INSTITUTE
OF ECONOMICS

Scuola Superiore
Sant'Anna

LEM | Laboratory of Economics and Management

Institute of Economics
Scuola Superiore Sant'Anna

Piazza Martiri della Libertà, 33 - 56127 Pisa, Italy
ph. +39 050 88.33.43
institute.economics@sssup.it

# LEM

# WORKING PAPER SERIES

## Non-performing loans, systemic risk and resilience in financial networks

Giulio Bottazzi °
Alessandro De Sanctis *
Fabio Vanni °

° Institute of Economics, Scuola Superiore Sant'Anna, Pisa, Italy
* Paris School of Economics, Paris, France

2016/08 February 2018
ISSN(ONLINE) 2284-0400

# Non-performing loans, systemic risk and resilience in financial networks*

Giulio Bottazzi† Alessandro De Sanctis‡ Fabio Vanni§

First version: March 2016
This version: February 2018

**Abstract**

After the outbreak of the financial crisis in 2007-2008 the level of non-performing loans (NPLs) in the affected economies has significantly increased. However, while in some countries this has been a transitory phenomenon, in others it still represents a major threat for financial stability and economic recovery. The present work investigates the relationship between non-performing loans, systemic risk and resilience of the financial system using a network-based approach. We develop a model with two types of agents, banks and firms, linked one another in a two-layers structure by their reciprocal claims. The model is studied analytically and via numerical simulations, and it allows to derive a synthetic measure of systemic risk and to identify the maximum level of NPLs sustainable by the financial system before it collapses. Finally, for illustrative purposes, we present an application of the model to Italy, Germany, and United Kingdom, using firm-level data for the three countries.

**Keywords:** financial crisis, network theory, non-performing loans, resilience, systemic risk

**JEL codes:** G21, C63, G01, D85

*This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 640772 - DOLFINS and grant agreement No 721846 - Marie Skłodowska-Curie.

†Scuola Superiore Sant'Anna, Pisa, Italy. E-mail: giulio.bottazzi@sssup.it. Corresponding author
‡Paris School of Economics, Paris, France. E-mail: alessandro.desanctis@psestudent.eu
§Scuola Superiore Sant'Anna, Pisa, Italy. E-mail: fabio.vanni@sssup.it

# 1 Introduction

Over the last 30 years world's economy has experienced an unparalleled process of globalization, which has lead to a reduction of the world's effective size "from XXL to Small" (Friedman, 2006). As for the financial sector, this phenomenon has been the consequence of the pursuit for new investment opportunities and of a strain to diversification which have increased the level of connectivity and complexity of the financial system, with the result that today's financial institutions are directly or indirectly much more connected than ever before.

A non-negligible role in fostering this process has been played by the belief that interconnection of financial markets would have lead to a greater financial stability, as risk, for any single institution, would have been reduced via its spreading around the world. However, the recent crisis has cast doubts on this idea (Stiglitz, 2010; Battiston *et al.* , 2012a) and showed how diversification, rather than curtailing the overall level of risk, has simply dispersed it, transforming risk from idiosyncratic into systemic. As pointed out by Stiglitz, [1] the high number of interconnections between financial intermediaries "facilitated the breakdown" and became "part of the problem". Similarly Yellen highlighted that "interconnections among financial intermediaries are not an unalloyed good. Complex interactions [...] may serve to amplify existing market frictions, information asymmetries, or other externalities" (Yellen, 2013).

The crisis has forced scholars and policy makers to rethink financial stability focusing on the role of interconnections among financial institutions, whose analysis is now considered crucial to gauge systemic risk and to prevent, or at least to dampen, future economic meltdowns (Schweitzer *et al.* , 2009). This has led to an intense research activity aimed at better understanding the role of pairwise interactions between the different actors of the financial system in propagating and amplifying negative shocks. This field of research goes back to Allen & Gale (2000) and Eisenberg & Noe (2001) and, over the past few years, developed along essentially two complementary directions: part of the literature focused primarily on theoretical models of networks (Gai & Kapadia, 2010; Gai *et al.* , 2011; Battiston *et al.* , 2012b; Montagna & Lux, 2013; Elliott *et al.* , 2014; Acemoglu *et al.* , 2015a,b; Chinazzi *et al.* , 2015)[2] while another part devoted its attention to the empirical analysis of financial networks (Soramäki *et al.* , 2007; Iori *et al.* , 2008; Bech & Atalay, 2010; Beltran *et al.* , 2015).

The works from Gai & Kapadia (2010) and Montagna & Lux (2013) are the closest to ours. Both studies analyze how unexpected exogenous shocks propagate through a complex financial network where shocks can force a financial firm to default and not to repay its debts. Gai and Kapadia develop a contagion model with homogeneous banks and a random network structure. Their main conclusions are that the probability of experiencing a default cascade is non-monotonic in connectivity (namely higher diversification is not always good) and that the same shock can have very different consequences depending on the point in which it hits the network. Montagna and Lux study systemic risk in a scale-free network with heterogeneous banks. They use a fitness model to generate the network with the aim of reproducing some of the frequently documented features of

[1] IMF conference "Interconnectedness: Building Bridges between Research and Policy", May 2014, http://www.imf.org/external/pubs/ft/survey/so/2014/RES052314A.htm (accessed on 1 July 2015)

[2] See Chinazzi & Fagiolo (2013) for a survey.

interbank markets in terms of assortativity, degree distributions and size distributions.

Building on these works, our paper develops a theoretical model of contagion in financial network and attempts to bring this framework to reality by calibrating it with firm-level data. More in detail, we develop a network model which simulates how exogenous shocks can affect the stability of a financial system in which two types of agents, banks and firms, are linked by their claims and obligations. We first focus and study analytically a simple version of the model with homogeneous agents, then by means of computer simulations, we investigate the relationship between non-performing loans, systemic crisis and resilience in the more realistic case of fully heterogeneous agents.

Unlike most of the literature in this field, we represent exogenous shocks as increases in the aggregate level of non-performing loans (NPLs)[3]. We focused on NPLs for several reasons. First, as repeatedly shown since the beginning of the crisis, NPLs are one of the main causes for banks' default. Moreover, using NPLs allows to measure the intensity of the shock and anchor it to an easily observable variable, making our modelling framework potentially useful for policy applications. Finally, in the countries most affected by the crisis, the level on NPLs is still nowadays very high and constitutes a major treat to economic growth and financial stability. Indeed during and after the financial crises, the dynamic of the NPLs displayed different patterns among countries. Figure 1 shows the percentages of non-performing loans to total gross loans granted by banks in several European countries from 1997 to 2014. By looking at the data it is possible to distinguish two groups of countries: a first group, shown in the left panel, where the financial crisis has had only transitory effects of the level of NPLs, which increased right after the outbreak of the crisis in 2008 and went back thereafter; and a second group, represented in the right panel and coinciding mainly with the periphery countries of the Eurozone, where the level of NPLs boomed after the crisis and remained well above levels prior to 2008. In particular, it is worth noting how in countries like Italy, Greece and Portugal, still in 2014 NPLs are marked by an upward trend.

In order to provide a closer matching with reality, we calibrate the model with firm-level data for Italy, Germany and United Kingdom, which are characterised by different economic structure in terms of banks and firms size distribution, as well as different level of capitalization of the banking system. Due to the lack of publicly available data on bilateral exposures between banks and between banks and firms, we have to adopt simplifying assumptions so that the results of the simulations should be intended only as illustrative of the usefulness of the framework. However, as detailed in the next section, the assumptions made and rooted into empirically observed regularities about the lending behaviour of banks, therefore in our view they represent a plausible approximation of the distribution of credits and debits among banks and firms, and hence an alternative way to "fill the blanks" of the adjacency matrices representing the financial network[4].

[3]Following the definition of the IMF "a loan is non-performing when payments of interest and principal are past due by 90 days or more, or at least 90 days of interest payments have been capitalized, refinanced or delayed by agreement, or payments are less than 90 days overdue, but there are other good reasons to doubt that payments will be made in full" (Clarification and Elaboration of Issues Raised by the December 2004 Meeting of the Advisory Expert Group of the Intersecretariat Working Group on National Accounts, International Monetary Fund, June 2005).

[4]On other ways to fill the adjacency matrix see for example Anand *et al.* (2015).

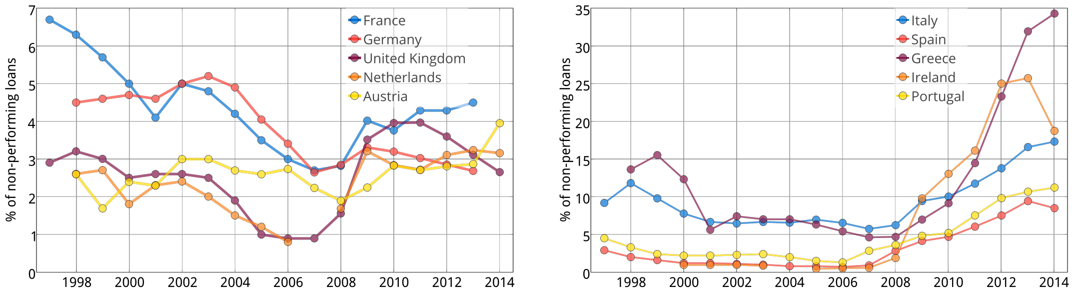



Figure 1: Non-performing loans divided by the total value of the loan portfolio (including non-performing loans before the deduction of specific loan-loss provisions). The loan amount recorded as non-performing should be the gross value of the loan as recorded on the balance sheet, not just the amount that is overdue (World Bank). Data are from 1997 to 2014 and cover different European countries. The left panel shows the countries where the level of NPLs remained relatively stable before and after the crisis. The right panel displays the countries where NPLs exploded after 2008 and are still today at levels higher than the pre crisis period. In particular, NPLs in Italy, Portugal and Greece in 2014 still show an upward trend. *Source: elaboration of the authors on data from the International Monetary Fund, Global Financial Stability Report.*

# 2 The model

Consider an economy composed by $N$ banks and $M$ firms. They are nodes in a network, linked one another through their balance sheets by credits and debits which result from financial transactions: for any node, be it bank or firm, an incoming link is a credit and an outgoing link is a debit. We assume that banks can lend and borrow from other banks, but can only lend to firms; moreover we assume that firms cannot borrow from each other, but only from banks. These assumptions imply that banks can have both incoming and outgoing links with other banks, but only incoming links form firms. On the other hand, firms can only have outgoing links toward banks and no links with other firms. With this assumption, the two sets of nodes form a bipartite network organized in two interconnected layers, where one comprises banks and the other firms, as shown in Figure 2.

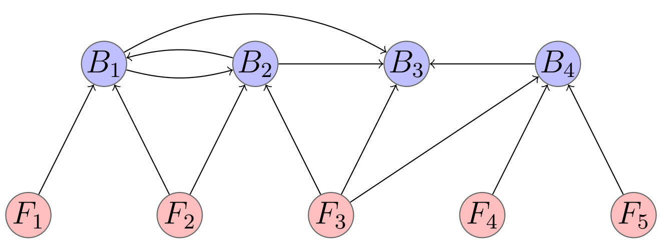



Figure 2: Stylized example of a network.

Blue nodes and red nodes in Figure 2 represent respectively banks and firms. Nodes' size and links' weight are ignored for simplicity. Following the literature and consistently with bankruptcy law, we do not net interbank positions, so two banks can be linked with each

| Assets $\longrightarrow$ | Liabilities $\longrightarrow$ |
|---|---|
| Interbank assets ($A_i^{IB}$) | Interbank liabilities ($L_i^{IB}$) |
| External assets ($A_i^F$) | Deposits ($D_i$) |
| | Capital ($K_i$) |
| Total assets ($A_i^{TOT}$) | Total liabilities ($L_i^{TOT}$) |

Table 1: Balance sheet structure. The arrows on the top of the figure indicate the direction of links: incoming links represent an asset, outgoing links a liability. The shock affects the portion of the assets held against firms. To cause the default of a node, the initial shock must be higher than the net worth owned by that node. Elaboration of the authors adapted from Haldane & May (2011).

other in both directions, as in the case of $B_1$ and $B_2$. In this example all banks and firms have at least a link, however as in our model the formation of links is probabilistic, it is possible to have completely disconnected banks or firms. The only constraint on the structure of the network is that self-loops, namely links starting and ending in the same node, are not allowed.

## 2.1 Balance sheet structure

Following Nier *et al.* (2007), we represent each bank via a simplified balance sheet structure as the one depicted in Table 1. The total assets of bank $i$ are composed by the interbank assets $A_i^{\mathrm{IB}}$, that is the total money lent to other banks, and by the external assets, $A_i^{\mathrm{F}}$, that is the total money lent to firms, so that $A_i^{\mathrm{TOT}} = A_i^{\mathrm{IB}} + A_i^{\mathrm{F}}$. At the same time, the banks liabilities are composed by money borrowed by other banks, $L_i^{\mathrm{IB}}$ and deposit, $D_i$. Due to the double-entry bookkeeping system, the total assets are equal to the total liabilities so that the capital (equity) of the bank, $K_i$, is defined as

$$K_i = A_i^{\mathrm{IB}} + A_i^{\mathrm{F}} - L_i^{\mathrm{IB}} - D_i \ .$$

A generic firm $j$ is uniquely described by its total assets $F_j$.[5]

While banks in our model posses heterogeneous inter-bank assets, we make the simplifying assumption that they share a constant portfolio composition and leverage ratio. More precisely, we assume that a fraction $\theta \in (0, 1)$ of assets comes form inter-bank lending and a fraction $1 - \theta$ from lending to firms, while, at the same time, the leverage, i.e.

[5]We do not try to precisely model the balance sheet structure of firms since our focus is on the consequences of shocks for the financial system.

the asset equity ratio, is fixed to be $1/\eta$. Formally one has

$$A_i^{\mathrm{IB}} = \theta A_i^{\mathrm{TOT}} \ , \quad A_i^{\mathrm{F}} = (1-\theta) A_i^{\mathrm{TOT}} \quad \text{and} \quad K_i = \eta A_i^{\mathrm{TOT}} \ . \tag{1}$$

With the assumptions above, the bank deposits are computed as a difference between $A_i^{\mathrm{TOT}}$ and $L_i^{\mathrm{IB}} + K_i$. Bank deposits can be both positive or negative. In the latter case, they should be considered assets owned by the bank. In any case, they represent positive or negative bank exposure to the risk-free part of the economy.[6]

## 2.2 Network creation and initialization

Each bank $i = 1, ..., N$ is initialized with an amount of interbank assets $A_i^{\mathrm{IB}}$ and each firm $j = 1, ..., M$ is assigned a value of total assets $F_j$. We take $A_i^{\mathrm{IB}}$ and $F_j$ as fitness parameters of the linking function that we use to generate the network, according to the general idea that larger banks and larger firms have an higher number of links. Specifically we assume that the probability $p_{i,j}^{\mathrm{IB}}$ to generate a link between bank $i$ and bank $j$ is proportional to a certain power of their inter-bank assets

$$p_{i,j}^{\mathrm{IB}} \sim \left(A_i^{\mathrm{IB}}\right)^{\alpha} \left(A_j^{\mathrm{IB}}\right)^{\beta} \quad \text{with} \quad \alpha, \beta > 0 \ , \tag{2}$$

while the probability $p_{i,j}^{\mathrm{F}}$ to generate a link between bank $i$ and firm $j$ is proportional to a certain power of the total assets of the bank and of the total assets of the firm. Since we have assumed that total asset $A^{\mathrm{TOT}}$ is proportional to interbank asset $A^{\mathrm{IB}}$ we can express the probability in terms of the latter

$$p_{i,j}^{\mathrm{F}} \sim \left(A_i^{\mathrm{IB}}\right)^{\phi} (F_j)^{\chi} \quad \text{with} \quad \phi, \chi > 0 \ . \tag{3}$$

The value of the exponents in 2 and 3 govern the degree of assortativity of the network in terms of nodes' size. In what follows we assume values for these parameters that allow us to reproduce the frequently documented observation of disassortative behavior[7] in the interbank network (Iori *et al.* (2006), Soramäki *et al.* (2007)) and to create an assortative behavior in the firms-banks network. The latter reflects the assumption that bigger firms have higher possibilities to access credit and that, on average, tend to have more links (i.e. more credit lines) (De Masi & Gallegati, 2007), see panel (c) in Figure 3.

In order to have a better control on the simulation parameters, we randomly generate the network in a kinetic way: we start with $N$ isolated nodes and connect two nodes at time, according to the probabilities in (2) and (3). The obtained directed network can be described by variables $l_{i,j}^{\mathrm{IB}}$ and $l_{i,k}^{\mathrm{F}}$ with $i, j = 1 \dots N$ and $k = 1 \dots M$: the first takes value one if a link exists from bank $j$ to bank $i$ and zero otherwise, the second takes value one if a link exists from firm $k$ to bank $i$ and zero otherwise. By stopping the procedure at the appropriate time, we can exogenously fix the final average degree of the interbank network $AD_{B,B} = \sum_{i=1}^{N} \sum_{j=1}^{N} l_{i,j}^{\mathrm{IB}}/N$, defined as the number of interbank links over the number

[6] In accordance with the current literature we label this variable "deposits", however it must be noted that this is just a convention and this variable has no direct relation with real banks' deposits.

[7] Notice that for our factorized linking probability, the disassortative behavior is due to the finite size of the network. This is the so-called structural cutoff in Catanzaro *et al.* (2005). According to Caldarelli (2007), in the asymptotic case of an infinite number of nodes, in our model the average degree of nodes linked to a specific node is expected to be independent upon the degree of the node itself.

of banks, and the average degree of the bank-firm network $AD_{B,F} = \sum_{i=1}^{N} \sum_{j=1}^{M} l^{\mathrm{F}}_{i,j}/N$ , defined as the number of links between banks and firms over the number of banks.[8]

After having generated the network, we assign a weight to each link. The link represents the amount of money which is borrowed by a node (bank or firm) and lent by another node (bank). For interbank links, the weight depends on the amount of interbank assets of the creditor ($A^{\mathrm{IB}}_i$) and of the debtor ($A^{\mathrm{IB}}_j$), as well as on the number of incoming links of the creditor. Formally, the amount of money bank $j$ owns to bank $i$ is

$$w^{\mathrm{IB}}_{i,j} = l^{\mathrm{IB}}_{i,j}\, A^{\mathrm{IB}}_i \, \frac{A^{\mathrm{IB}}_j}{\sum_{k=1}^{N} l^{\mathrm{IB}}_{i,k} A^{\mathrm{IB}}_k} \ . \tag{4}$$

Similarly, the weights of bank-firm links depend on the external assets of each bank $A^{\mathrm{F}}_i$, on the level of total assets of firms $F_j$ and on the number of incoming links of the bank. Formally the amount of money firm $j$ owns to bank $i$ is

$$w^{\mathrm{F}}_{i,j} = l^{\mathrm{F}}_{i,j}\, A^{\mathrm{F}}_i \, \frac{F_j}{\sum_{k=1}^{M} l^{\mathrm{F}}_{i,k} F_k} \ . \tag{5}$$

The weight above preserve the previously determined amount of interbank and external assets, $\sum_{j=1}^{N} w^{\mathrm{IB}}_{i,j} = A^{\mathrm{IB}}_i$ and $\sum_{j=1}^{M} w^{\mathrm{F}}_{i,j} = A^{\mathrm{F}}_i$ and, at the same time, guarantees the very natural condition that the amount involved in a financial transaction increases proportionally with the size of the involved parties. This reflect the assumption that bigger firms are able to get more credit from banks, which consider them as more trustworthy and less risky. In other words, we distribute the amount of interbank assets and of external assets of each bank proportionally to the size of the debtor.

Following this procedure we obtain a network that is bipartite, directed, and weighted and whose structure depends on the distribution of banks' and firms' size[9]. Figure 3 shows an example of the typical simulated network obtained using the previous procedure.

In our procedure, interbank liabilities of each bank are endogenously determined and we are able to compute all the elements of banks' balance sheet in Table 1. In particular the interbank liabilities of bank $i$ rest defined as

$$L^{\mathrm{IB}}_i = A^{\mathrm{IB}}_i \sum_{j=1}^{N} \frac{l_{j,i} A^{\mathrm{IB}}_j}{\sum_{k=1}^{N} l^{\mathrm{IB}}_{j,k} A^{\mathrm{IB}}_k}$$

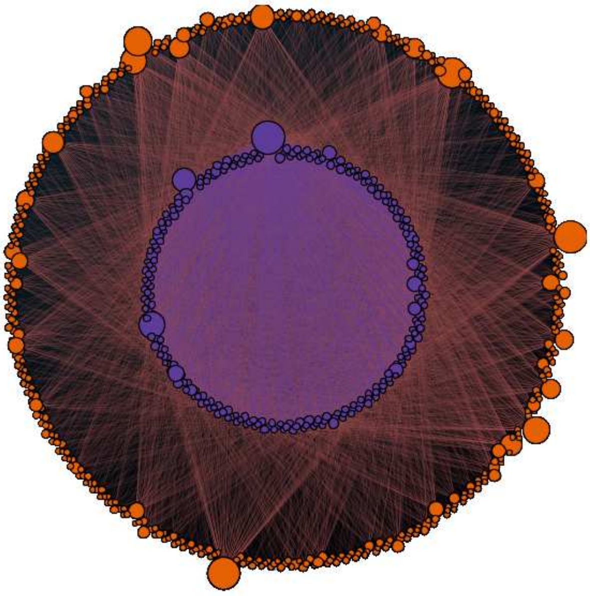
and the deposit

$$D_i = A^{\mathrm{IB}}_i \left( \frac{1-\eta}{\theta} - \sum_{j=1}^{N} \frac{l_{j,i} A^{\mathrm{IB}}_j}{\sum_{k=1}^{N} l^{\mathrm{IB}}_{j,k} A^{\mathrm{IB}}_k} \right) \ .$$

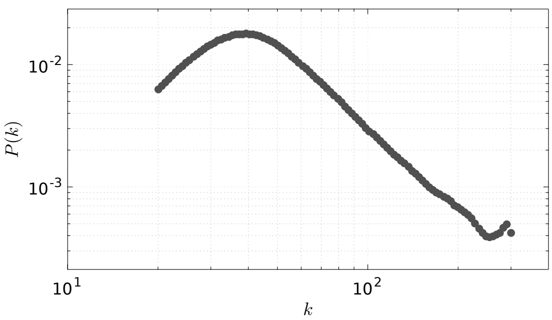
The latter expression can be interpreted as deposit if positive, or risk-free assets, for instance household mortgages, if negative. The total exposition of the banking system

[8]This approach is alternative with respect to the static procedure described in Caldarelli *et al.* (2002) and Caldarelli (2007) and adapted to interbank networks in Montagna & Kok (2013). The advantage of the kinetic approach is the much lower number of replications necessary to collect a sufficient statistics for the less probable values of average degree.

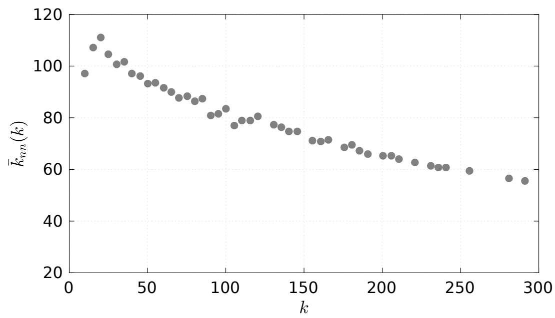
[9]For example, a power law distribution generates a scale-free type of network, as in the "Heterogeneous" case discussed in Section 3.



(a) Bank-firm network representation




(b) Degree distribution of the interbank network




(c) Disassortativity of the interbank network

Figure 3: In (a) Blue circles represent banks ($N = 300$), red circles firms ($M = 500$). The size of the circles is proportional to the amount of interbank assets for banks and total assets for firms. Red links represent links from firms to banks, blue links represent links from banks to other banks. The exponents of the linking functions are as in Table 2. For visualization purpose, it is useful to consider a rather dense network, then we set the average degree of the interbank network at $AD_{B,B} = 50$ and that of the bank-firm network at $AD_{B,F} = 100$. Notes that there are no links between firms and that, although it is not possible to (clearly) show in the figure the direction of links, red links can only be directed from firms (out) to banks (in), while blue links among banks can go in both directions (in and out). In (b) and (c) we report the out-degree distribution $P(k)$ and the average out-degree of nearest neighbor $\bar{k}_{nn}(k)$. The power law structure of the distribution and the disassortative behavior generated by the finite-size effect are evident.

to the riskless part of the economy is however positive and proportional to the overall amount of interbank assets

$$\sum_{i=1}^{N} D_i = \frac{1-\eta-\theta}{\theta} \sum_{i=1}^{N} A_i^{\mathrm{IB}} \,.$$

### 2.3 Solvency and bankruptcy cascades

After having initialized the model, we perturb the system at time $t = 1$ with an exogenous shock consisting in an increase in the level of NPLs due to firms default. The idea is that when a firm defaults it is no longer able to repay its debt, so the banks who granted it a loan mark it as non-performing and write-down the corresponding book value. Banks exposed toward defaulted firms incur in a loss which erodes their capital, potentially forcing them to default. In practice some of the credits provided by banks to firms are transformed in bad loans and their value is set to zero. We do so by selecting firms at random and assuming that they become unable to meet their obligations until we reach the desired amount of NPLs. More in detail, given an amount $x$ of NPLs, we select a firm at random; again at random we go through its outgoing link one by one; we set the value of the selected link equal to zero and we repeat until an amount of debt equal to $x$ is cancelled. If the total debt of the firm is greater than $x$, the last link considered is simply reduced by the amount necessary to reach $x$. If instead the total debt of the firm is lower than $x$, the procedure continues with another randomly selected firm, until the debts of all the link cancelled is equal to $x$. Notice that this way of selecting firms at random makes the distribution of firms' size relevant. In general, small firms have an higher default (or mortality) probability than big firms, a characteristic that matches empirical reality.

After this initial process is over, one or more banks have their external assets reduced below the initial level, $A_{i,1}^{\mathrm{F}} < A_i^{\mathrm{F}}$. If the amount of capital of bank $i$ is reduced such that it is unable the to meet the solvency condition

$$K_i = A_i^{\mathrm{IB}} + A_{i,1}^{\mathrm{F}} - L_i^{\mathrm{IB}} - D_i > \rho A_i^{\mathrm{TOT}} \quad \text{with} \quad \rho \geq 0 \,, \tag{6}$$

the bank becomes insolvent and it is set to default. Essentially we impose a minimum capital requirement on banks expressed as a fraction $\rho$ of their total assets. All defaulted banks are assumed to default on all their liabilities and for all the amount (*no partial recovery*).[10]

At this point a new round $t = 2$ starts. Assets corresponding to loans to previously defaulted banks are set to zero and the interbank assets $A^{\mathrm{IB}}$ of the creditor banks are accordingly reduced. The solvency condition (6) is checked again and the banks that are now unable to fulfill it are set to default. This process continues to iterate until no further bank failures occur. In this way the initial firm-level shock transmits at interbank level where failed banks are assumed to default on all of their interbank liabilities, eventually

[10] As pointed out in Gai & Kapadia (2010) "this assumption is likely to be realistic in the middle of a crisis: in the immediate aftermath of a default, the recovery rate and the timing of recovery will be highly uncertain and banks' funders are likely to assume the worst-case scenario". Anyway it would be possible to relax this assumption and allow for a partial recovery, so that when a linked bank defaults, the creditors do not lose all their asset, but get some fraction of it, for example a share of the remaining assets proportional to the weight of creditors' asset over all other liabilities of the defaulted bank.

pushing neighbour banks into default. The initial shock can be either absorbed or amplified, eventually triggering a cascade of defaults able to cause a systemic crisis within the financial network. Notice that liabilities are not adjusted following banks default. This assumption reflects the consideration that the process of deleveraging is typically much longer that the development of the default cascade and generally cannot help in preventing the bankruptcy.

It is useful to analyze the role played by the portfolio composition parameter $\theta$, and the leverage parameter $\eta$, in starting and sustaining a default cascade. Consider a specific bank $i$ and let $\delta$ be the fraction of external assets that have become non-performing after the initial shock, so that $A^{\mathrm{F}}_{i,1} = (1-\delta)A^{\mathrm{F}}_i$. Since initial total assets are equal to total liabilities, it is $L^{\mathrm{IB}}_i + D_i = A^{\mathrm{TOT}}_i - K_i$ and substituting (1) in (6) after little algebra one has that the solvency condition simply becomes

$$\delta < \delta^* \quad \text{with} \quad \delta^* = (\eta - \rho)/(1-\theta)\ . \tag{7}$$

The critical threshold $\delta^*$ is the minimum level of initial shock on external assets that sets a bank $i$ into default. As expected, this is a decreasing function of the leverage $1/\eta$ and an increasing function of the share of inter-bank assets $\theta$. In subsequent rounds, assume that bank $i$ has lost a fraction $\delta^{\mathrm{IB}}$ of its interbank assets. Using the same procedure, one can rewrite the solvency condition as

$$\frac{\theta}{1-\theta}\,\delta^{\mathrm{IB}} + \delta < \delta^*. \tag{8}$$

A few comments are in order. First, notice that the parameter $\rho$ enters into the definition of $\delta^*$ only as a modifier of the leverage $\eta$, thus without loss of generality we can assume $\rho = 0$. This corresponds to the case in which a bank is considered insolvent when its equity becomes zero or negative. Second, the role of the leverage is obvious, as it decreases the resilience of the network: higher leverage levels (lower $\eta$) makes banks more exposed to the failure of both firms and other banks. Conversely, the portfolio composition has two opposite effects on the default cascade. On the one hand, a higher value of $\theta$ increases $\delta^*$ and shields banks from possible default in round 1, when firms loans becomes non-performing. On the other, it increases the exposition of banks to the failure of other banks in the successive rounds.

## 2.4 Homogeneous fully-connected case

To understand how the model works it is useful to study the case of fully connected networks and homogeneous nodes, that is $A^{\mathrm{IB}}_i = A^{\mathrm{IB}}$ and $A^{\mathrm{F}}_j = A^{\mathrm{F}}$ for $i = 1 \dots N$ and $F_j = F$ for $j = 1, \dots, M$. In this case each firm owns an amount $A^{\mathrm{F}}/M$ to each bank and the aggregate external debt of banks is $NA^{\mathrm{F}}$. Let $[x]$ denotes the integer part of $x$ and $\{x\} = x - [x]$ its fractional part. One has the following

**Proposition 2.1.** *Let $\delta$ be the fraction of initially defaulting loans, then*

- *if $[\delta M] \geq \delta^* M$ all banks initially default.*
- *if $\delta^* M - 1 \leq [\delta M] < \delta^* M$ exactly $[N\{\delta M\}]$ banks initially default;*

- *if* $\delta^* M - 1 - \{N\{\delta M\}\} \leq [\delta M] < \delta^* M - 1$ *a single bank initially default;*
- *if* $[\delta M] < \delta^* M - 1 - \{N\{\delta M\}\}$*, no banks initially default;*

*Proof.* The number of completely defaulting firms is $[\delta M]$. Their NPLs generate a loss equal to $[\delta M] A^{\mathrm{F}}/M$ for each bank. According to (7), if $[\delta M]/M \geq \delta^*$ then all banks initially default and the first point is proved.

The following points can be proved analogously by observing that after the complete default of the $[\delta M]$ firms, a faction of NPLs equal to $N A^{\mathrm{F}}\{\delta M\}/M$ still has to default. This generate a further loss of $A^{\mathrm{F}}/M$ for $[N\{\delta M\}]$ banks.

Finally, the last loan is affected by a partial default equal to $A^{\mathrm{F}}\{N\{\delta M\}\}$. □

If some banks survive the initial NPLs shock, since they are all identical, for symmetry argument, only two possibilities arise: or they all default in the first round of the bankrupcy cascade or they never default. Specifically one has the following

**Proposition 2.2.** *If*

$$[\delta M] \geq \delta^* M - \frac{\theta}{1-\theta} \frac{M}{N-1} [N\{\delta M\}]$$

*then all banks surviving the initial NPLs shock will default. Otherwise, no banks further default after the first NPLs shock.*

*Proof.* All banks not defaulting for the initial shocks absorb a further loss due to the failing banks equal to $A^{\mathrm{IB}}[N\{\delta M\}]/(N-1)$, that is

$$\delta^{\mathrm{IB}} = \frac{[N\{\delta M\}]}{N-1}$$

using (8) the statement immediately follows. □

The joint effect of Propositions 2.1 and 2.2 is that when $N, M \to \infty$, the fully connected model has an abrupt phase transition: for $\delta \geq \delta^*$ all banks default, while for $\delta < \delta^*$ none does. Introducing heterogeneity will change the picture, however, as discussed through numerical examples in the next section.

# 3 Simulation results

We start the numerical investigation of the model considering two cases: one with homogeneous banks and firms and one with fully-fledged heterogeneity. The values of the parameters are reported in column "Homogeneous" and column "Heterogeneous" of Table 2 respectively. They are derived from the ones usually assumed in the literature (Gai & Kapadia (2010), Montagna & Lux (2013), Upper (2007), Nier *et al.* (2007)).

As seen in the previous section, in the homogeneous fully-connected case, if $\delta < \delta^* - 2/M$ no bank default may occur, while if $\delta > \delta^*$ all banks default. Given the value of the parameters it is $\delta^* = 0.1$, so that the whole transition from total safety to financial mayhem happens when $\delta$ increases from 0.096 to 0.1. In order to investigate the role of network topology, we consider instead the scenario in which all banks and all firms still have the same size, that without loss of generality we can set equal to 1, but random networks are generated with different average degree of the bank-bank and bank-firm networks. Remember that we adopt a kinetic approach to the generation of networks that allows for a direct control of the number of links. The average degree of the bank-bank and bank-firm network is a key parameter. It gives the average number of counterparts of a node and, as such, it is a proxy for both the level of interconnectedness of a system and the portfolio diversification of its node. We consider different combinations of $AD_{B,B}$ and $AD_{B,F}$, exploring all the range of possibilities from a collection of isolated nodes to a fully connected network. For every pair of values we generate 200 realizations of the network. Then, for each realization of the network, we shock the system by increasing the level of NPLs as described by the algorithm in Section 2.

Since we are interested in the risk of a systemic crisis we want to exclude small chain of defaults. For this reason, following Gai & Kapadia (2010), we define a systemic crisis as the occurrence of the default of more than 5% of banks in the network. Given this definition, we compute the frequency of a systemic crisis ($F$) as the number of times in which more than 5% of banks default over the 200 drawings and the extent of a systemic crisis ($D$) as the fraction of defaulted banks conditional on contagion over the 5% threshold breaking out, which is therefore a measure of the magnitude of the systemic crisis. These two quantities allow to define a synthetic statistics for measuring systemic risk $R = F \times D$, computed as the product between the frequency of contagion, $F$, and the extent of contagion, $D$.

Figure 4 reports the result of the simulation in the homogeneous case. Even if we consider random networks, the value $\delta^*$ remains a relevant upper bound, because if $\delta > \delta^*$ all banks fail and we have a systemic event irrespective of the network average degree. We then probe the model with $\delta$ in the interval $(0, 0.1)$. On the x-axis and y-axis we report respectively the bank-firm average degree $AD_{B,F}$ and the interbank average degree $AD_{B,B}$. Low values of $AD_{B,F}$ correspond to a poorly connected bank-firm network, while higher values correspond to an highly connected network. The same applies for the values of $AD_{B,B}$ in the interbank network. Different colors represent different levels of systemic risk: as shown by the vertical bar on the right-hand side of the heat-map, colors towards blue correspond to low levels, while colors toward red to high levels. Since we explore cases for $\delta < \delta^*$, we know that if the connectivity of the netwoek increases enough, the risk goes to zero. But for less connected network, the risk increases noticeably also for a relatively small fraction of NPLs. What seems to play the major role, at least for relatively large values of $\delta$, is the bank-firm topology. This is not surprising as the average degree of the bank-firm network is a measure of diversification of bank external assets. So we simply observe that more diversified banks are more resilient to an abrupt increase in NPLs. Conversely, the degree of connectivity in the bank-bank network appears to have a marginal role, at least for $\delta \geq 0.4$. If banks are not diversified enough in their exposition toward the risky firms, increasing the average degree of the interbank market will not save them from default.

We then move to the case of heterogeneous nodes. Each bank $i = 1, ..., N$ is initialized with an amount of interbank assets $A_i^{\mathrm{IB}}$ randomly drawn from a truncated power law distribution with bounded support $[5, 100]$ and exponent equal to 2.[11] Similarly, to each firm $j = 1, ..., M$ is assigned a value of total assets $F_j$, distributed according to a truncated power law with support $[5, 100]$ and exponent 2. In this case, also to make the finer

[11] The truncated power law with exponent $\tau$ and support $[a, b]$ has a distribution function $F(x) = (1 - a^{\tau} x^{-\tau+1})/(1 - (b/a)^{\tau})$.

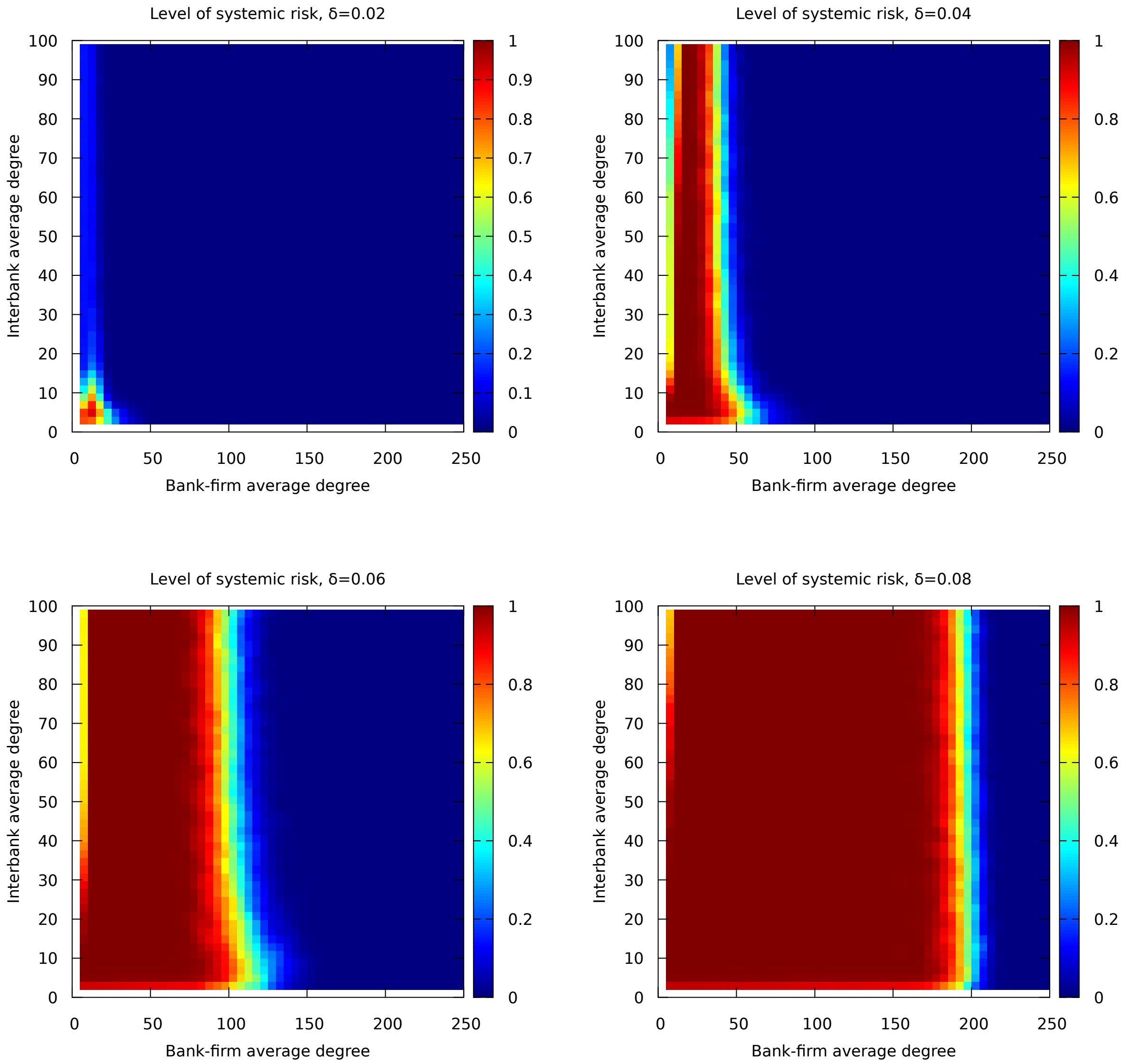



Figure 4: The figure shows the level of systemic risk associated to different levels of initial shock ($\delta$) in the homogeneous case, as a function of the average degrees of the inter-bank network ($AD_{B,B}$) and of the bank-firm network ($AD_{B,F}$). Estimates are obtained averaging over 200 independent realization of the model. Parameter values reported in Table 2.

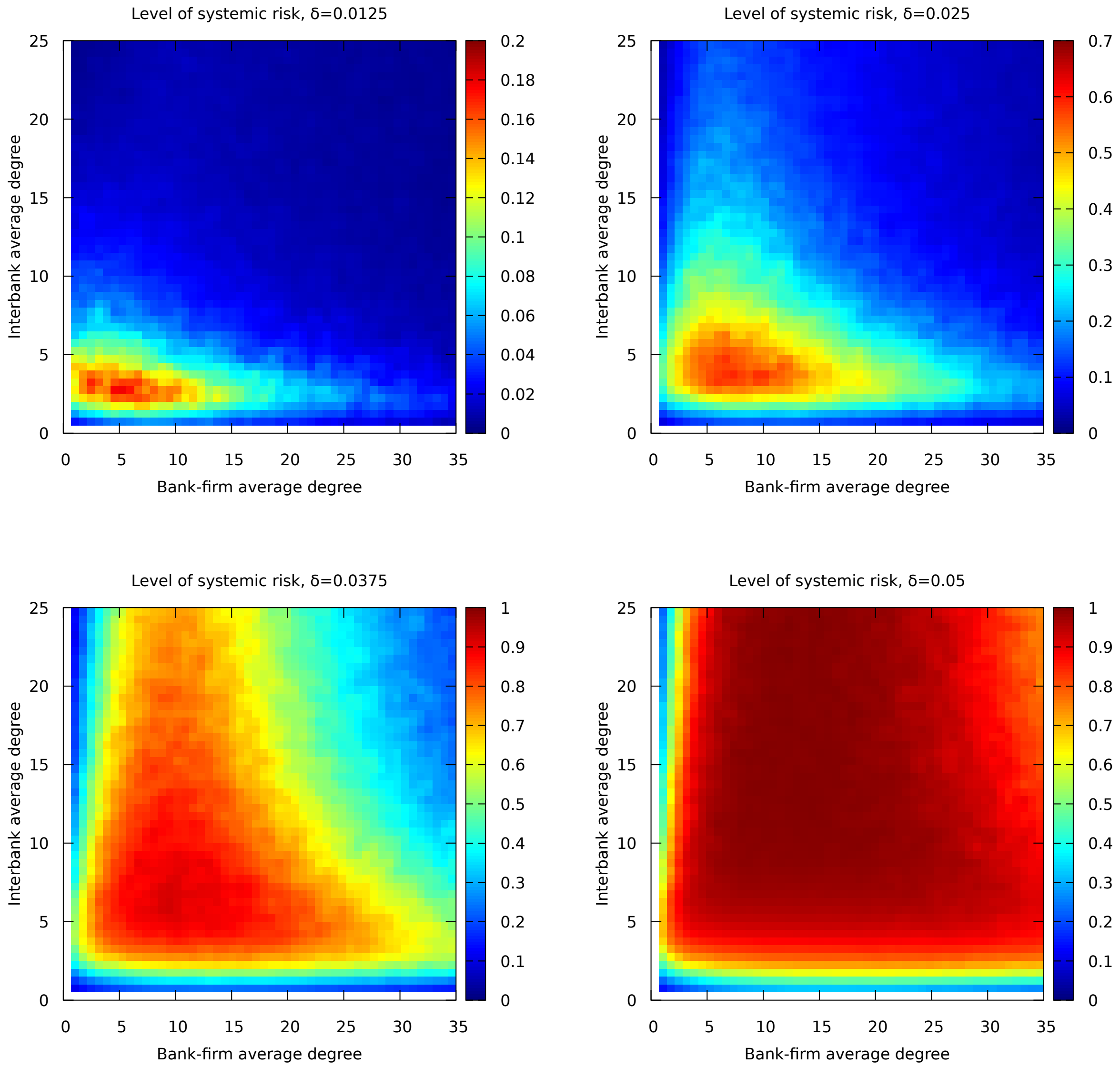



Figure 5: The figure shows the level of systemic risk associated to different levels of initial shock ($\delta$) in the heterogeneous case, as a function of the average degrees of the interbank network ($AD_{B,B}$) and of the bank-firm network ($AD_{B,F}$). Estimates are obtained averaging over 200 independent realization of the model. Parameter values reported in Table 2.

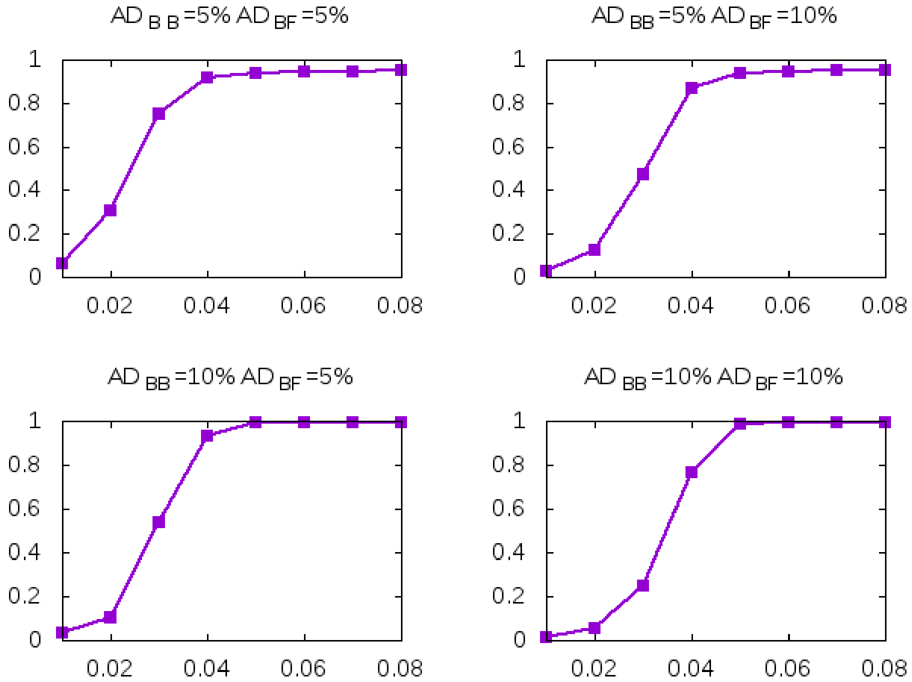



Figure 6: Fraction of defaulted banks for different levels of initial shock and different combinations of average degrees in the heterogeneous benchmark case. Estimates are obtained averaging over 200 independent realizations of the model. The average degree is expressed as a fraction of bank nodes in the case of $AD_{B,B}$ and as a fraction of firm nodes in the case of $AD_{B,F}$.

details of the models more clearly apparent, we focus on a range of values for the average degrees which is economically more reasonable, even if we probe a relatively large extent of possible values. In fact, the real value of $AD_{B,B}$ and $AD_{B,F}$ is in general not know and the few estimates of the interbank average degree present in the literature often refer only to short term lending. For example Anand *et al.* (2015) finds that the average degree for the German interbank network is 10.5, while Soramäki *et al.* (2007) finds an average degree of 15.2 for the Fedwire interbank payment network.

The four panels in Figure 5 show the level of systemic risk associated with different increases in the percentage of NPLs over total gross loans. The values of $\delta$ tested go from 1.25% to 5%, well below the critical threshold $\delta^*$. As can be seen, when heterogeneity is fully taken into consideration, the topology of the bank-bank network becomes more relevant. In fact, the figure shows that, on both the axes, the levels of systemic risk first increases and then decreases, showing a non-monotonic behaviour and peaking in the bottom-left area of all the panels. Notice that the scale of these plots is reduced. When $AD_{B,B}$ and $AD_{B,F}$ become larger the system converge to the fully connected case and the systemic risk goes to zero.[12] Also notice that an higher initial shock does not change the general shape of the plot, but rather increases the overall level of risk for a maximum of 0.7 when $\delta = 0.025$ to a maximum of 1 when $\delta = 0.05$.

[12]Given the purely illustrative intent of the work we show only some selected charts in order not to overload the reading. A full set of charts is available upon request.

The role of the size of the initial NPLs shock can be better assessed by varying $\delta$ while keeping fixed the average degrees of the interbank and bank-firm networks. Figure 6 shows the fraction of defaulted banks as a function of the level of the shock. In all the panels is evident the presence of a relatively steep phase transition, which implies that for certain level of NPLs, a small change in the magnitude of the initial shock can have very different consequences in terms of banks' defaults. These transitions allow to identify a threshold level of $\delta$, which is an important measure of the resilience of the system to external shocks. As expected this value is below $\delta^*$ but the exact position of the transition varies slightly depending on the values of $AD_{B,B}$ and $AD_{B,F}$. Comparing the four panels of Figure 6 it is possible to see that, in line with the previous analysis, an increase in $AD_{B,F}$ moves the transition to the right, making the banks more resilient to external shocks, while an increase of $AD_{B,B}$ makes the curve higher, thus enhancing the disruptive effect of bankrupcy cascades. It is worth highlighting that in the two top panels of Figure 6 the number of defaulted banks tends to 1 without reaching it. This is due to the fact that for very low values of the interbank average degree, some banks are completely disconnected and hence, if they survive the initial shock, they never fail since they cannot be reached by the contagion cascades.

# 4 Model calibration

In this Section we calibrate the model described in Section 2 and simulate it for the cases of Italy, Germany and United Kingdom. We use firm-level data from *Bankscope* and *Amadeus*, two databases produced by *Bureau van Dijk* collecting balance sheet informations on financial institutions and on public and private companies respectively. From *Bankscope* we extract informations on banks' interbank assets ($A^{\mathrm{IB}}$), total assets ($A^{\mathrm{TOT}}$) and capital ($K$), while from *Amadeus* we obtain data about firms' total assets ($F$).

*Bankscope* contains data both on banks and other kinds of financial institutions, such as funds and asset management companies. From the informations available it is not possible to immediately identify the type of institution, therefore in order to extract a sub-sample consisting only of banks, we match the data from *Bankscope* with the list of authorised credit institutions (excluding branches) published by the European Banking Authority (EBA Register of Credit Institutions)[13].

As far as *Amadeus* is concerned, we restrict our focus on firms with more than 50 employees[14], excluding NACE sectors 64.1 (Monetary intermediation) and 64.2 (Activities of holding companies), so as not to include the banks among other firms and to focus on firms whose activity is not just to own shares of other companies.

In the simulations that follows, the values of the parameters are calibrated on data from 2013[15]. While our dataset goes from 2000 to 2014 for what concerns *Bankscope* and from 2005 to 2014 for what concerns *Amadeus*[16], we use 2013 as the reference year so as to have data which are both quite recent and with a sufficiently high coverage. Indeed,

[13] For more informations visit http://www.eba.europa.eu.

[14] We do so both because the quality of data for firms with less than 50 employees is lower, both because small firms are likely to have small credit lines with banks and therefore a negligible impact.

[15] This also holds for the list of authorised credit institutions published by the EBA which we use to filter the banks.

[16] Both for *Bankscope* and for *Amadeus* the cut off date for the observations is 31 March 2015.

| Parameters | Homogeneous | Heterogeneous | Italy | Germany | United Kingdom |
|---|---|---|---|---|---|
| N | 100 | 100 | 100 | 100 | 100 |
| M | 250 | 250 | 4500 | 1500 | 23600 |
| $\eta$ | 8% | 8% | 9.2% | 8.4% | 18.8% |
| $\theta$ | 20% | 20% | 6.5% | 7.8% | 8.6% |
| $\delta^*$ | 10% | 10% | 9.8% | 9.1% | 20.5% |

Table 2: Parameters used in the numerical simulations. $N$ and $M$ are respectively the number of banks and firms in the network; $\eta$ is the fraction of capital with respect to total asset; $\theta$ is the fraction of interbank assets with respect to total assets. $\delta^*$ is the solvency critical threshold assuming the corresponding values of $\eta$ and $\theta$. For the exponents used in the linking functions in equations 2 and 3 we keep the same values in all four scenarios, namely $\alpha = 1$, $\beta = 0.25$, $\phi = 1$ and $\chi = 1$.[21]

after filtering the banks using the EBA list, the data for 2013 are the most complete in our dataset, and the coverage of banks' interbank assets, total assets, capital and firms' total assets are respectively 0.92, 0.92, 0.92 and 0.88 for Italy, 0.97, 0.97, 0.97 and 0.50 for Germany, 0.92, 0.97, 0.97 and 0.93 for the United Kingdom[17]. Based on the informations on assets and equity, for each country we obtain the empirical distribution of the capital/total asset ratio $\eta$ and the interbank assets/total assets ratio $\theta$.

It should further be noted that *Bankscope* and *Amadeus* report financial information about banks and firms at the consolidated level. In order to avoid double counting issues and keep banking groups as much aggregated as possible, we selected data associated to consolidation codes *U1*, *C2* or *C1*[18]. After this procedure, and considering only banks and firms which have a value greater than 0 for all the variables considered, we finally have 1665 banks and 25855 firms for Germany, 527 banks and 23929 firms for Italy and 163 banks and 38521 firms for U.K. Notice that, as pointed out by Duprey (2012) in relation to *Bankscope*, and the same holds for *Amadeus*, even after having considered consolidated entities, it is still possible to incur in some double counting. This problem can be solved only with information about firm ownership for all the years corresponding to the date at which the observations have been recorded. These data are not available to us.[19] In any case, for the purposes of the present work, essentially based on distributional properties, the possible occurrence of some double counting in the banks or firms sample is likely to have a small and negligible effect. However, we observed that the number of banks and firms in these national economies is too large to be simulated effectively. In order to maintain the same proportionality observed in real data, the number of nodes in the inter-bank sector is set to 100 and the number of firms proportionally adjusted.

Table 2 reports the adjusted number of banks and firms used in the simulations, as well

[17]The estimates for the other years in our dataset are anyway relatively similar to those obtained for 2013.

[18]See the guide provided by *Bureau van Dijk* for more information.

[19]Ownership data are provided by *Bankscope*, but they require an extra license; moreover ownership data are in the cross-section for the current years, therefore in order to get the evolution over time of ownership structure it is necessary to use the updated version of the database at that time.

[21]For simplicity the values of $\alpha, \beta, \phi$ and $\chi$ are taken from the literature (see for example Montagna & Lux (2013)) and chosen so as to replicate the often documented features of assortativity and disassortativity, as discussed in Section 2.2. However, in presence of data on bilateral exposures they could be empirically estimated.

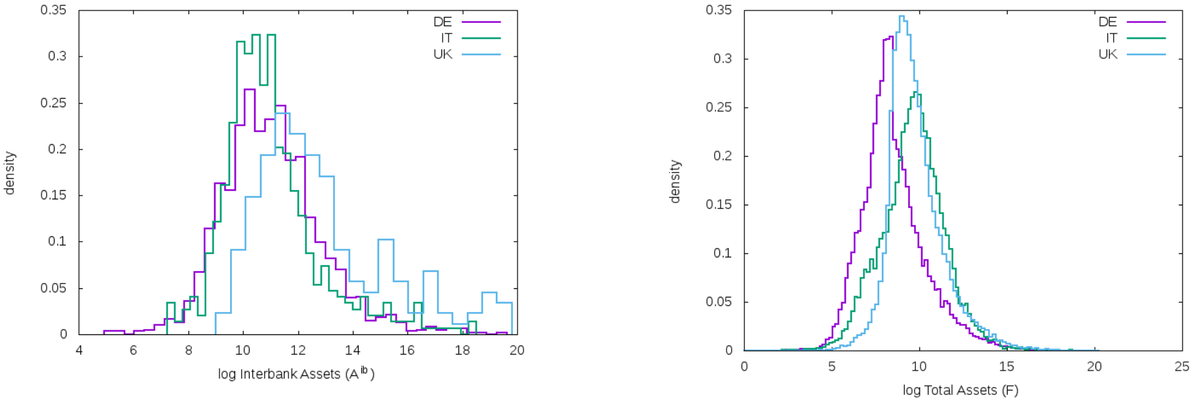



Figure 7: Empirical density (histogram) of interbank assets (panel a) and firms' total assets (panel b) for Italy, Germany and United Kingdom in 2013. Source: computations of the authors based on data from *Bankscope* and *Amadeus*.

as the values of $\eta$ and $\theta$ which have been computed as the modal values of the respective distributions. As for the high value of the estimate of the capitalization level in UK, this is roughly in line with the level of regulatory capital – Tier 1 and Tier 2 – reported by the Bank of England. The annual average of the banking sector regulatory capital is indeed about 17% in 2014, 18% in 2015 and 20% in 2016[22]. Finally, the critical value $\delta^*$ is 9.8% for Italy, 9.1% for Germany and 10% for UK. The relatively higher value for the latter is due to both a portfolio effect, with UK banks having a larger share of their portfolio invested in interbank assets, and to a leverage effect, with UK banks being more capitalized than Italian or German banks for the same level of total capital.

The empirical distributions of $A^{\mathrm{IB}}$ and $F$ in 2013 are reported in Figure 7. The initial values for inter-bank assets and firm assets are randomly sampled without replacement form these distributions.

For illustrative purposes in Figures from 8 to 10 we show the results of the simulations for the cases of Italy, Germany and United Kingdom.

For each country, we probed a range of values for the initial NPLs shock $\delta$ from zero to the respective $\delta^*$. As expected when the size of the initial shock approach the critical value, the risk becomes higher irrespective of the network structure. For lower values of $\delta$, the results obtained for the three countries are more "noisy" that the ones obtained for the heterogeneous case. Given the high firms to banks ratio, in Italy and UK the diversification aspect of banks portfolio has a central role. The average degree of the bank-firm network is what mainly decides the level of risk, while the interbank network is less relevant. In Germany this ratio is lower and the topology of the interbank network has a more prominent role. What is common in all three cases is that the level of risk cannot be, in general, effectively reduced increasing only the interbank or the bank-firm avergae degree. The most efficient strategy is always to have banks that are at the same time sufficiently diversified in their external lending and have sufficient connections with other banks.

In Figure 11 we report the fraction of defaulted banks for different levels of initial

[22]Unfortunately no value for 2013 is available as the reports of the Bank of England start in 2014. For more information visit http://www.bankofengland.co.uk/pra/Pages/regulatorydata/default.aspx

shock and different combinations of average degrees in the case of the three countries. As it can be seen, the overall effect of the average degree in the interbank and bank-firm network is different for the different countries. Nevertheless, the same considerations made for the heterogeneous apply also to the other three cases, both in terms on non-monotonic behavior of the systemic risk (with levels and position varying in the $AD_{B,B}$, $AD_{B,F}$ space according to country characteristics) and for what regards the effects of an increase in the average degrees on the fraction of defaulted banks over the total number of banks. Moreover, heterogeneity reduces the amount of NPLs for which the critical threshold occurs, as the transition to the default of the entire financial system occurs for lower values of $\delta$ compared to the homogeneous fully-connected case.

Comparing the curves in Figure 11 Germany appears to be structurally the weakest of the three countries, as the phase transition from a situation of low distress to a situation of high distress occurs earlier (namely for lower values of $\delta$) than the other two countries, for all the combinations of average degrees. The United Kingdom appears to be the most resilient among the countries considered for all the levels of connectivity, while Italy places itself in the middle. These results reflect the structural characteristics of the different economies and are based on the assumption that all the three countries are exposed to the same exogenous shock. Clearly in practice the magnitude of the shock to which countries are exposed is different and in this light the weakest country is obviously Italy, as it is the only one of the three which experienced a sustained increase in the level of NPLs, while Germany and United Kingdom registered a decline to levels equals or lower than the pre-crisis period. To give and idea of the magnitude of the issue, in Italy just from December 2012 to December 2013, NPLs increased from 124.973 millions euro to 155.885 millions euro (+24,7%)[23]. The total stock of loans made by Italian banks to the private and public sector (excluding interbank loans) at the end of 2012 was 1.927.861 millions euro, so the aggregate shock suffered by the banking system in 2013 was about $\delta = 1.6\%$[24].

# 5 Conclusions

In this paper we studied the relationship between NPLs, systemic risk and resilience of the financial system in a network perspective. We developed a model with two types of agents, banks and firms, linked one another in a network of credits and debits, and we analyzed how an exogenous shock, represented by an increase in NPLs at firm level, affects the financial system.

We first focused on a simple version of the model with homogeneous agents, which we analyzed both analytically and numerically. Then, by means of computer simulations, we studied the more realistic case of fully heterogeneous agents.

[23] Elaboration of the authors based on data from Bank of Italy and Italian banking association (ABI). For more information see https://www.abi.it/DOC_Info/Comunicati-stampa/Rapporto_mensile_maggio2014.pdf (in italian).

[24] This is clearly a rough estimate, as it assumes that all the new NPLs come from loans made before the end of 2012 (a fraction of them could come from new loans made during 2013, although likely very small), but still it gives a sense of the magnitudes at play. It would also be possible to extend the horizon to the post-crisis period and consider the cumulated increase in NPLs, for example from 2007 to 2013, but more detailed data would be required in order to give an estimate reasonably accurate.

To bring the model to reality and show its practical application we also calibrated it using firm-level data on banks and firms for the cases of Italy, Germany, and United Kingdom.

We found that the level of systemic risk varies with the level of interconnectedness of the financial network in a non-monotonic way and that in order to effectively reduce the risk, banks should at the same time diversify their external portfolio and increase the number of their neighbours in the interbank market. In terms of resilience, we showed how it is possible to derive analytically the maximum amount of NPLs sustainable by a system of homogeneous and fully connected banks. Instead when we introduced heterogeneity it was not possible to find a closed form solution and we had to rely on numerical approximations. The simulations confirmed the existence of a phase transition effect, so that small variations in the magnitude of the initial shocks can have very different consequences in terms of fraction of defaults. The results of the simulations also showed that heterogeneity reduces the amount of NPLs for which the critical threshold occurs.

Although the model presented is a simplified representation of the dynamics that lead to the emergence of systemic risk, we argue that in presence of data on bilateral exposures between banks and firms it can help assess the level of risk to which the financial system is exposed and evaluate its resilience, providing useful guidance to policy makers. For example, it would be possible to quantify whether the level of NPLs in an economy is getting critical and actions to preserve financial stability should be taken; whether the structure of the economy is such that it exposes the system to excessive risk and incentives/disincentives for diversification are advisable so as to modify the location of the system in the average degrees space; whether the level of capitalization of banks is too low or whether the amount of assets which have other financial institutions as counterpart are too high and create negative externalities for the whole system.

This paper represents a first attempt to link together the financial and the real side of the economy. We believe that the theoretical framework provided can be extended in several ways. First, a deeper investigation of the process of network formation is certainty an interesting area of research. Second, an empirical analysis of whether the network generated by our algorithm is a good approximation of reality is advisable and it might be facilitated by the growing amount of data on banks' exposures collected by regulators at international level. Third, our results abstract from the international dimension, which is obviously crucial and was central in spreading globally the crisis in 2007-2008. Including cross-countries credit relationships would allow to take into account also the international propagation of shocks. One relatively straightforward way to do so, is to draw from the gravity models developed in the trade literature and modify the linking functions used to generate the network accordingly. In our view this is a promising and potentially fruitful direction of research, which so far has not been investigated in the financial network literature and whose exploration is left for future work.

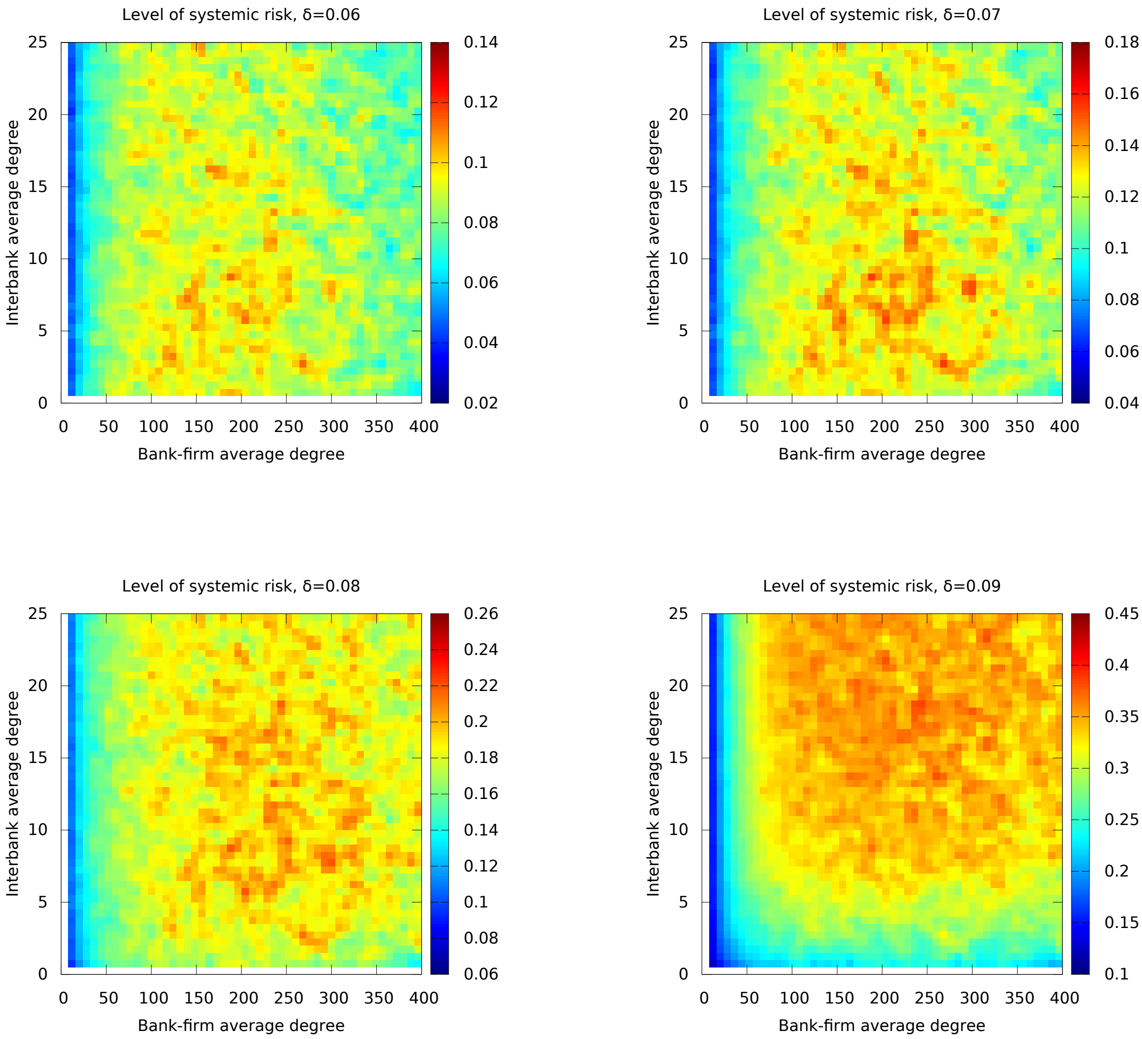



Figure 8: The figure shows the level of systemic risk associated to different levels of initial shock ($\delta$) in the Italy case, as a function of the average degrees of the inter-bank network ($AD_{B,B}$) and of the bank-firm network ($AD_{B,F}$). Estimates are obtained averaging over 200 independent realizations of the model.

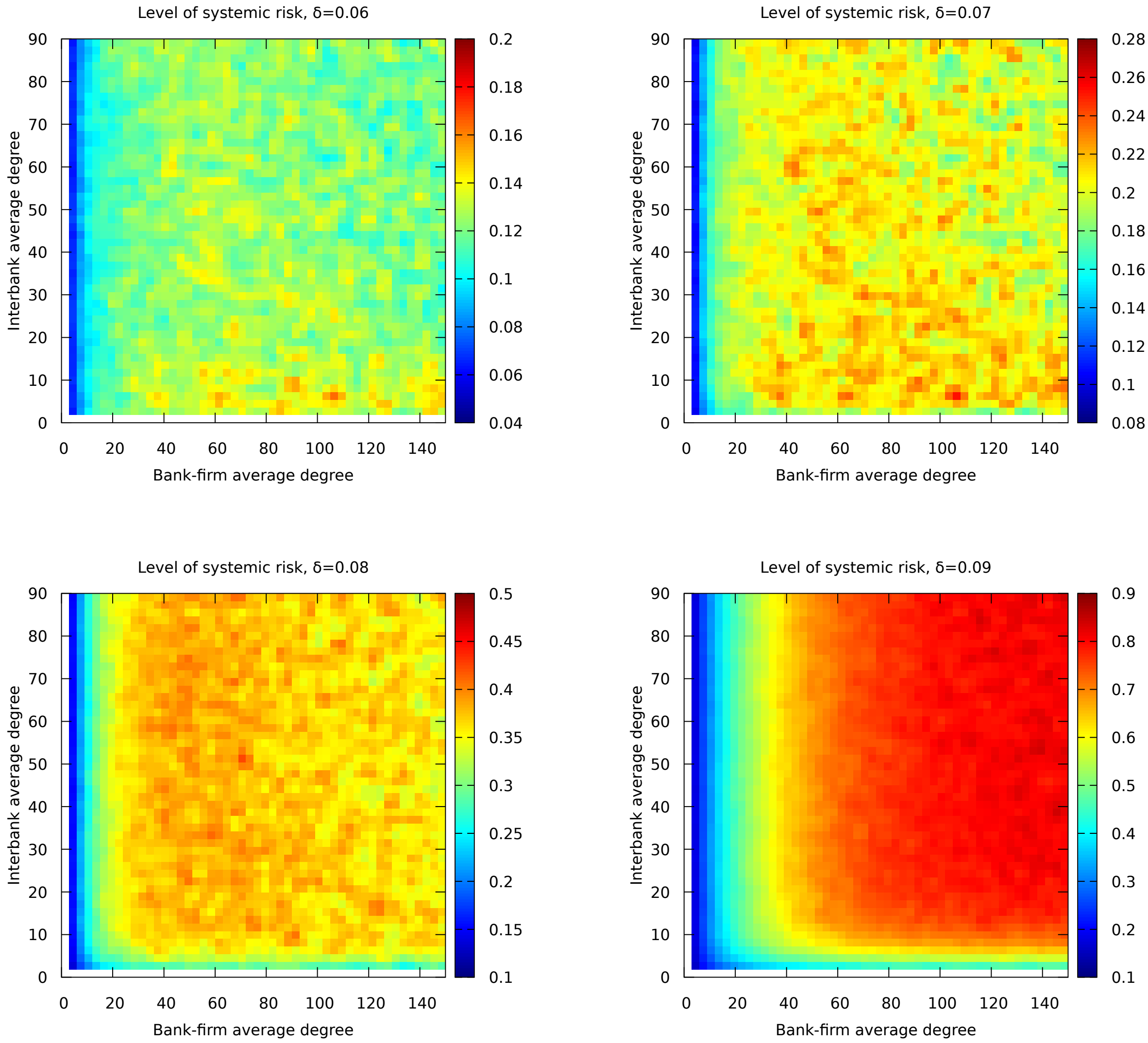



Figure 9: The figure shows the level of systemic risk associated to different levels of initial shock ($\delta$) in the Germany case, as a function of the average degrees of the interbank network ($AD_{B,B}$) and of the bank-firm network ($AD_{B,F}$). Estimates are obtained averaging over 200 independent realizations of the model.

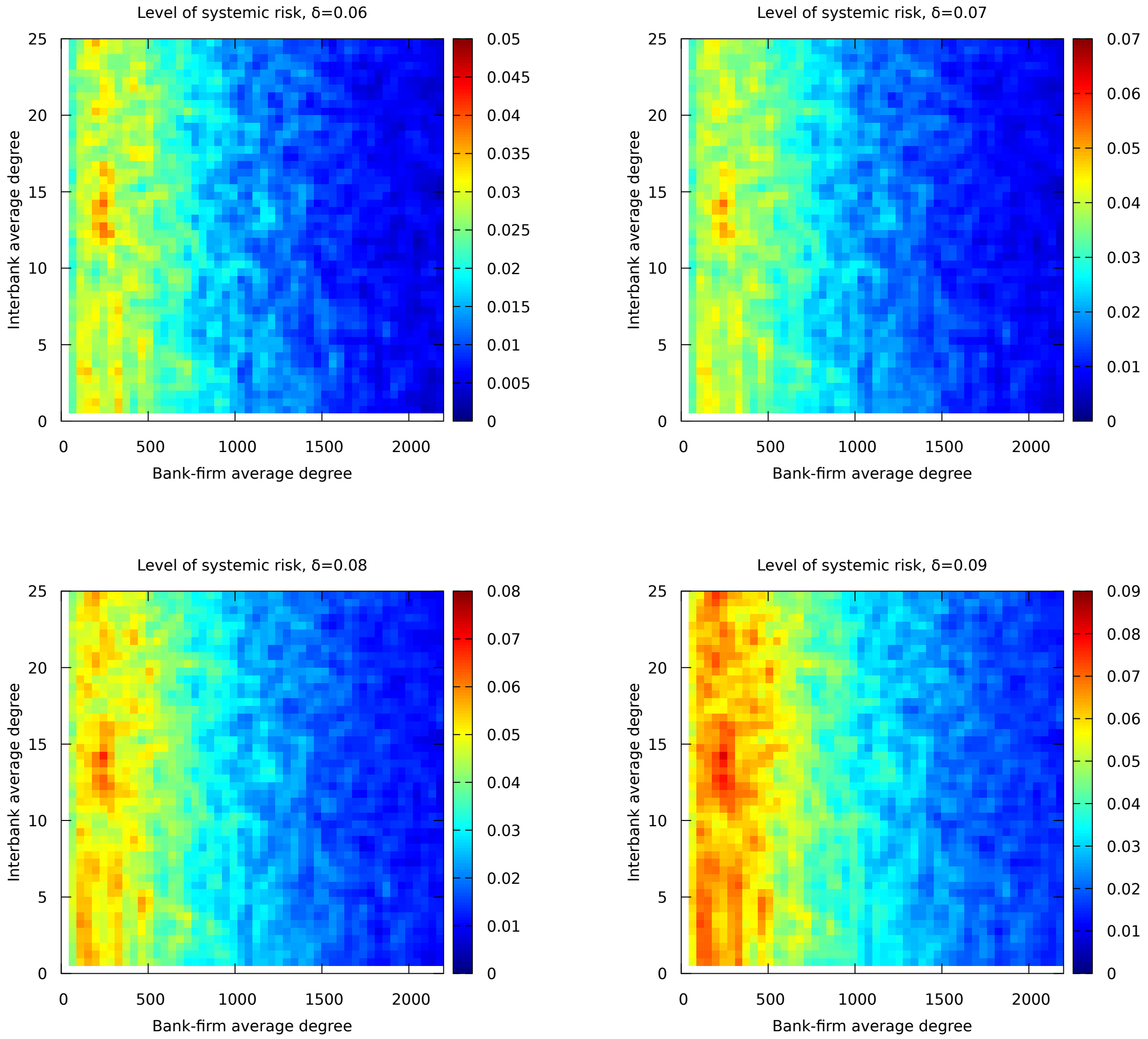



Figure 10: The figure shows the level of systemic risk associated to different levels of initial shock ($\delta$) in the U.K. case, as a function of the average degrees of the inter-bank network ($AD_{B,B}$) and of the bank-firm network ($AD_{B,F}$). Estimates are obtained averaging over 200 independent realizations of the model.

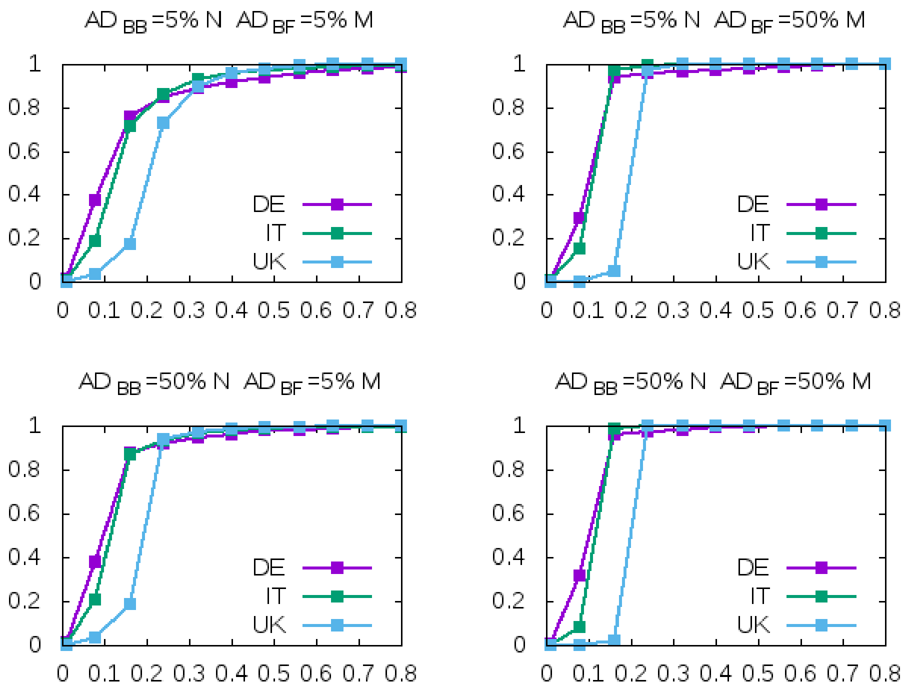



Figure 11: Fraction of defaulted banks for different levels of initial shock and different combinations of average degrees. The average degree is expressed as a fraction of bank nodes in the case of $AD_{B,B}$ and as a fraction of firm nodes in the case of $AD_{B,F}$.

# References


Acemoglu, Daron, Ozdaglar, Asuman, & Tahbaz-Salehi, Alireza. 2015a. Systemic risk and stability in financial networks. *The american economic review*, **105**(2), 564–608.

Acemoglu, Daron, Ozdaglar, Asuman E, & Tahbaz-Salehi, Alireza. 2015b. Systemic risk in endogenous financial networks. *Available at SSRN 2553900.*

Allen, Franklin, & Gale, Douglas. 2000. Financial contagion. *Journal of political economy*, **108**(1), 1–33.

Anand, Kartik, Craig, Ben, & Von Peter, Goetz. 2015. Filling in the blanks: Network structure and interbank contagion. *Quantitative Finance*, **15**(4), 625–636.

Battiston, Stefano, Gatti, Domenico Delli, Gallegati, Mauro, Greenwald, Bruce, & Stiglitz, Joseph E. 2012a. Default cascades: When does risk diversification increase stability? *Journal of Financial Stability*, **8**(3), 138–149.

Battiston, Stefano, Delli Gatti, Domenico, Gallegati, Mauro, Greenwald, Bruce, & Stiglitz, Joseph E. 2012b. Liaisons dangereuses: Increasing connectivity, risk sharing, and systemic risk. *Journal of Economic Dynamics and Control*, **36**(8), 1121–1141.

Bech, Morten L, & Atalay, Enghin. 2010. The topology of the federal funds market. *Physica A: Statistical Mechanics and its Applications*, **389**(22), 5223–5246.

Beltran, Daniel O, Bolotnyy, Valentin, & Klee, Elizabeth. 2015. Un-Networking: The Evolution of Networks in the Federal Funds Market.

Caldarelli, Guido. 2007. Scale-free networks: complex webs in nature and technology. *OUP Catalogue.*

Caldarelli, Guido, Capocci, Andrea, De Los Rios, Paolo, & Munoz, Miguel A. 2002. Scale-free networks from varying vertex intrinsic fitness. *Physical review letters*, **89**(25), 258702.

Catanzaro, Michele, Boguñá, Marián, & Pastor-Satorras, Romualdo. 2005. Generation of uncorrelated random scale-free networks. *Phys. Rev. E*, **71**(Feb), 027103.

Chinazzi, Matteo, & Fagiolo, Giorgio. 2013. Systemic risk, contagion, and financial networks: A survey. *Contagion, and Financial Networks: A Survey (April 2, 2013).*

Chinazzi, Matteo, Pegoraro, Stefano, & Fagiolo, Giorgio. 2015. Defuse the Bomb: Rewiring Interbank Networks. *LEM Papers Series*, **16**.

De Masi, Giulia, & Gallegati, Mauro. 2007. Debt-credit economic networks of banks and firms: the Italian case. *Pages 159–171 of: Econophysics of Markets and Business Networks.* Springer.

Duprey, Thibaut. 2012. Bank Ownership and Credit Cycle: the lower sensitivity of public bank lending to the business cycle.

Eisenberg, Larry, & Noe, Thomas H. 2001. Systemic risk in financial systems. *Management Science*, **47**(2), 236–249.

Elliott, Matthew, Golub, Benjamin, & Jackson, Matthew O. 2014. Financial Networks and Contagion. *American Economic Review*, **104**(10), 3115–53.

Friedman, Thomas L. 2006. *The world is flat [updated and expanded]: A brief history of the twenty-first century*. Macmillan.

Gai, Prasanna, & Kapadia, Sujit. 2010. Contagion in financial networks. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Science*, rspa20090410.

Gai, Prasanna, Haldane, Andrew, & Kapadia, Sujit. 2011. Complexity, concentration and contagion. *Journal of Monetary Economics*, **58**(5), 453–470.

Haldane, Andrew G, & May, Robert M. 2011. Systemic risk in banking ecosystems. *Nature*, **469**(7330), 351–355.

Iori, Giulia, Jafarey, Saqib, & Padilla, Francisco G. 2006. Systemic risk on the interbank market. *Journal of Economic Behavior & Organization*, **61**(4), 525–542.

Iori, Giulia, De Masi, Giulia, Precup, Ovidiu Vasile, Gabbi, Giampaolo, & Caldarelli, Guido. 2008. A network analysis of the Italian overnight money market. *Journal of Economic Dynamics and Control*, **32**(1), 259–278.

Montagna, Mattia, & Kok, Christoffer. 2013. *Multi-layered interbank model for assessing systemic risk*. Tech. rept. Kiel Working Paper.

Montagna, Mattia, & Lux, Thomas. 2013. *Hubs and resilience: towards more realistic models of the interbank markets*. Tech. rept. Kiel Working Paper.

Nier, Erlend, Yang, Jing, Yorulmazer, Tanju, & Alentorn, Amadeo. 2007. Network models and financial stability. *Journal of Economic Dynamics and Control*, **31**(6), 2033–2060.

Schweitzer, Frank, Fagiolo, Giorgio, Sornette, Didier, Vega-Redondo, Fernando, Vespignani, Alessandro, & White, Douglas R. 2009. Economic networks: The new challenges. *science*, **325**(5939), 422.

Soramäki, Kimmo, Bech, Morten L, Arnold, Jeffrey, Glass, Robert J, & Beyeler, Walter E. 2007. The topology of interbank payment flows. *Physica A: Statistical Mechanics and its Applications*, **379**(1), 317–333.

Stiglitz, Joseph E. 2010. Risk and Global Economic Architecture: Why Full Financial Integration May Be Undesirable. *American Economic Review*, **100**(2), 388–392.

Upper, Christian. 2007. Using counterfactual simulations to assess the danger of contagion in interbank markets.

Yellen, Janet L. 2013. *Interconnectedness and Systemic Risk: Lessons from the Financial Crisis and Policy Implications: a speech at the American Economic Association/American Finance Association Joint Luncheon, San Diego, California, January 4, 2013*. Speech 631. Board of Governors of the Federal Reserve System (U.S.).