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Institute of Economics
Scuola Superiore Sant'Anna

Piazza Martiri della Libertà, 33 - 56127 Pisa, Italy
ph. +39 050 88.33.43
institute.economics@sssup.it

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Theories of market selection: a survey

Luca Fontanelli [a]

[a] University of Brescia, Italy.

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Theories of market selection: a survey

L. Fontanelli*

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Abstract

We provide a survey of the main mechanisms of market selection used in economics. We gather them in three theoretical paradigms (rational equilibrium, Simonesque and evolutionary), that we try to reconcile in terms of underlying laws of selection. We show that the three paradigms have been converging in their focus on firm heterogeneity and increasing returns. These selection mechanisms are however fostered by theories which differ in terms of sources of increasing returns, generating mechanisms of firm heterogeneity, firm rationality and emphasis on equilibrium states vis-à-vis out-of-equilibrium dynamics. Our discussion suggests that the convergence between the three theoretical paradigms is taking place in the direction of research, which is aimed at the replication of empirical patterns related to firm heterogeneity, rather than in the theory underlying selection mechanisms.

Keywords: selection, competition, monopolistic competition, quasi-replicator, Gibrat's Law
JEL Codes: L10, D20, D40

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*University of Brescia
✉ Luca Fontanelli – luca.fontanelli@unibs.it.

1 Introduction

In this survey we discuss the main selection mechanisms used in economics by grouping them in three theoretical paradigms. Our definition of selection mechanism encompasses all the market devices that determine firms' size, growth rate and survival as the outcome of competition among firms.

Economic theories of industrial organization have been influenced by two concepts in their description of the relation between firms behaviour and market structures: perfect competition and perfect selection. In last decades, economics has experimented a widespread deviation from perfect competition thanks to the emergence of theories encompassing firm heterogeneity and increasing returns (see Nelson and Winter, 1982; Dixit and Stiglitz, 1977; Melitz, 2003). In contrast, perfect selection (i.e., the fact that only the best firms are allowed to survive) is still assumed by most theoretical models. In these models, non-rational behaviour by firms is not taken into account, because, it is argued, only rational firms would survive to market selection (Friedman, 1953). However, several empirical findings show that real-world markets are not able to select the best out of existing firms. First, there is no evidence of convergence to a "best" or representative firm because the width of firms' productivity differentials is large and exceptionally persistent in time (Bartelsman and Dhrymes, 1998; Syverson, 2011). Second, productivity decomposition exercises have found that aggregate productivity growth is mostly governed by firm learning, and attribute a residual role to selection (Foster et al., 2001; Bottazzi et al., 2010; Dosi et al., 2015). Third, selection also imperfectly works in terms of firms' entry in and exit from markets, as highlighted by the imperfect productivity sorting in foreign markets (non-productive exporters cohexist with productive non-exporters, see e.g., Mayer and Ottaviano, 2008; Melitz and Trefler, 2012). Finally, empirical findings have shown that resources allocation to firms may be highly imperfect when compared to optimal equilibrium allocations (Hsieh and Klenow, 2009, 2018).

Many theories of imperfect competition and imperfect selection have been developed with the aim of explaining some of the above findings. The main contribution of this survey is to rationalize these theories in three broad theoretical classes based on the adopted selection mechanism: rational equilibrium models, models inspired by the work of Herbert Simon on firm dynamics (also Simon-inspired or Simonesque henceforth) and evolutionary models. Next, we discuss how selection mechanisms are represented in each of them, with a special focus on the role of firm heterogeneity and increasing returns. We define as rational equilibrium models the theoretical contributions making use of both monopolistic competition (Dixit and Stiglitz, 1977) and rational expectations on production, entry and exit decisions (Hopenhayn, 1992). These models are characterized by optimizing firms, who take economic decisions with a high degree of rationality and are selected on the basis of productivity, which is in turn perfectly mapped into the firm size distribution (see Melitz, 2003). In Simonesque models, selection is modelled via reduced form stochastic processes inspired by the work of Gibrat (Gibrat, 1931), wherein firms are hit by random growth shocks independent from size.

These models stress the role of firms' interaction in reduced form and, differently from the original Gibrat's work, are generally based on the presence of *dynamic increasing returns* stemming from the allocation of market opportunities to firms (see Simon, 1955; Simon and Bonini, 1958).[1] Finally, evolutionary models study the emerging properties of the firm population as the result of the out-of-equilibrium competitive interaction between boundedly rational firms (see Nelson and Winter, 1982). Selection in evolutionary economics is generally driven by both firms' size and productivities, and it is represented as the outcome of a *dynamic* process, whose focus is indeed on firms' growth rates, rather than on firm size. Most of evolutionary models employs the quasi-replicator equation (see Silverberg et al., 1988; Metcalfe, 1994, 1998) as a selection mechanism. One conclusion of our survey is that the above three families of model have been converging to similar research directions, aimed at the replication of empirical patterns mostly related to firms' micro-data, and heuristics, which are based on firm heterogeneity and increasing returns.

Increasing returns and firm heterogeneity play indeed a fundamental role in all the three research paradigms. The importance of increasing returns has been acknowledged since Adam Smith's and Alfred Marshall's contributions.[2] The evidence in favour of decreasing returns is relatively weak if compared to the ones supporting increasing returns, and may apply to limited and unrepresentative cases. It is indeed very unlikely that the presence of decreasing returns, stemming for example from communication and organizational problems arising when the productive scale of companies increases, can offset the strength of increasing returns arising from presence of re-usable knowledge and information embedded in organizations (Arrow, 1996), the processes associated to dynamic increasing returns (Simon and Bonini, 1958) and to cumulative technical change (Nelson and Winter, 1982) and economies of scale (Dixit and Stiglitz, 1977) or of scope (Teece et al., 1994; Breschi et al., 2003), between the others. However, the use of increasing returns can hardly be reconciled with the formal structure of perfect competition because, in their presence, prices cannot equate marginal costs.

Firm heterogeneity has attracted the attention of scholars of different streams as well, first in the Simonesque and evolutionary tradition and more recently also in the rational equilibrium paradigm. A multitude of empirical studies unveils how heterogeneity concerns several dimensions of firms at once. Inter alia, markets are populated by firms with heterogeneous size (Simon and Bonini, 1958; Axtell, 2001), productivity (Baily et al., 1992; Bartelsman and Dhrymes, 1998; Doms and Bar-

[1]Dynamic increasing returns are also referred to as preferential attachment models or success-breed-success mechanism.

[2]The most well known concept from "The Wealth of Nations" is probably the relation between the division of labour and the learning by doing, which entails increasing returns in production. Alfred Marshall argued in favour of the general validity of increasing returns in its "Principle of Economics" (p. 265): *In other words, we say broadly that [...] the part in which man plays shows a tendency to increasing return. The law of increasing returns may be worded thus: an increase of labour and capital leads generally to improved organization, which increases the efficiency of the work of labour and capital.*

telsman, 2000), age (Coad, 2010b,a; Calvino et al., 2020) and growth (Hymer and Pashigian, 1962; Evans, 1987; Stanley et al., 1996; Sutton, 2002; Calvino et al., 2018). Such a widespread heterogeneity raises a number of issues related to selection mechanisms. How comes that several domains of firms' characteristics are heterogeneous at once and how are these linked? How can firms with huge differentials in size and productivity survive together within same markets? Which mechanisms are able to reconcile firms' performance and productivity differentials to selection effects of competition? Economists have not yet reached a consensus in answering these questions. What is self-evident is that the existence of wide differentials in firm characteristics sits uneasily with models of perfect competition and selection because firms with very different scale, performance, productive techniques and age survive together within the same competitive environment, in disagreement with both the representative and rational agent assumption.

At the same time, even though the three paradigms acknowledge the role of firm heterogeneity and increasing returns, they markedly differ in terms of key assumptions. These differences pertain the sources of increasing returns, the mechanisms generating firm heterogeneity, firm rationality and the focus on equilibrium (either static or steady) states vis-à-vis dynamic out-of-equilibrium outcomes.

First, even though increasing returns are common to all the three research paradigms, their sources are different. Returns in rational equilibrium models are based on the existence of fixed costs and are therefore static. In evolutionary and Simonesque models, instead, they are part of the selection process and are thus dynamic. As a consequence, the effect of increases in the strength of returns on concentration levels is mediated by different effects of competition on the basis of the selection paradigm considered. In rational equilibrium models an increase in fixed costs tends to positively affect concentration by lowering the level of competition, whereas in Simonesque and evolutionary models higher levels of concentration are the natural consequence of increases in competitive pressures induced by stronger dynamic increasing returns. It falls that higher concentration levels are induced by less tough competition policies in rational equilibrium models and, conversely, by harsher ones in the Simonesque and evolutionary traditions.

Second, all the three research paradigms are characterised by the presence of heterogeneous firms. Firm heterogeneity in rational equilibrium models can be to a great extent explained by their assumptions, whereas in evolutionary and Simonesque models it endogenously emerge from the interaction between firm learning and market selection.

Third, the strength of firms' rationality remarkably varies across models. Firms in rational equilibrium models are perfectly rational. They are able to forecast future profits and productivities, and information is generally perfect. Simonesque models are neutral on firm rationality because firms do not take economic decisions in those models. Firms in evolutionary models are boundedly rational, as they cannot make predictions and optimize their production decisions due to the assumption of Knightian uncertainty. The differences in how selection takes place in models are reflected by the choice of the firm rationality type. In rational equilibrium models entry, exit and production are optimal decisions by firms, *"as if"* a competitive interaction actually took place and drove out of the market non-rational firms. In Simonesque and evolutionary models selection is based on a competitive process of firms' market interaction, which comes in a more reduced form in Simonesque models with respect to evolutionary ones.

Finally, another distinguishing trait in the three research paradigms concerns the presence of a (static or steady state) equilibrium vis-à-vis the focus on out-of-equilibrium dynamics. Rational equilibrium models are characterised by equilibrium outcomes wherein the characteristics of firms and industries are fixed. Similarly, Simonesque models converge to invariant distributions of firms characteristics, that remain therefore fixed. In evolutionary models the use of quasi-replicators is usually coupled with firms' learning. This prevents the convergence of the quasi-replicator to monopoly, allowing out-of-equilibrium patterns to emerge.

In the remainder of this paper we further develop our arguments by discussing separately which features characterize rational equilibrium, Simonesque and evolutionary models and how they deal with imperfect competition devices respectively in sections 2, 3 and 4. In section 5, we compare the three paradigms and we search for elements of convergence and divergence among them by dividing the discussion in 4 main blocks (heterogeneity, rationality, returns and heuristic of research). Finally we draw conclusions in section 6.

2 Firm selection in rational equilibrium models

We start by surveying the main contributions related to rational equilibrium models. These models are based on two pillars, monopolistic competition (Dixit and Stiglitz, 1977) and the threshold approach to firm heterogeneity (Hopenhayn, 1992), that have been merged together in the workhorse model of industrial dynamics and international trade by Melitz (2003). Monopolistic competition introduces increasing returns at the firm level by including fixed costs. The threshold approach to firm heterogeneity introduces firm heterogeneity by modelling entry and production fixed costs, that generate productivity thresholds to market entry and exit for firms. As a consequence, only firms that are enough productive decide to enter into the market and to produce. The early development of this strand of models encompasses both a strong form of firm rationality and evolutionary selection features (see Jovanovic, 1982).[3] However, the latter have been abandoned in subsequent development of rational equilibrium models. Finally, the model by Melitz (2003) merge together the two approaches in a framework encompassing both increasing returns to scale and firm heterogeneity.

[3]Evolutionary selection focuses on firm learning and out-of-equilibrium patterns, with on average more productive firms expanding at the expenses of less productive ones.

2.1 Monopolistic competition

Alfred Marshall has probably been the first economist to conjecture the existence of markets not governed by the laws of perfect competition or of monopoly. In its 'Principle of Economics' he acknowledges the existence of market niches, or *"special markets"*, in which each producer deals with its own specific demand. The notion of Marshall's special markets unveils a competition of monopolistic type, in which the characteristics of the market and of its niches overlap to some extent. From the Principle of Economics (p. 415):

"We must take account of the fact that [..] the relations between the individual producer and his special market differ in important respects from these between the whole body of producers and the general market."

Subsequent attempts to formalize monopolistic competition have been equally unsuccessful (Chamberlin, 1933; Robinson, 1933). Also, it is a well-spread opinion among historians of economic thought (Archibald, 1987; Brakman and Heijdra, 2003) that the theory of Marshall, Chamberlin and Robinson were not analytically solid enough to stand against the Arrow-Debreu workhorse model of perfect competition. During the Seventies, a revival of imperfect competition theories was fostered by the contributions of Spence (1976) and Dixit and Stiglitz (1977) (henceforth SDS). Since then, the monopolistic competition of the SDS framework has been adopted as the backbone of hundreds of models, spanning from industrial dynamics and international trade to Dynamic Stochastic General Equilibrium (DSGE) models.[4] Monopolistic competition is characterised by a general equilibrium outcome in which a large number of optimizing firms produce horizontally differentiated goods and levy a price higher than marginal costs, behaving non-strategically and making zero profits. In equilibrium, each firm produces a different, although imperfect substitute product/variety, and consumers buy a small, but positive quantity of each variety. The main assumptions of the model are consumers' love for variety, imperfect substitutability between products and increasing returns to scale. Love for variety depends on Constant Elasticity of Substitution (CES) utility function U:

$$U = \left(\int_0^N q(\omega)^\rho d\omega \right)^{\frac{1}{\rho}}, \quad \rho = \frac{\sigma - 1}{\sigma}, \quad \rho \in (0, 1) \tag{1}$$

where $q(\omega)$ is quantity of the variety ω, σ is the elasticity of substitution of products and captures the level of firm competition. Love for variety depends on U being concave in individual varieties ($0 < \rho < 1$) and thus on consumers preferring differentiated bundles over more quantity of same

[4]Other attempts at modelling imperfect competition through product differentiation were not equally successful. The framework by Lancaster (1979), based on Hotelling (1976), shares the same scope and predictions of Dixit and Stiglitz (1977). Consumers choose their optimal consumption on the basis of the distance of goods from their preferred characteristics. Lancaster's *"proved less tractable and hence less fruitful than the Dixit-Stiglitz specification"* (Neary, 2003, p. 160) and, consequently, the Dixit and Stiglitz (1977) became the workhorse model of imperfect competition.

goods. The maximized utility takes the following form:

$$U = N^{\frac{1}{1-\sigma}} \frac{I}{P} \tag{2}$$

where N is the number of products/varieties/firms, I is consumer income and P is the general level of prices. Thanks to CES preferences, U is an increasing function of the number of products in the market (love for variety), and a small, but positive quantity of each good is purchased by each consumer. Labour is the only productive input. The labour demand function is linear in the quantity (wage w is the numeraire) and constitutes thus the only source of costs for firms:

$$L = F + cq \tag{3}$$

where F are the fixed costs and c are the marginal costs. Returns to scale are increasing thanks to fixed costs, and average costs cannot be minimized. The will of firms to produce more and charge lower prices is counterbalanced by the consumers desire to buy the most different variety of products. In equilibrium, price, quantity and number of firms are determined by the following relations:

$$P = \frac{c}{\rho}, \quad Q = \frac{F}{c}(\sigma - 1) = \frac{F}{P}\sigma, \quad N = \frac{L}{F\sigma} \tag{4}$$

Changes in the level of substitutability σ cause the ones of prices, quantities and number of varieties. Since each good is an imperfect substitute of the other, firms can sell their products at a price higher than marginal cost c by levying the positive mark-up $\frac{1}{\rho}$. Increases in σ entails a lower level of price and a decrease in number of varieties/firms because products will be more similar. Concerning fixed costs F, they only affect firm production and the number of varieties. Higher fixed costs are associated to higher optimal production and to a lower number of firms/varieties because a higher amount of production will be necessary to cover fixed cost in order to stay in the market. Accordingly, an increase in fixed costs also leads to a decrease in variety, because a lower number of firms can afford to stay in the market. Finally, notice that there is a net distinction between the role of costs, as fixed costs only affect production, whereas variable costs affect both prices and quantities.

2.2 Firms heterogeneity and fixed costs

Another building block of modern rational equilibrium models is firm heterogeneity, that was however first modelled in Simonesque and evolutionary models (see Simon and Bonini, 1958; Nelson and Winter, 1982). The two seminal contributions that inspired subsequent rational equilibrium models are Jovanovic (1982) and Hopenhayn (1992). They both aim at explaining new empirical evidence on firm dynamics as stemming from processes of firm learning. In addition, they are both models of perfect competition belonging to the tradition of "passive" (i.e., non-intentional) learning models, as well as to models of evolutionary selection, wherein more productive firms expand and less productive firms shrink.[5] In both models, the supplied goods are homogeneous, but productivities are heterogeneous. An initial productivity distribution must be assumed and its subsequent dynamics governs both market selection and the evolution of firms' characteristics. Both models are characterised by a transitional dynamics that converges to a steady state. Firms are rational because they take decisions on production, entry and exit based on future expectations on productivity, whose distribution is known to firms.

Jovanovic (1982) introduces firm heterogeneity to motivate the higher volatility of smaller firms (see Hymer and Pashigian, 1962).[6] In Jovanovic (1982) firms are uncertain about their own "true" productivity a_i and this generates a dynamic pattern, wherein incumbents acquire information on their productivity from past realisations of productivity shocks and thus realize whether they are enough productive to stay in the market. Firms' productivities are drawn from the distribution $a_i \sim N(\bar{a}, \sigma_a^2)$, which is known to firms. At the time of entry, firms believe to be $\bar{a}$ (see the average of the productivity distribution) productive and they produce $q(\bar{a})$. At each time step the firm productivity is subject to an idiosyncratic 0-centred shock $\epsilon_{i,t} \sim N(0, \sigma^2)$. After entry, firms believe to be $\bar{\eta}_{i,t}$ productive, where $\bar{\eta}_{i,t}$ is the average of past productivity realisations. The source of firm heterogeneity lies in the imperfect information over firms' own productivities. As time goes by, firms learn their true productivity levels. Intuitively, by taking the limit for time going to infinity ($T \to \infty$) of the following equation, one retrieves the initial level of productivity:

$$\bar{\eta}_{i,T} = \frac{\bar{a} + \sum_{t_0}^{T} (a_i + \epsilon_{i,t})}{T+1} = \frac{\bar{a}}{T+1} + \frac{T a_i}{T+1} + \frac{\sum_{t_0}^{T} (\epsilon_{i,t})}{T+1} \tag{5}$$

Firms' exit happens when the expectations of a firm's future value is lower than the one of a fixed exogenous option. Firms with lower "true" productivities tend to shrink exit on average before firms with higher "true" productivity.[7] The productivity and size distributions tend to diverge in time because surviving firms are more productive, and thus produce more, on average, than failing firms and entrants. The variance of growth rates decreases with age, and thus with size conditional on survival, because productivity beliefs become more precise in time and survivors are on average larger and more productive. Finally, the model converges to steady state equilibrium in which price predictions by firms are met, firm productivity converges to a constant and entry and exit do not take place anymore.

Hopenhayn (1992) generalizes the baseline model of Jovanovic (1982) and introduces two main novelties to explain the evidence on firm entry and exit (see Dunne et al., 1988). First, the evolution of productivities is modelled as a Markov process characterised by persistent shocks. Second, the

[5]In passive learning models, firms' productivity is determined as a stochastic process and firms cannot invest. Conversely, in "active" learning models (Ericson and Pakes, 1995) firms can invest in R&D to boost their productivity.

[6]As we discuss more in depth in section 3, this stylised fact contrasts with the predictions of Gibrat's Law, that postulates the independence between growth rate and size.

[7]This feature of (Hopenhayn, 1992) reminds the selection mechanisms of evolutionary models.

inclusion of fixed costs of entry and production allows for entry/exit dynamics also in equilibrium. Firms face a fixed cost of entry F_e, and incur a fixed cost of production F to produce. The productivity level of entrants is drawn from a general distribution, whose density function we label with $g(a)$, after paying the entry fixed cost.[8] The productivity of incumbents is subject at each time step to a permanent shock whose distribution is known to firms. The expectations on the future value of firms depend on the one-step ahead productivity level (i.e., the productivity level times the average shock). Differently from Jovanovic (1982), shocks are also persistent, meaning that the more positive shocks hit a firm in time, the higher will be the probability of positive shocks in the future.[9] The presence of fixed costs of entry lowers the level of expected profits for entrants and thus the average firm future value, making entry more challenging for firms. Entry happens when:

$$F_e \leq \sum_a v(a, \mu) \cdot g(a) \tag{6}$$

where $v(a, \mu)$ is the present value of a firm with productivity a at industry state μ. In other words, entrants decide to pay the fixed cost of entry if the average value of the firm at the industry state μ is greater than the fixed costs of entry. Firms' exit depends on whether future profits expectation will cover the fixed cost of production F. Incumbents decide not to cease their production if their current productivity $a_{i,t}$ is greater than the minimum productivity level ensuring a non-negative firm value, given one-step ahead change in productivity. They exit according to a productivity threshold rule:

$$a_i < a^*, \;\; a^* \; s.t. \sum_a v(a, p) \cdot M(a|a^*) = 0 \tag{7}$$

where the exit productivity threshold a^* is the value of productivity such that the expected value of firms based on the one-step ahead productivity distribution (as represented by the Markov chain notation $M(a|a^*)$) equalizes the firm value to 0. An increase in the fixed costs of entry leads to a general decrease in market toughness in the model. The entry rate decreases, the price level and expected profits increase. Consequently, an increase in F_e also causes the decrease in the exit productivity threshold a^* and in the exit rate via $v(a, p)$. Accordingly, a change in the fixed costs of entry has ambiguous effects on the firm size distribution and on concentration. The increase in the fixed cost of production F induces higher firm values and profits by increasing the exit productivity threshold a^*. Furthermore, under certain conditions an increase in fixed cost of production leads to stochastically larger firm size distributions, meaning that less competitive markets are associated with higher concentration induced by higher market toughness.[10] When fixed costs of production are greater than the ones of entry, the model converges to an equilibrium in which the mass of entrants is equal to the one of exiters, in line with the empirical evidence of Dunne et al. (1988).

[8] This mechanism is often described as a lottery. Firms buy the lottery ticket for F_e (i.e., the fixed costs of entry) and they get a productivity as a prize.

[9] In this respect, this model also encompasses dynamic increasing returns in learning as many other evolutionary models (see Dosi et al., 2010).

[10] See Proposition 6 from Hopenhayn (1992).

In Jovanovic (1982) and Hopenhayn (1992), firms that are on average more (respectively, less) productive are more (respectively, less) likely to grow over time and to survive competition. The models emphasize thus how firm selection depends on a learning process, similarly to the evolutionary paradigm (see section 4). They retain however a series of features which are at odds with the evolutionary paradigm and does not allow to classify them as such. Firms are indeed rational maximizers and distributions of productivities and of their shocks are known to firms, differently from evolutionary models wherein firms are boundedly rational and take decision based on rules of thumb and past information.[11]

2.3 Blending all the ingredients: the Melitz (2003) model

The first contribution to model heterogeneous agents in a monopolistic competition environment is the work of Melitz (2003), which aims at providing a theoretical intra-industry trade framework consistent with the evidence on exporters self-selection (Bernard and Jensen, 1995, 1999) and on the reallocation effects of trade liberalization (Pavcnik, 2002; Trefler, 2004). The Melitz (2003) model is based on new trade theory models (see Krugman, 1980; Helpman, 1981), which employs monopolistic competition to explain the evidence on intra-industry trade between similar countries (Balassa, 1966; Grubel and Lloyd, 1975).[12] The most notable theoretical results of the new trade theory are the so called new gains from trade (as opposed to the new new gains from trade of Melitz, 2003). In an open economy consumers buy a small amount of each domestic and foreign product, inducing the availability of an increased number of horizontally differentiated varieties in each economy. This result is achieved because all firms in all countries export thanks to consumers love for variety.

[11]The models by Hopenhayn (1992) and by Dixit and Stiglitz (1977) have been employed to micro-found macroeconomic models and business cycle theories. On the one hand, the structure of Hopenhayn (1992) has been used to study the effects of firm productivity shocks on the business cycle and on aggregate volatility (see .g., Clementi and Palazzo, 2016; Carvalho and Grassi, 2019). On the other hand, DSGE models (see e.g., Woodford, 2003; Gali, 2015) build upon a Real Business Cycle (Long and Plosser, 1983; Kydland and Prescott, 1982) structure, that is augmented with a number of distortion, such as nominal rigidities (see e.g., Calvo, 1983; Blanchard and Kiyotaki, 1987). Firm selection is based on monopolistic competition, that also serves to introduce demand shocks through preferences. The DSGE micro-foundation generates a vector auto-regressive (VAR) macro-structure that allow one to study business fluctuations as the outcome of different types of exogenous structural shocks, including the ones to fiscal and monetary policy. DSGE models have been harshly criticized in the wake of the Great Recession for their inability to predict and deal with severe recessions. On this topic of discussion, there is a long list of contributions. The interested reader may want to look into Reis (2017); Ghironi (2018); Blanchard (2018); Lindè (2018) for arguments in favour of DSGE and into Fagiolo et al. (2008a); Romer (2016); Stiglitz (2018) for arguments against them.

[12]Almost all the contributions attempted to explain this new evidence on trade flows via a monopolistic type of competition (see also Lancaster, 1980). Ethier (1979) is a notable exception. In Ethier (1979) intra-industry trade is vertically integrated in a Smithian fashion by assuming that countries specialise either in intermediate or in final goods thanks to external economies of scale.

Melitz (2003) nests the intra-industry trade model of Krugman (1980) with the fixed cost and productivity heterogeneity framework of Hopenhayn (1992). Firms decide to produce and to export on the basis of expected profits, that decrease with fixed costs of production and increase with firms' productivities. Productivities are heterogeneous, fixed in time and drawn from a distribution $g(a)$ upon the payment of the fixed cost of entry F_e. As there are local and foreign markets, there are both fixed costs of production F and of export F_x. Firms enter in the local market if expectations on future firm value, conditional on drawing a level of productivity high enough to generate non-negative profits, is greater than the fixed cost of entry F_e. After entry, firms serve each market that grants them non-negative profits conditional on productivities and fixed costs and exit with a fixed exogenous probability. When the economy is open, two productivity thresholds (similar to equation 6 from Hopenhayn, 1992) are generated, one for the domestic and one for the foreign market, below which firms will not enter the respective market. This leads to the two main results of the model. First, more productive firms become exporters, in line with the empirical evidence (Bernard and Jensen, 1995, 1999).[13] Second, in an open economy the productivity threshold relative to the domestic market increases as selection becomes tougher with the entry of foreign exporters in the domestic market, implying the exit of incumbent firms with productivity lower than the new thresholds and the reallocation of resources from less to more productive firms.[14] Trade liberalization induces thus a selection effect similar to the decrease in fixed costs of entry in Hopenhayn (1992) and imply higher industry concentration.[15] In this context, new new gains from trade arise in terms of higher aggregate productivity, that reduces price level and increases real wages.[16] More formally, the productivity threshold in an open economy a_o^* is higher than the one in closed economy a_c^* (i.e., $a_o^* > a_c^*$). Equilibrium prices in closed and open economy are very similar to the ones of the baseline monopolistic competition (see equation 4) and read as follow:

$$P^c = (N)^{\frac{1}{\sigma-1}} \frac{1}{\rho a_c^*} > P^o = (N)^{\frac{1}{\sigma-1}} \frac{1}{\rho a_o^*} \tag{8}$$

where P^c and P^o are the aggregate price levels in closed and open economy, N is the (endogenous) number of firms and σ and ρ defines the degree of product substitutability (see section 2.1). Price equations in the Melitz model are very similar to the one of Dixit and Stiglitz (1977) (see equation 4 in section 2.1) and differences between them are due to the selection at entry induced by the Hopenhayn (1992) structure, that implies the presence of the term $(N)^{\frac{1}{\sigma-1}}$ and the use of productivity thresholds

[13]This result depends on the fixed cost of export being greater than the fixed cost of entry in the domestic market, since export activity induces additional entry cost, related for example to transportation or to product readaptation.

[14]On empirical grounds, despite the weak evidence on the existence of selection (Foster et al., 2001) and reallocation of resources to more productive firms (Hsieh and Klenow, 2009, 2018), some studies found positive effects of trade liberalization on productivity taking place thanks to reallocations induced by selection effects (Pavcnik, 2002; Trefler, 2004).

[15]It is not however univocally determined the relation between the reduction of fixed costs of exports and the change in concentration.

[16]This result depends on CES preferences (Bertoletti and Epifani, 2014).

instead of marginal costs.

The research stream started by Melitz (2003) is still very active. Ghironi and Melitz (2005, 2007) nest the (Melitz, 2003) framework in a DSGE model to explain the stylised facts of trade related to the business cycle. Bernard et al. (2007) integrates Melitz' model in a comparative advantage framework and shows how reallocation effects are stronger in the sector with comparative advantage. Melitz and Ottaviano (2008) introduces variable mark-ups function of market size and Mayer et al. (2014) expand the model to multi-product firms and find a competition effect that induces firms to skew their exports toward best performing products in markets with higher competition. di Giovanni and Levchenko (2012) use the Melitz model with Pareto distributed productivity to estimate the effect of productivity shocks in granular open economies.[17] Openness increases aggregate volatility because the availability of external markets skew the firm size distribution.

Finally, a recent research stream has been analysing the effect of learning within the Melitz framework. In these models exit is an endogenous decision related to the dynamics of productivity. Luttmer (2007) studies the consequences of firm selection over long-run growth in a dynamic closed-economy framework based on monopolistic competition, firm heterogeneity and learning. The main technical innovation of the paper regards the dynamics of productivity, that is modelled as a Gibrat's Law (Gibrat, 1931) for incumbents and is augmented with entry and exit dynamics.[18] Such an implementation entails a Pareto distribution of productivity, that in turn generates a Pareto firm size distribution via monopolistic competition. Notice that in the context of Hopenhayn (1992) the Pareto distribution defines a good technical solution for productivity because the equilibrium productivity distributions in models à la Hopenhayn (1992) are distributions truncated by the entry productivity threshold.[19] Indeed a truncated Pareto distributed variable remains Pareto distributed, differently from other distributions. Similar results in terms of Pareto distribution of productivity and size arise in Atkeson and Burstein (2010); Burstein and Melitz (2011), where firms choose the level of innovation intensity to maximize future profits. In Bustos (2011) firms optimally decide whether to innovate or not depending on the level of fixed cost of innovation. This generates a cut-off level of productivity above which it is convenient to innovate. Lower trade costs positively impact on firm innovation choice, increasing the productivity of incumbents and lowering entry rates as in Melitz (2003). Furthermore, Costantini and Melitz (2007) uses a similar model to study

[17]By granular economies we mean ones wherein the size is so skewed that idiosyncratic firms' shocks do not vanish in the aggregate (see Gabaix, 2011). Such a result is generated when the variance of size does not exist (i.e., the size is Pareto distributed with coefficient lower than 2).

[18]Simonesque models use similar stochastic processes to generate Pareto distributed size, see the discussion in section 3.

[19]The use of Pareto productivity distribution is widespread in the rational equilibrium models literature. In Melitz (2003) the Pareto productivity assumption delivers easy-to-use tractable solutions (see Baldwin, 2005; di Giovanni and Levchenko, 2012). Notice however that the evidence on Pareto distributed productivity is far from being robust. The empirical literature finds the productivity distribution to be instead close to a Log-Normal (Baily et al., 1992; Bartelsman and Dhrymes, 1998).

the effect of trade liberalization. Impullitti et al. (2013) proposes an open economy version of the model in Luttmer (2007). Firms are subject to stochastic productivity shocks and decide to enter in a market even when the profits are negative, but not too low. Accordingly, they are characterised by hysteresis in their export behaviour, as they do not exit from the market as soon as their profits become negative. Thanks to this assumption, the model generates overlaps in the productivity distribution of exporters and the one of non-exporters in line with the empirical evidence (Mayer and Ottaviano, 2008; Melitz and Trefler, 2012).

2.4 Recent developments in the rational equilibrium paradigm

The research stream dealing with technical improvements of the imperfect competition framework has seen a revival in the last decade due to the new empirical evidence on mark-ups and firm size heterogeneity.

Mark-ups have been found to be heterogeneous and affected by a long list of different factors (see e.g., De Loecker and Warzynski, 2012; Bellone et al., 2010). These findings are at odds with the standard monopolistic competition framework, where mark-ups are constant due to the CES utility assumption. Recent contributions in the rational equilibrium paradigm are aimed at relaxing the assumption of constant elasticity of substitutions by using different utility functions (see Parenti et al., 2017). For instance Melitz and Ottaviano (2008) uses quadratic preferences in a two-sector economy producing an homogeneous and a continuum of differentiated goods. In this context, market size and trade liberalization lower mark-ups and increase aggregate productivity. Zhelobodko et al. (2012) generalizes the variable mark-up framework of Krugman (1979).[20] The demand elasticity of substitution $\sigma(q)$ is assumed to be a function of the level of consumption. When $\sigma(q)$ decreases (respectively, increases) with consumption q, a decrease (respectively, increase) in fixed costs induce lower (respectively, higher) price levels. Behrens and Murata (2007) use non-additive utility functions to show that a reduction in fixed costs have pro-competitive effects in reducing mark-ups. In Bertoletti and Etro (2017), the elasticity of substitution negatively depends on individual income, thanks to indirectly additive utility functions. Mark-ups are higher in per capita richer countries. In Osharin et al. (2014) the elasticity of substitution to CES is heterogeneous. The correlation between individual income and substitutability determines the change in mark-up after income redistributions. In Parenti et al. (2017) the elasticity of substitution is function of individual consumption and of the number of firms in the market, each producing one variety. In this context, the effect of changes in the market size and in the fixed costs have not a univocal effect (similarly to Zhelobodko et al., 2012), and depends on the sign and on the relation between the part of elasticity due to consumption

[20] The main difference between Krugman (1979) and Krugman (1980) lies in the utility function. In Krugman (1979) mark-ups are variable as demand elasticity decreases with consumption ($\sigma = f(.,q)$, $\frac{\partial\sigma}{\partial q} < 0 \Rightarrow \frac{\partial\sigma}{\partial p} > 0$), thus there are additional gains from trade from a positive scale effect thanks to the price reduction in an open market due to the elasticity assumption. Krugman (1979), differently from Krugman (1980), also entails new new gains from trade.

and the one due to the number of firms. In Mrázová and Neary (2017) the local association between the elasticity of substitution (slope) and convexity (curvature) of demand functions is used to infer local market outcomes when a change in the primitives of the model occurs. Kokovin et al. (2023) provides a model of heterogeneous firms and heterogeneous consumer preferences. Preferences are described by a distribution over the support of an horizontal product space, whose extremes are less popular products and values closer to the mode are more popular. Product space is modelled as in Hotelling (1976). Consumers can buy products different from the preferred ones, but their utility is decreased in proportion to the distance between preferred and bought product. Firms serve different niches by locating in different parts of the product space. However, there is a trade-off, as more popular markets also leads to more competition. As a result, more productive firms charge higher prices and produce larger volumes. Mark-ups are non-monotonic. They are high in most and least popular locations where demand and competition are respectively very high or very low.

Another strand of contributions in the rational equilibrium framework has focused on the properties of the firm size distribution. The firm size distribution at the country level can be well approximated by a Pareto Law with non-finite variance (Axtell, 2001).[21] The seminal contribution of Gabaix (2009) investigates what are the effects of this stylised fact (i.e., the *granular hypothesis*) by studying how micro shocks to large firms contribute to aggregate fluctuations as compared to a size distribution with finite variance. The theoretical and empirical evidence discussed in the paper suggests that aggregate fluctuations of GDP are affected by the production activities of big firms because their growth shocks do not vanish in the aggregate, as it was previously assumed by macroeconomists (see Long and Plosser, 1983). This new evidence, also spurred by the empirical relevance of large exporters (Bernard et al., 2007; Mayer and Ottaviano, 2008; di Giovanni et al., 2011; Freund and Pierola, 2015), paved the way to several contributions on the effect of large exporters growth on both the aggregate volatility and the international business cycle (di Giovanni and Levchenko, 2012; di Giovanni et al., 2017, 2020; Gaubert and Itskhoki, 2021). Also, a new generation of international trade models mixing monopolistic competition with oligopolistic features is growing motivated by these empirical stylised facts. The empirical relevance of large exporters suggests indeed that atomistic competition models, such as the ones endowed with a monopolistic competition structure, may fail to account for firm interactions and market power on individual and aggregate prices (e.g. Ciliberto and Jäkel, 2021). Neary (2016) assumes that firms competition effects are industry specific, but that their decisions do not affect the consumer income, as they take as given income, prices and wages in other sectors.[22] Similarly, in Bernard et al. (2018) preferences are CES, but firms are allowed to be large in their own sector. Accordingly firms influence the sectoral price and take wages as given. This modelling structure is also adopted by Gaubert and Itskhoki (2021) to study the effect of gran-

[21]Notice that at the sectoral level the firm size distribution may not be well approximated by Pareto (see Bottazzi and Secchi, 2003).

[22]Using the authors words, firms are "large in their own markets, but small in the economy as a whole"

ularity on comparative advantages. In Parenti (2018) oligopolistic and monopolistic competitive firms coexists. In this model trade liberalization entails a competition effects for small firms, that are forced out by the entry of large oligopolistic firms. The predictions of this model are in line with the evidence on negative competitive effects by large firms provided (see Ciliberto and Jäkel, 2021).

3 Simon-inspired firm selection models

Firms' heterogeneity and increasing returns are central in rational equilibrium. However, these were anticipated by the first contribution belonging to the Simonesque research paradigm (Simon and Bonini, 1958), which builds upon the results of Gibrat (1931) to explain the emergence of the Pareto firm size distribution. Simonesque models aim at reproducing one or more stylised facts of firm dynamics by means of reduced form stochastic processes which fold together firms' learning and market selection and introduce dynamic increasing returns via the sequential assignment of market opportunities with probability proportional to size. These processes emphasize the dynamics of firms' competition, which is based on very stylised form of firms' market interaction, do not encompass assumptions on firms' rationality and generate firm heterogeneity in size and growth rates. Admittedly and differently from both the rational equilibrium and the evolutionary traditions, Simonesque models are always purely descriptive, as they do not include variables that can generate policy prescriptions, other than the ones regulating the strength of returns.

3.1 Simonesque models, the Gibrat's Law and the firm size distribution

The ancestor of Simonesque models is the Gibrat's Law of Proportionate Effects (Gibrat, 1931), wherein we find the first stochastic process able to address the emergence of skewed firm size distributions in economics. Gibrat's model describes size as the aggregation of several shocks whose level is proportional to size and rate is independent from it. Consider firm size at time 0, S_0, and hit it with a multiplicative (i.e., proportional to size) and i.i.d. (i.e., independent from size) shock ϵ_τ at each time step. The logarithm of size evolves as a random walk and, after t time steps, it is characterised by the following equations:

$$S_t = S_{t-1}(1+\epsilon_t) = S_0 \prod_{\tau=1}^{t}(1+\epsilon_\tau) \quad \Rightarrow \quad \log S_t = \log S_{t-1} + \epsilon_t = \log S_0 + \sum_{\tau=1}^{t} \epsilon_\tau \tag{9}$$

By the Central Limit Theorem the sum of the i.i.d. shocks $\sum_{\tau=1}^{t} \epsilon_\tau$ is Normal. Accordingly, the limit distribution of size is Log-Normal with diverging variance and the one of growth rates (computed on appropriately long time period) is Normal.[23]

[23]Kalecki (1945) shows that, when the covariance between size and growth is negative and growth shocks ϵ_i decrease in time, the final size distribution remain Log-Normal and its variance is finite.

Simonesque models can be seen as extensions of the original Gibrat's framework. They maintain indeed the proportionality of size growth to past size, but introduce mechanisms of dynamic increasing returns that violate the i.i.d. property of Gibrat's shocks and enable the emergence of distributions characterised by fat tails. Thanks to this property, Simonesque models account for three broad classes of stylized facts, that Gibrat's Law cannot explain. First, the aggregate size distribution is more skewed than a Log-Normal (see Simon and Bonini, 1958; Axtell, 2001) and it can be approximated by a Pareto distribution. Second, the growth rate distribution is Laplace (or tent-shaped, see Stanley et al., 1996; Bottazzi and Secchi, 2003) and, even when one takes into account time periods larger than one year, its shape deviates from the Normal distribution (Bottazzi and Secchi, 2006a). Third, it has been shown that the growth rates are not independent from size, as smaller firms grow faster than bigger firms (see Hymer and Pashigian, 1962; Stanley et al., 1996; Calvino et al., 2018).

The first model addressing one of these properties is Simon and Bonini (1958) (see also Ijiri and Simon, 1975, 1977), which explains the fat-tailed nature of the firm size distribution by using the statistical properties of the stochastic process described in Simon (1955). The later framework developed by Herbert Simon paved the way to other Simonesque models explaining stylised facts related to firms' growth rates and to their relation with size. More in detail, Simon and Bonini (1958) model firm size as a dynamic increasing returns process (i.e., a Pólya urn) that generates a Pareto firm size distribution under appropriate conditions (see also below and Simon, 1955; Ijiri and Simon, 1975, 1977). Imagine an urn filled with N balls, each of different colour. Each colour represents a firm and the urn represents the market where firms compete. The size of each firm at time t ($S_{i,t}$) is equal to the number of balls belonging to the firm. Now imagine to repeat t times the following iteration, which represents the assignment of a market opportunity to a firm: a ball is drawn from the urn and is put back in the urn together with another ball of the same colour. The share of each type of ball $s_{i,t}$ represents the firms' market share and reads as:

$$s_{i,t} = \frac{S_{i,t}}{NS_0 + t} \tag{10}$$

The firm's market share also determines the firms' probability to grow by getting the next ball, and larger firms are thus more likely to grow. This process can be formalized as follows:

$$S_{i,t} = S_0 + \sum_{\tau=1}^{t} \epsilon_\tau(s_{i,\tau-1}) = S_{i,t-1} + \epsilon_t(s_{i,t-1}) \tag{11}$$

Where $\epsilon_\tau(s_{i,\tau-1})$ takes value 1 with probability proportional to the market share of firm in $\tau - 1$, and value 0 otherwise. The above dynamic increasing returns mechanism captures the idea that firms getting the market opportunities today will be more likely to get other ones tomorrow. Furthermore, the above mechanism of market opportunities assignment also accounts for firms' market interaction, because what is gained by a firm is lost by another.

It is easy to show that the above mechanism implies a proportionality between size and its growth in expected terms. More precisely, from equation 11 we get:

$$E(S_{i,t}) = S_{i,t-1} + E(\epsilon_t(s_{i,t-1})) = S_{i,t-1} + s_{i,t-1} = S_{i,t-1} \cdot \left(1 + \frac{1}{NS_0 + t - 1}\right) \tag{12}$$

In other words, the process respects Gibrat's proportionality in a stochastic sense, because a firms' probability to be hit by a shock (the additional ball) is proportional to firm size. However, the shocks $\epsilon_t(s_{i,t-1})$ are auto-correlated (i.e., non-i.i.d.), because a positive realization today affects future ones, generating dynamic increasing returns. This dynamic increasing returns mechanism generates a fat-tailed distribution of size (i.e., the Bose-Einstein distribution) when firms' initial size S_0 is homogeneous (see Feller, 1968). By contrast, when the probability to get the next ball is constant (i.e., in absence of the dynamic increasing returns mechanism), the distribution of the cumulated shocks $\sum_{i=1}^{t} \epsilon_\tau$ is Binomial, while the firm size distribution Normal because of the Central Limit Theorem. When boundary conditions are added to the mechanism described above, a Pareto firm size distribution emerges. In Simon and Bonini (1958) each iteration gives rise to the assignment of a market opportunity to incumbents firms with probability p (i.e. a Pólya urn step, see above), and to an entrant firm with probability $1 - p$. The second case equates to adding a ball with a new colour to the urn. In this setting, the firm size distribution converges to a steady-state Pareto distribution. Notice that this result is generated from a initial setting of size homogeneity, meaning that the model is able to explain the emergence of the size distribution without assuming it in the first place.

The literature in the Simonesque approach has built upon the results of Simon and Bonini (1958) described above.[24] Steindl (1968) considers firms' size as a collection of costumers, that join and abandon the firm in proportion to its size. Again, we have a Gibrat's Law as in Simon and Bonini (1958). Firms enter in the market at a constant rate. If new firms appear and incumbents disappear at a constant rate with the entry and exit rates lying within given boundaries, the distribution of firm costumers conditional on age asymptotically converges to a Pareto. Sutton (1997) generalizes Simon's framework to the case in which the probability of taking the next opportunity is non-decreasing in firm size. As a result the firm size distribution is less skewed than a Pareto, in line with data at the industry level (see the evidence in Bottazzi and Secchi, 2003; Bottazzi et al., 2015).

3.2 The firms' growth rates distribution in Simonesque models

The pivotal importance of introducing dynamic increasing returns in Simonesque models does not only have consequences on firm size. Indeed, Pólya urns have also been employed to model the firms' growth rates dynamics (Bottazzi and Secchi, 2006b; Fu et al., 2005), whose empirical distribution is approximated by a Laplace (tent or double-exponential) shape (see Lee et al., 1998; Bottazzi and Secchi, 2003; Lunardi et al., 2014).

[24] Mechanisms similar to Simon and Bonini (1958) have also been adopted in works outside of the Simonesque literature (see Luttmer, 2007)

Fu et al. (2005) build a model of growth in which units are composed by sub-units, whose number is driven by a Pólya urn and growth is proportional to their size. The metaphor of units and sub-units can be applied to different economic aggregates, in particular firms and their products or sectors/countries and their firms. This key characteristic ensures the scale invariance of its properties to different aggregation levels (i.e., firms, sectors, countries), in line with the empirical evidence (Lee et al., 1998; Fagiolo et al., 2008b; Castaldi and Sapio, 2008). When considering firms as units composed by products, the total size of a firm is the sum of the sales of its products. Consider a firm with a given initial number of products. At each time step, a new product is added to the market. With a given probability p, it is commercialized by a new firm (i.e., a new firm enter in the market). With the complementary probability $1 - p$ instead, the new product is commercialized by one of the incumbent firms. If the new product is assigned to incumbents, it is assigned to one of them with probability proportional to their size, i.e. similarly to the Pólya urn process described above in section 3.1. Firms become monopolist in each of their products, whose sales evolve according to the Gibrat's equation in 9. The distribution of firm growth rates generated by the model is Laplace with Power Law tails. This result holds at different level of aggregation (firms, sector, countries), and is generated thanks to the presence of the dynamic increasing returns captured by the Pólya assignment, that generates a Power Law distribution of firms' number of products. Moreover, and not surprisingly, the size distribution generated by the model is more skewed than a Log-Normal (Growiec et al., 2008).

Bottazzi and Secchi (2006b) consider a firm growth process consisting of two-steps (assignment and realization of market opportunities). In the assignment phase, firms compete to gain a series of market opportunities through a Pólya urn. In the realisation phase, market opportunities become shocks, whose sum determines firms' growth rates. In other words, growth rates are modelled as a random sum of random variables. In the context of equation 9, imagine each shock ϵ_t to be composed of a certain number $h(H_t, s_{i,t})$ of market opportunities x, where $h(H_t, s_{i,t})$ is determined by the Pólya urn assignment rule. More formally:

$$\epsilon_t = \sum_{h=0}^{h(H_t, s_{i,t})} x \tag{13}$$

In this augmented version of Gibrat's Law, the presence of dynamic increasing returns captured by the Pólya urn induce a violation of Central Limit Theorem assumptions, that would otherwise lead to a Normal distribution of growth rates by summing up i.i.d. shocks x. Indeed, the i.i.d. specification of ϵ_t in Gibrat's equation 9 does not hold for Pólya urns because, by construction, market opportunities are not independently distributed for the same firm.[25] As a consequence, a firms' number of shocks $h(H_t, s_{i,t})$ distribution is fat-tailed (i.e., Bose-Einstein distributed) and the one of firm growth rates

[25] This is due to the auto-correlation structure of market opportunities (i.e., to dynamic increasing returns).

ϵ_t converges to a Laplace, in line with the empirical evidence (Stanley et al., 1996).[26]

In Fu et al. (2005) and Bottazzi and Secchi (2006b) the growth rates of firms are determined by the aggregation of a large number of small shocks. The extreme events characterising the firm growth rates distribution are therefore due to the presence of dynamic increasing returns, which generate a fat-tailed distribution of firm-level market opportunities. Arata (2019) consider instead a generalization of the Gibrat's Law, where large shocks account for the existence of a Laplace distribution of firm growth rates. More specifically, in the model a Variance Gamma stochastic process governs the firm size, which is therefore driven by a jump process as opposed to the continuous process determined by the Gibrat's Law, Fu et al. (2005) and Bottazzi and Secchi (2006b).

3.3 The growth rates-size relation in Simonesque models

Another relevant stylised fact of firm dynamics is the negative relation between the firms' growth rates volatility and their size. It is well known (at least since Hymer and Pashigian 1962 and all the way to Calvino et al. 2018) that small firms' growth rates are more volatile than the ones of large firms. Empirically, it has been found that the logarithmic relation between growth rates volatility and size is linear and decreases with a -0.2 slope, which is also referred to as the scaling coefficient.[27]

This stylised fact bears relevant consequences for the Gibrat's model in equation 9. First, the Law of Proportionate Effect does not hold. The shocks ϵ_t are indeed not identically distributed as smaller firms' growth rates are on average more volatile. Second, the scaling coefficient is higher than -0.5, which is the prediction generated by processes driven by the Central Limit Theorem (see Gabaix, 2011; Calvino et al., 2018). Thus big firms are less volatile than small firms, but more volatile than what the Central Limit Theorem would predict. Third, this evidence constitutes a mitigating factor for granular hypothesis (see Gabaix, 2011). As firms become larger, their variance will ultimately converge to 0, dampening the relation with aggregate volatility.

Several models in the Simonesque paradigm have tried to explain the scaling growth variance relation. Stanley et al. (1996) and Amaral et al. (1997) model the firm as an ensemble of sub-units organized in a hierarchical structure, wherein shocks to the top propagates to sub-units below. The hierarchical structure is modelled as a branching tree. Shocks hitting units at the top of the tree propagate with given probability to other units down in the structure. The propagation mechanism introduces correlation in the growth shock across different sub-units that induces a violation of the Central Limit Theorem i.i.d. assumption. There are two limiting cases. When the shocks propagate

[26] In Bottazzi and Secchi (2006b) model this result is generated under the reasonable assumption that the number of shocks asymptotically dominates the one of firms.

[27] Generally, the scaling coefficient is estimated as follows. First, firms are ordered by size and binned in equally populated groups. Second, the average size and the standard deviation of growth is computed for each group. Third, the scaling coefficient is estimated by regressing the logarithm of growth rate standard deviation on the logarithm of size. The scaling coefficient is the estimated size coefficient. Other estimation procedures confirm the -0.2 estimation (see Yeh, 2021).

with probability 0 or 1, the scaling coefficient is respectively equal to -0.5 and to 0. For probabilities between 0 and 1, the model can be calibrated to produce a scaling coefficient of -0.2 on the basis of the number of hierarchical levels and of the number of nodes at each level. Sutton (2002) show that the correlation of the sub-units growth rate is empirically negligible and that a simple model of firm size partition in H products of heterogeneous size can account for the negative scaling growth variance of size for certain values of H. When products are hit by i.i.d. growth shocks, the model generates a scaling coefficient in line with the empirical evidence.[28] However, in Sutton (2002) the scaling property changes at higher level of aggregations. This is in contrast with empirical evidence, which has shown how the scaling growth variance relation also holds at the sector (Castaldi and Sapio, 2008) and country (Lee et al., 1998; Canning et al., 1998; Castaldi and Dosi, 2009) levels.[29] In this respect, the model by Fu et al. (2005) provide a good theoretical benchmark because, as discussed above, it generates properties that are invariant to the scale of aggregation. Riccaboni et al. (2008) show that Fu et al. (2005) generates a scaling growth variance relation whose coefficient depends on both the number and size of sub-units and ranges accordingly between 0 and -0.5. Similarly, Bottazzi and Secchi (2006b) shows that, if the probability of entering a new sub-market k_t increases with the number of sub-markets H (i.e., there are economies of scope), the scaling growth variance relation emerges from the positive correlation between a firm's size and the number of its sub-markets. The theoretical findings on the scaling growth variance relation hints once again at the importance of dynamic increasing returns for the generation of firm-level stylised facts.

The most recent empirical evidence on the shape of the relation between firms' size and growth rates volatility leverages firm-level data on employment (Calvino et al., 2018; Yeh, 2021) or specific sectors' sales (the global pharmaceutical sector, Bottazzi and Secchi, 2006b; Riccaboni et al., 2008). Instead, Fontanelli et al. (2022) show that, when firms' sales are examined in comprehensive datasets (i.e. the population of French manufacturers), the negative relation between growth rates variance and size is non-linear. It is very steep for small firms and flat for large ones, suggesting that large firms are Gibrat's.[30] This new finding is explained by a tractable model of imperfect selection encompassing firms competing on the basis of both size and productivity (see also Fontanelli et al., 2023) in a two-step sequential process of firm growth (see also Bottazzi and Secchi, 2006b). First, a number of market opportunities is randomly assigned to firms on the basis of their relative sizes

[28] Wyart and Bouchaud (2003) shows that in Sutton (2002) the shocks to products induce a Normal growth rates distribution through the Central Limit Theorem, unless when the size of products is Pareto distributed with a coefficient bounded between 1 and 2. In that case, the growth rates distribution becomes fat-tailed. When the Pareto coefficient is indeed equal or smaller than 2, the variance is infinite, in violation of one of the assumptions leading to the Central Limit Theorem.

[29] Notice that the scale invariance property does not hold in all firms' stylised facts. It holds for the growth rates distribution, that is Laplace for countries (Lee et al., 1998; Fagiolo et al., 2008b) and for sectors (Castaldi and Sapio, 2008). It does not for size distributions. The country GDP distribution is bimodal (Bianchi, 1997; Quah, 1996; Henderson et al., 2008), differently from the Pareto distributed size of firms at the country level (Axtell, 2001).

[30] Notice that this inverse J-shaped relation is found at both the aggregate and the sector levels.

(i.e., market shares, see also Ijiri and Simon, 1977). Second, once that opportunities have been assigned, firms' relative productivities (i.e., the Cumulative Distribution Function, or CDF, of firms' productivities) determine whether the assigned market opportunities become positive or negative growth shocks, in line with evolutionary selection (see section 2.2 above and 4.2 below). The results suggest that the inverse-J shape of the firms' growth rates variance-size relation can be explained by imperfect selection mechanisms whose outcomes are mediated by both the strength of shares reallocation and firms' *joint* heterogeneity in size and productivity.

4 Firm selection in evolutionary models

The third paradigm we consider is represented by evolutionary economics. The book by Nelson and Winter (1982) laid the foundation of evolutionary economics in a first attempt at showing the advantages in terms of a micro-foundation encompassing firms which behave according to simple heuristics of learning and investment resembling ones actually employed by firms. Thanks to their seminal contribution and its successive developments, evolutionary economics has become an influential research stream. The set of assumptions and techniques first employed in Nelson's and Winter's model are today at the basis of several evolutionary Agent-Based Models (ABM) in economics and the past decades have witnessed a blossoming in the use of evolutionary ABMs and their diffusion to many different fields of economic research.[31]

The evolutionary paradigm incorporates the insights on competition among heterogeneous firms from Simonesque models. The main selection mechanism of evolutionary economics, the quasi-replicator equation, will be the object of the next section. The quasi-replicator emphasizes *changes* in market shares, and not market shares themselves, as the outcome of an explicit competitive process. Furthermore, it takes a functional form which is very close to the law of motions generated by Simonesque models (see equation 9). However, differently from Simonesque contributions, in evolutionary models selection is decoupled from learning. This feature requires evolutionary models to describe firms' behaviour, that is assumed to be boundedly rational.

4.1 The principles of evolutionary economics

In this section we briefly summarize the main characteristics of evolutionary models (see also Nelson and Winter, 1982; Dosi and Nelson, 2010; Fagiolo et al., 2008a). This discussion is propedeutic to the understanding of the theoretical foundation underpinning the quasi-replicator equation and its use as a selection mechanism in evolutionary models of industry dynamics.

[31] ABM, also referred to as Agent Based Computational Economics (ACE), are simulation models. Economics has not been the first field to use this modelling tool (see Turrell, 2016; Bonabeau, 2002; Orbell et al., 2004). ABM contributions was particularly central to the policy debate that followed the Great Recession (see section 2.2 and, among the others, Farmer and Foley, 2009; Kirman, 2010, 2016; Fagiolo et al., 2008a; Gaffard et al., 2018).

One of the main distinctive traits of evolutionary economics is that agents are boundedly rational. Bounded rationality is grounded in the concept of Knightian uncertainty (see Dosi and Egidi, 1991). The future states of the economy are unknown and unpredictable to firms, that cannot reduce the odds of future events to probabilities. Firms' decision making does not hinge on expectations of future market/productivity states and relies instead on past information. It is also assumed that firms cannot process too complex sets of information and thus infer the future state of the world through the knowledge of past events: the economy is indeed conceived as a continuously evolving system and firms are not endowed with the processing skills necessary to transform past information into perfect predictions. As a consequence, firms cannot take maximizing decisions. Rather, firms behave based on simple rules, or heuristics, and on a limited information set, in line with behavioural theories (Cyert and March, 1963; Kahneman and Tversky, 2000; Gigerenzer, 2007). This is relevant for firms' selection in evolutionary models, because firm exit is the outcome of the market interaction with other firms. Accordingly, selection is *competitive*, because it is explicitly based on a *"dynamic process that in some way captures the general idea that profitable firms tend to survive and grow, while unprofitable ones tend to decline and fail"* (Winter, 1991), as compared to rational equilibrium models where selection are black-boxed in voluntary decisions of firms, *as if* a competitive process took place.

In the context of evolutionary models, firm heterogeneity endogenously emerges via the interaction between firms' learning and selection mechanisms.[32] Firms' learning increases the level of heterogeneity in the economy by allowing laggard firms to survive thanks to productivity-enhancing innovations. Conversely, market selection governs the evolution of shares, driving firms with below-average productivity out of the market and reducing thus firm heterogeneity. Notice that firms' productivities are mapped via selection into firms' growth rates, rather than into size. Market selection emphasizes thus the *dynamics* of firm competition and its path-dependency.

Firm heterogeneity is both the input and the output of the selection mechanism, that determine which firms survive based on their relative productivity. This feedback mechanism is a typical evolutionary feature and is at the basis of increasing returns. Returns are an inherent characteristic of the interaction between learning and selection, and realize thus *dynamically* via selection. Through learning, firms increase their productivity and ameliorate their growth rate and their chance of survival via the replicator. In turn, firms will have higher chances of increasing their productivity, survive and grow in the future.[33]

Thanks to the assumption of firm bounded rationality and to the learning-selection dynamics, evolutionary models are able to investigate economies characterised by evolving characteristics and

[32]Firms in evolutionary models, similarly to Simonesque ones, generally start from homogeneous firms' size and productivity.

[33]Increasing returns may also be introduced more explicitly through cumulative mechanisms of learning, for example the Schumpeter Mark II regime (see Dosi et al., 1995, 2017), or via dynamic increasing returns mechanisms embedded in selection (see Fontanelli et al., 2023).

out-of-equilibrium dynamics, which are typically studied via numerical simulation techniques (i.e., Monte Carlo simulations).[34][35] The economy is considered as a complex system, whose characteristics cannot be reduced to a static or steady state equilibrium because continuously evolving thanks to learning-selection feedbacks. These generate a population-wide convergence to a set of micro and macro statistical properties by keeping economies far away from monopoly, which is the ultimate state of the market in dynamics governed by quasi-replicator types of equations (see section 4.2 below and Metcalfe, 2007). The inherent (and voluntary) absence of equilibrium states also has two practical advantages. First, evolutionary models are very flexible. Second, the mechanisms of evolutionary models can be built to resemble ones by real-world firms. This particular characterization allows evolutionary models to be easily parametrized, tested and calibrated against data (for a survey see Fagiolo et al., 2019).[36]

4.2 The quasi-replicator

As we have already mentioned, any evolutionary model builds upon two main pillars: learning and selection. Learning is typically modelled as a stochastic process, in one (Dosi et al., 1995) or two (Nelson and Winter, 1982) steps, that determines the evolution of firm productivities as a cumulative process.[37] Furthermore, a significantly growing number of models employs the quasi-replicator equation as the driver of market selection. First formalized by Taylor and Jonker (1978) in population biology, the replicator is probably the most popular differential equation of evolutionary game theory.[38] Before docking to firm selection studies, one of the first contribution being Metcalfe (1994), the replicator equation has been employed in economics to model changes in technology shares on the basis of their relative fitness and to inquire the diffusion and the selection of competing technologies (see Silverberg et al., 1988; Silverberg and Lehnert, 1993; Saviotti and Mani, 1995). Similarly, in a firm selection framework the quasi-replicator equation redistributes market shares to firms on the

[34] In a Monte Carlo analysis the target statistics are computed as averages in multiple instances of the same model with same parameters and different pseudo random draws. For a comparison between the scenarios, parameters are varied while the pseudo random draws are kept constant. This adds up to fixing the same seeds in computer simulations. That is to say, the output of the model under different parametrization is a sort of *ceteris paribus* comparative exercise in which the same shocks hit the model through a system governed by different initial parameters.

[35] The use of simulated models to study firm dynamics was previously advocated by Cyert and March (1963).

[36] The most used validation mean against data consists in the reproduction of a set of empirical stylised facts. The idea is that, if the model can robustly and jointly reproduce them, then it can plausibly be considered a possible explanation of real-world data generating processes.

[37] Firm learning may not only relate to productivity, but also to other aspects of the firms, for instance the routines that firms use to set the level of R&D and capital investments.

[38] A survey effort on evolutionary game theory is out of the scope of this work, whose main focus is firm selection. In evolutionary game theory, the replicator dynamics drives the change in the share of players adopting a certain strategy, that is proportional to the difference between the average payoffs under that strategy and the average payoffs of the other players.

basis of the level of fitness of individual firms relative to the market. The concept of fitness embodies the firm characteristics driving the change in market shares (e.g., productivity or quality), whose accurate definition depends on the model. Fitness differentials among firms drive the performance of firms. More fitting firms tend to grow, less fitting firms tend to shrink. The quasi-replicator reduces thus firm heterogeneity, reallocating the shares towards fittest firms and driving least fit firms out of the market.[39] The quasi-replicator embodies indeed the concept of growth of the fitter and the Fisher principle (Metcalfe, 1994).[40]

The typical functional form of the quasi-replicator in firm dynamics studies reads as follows:

$$s_{i,t+1} = s_{i,t} \cdot (1 + \chi \frac{a_{i,t} - \bar{a}_t}{\bar{a}_t}), \quad \bar{a}_t = \sum_{i=1}^{N} a_{i,t} \cdot s_{i,t} \tag{14}$$

where $s_{i,t}$ is the ith firm share in time t, χ is the parameter driving the strength of the replicator, $a_{i,t}$ is the chosen fitness measure of the ith firm and $\bar{a}_t$ is firms' average fitness measure weighted on market shares. Even though the quasi-replicator and Gibrat's Law (see equation 9 and section 3.1) may seem similar in a first approximation, Gibrat's assumptions fail to hold in two respects in equation 14. First, the size of the growth shocks for the ith firm $\chi \frac{a_{i,t} - \bar{a}_t}{\bar{a}_t}$ is a function of the average productivity $\bar{a}_t$, which is weighted on firm size. Accordingly, as argued in Dosi et al. (2017), the larger it is a firm, the closer will be $a_{i,t}$ to $\bar{a}_t$ and the lower the growth rate, in line with the empirical evidence on the negative relation between size and growth rates volatility of firms (Hymer and Pashigian, 1962; Bottazzi and Secchi, 2006b; Calvino et al., 2018). Second, as the quasi-replicator characterises the reallocation of market shares across firms, a firm gain is another firm loss. In other words, the sum of individual growth rates in levels equals 0:

$$\sum_{i=1}^{N} s_{i,t} \frac{a_{i,t} - \bar{a}_t}{\bar{a}_t} = 0 \tag{15}$$

The latter two properties entails a full deviation from the i.i.d. characterization of shocks in the Gibrat's Law. Other two factors influence the magnitude of growth shocks. First, wider fitness differentials cause more extreme growth rates because the rate of change is driven by the firm fitness relative to the market mean (i.e., the term $a_{i,t} - \bar{a}_t$). This relation is at the basis of the so called Fisher principle (see Metcalfe, 1994). Second, the parameter χ determines the strength of the competitive pressure, and thus the width of growth rates, by increasing the magnitude of growth shocks. The parameter χ can be interpreted in several ways. It may be considered as a proxy of market frictions such as search and switching costs (see Bernard and Moxnes, 2018; Lenoir et al., 2022; Steinwender,

[39]Notice that the quasi-replicator is often coupled with a positive lower boundary condition below which firms go bankrupted. The boundary condition is defined in terms of market share and can be interpreted as the minimum size relative to the market that allows to sustain a certain level of fixed costs of production. This is the reason why it is called quasi-replicator, as a standard replicator would span all over the unit simplex.

[40]Metcalfe (1994) defines the Fisher principle as follows: *"[...] the rate of change of average behaviour within a population of competing firms is governed by the degree of variety in behaviour within a population."*

2018; Bernard et al., 2019; Akerman et al., 2022), or of imperfect information of costumers relative to the fitness (e.g., prices) distribution of the products sold in the market, as in a Phelps-Winter models of consumer flows (that is indeed based on a replicator dynamics, see Phelps and Winter, 1970; Greenwald and Stiglitz, 2005). In the same vein, it represents the strength of returns to market selection. The higher it is, the faster will be shares reallocation towards the most productive firms (and the higher will be returns from firms' learning). Finally, notice that the quasi-replicator equation has 2 equilibrium points in which growth shocks are null. In the first equilibrium, firm productivity is homogeneous, i.e., $a_{i,t} = \bar{a}_t \; \forall i$. In the second equilibrium, the most productive firm i, such that $a_{i,t} > a_{j,t} \;\; \forall j \neq i$, controls the whole market and is thus a monopolist. Evolutionary models employing the quasi-replicator equation typically focus on out-of-equilibrium outcomes, which are thus far from the two equilibrium cases above. Firms' learning is needed to obtain such outcomes. The tendency of selection to destroy inferior productive techniques and converge to monopoly is counterbalanced by learning opportunities for incumbents, that reshuffle the ranking of fitness, and by the increase in the number of available productive techniques due to firms' entry (also see Fontanelli et al., 2023).

4.3 Empirical evidence on the quasi-replicator dynamics

Given its simplicity, the quasi-replicator equations have been tested several times against data. Notwithstanding the increasing trend in industry concentration in Europe and in the U.S. (see Bajgar et al., 2023) may indicate that the principle of growth of the fitter is at work, the empirical test provide little support in favour of this principle (Cantner, 2016). Productivity and growth rates do not seem to be related at the firm level (Bottazzi et al., 2008, 2010). Aggregate productivity growth is mainly driven by within firm changes in productivity (i.e., learning) and net entry, whereas selection forces seem to play a marginal role (Foster et al., 2001; Bottazzi et al., 2010; Dosi et al., 2015; Yu et al., 2015). A significant, positive, but small effect has been estimated employing firms' profit rates as the fitness measure (Coad, 2007). Some encouraging evidence on selection playing a major role has been found at the level of individual sectors (Metcalfe and Calderini, 2002; Cantner and Krüger, 2008; Cantner et al., 2012). The weak empirical performance of the replicator in reproducing real world selection may have several explanations. One reason, advocated by Holm et al. (2016) and Hölzl (2015), may lie in the existence of confounding factors affecting firms' fitness $a_{i,t}$ beyond productivities and profit rates. In this respect, "one size fits all" studies using the same fitness measure (e.g., productivity) for all sectors, may fail to properly identify the type of competition taking place in specific sectors (e.g., based on price or on differentiation), or other relevant sectoral characteristics governing firm fitness. For instance, Cantner et al. (2019) shows that value chains may play a role as a confounding variable. Low fitting firms may entertain commercial relations with highly fitting ones through value chains, corrupting the empirical estimates of selection based on productivity only. Another possible explanation for the weak empirical performance of the replicator lies the mismatch between the level of aggregation at which competition takes place in real world dynamics and the one considered in empirical studies. On the one hand, Western countries are open to international trade. Firms thus participate and are subject to foreign competition. As domestic exporters may not compete on same markets, measuring selection based on sales not cleansed from exports may bias selection effect estimates, but controlling for the source of bias due to foreign exporters' competition against domestic firms may prove very challenging due to the absence of proper data. Trade openness should not be underestimated as a bias source since by construction larger and more productive firms, which export with higher probability (Mayer and Ottaviano, 2008), are also more likely to heavily affect the quasi-replicator estimates. On the other hand, it may be misleading to use firms' productivities and sales at the industry level to estimate selection effects, as competition takes place at the market level. Consider the following examples from the "Combined Nomenclature".[41] Firms in sector 9202 "Other string musical instruments" may produce violins (92021010) or guitars (92029030), that are clearly not substitute goods: firms producing violins are not likely to compete with guitar producers. However, firms in sector 2204 "Wine" may produce wines from Tuscany (22042126) or Piedmont (22042162) regions, which are obviously substitutes. Finally, another relevant factor that affects firm competition, nonetheless not taken into account by existing empirical studies due to the uncommon availability of firm-level data on sub-markets, is firm diversification. Indeed, multi-product firms are classified according to their most representative industry, but a large share of their sales may come from other industries, inducing a bias in estimations that is likely to affect larger firms, which are more diversified.

To conclude, the empirical literature on the estimation of selection effects calls for firm level measures aimed at providing a better approximation of the boundaries of competition among firms. In other words, the studies investigating the empirical consistence of the quasi-replicator would benefit from a definition of sectors overlapping to a higher extent with the one of markets. [42]

4.4 Evolutionary models of industrial dynamics

In this section we describe the main contributions of evolutionary modelling, focussing in particular on models including the quasi-replicator.

The theory discussed in the book *"An Evolutionary Theory of Economic Change"* (Nelson and Winter, 1982) is the cornerstone of evolutionary economics. Even though they do not use the quasi-replicator equation to model selection it is relevant to briefly discuss two models from (Nelson and Winter, 1982), as they are among the first examples of models encompassing the evolutionary triad

[41] More information on the Combined Nomenclature can be found here and here.

[42] In this respect an interesting methodology has been developed in Hoberg and Phillips (2016), wherein the authors classify competitor firms on the basis of their self-provided descriptions.

of learning, selection and their interaction.[43] These models aim at studying economic outcomes as properties emerging from the interaction between boundedly rational and heterogeneous firms. In the models, firms have limited information sets, which do not allow them to forecast future productivities and profits. They thus take decisions according to simple behavioural rules of innovation and investment, which represent firms' learning and selection. These mechanisms are inherently *dynamic*, as they stress changes in firms' productivities and sizes (i.e., capital) respectively, rather than their levels, and generate a feedback mechanism between firms' learning and competition vis-à-vis other firms, which jointly determine firms' willingness and ability to further invest and ultimately govern their survival. In these models, the selection of firms depends indeed on firms' capital stocks, which determines firms' exit when it reaches 0. A distinguishing characteristic of these models are positive profits, that firms can use to finance their capital investments and R&D expenditures.

The first model stresses aggregate outcomes by providing a micro-founded behavioural explanation to the macroeconomic time series relative to GNP, capital, labour and factor prices (see Solow, 1957). Profits, that depend on firms' capital and production technique, are the drivers of firm investment and learning decisions. Positive profits are indeed reinvested in new capital. If profits of a firm fall below a given threshold, the firm tries to innovate or imitate other incumbents to ameliorate its production technique in an attempt to raise its odds of survival.

The second model encompasses a more complex innovation structure and focuses on outcomes of industries where all firms are imitator and only some of them are innovator, and is aimed at studying how firms' learning affects extant industrial structures. The higher are capital and productivity, the higher are profits and the part of them which will be invested in new capital and R&D expenditures, generating a dynamic increasing returns mechanism. Innovation and imitation are two-steps processes wherein first it is randomly determined if the firm innovates/imitates with probability increasing in the amount of R&D invested. If firms successfully innovate, a new random technique is developed.[44] If the firm successfully imitates, it acquires the most productive technique in the industry. In this model, the industrial outcome depends on the survival of innovators and imitators, which in turn depends on how easy it is to imitate other firms, on the strength of investment by firms and on the variability of innovation outcomes. Interestingly, more aggressive investments rules, which also entails stronger selection, induce higher levels of firm concentration.

We now discuss a series of contributions employing the quasi-replicator. One of the first applications of the quasi-replicator equation to firm selection can be found in Metcalfe (1994), which discusses some properties of the quasi-replicator. A fixed number of heterogeneous firms compete over homogeneous good with given price and invest a share of their profits to increase their productive capacity. Firms' growth rates depend on the distance between the firm-level productivity and

[43] See chapters IV and V from the book.

[44] A new technique may also be inferior to the incumbent one. In that case it will not be adopted by the firm, that will thus rely on past one.

its industry (market shares weighted) average. Two properties related to the the Fisher principle are proved to hold. First, the rate of change in the average productivity due to selection is equal to the covariance between the productivity and the growth rate of firms, which is proportional to the productivity variance. Second, the rate of change of productivity variance is proportional to the skewness of the productivity distribution. Higher skewness means higher firm differentials in productivity, and thus faster growth, polarizing the shares associated to firms more and less productive than the mean. Montobbio (2002) expands the results of Metcalfe (1994) to a multi-sector framework, wherein sector proximity in terms of product substitutability affects selection and sectoral growth. Higher product substitutability increases the strength of selection as firms in a sector also compete with firms producing substitutes in other sectors.

Mazzucato (1998) explores the determinants of concentration and market shares instability in an industry life cycle model of industrial dynamics wherein a population of heterogeneous firms compete via the quasi-replicator equation. The paper is aimed at explaining why younger industries are characterised by higher instability and concentration of firms' market shares (Hymer and Pashigian, 1962; Klein, 1977; Klepper, 1997). According to Mazzucato (1998), this change is due to different types of dynamic returns to scale characterizing different industry stages. In early stages, increases in shares are associated to lower increases in productivity (dynamic *decreasing* returns), vice versa in mature industries to greater increases in productivity (dynamic *increasing* returns) because firms in mature industries can use their earnings to consolidate their competitive position, differently from ones in young industries (Acs and Audretsch, 1987). The results of the model confirm indeed the switch in economies of scale as the possible break defining the change in regime in shares instability and concentration at different stages of the industry life cycle.

Furthermore, the work of Dosi et al. (1995) aims at reproducing some of the most important empirical regularities of industrial dynamics by the means of a model incorporating multiplicative random process of firm learning and a quasi-replicator equation driving market shares. The model is composed of a sector populated by heterogeneous firms. The sector can grow in two respects. New micro-sectors (i.e., sub-markets) are randomly generated with given probability. Within each micro-sector, the number of firms is variable due to entrants, that may be either new to the economy or incumbents from other sectors. Learning and selection take place at the micro-sector level. The model is able to reproduce a skewed firm size distribution at the aggregate level and the presence of productivity differentials.[45] Significant levels of entry, exit and turbulence rates are generated at the level of micro-sectors. These properties change with the parametrization of learning and selection. Within micro-sectors, industry concentration increases with the strength of selection and with incumbents learning opportunities, and decreases with the ones of entrants. Conversely, the degree of productivity asymmetries and of turbulence both decrease with incumbents learning

[45] In this model firms' size is the sum of firms' sales in each micro-sector. Sales in the micro-sectors are the share of micro-sector demand, that is exogenously determined through a S-shaped cycle.

and increase with selection and higher entrants opportunities. Finally, the number of firms, that is function of the net effect of entry and exit rate, decreases with both incumbents learning and selection. The results of the model confirm again the relevance of the interaction between firms learning and market selection in determining industrial structures.

Similarly to Dosi et al. (1995), the work of Dosi et al. (2017) focuses on the properties of firms and industrial dynamics as stemming from the learning-selection interaction.[46] Learning is random and multiplicative and market shares are driven by a quasi-replicator equation, as in Dosi et al. (1995).[47] Despite its simplicity, the model is able to match several stylised facts of firm dynamics, including large and persistent productivity differentials, the tent-shaped growth rate distribution, the scaling growth variance relation with size and the skewed firm size distribution.[48] The model is tested under three different learning scenarios, Schumpeter Mark I, II and an intermediate regime.[49] In Schumpeter Mark I incumbents do not learn (i.e., they enter with a productivity level that remains constant). In the intermediate regime, incumbents learn. In the Schumpeter Mark II regime, incumbents productivity shocks are cumulative and endogenous on firms' past productivities. The Schumpeter Mark II regime corresponds to an increasing returns scenario thanks to its *success-breeds-success* dynamics. The level of concentration increases with the degree on incumbents' learning. As a result, the support of the growth rate distribution becomes larger and the scaling growth variance relation shifts upward, indicating a higher level of volatility in the system. The level of concentration is higher when incumbents learn, even more when they do that endogenously in the Schumpeter Mark II regime.

[46]The use of a micro-foundation based on the interaction between learning and selection is not limited to studies of industrial dynamics, as it has also been employed to investigate the emergence of stylised facts related to business cycles and macroeconomic policies in the Keynes meeting Schumpeter, or K+S, models (Dosi et al., 2010). In these models the Schumpeterian structure of learning encompassing both innovation and imitation (see Nelson and Winter, 1982) is coupled with household and firm demand in a two-sectors (capital and consumption goods) economy. The model is able to replicate a number of macro economic stylised facts relative to business cycle fluctuations (e.g., the presence of large recessions), other than empirical properties of firm dynamics discussed in previous sections. Furthermore, the baseline structure of the model has been extended by several contributions aimed at studying problems of fiscal and monetary (see Dosi et al., 2015), green transition (see Lamperti et al., 2018) and labour market policies (see Dosi et al., 2017). The same model has also been used to understand the effect of different types of firms' expectations on macroeconomic series Dosi et al. (2020) and to explain countries development patterns and the emergence of income clubs (Dosi et al., 2019).

[47]As above, learning outcomes can lead to techniques which may also be inferior to extant ones. In that case, the firm will keep on relying on past technique.

[48]Persistent differentials are an empirical regularity that hint at the existence of what in (Metcalfe, 1994) is referred to as the inertia in the degree of variety of firm behaviour. Rules, routines and heuristics adopted by an organization change slowly with respect to the process of selection.

[49]The three scenarios bring to the extreme the patterns of innovation that may characterise the state of different technologies (see Malerba and Orsenigo, 1995).

5 Similarities and differences between research paradigms

In the previous sections we have discussed the properties of selection mechanisms in the rational equilibrium, Simonesque and evolutionary paradigms and briefly surveyed the main models employing them in sections 2, 3 and 4 respectively. We now compare the above three theoretical paradigms by highlighting their similarities and differences and whether there has been convergence in terms of assumptions and results. The discussion is divided in four main blocks (increasing returns, firm heterogeneity, firm rationality and focus on equilibrium states vs out-of-equilibrium dynamics). Via these blocks we want to represent the main characteristics of the paradigms at stake, and inquire thus the differences among models within blocks. These are summarised on table 1.

	Rational Equilibrium	**Simonesque**	**Evolutionary**
Returns	Increasing from Fixed Costs	Dynamic Increasing Returns	Dynamic Increasing Returns
Firms' Heterogeneity	Yes, Exogenous	Yes, Endogneous	Yes, Endogneous
Firms' Rationality	Perfect	Neutral	Bounded
Emphasis on	Equilibrium	Invariant Distributions	Out-of-Equilibrium Dynamics

Table 1: The main characteristics of the three paradigms in terms of returns, heterogeneity, rationality and type of outcome.

5.1 Increasing returns

The use and relevance of increasing returns is an element of convergence between selection paradigms. However, returns take a different form depending on the theory considered. This may generate different policy predictions concerning the relation between increasing returns, competition and concentration of shares, which is key given the recent discussions on industry concentration (see e.g., Bajgar et al., 2023; Grullon et al., 2019; Autor et al., 2020; Bessen, 2020; Bajgar et al., 2021).

In rational equilibrium models of monopolistic competition, increasing returns are generated by the presence of fixed costs (see equation 3). As a consequence, the average cost curve monotonically decreases with production and firms find it convenient to expand their production to the highest level possible. The strength of returns depends on the entity of fixed costs and returns are *static*.[50] Higher returns (i.e., higher fixed costs) are associated to higher productivity thresholds (see Hopenhayn, 1992; Melitz, 2003).[51] In turn, industry concentration increases because the number of firms is lower due to the increase in entry barriers and the size distribution shifts to the right. In Simonesque models, increasing returns derive from the use of Pólya types of processes. In this case,

[50] Cost curves are fixed in time. However, firms may move on cost curves in rational equilibrium models due to the presence of stochastic disturbances.

[51] Increasing fixed costs, firms get higher returns to scale because the decrease in average costs due to an increase in production increases as well.

returns are *dynamic* because they are not fixed in time, and they are an inherent feature of the selection process, as firms experiencing higher growth today will be more likely to grow in the future. Dynamic increasing returns have a fundamental role in allowing Simonesque models to generate the stylised facts relative to firms' size, growth rates and to the scaling relation existing between the two of them. Moreover, when returns are stronger in Simonesque models of the firm size distribution, market opportunities are more likely to be allocated to larger firms, inducing increases in concentration (see Sutton, 1997). Finally, in evolutionary models increasing returns stem from the interaction between learning and selection and, like in Simonesque models, they are *dynamic*, because firms growing more today will be more likely to grow in next periods as well. In addition, returns in evolutionary models cannot be dissociated from the selection process, because they allow larger and more productive firms to increase their shares with higher probabilities. Finally, when returns are higher in these models, selection mechanisms induce a faster reallocation of shares to more competitive firms and thus a higher industry concentration.

Overall, higher returns entail higher concentration via firms' competition in all the three paradigms. However, the relation between returns and concentration levels has different implications for competition policy. In rational equilibrium models, higher returns (i.e., higher fixed costs of production) induce less competitive markets with higher levels of concentration. The increase in fixed costs induces indeed a higher productivity threshold to survive, which lowers the number of firms in the market, and, on average, a larger firm size (see section 2.2 above). In contrast, in Simonesque and evolutionary models higher returns and concentration arise because of the higher competition levels induced by the increase of the velocity of reallocation toward more productive firms. Accordingly, in rational equilibrium models a higher level of competition entails a market which is less selective towards firms and where concentration is lower, and, conversely, in Simonesque and evolutionary models an increase in competition between firms generates a higher level of selection and eventually of concentration. In this respect, this difference in policy prescriptions is induced by the way in which market selection takes place (also see 5.3 below). In the rational equilibrium framework, an equilibrium state characterizes selection, *as if* a competitive process of survival of the fittest took place. In the Simonesque and evolutionary cases, this process is *explicitly* represented by the dynamics of selection, which can therefore take into account the increase in competition inducing a reallocation mechanisms toward more productive firms.

5.2 Firm heterogeneity

The widespread presence of firms' heterogeneity in the three research paradigms is a second element of convergence among them. At the same time, the mechanisms at the basis of its generation differ remarkably across them. In rational equilibrium models an underlying productivity distribution

$g(a_t)$ is assumed.[52] Firms' productivities are randomly drawn from this distribution, after paying the fixed cost of entry. The equilibrium distribution of productivity is the assumed distribution $g(a_t)$ truncated at the entry productivity threshold induced by fixed costs and full rationality assumption (see Hopenhayn, 1992; Melitz, 2003). The monopolistic competition structure perfectly maps the productivity into size, that will be characterised by the same distribution of the productivity. Conversely, in Simonesque and evolutionary models, firms' are homogeneous at the beginning of the competition process and heterogeneity is an endogenous outcome of the model. In Simonesque models, size heterogeneity is generated by the Gibrat's type of stochastic processes used in this research stream. The process underlying the selection of firms assumes a learning process in which firms' size and productivities develop at the same pace and maps thus one into the other. In evolutionary models shocks hit firms' productivities, and their effect is mediated into size growth via the quasi-replicator equation. The degree of firms' heterogeneity will depend on the interaction between the creation and destruction of varieties fostered by learning and selection respectively.

The above differences in modelling of firm heterogeneity have consequences for the ability of different models to explain the stylised facts of firm dynamics. On the one hand, the degree of size and productivity heterogeneity are largely exogenous in rational equilibrium models, as in the equilibrium the productivity distribution is the one assumed by the modeller truncated at the threshold induced by fixed costs and is perfectly mapped into size. For this reason, a Pareto productivity distribution must be assumed (or generated, as in Luttmer, 2007) in order to provide a tractable estimation of the model and to generate size distributions in line with the empirics (see the discussion in section 2). This assumption clashes however against the empirical evidence on the productivity distribution, which is better approximated by a Log-Normal (Doms and Bartelsman, 2000), and on its relation with size, which is not perfect (see Fontanelli et al., 2022; Berlingieri et al., 2018; Hsieh and Klenow, 2009, 2018). Notice that rational equilibrium models mostly rely on static mechanisms, which give rise to non-dynamic outcomes. As such these models may have difficulties in replicating stylised facts relative to firms' growth rates distribution and its relation with size.[53] On the other hand, evolutionary models are not affected by the choice of an underlying productivity distribution, nor by the perfect map between size and productivity. Firms' heterogeneity endogenously emerges from the model dynamics and the productivity distribution is not automatically mapped into size thanks to the frictions induced by the quasi-replicator. However, the modeller must tune productivity shocks in order to "keep the pieces together". The quasi-replicator fosters the destruction of heterogeneity pushing firms out of the market. Accordingly, the learning by incumbents and entrants must be combined with the quasi-replicator to prevent the convergence to monopoly (see Metcalfe, 2007). Moreover, as discussed in section 4.3, the empirical validity of the quasi-replicator

[52] There are few exceptions in which a Pareto distributed productivity endogenously emerge from the mechanics of the model (Luttmer, 2007; Impullitti et al., 2013).

[53] To our knowledge, there are no rational equilibrium models that attempt at replicating these facts.

is still outstanding.

Finally, notice that the change in heterogeneity when the initial parametrization and returns vary across differs across research paradigms. In rational equilibrium models, given the assumption of the initial productivity distribution, firm heterogeneity only changes by moving the truncation point, but the underlying distribution $g(a_t)$ and the one originating from it (e.g., the size distribution) remains unaltered. In Simonesque and in evolutionary models, the final degree of heterogeneity hinges on the initial parametrization and on the type of processes characterizing the dynamics of the model. In particular, in Simonesque ones the moments of the equilibrium distribution depend on the parametrization of the size shocks, whereas its shape on the type of returns assumed by the process (see the discussion in 3 and Sutton, 1997). In evolutionary models both the parametrization of learning and selection mechanisms and the type of returns impact on the degree and type of heterogeneity. Not only the moments, but also the shape of the distribution may be affected accordingly.

5.3 Firm rationality

Even though firms' heterogeneity is present in all the three paradigms, we argued how the assumptions over its generating process markedly differ across them. In particular, firms' heterogeneity is fostered by the process of selection, which is based on firms' decisions about entry, exit, learning and production, that depend in turn on the degree of rationality of firms. In this respect, the differences between rational equilibrium and evolutionary selection are marked because they originate from completely different theoretical "primitives". In rational equilibrium models entry, exit and production are voluntary decisions by firms. Firms take economic decisions by optimizing their profits because they can forecast future states of the economy by exploiting distributions and generating processes of the model. Firms in evolutionary model are boundedly rational. Information is highly imperfect and firms' decisions are governed by rules of thumb, whose relevant set of information is the past one only. In evolutionary models the uncertainty is Knightian and the economy is conceived as a complex system, whose amount of information can only be imperfect in comparison to the one that would be needed to be processed to forecast future states of the economy. Accordingly, economic decisions by firms are inherently imperfect, may not maximize the current or the future performance of the firm and even lead to its exit from the market. In the same vein, future learning shocks do not affect current firms' decisions because innovation outcomes cannot be forecasted by firms. Finally, the reduced form stochastic processes used in Simonesque models do not require agents to take economic decisions. Firms' selection and learning are a unique random process, entry (when present) is random as well and firm exit is generally not modelled. Simonesque models are thus neutral on firm rationality as they black-box firms' decisions into a reduced form stochastic process.

The assumptions on rationality have relevant consequences for the economic logic underpinning firms' selection. In rational equilibrium models firms are allowed to optimize their entry, production and exit processes. Based on future profit expectations and on the level of fixed costs, they decide if it is convenient or not to start or to continue producing. In dynamic rational equilibrium models, firms also know the distribution of innovation shocks, and their future profit expectations include the effect of future productivity shocks. Notice that this mechanism does not envisage a dynamic process of competition, wherein the least productive firms are forced out of the market. Conversely, this mechanism is conceived "as if" it took place because most productive firms will survive by optimizing their production processes. In evolutionary models, the focus is on firms' growth rates and on the consequent evolution of market shares as driven by the quasi-replicator equation. Firms' size is the sum of present and past shocks, that depend on relative productivities of firms, and is directly affected by other firms' dynamics, that are based on past information and mainly relate to innovation and investment. Firms do not take voluntary exit decisions. Rather they are forced out of the market when their share falls below a positive threshold. Selection is therefore competitive because firms' growth is driven by relative productivities and, ultimately, by firm interaction. Entry does not envisage a rational decision by firms. Bankrupted firms are mechanically substituted by entrants, whose productivity level is randomly drawn in function of incumbents productivities.[54] Finally, firms in Simonesque models do not take optimizing decision and selection only relates to firm growth and entry, as exit is generally not modelled. Firms are randomly hit by shocks with probability proportional to their market shares and cannot govern the processes related to their production and efficiency. In Simonesque models selection is thus competitive, as it is the output of firm interaction based on market opportunities assignment and is not black boxed by "as if" assumptions.

5.4 Equilibrium vs out-of-equilibrium dynamics

The type of models outcome constitutes a major discrepancy between different paradigms and are at the basis of a theoretical trade-off. Selection mechanisms have a relevant role on this matter, as they determine the convergence to a final state of equilibrium outcome or to out-of-equilibrium dynamics.

Rational equilibrium models are characterised by the convergence to an equilibrium, that can be static or encompass a dynamic toward a steady-state, and are tractable.[55] The concept of equilibrium in models with heterogeneous firms is intimately related to the assumption of firms' rationality.

[54] In most of evolutionary models, the number of entrants is assumed to be equal to the one of exiters in line with the evidence on entry and exit (Dunne et al., 1988).

[55] A model is tractable if it can be solved or approximated by a closed form solution. The causal mechanisms and the predictions of tractable models become thus explicit. In rational equilibrium models, this implied the existence of an equilibrium. Analytical tractability is a primary concern of rational equilibrium models, to the extent that it has often determined the emergence of the dominant selection mechanism in the history of the rational equilibrium theories (see the discussions in Archibald, 1987; Brakman and Heijdra, 2003; Neary, 2003, 2010).

Firms take optimizing production, entry and exit decisions based on the future state of the system, that are known to them thanks to the rationality assumption. Therefore an equilibrium arises because expectations by firms match with ones of the model. Equilibrium can be characterised by static or steady-state outcomes. However, in both the cases firms in equilibrium will be characterised by same average characteristics and distributions of productivities and size. These consists in the comparison of equilibrium outcomes of the same model under different parametrizations. Simonesque models converge to invariant distribution of one or more firm characteristics which are defined by the dynamics of firm selection. Differently from the rational equilibrium tradition, Simonesque models do not need to make assumptions on rationality to achieve this result because selection is represented by highly simplified processes which only encompasses firm size. However, once that models have converged to the invariant distributions, the characteristics of firms and industries do not change anymore, differently from evolutionary models. Evolutionary models generate indeed a dynamic outcome characterised by continuously evolving firms and industries characteristics thanks to the out-of-equilibrium dynamics fostered by replicator processes. These indeed converge to monopoly in presence of firm heterogeneity and absent learning (Metcalfe, 2007). However, firms' learning counterbalances the natural tendency to monopoly characterising selection, determining the existence of firms' heterogeneity as an out-of-equilibrium process. In this context, tractability can be difficultly reached and closed form solutions may be approximated only for sub-sections of the model.

The necessity for tractability may circumscribe the ability of the modeller to explain empirical stylised facts and to generate the non-Gaussianity typical of many economic relations and firm-level characteristics. On the one hand, the inclusion of further mechanisms of interaction to the standard selection structure of rational equilibrium and Simonesque models may easily entail the violation of the conditions which characterize their equilibrium outcomes. On the other hand, evolutionary models are not affected by the necessity of a closed form solution. This characteristic makes evolutionary models the best modelling environment to deal with dynamics, complexity, non-linear behaviours and non-Gaussian outcomes, specially when policy exercises and transition dynamics are involved. However, the absence of tractability may cause difficulties in understanding the statistical properties of the model, its data generating mechanisms. Still, it is possible to leverage statistical testing to understand the stationarity/ergodicity of models statistical outcomes and its generating processes (Fagiolo et al., 2019).

6 Concluding remarks

In this survey we have discussed the main theories of selection in the economic literature. We divided them in three main research paradigms (rational equilibrium, Simonesque and evolutionary models), which we described by focusing on the role of firm heterogeneity and increasing returns.

We have argued that these two elements have had a pivotal role in the recent development of the three theories. Increasing returns from fixed costs (Dixit and Stiglitz, 1977) and the heterogeneity stemming from fixed cost cum rationality (Hopenhayn, 1992) enables rational equilibrium models to replicate several stylised facts (see Melitz, 2003, and expansions). Firms' heterogeneity is the main object of inquiry in Simonesque models, and increasing returns are the key to reproduce stylised facts of firm dynamics (see Ijiri and Simon, 1977; Bottazzi and Secchi, 2006a; Fu et al., 2005; Riccaboni et al., 2008). Evolutionary models were born to explain firm heterogeneity as the emergent property of a Schumpeterian process of learning and selection. Dynamic increasing returns are at the basis of several evolutionary models and heavily affect the industrial structures emerging from these models.

We have discussed the similarities and differences in selection between the three research paradigms in relation to four main blocks (heterogeneity, returns, rationality and type of outcome). The use of increasing returns concerns and the presence of firm heterogeneity characterize the three research paradigms, albeit hinging on different theoretical assumptions. Conversely, the three paradigms remarkably differ in terms of firms' rationality assumptions of their outcome types (static or steady state equilibrium or by out-of-equilibrium dynamics).

To conclude, the history of selection mechanisms in different research paradigms and their characterising features suggest that the only type of convergence witnessed between them concerns the direction of research, that has focused on the replication of empirical stylised facts related to firm heterogeneity. In this respect, the paradigms also employ similar research heuristics by modelling firm heterogeneity and increasing returns, whose use was fostered by the need to incorporate in models selection mechanisms capable of generating empirical results that previous models could not replicate. The convergence between the models has thus been driven by instrumental, rather than theoretical concerns. The differences in the theoretical assumptions underlying selection in the three research paradigms remain wide (see the discussion in section 5).

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