w_{ij} \quad \text{and} \quad s_i^{out} = \sum_{j=1}^{n} w_{ji}.$$

The definition parallels that of node in- and out-degree and, in our framework, can be interpreted as the total amount of interbank assets and liabilities.

Connectivity Connectivity is the fraction of possible links that the network actually displays. Calling l the number of existing edges, in a directed graph, connectivity is given by

$$c = \frac{l}{n(n-1)}.$$

Connectivity of thus a measure of the fraction of possible interbank relations which actually exist. It thus provide a measure of diversification and also of the channels of transmissions through which a shock may flow. A closely related concept is that of average degree

Average Degree The average degree is the average in-degree, or, equivalently, the average out-degree, of the n nodes in the network

$$\bar{k} = \sum_{i=1}^{n} k_i^{in} = \sum_{i=1}^{n} k_i^{out}$$

Which thus represents the average number of counterparties a bank has, and thus even more clearly represents both the average level of diversification and the average number of possible sources of shock.

Average Path Length A path is a sequence of vertices such that each pair of consecutive vertices in the sequence are connected by an edge. The number path of length r from i to j is the element i, j of A^r. The average path length is the average shortest path between any two nodes.

Despite most studies on interbank contagion have, so far, focused on connectivity, also the average path length should be taken into consideration when exploring the resilience of an interbank markets. Indeed it represents the average number of connections separating two banks and may thus be relevant for the timing and the severity of a default cascade.

Reciprocity In directed networks, reciprocity is the fraction of links for which a link in the opposite direction exists. An expression for reciprocity is

$$r = \frac{TrA^2}{l}.$$

Reciprocity represents the frequency of reverse lending relationships. Certainly, from an empirical point of view, it is interesting to note how several contributions have found high levels of reciprocity in real interbank networks (Bech and Atalay, 2010; Soramäki et al., 2007). This possibly reflects the role of what Cocco et al. (2009) define preferential lending, i.e. the importance of non-economic foundations for interbank lending.

Clustering In an undirected network, the clustering coefficient is defined as the probability that two nodes, which are connected with another node, are connected between themselves:

$$C = \frac{\text{number of triangles} \times 3}{\text{number of connected triples}} = \frac{1}{n} \sum_{i=1}^{n} \frac{(A^3)_{ii}}{k_i(k_i - 1)}.$$

In the definition of the clustering coefficient, we consider, for simplicity, the case of an undirected network, which can be derived from a directed one if the directionality of a link is neglected. k_i indicates the (undirected) degree of node i, i.e. the number of connections i has, and $(A^3)_{ii}$ represents the i-th element in the diagonal of A^3. The clustering coefficient is a measure of how tight interbank relations are at local level. An high clustering coefficient indicates that the counterparties of a given bank are very likely to make transactions also between themselves.

Assortativity and Disassortativity A network is said to be assortative if nodes with a certain degree are more likely to be connected with nodes with similar degree. It is said to be disassortative if the opposite holds. A simple measure of assortativity in undirected networks is

$$m = \frac{cov(k_i, ANND_i)}{\sigma(k_i)\sigma(ANND_i)} \in [-1, 1]$$

where $ANND_i$ is the average nearest neighbor degree, i.e. the average degree of node i's neighbors.

As for the clustering coefficient, we presented only the undirected version of the assortativity coefficient. Interbank markets tend to be disassortative, in the sense that small banks tend to trade with large banks and viceversa. This may be symptomatic of the presence of banking groups, in which small subsidiaries preferentially trade with the parent company.

The previous statistics are either referred to single nodes, or are synthetic network-level measures that summarize, in a single number, a series of local features. Other relevant information may instead come from the statistical *distribution* of certain local characteristics. First and foremost, the distribution of the degree, i.e. the number of counterparties, provides important information regarding the structure of an interbank network, since it is able to quantify the level of heterogeneity of its nodes.

Degree Distribution Given a network, construct a sequence of possible degrees $\{1, 2, \dots\}$ and a sequence of probabilities $\{p_1, p_2, \dots\}$, where p_k is the frequency of nodes with degree k. The quantities $\{p_1, p_2, \dots\}$ thus define a probability distribution over degrees $\{1, 2, \dots\}$, which is defined as *degree distribution.*

More appropriately, in the context of directed graphs, we would dealt with a *joint degree distribution* $\{p_{k^{in}k^{out}}\}$ representing the probability that a node have in-degree k^{in} and out-degree k^{out}.

B Models of Network Formation

The previous overview of definitions can be applied to any arbitrary network. However, when one seeks to build a network displaying some desired features, he has to confront with the theory of network formation, which provides a set of models that, because of different assumptions on the mechanism of link formation, are able to generate a corresponding set of networks with specific statistical peculiarities. Here, we intend to provide a brief description of the two network formation models employed in our analysis, namely the random graph model by Erdős and Rényi (1960) and the fitness model[1].

B.1 Erdős-Rényi Model

The random graph model due to Erdős and Rényi (1960) is a model in which, given a set of N nodes, a link from node i to node j exists with probability p, which is constant for each pair of nodes. In the network there are $N(N-1)$ possible directed links to be created, resulting in an expected number of edges in the network equal to $pN(N-1)$, so that the (expected) average degree is $p(N-1)$. Indeed each node has $(N-1)$ nodes to which it can connect. It follows that both the distribution of the in- and the out-degree follows a binomial distribution:

$$p^{in}(k) = p^{out}(k) = \binom{N-1}{k} p^k (1-p)^{N-k-1}.$$

If c denotes the average degree of a random graph, asymptotically, as $N \to \infty$, the degree distribution converges to a Poisson(c)

$$p(k) = \frac{e^{-c}c^k}{k!},$$

[1] For a more complete overview of network formation models one may refer to standard textbooks as Newman (2010) or to the reviews by Albert and Barabási (2002) and Chakrabarti and Faloutsos (2006).

which is the reason why the model is sometimes referred to as Poisson random graph.

Since the probability of forming a link is homogeneous, the resulting network structure does not present marked heterogeneity. In a Poisson distribution the dispersion around the mean is limited and deviations from it are exponentially rare. An interbank network generated using this model will thus provide of a homogeneous market, in which banks tend to have similar levels of connectivity, i.e. their specific number of counterparties does not significantly vary from the average.

The Erdős and Rényi (1960) graph is also said to be small-world, since it presents a short average path length and its diameter, i.e. the longest of the shortest paths linking two nodes, grows at a much lower rate than N, precisely as $\log(N)$. The clustering coefficient is equal to the probability of a link's existence, p.

This model has been extensively applied for the study of contagion in financial networks, e.g. in the contributions from Nier et al. (2007), Gai and Kapadia (2010), Iori et al. (2006) and Montagna and Kok (2013).

B.2 Fitness Model

The fitness model is a very flexible model of network formation, which is able to generate a wide range of structural features. Every node i is endowed with a fitness parameter, x_i, which is a measure of its "attractiveness", and links are formed between nodes with a probability which is a function of the fitness of the nodes. More formally, if we define p_{ij} as the probability that a link exists from i to j, this probability is given by

$$p_{ij} = f(x_i, x_j)$$

for a generic function f.

Depending on the shape of the function f and on the probability distribution ρ of the fitness, various properties may emerge.

In general the expected in-degree for a node with fitness x is

$$k^{in}(x) = n \int_{-\infty}^{+\infty} f(t, x)\rho(t)dt \equiv nF_{in}(x),$$

while the expected out-degree is

$$k^{out}(x) = n \int_{-\infty}^{+\infty} f(x, t)\rho(t)dt \equiv nF_{out}(x).$$

Clearly, the two expressions coincide if f is symmetric, i.e. $f(x_i, x_j) = f(x_j, x_i)$, meaning that the fitness parameter represents the attractiveness of the node irrespective of the direction of the relation to be established.

Under the assumption of invertibility of F^{in} and F^{out} and of differentiability of their inverse, one can derive an analytical expression for the probability of observing nodes with in-/out-degree equal to a generic k:

$$P^{in}(k) = \rho\left[F_{in}^{-1}\left(\frac{k}{n}\right)\right] \cdot \frac{d}{dk}F_{in}^{-1}\left(\frac{k}{n}\right)$$
$$P^{out}(k) = \rho\left[F_{out}^{-1}\left(\frac{k}{n}\right)\right] \cdot \frac{d}{dk}F_{out}^{-1}\left(\frac{k}{n}\right).$$

One may also compute higher order properties of networks built via fitness models. Focusing on undirected models, where $p_{ij} = f(x_i, x_j) = f(x_j, x_i)$ represents the probability of an undirected link between i and j, closed form solutions are available for the clustering coefficient of nodes with fitness x

$$C(x) = n^2 \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,t)f(t,s)f(s,x)\rho(t)\rho(s)dtds}{k(x)^2}$$

and for the average degree of their neighbors

$$ANND(x) = \frac{n}{k(x)}\int_{-\infty}^{+\infty} f(x,t)k(t)\rho(t)dt.$$

Despite this is just a brief summary of the properties of a fitness model[1], it should be clear enough that its flexibility has the potential to take into account a number of target properties. This is the reason why authors as De Masi et al. (2006) and Montagna and Lux (2013) suggest the fitness model in order to match the empirical features of real interbank networks.

In our (directed) interbank network we use a fitness model with an additive linking function

$$p_{ij} = f(x_i, x_j) = c(x_i + x_j), \tag{21}$$

where c is a constant that we tune in order to obtain the desired average degree.

We then draw a series of n fitness parameters $\{x_1, x_2, \ldots, x_n\}$, one for each bank from a power-law distribution with exponent $\beta > 2$ and minimum value x_0

$$P(x) = ax^{-\beta}, \quad x > x_0. \tag{22}$$

Solving the integration for the expected in- and out-degree we find that

$$F_{in}(x) = F_{out}(x) = \frac{cx_0^{2-\beta}}{\beta - 2} + \frac{acx_0^{1-\beta}}{\beta - 1}x.$$

Inverting these functions and using the formulas for the degree distribution we see that

$$P^{in}(k) = P^{out}(k) \propto (k - \xi)^{-\beta}, \tag{23}$$

where ξ is a positive constant that depends on the parameters of the fitness model. This means that our model is able to replicate a power-law tail decay of the degree distribution, which is a feature often observed in real networks (Caldarelli, 2007; Newman, 2010), including in interbank markets (Bech and Atalay, 2010; Boss et al., 2004; Cont et al., 2013; Iazzetta and Manna, 2009; Iori et al., 2008; Soramäki

[1]For a more detailed description of its properties we invite the reader to refer to Caldarelli et al. (2002), Caldarelli (2007) and Servedio et al. (2004).

et al., 2007) and which is thus symptomatic of high levels of heterogeneity in the connectivity of financial institutions. The model is flexible enough to allow us to tune the exponent of the tail decay.

C Scenarios

The models presented in Section 2 are tested against five different architectures of the financial system. Three employ the Erdős and Rényi (1960) network model (cases *ER1*, *ER2* and *ER3*), while two of them use the fitness model (De Masi et al., 2006) (cases *FIT1* and *FIT2*) to generate the interbank network[1].

ER1: Homogeneous banks with homogeneous exposures. The Erdős and Rényi (1960) network model is used to generate interbank connections and banks are assumed to have the same asset size. In the *benchmark model*, just one network is generated; in the *extended model*, two Erdős and Rényi (1960) graphs are generated (on for the short-term interbank market and one for the long-term interbank market). By using - in the *extended model* - two independent graphs to assign short and long term exposures, we allow cases in which a bank can lend (or borrow) from another financial institution at both maturities. Interbank claims are evenly distributed among the outgoing links, so that there is no single exposure which is more dangerous than the others. In the *extended model*, they are evenly distributed within each maturity class.

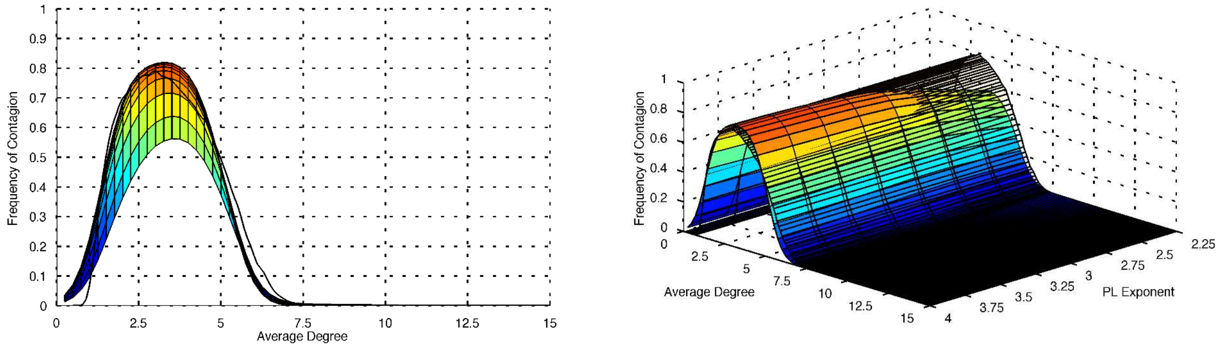
ER2: Homogeneous banks with heterogeneous exposures. As in the previous case, all banks have the same asset size and the network(s) is Erdős and Rényi (1960). However, we now allow banks to unevenly distribute their exposures across creditors, in such a way the link weight is power-law distributed. For each bank we extract a number of weights equal to its out-degree from a power-law distribution, we then assign interbank claims to the links proportionally to the respective weights. This represents a scenario in which over-exposures may be present, implying the existence of contagious links. The assumption about the distribution of link weights has been made in accordance to empirical findings (Cont et al., 2013; Soramäki et al., 2007).

ER3: Heterogeneous banks with heterogeneous exposures. In this case we allow for heterogeneity also in the asset size. First, Erdős and Rényi (1960) network(s) is generated. Then, in the *benchmark model*, link weights are drawn from a power-law distribution and assigned the links. Total assets are assigned to banks proportionally to their interbank exposures ($A^{IB} + L^{IB}$) in such a way that, on average, interbank assets represent 20% of total assets. The result is a network in which link weights are power-law distributed and asset sizes are power-law distributed as well. The presence of heterogeneity in balance sheet sizes implies the presence of money center hubs, whose ambiguous role as shock absorbers or shock amplifiers will be assessed. Instead, in the *extended model*, the constraints set by our model (i.e. ι, τ, λ, e^* and β) are verified exactly. For instance, the constraint that interbank assets must be 20% of total assets is satisfied only on average in the *benchmark model*; instead, in the *extended model*, this constraint is verified for each individual bank. This is obtained in the following way. First, we generate link weights (in the two graphs) extracting their values from a Pareto distribution. Second, we reshuffle part of the links so that the constraints imposed on the balance-sheets are satisfied. The same reshuffling procedure - for the *extended model* - is applied also in the following scenarios: *FIT1* and *FIT2*.

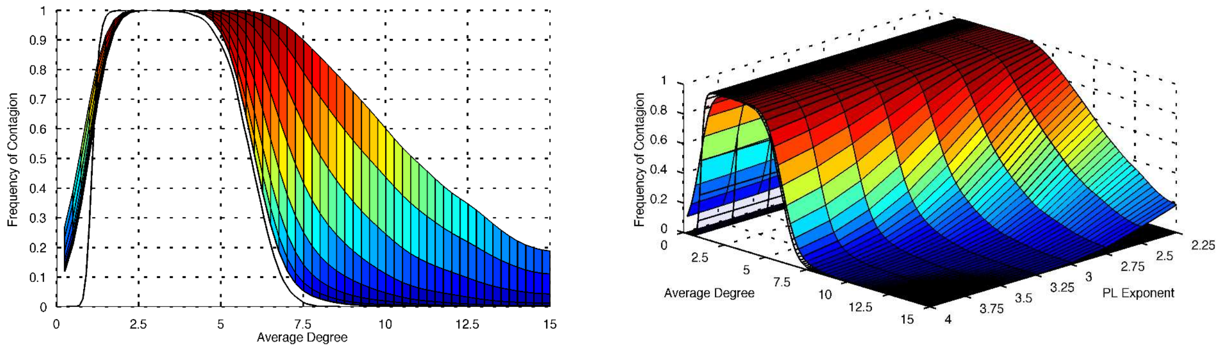
[1] Appendix A explains the differences between the two models and their main features.

FIT1: Heterogeneous banks with homogeneous exposures. We move towards a more realistic architecture for the financial system, which shows a fat-tail degree distribution. We generate a network using a fitness model (De Masi et al., 2006), with an exogenous distribution of the fitness parameter that follows a power-law. In order to close the model, the high level of heterogeneity in the connectivity imposes heterogeneity also in the asset size. However, in this case, exposures remain homogeneous, so that no link is more dangerous than any other. In the *benchmark model*, we first build the network and then assign assets to bank proportionally to their interbank exposures ($A^{IB} + L^{IB}$), the distribution of the asset size is power-law and the fixed ratio of interbank assets to total assets is maintained on average. In the *extended model*, even though the two graphs are generated independently, the "fitness sequence" is the same in the two networks. That is, the bank with the highest fitness in the short-term interbank market will also be the bank with the highest fitness in the long-term interbank market, and so on. As done in *FIT1*, exposures are evenly distributed (within each maturity class).

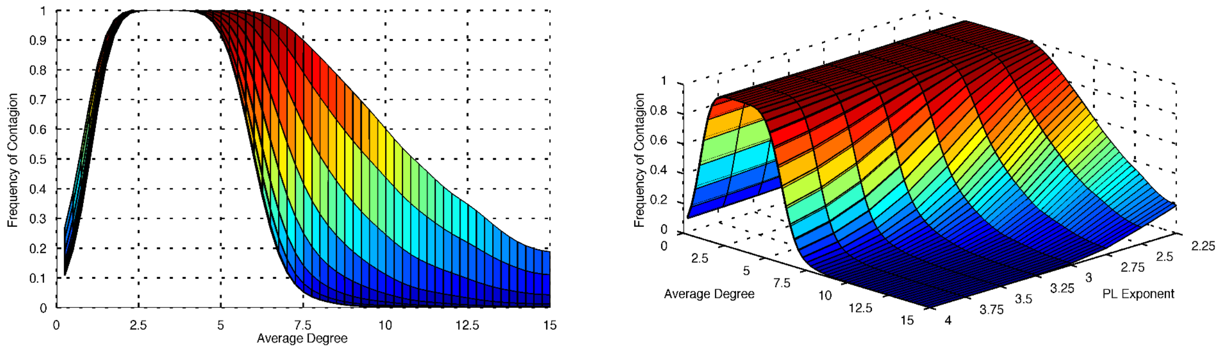
FIT2: Heterogeneous banks with heterogeneous exposures. Here, we allow also for heterogeneous exposures so that, once the network is generated using the fitness model, we draw the value of the exposures from a power-law distribution with the same exponent of the distribution of the fitness parameter. In the *benchmark model*, total assets are then assigned proportionally to total interbank exposures ($A^{IB}+L^{IB}$) in order to maintain the interbank ratio fixed at 20% on average. In the *extended model*, link weights are generated using the same procedure used in cases *ER2* and *ER3*.



(a) Random Default

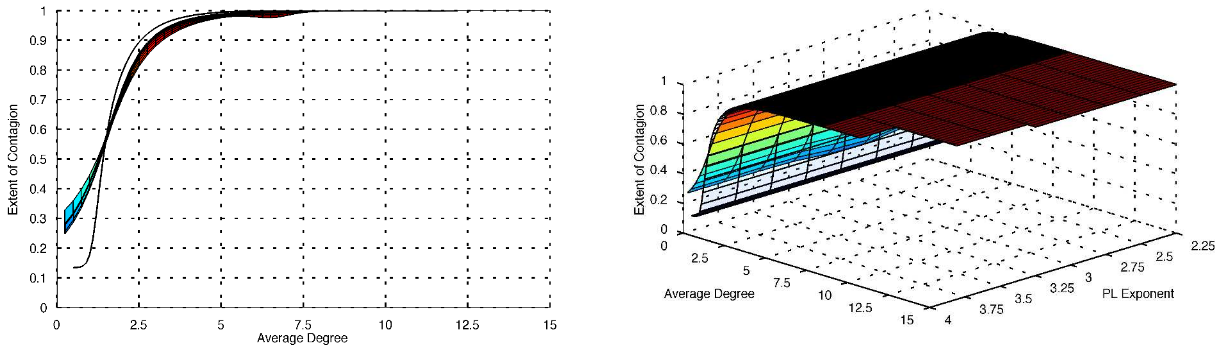


(b) Too-Connected-To-Fail

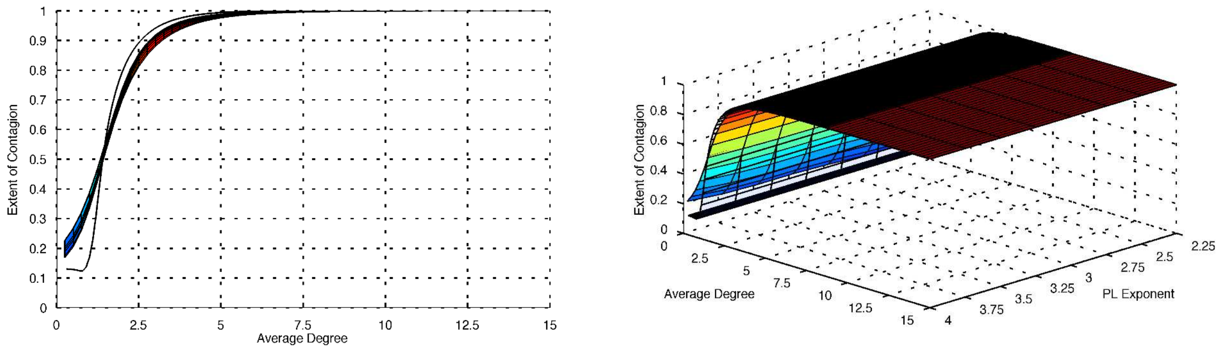


(c) Too-Big-To-Fail

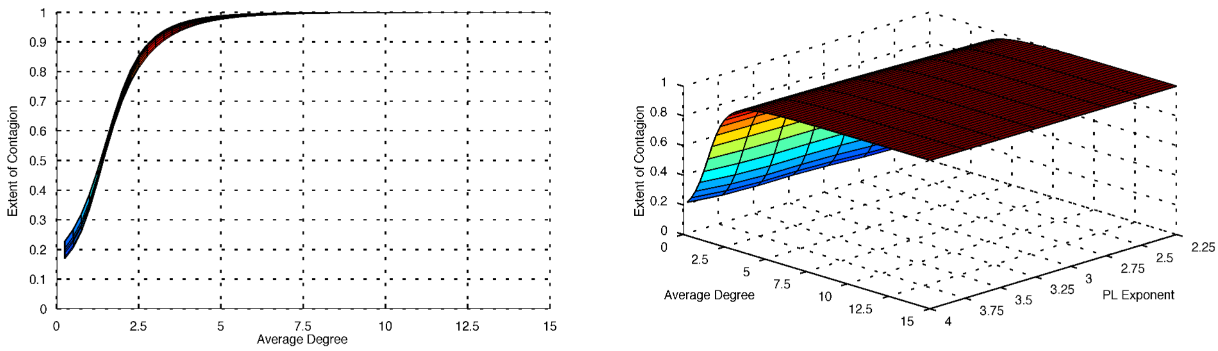
Figure 19: **Benchmark model - [FIT1] | Frequency of Contagion - Threshold: 10%** Frequency of contagion as a function of the average number of interbank connections of the banks in the system and of the exponent of the power-law distribution used to generate the banks' fitness parameter.



(a) Random Default

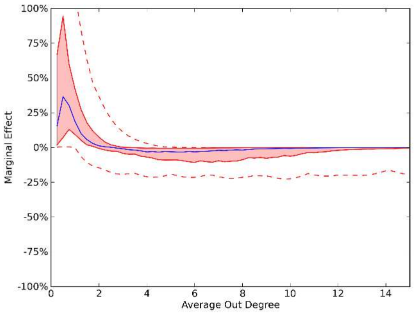


(b) Too-Connected-To-Fail

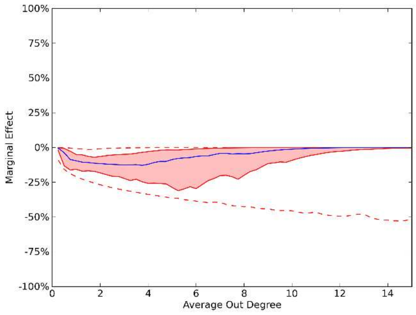


(c) Too-Big-To-Fail

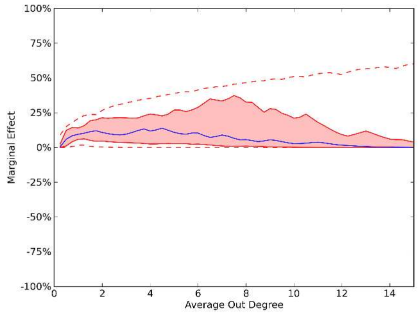
Figure 20: **Benchmark model - [FIT1] | Extent of Contagion - Threshold: 10%**
Extent of contagion as a function of the average number of interbank connections of the banks in the system and of the exponent of the power-law distribution used to generate the banks' fitness parameter.



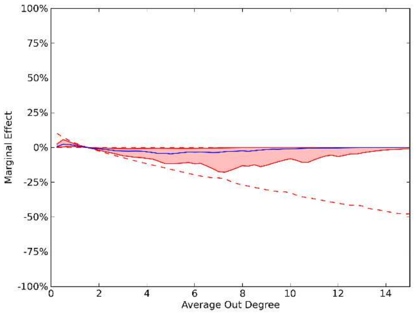
(a) Average degree



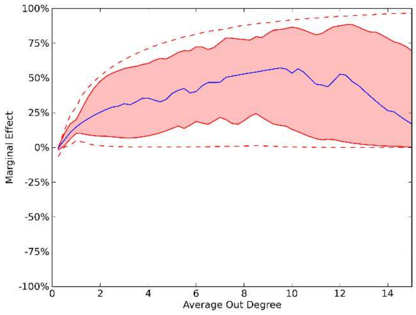
(b) High min capital ratio



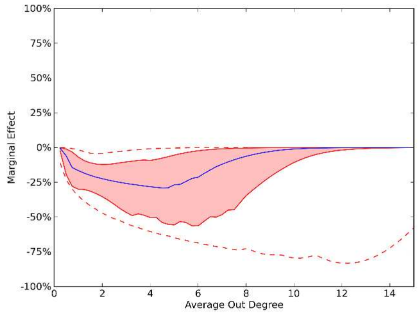
(c) Low min capital ratio



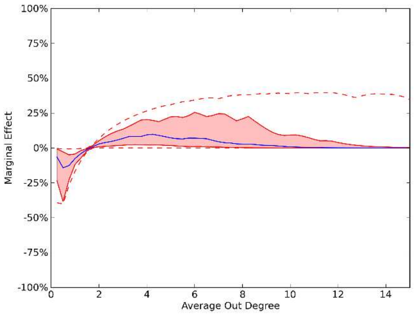
(d) Sales at Loss



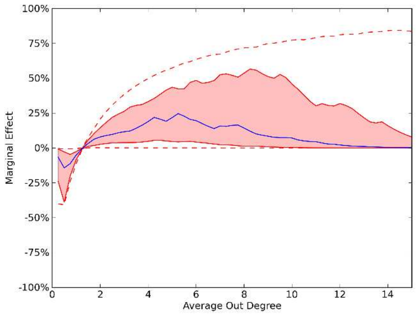
(e) High liquidity



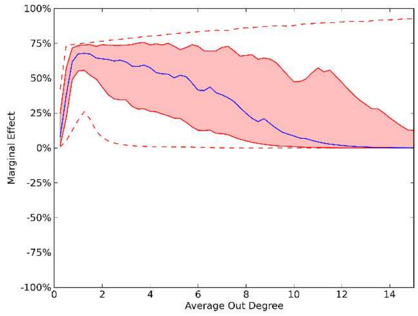
(f) Low liquidity



(g) Large short-term market

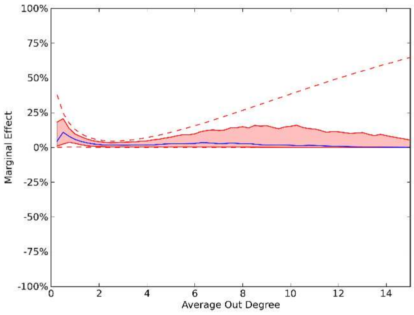


(h) Large long-term market

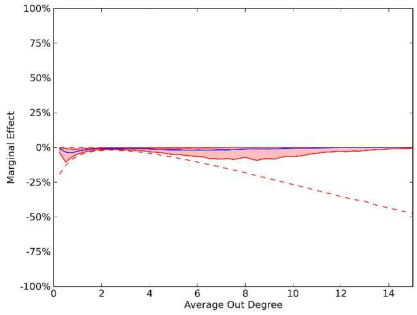


(i) Too-connected-to-fail

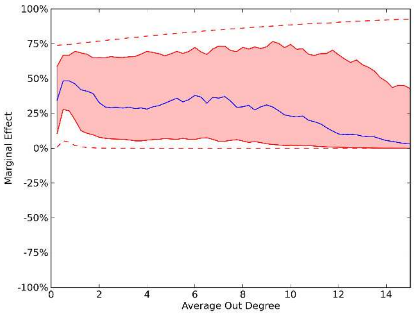
Figure 21: **Extended model - [FIT1] | Marginal Effects - Threshold: 10% (A)**
Panels (a)–(i) show the *marginal effects* for the different parameters: the blue line represents the median marginal effect; the shaded area marks the value of the marginal effect between the 25^{th} and the 75^{th} percentiles; while the upper and lower dashed red lines correspond, respectively, to the maximum and minimum estimated marginal effect for each specific regressor.



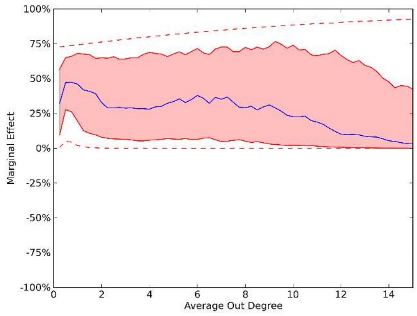
(a) High heterogeneity



(b) Low heterogeneity

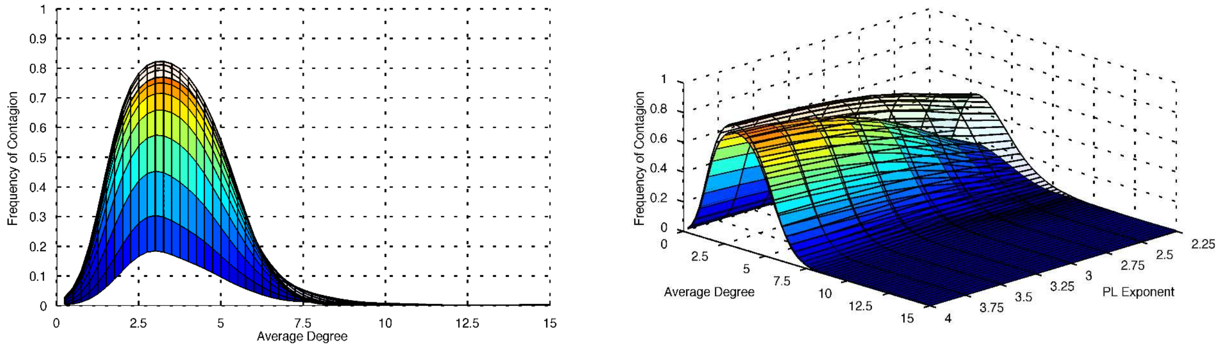


(c) Too-exposed-to-fail

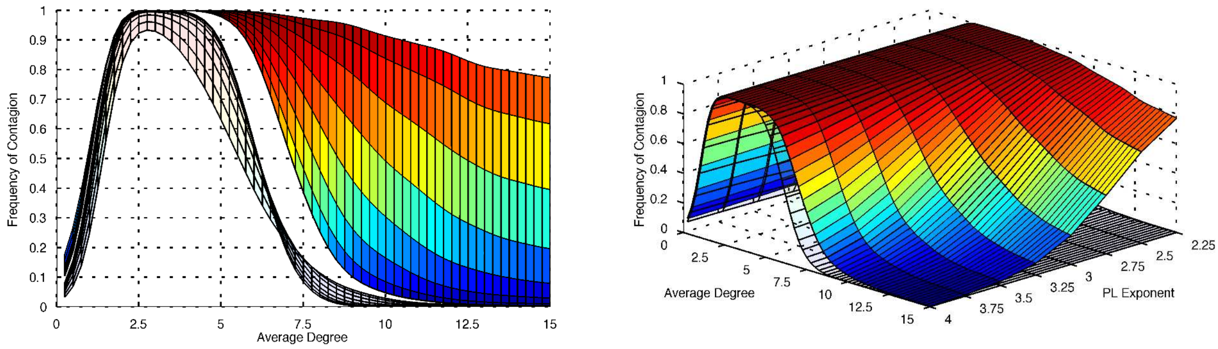


(d) Too-big-to-fail

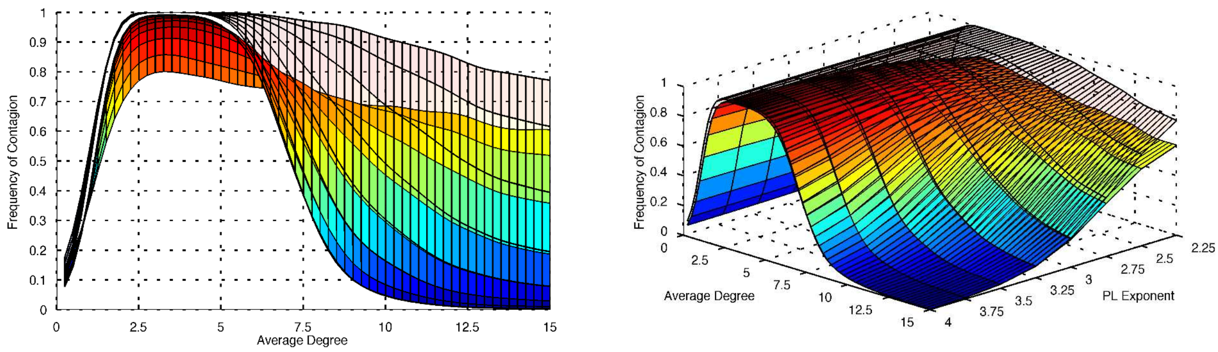
Figure 22: **Extended model - [FIT1] | Marginal Effects - Threshold: 10% (B)**
Panels (a)–(c) show the *marginal effects* for the different parameters: the blue line represents the median marginal effect; the shaded area marks the value of the marginal effect between the 25^{th} and the 75^{th} percentiles; while the upper and lower dashed red lines correspond, respectively, to the maximum and minimum estimated marginal effect for each specific regressor.



(a) Random Default

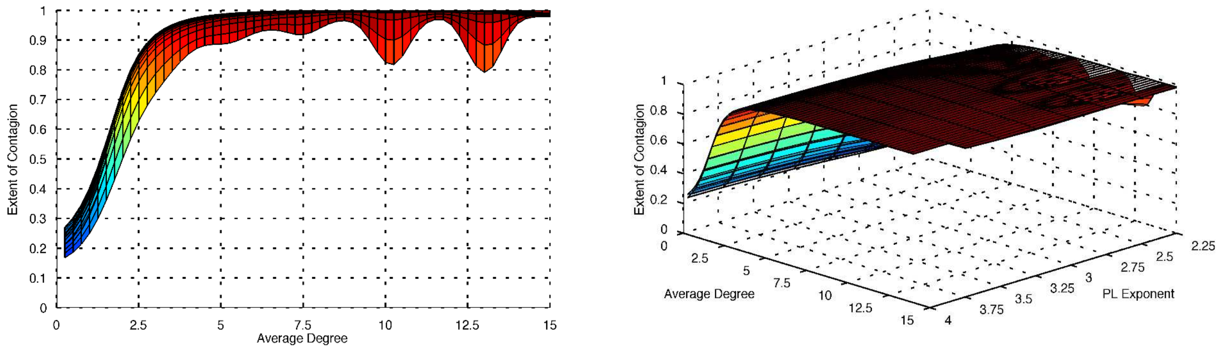


(b) Too-Connected-To-Fail

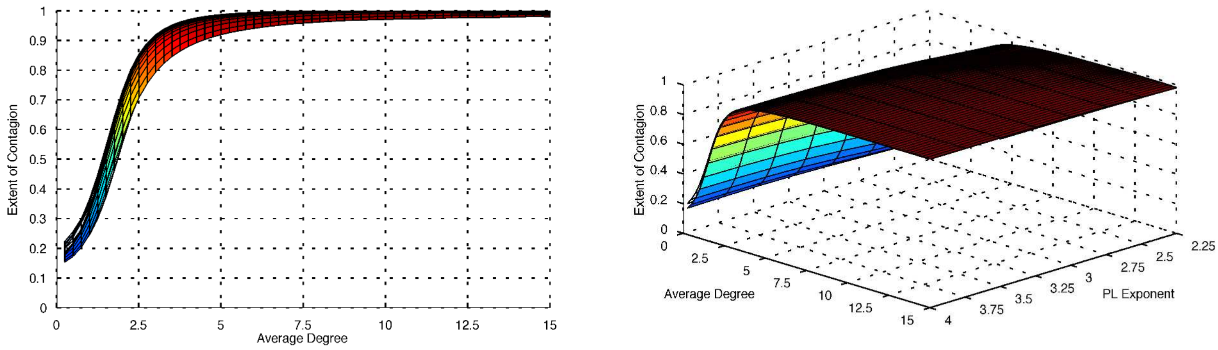


(c) Too-Big-To-Fail

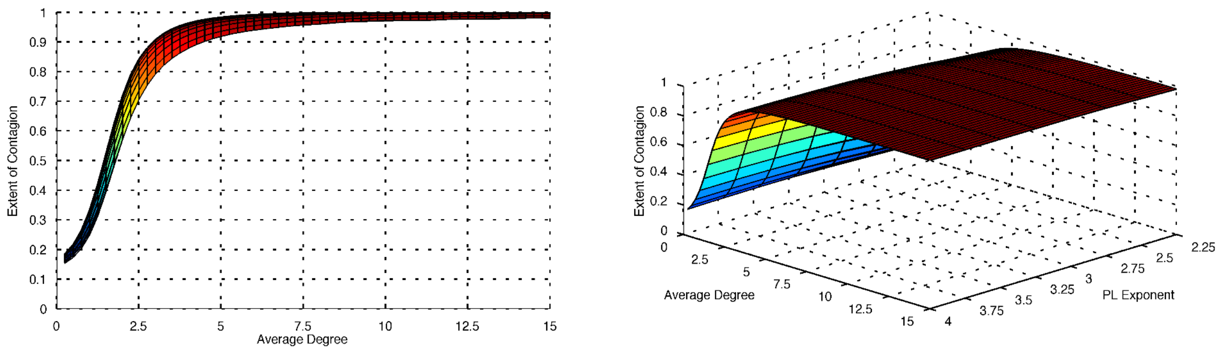
Figure 23: **Benchmark model - [FIT2] | Frequency of Contagion - Threshold: 10%**
Frequency of contagion as a function of the average number of interbank connections of the banks in the system and of the exponent of the power-law distribution used to generate the banks' fitness parameter.



(a) Random Default

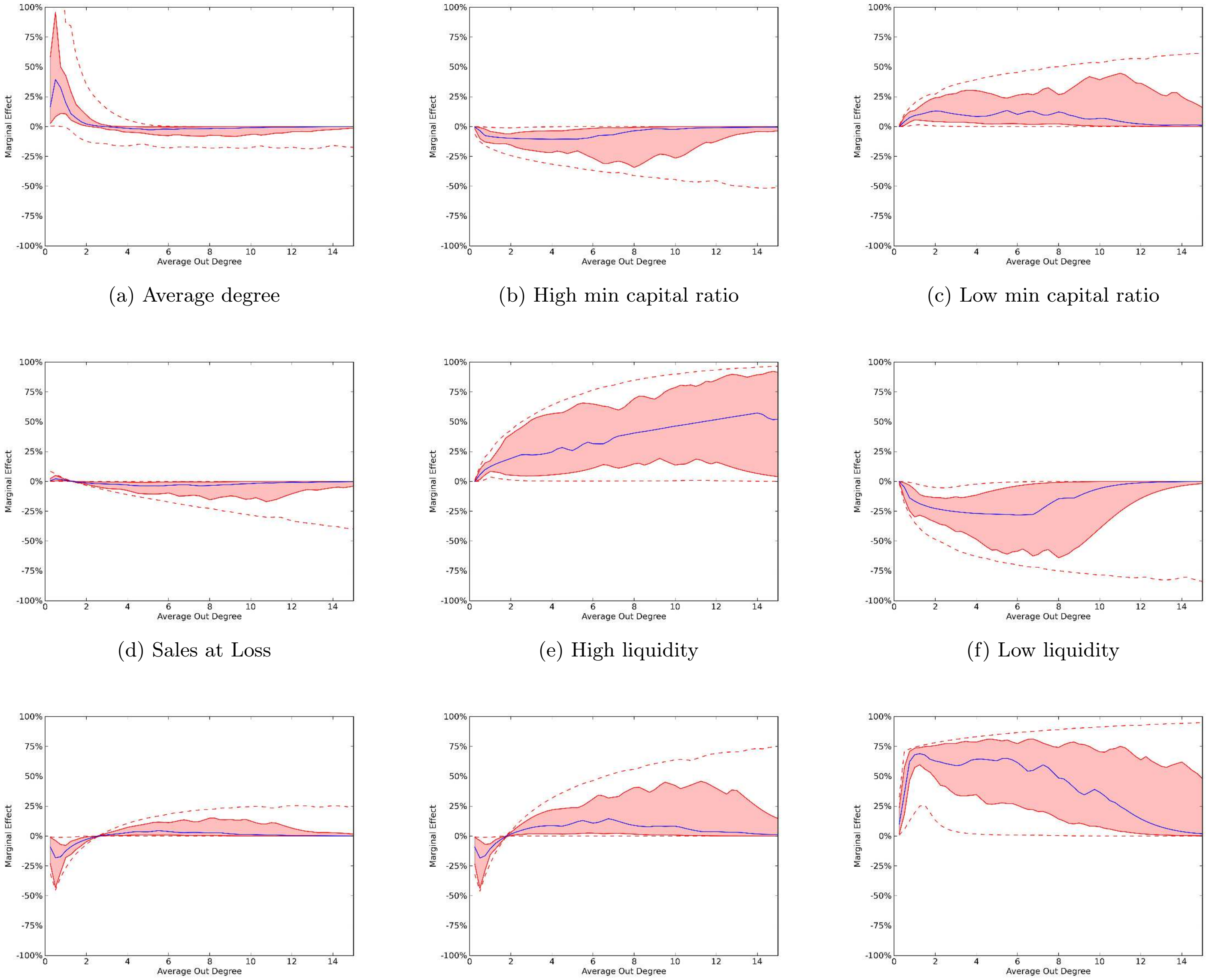


(b) Too-Connected-To-Fail



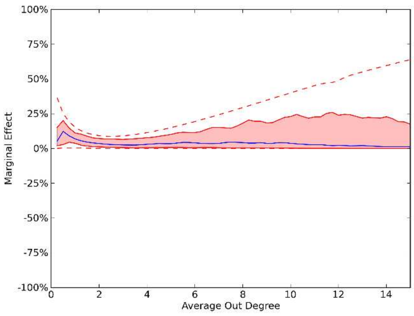
(c) Too-Big-To-Fail

Figure 24: **Benchmark model - [FIT2] | Extent of Contagion - Threshold: 10%**
Extent of contagion as a function of the average number of interbank connections of the banks in the system and of the exponent of the power-law distribution used to generate the banks' fitness parameter.

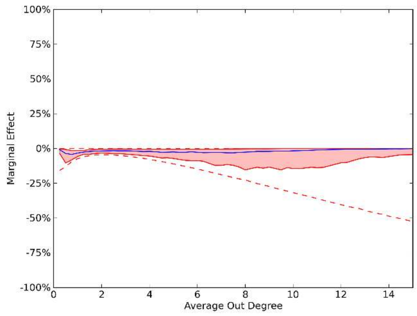


(a) Average degree
(b) High min capital ratio
(c) Low min capital ratio
(d) Sales at Loss
(e) High liquidity
(f) Low liquidity
(g) Large short-term market
(h) Large long-term market
(i) Too-connected-to-fail

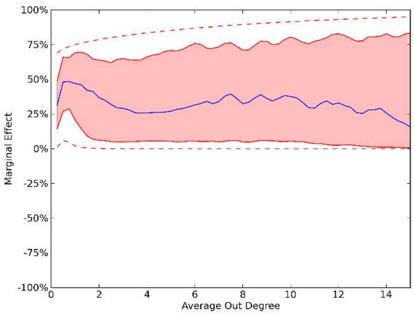
Figure 25: **Extended model - [FIT2] | Marginal Effects - Threshold: 10% (A)**
Panels (a)–(i) show the *marginal effects* for the different parameters: the blue line represents the median marginal effect; the shaded area marks the value of the marginal effect between the 25^{th} and the 75^{th} percentiles; while the upper and lower dashed red lines correspond, respectively, to the maximum and minimum estimated marginal effect for each specific regressor.



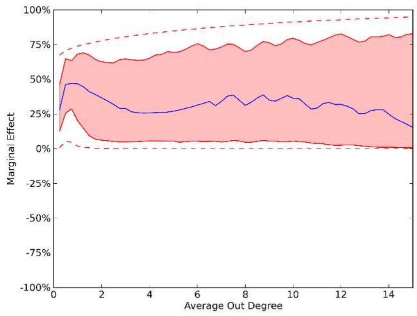
(a) High heterogeneity



(b) Low heterogeneity



(c) Too-exposed-to-fail



(d) Too-big-to-fail

Figure 26: **Extended model - [FIT2] | Marginal Effects - Threshold: 10% (B)**
Panels (a)–(c) show the *marginal effects* for the different parameters: the blue line represents the median marginal effect; the shaded area marks the value of the marginal effect between the 25^{th} and the 75^{th} percentiles; while the upper and lower dashed red lines correspond, respectively, to the maximum and minimum estimated marginal effect for each specific regressor.

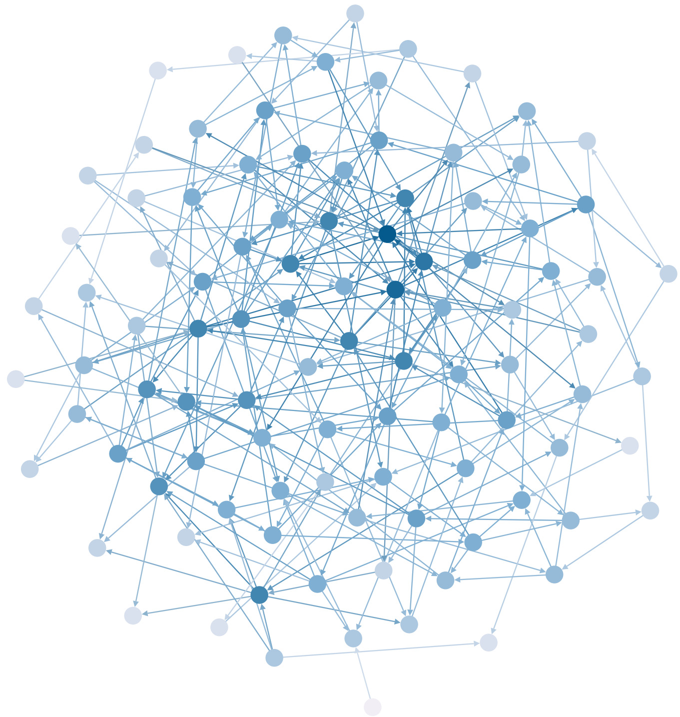


Figure 27: **Example of Erdős and Rényi (1960) random graph**: 100 nodes and average degree of 3.

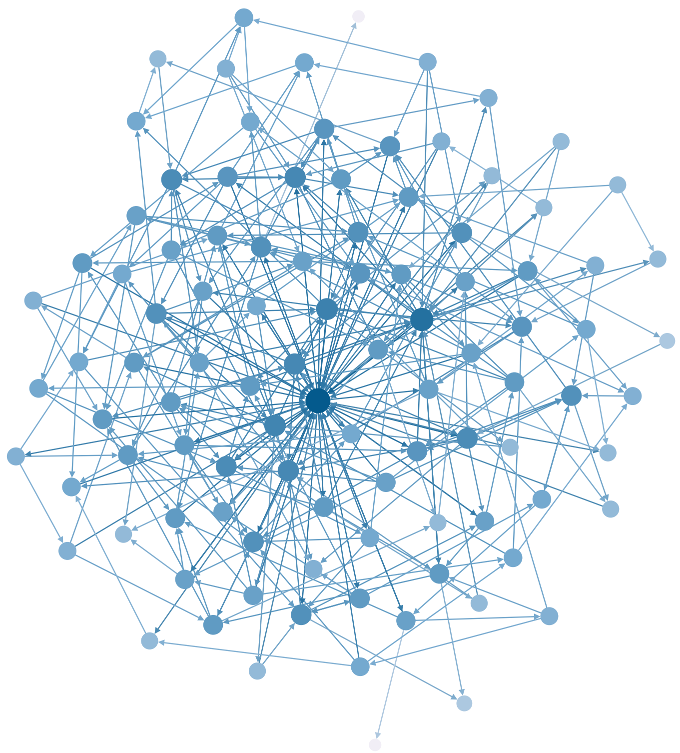


Figure 28: **Example of graph generated with a fitness model**: 100 nodes and average degree of 3. The distribution of the fitness is power-law with exponent 2.5.

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