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Social Learning and Higher Order Beliefs: A Structural Model of Exchange Rates Dynamics

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Abstract

This paper proposes a structural model of exchange rates where agents formulate their one-step ahead predictions based on social learning process and higher order beliefs. Individual choices are then aggregated and plugged into a rather standard macroeconomic model to derive the dynamics of exchange rates. Bayesian estimation of the structural parameters is implemented exploiting Foreign exchange Consensus Survey data of heterogeneous forecasts and fundamentals. Results show that higher order beliefs accounts for a large part of the total value, while public information play the most important role in determining individual expectations. Although the precision of the private signal is larger than the public one, information publicly revealed does exert a disproportionate influence, and differences in the estimated signals determine the equilibrium strategy of each agent as a combination between personal beliefs and higher order expectations.

Keywords: higher order beliefs, exchange rates, economic fundamentals, survey data

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1. Introduction

Are agent's expectations a reasonable reflection of the available information, and if so, do the choices those expectations lead to, drive the equilibrium outcome? Can other investors' choices matter in forming those expectations?

Systematic biases in individual expectations away from the truth have been empirically tested since Lucas (1972). Recent evidences have tried to better comprehend the limitations faced by agents in the acquisition process of information as stressed by Sims (2003) and Woodford (2002), amongst others. A growing body of the recent empirical literature focuses on the role information play in the expectation formation mechanism using macro and micro data. These contributions are based on estimates of the process that drives expectations (see, e.g., Andolfatto et al., 2008 and Del Negro and Eusepi, 2011), as well as investigations of whether survey responses conform to various theories, e.g., sticky information theories (Reis, 2006; Branch, 2007).

It's not yet clear to what extent different information sources are relevant for the equilibrium relationships between economic aggregates and individual choices and if heterogeneity among investors is important. In particular, one untested question is if there is perhaps a role for higher order expectations and which kind of information is important in making the optimal choice.

This paper aims to understand the decision process that takes place in a market where expectations are formed. Rather than blindly follow a simple individual maximization rule, investors work to sort out how choices of other investors match their need. Thus, an average evaluation of individual forecasts among investors is taken into account when individual expectations are derived, since agents rationally choose to learn more about the market behavior.

We focus on the exchange rates market where agents form their expectations by combining attention on economic fundamentals together with subjective knowledge. Bacchetta and Van Wincoop (2006) suggest that private information constitute relevant factors of predictions in the short run, whereas macroeconomic fundamentals play a larger role in the long run. Survey data on individual forecasts appear to be a reliable tool to recover private information for an econometric model. In our setup, we use survey data on subjective expectations on exchange rates associated with information on economic fundamentals to empirically measure how the combination of both public and private information helps agents to build their forecasts. We base our analysis on the Foreign Exchange Consensus Survey by looking at the eur/usd currency from 2006 to 2012. As confirmed by Bellemare and Manski (2011) and Engelberg et al. (2011), the use of expectations in survey has a strict comparative advantage especially for econometric applications. Jongen et al. (2012) exploit survey information on exchange rates in order to disentangle the effect of differences in agents' beliefs. Survey data have also been used by Coibion and Gorodnichenko (2012) to compare the impact of different information rigidities faced by economic agents. They find that, when agents observe noisy signals about the true current inflation, a learning process figures out as the most important determinant in explaining US inflation dynamics. They claim that investors are continuously updating their information set in order to make a correct inference. Different from Bacchetta and Van Wincoop (2006), who assume that uncertainty on exchange rates is due to some lack of knowledge related to future fundamentals, here instead, in line with Coibion and Gorodnichenko (2012), we hypothesize agents do not observe the current exchange rate, but receive noisy signals as in standard process of learning and consequently derives one-step-ahead expectations.[1]

We develop a simple, dynamic framework *à* la Morris and Shin (2002) that highlights interactions between higher order beliefs and learning considering the effect that individual's choices have on other participants. The key role of social learning is to help agents in the decision about her own individual belief, but also to evaluate others' beliefs in an optimal manner. Allowing agents to gather information selectively after observing market behavior has a fundamental effect on individual learning and on the resulting market dynamics. We embed this process of forecasting information into a macroeconomic framework of exchange rates in line with Bacchetta and Van Wincoop (2006).

Our main contribution consists in defining and estimating a theoretical model that describes the expectation process for exchange rates. The structural model we find is casted in a state-space form and estimated through Bayesian techniques. We estimate a social learning process where private and public information strategically interact by observing how the precisions of signals guide the equilibrium strategy. We gauge the relevant information used by agents and disentangle the effects of both public and private signals identifying the weights assigned to each source of information. The discrepancy between the two sources of information is motivated by the effect that higher order beliefs play in determining outcomes.

The analysis provides two main results. First, we find that higher order beliefs accounts for about 82% of the total value. Secondly, public information play the most important role in determining indi-

[1] In financial markets the assumption of noisy signals on risky assets has been used, amongst others, in Diamond and Verrecchia (1981). However, in our framework, it is easy to derive the dynamics of the exchange rates even without the presence of noise terms.

vidual expectations with more than 75% compared to private information. Although the precision of the private signal is larger than the public one, information publicly revealed does exert a disproportionate influence, while differences in the estimated signals drive a wedge between personal choices and higher order expectations-formation process.

A fuller review of related research is postponed until Section 6. To our knowledge, this paper represents the first attempt to estimate a structural learning model on exchange rates expectations demonstrating that public information, and the consequent transparency among investors, are the main features of the optimal process. Our theoretical framework follows in the tradition the literature on social value of information firstly popularized by Morris and Shin (2002) and afterwards analyzed by several contributions, see among others Colombo et al. (2014) and Myatt and Wallace (2014, 2015). The game analyzed has a similar structure to that found in the literature concerned with information sharing (e.g., Vives, 1997; Angeletos et al., 2004, 2007), where the Gaussian-quadratic model and the linear solutions are common hypothesis. This choice represents a simple and flexible tool to link the theoretical part to the empirical structural model. This close relationship allows to derive a reduced form parametrization that allows to get estimates of the parameters in the individual utility function. The role of private information and its relationship in coordinating settings was investigated by Hellwig and Veldkamp (2009) who show that adopting strategic complementarities in actions incentive the role that higher order beliefs play in determining exchange rates. We validate this point observing a reassessment of the information. The insight is that the optimal choices are closed by averagely public signals, although contrary to Morris and Shin (2002, 2005) is never optimal to release just a purely public or a private signal.

The remainder of this paper is as follows: section 2 introduces the theoretical set-up and equilibrium solutions. Section 4 presents the empirical estimation of the structural model, while some comments are proposed on Section 5 about the posterior estimates and policy implications. Section 6 encompasses the relevant literature in the field, plus some concluding remarks.

2. A social learning process and exchange rate dynamics

Let us assume an economy populated by a finite series of predictors, $n = \{1, \ldots, N\}$. In period t, each agent i observes noisy private and public signals about the exchange rate s_t which belongs to a set

$\Psi : s \in \Psi$ and evolves stochastically:

$$s_t = s_{t-1} + \gamma_t \quad \text{where} \quad \gamma_t \sim N(0, \sigma_\gamma^2) \tag{1}$$

where the shock γ_t occurring at the beginning of period t is normally distributed with mean 0, variance σ_γ^2, and precision $\rho_s \equiv \sigma_\gamma^{-2}$.

After the occurrence of a shock, each agent i receives a common public signal about the fundamental f_t as a function of the exchange rate:

$$f_t = s_t + \eta_t \qquad \eta_t \sim N(0, \sigma_\eta^2) \tag{2}$$

and a private personal signal:

$$x_{it} = s_t + \epsilon_{it} \qquad \epsilon_{it} \sim N(0, \sigma_\epsilon^2) \tag{3}$$

So while information about the fundamental f_t is common knowledge among agents, the private signal x_{it} is specific to agent i and not observed by other predictors. The common posterior about s_t, taking into account public information, is normally distributed with mean $\tilde{y}_t = \mathbb{E}_t[s_t|f_t]$ and precision $\rho_{\tilde{y}} = \rho[s_t|f_t]$, such that $\mathbb{E}_t[s_t|f_t] = (\rho_s s_{t-1} + \rho_f f_t) / (\rho_s + \rho_f)$ and $\rho_{\tilde{y}} = \rho[s_t|f_t] = \rho_s + \rho_f$. Private posteriors are gaussian and identified by mean $\mathbb{E}_t^i[s_t|f_t; x_{it}] = (\rho_{\tilde{y}}\tilde{y}_t + \rho_x x_{it}) / (\rho_{\tilde{y}} + \rho_x)$ and precision $\rho[s_t|f_t; x_{it}] = \rho_{\tilde{y}} + \rho_x$. The weight of the public signal in the Bayesian projection s on the information set $H_i(t) = \{f_t; x_{it}\}$ is $\alpha_{\tilde{y}} = \rho_{\tilde{y}} / (\rho_{\tilde{y}} + \rho_x)$, while the weight of the private signal is $\alpha_{x_i} = \rho_x / (\rho_{\tilde{y}} + \rho_x)$. The posterior mean for each agent i is then derived, i.e., $\mathbb{E}_t^i[s_t|f_t; x_{it}] = \alpha_{x_i} x_{it} + \alpha_{\tilde{y}}\tilde{y}_t$. Thus, based on public and private signals at time t, the expected prediction for s_{t+1} is easily obtained through the recursion in eq. (1), i.e. $\mathbb{E}_t^i[s_{t+1}] = \mathbb{E}_t^i[s_t|x_{it}; f_t]$.[2]

We denote the individual expectation of subject i at time t, i.e., $e_{it} = \mathbb{E}_t^i[s_{t+1}]$, while $\bar{e}_t \equiv \int_j e_{jt}(.)dj$ and $\sigma_e^2 \equiv \int_j [e_{it}(.) - \bar{e}_t(.)]^2 dj$ are respectively the average or consensus, and the dispersion of investors' expected evaluations in the economy. Preferences of agents are explicitly characterized by a concave increasing function $U(e_{it}, \bar{e}_t, \sigma_e^2, s_{t+1})$. As generally as possible, we assume that the dispersion σ_e has only a second-order non-strategic effect, i.e., $U_{e\sigma} = U_{s\sigma} = 0$, while $U_\sigma(e_{it}, \bar{e}_t, 0, s_{t+1}) = 0$, $\forall e_{it}; \bar{e}_t; s_{t+1}$. Under perfect information about the exchange rate s_{t+1}, due to symmetry ($e_{it}(.) = \bar{e}_t(.) = s_{t+1}$, $\forall i$),

[2] This is customary in the Kalman filter one-step-ahead prediction step, as stressed by Coibion and Gorodnichenko (2012).

the best response is given by the unique equilibrium characteristics where the predictors' choice exactly coincides with their expectation. In case of imperfect information, in contrast, optimality is required for any $(x_{it}; f_t)$ in the predictor's choice. For a finite number of investors (as in Marinovic et al., 2011), the individual's expected utility assumes the following form:

$$U(e_{it}, \bar{e}_t, \sigma_e^2, s_{t+1}) = -(1-\delta)(e_{it} - s_{t+1})^2 - \delta(e_{it} - \bar{e}_t)^2 \tag{4}$$

The first component is a quadratic loss in the distance between the expectation and the actual exchange rate, while the second component is a quadratic loss in the distance between the expectation and the consensus. Each predictor aims to minimize the expected distance between their evaluation and the average. The parameter $\delta \in (0, 1)$ is a scalar that describes the intensity of the coordination motive, i.e. the importance that agent i attaches to the expectations of other market predictors.

More intuitively, eq. (4) describes the predictor's decision-making process in terms of her choice between two incentives which constitute the reward rule assigned to agent's forecast success. The first incentive induces the agent to anchor her predictions on the fundamentals. It relies on the distance between the expectation of the agent and the one-step-ahead spot, and represents the cost of the forecast error, i.e. the cost of making a mistake with respect to the fundamental. The second incentive, in contrast, captures the cost of diverging from the consensus prediction. This is the factor related to the presence of higher order beliefs and whose weight is expressed by the parameter δ. The quadratic specification of the utility function ensures the linearity of the predictors' best responses and efficient allocations. To keep the algebra simple, we assume the best prediction of each agent is based on current information, that is, x_{it} and f_t, even though, a generalization that includes past information can be easily derived, as suggested by Coibion and Gorodnichenko (2012).

Solving for e_{it}, we obtain that:

$$e_{it}(x_{it}; f_t; \rho_{\tilde{y}}; \rho_x) = (1-\delta)\mathbb{E}_t^i[s_{t+1}|x_{it}; f_t; \rho_{\tilde{y}}; \rho_x] + \delta\mathbb{E}_t^i[\bar{e}_t|x_{it}; f_t; \rho_{\tilde{y}}; \rho_x] \tag{5}$$

which can be rewritten as:

$$e_{it}(x_i; f; \rho_{\tilde{y}}; \rho_x) = (1-\delta)\mathbb{E}_t^i[s_{t+1}|x_{it}; f_t; \rho_{\tilde{y}}; \rho_x] + \delta\frac{e_{it}}{n} + \delta\frac{n-1}{n}\mathbb{E}_t^i[e_{-it}|x_{it}; f_t; \rho_{\tilde{y}}; \rho_x] \tag{6}$$

where $\mathbb{E}_t^i[e_{-it}|x_{it}; f_t; \rho_{\tilde{y}}; \rho_x] = \mathbb{E}_t^i[(\frac{e_{1t}+...+e_{i-1t}+e_{i+1t}+...e_{nt}}{n})|x_{it}; f_t; \rho_{\tilde{y}}; \rho_x]$. In the unique equilibrium with heterogeneous information, each individual $i \neq j$ at time t follows a linear strategy as:

$$e_{it}(x_{it}; f_t; \rho_{\tilde{y}}; \rho_x) = \varphi_x x_{it} + \varphi_{\tilde{y}} \tilde{y}_t \tag{7}$$

According to this strategy, the predictor's expectation about the other $(n-1)$ agents is linear in $(f_t; s_{t+1})$ and is given by:

$$\mathbb{E}_t^i[e_{-it}|x_{it}; f_t; \rho_{\tilde{y}}; \rho_x] = \varphi_x \mathbb{E}_t^i[x_{-it}] + \varphi_{\tilde{y}} \tilde{y}_t$$

then according to eq. (3), $\mathbb{E}_t^i[x_{-it}] = \mathbb{E}_t^i[s_t + \epsilon_{-it}]$. Moreover since $\epsilon_{it} \sim N(0, \sigma_\epsilon^2)$ and using it into eq. (1), $\mathbb{E}_t^i[s_t + \epsilon_{-it}] = \mathbb{E}_t^i[s_{t+1}]$. Therefore,

$$\mathbb{E}_t^i[e_{-it}|x_{it}; f_t; \rho_{\tilde{y}}; \rho_x] = \varphi_x \mathbb{E}_t^i[s_{t+1}] + \varphi_{\tilde{y}} \tilde{y}_t \tag{8}$$

Plugging eq. (8) and eq. (7) into eq. (6),

$$e_{it} = (1-\delta)\mathbb{E}_t^i[s_{t+1}|x_{it}; f_t; \rho_{\tilde{y}}; \rho_x] + \delta \frac{e_{it}}{n} + \delta \frac{n-1}{n} \mathbb{E}_t^i[\varphi_x s_{t+1} + \varphi_{\tilde{y}} \tilde{y}_t | x_{it}; f_t; \rho_{\tilde{y}}; \rho_x] \tag{9}$$

while rearranging it, we obtain:

$$e_{it} = (1 - \varrho + \varrho \varphi_x) \left[\frac{\rho_x}{\rho_{\tilde{y}} + \rho_x} x_{it} + \frac{\rho_{\tilde{y}}}{\rho_{\tilde{y}} + \rho_x} \tilde{y}_t \right] + \varrho \varphi_{\tilde{y}} \tilde{y}_t \tag{10}$$

where $\varrho = \frac{n\delta - \delta}{n - \delta}$. According to eq. (7), the coefficients $(\varphi_x; \varphi_{\tilde{y}})$ for the optimal linear strategy must therefore satisfy:

$$\varphi_x = \frac{(1-\varrho)\rho_x}{(1-\varrho)\rho_x + \rho_{\tilde{y}}} \quad \text{and} \quad \varphi_{\tilde{y}} = \frac{\rho_{\tilde{y}}}{(1-\varrho)\rho_x + \rho_{\tilde{y}}} \tag{11}$$

as the unique solution of the system. Therefore, the optimal solution of the social learning game relies on the individual expectation about the next-period exchange as:

$$\mathbb{E}_t^i(s_{t+1}) = \varphi_x x_{it} + \varphi_y \tilde{y}_t, \tag{12}$$

where the sensitivity of the predictor's expectations to exchange rates is driven by two factors. First, the weight of the beauty contest factor, i.e. δ, identifies the importance attached to the expectations of other predictors. Note that when $\delta = 0$, the predictor's optimal choice coincides with her personal expectation.

Higher values of δ induce the agent to take mainly into account public sources of information when making her own prediction. Second, the sensitivity of the predictor's expectations to the exchange rate depends on the quality of private and public signals in terms of precision. Agents assign lower weights to private signal while public source acts as a coordinating mechanism for prediction of others' action.

Following Bacchetta and Van Wincoop (2006), the aggregate expectation is implemented in a standard equilibrium model for exchange rates with heterogeneous agents, such that:

$$s_t = \lambda \bar{\mathbb{E}}_t[s_{t+1}] + (1-\lambda) f_t - \lambda \psi_t \tag{13}$$

in which α is the parameter describing interest rates in equilibrium, $\lambda = \alpha/(1+\alpha)$, ψ_t is the liquidity premium whereas the observable fundamental f_t follows

$$f_t = m_t - m_t^* - \phi(y_t - y_t^*), \tag{14}$$

where m_t and m_t^* are the logs-money supplies, while y_t and y_t^* are the logs-output levels respectively for the home and the foreign countries.[3] The combination of learning process in a macroeconomic setup allows to easily derive a statistical model necessary to make inference on utility function's parameters. Equation (13) explains how the current exchange rate is related in a simple way to the heterogeneous expectations of investors, $\bar{\mathbb{E}}_t[s_{t+1}]$, a commonly observed fundamental, f_t and the value of liquidity trade, ψ_t.[4]

Therefore, according to the individual solution of social learning game, eq. (12), we aggregate the individual predictions of n investors as,

$$\bar{\mathbb{E}}_t[s_{t+1}] = \varphi_x \bar{x}_t + \varphi_y \tilde{y}_t, \tag{15}$$

while, by substituting it in eq.(13)

$$s_t = \lambda(\varphi_x \bar{x}_t + \varphi_{\tilde{y}} \tilde{y}) + (1-\lambda) f_t - \lambda \psi_t. \tag{16}$$

[3] The steps to derive equation (13) are available in AppendixA.

[4] Note that the exchange rate proposed in eq. (13) is one of the possible solutions in macroeconomic modeling. Alternative aggregations can be proposed and estimated upon requests.

and rearranging, we get:

$$s_t = (1 - \lambda + \lambda\tau_2\varphi_{\tilde{y}})f_t + \lambda\varphi_x\bar{x}_t + \lambda\tau_1\varphi_{\tilde{y}}s_{t-1} - \lambda\psi_t, \tag{17}$$

with $\tau_1 = \frac{\rho_s}{\rho_s+\rho_f}$ and $\tau_2 = \frac{\rho_f}{\rho_s+\rho_f}$ which can be reduced to

$$s_t = \beta_1 f_t + \beta_2\bar{x}_t + \beta_3 s_{t-1} + \beta_4\psi_t, \tag{18}$$

where $\beta_1 = (1 - \lambda + \lambda\tau_2\varphi_{\tilde{y}})$, $\beta_2 = \lambda\varphi_x$, $\beta_3 = \lambda\tau_1\varphi_{\tilde{y}}$ and $\beta_4 = -\lambda$. The first term indicates the role of fundamentals in the determination of exchange rate, the second term indicates the role of the strategic interaction of higher order beliefs, the third term represents the role of persistence, and the fourth term, finally, represents the role of liquidity trade[5].

3. Foreign exchange Consensus Survey data

We consider data on expectations obtained from the Foreign Exchange Consensus Forecasts ($FECF$) survey by *Consensus Economic of London*. This dataset has been recently analyzed in Fratzscher et al. (2015) and in Jongen et al. (2012).

In this survey, at the second Monday of each month, panelists are asked to forecast spot rates at different maturities. The sample is composed by almost 250 panelists spread all over the world and 40 of them are personally identifiable with their names. Some panelists provide systematically predictions in each publication, while some others appear with a lower frequency. There are also cases in which the panelist is included in the list although its prediction is not indicated. This is depicted by the term, *na*, in its forecast value. Albeit the survey refers to different currencies, we focus on US$ vs euro one-month-ahead forecasts consisting of 78 observations per agent from January 2006 to June 2012.

Our analysis is conducted by taking into account individual forecasts, i.e., forecasts reported by personally identifiable panel members in the publication. The average expectation of all members is also reported and indicated by *consensus forecast*.

We built our dataset as follows. First we collected all individual forecasts from 2006 to 2012 and we recorded only forecasts from panelists with a response rate higher than 40 percent. We selected 15

[5]Note that it is also possible to derive a similar dynamics in case s_t is measured without noise

institutions, i.e., 15 time series of expectations. We observed that 9 of them have just from 0 to 5 missing data and only in one case we noticed a response rate smaller than 50 percent. On average, the response rate for the US$ vs euro one-month-ahead forecasts is around 90 percent. It is worth noting that for each month we observe at least 11 individual expectations, and the average number of respondents per month is larger than 13, i.e., a value sufficient for cross sectional heterogeneity among forecasters. Figure 1 evidences that average one-month-ahead forecasts approximates fairly well actual exchange rates. As stressed by Jongen et al. (2012) expectations are dispersed, thus indicating heterogeneity among panelists. In particular, lower panel of Figure 1 show that dispersion is relatively moderate from January 2006 to September 2007, then it increases until reaching a peak in January 2009 and finally decline and stabilizes from November 2009 to June 2012.

Figure 1: Upper panel: Actual vs average forecasts together with a 95% confidence interval from january 2006 to june 2012. Lower panel: Estimated dispersion over time

Regarding the representativeness of panelists taken into account, it is worth noting that some of them represent major dealing banks, whose names are reported in Table 1. In particular, 8 out of 15 institutions are included in the Top 10 currency traders of the Forex market and represent about 70 percent of market share of exchange rates. Furthermore, some of these institutions provide trading platforms to smaller banks. This procedure is called *white labeling* and is highly efficient for market dynamics, although it induces a concentration of information. In fact, large banks can observe directly small banks' trading flows and extract from these data possibly relevant information at lower costs (King et al., 2012). The list of panelists, the actual sample size and their relevance in term of market size are described in Table 1.

Table 1: Predictions of individual forecasters: missing observations and percentage on the total number of observations in the sample. Last two columns: companies with proprietary trading platforms and if they are in top 10 currency traders list.

	n missing	% on total	white label	Top 10
Bank of Tokio - Mitsubushi	1	1		
Barclays Capital	31	40	yes	yes
BNP Paribas	7	9		yes
BoA - Merril Linch	4	5		yes
Citigroup	45	57	yes	yes
Commerzbank	4	5		
Deutsche Bank Research	18	23	yes	yes
General Motors	0	0		
HSBC	1	1	yes	yes
IHS Global Insight	0	0		
J.P. Morgan	10	12	yes	yes
Oxford Economics	7	9		
Royal Bank of Canada	0	0		
UBS	3	4	yes	yes
WestLB	2	4		

4. Empirical Model

4.1. Methods and Data

We consider a state-space model to describe the economic environment as a combination of learning mechanism of Ottaviani and Sørensen (2006) and the equilibrium model for exchange rates à la Bacchetta and Van Wincoop (2006). We define our empirical strategy to closely mimic the theoretical framework described in Section 2. Our empirical model reads as follows:

$$
\begin{aligned}
s_t &= \beta_1 f_t + \beta_2 \bar{x}_t + \beta_3 s_{t-1} + \beta_4 \psi_t + \epsilon_{s,t} && (19)\\
f_t &= \alpha_0 + \alpha_1 f_{1,t} + \alpha_2 f_{2,t} + \epsilon_{f,t} && (20)\\
f_{1,t} &= \phi_{01} + \phi_{11} f_{1,t-1} + \phi_{12} f_{2,t-1} + \epsilon_{f_1,t} && (21)\\
f_{2,t} &= \phi_{02} + \phi_{21} f_{1,t-1} + \phi_{22} f_{2,t-1} + \epsilon_{f_2,t} && (22)\\
\tilde{y}_t &= \frac{\rho_f}{\rho_f + \rho_s} f_t + \left(1 - \frac{\rho_f}{\rho_f + \rho_s}\right) s_{t-1} && (23)\\
\psi_t &= \rho_\psi \psi_{t-1} + \epsilon_{\psi,t} && (24)\\
\mathbb{E}_t^i[s_{t+1}] &= \varphi_{\tilde{y}} \tilde{y}_t + \varphi_x x_{it}, \qquad i = 1, \ldots, N && (25)\\
x_{it} &= x_{it-1} + \epsilon_{x_i,t}, \qquad i = 1, \ldots, N && (26)
\end{aligned}
$$

Equation (19) is the empirical counterpart of eq. (18) that describes in equilibrium the dynamics of exchange rates as a function of private information, i.e., $\bar{x}_t$, of economic fundamentals f_t and past exchange rates. It is worth noting that the coefficients β_i are explicit functions of the structural parameters. Equations (20) to (22) defines f_t as a linear combination of two observable fundamentals in line with eq. (14). In particular, $f_{1,t} = m_t - m_t^*$ and $f_{2,t} = y_t - y_t^*$, are the differential of the logs-money supplies, and the differential of logs-output levels respectively for the home and the foreign countries. To be consistent with Bacchetta and Van Wincoop (2006), we assume $f_{i,t}$ as random walks, by setting $\phi_{11} = \phi_{22} = 1$ and $\phi_{21} = \phi_{12} = 0$. This assumption could be relaxed in principle although it is reasonable in this setup due to the non-stationary nature of the exchange rates, their expectations and most of their determinants (Engel and West, 2005). Furthermore, eq. (23) represents the dynamics of the public information defined as a convex combination of past exchange rates and fundamentals as derived in Section 2. In turn, eq. (25) identifies the mechanism that forms individual expectations as a mixed effect of private and public information, weighted respectively by φ_x and $\varphi_{\tilde{y}}$ defined in eq. (11). The process ψ_t is an autoregressive process. However, in our empirical exercise, we set $\rho_\psi = 0$ to be consistent with the theoretical setup of Bacchetta and Van Wincoop (2006). We assume the dynamics of x_{it} in Eq. (26) are non observable random walks in order to model the expected high persistence in the subjective private information.

The shocks $\epsilon_t = (\epsilon_{s,t}, \epsilon_{f,t}, \epsilon_{f_1,t}, \epsilon_{f_2,t}, \epsilon_{\psi,t}, \epsilon_{x_i,t})$, $i = 1, \ldots, N$ are all Gaussian with mean zero and standard deviation, respectively, $\sigma_s, \sigma_f, \sigma_{f_1}, \sigma_{f_2}, \sigma_\psi$ and σ_{x_i}, whereas N is the number of informed agents that make predictions on exchange rates.

The model defined in eqs. (19-26) can be rewritten in compact form as

$$\Gamma_0 \mathbf{x}_t = c_x + \Gamma_1 \mathbf{x}_{t-1} + \Gamma_\epsilon \boldsymbol{\epsilon}_t \tag{27}$$

and in particular $\mathbf{x}_t = \left(s_t, f_t, f_{1,t}, f_{2,t}, \tilde{y}_t, \psi_t, \mathbb{E}_t^i, x_{it}\right)$, $i = 1, \ldots, N$, while Γ_0, Γ_1 and Γ_ϵ are appropriate square matrices of parameters that define the system (19-26). By pre-multiplying eq. (27) with Γ_0^{-1} we get

$$\mathbf{x}_t = \Theta_c + \Theta_x \mathbf{x}_{t-1} + \Theta_\epsilon \boldsymbol{\epsilon}_t. \tag{28}$$

Some of the variables described through eq. (27) are potentially unobservable. For our empirical analysis, we consider as observables the current exchange rates, s_t, the expectations $\mathbb{E}_t^i[s_{t+1}]$ which are represented by our dataset on heterogeneous survey forecasts concerning the actual exchange rates and the two fundamentals $f_{i,t},\ i = 1, 2$, that is, $\hat{\mathbf{y}}_t = (\hat{s}_t, \hat{f}_{j,t}, \hat{\mathbb{E}}_t^i[s_{t+1}])$, with $j = 1, 2$ and $i = 1, \ldots, N$.[6]

We consider observable expectations $\mathbb{E}_t^i[s_{t+1}]$ for $N = 15$ institutions which represent the most influential companies providing predictions for exchange rates in the whole market as stressed in Section 3.

Data on macroeconomic fundamentals have been obtained from Datastream. The money supply for the two countries is measured by the variable $M2$ at a monthly frequency. Regarding the output of the two countries, quarterly data on GDP were disaggregated to a monthly frequency using the methodology described by Proietti (2006). Following Golinelli and Parigi (2008), as the leading indicators, we use long-term interest rates (per cent per annum), harmonized unemployment rates, retail trade and industrial production. Finally, exchange rates have been observed the *business* day before the survey was conducted as suggested by Fratzscher et al. (2015).

We thus consider the following measurement equations to link our theoretical model to the real-world

[6]We use the symbol ˆ to distinguish observed from theoretical variables.

economy:

$$\hat{\mathbf{y}}_t = S\mathbf{x}_t \tag{29}$$

where S is a selection matrix that links the actual data set to the macroeconomic structure. Equations (28-29) represent a linear and Gaussian state-space system for which the likelihood can be computed in closed form through the Kalman filter. In particular, eq. (28), called transition equation, relies on the latent structure of the model, while (29) is the so called measurement equation. It is worth noting that our database on subjective forecasts is affected by missing values. This is not a significant problem since the Kalman filter predicts missing data and allows for the computation of the likelihood function in a natural way (see Koopman et al., 1999 for a treatment on this point).

In order to deal with non-stationary observed data, it is appropriate to use first differences. To take this transformation into account, the model can easily be redefined as follows:

$$\Delta\hat{\mathbf{y}}_t \equiv \begin{bmatrix} \Delta\hat{s}_t \\ \Delta\hat{f}_{1,t} \\ \Delta\hat{f}_{2,t} \\ \Delta\hat{\mathbb{E}}_t^i[s_{t+1}] \end{bmatrix} + \begin{bmatrix} s_t - s_{t-1} \\ f_{1,t} - f_{1,t-1} \\ f_{2,t} - f_{2,t-1} \\ \mathbb{E}_t^i[s_{t+1}] - \mathbb{E}_{t-1}^i[s_t] \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \tilde{\gamma}_i \end{bmatrix}, \ i = 1, \ldots, N \tag{30}$$

in which we also added some Gaussian measurement errors $\tilde{\gamma}_i$ with standard deviation σ_{E_i} that might affect the observables. Notice that in eq. (30) we also include the lags in the observed variables which are not specified in the vector $\mathbf{x}_t$. To fix the problem it is possible to generalize the vector of the variables of the model as follows:

$$\tilde{\boldsymbol{x}}_t = \left(s_t, f_t, f_{1,t}, f_{2,t}, \tilde{y}_t, \psi_t, \mathbb{E}_t^i[s_{t+1}], x_{it}, s_{t-1}, f_{1,t-1}, f_{2,t-1}, \mathbb{E}_{t-1}^i[s_t]\right), \quad i = 1, \ldots, 15 \tag{31}$$

to finally obtain the reduced form

$$\begin{aligned} \Delta\hat{\mathbf{y}}_t &= \tilde{S}\tilde{\boldsymbol{x}}_t + \tilde{\boldsymbol{\gamma}}_t \\ \tilde{\boldsymbol{x}}_t &= \tilde{\Theta}_c + \tilde{\Theta}_x\tilde{\boldsymbol{x}}_{t-1} + \tilde{\Theta}_\epsilon\tilde{\boldsymbol{\epsilon}}_t. \end{aligned} \tag{32}$$

4.2. Prior distributions and inferential methods

To make inference on structural parameters, we recur to Bayesian estimation methods here, and in particular to Markov chain Monte Carlo algorithms[7] ($MCMC$), which have proved to be successful in the empirical macroeconomic literature (Kim and Pagan, 1995; Canova, 2007). In particular, as standard practice for DSGE models (An and Schorfheide, 2007), we update the structural parameters through a Random Walk Metropolis Hastings algorithm and then, for each draw, we compute likelihood and acceptance probabilities using the state-space representation of eq. (32).

Our first interest is to capture the effect of higher order beliefs on the dynamics of the rate. This is identified by the weight δ in the decision process of the individual predictor. Our second task is to measure the role of private and public information to determine the actual expectation. We need to explore the coefficients φ_x and $\varphi_{\tilde{y}}$ obtained from eq. (11).

In the theoretical model of Section 2, we show that the coefficient φ_x measures the relevance of private information in the formation process of expectations, while, $\varphi_{\tilde{y}}$ indicates the relevance of public information. There is also an influence of the value of δ on the dimensions of φ_x and $\varphi_{\tilde{y}}$. The larger the value of δ, the greater the weight associated to the public signal with respect to the private one. Our prior choices on the structural parameters are summarized in Table 2 [8] . Overall, we considered prior densities that match the domain of the structural parameters. In particular, we select a prior distribution for δ with average 0.5 (and standard deviation 0.1), consequently assigning an equal weight to the two incentives present in the decision-making function of our predictors (eq. 4).

A priori, we assume that public and private information play the same role when agents form their own expectations, i.e., without forcing the model to privilege certain sources of information. This guess is consistent with the hypothesis that φ_x and $\varphi_{\tilde{y}}$ are equal. Since these weights depend on the precision coefficients ρ_f, ρ_s and ρ_x, we need to find prior distributions for them that at least on average, give $\mathrm{E}[\varphi_x] = \mathrm{E}[\varphi_{\tilde{y}}] = 0.5$. To obtain this result, we set the prior distributions for ρ_f and ρ_s as Gamma with mean 1 and standard deviation 0.1, whereas ρ_x is still Gamma, but with larger expected value, namely, 4 and standard deviation 0.4.[9] The discount factor λ is a Beta variable with mean 0.5 and standard deviation 0.1. Furthermore, we assume a weakly informative prior for α_1 and α_2 that are both Gaussian

[7]See Robert and Casella (1999, ch. 6-7) for a general treatment on MCMC algorithms and Monte Carlo methods in general.

[8]Measurement errors and private information's precision prior choices are summarized in Tables B.3 and B.4 in AppendixB.

[9]An extensive sensitivity analysis suggests that posterior estimates of φ_x and $\varphi_{\tilde{y}}$ are robust with respect to this choice.

with mean 0 and rather large variance with respect to the mean, i.e., 1. Finally the standard deviations of the shocks, including standard deviations of the measurement errors, are relatively dispersed. Their standard deviations in particular are quite large with respect to the corresponding expected values. They are Inverse Gamma variables with mean 0.6 and standard deviation 0.2.

5. Posterior estimates and policy implications

All computations are based on software written using the Ox© 7.0 language of Doornik (2001) combined with the state space library `ssfpack` of Koopman et al. (1999). Posterior estimates were obtained by running 200,000 iterations of the *MCMC* algorithm with a burn-in of 10,000, which is a sufficient number of iterations to remove dependence on initial conditions. As standard practice in macro-econometrics, initial conditions were obtained by maximizing the posterior mode for the parameters. Results are summarized in Table 2 and in Figure 2. Other parameter's estimates are reported in Table B.3 and B.4 in

İn particular, Table 2 includes posterior estimates of the structural relevant parameters, namely posterior averages and credibility intervals, whereas Figure 2 displays prior versus posterior comparisons. Figure 2 shows the comparison between prior and posterior distributions. Overall the two sets of distributions differ substantially, thus suggesting that the contribution of the data/likelihood is relevant and the relevance of the prior assumptions do not drive the posterior results[10].

The first interesting result relates to the value of the coefficient δ. A noticeable shift to the right of the posterior is observed in the comparison with its prior distribution, thus confirming the important role of the *beauty contest* mechanism in the predictor's evaluation process. Individuals assign a higher weight than expected (82%) to correctly interpret other predictors' beliefs and a lower weight (18%) to the cost of making forecast errors with respect to the fundamental. The rational incentives of predictors are therefore distorted. The weight assigned to consensus forecasts is definitely higher than the option of making the right choice on the basis of their own private information.

A second important factor is the role that public and private information play in individual forecasts. Our analysis is based on the coefficients φ_x and $\varphi_{\tilde{y}}$. Results suggest that public information accounts for about 75% of predictions, whereas just 25% are based on private information, see Table 2 for details. This is coherent with the previous theoretical results related to the weights associated with higher order beliefs. The combination of higher order beliefs and information structure ensures relatively rational behavior in

[10]This evidence is supported also by many robustness checks available upon request

Table 2: **Posterior computation (MCMC) - Structural parameters**

	Posterior distribution			Prior information		
	Parameter value	Mean	95% Cred. Int.	Mean	S.E.	Type
$p(\beta_1\|\hat{\boldsymbol{y}})$	$\beta_1 = 1 - \lambda + \lambda\varphi_{\tilde{y}}\frac{\rho_f}{\rho_f+\rho_s}$	0.2302	[0.170, 0.301]			
$p(\beta_2\|\hat{\boldsymbol{y}})$	$\beta_2 = \lambda\varphi_x$	0.6489	[0.559, 0.724]			
$p(\beta_3\|\hat{\boldsymbol{y}})$	$\beta_3 = \lambda\varphi_{\tilde{y}}\frac{\rho_s}{\rho_f+\rho_s}$	0.1209	[0.070, 0.177]			
$p(\beta_4\|\hat{\boldsymbol{y}})$	$\beta_4 = -\lambda$	-0.8561	[-0.916, -0.779]			
$p(\alpha_1\|\hat{\boldsymbol{y}})$		0.0842	[-0.716, 0.851]	0	1	Normal
$p(\alpha_2\|\hat{\boldsymbol{y}})$		-2.5237	[-3.724, -1.289]	0	1	Normal
$p(\rho_f\|\hat{\boldsymbol{y}})$		0.8764	[0.706, 1.070]	1	0.1	Gamma
$p(\rho_s\|\hat{\boldsymbol{y}})$		1.2342	[1.029, 1.454]	1	0.1	Gamma
$p(\rho_x\|\hat{\boldsymbol{y}})$		3.618	[2.910, 4.394]	4	0.4	Gamma
$p(\varphi_x\|\hat{\boldsymbol{y}})$	$\varphi_x = \frac{(1-\varrho)\rho_x}{(1-\varrho)\rho_x+(\rho_s+\rho_f)}$	0.2412	[0.148, 0.352]			
$p(\varphi_{\tilde{y}}\|\hat{\boldsymbol{y}})$	$\varphi_{\tilde{y}} = \frac{\rho_f+\rho_s}{(1-\varrho)\rho_x+(\rho_s+\rho_f)}$	0.7588	[0.648, 0.852]			
$p(\sigma_s\|\hat{\boldsymbol{y}})$		1.019	[0.468, 1.455]	0.6	0.16	Inv. Gamma
$p(\sigma_f\|\hat{\boldsymbol{y}})$		9.2763	[6.986, 12.625]	0.6	0.16	Inv. Gamma
$p(\sigma_{f1}\|\hat{\boldsymbol{y}})$		0.5854	[0.503, 0.683]	0.6	0.16	Inv. Gamma
$p(\sigma_{f2}\|\hat{\boldsymbol{y}})$		0.9472	[0.812, 1.116]	0.6	0.16	Inv. Gamma
$p(\sigma_\phi\|\hat{\boldsymbol{y}})$		0.8621	[0.408, 1.588]	0.6	0.16	Inv. Gamma
$p(\lambda\|\hat{\boldsymbol{y}})$		0.8561	[0.779, 0.916]	0.5	0.1	Beta
$p(\delta\|\hat{\boldsymbol{y}})$		0.8202	[0.694, 0.902]	0.5	0.1	Beta

the decision-making process. When agents attach more importance to the consensus prediction than to their own personal assessment, they implicitly reduce the importance of their private signal. This finding is consistent with different choices for the prior distributions. Although private signals ρ_x are more accurate than public signals, i.e., ρ_f and ρ_s, the role of higher order beliefs is largely confirmed. Public information, therefore, in accordance with the beauty contest analogy acts as a coordinating mechanism. This is a central result, firstly, proposed by Morris and Shin (2002). We have intentionally integrated it into our framework to test its presence and intensity in the context of the exchange rate market. Furthermore, it also furnishes a complementary result to the empirical test of the scapegoat model posited by Bacchetta and Van Wincoop (2004) and implemented by Fratzscher et al. (2015), who find that using survey predictions on fundamentals as proxies for scapegoat effects improves our ability to explain exchange rate movements. Public information is, therefore, capable of capturing changes in actual exchange rate dynamics. This estimation discovers that the importance of the higher order beliefs is associated with a larger weight to public information. On a rational level, predictors seek information on fundamentals. However, they end up attributing excess weight to public information which is not

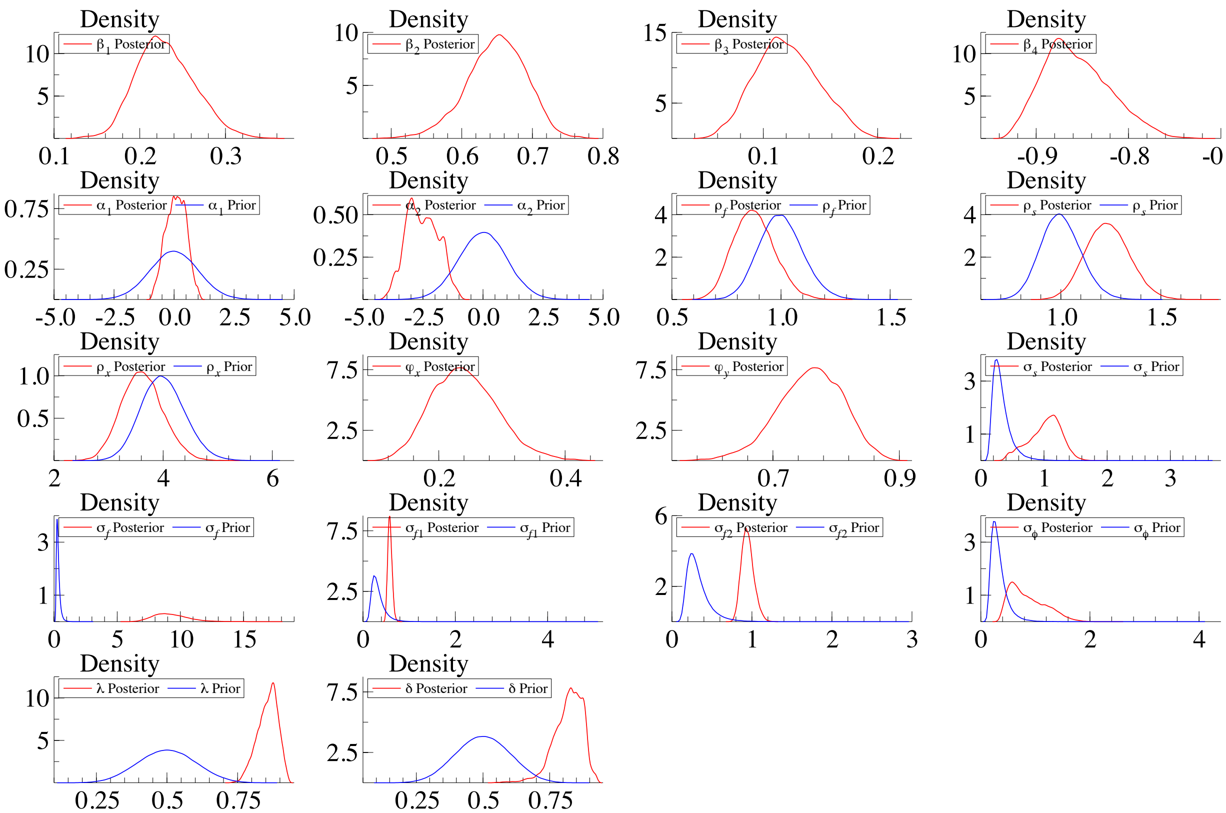

Figure 2: Prior vs Posterior distributions of the structural parameters

informative, at least in the short run. This is clearly due, on one side, to the presence of higher order beliefs, and, on the other side, to the uncertainty entailed by the heterogeneity of expectations. The fundamental is therefore transformed into a scapegoat in the event of uncertainty regarding structural parameters. In particular, the higher value of the conditional variance of the fundamental, σ_f (around 9) in Table 2 suggests that the short-term uncertainty proposed by Fratzscher et al. (2015) is congruent with uncertainty stemming from changes in fundamentals, specifically generating the scapegoat effect discussed by Bacchetta and Van Wincoop (2004).

5.1. Robustness checks and goodness-of-fit

In this section we evaluate the performance of our model against the data. Our first task is to check if the model provides results consistent with the current literature on exchange rates dynamics, especially in term of its ability to produce reliable long and short run predictions.

As a first result, by studying the rank of the autoregressive matrices of eq. (28), we found that variables are cointegrated. This result suggests a systematic relation between exchange rates and fundamentals and indicates that fundamentals are reliable predictors for exchange rates in the long run. This result is consistent with Mark (1995) and Bacchetta and Van Wincoop (2006) amongst others.

As a side result, we evaluated the forecast error variance decomposition (FEVD) for the returns on exchange rates. To compute the FEVD, we made use of the cointegration relation of the system to derive its MA representation of $\Delta \boldsymbol{x}_t$.[11] This representation was obtained through the Smith-McMillan factorization of the polynomial matrix associated with the model as proposed in Engle and Yoo (1991).

Our analysis suggests that shocks involving fundamentals constitute the main factor explaining Δs_t. Specifically, fundamentals account for about 90% of the variance in returns. Exchange rate shocks account for approximately 6% of variability, while liquidity appears to be less significant accounting for about 3%. Similar results are obtained by looking at the one-step-ahed predictions. A more detailed analysis on the short run predictions is displayed in Figure 3, in which one-step-ahead forecasts for exchange rates and their returns have been computed.

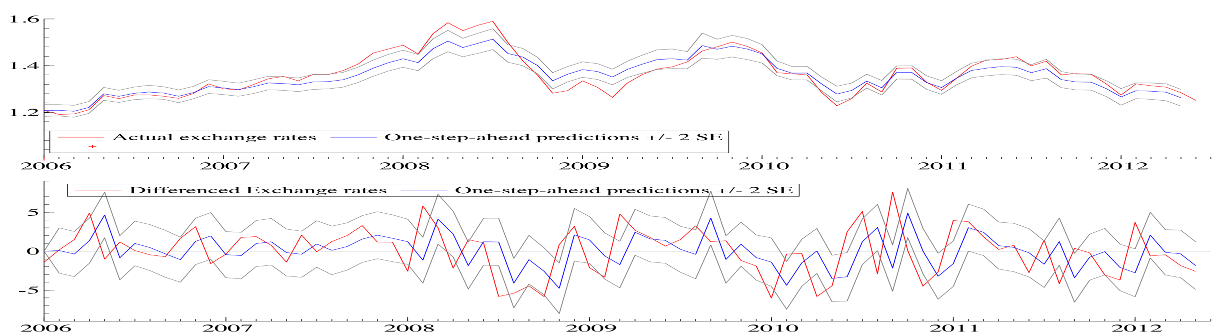

Figure 3: Upper panel: Actual exchange rates (red line) vs. predicted exchange rates (blue line) together with 95% credibility bands. Lower panel: Actual exchange rates returns (red line) vs. predicted exchange rates returns (blue line) together with 95% credibility bands.

In this case, the model replicates the actual data fairly closely. Indeed, more than 70% of actual exchange rates fall within the 95% credibility interval. Specifically, predicted estimates provide a good proxy for the behavior of the actual data.

[11]See AppendixC for some technical details.

This analysis, together with the FEVD reported above, suggests that fundamentals are reasonable predictors even in the short run. Apparently, private information is not relevant for forecasting purposes. This finding is somewhat counterintuitive with respect to the literature since the work of Meese and Rogoff (1983). This performance can be explained by the indirect role that private information play in the model. In our setup private informations enter the model indirectly through individual expectations as a combination of public and private information.

To confirm this analysis, we also evaluated the short run prediction by using a *naive* rational expectation model with no private signals that closely mimics the dynamics defined in eq. (13). In particular we consider

$$s_t = \lambda \mathbb{E}_t[s_{t+1}] + (1-\lambda) f_t - \lambda \psi_t + \epsilon_t, \tag{33}$$

in which f_t is described in eq. 20, while ψ_t is an independent and identically distributed sequence. Furthermore, rational expectations are defined so that $\mathbb{E}_t[s_{t+1}] = s_t + \eta_t$, where η_t is a Gaussian shock with mean zero and constant variance. We estimated exchange rate dynamics according to the rational expectation model using MCMC. Specifically, for each posterior draw of the parameters, we solved the rational expectation system using Sims (2002) and implementing it with the Ox package `LiRE` developed by Mavroeidis and Zwols (2007). Then, for each parameter, we simulated the one-step-ahead prediction produced by the rational expectation model. Figure 4 compares the average rational expectations tra-

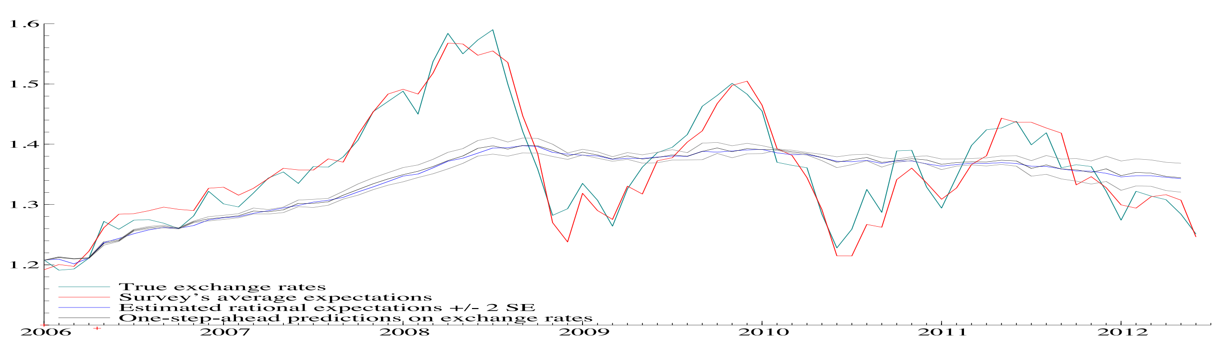

Figure 4: Actual exchange rates (green line), average from the survey (red line) together with an estimate of the rational expectation on exchange rates $\mathrm{E}_t[s_{t+1}]$ (blue line) and the one-step-ahead prediction on s_t (black line).

jectory and the average one from the survey.[12] Figure 4 still points out predicted average expectations, which differ significantly from observed expectations.

Figures 3 and 4, together with the FEVD analysis confirm that fundamentals represent the most important device for forecasting purposes. However, the role of private signals is crucial and make fundamentals to be reliable predictors. This empirical evidence can be considered as a symptom that the private signal is a necessary condition for a correct specification of the model especially for short run analysis. From an econometric point of view, excluding private information may induce mis-specification on the exchange rates dynamics. This is the reason why using survey data appears to be a viable strategy to provide a better description of observed exchange rates.

We also investigate the role of measurement errors in the model by evaluating the goodness-of-fit through the marginal likelihood estimated as the harmonic mean of the likelihood function evaluated for each posterior draw of the parameter vector (An and Schorfheide, 2007). The log-marginal likelihoods for the model with and without measurement errors are -2349.2 and -2359.7 respectively, which imply a Bayes factor of about e^{10} in favor of the former model. This evidence suggests a strong rejection of the model with no measurement errors.

Finally, to study the dynamic interactions between the variables expressed in level, it is useful to analyze how exchange rates react to structural shocks. This assessment is illustrated in Figure 5. We show the impulse response functions (IRFs) for exchange rates resulting from positive economic shocks, related to exchange rates, fundamentals, liquidity and private information. For each shock, the impulse-response functions are shown along with the 95% credible intervals. Despite the non-stationary nature of the dynamics of the model, impulse response functions can still be computed (Lütkepohl, 2005, ch. 6.7).

In the graphs shown in Figure 5, the red line relies on the IRF, while the grey band is the 95% credible interval for the IRF. The top left panel shows how an exchange rate shock affects the overall dynamics of the exchange rate at short horizons, fading away in few months. The same happens for the shock to the linear combination of the two fundamentals (top right panel), which also affects the dynamics of the rate. Such a result is consistent with the standard monetary model with flexible prices where an anticipated monetary shock affects the exchange rate only temporarily. The bottom left panel shows how a shock to liquidity has a substantial effect on exchange rate dynamics, in line with Evans and Lyons (2002) which

[12] Here rational expectations have been computed as the average trajectory compared to the posterior draws based on the model's parameters.

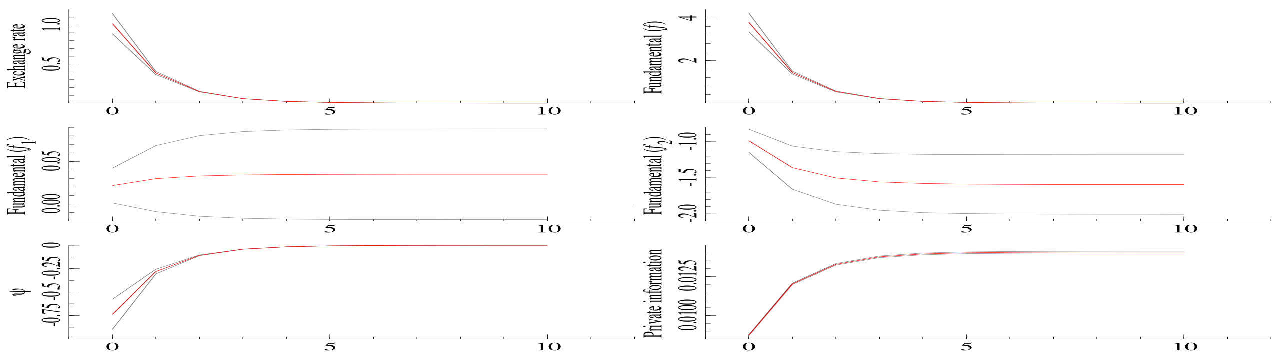

Figure 5: Impulse Response Functions and the 95% credible intervals

have shown the importance of order flow in the determination of the exchange rate. However, this effect is absorbed with time as a result of the decision to model liquidity as white noise. The final graph of Figure 5 illustrates the increased role of private information, particularly in the short run, although the results are rather small in magnitude.

6. Related literature and concluding remarks

Our contribution relates to the whole fast-growing literature on social learning theory which considers the effect that the diffusion of information and the choice of agents have on subsequent market participants (Banerjee, 1992; Bikhchandani et al., 1992; Chamley and Gale, 1994). In the spirit of Keynes (1936), Morris and Shin (2002) study a static coordination game demonstrating how agents' choices are mainly influenced by public source. This result is induced by the role of higher order beliefs which selectively reduce the importance of highly informative private information. In the last decade, Morris and Shin equilibrium strategy was further proposed and enriched in many settings with asymmetric information, including financial markets (Allen et al., 2006), business cycle models, (Angeletos and La'O, 2009), oligopolistic competition (Myatt and Wallace, 2015).

Initially, the literature slightly distinguished between public and private information by looking at two signals. We intentionally make the same choice to test our structural model for the sake of simplicity. Of course multiple information sources may enrich the design and constitute a fertile ground for follow-up research. Alternative studies that theoretically verified the effects of multiple information sources are among others, Angeletos et al. (2004, 2007) who consider an investment game evaluating welfare as aggregation of agents' outcomes. Similar information structure is observed in Dewan and Myatt (2008)for political leadership or in a Lucas-Phelps island setting in Myatt and Wallace (2014). In particular the last one focuses on the island-economy analysis in a beauty contest where each source (and it consequent signal) can be perceived by its quality and its clarity, respectively as measures of accuracy and transparency. Thus a signal with imperfect quality (but perfect clarity) imperfectly identifies the fundamental but is observed by all agents. The sharp distinction between private and public sources is thinned down and the results are driven by sources relatively far from averagely public. Information acquisition is also studied by Colombo et al. (2014) who take into account the sources of inefficiency in the acquisition of private information studying how this effect slightly reduce the importance of the public source. The incentive for agents to acquire information can be enhanced when others acquire information and when actions are complementary as in Hellwig and Veldkamp (2009).

Our investigation also refers to a strand of macroeconomic theory modeling the behaviour of exchange rates. Building upon work by Bacchetta and Van Wincoop (2004, 2006) who have explored the implication of heterogeneity in expectations, we differentiate by proposing a learning process on the current exchange rate and then derives one-step-ahead expectations through Kalman filter (Coibion and Gorodnichenko, 2012). Bacchetta and Van Wincoop (2004) instead develop a framework where agents observe current exchange movements, basically inconsistent with their future expectations. Searching for an explanation of this inconsistency, a weight, higher than average, is assigned to some fundamentals chosen as *scapegoats*. Fratzscher et al. (2015) develop an empirical test of this theory using as a proxy of scapegoat fundamentals, Consensus Economics of London surveys of predictors. The authors find that the inclusion of these expectations improves the explaining power of the fundamentals.[13] Bacchetta and Van Wincoop (2006) focus instead on the order flow and introduce a possible explanation for the empirical results verified by Evans and Lyons (2002), Payne (2003), and Froot and Ramadorai (2005).[14] Assuming that agents are risk averse, the authors show that due to the imperfect correlated signals among investors, transitory shocks

[13]In the short run the heterogeneity in the individual evaluation may lead to overrate the random macroeconomic fundamental.

[14]Evans and Lyons (2002) exploit data pertaining to bilateral transactions among FX dealers via Reuters Dealing 2000-1 electronic trading system. They follow Meese and Rogoff (1983)'s methodology to investigate the out of sample forecasting ability of their linear model. Unfortunately, they do not take into account potential issue of simultaneity bias emerging when exchange rate movements cause order flow. In order to evaluate the possible feed-back effects of exchange rates on order flow, an alternative methodology was suggested by Payne (2003). He elaborates a VAR model estimating information on the size of transactions. This methodology allows for a more precise estimation of the information provided by the order flow. Froot and Ramadorai (2005) extends the framework of Payne (2003) considering inflation and interest rate differentials alongside order flow and excess returns. They also estimate long-run effects of international flows on exchange rates and their relation to fundamentals proposing a decomposition of permanent and transitory components of asset returns.

may continuously influence the dynamics of exchange rate. These results originate from a counterbalance effect of risk aversion and uncertainty of information. On one side, the risk-sharing impact is justified since agents must be compensated for the *extra* risk assumed as a consequence of their actions. On the other side, due to information uncertainty, investors may confuse variations of the exchange rate caused by a liquidity shock with that induced by information on fundamentals. In our micro analysis instead, empirical results show that higher order beliefs mainly form agents' expectations. Public source is as a coordinating device among investors which generate the process of expectations in accordance with the aggregations of their signals. From a macro perspective, however, similar to Bacchetta and Van Wincoop (2006), our structural model describes accurate predictions both in the short and long run through a cointegration relationship between exchange rates and fundamentals.

Our empirical estimation finally takes into account the extensive empirical literature on rationality and inefficiency of heterogenous surveys. The seminal paper in this field is Ito (1990) which tests individual biases and idiosyncratic effects for a set of disaggregate expectations about the $1-$, $3-$ and $6-$month-ahead JY/US rate from the JCIF survey over the period $1985 - 1987$ finding substantial heterogeneity among predictors.[15] Similar results can be confirmed by Elliott and Ito (1999) and Benassy-Quere et al. (2003). MacDonald and Ian (1996) replicate Ito's test for $3-$ and $12-$month- ahead estimates of the BP/usd, DM/usd and JY/usd rates from the 1989-1992 and find significant evidence of heterogeneous expectations. Extending the Consensus data set to 1995, Chionis and MacDonald (1997) confirm the presence of individual effects for predictors. Mitchell and Pearce (2007) conducted a thorough analysis of unbiasedness and success rate of predictions along with tests for heterogeneity and strategic forecasting, finding systematic heterogeneity in predictions.[16]

[15] This literature stems from the recent availability of individual survey-predictions of exchange rates. Previously, a long strand of literature has studied inefficiency and irrationality of exchange rates forecasts. Dominguez (1986) tests the efficiency of foreign exchange market showing that predictors systematically fail in forecasting in the magnitude and the direction of exchange rates movements. Avraham et al. (1987) test the same hypothesis in a high inflationary Israel of the eighties rejecting the notion of rationality of exchange rate expectations. Cavaglia et al. (1993) find that exchange rates forecasts in the EMS are biased. Chinn and Frankel (1994) propose a test rejecting the hypothesis of efficiency and unbiasedness of exchange rate predictions.

[16] For a complete review of the tests of heterogeneity hypothesis using disaggregated survey expectations of professional forecasters, see Jongen et al. (2008).

AppendixA. A simple model of Exchange Rates

Let us start by a standard dynamic two-country model of exchange rates with the following basic relationships. Define, p_t and p_t^*, as the logs of the Home and the Foreign price level, respectively and s_t is the log-exchange rate. The purchasing power parity equation is then ensured as:

$$p_t = p_t^* + s_t \tag{A.1}$$

while the money market equilibrium for the two countries is depicted by:

$$m_t - p_t = \phi y_t - \alpha r_t \tag{A.2}$$

$$m_t^* - p_t^* = \phi y_t^* - \alpha r_t^* \tag{A.3}$$

where m_t and m_t^* are the logs-money supplies, y_t and y_t^* are the logs-output levels, while r_t and r_t^* are the interest rates, respectively for Home and Foreign countries.[17] Following Bacchetta and Van Wincoop (2006), the demand for foreign bonds by investor i, namely, $b^i_{F_t}$, is:

$$b^i_{F_t} = \frac{\mathbb{E}_t^i[s_{t+1}] - s_t + r_t^* - r_t}{\gamma \sigma_t^2} - b_{it} \tag{A.4}$$

where the individual expectation of the next period exchange rate is $\mathbb{E}_t^i[s_{t+1}]$, while the conditional variance is σ_t^2 and b_{it} is the hedge against non-asset income. Since bonds are in zero net supply, market equilibrium is given by

$$\int_0^1 b^i_{F_t}\, di = 0 \tag{A.5}$$

The exchange rate exposure is assumed to be equal to the average exposure plus an idiosyncratic term, i.e., $b_{it} = b_t + \epsilon_{it}$ where $\epsilon_{it} \sim N(0, \sigma_\epsilon^2)$. Market equilibrium, therefore, determines the following interest rate arbitrage condition:

$$\bar{\mathbb{E}}_t[s_{t+1}] - s_t = r_t - r_t^* + \psi_t \tag{A.6}$$

where $\bar{\mathbb{E}}_t[s_{t+1}]$ is the average expectation across all investors, while, $\psi_t = \gamma \sigma_t^2 b_t$ is defined as the expectational error or a risk premium associated with liquidity or hedge trade.

[17] We assume that there is no a-priori differences in the structure of the two countries, then ϕ and α are equal among them.

AppendixB. Other parameters estimates

In this Section are reported the results related to other parameters that are not relevant in our analysis. In particular Table B.3 summarizes the posterior output of the precision of the private signal for the 15 forecasters considered as in eq. (26), whereas Table B.4 are related to the standard errors of the measurement errors of eq. (30).

Table B.3: **Posterior computation (MCMC) - Other parameters**

	Posterior distribution		Prior information		
	Mean	95% Cred. Int.	Mean	S.E.	Type
$p(\sigma_{x_1}\|\hat{\boldsymbol{y}})$	0.6314	[0.395, 1.081]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_2}\|\hat{\boldsymbol{y}})$	0.6096	[0.393, 0.972]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_3}\|\hat{\boldsymbol{y}})$	0.6044	[0.375, 0.975]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_4}\|\hat{\boldsymbol{y}})$	0.6098	[0.392, 0.977]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_5}\|\hat{\boldsymbol{y}})$	0.6467	[0.393, 1.104]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_6}\|\hat{\boldsymbol{y}})$	0.5871	[0.376, 0.958]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_7}\|\hat{\boldsymbol{y}})$	0.5902	[0.372, 1.125]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_8}\|\hat{\boldsymbol{y}})$	0.6077	[0.371, 1.104]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_9}\|\hat{\boldsymbol{y}})$	0.5806	[0.372, 0.934]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_{10}}\|\hat{\boldsymbol{y}})$	0.6039	[0.386, 0.963]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_{11}}\|\hat{\boldsymbol{y}})$	0.6247	[0.381, 1.017]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_{12}}\|\hat{\boldsymbol{y}})$	0.6142	[0.380, 1.056]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_{13}}\|\hat{\boldsymbol{y}})$	0.5919	[0.391, 0.913]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_{14}}\|\hat{\boldsymbol{y}})$	0.6243	[0.404, 0.983]	0.6	0.16	Inv. Gamma
$p(\sigma_{x_{15}}\|\hat{\boldsymbol{y}})$	0.6564	[0.406, 1.183]	0.6	0.16	Inv. Gamma

Table B.4: **Posterior computation (MCMC) - Measurement Errors**

	Posterior distribution		Prior information		
	Mean	95% Cred. Int.	Mean	S.E.	Type
$p(\sigma_{E_1}\|\hat{\boldsymbol{y}})$	1.4055	[1.170, 1.686]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_2}\|\hat{\boldsymbol{y}})$	1.7511	[1.386, 2.204]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_3}\|\hat{\boldsymbol{y}})$	2.5709	[2.162, 3.071]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_4}\|\hat{\boldsymbol{y}})$	3.9357	[3.343, 4.640]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_5}\|\hat{\boldsymbol{y}})$	0.867	[0.604, 1.218]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_6}\|\hat{\boldsymbol{y}})$	3.3454	[2.844, 3.961]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_7}\|\hat{\boldsymbol{y}})$	2.8154	[2.346, 3.380]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_8}\|\hat{\boldsymbol{y}})$	1.5442	[1.293, 1.841]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_9}\|\hat{\boldsymbol{y}})$	2.6808	[2.277, 3.157]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_{10}}\|\hat{\boldsymbol{y}})$	1.9843	[1.693, 2.337]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_{11}}\|\hat{\boldsymbol{y}})$	2.9299	[2.456, 3.525]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_{12}}\|\hat{\boldsymbol{y}})$	2.9091	[2.450, 3.444]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_{13}}\|\hat{\boldsymbol{y}})$	2.5205	[2.148, 2.969]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_{14}}\|\hat{\boldsymbol{y}})$	2.9644	[2.542, 3.491]	0.6	0.16	Inv. Gamma
$p(\sigma_{E_{15}}\|\hat{\boldsymbol{y}})$	1.687	[1.410, 2.014]	0.6	0.16	Inv. Gamma

AppendixC. Forecast Error Variance Decomposition for $\boldsymbol{\Delta x_t}$

To obtain the FEVD for the observed data, we recur to the following MA representation of the economic system. The reduced form model is

$$\boldsymbol{x}_t = \Theta_x \boldsymbol{x}_{t-1} + \epsilon_t, \qquad \mathrm{E}[\epsilon_t \epsilon_t'] = \Theta_\epsilon \Theta_\epsilon'$$

or, using a polynomial notation with lag operator L,

$$\Theta_x(L)\boldsymbol{x}_t = \epsilon_t.$$

In particular some of the processes involved are random walks. In the following, n is the dimension of $\boldsymbol{x}_t$ whereas $n-r$ is the number of random walks.

By looking at the rank of Θ_x, we observe that for all the posterior parameters from the MCMC algorithm, there exist r cointegration relationships, then an MA representation of $\Delta\boldsymbol{x}_t$ exists. Consider that,

$$\Theta_x(L) = \begin{bmatrix} (1-L)I_{n-r} & 0_{n-r,r} \\ -A_{r,n-r}L & I - LB_{r,r} \end{bmatrix}$$

Following Engle and Yoo (1991), the autoregressive polynomial can be factorized as $\Theta_x(L) = U(L)M(L)V(L)$, to disentangle the stationary from the non-stationary part of the model. In particular we get

$$\Theta_x(L) = \begin{bmatrix} I_{n-r} & 0_{n-r,r} \\ 0_{r,n-r} & I_{r,r} \end{bmatrix} \begin{bmatrix} (1-L)I_{n-r} & 0_{n-r,r} \\ 0_{r,n-r} & I_{r,r} \end{bmatrix} \begin{bmatrix} I_{n-r} & 0_{n-r,r} \\ -A_{r,n-r}L & I - LB_{r,r} \end{bmatrix}$$

and the corresponding matrix

$$V = \begin{bmatrix} I_{n-r} & 0_{n-r,r} \\ A_{r,n-r}L & B_{r,r} \end{bmatrix}$$

Some tedious algebra allows to get

$$\Delta \boldsymbol{x}_t = \begin{bmatrix} I_{n-r} & 0_{n-r,r} \\ \sum_{i=1}^{\infty} \Theta_{21}^{(i)} L^i & \sum_{i=1}^{\infty} \Theta_{22}^{(i)} L^i (1-L) \end{bmatrix} \epsilon_t$$

in which $\Theta_{21}^{(i)}$ and $\Theta_{22}^{(i)}$ are respectively the lower left and right blocks of V^i.

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