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The DSK-SFC stock-flow consistent agent-based integrated assessment model

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The DSK-SFC stock-flow consistent agent-based integrated assessment model

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ABSTRACT

We present an updated, stock-flow consistent version of the 'Dystopian Schumpeter meeting Keynes' agent-based integrated assessment model. By embedding the model in a fully specified accounting system, all balance sheet items and financial flows can be explicitly and consistently tracked throughout a simulation. This allows for an improved analysis of climate change and climate policy scenarios in terms of their systemic implications for agent and sector-level balance sheet dynamics and financial stability. We provide an extensive description of the updated model, representing the most detailed outline of a model from the well-established 'Keynes + Schumpeter' family available to date. Following a discussion of calibration and validation, we present a range of example scenarios.

1. Introduction

This paper presents an updated, explicitly stock-flow consistent (SFC) version of the *Dystopian Schumpeter meeting Keynes* (DSK) agent-based integrated assessment model (Lamperti et al., 2018, 2019a, 2020, 2021) representing the first integrated assessment model featuring full stock flow consistency, agents' heterogeneity, bottom-up climate impacts and endogenous GDP growth as well as cyclical fluctuations. Over the past 15 years, the use of agent-based models (ABMs) in macroeconomic analysis has increased in popularity, proposing a wide array of frameworks for the analysis of business cycle fluctuations, long-term growth, financial crises and their interplay (see e.g. Fagiolo and Roventini, 2017, Dawid and Delli Gatti, 2018 and Dosi and Roventini, 2019 for an overview). Somewhat more recently, complexity approaches in general and AB modelling in particular have also seen applications within the literature on environmental/ecological economics (Balint et al., 2017), with several macroeconomic frameworks which incorporate environment-energy-economy interactions being proposed (see Naumann-Woleske, 2023, for a recent review). The DSK model, originally proposed by Lamperti et al. (2018), is the first agent-based integrated assessment model (IAM), providing an alternative and complementary perspective to the analyses produced by more conventional IAM frameworks (e.g. Bosetti et al., 2006; Leimbach et al., 2010; Huppmann et al., 2019), both for what concerns impact assessment (Lamperti et al., 2019a, 2021) and climate policy (Lamperti et al., 2021).

Several features can make an agent-based approach a valuable complement or alternative to more conventional approaches to integrated assessment modelling (see also Farmer et al., 2015; Lamperti et al., 2019b). Most obviously, ABMs are inherently well-suited for the incorporation of agent heterogeneity along multiple dimensions and their interactions. Suitably specified ABMs can hence be used to investigate issues such as changes in the distribution of income or the sectoral composition of economies as a consequence of climate change and climate policy, as well as interactions occurring on goods and financial markets. Moreover, ABMs allow for a more detailed depiction of institutional frameworks and policy measures than is typically the case in general equilibrium frameworks, meaning that a richer set of climate policy packages can be explored. For both methodological and computational reasons, ABMs also do not make of the usual assumptions on agent rationality and perfect foresight or rational expectations typically underlying general equilibrium models, instead positing that model agents follow a set of more or less sophisticated heuristics and rules of thumb in their decision-making, which may adapt and evolve over time. This use of alternative behavioural assumptions can provide an important comparative perspective to the optimisation-based results obtained from standard IAMs. Though this is neither a unique nor a strictly necessary characteristic, virtually all existing macroeconomic ABMs, including the macroeconomic framework underlying the DSK model, feature short-run endogenous business-cycle dynamics, including periods of deep crisis. This means that an agent-based IAM can be used to study not only the long-run implications of climate change and climate policy, but also their short-run impacts which are typically of great interest to policy-makers. Finally, ABMs have traditionally strongly emphasised the modelling of the financial sector and real-financial interactions. By contrast, this dimension

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is under-developed in conventional IAMs (Sanders et al., 2022), meaning that an AB approach can provide useful and unique insights on issues such as the financial stability implications of climate change and climate policy.

This paper represents an advancement in particular regarding the latter dimension. It embeds the DSK model in a fully consistent accounting framework, following the paradigm of stock-flow consistent modelling (Godley and Lavoie, 2007) which has become an important component of AB macroeconomics (Caiani et al., 2016). This makes it possible to explicitly and consistently track all balance sheet items and financial flows in the model, allowing for a detailed analysis of the systemic implications of particular scenarios and trajectories in terms of changes in financial ratios and balance sheet positions at the agent and/or sector level. It is also an important building block for planned future extensions of the framework which we briefly discuss in the conclusions. Additionally, a range of usability and accessibility updates were made to the code during the construction of the model version presented here. The paper also contains the most detailed description of a model of the 'Keynes + Schumpeter' (K+S)/DSK family available to date. Since this is one of the most widely applied macroeconomic ABM frameworks in the literature, we make an important contribution to improving the transparency and reproducibility of macroeconomic agent-based modelling (cf. Dawid et al., 2016).

The paper proceeds as follows: Section 2 presents a broad overview of the model's main features and its accounting structure. Section 3 contains a brief model description, with a fully exhaustive one being provided in Appendix A. Section 4 describes the main accessibility and usability upgrades which were made to the model code. Section 5 describes the calibration and validation of the model, with additional details being given in Appendix A. Section 6 presents some example simulation experiments. Section 7 concludes. Appendix B contains tables giving a full list of all model parameters and initial values, along with the respective values used in the simulations shown in the paper.

2. Overall structure

With the exception of the newly added separate fossil fuel sector, the sectoral structure of DSK-SFC is identical to that of previous versions of the DSK model (Lamperti et al., 2018, 2019a, 2020, 2021), with the difference that all balance sheet items and transaction flows are explicitly modelled and tracked during simulations.

The DSK framework is part of the *Schumpeter meeting Keynes* (K+S) family of models Dosi et al. (2010, 2013, 2015, 2017b). As such, the economic core of the model is formed by an agent-based firm sector, differentiated into consumption good firms and capital good firms (C-Firms and K-Firms hereafter). K-Firms produce capital goods which possess heterogeneous characteristics in terms of the labour productivity, energy efficiency and emission intensity. To produce them, K-Firms use production techniques which are also heterogeneous in terms of labour productivity, energy efficiency and emission intensity. New and superior vintages of capital goods as well as novel techniques emerge as the outcome of an innovation process driven by K-Firms' endogenous R&D expenditure. This ultimately drives long-term growth in the model. K-Firms sell capital goods to C-Firms, which use them (alongside labour and energy) to produce a homogeneous consumption good. The investment of C-Firms depends on their expected demand and on the process of technological change. This can trigger Keynesian endogenous business cycles. Firms' activities are financed through retained earnings and, in the case of C-Firms, loans from a banking sector. Households consume and receive income in the form of wages for supplied labour, interest on deposits, dividends from firms, banks and the energy sector, as well as unemployment benefits. The government collects taxes and spends on unemployment benefits as well as, possibly, the bailout of failing banks. The DSK model also includes an energy sector which supplies the firm sector with energy needed for production and which also engages in endogenous R&D. Moreover, it features a climate module which receives emissions from the economic model and can feed back on the latter through climate shocks. Finally, the version of the model presented here also includes a separate fossil fuel sector which sells fossil fuels as an input to the energy sector.

Figure 1 provides an overview of the model, including market and non-market interactions between sectors and the role of the climate module. Table 1 gives more detail on the accounting structure, showing the balance sheet matrix including the assets and liabilities held by each sector. The consumption and capital good sectors, as well as the banking sector, consist of multiple and heterogenous agents, while the other sectors (including energy and households) can be considered as aggregate entities. Table 2 shows the transactions flow matrix of the model, summarising the transactions between sectors and showing how these are financed.

The balance sheet and transaction flow matrices can be used to derive the accounting identities that must be satisfied for the model to be formally stock-flow consistent. To ensure stock-flow consistency during simulations of the model, all transaction flows and balance sheet items are explicitly tracked. At the end of each simulation period, the model performs a series of checks at the agent, sectoral and aggregate levels to ensure that no accounting identities have been violated during the period.

3. The model

The present section provides a compact overview of the DSK-SFC model, describing agent types and their behavioural rules. A fully exhaustive model description, including the sequence of events taking place within each simulation period, is provided in appendix A.

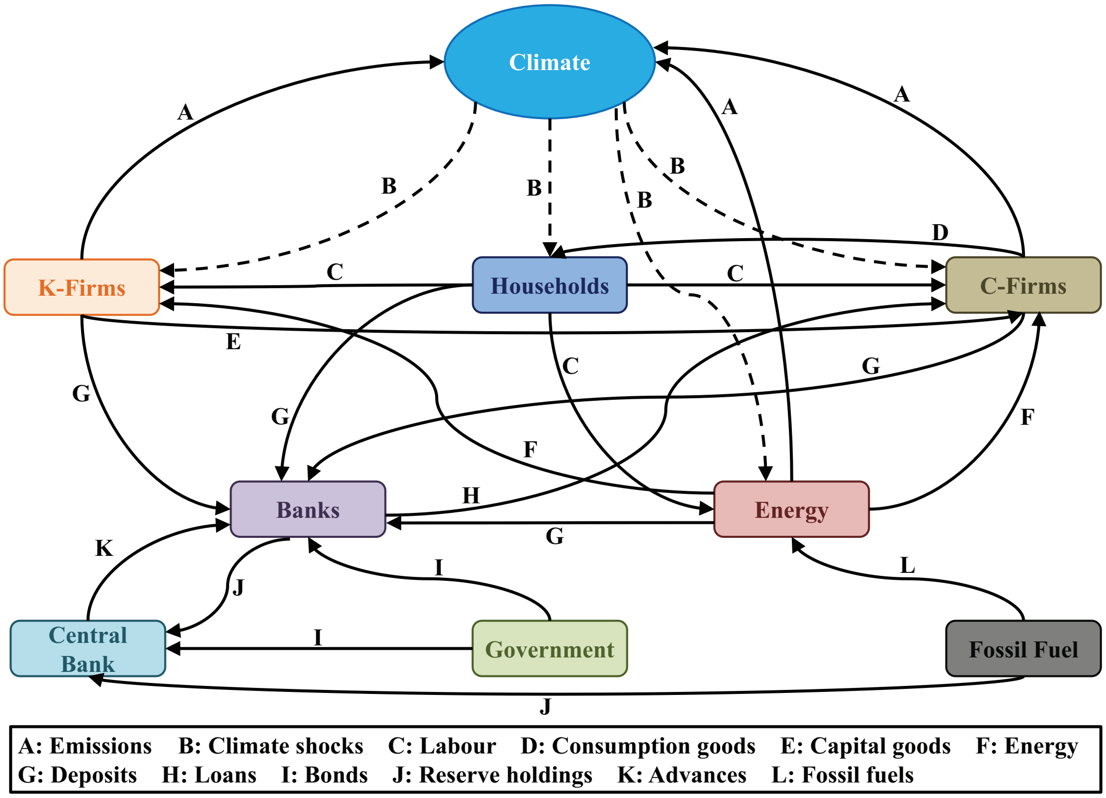

Figure 1: Overview of the DSK-SFC model. Arrows represent market or non-market interactions between sectors/model components as detailed in the legend.

Table 1
Balance Sheet Matrix

	Households	C-Firms	K-Firms	Banks	Gov.	CB	Energy	Fossil	Σ
Bank Deposits	$+D_h$	$+D_c$	$+D_k$	$-D$			$+D_e$		0
Gov. Bonds				$+GB_b$	$-GB$	$+GB_{cb}$			0
Loans		$-L$		$+L$					0
CB Reserves				$+R_b$		$-R$		$+R_f$	0
CB Advances				$-A$		$+A$			0
Fixed Capital		$+K$					$+K_e$		$K+K_e$
Inventories		$+Inv$							Inv
Σ	NW_h	NW_c	NW_k	NW_b	NW_g	NW_{cb}	NW_e	NW_f	$K+K_e+Inv$

3.1. Households

The household sector is modelled as an aggregate entity which earns wages (in exchange for supplying labour), unemployment benefits, as well as dividends from firms, banks and the energy sector. The maximum aggregate labour supply of households (representing the labour force) changes at an exogenous rate reflecting population growth (or decline); up to this maximum, households will supply any amount of labour demanded at the current nominal wage rate. For any part of the aggregate labour supply which is not employed, households receive an unemployment benefit payment.

Households' nominal consumption demand is given by

$$C_{d,t} = \alpha_1\left(W_t + UB_t - Tax_t^H\right) + \alpha_2\left(Div_{t-1} + iD_{h,t}\right) + \alpha_3 D_{h,t-1} \tag{1}$$

where $\left(W_t + UB_t - Tax_t^H\right)$ is wage and benefit income net of taxes, $\left(Div_{t-1} + iD_{h,t}\right)$ is income from dividends and interest on households' bank deposits, and $D_{h,t-1}$ is the stock of deposits held by households. Households hence have different propensities to consume out of wage and benefit income, dividend and interest income, and wealth. This functional form is very similar to what is found in many other macroeconomic ABMs (Dawid and Delli Gatti,

Table 2
Transactions Flow Matrix

	Households	**C-Firms**	**K-Firms**	**Banks**	**Government**	**Central Bank**	**Energy**	**Fossil**	Σ
Consumption	$-C$	$+C$							0
Investment		$-I$	$+I$						0
Benefits	$+UB$				$-UB$				0
Taxes	$-Tax_h$	$-Tax_c$	$-Tax_k$	$-Tax_b$	$+Tax$		$-Tax_e$		0
Wages	$+W$	$-W_c$	$-W_k$				$-W_e$		0
Fuel							$-FF$	$+FF$	0
Energy		$-E_c$	$-E_k$				$+E$		0
Dividends	$+Div$	$-Div_c$	$-Div_k$	$-Div_b$			$-Div_e$	$-Div_f$	0
Interest Loans		$-iL$		$+iL$					0
Interest Deposits	$+iD_h$	$+iD_c$	$+iD_k$	$-iD$			$+iD_e$		0
Int. Gov. Bonds				$+iGB_b$	$-iGB$	$+iGB_{cb}$			0
Int. Reserves				$+iR$		$-iR$			0
Int. Advances				$-iA$		$+iA$			0
Transfer CB					$+T_{cb}$	$-T_{cb}$			0
Transfer Entry	$-T_h$	$+T_c$	$+T_k$	$-T_b$	$-T_g$				0
Bailout				$+Bail$	$-Bail$				0
Saving	(Sav_h)	(Sav_c)	(Sav_k)	(Sav_b)	(Sav_g)	(Sav_{cb})	(Sav_e)	(Sav_f)	0
Δ Deposits	$-\Delta D_h$	$-\Delta D_c$	$-\Delta D_k$	$+\Delta D$			$-\Delta D_e$		0
Δ Gov. Bonds				$-\Delta GB_b$	$+\Delta GB$	$-\Delta GB_{cb}$			0
Δ Loans		$+(\Delta L)$		$-(\Delta L)$					0
Δ Reserves				$-\Delta R_b$		$+\Delta R$		$-\Delta R_f$	0
Δ Advances				$+\Delta A$		$-\Delta A$			0
Σ	0	0	0	0	0	0	0	0	

2018), as well as in aggregate SFC models (Nikiforos and Zezza, 2017). All household savings are held in the form of bank deposits and households cannot borrow to finance consumption.

The nominal wage rate follows a Phillips curve-type rule (see equation A.3 in the detailed model description), being a decreasing function of the unemployment rate and increasing in current inflation. In addition, the nominal wage is pegged to long-run labour productivity growth.

3.2. Capital good firms

The sector of capital goods firms (K-Firms) consists of $N1$ individual firms. Each firm produces a capital good with unique characteristics in terms of the embedded labour productivity, energy efficiency, and environmental friendliness. To do so, the firm uses a unique Leontief production technique characterised by a specific set of technical coefficients, with labour and energy as inputs. Capital good vintages and production techniques are both subject to endogenous technological change.

K-firms receive orders for capital goods from C-Firms and produce on demand. Their demand for energy and labour, as well as the emissions arising from the production of capital goods are computed based on K-Firms' current production techniques. K-firms price capital goods by applying a fixed and homogeneous markup over unit cost of production. Since K-Firms' production techniques are heterogeneous, so are their unit costs and hence the selling prices of capital goods. All K-Firms begin each simulation with an equal number of C-Firm customers, but subsequently compete to attract additional ones by sending brochures to C-Firms, which detail both the selling price and characteristics of the capital goods offered. Every C-Firm can switch to a new supplier of capital goods in every period, using the attractiveness measure described in Section 3.3 to compare brochures.

K-firms aim to improve their production technique and to offer improved vintages of capital goods through innovation of new technologies and imitation of technologies of competitors. The process of technological change, drawing on the work of Nelson and Winter (1982) and Dosi (1988), consists of two steps. First, K-Firms allocate resources for innovation and imitation, investing a fixed fraction of revenues into research and development. These resources are used to hire labour and split between efforts directed toward innovation and imitation. The size of these R&D investments determines the likelihood of innovation and/or imitation being successful, i.e. the probability for a given K-Firm to innovate/imitate in some period t is increasing in the respective R&D inputs, $RD^{in}_{k,t}$ for innovation and $RD^{im}_{k,t}$ for imitation:

$$\begin{aligned} P(Innovate)_{k,t} &= 1 - exp\left(-\mathfrak{b}^K_1 RD^{in}_{k,t}\right) \\ P(Imitate)_{k,t} &= 1 - exp\left(-\mathfrak{b}^K_2 RD^{im}_{k,t}\right) \end{aligned} \tag{2}$$

with $\mathfrak{b}_1^K$ and $\mathfrak{b}_2^K$ being fixed parameters. Conditional on innovation and/or imitation being successful, the characteristics of the resulting technology or technologies are determined stochastically.

Innovation is depicted as a random simultaneous change to all characteristics (labour productivity, energy efficiency and environmental friendliness) of the capital goods produced by, as well as the production technique used by the innovating K-Firm. Importantly, these stochastic innovations need not all be positive, i.e. the innovation process may result in a technology that is worse than the existing one along one or more dimensions. This accounts for the trial and error nature of the innovation process.

Imitation, by contrast, is based on a measure of technological proximity, derived by comparing the labour productivities, energy efficiencies and emission intensities of the capital vintages and production techniques of all K-Firms (see Appendix A). K-Firms are assumed to be more likely to imitate the technology of competitors whose technology is more similar to their current one. Here, too, the model allows for the possibility that an imitated technology is inferior to the one already possessed by the imitating firm.

To decide which new technology (if any) to adopt, a K-Firm k compares the innovated and imitated technologies to one another, as well as to its existing technology. To do so, it uses a measure of attractiveness (which is also used by C-Firms in choosing suppliers and deciding on substitution investment; see Section 3.3), which it computes for its existing technology, as well as the innovated and imitated ones:

$$\begin{aligned} A_{\kappa,t} &= p_{k,t} + uc_{\kappa,t} b \\ A_{\kappa_{in},t} &= p_{in,k,t} + uc_{\kappa_{in},t} b \\ A_{\kappa_{im},t} &= p_{im,k,t} + uc_{\kappa_{im},t} b \end{aligned} \tag{3}$$

where $p_{k,t}$ is the price which k currently charges for one unit of capital good, while $p_{in,k,t}$ and $p_{im,k,t}$ are the prices which k would charge when producing using the innovated or imitated production technique, respectively. The uc terms denote the unit cost of production which C-Firms would incur when using a machine of the current, innovated, and imitated vintage, respectively. b is a fixed and homogeneous payback parameter. The K-Firm chooses the technology for which A takes the lowest value. Note that this technology need not be superior along all dimensions, and the values of the A's also depend on the wage rate, the energy price, and the emission tax rate, and hence economic conditions and the policy environment.

Once K-Firms have produced and sold the capital goods demanded in t, their gross profits are computed. If these are positive, they pay profit taxes at a flat rate. In addition, K-Firms distribute a fixed share of after-tax profits as dividends to households. Retained earnings are held in the form of bank deposits, with each K-Firm being a customer of one of the banks in the model. For simplicity, it is assumed that K-Firms cannot borrow from the banking sector.

As explained in the detailed model description in appendix A, a K-Firm may be unable to fulfill all of its payment obligations for energy input or wages. If this is the case, the firm in question exits the model. K-Firms which lose all of their clients also exit the model. Exiting K-Firms are replaced according to the mechanism described in appendix A and section 3.4.

3.3. Consumption good firms

The consumption good sector consists of $N2$ individual firms producing a homogeneous final consumption good using capital, labour, and energy. The capital stocks of consumption good firms (C-Firms) are heterogeneous in terms of vintages, such that each C-Firm has an individual labour productivity, energy efficiency, and environmental friendliness. Since consumption goods are homogeneous, C-Firms compete for market shares through price and their ability to satisfy demand.

C-Firms' desired production is calculated based on expected demand (which follows adaptive expectations)[1] and desired inventory holdings. If a C-Firms' productive capacity in terms of capital stock is insufficient to carry out the desired production, the latter is scaled back accordingly. In order to expand their productive capacity, C-Firms may invest to expand their capital stock (expansion investment).

C-Firms aim to maintain their capacity utilisation at a target level u; for a given desired production $Q^d_{c,t}$, the desired capital stock of firm c, expressed in terms of productive capacity, is hence:

$$\mathfrak{K}^d_{c,t} = \frac{Q^d_{c,t}}{u} \tag{4}$$

Desired expansion investment is then given by the difference between the desired and the current capital stock (net of depreciation) of c if this difference is positive (i.e. we do not allow for disinvestment).

In addition, a C-Firm may decide to replace capital goods which have not depreciated but are technologically obsolete relative to newly available vintages (substitution investment). A detailed description of this procedure is provided in appendix A.

To pick a capital good supplier, C-Firms compare the brochures they have received (see Section 3.2). This comparison is made using the same attractiveness measure employed for technology selection in the K-Firms' innovation/imitation process (see Equation (3)). For a given vintage κ produced by a K-Firm k, this measure is given by

$$A_{\kappa,t} = p_{k,t} + uc_{\kappa,t} b \tag{5}$$

with b being the same payback parameter used in Equation (3). C-Firm c then chooses the observed supplier of capital goods whose vintage offers the lowest value of A.

C-Firms set the price for their output by applying a mark-up on unit cost of production. Mark-ups are heterogeneous and change as a function of C-Firms' market shares. The

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[1] See Dosi et al. (2020) for an exploration of alternative expectation formation mechanism, showing that the underlying K+S macroeconomic framework is robust to such variations.

unit cost of production is also heterogeneous across C-Firms as it depends on the composition of an individual C-Firm c's capital stock in terms of capital vintages. C-Firm c's effective labour productivity ($Pr^{e}_{c,t}$), energy efficiency ($EE^{e}_{c,t}$) and emission intensity/environmental friendliness ($EF^{e}_{c,t}$) are a weighted average of the labour productivities, energy efficiencies and emission intensities embedded in the various vintages capital goods used by c. Based on these, C-Firms' demand for labour and energy, as well as the emissions resulting from the production of consumption goods are computed.

C-Firms finance their production and investment using retained earnings, held in the form of bank deposits, and loans from the banking sector. Besides possibly being credit-rationed (see below), each C-Firm has an internal constraint giving a maximum amount of credit it is prepared to take on for the purpose of investment, given by a multiple of its net revenue (sales revenue minus production cost) in the previous period. If the the nominal value of desired investment exceeds the sum of internal funds and this maximum amount of credit, the C-Firm will scale its investment back accordingly. Additionally, a C-Firm may also be subject to an external credit constraint if its bank is unwilling to extend all the credit which the firm demands (Stiglitz and Weiss, 1981). In this case, too, planned expenditures (possibly also including planned production) must be reduced until they can be financed out of internal funds plus the maximum amount of credit available.

Households' aggregate demand for consumption goods is distributed to C-Firms according to market shares. As the consumption good market is characterised by imperfect information, the market share of each firm follows a quasi-replicator dynamic (cf. Dosi et al., 2010) and is a function of its competitiveness, $E_{c,t}$. The latter is in turn computed based on the firm's price relative to the average across C-Firms $\left(\frac{p_{c,t}}{\hat{p}_t}\right)$ and its relative ability to satisfy the demand received in the previous period $\left(\frac{l_{c,t}}{\hat{l}_t}\right)$:[2]

$$E_{c,t} = -\left(\frac{p_{c,t}}{\hat{p}_t}\right)^{\omega_1} - \left(\frac{l_{c,t}}{\hat{l}_t}\right)^{\omega_2} \tag{6}$$

Market shares $f_{c,t}$ are then computed using this measure of competitiveness and normalised to ensure that they sum to 1:

$$\tilde{f}_{c,t} = f_{c,t-1}\left(\frac{2\omega_3}{1+e^{\left(-\chi\frac{E_{c,t}-\hat{E}_t}{\hat{E}_t}\right)}} + \left(1-\omega_3\right)\right) \tag{7}$$

$$f_{c,t} = \frac{\tilde{f}_{c,t}}{\sum_{i=1}^{N2}\tilde{f}_{i,t}} \tag{8}$$

Consumption demand is then distributed using these market shares over multiple rounds, until either all consumption demand has been satisfied or all C-Firm output has been sold.

To ensure stock-flow consistency, C-Firms' profit is computed taking into account both revenues and expenditures, as well as revaluations of capital and inventory stocks. If profits are positive, firms pay taxes at a flat rate τ^{C}. Moreover, they distribute a fixed share of post-tax profits as dividends to the household sector. In addition to interest payments on loans (which enter the profit calculation), C-Firms must also repay a share of outstanding loans at the end of each period.

C-Firms exit the model if they are unable to make a due payment, if they are unable to roll over outstanding loans, if their market share falls below a fixed lower threshold, or if their net worth becomes negative.

3.4. Firm exit and entry

Every exiting firm is replaced by a new one operating in the same sector at the end of each period (Bartelsman et al., 2005). The firm replacement mechanism is designed to ensure stock-flow consistency.

When a K-Firm exits, any remaining balance of deposits is transferred to the household sector. The new K-Firm replacing the exiting one receives a transfer of deposits from the household sector. Its initial technology is a random copy of an incumbent in the capital good sector.

When a C-Firm exits, any remaining bank deposits are used to pay off outstanding loans, with the rest being transferred to households. In addition, the banks may sell the remaining capital stocks of exiting firms on a second-hand market to compensate for losses on remaining unpaid loans. Entering C-Firms receive a transfer of deposits from the household sector. In addition, they receive an initial stock of capital, made up of second-hand capital goods stemming from exiting firms which were previously transferred to households or sold to them by the banks. Appendix A contains details on how entering firms are initialised.

3.5. Banks

The banking sector consists of NB individual banks which differ in the number of individual firm customers, implying that the size and composition of bank balance sheets is heterogeneous. At the beginning of a simulation, each bank is assigned a number of K-Firm and C-Firm customers drawn from a truncated Pareto distribution to produce a right-skewed size distribution (Ennis, 2001). Thereafter, the firm-bank networks remain static unless a bank fails and is not bailed out (see below). While each firm is hence linked to a single bank, the deposit holdings of households and the energy sector (which are aggregates) are distributed across all banks in proportion to the number of firm customers of the respective banks.

Deposits held by firms, households and the energy sector are the banks' main liability. The interest rate on deposits is identical across banks and given by a markdown on the central bank deposit interest rate. On the asset side, banks provide loans to C-Firms. The maximum amount of credit a bank b is prepared to extend is given by a multiple of its net worth. This 'credit multiplier' changes endogenously as

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[2] The computation of $l_{c,t}$ is described in appendix A.

a function of the financial fragility of b, defined as losses from defaults taken in the previous period as a share of net worth. Specifically, the maximum amount of credit supply a bank provides in each period is:

$$\mathfrak{C}^{s}_{b,t} = \frac{NW_{b,t}}{buffer_{b,t}} \tag{9}$$

where NW represents net worth and $buffer$ a credit multiplier depending on a parameter representing prudential regulation, as well as the bank's financial fragility (see also Appendix A).

Banks rank their C-Firm customers in ascending order according to their debt service to revenue ratio. The loan interest rate charged by bank b to customer C-Firm c is then given by:

$$r^{l}_{b,c,t} = r^{l}_{b,t}\left(1 + \left(rank_{c,t} - 1\right)\mathfrak{M}\right) \tag{10}$$

where $r^{l}_{b,t}$ is a baseline loan rate given by a constant and homogeneous mark-up over the central bank lending rate, $\mathfrak{M}$ is a parameter, and $rank_{c,t}$ is the quartile of the distribution of debt service-to-revenue ratios among b's customers to which c belongs. The bank hence charges a higher loan rate to customers with a higher debt service to revenue ratio. In addition, this ranking is also used in the allocation of loans, with the banks satisfying firm credit demand in the order of the ranking of their customers, meaning that firms with high debt service to revenue ratios are more likely to be credit rationed (Bernanke et al., 1996).

In addition to lending to C-Firms, banks also invest in government bonds, with each bank's demand for government bonds being given by a fraction of its stock of loans to the private sector.

When a bank needs to make an interbank payment, it uses central bank reserves, which it can borrow at the lending rate set by the central bank. Stocks of reserves are remunerated at the central bank deposit rate.

Bank profits are calculated taking into account all interest income and expenditures, as well as possible losses from defaults of C-Firms. If profits are positive, banks pay a fraction τ^{B} of them in taxes. In addition, they pay a fixed share of profits as dividends to the household sector.

A bank fails if its net worth becomes negative. Depending on the simulation setting, failing banks are either always bailed out by the government (this is the case in the simulations shown below) or purchased by the surviving bank with the highest net worth (in this case the government only provides a bailout if that latter bank is unable to purchase the failing one or if there is no surviving bank).

3.6. Government

The government collects taxes on wages, as well as on the profits of firms, banks and the energy sector. These taxes are levied at a flat rate. In addition, the government may collect a carbon tax on firm and/or energy sector emissions which is charged per unit of emission produced and may change at differing rates depending on the simulated scenario (see Appendix A). Note that while such a mechanism can easily be implemented, the DSK-SFC model as shown here does not include a dedicated 'recycling' mechanism for carbon tax revenues. Instead, revenue from carbon taxation enters the government budget in the same way as other tax revenue. Finally, any profits made by the central bank are paid to the government (but central bank losses are also compensated by the government).

On the expenditure side, the government pays unemployment benefits to households, given by:

$$UB_{t} = \zeta w_{t}\left(LS_{t} - L_{t}\right) \tag{11}$$

where w_t is the current nominal wage rate, ζ is a parameter, and $\left(LS_{t} - L_{t}\right)$ is the difference between the total labour force and the amount of labour employed in t. As explained in appendix A, the government may also have expenditures to finance the entry of new firms (if households are unable to finance entry) and for bailing out failing banks.

Additionally, the government must make interest payments on the stock of outstanding government bonds. The interest rate on government bonds is determined by marking down the central bank lending rate.

Finally, in every period, the government must repay a share of outstanding government bonds (but can repay more if it has a sufficiently large surplus).

If tax revenues are insufficient to finance all payments the government must make, it issues new bonds to cover the difference. These bonds are in the first instance offered to banks, with any unsold remainder being purchased by the central bank.

3.7. Central bank

The central bank conducts monetary policy by setting the base interest rate. Its lending rate follows a Taylor rule (Taylor, 1993) of the form:

$$\begin{aligned} r^{l}_{CB,t} = max\Big(&\underline{r}, \iota_{1} r^{l}_{CB,t-1} + \left(1 - \iota_{1}\right) \\ &\times \left(r + \iota_{2}\left(\pi^{a}_{t} - \pi^{*}\right) + \iota_{3}(U^{*} - U_{t})\right)\Big) \end{aligned} \tag{12}$$

where ι_1 is an interest rate smoothing parameter, r is a fixed intercept, π^{a}_{t} is the current year-on-year inflation rate with π^{*} being the year-on-year inflation target, U_t being the current unemployment rate and U^{*} the central bank's target unemployment rate. $\underline{r}$ is a fixed lower bound close to 0. The central bank's deposit rate is given by a fixed percentage markdown on the lending rate.

In addition, the bank can be thought of as the prudential policy-maker setting the banks' capital requirement as detailed in appendix A. Finally, the central bank also enables interbank payments by supplying the reserves required to settle such transactions. All inflows and outflows of reserves occurring for each bank during a period are recorded to calculate net flows at the end of a period. Banks recording a net outflow either use existing stocks of reserves to cover this or take advances from the central bank, which the latter provides on demand at the current lending rate. We do not model an interbank market for reserves.

3.8. Energy Sector

The energy sector consists of a single agent which sells energy to K-Firms and C-Firms. Energy is produced using both 'green' and 'brown' technologies of various vintages.

We assume that the total amount of energy produced is always demand-determined and the energy sector can instantaneously expand its productive capacity by erecting new energy plants to meet demand if necessary. The productive capacity of the energy sector, $\mathfrak{K}_{e,t-1}$, is expressed in units of energy producible and can be divided into 'brown/dirty' $\left(\mathfrak{K}^{de}_{t-1}\right)$ and 'green/clean' $\left(\mathfrak{K}^{ge}_{t-1}\right)$ capacity. Green and brown energy technologies are highly stylised. Brown energy production gives rise to emissions while green energy production does not. Energy production using green technologies is assumed to be costless, while brown energy production requires a costly fossil fuel input, as well as incurring the carbon tax if one is implemented. Conversely, the expansion of brown energy capacity is assumed to be costless, while green energy capacity expansion carries a positive cost.

When expanding capacity, the energy sector must choose whether to invest in green or brown capacity, considering only the most recent vintage of each technology. To do so, it compares the cost of production of one unit of brown energy using the most recent vintage to the cost of installing one unit of green capacity, divided by a payback period parameter. The actual composition of investment carried out then depends on the simulation setting (e.g. the maximum per-period green capacity expansion can be exogenously constrained or not, or investment shares in green and brown technologies can be completely exogenously fixed, see appendix A).

If the energy sector invests in green capacity, the cost for this takes the form of labour input which is hired from the household sector at the current wage rate. The cost is staggered over the payback period b^e of the investment, but the constructed capacity comes online instantaneously. The model is calibrated such that the energy sector can always internally finance capacity expansion. All energy production plants are assumed to have a fixed lifetime of $\aleph^E$ periods after which they are written off and scrapped.

When producing energy, plants of different technologies and vintages are activated following a 'merit-order' principle (Sensfuß et al., 2008). Since the production cost for green energy is assumed to be zero, green plants are always activated first. If the existing green capacity is insufficient to satisfy all energy demand, brown plants are activated starting from the one with the lowest unit cost of energy production. The uniform price of energy is determined as a mark-up ($\mu_{e,t}$) over the unit cost of energy production of the last plant activated ($mc_{e,t}$):

$$p_{e,t} = \mu_{e,t} + mc_{e,t} \tag{13}$$

where $\mu_{e,t}$ is assumed to grow over time following a weighted average of past changes in the nominal wealth to keep the energy price in line with the nominal size of the rest of the economy. If brown energy is produced, the energy sector purchases the required fossil fuel input from the fossil fuel sector (described below) and emissions from energy production are calculated.

Similarly to K-Firms, the energy sector engages in R&D in order to develop new and superior vintages of green and brown energy production technology. Since there is only one agent in this sector, no imitation of technologies takes place and all R&D efforts are directed toward innovation. The energy sector invests a fixed share of its revenue into R&D activities which are divided between research on green and brown technologies either in fixed proportions or endogenously (i.e., either according to the shares of the two technologies in total capacity or current energy production, see appendix A).

R&D is carried out using labour as an input, and innovation proceeds in similar fashion as for K-Firms. The probability of an innovation taking place in green/brown energy technology is increasing in the amount of R&D activity directed toward the respective technology. Innovations are modelled as random changes to the characteristics of the existing technologies. In the case of brown energy, the new technology is characterised by a different fossil fuel input requirement and emission intensity, i.e. a different unit cost of energy production. In the case of green technology, innovation takes the form of a different unit cost of capacity expansion. As for K-Firms, we allow for the possibility of an innovated technology to be inferior to the current one, in which case it is not adopted.

Having carried out investment, production and R&D activities, the energy sector calculates its profit and pays a fixed share of positive profits as dividends to the household sector. Retained earnings are held in the form of bank deposits, distributed across all banks in the same way as household deposits.

3.9. Fossil fuel sector

The present version of the model adds a simplified separate fossil fuel sector which sells fossil fuels as input to the energy sector. It is modelled in such a way that it can largely be separated from the rest of the model to mimic an external fossil fuel supplier when the model is calibrated to represent a single region (e.g., the European Union).

The sector is not connected directly to the commercial banking system but instead holds a non-remunerated reserve account at the central bank which it uses to make and receive payments. It sells a costlessly produced fossil fuel at a price which in every period is updated in line with the weighted average of past changes in the nominal wage rate which is also used to update the mark-up of the energy sector, to keep the fossil fuel price in line with other nominal quantities unless otherwise specified. Given that the fossil fuel sector has zero cost, its revenue coincides with its profit, which is added to its reserve balance. It pays a fixed share δ^F of its accumulated wealth to households in every period. By setting δ^F to a very small (but positive) value, it can be ensured that payments from the fossil fuel sector have a very limited effect on the rest of the model.

3.10. Climate

The model incorporates two climate modules between which the user can switch. Both are calibrated to run at annual frequency. If the economic parts of the model are calibrated to run at quarterly frequency, the activated climate module is called every four periods.

The emissions being used as inputs for the climate module can either be all endogenous (if the economic model is calibrated to the global level) or partly exogenous (if the economic model is calibrated to some region). Endogenous emissions are the sum of emissions from the production of capital and consumption goods, and emissions from the energy sector.

The simpler climate module is based on cumulative emissions (cf. Matthews et al., 2009, 2012), deriving a global temperature anomaly as:

$$Temp_t = \Upsilon_1 + \Upsilon_2 \mathscr{E}_t^{\Sigma} \tag{14}$$

where $\mathscr{E}_t^{\Sigma}$ are cumulative emissions up to period t.

The more complex climate module, based on Sterman et al. (2013), depicts a carbon cycle in which the atmospheric carbon content depends on anthropogenic emissions and carbon exchange between the atmosphere, oceans and biomass. Via several steps described in detail in appendix A, the module derives the heat content and temperature anomaly of ocean layers, with the global temperature anomaly being assumed equal to that of the top ocean layer.

Whichever climate module is active in a given simulation will return a global temperature anomaly which forms the basis for the determination of climate impact shocks. These shocks may enter the model economy through various channels at the micro- and macroeconomic level depending on simulation settings. Among others, model allows for a simulation of climate impact shocks to current output, capital stocks, productivity and R&D effectiveness, individually or jointly, with a range of specifications. A full description of all shock channels and specifications is provided in appendix A.

When shocks through some channel s are determined directly at the individual agent level, we posit that they are drawn for each agent in each period from a beta distribution:

$$shock_t^s \sim Beta\left(\mathcal{S}_{1,t}^s, \mathcal{S}_{2,t}^s\right), \tag{15}$$

The shape parameters of this distribution are a function of the temperature anomaly, being given by

$$\begin{aligned} \mathcal{S}_{1,t}^s &= \mathcal{S}_{1,0}^s \left(1 + ln\left(\frac{Temp_{t-1}}{Temp_0}\right)\right)^{\Upsilon_3^s} \\ \mathcal{S}_{2,t}^s &= \mathcal{S}_{2,0}^s \left(\frac{Temp_0}{Temp_{t-1}}\right)^{\Upsilon_4^s} \end{aligned} \tag{16}$$

where $\Upsilon_3^s > 1$ and $\Upsilon_4^s > 1$. This implies that as the temperature anomaly increases, the mode of the distribution will shift upwards and the right tail of the distribution will become thicker, reflecting an increased frequency of extreme events (Katz and Brown, 1992). When aggregate shocks are required, we assume that these are given by the current mean of the beta distribution defined in Equation (15).

4. Code upgrades

In addition to the incorporation of a comprehensive accounting system to ensure stock-flow consistency, another major emphasis during the development of DSK-SFC has been on the usability and accessibility of the model code, with a view to lowering the entry cost for researchers interested in applying the model and enhancing the reproducibility of results (Janssen et al., 2020). Here, we briefly outline the main upgrades which were made to the code in this respect.

- As for previous versions, the model code is written in C++ and compiled using CMake. For DSK-SFC, we ensured that the model can be compiled and simulated out of the box on all common operating systems (Linux distributions, Windows, macOS).[3]
- Large amounts of unused legacy code have been removed, commenting has been improved and the code has been thoroughly cleaned to ensure easy readability.
- Parameters, initial values and flags were previously defined as global constants, with values hard-coded into the model scripts. They could hence not be changed at runtime. Parameters, initial values and flags are now externally supplied through a *.json* file and their values can be changed at runtime. Experiments can hence be performed without having to recompile the model executable.
- When invoking the model executable file through the command line, the user can now specify a range of arguments, including the path to the input file, a name for the run to be appended to all output files, and most importantly the seed for the pseudo-random number generator which was previously set in a loop within the model code itself. This in turn allows for the model executable to be called e.g. through a shell, Python or R script with different seeds on different cores, hence enabling the implementation of parallel model runs e.g. for calibration or large simulation experiments.

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[3] While the model can be compiled and simulated on all common operating systems, results of a given individual run may not be identical across machines and operating systems due to differences in the handling of floating point arithmetic. These may arise from the use of different compilers, compiler versions, compiler options, as well as differences in processor architecture. Through test runs on different machines and different operating systems, we have ensured that while results of individual runs for a given seed may exhibit differences, the distributions of results arising from a full scenario simulation (108 different seeds in our case) are almost identical. For precise reproducibility, works making use of the model code should exactly specify the compilation and simulation setup under which specific results were produced.

- The model code includes an increased number of checks for errors (e.g. variables that should be strictly positive but may become zero or negative when unusual parameter combinations are supplied) and unusual/undesired behaviour (e.g. households not being able to finance firm replacement), as well as exit conditions leading to a cancellation of degenerate runs. Additionally, the model code performs an extensive and rigorous set of stock-flow consistency checks and generates a warning when violations occur. All registered errors and warnings are recorded in a dedicated log file including the name of the run, the seed and the period in which an error or warning occurred to allow for easy reproducibility and tracing of the causes.

The next section briefly describes the model initialisation and calibration procedure followed in this paper.

5. Calibration and validation

The current initialisation of the model is simplified. Initial values are set such that stock-flow consistency is satisfied. The overall approach is similar to that common with other macroeconomic ABMs in the literature in that initial values, especially for economic variables, are set to fairly arbitrary levels. The model subsequently undergoes a ‘burn-in’ period (which is discarded in the analysis of model output) when simulated, before converging to its eventual trajectory. The initialisation procedure is described in more detail in appendix A. That appendix also explains how aggregate variables such as GDP, employment, emissions and the consumption price index are computed and how the checks for stock-flow consistency during simulation are performed.

The calibration of the climate modules is identical to that used in previous versions of the DSK model. We perform a small validation exercise on both modules by obtaining time-series on emission pathways and temperature anomalies for the period 2010 to 2100 from the IPCC AR6 scenario database (Byers et al., 2022), considering a range of scenarios with different carbon budgets (from 400 up to 3000 $GtCO_2$). We then feed these emission pathways into the climate modules used in our model to confirm that the temperature anomalies predicted by the modules lie within the range of those reported in the scenario database for the scenario in question. This is indeed the case for all scenarios we consider.

For the calibration of the economic model, we aim to arrive at a *quarterly* calibration under which the (filtered) simulated macroeconomic time-series exhibit qualitatively reasonable business-cycle dynamics. In addition, we aim to calibrate the long-run dynamics of the model such that the average growth rates of real GDP, energy use in industry and emissions broadly match those of a ‘Shared Socioeconomic Pathway 2’ (SSP2) scenario (Riahi et al., 2017) for the *European Union* from 2010 to 2100. We also make use of these scenarios in order to set the parameters governing the rate of change of the labour supply and the growth of exogenous (rest of world) emissions. The parameter values used to construct the simulations shown in this paper are listed in appendix B. Overall, our approach hence corresponds to the ‘indirect’ calibration method commonly used in agent-based modelling, as described e.g. by Chiarella and Di Guilmi (2019). As the model description indicated, the model code contains a number of indicator variables allowing the user to specify the simulation setting. Appendix A indicates how these variables are set in the simulations shown here. Most importantly, climate impact shocks are deactivated for the baseline run and activated in one of the experiments shown below.

Table 3
Simulated business cycle and growth statistics

Standard deviation of GDP	0.02580
Standard deviation of Consumption	0.01582
Standard deviation of Investment	0.15576
Standard deviation of Employment	0.01981
Standard deviation of Inflation	0.01649
Av. ann. GDP growth	0.01305
Av. ann. emissions growth	0.00125
Av. ann. growth of energy use in industry	0.00110

In Table 3 we report the main business cycle and growth statistics produced by the model as average values across 108 runs with different reproducible seeds for the pseudo-random number generator. Each of the 108 baseline simulation has a post-transient duration of 400 periods (quarters), i.e. 100 years. The length of the discarded transient is 200 periods. Standard deviations of macroeconomic time series are derived by first applying the Hamilton filter (Hamilton, 2018; Schüler, 2018) to the respective series and then calculating the standard deviation of the cyclical component. Growth rates are calculated on raw quarterly simulation data and then annualised. It can be seen that the model is able to produce reasonable volatilities for GDP, consumption and inflation. Investment, however, is much too volatile relative to GDP, as is common in macroeconomic ABMs. Similarly, the employment rate is too volatile relative to GDP compared to what is typically observed in empirical data. The growth rates of GDP, carbon emissions and energy use in industry are close to what is predicted for the European Union countries in SSP2 scenarios with current policy.

Figure 2 plots the autocorrelation functions of the main macroeconomic time-series, calculated on filtered data for the 108 baseline runs performed (the bands around the lines, which in this and many other figures are almost invisible due to being very narrow, represent 95% confidence intervals around mean values across seeds). In addition, the figure shows the cross-correlation functions of the main macroeconomic time-series with real GDP, demonstrating that the cyclicality of the series is qualitatively reasonable.

Appendix A contains further figures illustrating the cyclicatlity of important macroeconomic variables. Making use of the stock-flow consistent accounting strucutre, it also shows that the sectoral net worth to nominal GDP ratios

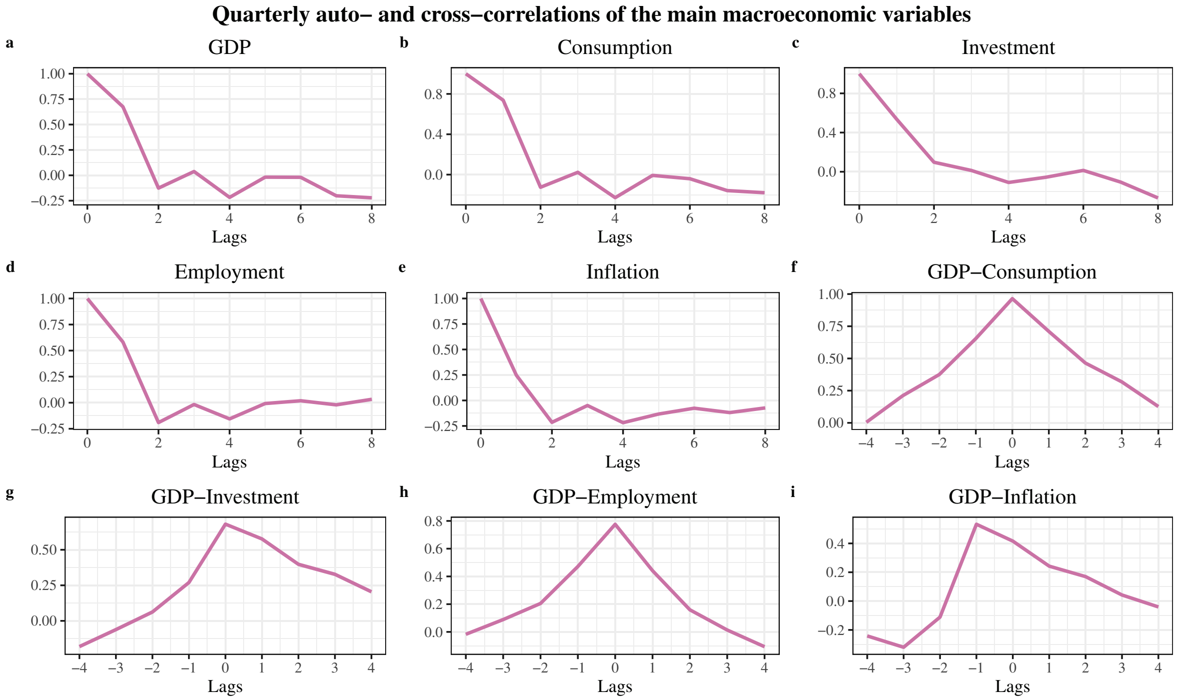

Figure 2: Panels (a-e): Simulated autocorrelations of the main macroeconomic aggregates; Panels (f-i): Simulated cross-correlations of real GDP and other macroeconomic aggregates. Auto- and Cross-correlations are calculated on filtered quarterly simulated time-series. Bold lines represent averages across 108 simulations with different seeds. Shaded bands represent 95% confidence intervals.

and sectoral financial balances do not exhibit long-term trends. Furthermore, it plots examples of variables which can be tracked more accurately using the stock-flow consistent structure. Finally, we use the simulated model data to inspect a range of characteristics and qualitative stylised facts (cf. Haldane and Turrell, 2019), as is usually done for models from the K+S family (see e.g. Dosi et al., 2010, 2015, 2017a; Lamperti et al., 2018). The relevant table along with references to the literature can also be found in appendix A.

6. Simulation experiments

To provide some example scenarios, we conduct several simulation experiments featuring climate policy, climate change impacts, as well as changes in macroeconomic policy:[4]

- A scenario with a higher carbon tax on the energy sector than in the baseline.
- A scenario with climate shocks to C-Firms' capital stocks, and one with shocks to both labour productivity and energy efficiency.
- Two scenarios in which the unemployment benefit ratio is increased by two different amounts w.r.t. the baseline.

The following sub-sections outline the results of these exercises.

6.1. Carbon tax

In the baseline scenario we assume a carbon tax on energy sector emissions, $\tau_t^{Em,E}$, which grows with nominal GDP starting from an initial value. This tax is re-set at an annual frequency, i.e. every four simulation periods. At the beginning of a period in which the carbon tax is adjusted, it is set as follows:

$$\tau_t^{Em,E} = \tau_0^{Em,E} \frac{GDP_{t-1}^n}{GDP_0^n} \tag{17}$$

where GDP^n denotes nominal GDP and $\tau_0^{Em,E}$ is the initial value of the tax. In the simulation experiment, we examine the effect of a sharp increase in the carbon tax on the energy sector. In particular, we assume that in the first four post-transient simulation 'years', the tax doubles every year (i.e. every four simulation periods), and subsequently continues to grow with nominal GDP as shown in Equation (17).

Panel a of figure Figure 3 shows the effect of this experiment on the energy price in the model. The bold lines represent averages across 108 simulations with different seeds while bands (which are too narrow to be visible in some plots) represent 95% confidence intervals. Recall from

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[4] The calibration and simulation runs shown in this paper were produced on the 'Zeus' High Performance Cluster of the Euro-Mediterranean Center on Climate Change (CMCC), running Linux CentOS 7.6 x86_64 on compute nodes with Intel Xeon Gold 6154 CPUs. Results were subsequently reproduced on the corresponding author's computer running Ubuntu 20.04.4 LTS in WSL2 on an Intel Core i7-1165G7 CPU. The executable was compiled on the corresponding author's computer using GNU GCC 9.4.0.

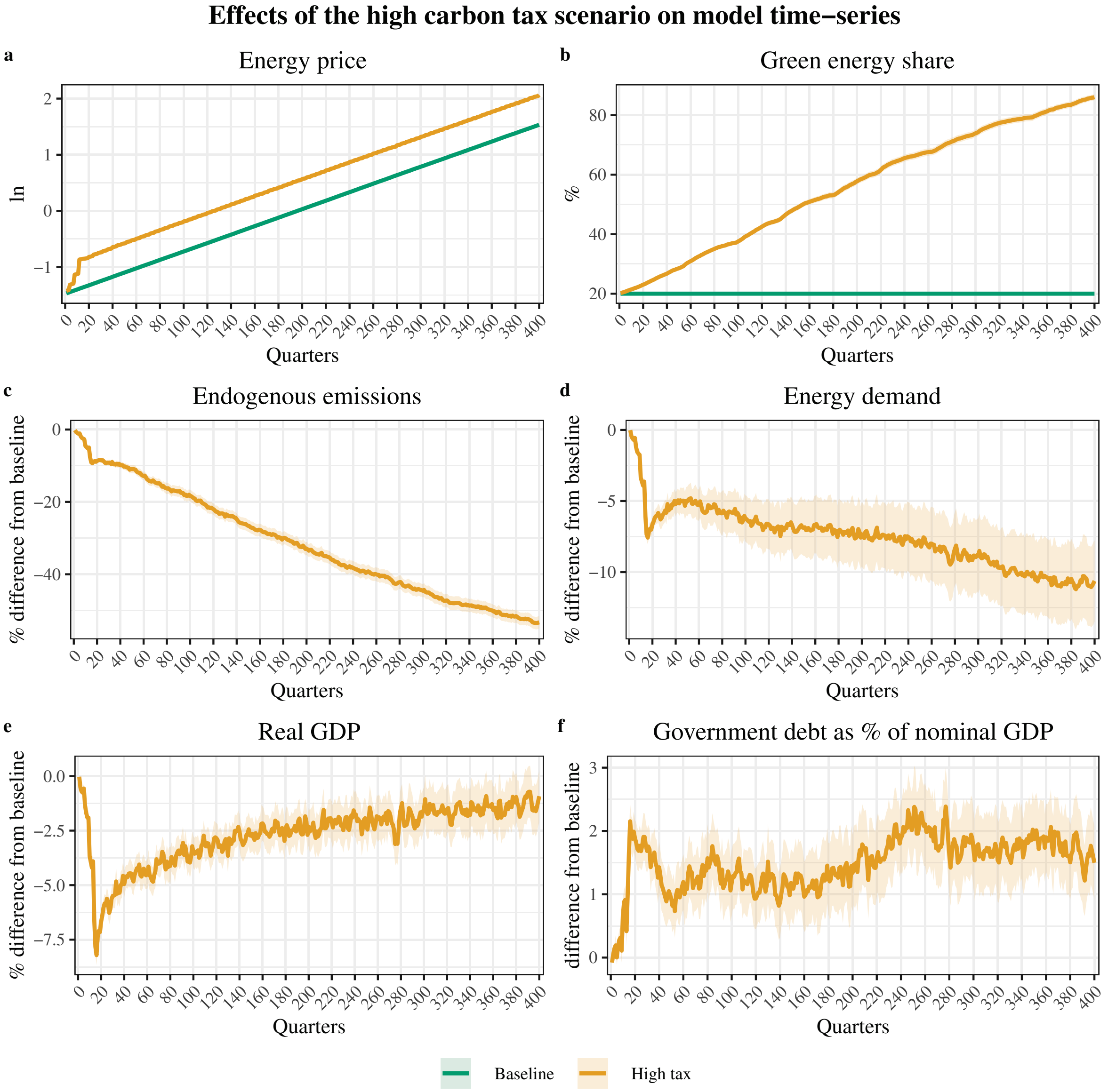

Figure 3: Panel (a): Natural logarithm of the energy price in the baseline scenario and the carbon tax experiment; Panel (b): Share of green energy capacity in total productive capacity of the energy sector in the baseline scenario and the carbon tax experiment; Panel (c): Endogenous per-period emissions in the carbon tax experiment, as % difference from the baseline; Panel (d): Per-period demand for energy from the firm sectors in the carbon tax experiment, as % difference from the baseline. Panel (e): Real GDP in the carbon tax experiment, as % difference from the baseline; Panel (f): Government debt as percentage of nominal GDP, difference from the baseline. Bold lines represent averages across 108 simulations with different seeds. Shaded bands represent 95% confidence intervals.

the description of the energy sector that the energy price is determined by the infra-marginal cost of energy production, to which a mark-up is added. The former is zero for green energy and positive for brown energy. As long as any brown energy capacity exists and is used to produce energy, a higher carbon tax will be fully passed on into the price of energy, meaning that the latter increases sharply as the shocks to the tax rate take place, and subsequently continues to grow in line with the baseline price.

Panel b of Figure 3 shows that the share of green energy in total productive capacity of the energy sector grows gradually, while it remains at the exogenously imposed lower bound of 20% in the baseline. The baseline carbon tax is deliberately calibrated such that it does not lead to green energy becoming cheaper than brown energy and endogenous R&D also does not lead to such a cost advantage in the calibration shown here. The higher carbon tax, by contrast, does confer a cost advantage to green energy technology, which subsequently becomes the preferred option for investment in the energy sector. Panel c of Figure 3 shows that the higher carbon tax affects *endogenous* emissions (exogenous ‘rest of the world’ emissions are of course not affected by the experiment), which decline by over 50% relative to the baseline by the end of the simulation. Since only the energy sector is subject to the tax, the decline in emissions is almost entirely due to reduced emissions from that sector. This reduction is in turn chiefly driven by the build-up of green energy production capacity, but as Panel d of Figure 3 shows, the demand for and hence production of energy also declines relative to the baseline. Initially this is due to the decline in GDP associated with the increase in the carbon tax (see Panel e of 3), but energy demand continues to decline relative to the baseline even as GDP gradually recovers, since the higher energy price incentivises C-Firms and K-Firms to adopt more energy efficient technologies, reducing the energy intensity of GDP. Due to its impact on endogenous emissions, the higher carbon tax also affects the simulated temperature anomaly. However, this effect is extremely limited due the fact that, under the calibration shown here, the vast majority of emissions are exogenous (representing rest of the world, i.e. non-EU emissions not subject to the modelled carbon tax).

Panel e of Figure 3 shows that a sharp increase in the carbon tax has significant implications for macroeconomic performance, in line with previous versions of the model (Lamperti et al., 2020; Lamperti and Roventini, 2022). GDP decreases strongly on impact and recovers only very gradually, remaining below its baseline value at the end of the simulation. At the same time, the ratio of government debt to nominal GDP (shown in Panel f of Figure 3) tends to increase. While the additional revenue from the carbon tax by itself does improve the budget balance, this is more than outweighed by declines in other tax revenue and increases in outlays for unemployment benefits. Indeed, Panel b of Figure 4 shows that under the higher carbon tax, the average unemployment rate is significantly lower than in the baseline scenario. Panel a of the same figure also indicates that the higher carbon tax tends to increase the volatility of GDP.

Panels c to f of Figure 4 examine the effects of the high carbon tax scenario on the average growth rate of real GDP, splitting the simulation into four phases of 100 periods. The figures indicate that the higher tax has a slightly negative effect on growth initially, while GDP subsequently grows slightly faster during the prolonged recovery phase (without, however, allowing the level of real GDP to fully catch up with its baseline value, as shown in Panel e of Figure 3).

Overall, these results indicate that carbon taxation can play a role in promoting green transitions, but that it may also lead to substantial transition risks (Lamperti and Roventini, 2022; Känzig, 2023). Recall from the model description that the model as described here does not include a 'recycling' mechanism whereby carbon tax revenue is directly redistributed to households or firms. In an ongoing application of the DSK-SFC model (Fierro et al., 2024), we show that the macroeconomic costs of carbon taxation can be effectively - though possibly not fully - mitigated through well-designed carbon revenue recycling schemes.

6.2. Bottom-up climate damages

Previous versions of the DSK model have been used to study the macroeconomic relevance of disaggregated, micro-level climate change impacts (Lamperti et al., 2018, 2019a). Building on such exercises, we illustrate the impact of climate change on the DSK-SFC model by simulating climate impacts through two channels: shocks to C-Firms' capital stocks and shocks to the labour productivity and energy efficiency of K-Firm production techniques and capital good vintages. In both scenarios, shocks are endogenously determined. As described in Section 3.10 the beta distribution from which shock values are drawn changes as a function of the global temperature anomaly.

Capital stock shocks follow specification 1 outlined in appendix A: a shock $shock^{cap}_{c,t}$ is drawn from the relevant beta distribution for each C-Firm c in each period. Firm c then loses a share $shock^{cap}_{c,t}$ of its capital stock prior to carrying out production in t. Shocks to labour productivity and energy efficiency follow specification 3 outlined in appendix A: for each K-Firm k, two shocks ($shock^{lp}_{k,t}$ and $shock^{ee}_{k,t}$) are drawn from the relevant beta distribution in each period. The labour productivity and energy efficiency of k's production technique as well as those of all capital good vintages currently and previously produced by k are then reduced by percentages given by $shock^{lp}_{k,t}$ and $shock^{ee}_{k,t}$, respectively. Table 4 illustrates how the two shock channels are initialised and parametrised. Note that productivity shocks are initially smaller and less dispersed than capital stock shocks by a large margin, and the shape parameters change much more slowly with the temperature anomaly.

Despite the fact that productivity shocks are calibrated to be much smaller and to grow more slowly with temperature than capital stock shocks, Panel a of Figure 5 shows that they nevertheless have a dramatic effect on GDP. While the long-term trajectory of real GDP is largely unaffected by capital stock shocks, productivity shocks have a pronounced growth effect which leads GDP to diverge from the baseline trajectory, predicting much larger physical risks from unmitigated climate change than what mainstream, equilibrium-based models usually suggest (Lamperti and Roventini, 2022; Stern et al., 2022). Panel b of Figure 5 shows the global temperature anomaly resulting from the baseline and the three shock scenarios. In the baseline scenario, the temperature anomaly arrives at just above 3.25 Degrees Celsius at the end of the simulation, and climate impact shocks do not have a significant impact on this trajectory.[5]

Panels e to f of Figure 5 show how the means and standard deviations of the theoretical distributions of capital stock and productivity shocks change over the course of the simulations as a result of the changes in the global temperature anomaly shown in Panel b of figure 5. While both the

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[5] Recall that, just as in the carbon tax experiment, exogenous rest of the world emissions are unaffected by anything taking place in the scenarios. In addition, as shown below, endogenous emissions do not change much in the shock scenarios relative to the baseline. As such, temperature trajectories are practically identical, making the trajectories of the climate shocks directly comparable.

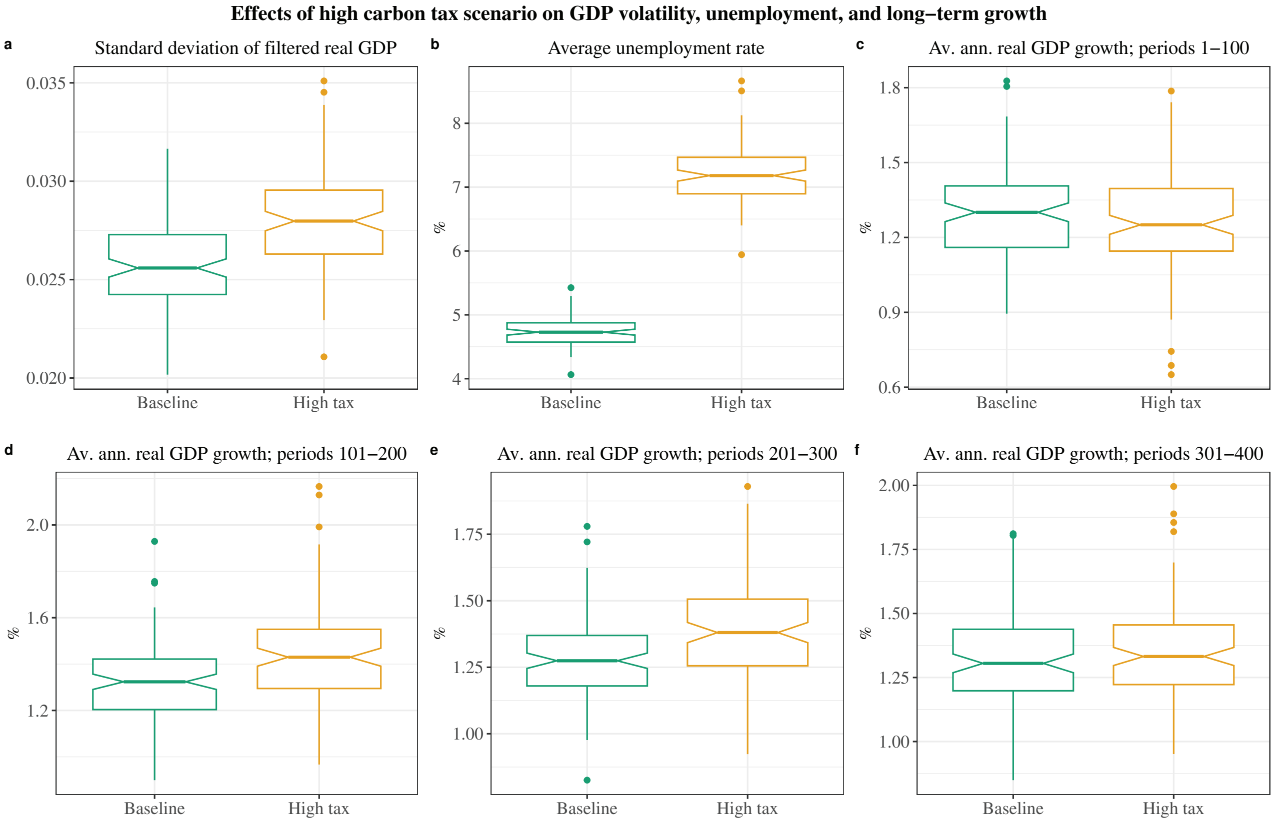

Figure 4: Panel (a): Standard deviation of filtered simulated quarterly real GDP in the baseline and under a high carbon tax; Panel (b): Average unemployment rate in the baseline and under a high carbon tax; Panels (c) to (f): Average annualised growth rate of simulated real GDP in the baseline and under a high carbon tax. Panel (c): Post-transient simulation periods 1-100; Panel (d): Post-transient simulation periods 101-200; Panel (e): Post-transient simulation periods 201-300; Panel (f): Post-transient simulation periods 301-400. The respective statistics are calculated for each of the 108 individual model evaluations for each scenario, so that boxplots illustrate their distribution in the respective scenarios.

Table 4
Initial values and parameters for climate shock scenarios

Description	Capital stock shocks	Productivity shocks
Initial value shape param. 1 ($S^s_{1,0}$)	1	0.005
Initial value shape param. 2 ($S^s_{2,0}$)	100	1000
Exponent Υ^s_3	3	0.25
Exponent Υ^s_4	8	0.25

mean and standard deviation of capital stock shocks grow substantially, the change for productivity shocks, which already start from far lower values, is much more modest. To understand the radically different effects on GDP, note that the productivity and energy efficiency shocks directly impact the labour productivities and energy efficiencies of all existing vintages of capital goods and K-Firm production techniques. The shocks are hence inherently persistent and compounding. In addition, by affecting the process of technical change and innovation diffusion, they hamper the underlying forces of growth in the model. This explains why even a series of stochastic shocks which is strongly concentrated at very small values may, over time, induce a sizeable decline in GDP growth. Such results call for a much deeper understanding of how climate change may affect productivity advancements and innovation diffusion.

Indeed, Panel b of Figure 6 shows that in the productivity shock scenario, the average growth rate of real GDP is significantly lower than in the baseline. Panel a of Figure 6 shows that under shocks to capital stocks, the volatility of filtered real GDP tends to increase relative to the baseline, indicating more pronounced fluctuations at business cycle frequency. Panel b of Figure 6 also suggests that the average growth rate of real GDP over the entire post-transient simulation is very slightly higher in the presence of capital stock shocks. This is due to attempts by C-Firms to replace the

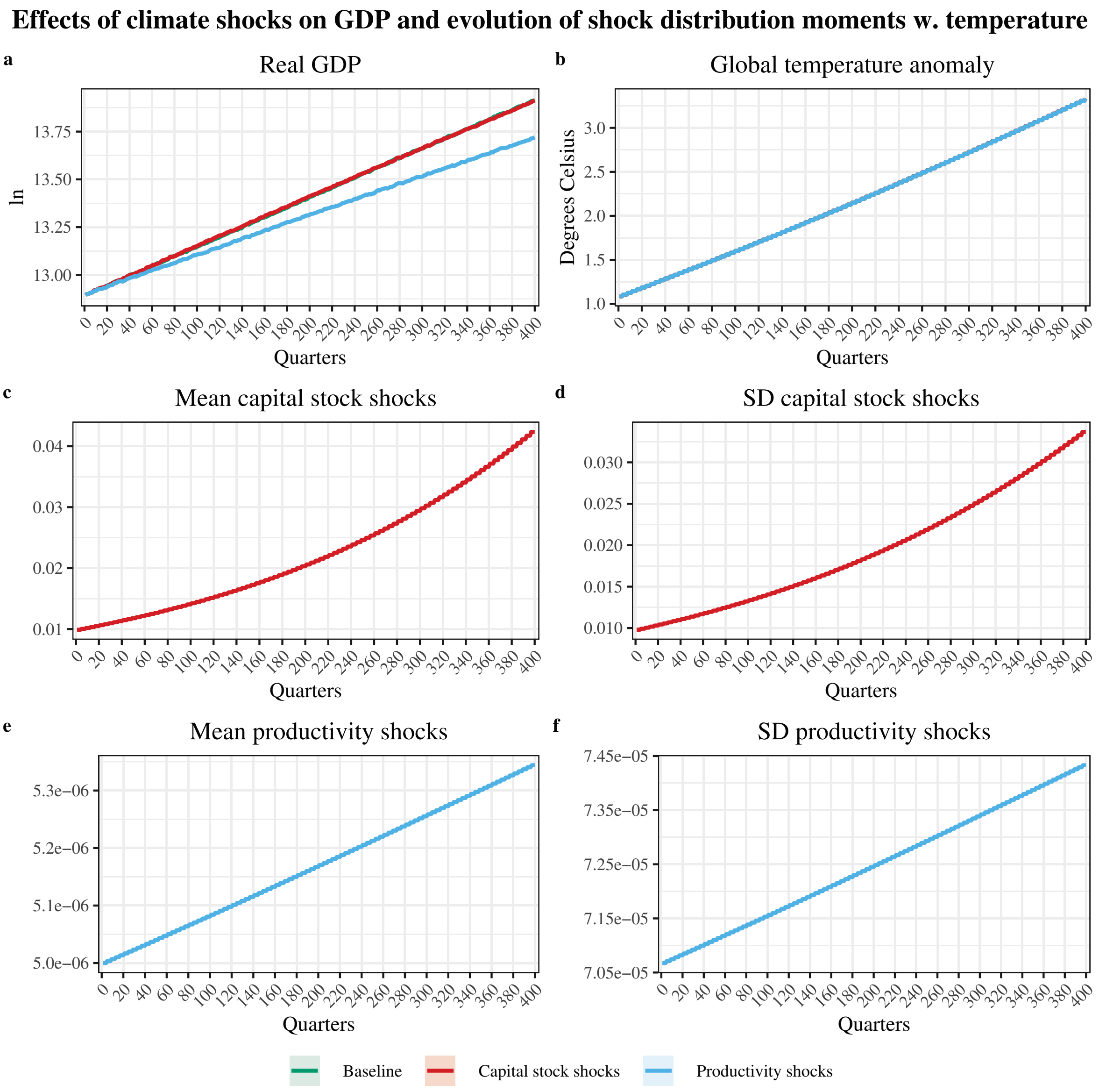

Figure 5: Panel (a): Natural logarithm of real GDP in the baseline and climate shock scenarios; Panel (b): Global temperature anomaly relative to pre-industrial temperature in the baseline and climate shock scenarios; Panel (c): Theoretical mean of the beta distribution from which capital stock shocks are drawn; Panel (d): Theoretical standard deviation of the beta distribution from which capital stock shocks are drawn; Panel (e): Theoretical mean of the beta distribution from which productivity shocks are drawn; Panel (f): Theoretical standard deviation of the beta distribution from which productivity shocks are drawn. Bold lines represent averages across 108 simulations with different seeds. Shaded bands represent 95% confidence intervals.

capital stocks destroyed by climate shocks, which leads to additional investment demand, employment and household income which in turn feeds back on consumption demand and the desired production of C-Firms. In the scenario considered here, this leads to a temporary increase in the average growth rate of GDP. Panels c to f of Figure 6 split simulations up into four phases of 100 periods, showing that the increase in GDP growth induced by capital stock shocks is largely limited to the first three phases, while in the last phase, average growth is lower than in the baseline. This is in line with results from previous versions of the DSK model (e.g. Lamperti et al., 2019a), which showed that capital stock shocks, as long as they are sufficiently small, can lead to a temporarily positive effect on GDP growth due to efforts to rebuild the lost capital stocks. As the average size of shocks as well as the frequency of extreme events increases with the temperature anomaly, however, negative effects on the rate of growth eventually come to dominate.

Despite their limited impacts on the rate of growth, shocks to capital stocks have important detrimental impacts which manifest themselves in the increase in macroeconomic volatility highlighted above. Figure 7 depicts a number of statistics further illustrating these impacts. Panel a shows that shocks to the capital stock of C-Firms increase

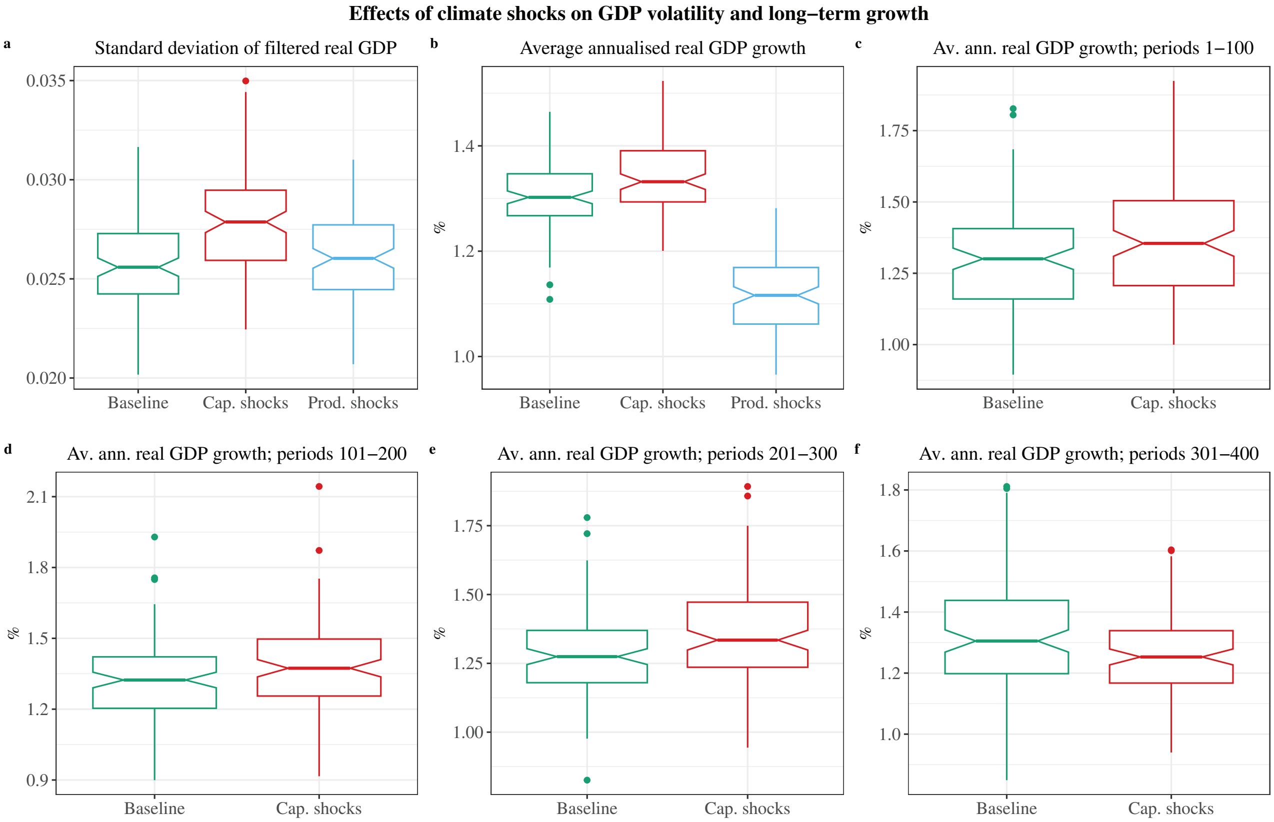

Figure 6: Panel (a): Standard deviation of filtered simulated quarterly real GDP in the baseline, under shocks to capital stocks, and under shocks to productivity; Panel (b): Average annualised growth rate of simulated real GDP in the baseline, under shocks to capital stocks, and under shocks to productivity; Panels (c) to (f): Average annualised growth rate of simulated real GDP in the baseline and under shocks to capital stocks; Panel (c): Post-transient simulation periods 1-100; Panel (d): Post-transient simulation periods 101-200; Panel (e): Post-transient simulation periods 201-300; Panel (f): Post-transient simulation periods 301-400.The respective statistics are calculated for each of the 108 individual model evaluations for each scenario, so that boxplots illustrate their distribution in the respective scenarios.

the incidence of bank failures. This effect can be explained through two channels. Firstly, the loss of a portion of its capital stock represents a loss to a C-Firm which may push it into bankruptcy, leading it to default on (part of) its outstanding loans. Secondly, the additional investment which C-Firms undertake to rebuild destroyed capacity must be financed, cet. par. making them more financially fragile. Indeed, Panels b and c of figure 7 show that both the number of C-Firm failures per run and the average per-period value of bad debt as a percentage of nominal GDP increase in the presence of capital stock shocks. Shocks to productivity and energy efficiency, by contrast, have only a slight effect on the statistics shown in Figure 7.

Finally, Figure 8 shows the impact of the two climate shock scenarios on endogenous emissions. In the case of shocks to the capital stock, losses of installed capital goods lead to an increase in production aiming to replace them, producing slightly higher emissions. While productivity shocks give rise to a very sizeable loss in real GDP, emissions remain almost constant. Recall that the productivity shocks also affect the energy efficiency of capital goods and K-Firm production techniques. This increases the amount of energy demanded per unit of output and hence prevents a decline in emissions alongside real GDP.

6.3. Unemployment benefits

As a last experiment, we simulate two scenarios in which the unemployment transfer (the payment received by households per unit of unemployed labour as a share of the current nominal wage) is permanently increased. In particular, the baseline value of 0.4 is moved to 0.45 in the first case, and to 0.6 in the second.

Panel a of Figure 9 shows that in both cases, the increase in benefit payments gives a one-off boost to real GDP, which permanently shifts to a higher trajectory relative to the baseline (without, however, subsequently growing faster), with the boost being larger in the scenario involving a larger increase in the benefit ratio. Importantly, as shown by Panel b of Figure 9, the increased payments per unit of unemployed labour do not lead to a significant change in the ratio of government debt to nominal GDP. The increase in the benefit ratio hence appears to be self-financing in both cases. While government outlays per unit of unemployed

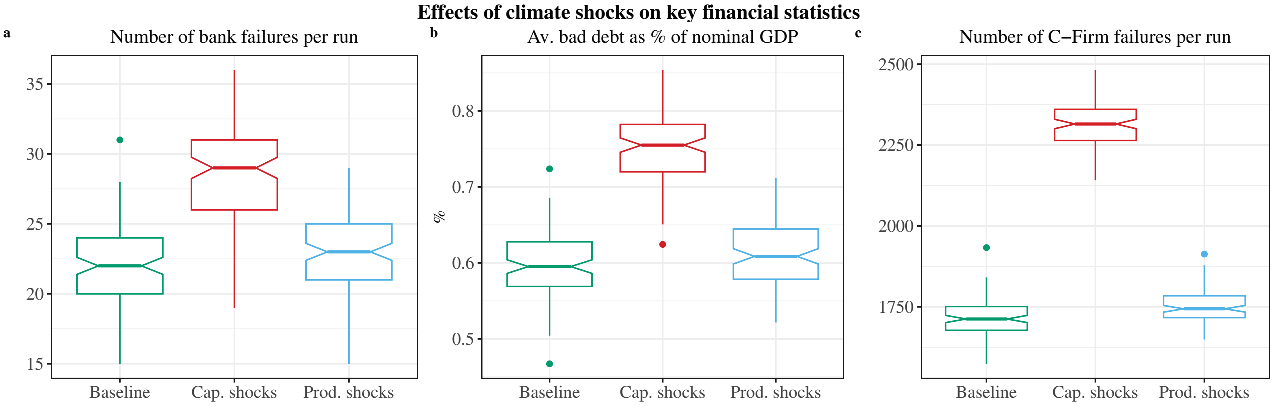

Figure 7: Panel (a): Number of bank failures over the post-transient duration of model runs in the baseline, under shocks to capital stocks, and under shocks to productivity; Panel (b): Average value of defaulting C-Firm debt as percentage of nominal GDP over the post-transient duration of model runs in the baseline, under shocks to capital stocks, and under shocks to productivity; Panel (c): Panel (a): Number of C-Firm failures over the post-transient duration of model runs in the baseline, under shocks to capital stocks, and under shocks to labour productivity and energy efficiency. The respective statistics are calculated for each of the 108 individual model evaluations for each scenario, so that boxplots illustrate their distribution in the respective scenarios.

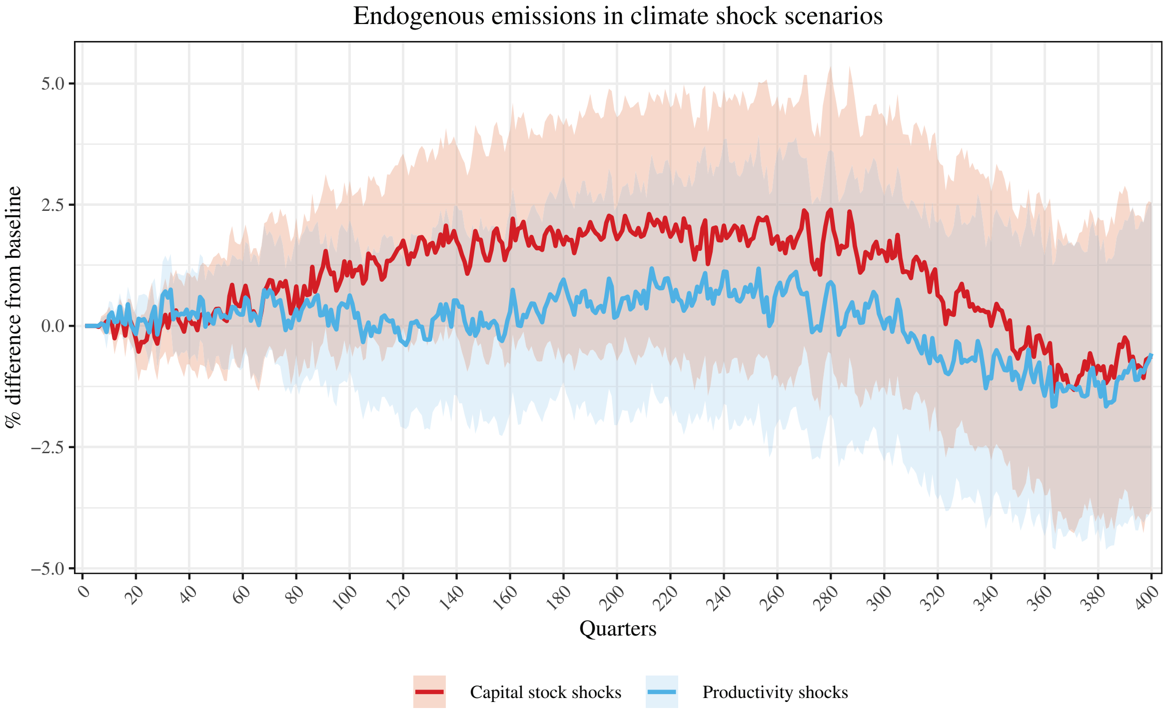

Figure 8: Endogenous per-period emissions under shocks to capital stocks, and under shocks to productivity, as % difference from the baseline. Bold lines represent averages across 108 simulations with different seeds. Shaded bands represent 95% confidence intervals.

labour are higher, the higher level of real GDP relative to the baseline is accompanied by a decline in the unemployment rate as well as an increase in tax revenue.

In line with previous results from works using a K+S framework (e.g. Dosi et al., 2010, 2013), Panel c of Figure 9 suggests that an increase in the benefit ratio also contributes to macroeconomic stability in both scenarios, showing that the standard deviation of filtered real GDP declines significantly in both scenarios. In addition, the higher levels of real GDP in the scenarios also result in lower average unemployment rates, shown in Panel d. Panel e confirms that, as can also be deduced from Panel a, the long-term growth rate of GDP is unaffected by the higher benefit ratios.

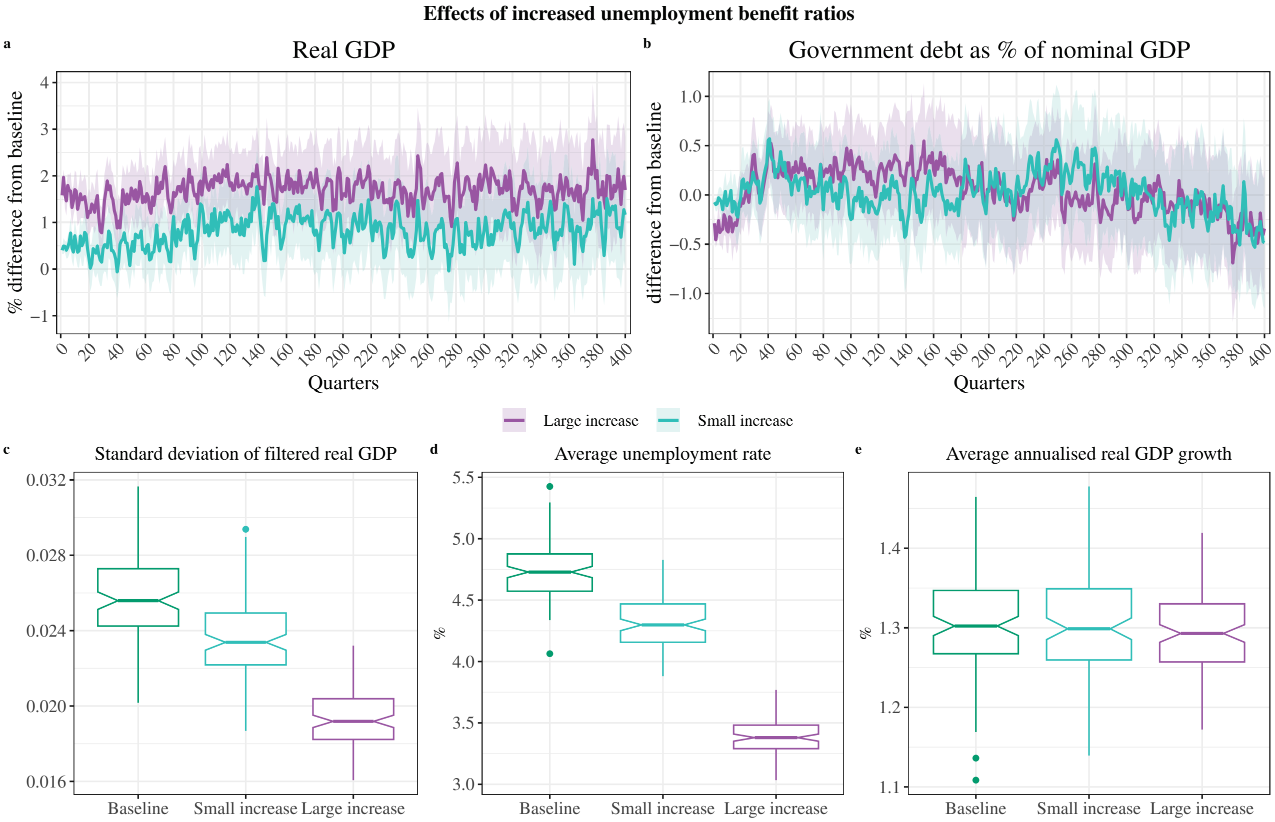

Figure 9: Panel (a): Real GDP with increased unemployment benefit ratios, as % difference from the baseline; Panel (b): Government debt as percentage of nominal GDP with increased unemployment benefit ratios, difference from the baseline. Bold lines represent averages across 108 simulations with different seeds. Shaded bands represent 95% confidence intervals. Panel (c): Standard deviation of filtered simulated quarterly real GDP in the baseline and under increased unemployment benefit ratios; Panel (d): Average unemployment rate over the post-transient duration of model runs in the baseline and under increased unemployment benefit ratios; Panel (e): Average annualised growth rate of simulated real GDP in the baseline and under increased unemployment benefit ratios. The respective statistics are calculated for each of the 108 individual model evaluations for each scenario, so that boxplots illustrate their distribution in the respective scenarios.

7. Conclusions

This paper presented a fully stock-flow consistent version of the 'Dystopian Schumpeter meeting Keynes' (DSK) agent-based integrated assessment model. This is the first integrated assessment model featuring full stock flow consistency, agents' heterogeneity, bottom-up climate impacts and endogenous growth and fluctuations. In this updated version, all balance sheet items and transaction flows are explicitly and consistently tracked throughout a simulation run. This ensures that simulation data correctly and fully capture the implications of simulated scenarios for agent and sector-level balance sheets and financial ratios.

The paper and its appendices also provide the most detailed description of a model from the widely used 'Keynes + Schumpeter'/DSK family (Dosi et al., 2010, 2017b; Dosi and Roventini, 2019) available in the literature to date, hence representing an important reference point for researchers in the sub-field. The paper also gave an outline of key improvements which have been made to the model code to ensure accessibility and usability. Following a description of the calibration and validation process, a range of example scenarios were presented, showing that the model can be used to address a variety of research questions related to macroeconomic and climate policy, as well as the economic impacts of climate change.

The new fully specified accounting framework lays an important foundation for planned future extensions of the model. Some major simplifications of the present version include the assumption of a fully static bank-customer network, the modelling of bank loans as credit lines which must be rolled over in every period, and the modelling of the household sector as an aggregate entity. The former two constrain the analysis of the financial stability implications of climate change and climate policy, while the third precludes analyses of the consequences for personal income distribution. The stock-flow consistent accounting framework now underlying the model will greatly ease the implementation of a dynamic credit network and multi-period loans, as well as the linking of an agent-based household sector to the financial system and the rest of the economy. Finally, the energy sector is simplified in many respects. A more detailed modelling of the energy sector, including a wider variety of green and brown technologies, its investment behaviour and the financial implications thereof, will be a priority in future extensions of the framework.

CRediT authorship contribution statement

Severin Reissl: Conceptualisation, Methodology, Software, Validation, Formal analysis, Investigation, Visualisation, Writing - Original Draft. **Luca E. Fierro:** Conceptualisation, Methodology, Software, Writing - Original Draft. **Francesco Lamperti:** Conceptualisation, Methodology, Writing - Review & Editing. **Andrea Roventini:** Conceptualisation, Methodology, Writing - Review & Editing.

Declaration of competing interest

All authors declare that they have no competing interests.

Data availability

The model code, including the input files used for the simulations shown in the present work, will be made publicly available on Zenodo and GitHub upon publication of a peer-reviewed version of the paper.

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Appendix A Full model description

The present appendix provides a fully exhaustive description of the DSK-SFC model, beginning with the sequence of events taking place in every simulation period and then moving on to a detailed explanation of all agent classes, behavioural rules and model components. In addition, it also describes the process used to initialise the model.

- Sub-section A.1 contains the sequence of events
- Sub-section A.2 describes the household sector
- Sub-section A.3 describes the K-Firm sector
- Sub-section A.4 describes the C-Firm sector
- Sub-section A.5 describes the firm exit and replacement mechanisms
- Sub-section A.6 describes the banking sector
- Sub-section A.7 describes the government
- Sub-section A.8 describes the central bank
- Sub-section A.9 describes the energy sector
- Sub-section A.10 describes the fossil fuel sector
- Sub-section A.11 describes the climate modules and climate impact shocks
- Sub-section A.12 describes the calculation of aggregate variables and stock-flow consistency checks
- Sub-section A.13 describes the initialisation procedure
- Sub-section A.14 provides additional details on the calibration

A.1 Sequence of events

In every period of a simulation of the model, the following sequence of events takes place:

1. If there is a non-zero interest rate on bank deposits, banks pay interest on the balances of deposits their customers held at the end of the previous period.
2. If a carbon tax regime is in place and the climate module will be updated in the current period (see below), the carbon tax is updated.
3. C-Firms receive capital goods ordered in the previous period.
4. C-Firms and K-Firms calculate unit cost and set their prices for the current period.

5. Banks determine the maximum amount of credit they are prepared to extend.
6. Banks set the loan interest rates offered to their customers.
7. K-Firms advertise their product to attract new clients; K-Firms and C-Firms update their network.
8. C-Firms calculate expected demand for their output and desired production.
9. C-Firms earmark worn-out machines for scrapping and decide which (if any) non-worn-out machines to replace with more recent vintages.
10. Based on scrapping and expected demand, C-Firms set a desired capital stock and desired expansion investment.
11. C-Firms calculate effective production cost based on desired production and vintages used.
12. C-Firms determine internal financing and the maximum amount they are willing to borrow. If necessary, desired investment is scaled back; the cost of desired investment is calculated.
13. Bank credit is allocated to C-Firms, which scale back investment and possibly production if credit-rationed. Firms which are unable to roll over existing loans become inactive and are prepared for exit.
14. C-Firms and K-Firms calculate the labour input required for production.
15. Total labour demand (also including labour demand for R&D from K-Firms and from the energy sector based on activities determined in the previous period) is calculated; if total labour demand exceeds the maximum labour supply, C-Firm and K-Firm production is scaled back.
16. The unemployment rate and consequent unemployment benefit payments are calculated.
17. Production takes place. Total energy demand and emissions from industry are calculated based on realised production.
18. Expansion investment, R&D, and energy production take place in the Energy sector.
19. C-Firms pay for investment.
20. C-Firms, K-Firms and the Energy sector pay wages; the Government pays unemployment benefits.
21. Machines are scrapped.
22. C-Firms' competitiveness and ex-ante market shares are calculated. C-Firms with very low market share become inactive and are prepared for exit.
23. K-Firm profit is calculated. K-Firms pay energy, taxes, and dividends. K-Firms which are unable to make energy payments become inactive and are prepared for exit.
24. Households pay taxes and calculate desired consumption.
25. Consumption expenditure is allocated to C-Firms.
26. K-Firm and C-Firms' profits are calculated.
27. C-Firms pay energy, loan service and taxes. C-Firms unable to pay for energy or loan service become inactive and are prepared for exit. C-Firms with negative equity become inactive and are prepared for exit.
28. Energy sector profits are calculated. The Energy sector pays fossil fuel input and taxes. The Fossil Fuel sector makes transfer/dividend payments to the households.
29. Macroeconomic aggregates and averages are computed.
30. The nominal wage rate is updated.
31. Exiting C-Firms and K-Firms are replaced by new entrants.
32. Bank profits are calculated. Banks pay taxes and dividends.
33. Banks with negative equity are bailed out by the Government or taken over by surviving banks.
34. The Government budget is calculated. Deficits are covered by bonds sold to Banks and possibly the Central Bank.
35. The Central Bank sets the policy interest rate for the following period.
36. Net inflows and outflows of reserves are calculated for Banks; if necessary, Banks take advances from the Central Bank.
37. Endogenous technological change takes place in the K-Firm sector.
38. The climate module is updated using emissions from the current period.[6]
39. The fossil fuel price and the mark-up in the energy sector are re-set for the next period.

A.2 Households

The household sector is modelled as an aggregate entity which has four sources of income: wage payments W_t, dividend payments Div_{t-1} (consisting of dividends from consumption good firms, capital good firms, the energy sector, banks, and the fossil fuel sector), unemployment benefits UB_t and interest payments on bank deposits iD_h (depending on whether there is a non-zero interest rate on bank deposits in the calibration used). Depending on whether or not there is a positive tax rate on wage income in the calibration used, households pay taxes to the government out of their wage income, making their disposable income:

$$YD_t = W_t + Div_{t-1} + UB_t + iD_{h,t} - Tax_t^H \tag{A.1}$$

The dividends paid by banks, firms, the energy and the fossil fuel sector are determined according to the rules set out below. $iD_{h,t}$ is given by the product of the interest rate on deposits and the amount of bank deposits held by households at the end of the previous period, $r_{d,t-1}D_{h,t-1}$.

Households will supply any amount of labour demanded at the current nominal wage rate w_t up to a maximum LS_t, which represents the current aggregate labour force and

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[6] The frequency with which this step is called depends on model settings and the length of time one period represents in a given simulation.

which changes at an exogenous rate, $LS_t = \left(1 + g_L\right) LS_{t-1}$. The amount of labour actually employed, L_t, depends on the labour demand of firms and the energy sector as described below. Labour income is then given by $W_t = w_t L_t$. Households receive an unemployment benefit payment given by $UB_t = \zeta w_t\left(LS_t - L_t\right)$, where ζ is the replacement rate. If there is a tax on wage income, this is given by $Tax_t^H = \tau^H W_t$.

Households' desired nominal consumption expenditure is given by

$$C_{d,t} = \alpha_1\left(W_t + UB_t - Tax_t^H\right) + \alpha_2\left(Div_{t-1} + iD_{h,t}\right) + \alpha_3 D_{h,t-1} \tag{A.2}$$

Households hence have different propensities to consume out of wage and benefit income $\left(\alpha_1\right)$, dividend and interest income $\left(\alpha_2\right)$ and accumulated wealth $\left(\alpha_3\right)$.[7] The actual consumption expenditure of households, C_t, is determined by households' interaction with consumption good firms described below. In addition to consumption, households may also make transfer payments T_h to firms in order to finance firm entry as described below. Household saving accumulates in the form of bank deposits, $D_{h,t}$. The rule used to distribute this aggregate quantity of deposits among individual banks is described in Section A.6.

At the end of a period t, the uniform nominal wage rate to be paid in $t+1$, is set. It is given by

$$w_{t+1} = \left(1 + \mathfrak{w}_t\right) w_t \tag{A.3}$$

$$\mathfrak{w}_t = min\left(\overline{\mathfrak{w}}, max\left(-\overline{\mathfrak{w}}, \pi^* + \psi_1 \hat{\pi}_t + \psi_2 \widehat{Pr}_t - \psi_3 \widehat{U}_t\right)\right) \tag{A.4}$$

where:

- $\overline{\mathfrak{w}}$ is an exogenous parameter limiting period-by-period variations in the wage rate
- π^* is the central bank's fixed inflation target
- $\hat{\pi}_t$ is the deviation of the current consumer price inflation rate from the inflation target
- $\widehat{Pr}_t$ is a weighted average of current and past percentage changes in the average labour productivity across firms (which, as described below, depends on the combination of vintages of capital goods owned by consumption good producers and the heterogeneous production techniques of capital goods producers).[8]
- $\widehat{U}_t$ is the change in the unemployment rate relative to $t-1$

A.3 Capital good firms

The sector of capital goods firms (K-Firms) consists of $N1$ individual firms, indexed by k, where $k = 1, ..., N1$. Each firm produces a capital good with unique characteristics, using a unique production technique (both of which evolve due to endogenous technological change) with labour and energy as inputs. K-firms compete on quality and price.

A.3.1 Production and labour demand

While at any given point in time, each K-firm produces one single 'vintage' of capital good, technological progress leads to the continuous emergence of new capital good vintages. A generic vintage is denoted using κ and is defined by the triple $\Sigma_\kappa = \left(Pr_\kappa, EE_\kappa, EF_\kappa\right)$, indicating, respectively, the embedded labour productivity, energy efficiency, and environmental friendliness (i.e. the amount of emissions generated per unit of energy used) implied by using a capital good of that vintage κ in the production of consumption goods. An existing unit of capital good/machine is defined by its vintage, i.e. Σ_κ, its age, i.e. how many periods have elapsed since its production, and its maximum lifespan. When the age of a machine exceeds $\aleph^K$, the machine can no longer be used in production. $\aleph^K$ is constant and homogeneous across machines.

In addition to producing capital goods with heterogeneous characteristics, K-firms also use heterogeneous production techniques. These are defined by the triple $\Sigma_k = \left(Pr_{k,t}, EE_{k,t}, EF_{k,t}\right)$, indicating, respectively, the labour productivity, energy efficiency, and environmental friendliness of a generic K-firm production process. Note that production techniques are also subject to technological innovation, hence they change over time.

K-firms produce on demand, i.e., they receive orders from clients in period t, produce all ordered machines in t, and deliver to clients in $t+1$. This implies that K-firms do not accumulate inventories, neither planned nor otherwise.

Once orders have been received, K-Firm labour demand is computed:

$$L_{k,t}^d = \frac{Q_{k,t}}{Pr_{k,t}\mathfrak{a}} \tag{A.5}$$

Where $Q_{k,t}$ is the quantity of machines ordered from k and $\mathfrak{a}$ is a uniform scaling parameter for the labour productivity of K-Firms.

Similarly, k's demand for energy is given by:

$$En_{k,t}^d = \frac{Q_{k,t}}{EE_{k,t}} \tag{A.6}$$

Production generates emissions, which we assume to be proportional to the amount of energy required in production:

$$Em_{k,t} = \frac{EF_{k,t}}{EE_{k,t}} Q_{k,t} \tag{A.7}$$

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[7] It is assumed that households cannot borrow for consumption, meaning that if desired consumption is greater than the stock of deposits currently held by households, desired consumption is reduced to the maximum amount which can be financed out of deposits.

[8] This is computed as $\widehat{Pr}_t = \eta \widehat{Pr}_{t-1} + (1-\eta)\frac{\overline{Pr}_t - \overline{Pr}_{t-1}}{\overline{Pr}_{t-1}}$ where $\overline{Pr}_t$ is the average labour productivity across C-Firms and K-Firms.

Section A.12.5 explains how emissions are aggregated and transformed into an input for the climate module.

A.3.2 Capital good market dynamics

K-firms set prices by applying a fixed and homogeneous markup, μ^K, over unit cost of production. For a generic K-firm, unit cost of production is given by:

$$uc_{k,t} = \frac{w_t}{Pr_{k,t}\mathfrak{a}} + \frac{p_{e,t-1}}{EE_{k,t}} + \tau_t^{Em,K}\frac{EF_{k,t}}{EE_{k,t}} \tag{A.8}$$

Where w_t is the nominal wage, $Pr_{k,t}$ is the labour productivity of k's production process, $p_{e,t}$ is the energy price, $EE_{k,t}$ is the energy efficiency, $\tau_t^{Em,K}$ is the carbon tax charged to the capital goods sector, and $EF_{k,t}$ is the environmental friendliness. The price charged by a generic K-firm can thus be written as:

$$p_{k,t} = \left(1 + \mu^K\right)uc_{k,t} \tag{A.9}$$

Each K-firm is endowed with an equal number of C-Firm clients at the beginning of a simulation. During the simulation, K-firms compete in order to increase their market share by sending brochures to potential new clients. Brochures contain all the relevant information which potential customers require to make their choice, that is, Σ_κ and the price charged, $p_{k,t}$. The number of brochures sent by a K-Firm k is assumed to be proportional to its size in terms of the number of existing clients:

$$BROCH_{k,t} = max\left(1, \left\lfloor \Gamma CLNT_{k,t-1} \right\rceil\right) \tag{A.10}$$

$BROCH_{k,t}$ is the number of brochures sent by k, $CLNT_{k,t}$ is the number of k's current clients, and Γ is an exogenous parameter. Brochures are sent to C-Firms which are randomly drawn from the full set of C-Firms (i.e. including those who are already customers of k). Each C-Firm, in each period and regardless of whether or not it plans to invest in the current period, compares the received brochures and chooses as its preferred supplier the K-Firm offering the best machine taking into account both the price charged for the machine and the unit cost of production implied by using the machine (this choice is described in detail in Section A.4.2 below).

A.3.3 Technological change

K-firms aim to improve their production technique Σ_k and the technology embedded in the capital vintage they produce, Σ_κ. In order to do so, they engage in technological innovation and imitation through research and development (R&D).

We assume a two-step process of technological change. First, K-Firms allocate resources for innovation and imitation. The size of these R&D investments determines the likelihood of innovation and/or imitation being successful. Conditional on innovation and/or imitation being successful, the characteristics of the resulting technology or technologies are determined stochastically. The innovating/imitating firm then determines whether a new technology is superior to the existing one and adopts it if this is the case.

The overall amount of resources which a K-Firm k wishes to devote to R&D is given by a fraction $\mathfrak{o}$ of its current revenue if k's current revenue is positive,[9] and equal to the resources devoted in the previous period otherwise:

$$RD_{k,t} = \begin{cases} \mathfrak{o}Sales_{k,t} & \text{If} \quad Sales_{k,t} > 0 \\ RD_{k,t-1} & \text{Otherwise} \end{cases} \tag{A.11}$$

$\mathfrak{o}$ is fixed and homogeneous across K-Firms. R&D activities are performed using labour as an input. Consequently, a K-Firm's demand for labour for R&D is given by

$$L_{k,t}^{rd} = \frac{RD_{k,t}}{w_t} \tag{A.12}$$

We assume that K-Firms' demand for labour used for R&D is never rationed.[10] The hired labour is subsequently divided between R&D activity devoted to innovation $\left(RD_{k,t}^{in}\right)$ and imitation $\left(RD_{k,t}^{im}\right)$:

$$\begin{aligned} RD_{k,t}^{in} &= \mathfrak{x}^K L_{k,t}^{rd} \\ RD_{k,t}^{im} &= \left(1 - \mathfrak{x}^K\right) L_{k,t}^{rd} \end{aligned} \tag{A.13}$$

$\mathfrak{x}^K$ is fixed and homogeneous across K-Firms. The model then determines whether a K-Firm k is successful in imitating and/or innovating a technology in period t. The probability of innovating/imitating is increasing in the respective R&D input:

$$\begin{aligned} P(Innovate)_{k,t} &= 1 - exp\left(-\mathfrak{b}_1^K RD_{k,t}^{in}\right) \\ P(Imitate)_{k,t} &= 1 - exp\left(-\mathfrak{b}_2^K RD_{k,t}^{im}\right) \end{aligned} \tag{A.14}$$

$\mathfrak{b}_1^K$ and $\mathfrak{b}_2^K$ are fixed and homogeneous across K-Firms. For each K-Firm k, two draws from a Bernoulli distribution are made. The first takes the value 1 with probability $P(innovate)_{k,t}$, and if this is the case, the firm k innovates. Similarly, the second takes the value 1 with probability $P(imitate)_{k,t}$, and if this is the case, the firm k imitates. Note that this implies that a K-Firm can both innovate a new technology and imitate the technology of a competitor in the same period. As described below, the technology actually adopted then depends on their respective characteristics.

If a K-Firm innovates, the characteristics of the new technology are determined stochastically. Recall that at each point in time, every K-Firm produces a single, unique vintage of capital good κ, characterised by the triple $\Sigma_\kappa = \left(Pr_\kappa, EE_\kappa, EF_\kappa\right)$ denoting the labour productivity, energy efficiency and environmental friendliness implied by using this vintage of capital good in the production of consumption

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[9] Since not all C-Firms invest in every period, an individual K-Firm with few customers may have zero sales in a period.

[10] If overall labour demand exceeds the size of the labour force, LS_t, only production activity is scaled back until aggregate labour demand equals the size of the labour force.

goods. In addition, each K-Firm has an individual technique for producing capital goods, defined by the triple $\Sigma_k = \left(Pr_{k,t}, EE_{k,t}, EF_{k,t}\right)$, denoting the labour productivity, energy efficiency and environmental friendliness of the production process. Innovation in the model is depicted as a random simultaneous change to all the components of Σ_κ and Σ_k, resulting in a new vintage of capital good κ_{in} and an associated technique for producing this type of capital good. In particular, the characteristics of κ_{in} are given by:

$$\begin{aligned} Pr_{\kappa_{in}} &= \left(1 + \mathfrak{I}_{1,k,t}\right) Pr_\kappa \\ EE_{\kappa_{in}} &= \left(1 + \mathfrak{I}_{2,k,t}\right) EE_\kappa \\ EF_{\kappa_{in}} &= \left(1 - \mathfrak{I}_{3,k,t}\right) EF_\kappa \end{aligned} \tag{A.15}$$

where:

- $\mathfrak{I}_{1,k,t}$ is a draw from a beta distribution with shape parameters $\mathfrak{b}_3^K$ and $\mathfrak{b}_4^K$, rescaled on the interval $\left(\mathfrak{b}_5^K, \mathfrak{b}_6^K\right)$.
- $\mathfrak{I}_{2,k,t}$ is a draw from a beta distribution with shape parameters $\mathfrak{b}_7^K$ and $\mathfrak{b}_8^K$, rescaled on the interval $\left(\mathfrak{b}_9^K, \mathfrak{b}_{10}^K\right)$.
- $\mathfrak{I}_{3,k,t}$ is a draw from a beta distribution with shape parameters $\mathfrak{b}_{11}^K$ and $\mathfrak{b}_{12}^K$, rescaled on the interval $\left(\mathfrak{b}_{13}^K, \mathfrak{b}_{14}^K\right)$.

Similarly, the production technique used to produce the innovated vintage κ_{in} is given by

$$\begin{aligned} Pr_{in,k,t} &= \left(1 + \mathfrak{I}_{4,k,t}\right) Pr_{k,t} \\ EE_{in,k,t} &= \left(1 + \mathfrak{I}_{5,k,t}\right) EE_{k,t} \\ EF_{in,k,t} &= \left(1 - \mathfrak{I}_{6,k,t}\right) EF_{k,t} \end{aligned} \tag{A.16}$$

where:

- $\mathfrak{I}_{4,k,t}$ is a draw from a beta distribution with shape parameters $\mathfrak{b}_{15}^K$ and $\mathfrak{b}_{16}^K$, rescaled on the interval $\left(\mathfrak{b}_{17}^K, \mathfrak{b}_{18}^K\right)$.
- $\mathfrak{I}_{5,k,t}$ is a draw from a beta distribution with shape parameters $\mathfrak{b}_{19}^K$ and $\mathfrak{b}_{20}^K$, rescaled on the interval $\left(\mathfrak{b}_{21}^K, \mathfrak{b}_{22}^K\right)$.
- $\mathfrak{I}_{6,k,t}$ is a draw from a beta distribution with shape parameters $\mathfrak{b}_{23}^K$ and $\mathfrak{b}_{24}^K$, rescaled on the interval $\left(\mathfrak{b}_{25}^K, \mathfrak{b}_{26}^K\right)$.

Note that the support of the various Beta distributions need not be confined to positive values. This implies that the firm may discover a new capital vintage or production technique which is inferior to the current one along one or multiple dimensions. This modelling choice mimics the trial and error process characterizing technological change.

Imitation, by contrast, is based on a measure of the technological proximity between two K-Firms. If a K-Firm k successfully imitates, the model computes the technological proximity between k and every other K-Firm j, comparing both the production techniques of k and j and the vintages κ and κ_{im} produced by k and j respectively:

$$\begin{aligned} Dist_{k,j,t}^1 &= \left(Pr_\kappa - Pr_{\kappa_{im}}\right)^2 \\ Dist_{k,j,t}^2 &= \left(EE_\kappa - EE_{\kappa_{im}}\right)^2 \\ Dist_{k,j,t}^3 &= \left(EF_\kappa - EF_{\kappa_{im}}\right)^2 \\ Dist_{k,j,t}^4 &= \left(Pr_{k,t} - Pr_{\kappa_{j,t}}\right)^2 \\ Dist_{k,j,t}^5 &= \left(EE_{k,t} - EE_{\kappa_{j,t}}\right)^2 \\ Dist_{k,j,t}^6 &= \left(EF_{k,t} - EF_{\kappa_{j,t}}\right)^2 \\ Prox_{k,j,t} &= \frac{1}{\sqrt{\sum_{i=1}^6 Dist_{k,j,t}^i}} \end{aligned} \tag{A.17}$$

The proximity measures are then normalised by dividing them by the sum of all proximity measures. They are then placed on the interval [0, 1] by iterating over all proximities and, for each j, modifying them to $Prox_{k,j,t} = Prox_{k,j,t} + Prox_{k,j-1,t}$. Next, a uniform random number ε is drawn. Firm k will imitate the technology of firm j if $\varepsilon \leq Prox_{k,j,t}$ and $\varepsilon > Prox_{k,j-1,t}$. This ensures that K-Firms are more likely to imitate the technology of competitors with a higher technological proximity. Note that the firm may imitate a technology which is inferior to its current one along one or multiple dimensions.

The final step in the process of endogenous technological change concerns the adoption decision. Recall that a new technology discovered by some K-Firm k may be inferior to the one currently used by k along one or more dimensions. Similarly, firm k may end up imitating a technology which is inferior along one or more dimensions. To decide which new technology (if any) to adopt, the firm compares the innovated and imitated technologies to one another, as well as to its existing technology. To do so, it uses the same rule which C-Firms use in choosing their capital goods supplier and in deciding whether an existing machine should be replaced with a more modern one (see Section A.4). In particular, k computes a measure of vintage attractiveness for its existing technology as well as the innovated and imitated technologies:

$$\begin{aligned} A_{\kappa,t} &= p_{k,t} + uc_{\kappa,t} b \\ A_{\kappa_{in},t} &= p_{in,k,t} + uc_{\kappa_{in},t} b \\ A_{\kappa_{im},t} &= p_{im,k,t} + uc_{\kappa_{im},t} b \end{aligned} \tag{A.18}$$

$p_{k,t}$ is the price which k currently charges for one unit of the capital good, computed as described in equations (A.8) and (A.9). $p_{in,k,t}$ and $p_{im,k,t}$ are the prices which k would charge when using the innovated and imitated capital good production techniques, respectively. The uc terms denote the unit cost of producing one unit of the consumption good using a machine of the current (κ), innovated $\left(\kappa_{in}\right)$ and imitated $\left(\kappa_{im}\right)$ vintages, respectively. b is a fixed and homogeneous payback parameter. The K-Firm then chooses the technology for which A takes the lowest value, i.e. that with best trade-off between price and quality. Note that a technology does not have to be superior along all dimensions in order to be adopted/retained by a K-Firm. For instance, a unit of the capital good of an innovated vintage may be more costly for the K-Firm to produce, implying a higher selling price $\left(p_{in,k,t} > p_{k,t}\right)$, but at the same time imply a a sufficiently lower unit cost for a C-Firm using that vintage to counterbalance the higher selling price. K-firms use the same payback parameter b[11] employed by C-firms in their investment decisions to evaluate whether to buy a machine with superior technology and when deciding whether to switch to a new supplier.

Finally, note that both the unit cost of producing a capital good of some innovated vintage and the unit cost of using that vintage in the production of consumption goods are functions of the wage rate, the energy price, and the carbon tax rates on K-Firms and C-Firms. Accordingly, a higher energy price may, for instance, induce K-Firms to more readily adopt technologies with a higher energy efficiency even if they are more costly along other dimensions (e.g. implying a higher labour input).

A.3.4 Profits and dividends

Once all K-Firm decisions and market interactions have taken place, gross profits can be computed: sales and interest on deposits enter the profit calculation with a positive sign; the wage and energy bills as well as carbon tax payments enter the profit calculation with a negative sign.

$$\Pi^{gross}_{k,t} = Sales_{k,t} + iD_{k,t} - W_{k,t} - En_{k,t} - CTAX_{k,t} \quad (A.19)$$

where:

$$\begin{aligned} Sales_{k,t} &= p_{k,t}Q_{k,t} \\ iD_{k,t} &= r^{d}_{t-1}Dep_{k,t-1} \\ W_{k,t} &= w_t L_{k,t} + w_{t-1} * L^{rd}_{k,t-1} \\ En_{k,t} &= p_{e,t}En^{d}_{k,t} \\ CTAX_{k,t} &= \tau^{Em,K}_{t} Em_{k,t} \end{aligned} \quad (A.20)$$

- $Sales_{k,t}$ ≡ nominal sales; $p_{k,t}$ ≡ price; $Q_{k,t}$ ≡ number of machines sold;
- $iD_{k,t}$ ≡ interest payments on deposit; r^{d}_{t-1} ≡ interest rate on deposits; $Dep_{k,t-1}$ ≡ stock of deposits;
- $W_{k,t}$ ≡ wage bill; w_t ≡ nominal wage; $L_{k,t}$ ≡ quantity of labour employed in production; $L^{rd}_{k,t-1}$ ≡ quantity of labour employed for R&D in $t-1$;
- $En_{k,t}$ ≡ energy bill; $p_{e,t}$ ≡ energy price; $En^{d}_{k,t}$ ≡ energy demand;
- $CTAX_{k,t}$ ≡ emission tax paid; $\tau^{Em,K}$ ≡ tax rate per unit of emission charged to the K-sector; $Em_{k,t}$ ≡ emissions;

If gross profits are positive, K-Firms pay profit taxes, which are charged at a flat rate τ^K:

$$\Pi^{net}_{k,t} = \left(1 - \mathbf{1}^{\left[\Pi^{gross}_{k,t}>0\right]}\tau^K\right)\Pi^{gross}_{k,t} \quad (A.21)$$

Where $\mathbf{1}^{\left[\Pi^{gross}_{k,t}>0\right]}$ is an indicator function taking the value 1 if $\Pi^{gross}_{k,t} > 0$ and 0 otherwise. If profits are positive, firms pay dividends, $Div_{k,t}$ to households:

$$Div_{k,t} = \mathbf{1}^{\left[\Pi^{net}_{k,t}>0\right]}\delta^K\Pi^{net}_{k,t} \quad (A.22)$$

where δ^K is the dividend payout rate, which is assumed to be constant and homogeneous across K-firms. Retained earnings are held in the form of bank deposits, and we assume that K-Firms cannot borrow from the banking sector.

A.3.5 Failure and exit

K-Firms may exit the model and be replaced by new ones for two reasons. First, a K-Firm will exit if it loses all of its customers, i.e. if all C-Firms for which it was the preferred supplier of capital goods switch to a different supplier.[12] Second, a K-Firm will exit if it is unable to meet payments for energy input or wages. Recall that K-Firms produce on demand and price their output at a mark-up over unit cost. In addition, as described below, C-Firms only invest if they are certain that they can pay for the capital goods ordered. However, while the current wage rate is known when unit cost is computed, the current price of energy is not, and hence its lagged value is used by K-Firms when setting prices. This means that an increase in the energy price may lead to one or more K-Firms being unable to (fully) pay for energy used as an input in production. In addition, wages for R&D paid in t are based on the amount of resources devoted to R&D in $t-1$. Hence, a situation may arise in which a K-Firm ‘over-invests’ in R&D in the sense that its revenues and accumulated deposits in the following period are insufficient to fully cover current production cost in addition to paying wages for R&D labour from the previous period. In these cases, a failing K-Firm will still produce the capital goods ordered by its customers in the current period but then exit the market after satisfying as many of its payment obligations as possible using all funds it still has available. The replacement of exiting K-Firms is described in Section A.5.

A.4 Consumption good firms

The model includes a consumption good sector consisting of $N2$ individual firms, each indexed as c, where $c = 1, ..., N2$. Each firm produces a homogeneous final consumption good using capital, labour, and energy as inputs. Production techniques are heterogeneous across C-Firms in terms of productivity, energy efficiency, and environmental

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[11] b is defined in terms of units of consumption goods and gives the number of units of consumption good which must be produced using a superior technology (i.e. one offering a lower unit cost of production) to justify investing in it.

[12] Note that this is distinct from having zero revenue. A K-Firm may have a set of existing customers but have zero revenue in a period since not all C-Firms invest in every period. Zero revenue in a given period does not necessarily lead to the exit of the K-Firm in question.

friendliness due to the composition of the capital stock of each C-firm being different in terms of vintages (see also Section A.3). C-Firms compete in the consumption goods market in order to capture as large a market share as possible. Since consumption goods are homogeneous, competition takes place along the dimensions of price and firms' ability to deliver the quantity demanded.

A.4.1 Desired production

C-Firms' desired production is set to match expected demand and achieve desired inventory holdings. The latter are kept in order to enable the firm to serve demand exceeding expectations. Actual production may fall short of desired production if a C-Firm is capital or labour-constrained or if it cannot finance the desired production. The desired production is determined as

$$Q^d_{c,t} = Dem^e_{c,t} + N^d_{c,t} - N_{c,t-1} \tag{A.23}$$

Where $Q^d_{c,t}$ is desired production, $Dem^e_{c,t}$ is expected demand, $N^d_{c,t}$ are desired inventories, and $N_{c,t-1}$ is c's existing stock of inventories. Desired inventories are proportional to expected demand:

$$N^d_{c,t} = \varphi Q^d_{c,t}, \quad \varphi \in [0,1] \tag{A.24}$$

Where φ is a fixed parameter that is homogeneous across C-Firms.

Demand expectations are assumed to be adaptive, i.e.:

$$Dem^e_{c,t} = \sigma Dem_{c,t-1} + (1-\sigma) Dem^e_{c,t-1} \tag{A.25}$$

Where $Dem_{c,t-1}$ is the actual demand received by c in the previous period and σ is an exogenous parameter that is homogeneous across C-Firms.

As indicated above, actual production $Q_{c,t}$ may differ from desired production if c has an insufficient stock of machines to carry out desired production, if c is constrained by labour availability, or if c cannot finance the desired level of production. In a first step, c checks whether its productive capacity in terms of available machine tools is sufficient to carry out its desired production. While, as outlined above, machine vintages differ in terms of labour productivity, energy efficiency and environmental friendliness, it is assumed that every machine can produce a maximum of $\mathfrak{Q}$ units of output when used at full capacity. $\mathfrak{Q}$ is constant and homogeneous across vintages. If the desired output of c exceeds its maximum productive capacity, c's desired output is scaled back to the maximum producible given its capital stock.

A.4.2 Investment

As described in Section A.3, C-Firms choose their current supplier of capital goods by comparing brochures received from their current and a subset of potential alternative suppliers, which specify the characteristics of the capital good vintages currently sold by these firms. C-Firms compute a measure of vintage attractiveness $A_{\kappa,t}$ for each observed vintage κ:

$$A_{\kappa,t} = p_{k,t} + uc_{\kappa,t} b \tag{A.26}$$

Where $p_{k,t}$ is the price charged by the K-Firm k which produces the vintage κ, $uc_{\kappa,t}$ is the unit cost of production implied by using vintage κ in the production of consumption goods, and b is a payback parameter. Note that this equation is identical to the one used by K-Firms in deciding whether or not to adopt an innovated/imitated technology. Each firm chooses the observed supplier whose offering implies the lowest $A_{\kappa,t}$.

We distinguish between two types of investment in capital goods: one is aimed at maintaining or expanding productive capacity in order to meet expected future production needs, the other is replacement investment and is aimed at substituting still usable but technologically obsolete machines with new ones situated at the technological frontier. Let us begin by describing the first type, which we simply term expansion investment:

C-Firms aim to attain a given level of capacity utilization $u < 1$, which is fixed and homogeneous across firms. Desired productive capacity, $\mathfrak{K}^d_{c,t}$, can therefore be written as:

$$\mathfrak{K}^d_{c,t} = \frac{Q^d_{c,t}}{u} \tag{A.27}$$

Desired expansion investment is set to achieve $\mathfrak{K}^d_{c,t}$. Expansion investment is constrained by an exogenous maximum level of addition to productive capacity achievable in a single period.[13] In addition, while consumption goods are assumed to be perfectly divisible, only integer units of capital goods can be purchased. Desired expansion investment is hence given by

$$EI^d_{c,t} = max\left(0, \left\lfloor \frac{min\left(\overline{EI}_{c,t},\ \mathfrak{K}^d_{c,t} - \mathfrak{K}_{c,t}\right)}{\mathfrak{Q}} \right\rfloor \mathfrak{Q}\right) \tag{A.28}$$

Where $\mathfrak{K}_{c,t}$ is c's current productive capacity from which machines reaching their maximum age in t (which the firm knows with certainty will be scrapped at the end of t) have already been removed. $\overline{EI}_{c,t}$ is the maximum possible expansion investment, which is defined as:

$$\overline{EI}_{c,t} = \left\lceil \frac{(1+\lambda)\mathfrak{K}_{c,t}}{\mathfrak{Q}} \right\rceil \mathfrak{Q} - \mathfrak{K}_{c,t} \tag{A.29}$$

Where λ is a homogeneous parameter.

Besides expansion investment, which covers both the replacement of machines which have reached their maximum

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[13] There is also a financial constraint, which becomes relevant when firms cannot or do not want to borrow a sufficient amount to finance their desired investment (see Sections A.4.4 and A.6)

age and the expansion of productive capacity, a C-Firm may also wish to substitute machines which have not reached their maximum age if they have become technologically obsolete *vis-a-vis* the vintage offered by its capital goods supplier. Machines owned by C-Firm c of some vintage κ are compared to the vintage currently offered by c's supplier of capital goods, κ^*, which is the most advanced technology known to c. c's machines of vintage κ are deemed to be technologically obsolete if:

$$\frac{p_{\kappa^*,t}}{uc_{\kappa,t} - uc_{\kappa^*,t}} \leq b \tag{A.30}$$

Where $p_{\kappa^*,t}$ is the price charged by c's current capital good supplier for the vintage κ^* and b is an exogenous payback parameter. $uc_{\kappa,t}$ is the current unit cost of production implied by the use of vintage κ, while $uc_{\kappa^*,t}$ is the corresponding unit cost arising from the use of κ^*. If vintage κ is deemed obsolete, firm c wishes to replace its entire stock of machines of vintage κ with machines of vintage κ^*. This comparison takes place in every period for all vintages currently operated by c. Unlike expansion investment, there is no exogenous constraint on the amount of substitution investment which can be carried out within a single period. Capital goods ordered by C-Firms in t, both for expansion and substitution investment are delivered at the beginning of $t + 1$. The nominal value of capital goods on C-Firms' balance sheets is given by their price at the time of purchase and subsequently remains constant until they are scrapped.

Consumption firms may reduce desired investment due to financial considerations. If the the nominal value of desired investment exceeds the sum of internal funds and the maximum amount of credit a firm is willing to take up after paying for production cost (see Section A.4.4), investment demand is reduced until it equals the amount of remaining potential liquidity. In addition, C-Firms may be constrained on the credit market if banks are not willing lend as much as C-Firms demand, in which case investment (and potentially also current production) will be (further) reduced.

A.4.3 Pricing and production costs

C-Firms set individual prices by applying a markup over unit cost of production. In every period, firm c is allowed to update its price with a given probability θ.[14] Therefore:

$$p_{c,t} = \begin{cases} \left(1 + \mu_{c,t}\right) uc_{c,t} & \text{If} \quad \varepsilon_{c,t} \leq \theta \\ p_{c,t-1} & \text{Otherwise} \end{cases} \tag{A.31}$$

Where $p_{c,t}$ is the price, $\mu_{c,t}$ is the mark-up, $uc_{c,t}$ is the unit cost of production (see below), and $\varepsilon_{c,t}$ is a random draw from a uniform distribution with support [0, 1].

The markup evolves following a simple adaptive rule: when demand for its own output is strong, C-Firm c revises its markup upward, and vice-versa.

$$\mu_{c,t} = \begin{cases} \mu_{c,t-1} \left[1 + \Delta^{\mu} \widehat{f_{c,t-1}}\right] & \text{if } f_{c,t-2} > 0 \\ \mu_{c,t-1} & \text{Otherwise} \end{cases} \tag{A.32}$$

Where $f_{c,t}$ is c's market share in the market for consumption goods at time t, Δ^{μ} is an exogenous parameter that is homogeneous across C-Firms and $\widehat{f_{c,t-1}} = \frac{f_{c,t-1} - f_{c,t-2}}{f_{c,t-2}}$. If the mark-up resulting from equation (A.32) is negative, it is set to zero instead.

The unit cost of production, $uc_{c,t}$, entering Equation (A.31) depends on the composition of c's capital stock. Recall that each capital vintage κ of which c currently owns one or more units implies a certain unique unit cost when used to produce consumption goods. $uc_{c,t}$ is hence a weighted average across all κ-specific unit costs of production, with the weights being given by the share of machine tools of each vintage κ in the capital stock of c. We can therefore compactly express $uc_{c,t}$ as:

$$uc_{c,t} = \sum_{\kappa \in \Phi_{\kappa,c,t}} uc_{\kappa,t} \frac{\mathfrak{K}_{\kappa,c,t}}{\mathfrak{K}_{c,t}} \tag{A.33}$$

Where κ is a generic capital vintage, $\Phi_{\kappa,c,t}$ is the set of vintages available to firm c, $uc_{\kappa,t}$ is the unit cost of production embedded in vintage κ, and $\mathfrak{K}_{\kappa,c,t}$ is the amount of production that firm c can achieve using technology κ. Note that $\frac{\mathfrak{K}_{\kappa,c,t}}{\mathfrak{K}_{c,t}}$ represents the weight applied to each vintage κ.

If the capacity utilisation implied by c's desired production is smaller than 1, c will use the most efficient combination of capital vintages allowing it to produce the desired level of output, meaning that its effective unit cost will differ from $uc_{c,t}$. Capital vintages are ranked according to their unit cost of production, from the lowest to the highest. Beginning from the most cost-efficient vintage, c activates machines until the desired scale of production has been reached, with all remaining capacity remaining idle. We can therefore write effective unit cost as

$$uc^{e}_{c,t} = \sum_{\kappa \in \Phi^{u}_{\kappa,c,t}} uc_{\kappa,t} \frac{\mathfrak{K}_{\kappa,c,t}}{\mathfrak{K}^{u}_{c,t}} \tag{A.34}$$

where $\Phi^{u}_{\kappa,c,t}$ denotes the subset of vintages available to firm c which is actually used in production in period t and $\mathfrak{K}^{u}_{c,t}$ denotes the part of the capital stock of c actually used in t.

Finally, the unit cost of production associated with a particular vintage κ, $uc_{\kappa,t}$, is given by the sum of labour cost, energy cost, and emission taxes:

$$uc_{\kappa,t} = \frac{w_t}{Pr_{\kappa}} + \frac{p_{e,t-1}}{EE_{\kappa}} + \tau^{Em,C}_{t} \frac{EF_{\kappa}}{EE_{\kappa}} \tag{A.35}$$

Where w_t is the nominal wage, Pr_{κ} is the vintage-specific labor productivity, $p_{e,t}$ is the price of energy, EE_{κ}

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[14] Note that in current applications, θ is set to 1, i.e., all C-Firms update their price in every period.

is the vintage-specific energy efficiency, $\tau^{Em,C}$ is the carbon tax applied to the consumption good sector, and EF_κ is the vintage-specific environmental friendliness.

By the same logic as Equation (A.34), we can write c's effective labor productivity, energy efficiency, and environmental friendliness as:

$$\begin{aligned} Pr^e_{c,t} &= \sum_{\kappa \in \Phi^u_{\kappa,c,t}} Pr_{\kappa,t} \frac{\mathfrak{K}_{\kappa,c,t}}{\mathfrak{K}^u_{c,t}} \\ EE^e_{c,t} &= \sum_{\kappa \in \Phi^u_{\kappa,c,t}} EE_{\kappa,t} \frac{\mathfrak{K}_{\kappa,c,t}}{\mathfrak{K}^u_{c,t}} \\ EF^e_{c,t} &= \sum_{\kappa \in \Phi^u_{\kappa,c,t}} EF_{\kappa,t} \frac{\mathfrak{K}_{\kappa,c,t}}{\mathfrak{K}^u_{c,t}} \end{aligned} \tag{A.36}$$

Using the effective labour productivity computed as shown above, C-Firms then calculate their labour demand as

$$L^d_{c,t} = \frac{Q^d_{c,t}}{Pr^e_{c,t}} \tag{A.37}$$

Similarly, c's demand for energy can be calculated as

$$En^d_{c,t} = \frac{Q^d_{c,t}}{EE^e_{c,t}} \tag{A.38}$$

Productive activity also generates emissions, which we assume to be proportional to the amount of energy input required for production:

$$Em_{c,t} = \frac{EF^e_{c,t}}{EE^e_{c,t}} Q^d_{c,t} \tag{A.39}$$

Section A.12.5 explains how emissions are aggregated and transformed into an input for the climate module. Note that the quantities calculated above are computed using the desired production of c, $Q^d_{c,t}$. As outlined below, actual output may be lower than desired output if c is unable to hire a sufficient amount of labour or if c cannot fully finance its desired production. In these cases, labour demand, energy demand and emissions are adjusted accordingly.

A.4.4 Credit

Besides possibly being credit-rationed by its bank (see Section A.6), each C-Firm c has an internal constraint in the form of a maximum increase in the amount of credit that it is willing to take up for the purpose of investment:

$$\mathfrak{c}^{d,max}_{c,t} = max\big(0, \phi NR_{c,t-1} - \mathfrak{l}_{c,t-1}\big) \tag{A.40}$$

Where ϕ is an exogenous parameter, $NR_{c,t-1}$ is previous revenue from sales of consumption goods net of production cost (wages and energy payments), and $\mathfrak{l}_{c,t-1}$ is c's stock of outstanding loans from the previous period.

In addition, c calculates a measure of available internal funds given by:

$$IF_{c,t} = max\big(0, D_{c,t} - \mathfrak{l}_{c,t-1} - uc^e_{c,t} Q^d_{c,t}\big) \tag{A.41}$$

where $D_{c,t}$ is c's stock of deposits and $uc^e_{c,t} Q^d_{c,t}$ is the cost of c's desired production.

In the first instance, C-Firms aim to finance investment in capital goods out of internal funds. If the latter are insufficient, they plan to take out additional loans up to a maximum $\mathfrak{c}^{d,max}_{c,t}$. If this is insufficient to finance desired investment, C-Firms first curtail substitution investment aimed at the replacement of functional but technologically obsolete machines, and subsequently also expansion investment aimed at the replacement of machines which have reached their maximum age and at the expansion of productive capacity.

In addition, even in cases in which planned investment has already been scaled back to meet a C-Firm's own readiness to borrow, it may still face an external financing constraint if its bank is not willing to extend as much credit as the firm demands (see Section A.6). In this case some planned expenditures must be (further) reduced. We assume a ranking of expenditures, whereby expenditures are sequentially reduced, up to the point at which internal resources plus the amount of credit which the bank is willing to extend are sufficient to cover the remaining expenditures. For this purpose, the C-Firm's expenditures are reduced in the following order:

1. Substitution investment due to technological obsolescence is reduced to 0
2. Expansion investment (including replacement of machines which have reached their maximum age) is reduced to 0
3. Production is scaled down until production costs can be met

If, following this process, available funds are still insufficient to roll over outstanding debt and finance a positive level of current production, the affected C-Firm does not produce any output and exits the market.

A.4.5 Competitiveness

Recall that since households are presently modelled as an aggregate, their demand for consumption goods is also an aggregate quantity. This aggregate demand is distributed across C-firms by applying a quasi-replicator equation. C-Firms supply a homogeneous good and hence do not compete on quality. Instead, the market share of each C-Firm is a function of its relative price. In addition, a C-Firm's market share will be negatively affected if it has been unable to satisfy all demand it has received in the previous period.

The process of consumption good market competition is split into two separate steps: First, a measure of competitiveness $E_{c,t}$ is computed for each C-Firm c. Second, this measure is used to update the market shares and distribute aggregate consumption demand across C-Firms. Competitiveness is defined as

$$E_{c,t} = -\left(\frac{p_{c,t}}{\widehat{p}_t}\right)^{\omega_1} - \left(\frac{l_{c,t}}{\widehat{l}_t}\right)^{\omega_2} \tag{A.42}$$

Where $p_{c,t}$ is the price charged by firm c, whereas $\widehat{p}$ is the average price across the whole consumption good sector. $l_{c,t}$ is the level of demand which c left unsatisfied in the previous period (computed as shown in Section A.4.6), with $\widehat{l}_t$ being the respective average across all C-Firms. ω_1 and ω_2 are exogenous parameters giving the relative importance of price and ability to fill demand in determining competitiveness. $E_{c,t}$ is then used in order to update the ex-ante share of aggregate consumption demand accruing to each individual C-Firm:

$$\tilde{f}_{c,t} = f_{c,t-1}\left(\frac{2\omega_3}{1+e^{\left(-\chi\frac{E_{c,t}-\hat{E}_t}{\hat{E}_t}\right)}} + \left(1-\omega_3\right)\right) \tag{A.43}$$

Where $\tilde{f}_{c,t}$ and $f_{c,t-1}$ are the ex-ante and lagged market shares of c respectively and $\widehat{E}_t$ is a weighted average of $E_{c,t}$, computed using $f_{c,t-1}$ as weights. χ and ω_3 are exogenous parameters. Note that the functional form chosen for Equation (A.43) implies that period-to-period percent changes in $f_{c,t}$ must fall within $\pm\omega_3$. The market shares of C-Firms which have already failed prior to the determination of market shares due to inability to finance their productive activities are re-set to zero. In addition, we assume that firms for whom $\tilde{f}_{c,t}$ becomes smaller than a lower threshold $\mathfrak{f}$ exit and their market shares are re-set to zero.

Finally, note that Equation (A.43) does not ensure that the ex-ante market shares sum to 1. The model therefore applies the following adjustment in order to normalise them:

$$f_{c,t} = \frac{\tilde{f}_{c,t}}{\sum_{i=1}^{N2}\tilde{f}_{i,t}} \tag{A.44}$$

A.4.6 Consumption good market

Following the determination of market shares, the distribution of households' consumption demand among C-Firms takes place. This distribution takes place over multiple rounds. In the first round, the consumption demand received by an individual C-Firm c is given by

$$Dem^1_{c,t} = \frac{C_{d,t}}{cpi_t} f_{c,t} \tag{A.45}$$

where $C_{d,t}$ is households' aggregate nominal consumption demand and cpi_t is a consumption price index computed using the market shares $f_{c,t}$ as weights. Given the demand received by c in the first round, two cases can result:

1. $Dem^1_{c,t} \leq Q_{c,t} + N_{c,t-1}$, i.e. the sum of the quantity produced by c and its remaining inventories is greater than the demand received in the first round. In this case, the current revenue of c, which is initialised to zero, is augmented by $Dem^1_{c,t} p_{c,t}$. $l_{c,t}$, which quantifies C-Firms' ability to meet demand, is set to 1. The quantity of goods produced by c still for sale in future rounds is set to $Q_{c,t} + N_{c,t-1} - Dem^1_{c,t}$. The market share of c for the second round is left unchanged; $f^2_{c,t} = f_{c,t}$.
2. $Dem^1_{c,t} > Q_{c,t} + N_{c,t-1}$, meaning that c cannot satisfy the demand received in the first round. In this case, the current revenue of c is augmented by $\left(Q_{c,t} + N_{c,t-1}\right)p_{c,t}$. $l_{c,t}$ is set to $1 + Dem^1_{c,t} - Q_{c,t} - N_{c,t-1}$. The quantity of goods produced by c still for sale in future rounds is set to zero. The market share of c for the second round set to zero; $f^2_{c,t} = 0$.

In both cases, $Dem_{c,t}$, which will enter into the determination of expected demand in $t + 1$, and which is initialised to zero, is augmented by $Dem^1_{c,t}$.

Following this first round of distribution of consumption demand, households' nominal consumption demand is reduced by the sum of sales which have taken place in the first round. Second round market shares (which have been set to zero for C-Firms which have already sold all that they have produced) are normalised again:

$$f^2_{c,t} = \frac{f^2_{c,t}}{\sum_{i=1}^{N2} f^2_{i,t}} \tag{A.46}$$

Then, a new consumption price index is computed using $f^2_{c,t}$ as weights. The second and further rounds of distribution of consumption demand proceed in a fashion similar to the first one, in each round using the updated market shares and consumption price indices to distribute the remaining household consumption demand among those C-Firms which still have some remaining goods to sell. The only difference between some round $n > 1$ and the first round are that:

- $l_{c,t}$ is left unchanged.
- If $Dem^n_{c,t}$ is smaller than the remaining stock of output of c, $Dem_{c,t}$ is still augmented by $Dem^n_{c,t}$ but if $Dem^n_{c,t}$ exceeds the remaining output stock of c, $Dem^n_{c,t}$ is only augmented by the quantity actually sold by c in round n, to avoid excessive over-production in $t + 1$.

The distribution of consumption demand continues until either households' consumption demand has been fully satisfied or until no C-Firm has any more output left to sell. Following this, the consumption price index is recomputed using actual sales. Any output remaining unsold is accumulated in the form of non-depreciating inventories if inventory dynamics are activated (this is determined by the setting of an exogenous indicator variable). Inventories on a C-Firm's balance sheet are valued at the respective C-Firm's current selling price. If inventory dynamics are deactivated, any unsold output is scrapped.

A.4.7 Profits and Dividends

Once all C-Firm decisions and market interactions have taken place, gross profits, on which taxes are paid, can be computed: Sales, interests on deposits, nominal changes in inventories, and nominal changes in the capital stock enter the profit calculation with a positive sign; nominal investment, the wage bill, the energy bill, interest on loans, and carbon tax payments enter the profit calculation with a negative sign.

$$\begin{aligned}\Pi^{gross}_{c,t} =& Sales_{c,t} + iD_{c,t} + \Delta Inv_{c,t} + \Delta K_{c,t} - I_{c,t} \\ & - W_{c,t} - En_{c,t} - iL_{c,t} - CTAX_{c,t}\end{aligned} \tag{A.47}$$

Where:

$$\begin{aligned} Sales_{c,t} &= p_{c,t} Q^s_{c,t} \\ iD_{c,t} &= r^d_{t-1} Dep_{c,t-1} \\ \Delta Inv_{c,t} &= p_{c,t} N_{c,t} - p_{c,t-1} N_{c,t-1} \\ \Delta K_{c,t} &= I_{c,t} - Scrap_{c,t} \\ I_{c,t} &= EI^n_{c,t} + SI^n_{c,t} \\ W_{c,t} &= w_t L_{c,t} \\ En_{c,t} &= p_{e,t} En^d_{c,t} \\ iL_{c,t} &= r^l_{c,t} \mathfrak{L}_{c,t} \\ CTAX_{c,t} &= \tau^{Em,C}_t Em_{c,t} \end{aligned} \tag{A.48}$$

- $Q^s_{c,t} \equiv$ quantity of output sold by c in t;
- $iD_{c,t} \equiv$ interest payments on deposits; $r^d_{t-1} \equiv$ interest rate on deposits; $Dep_{c,t-1} \equiv$ deposit stock of c;
- $\Delta Inv_{c,t} \equiv$ period-to-period change in the nominal value of inventories; $p_{c,t} \equiv$ price of output of c; $N_{c,t} \equiv$ real inventory stock of c
- $\Delta K_{c,t} \equiv$ period-to-period change in the nominal value of c's capital stock; $I_{c,t} \equiv$ nominal value of capital investment; $Scrap_{c,t} \equiv$ nominal value of scrapped capital goods;
- $EI^n_{c,t} \equiv$ Nominal value of expansion investment; $SI^n_{c,t} \equiv$ Nominal value of substitution investment;
- $W_{c,t} \equiv$ wage bill; $w_t \equiv$ nominal wage rate; $L_{c,t} \equiv$ quantity of labour employed by c.
- $En_{c,t} \equiv$ energy bill; $p_{e,t} \equiv$ price of energy; $En^d_{c,t}$ energy demanded by c
- $iL_{c,t} \equiv$ interest payments on debt; $r^l_{c,t} \equiv$ interest rate on loans charged to c; $\mathfrak{L}_{c,t} \equiv$ loan stock of c;
- $CTAX_{c,t} \equiv$ emissions tax paid by c; $\tau^{Em,c}_t \equiv$ emissions tax rate applied to C-Firms; $Em_{c,t} \equiv$ emissions produced by c;

In addition to paying interest on loans, each C-Firm c must also repay a fraction ξ_C of its outstanding stock of loans at the end of every period. Bank loans in the model can hence be interpreted as a type of credit line provided by the banks, with outstanding credit having to either be renewed/rolled over or repaid in full at the beginning of every period t. In addition, banks demand that borrowers reduce any debt taken on/rolled over at the beginning of t by a fraction ξ_C once they have received revenues at the end of t.

If gross profits are positive, firms pay profit taxes, which are charged at a constant and flat rate τ^C:

$$\Pi^{net}_{c,t} = \left(1 - \mathbf{1}^{\left[\Pi^{gross}_{c,t}>0\right]} \tau^C\right) \Pi^{gross}_{c,t} \tag{A.49}$$

Where $\mathbf{1}^{\left[\Pi^{gross}_{c,t}>0\right]}$ is an indicator function taking the value 1 if $\Pi^{gross}_{c,t} > 0$ and 0 otherwise.

If profits are positive, firms pay dividends, $Div_{c,t}$ to households:

$$Div_{c,t} = \mathbf{1}^{\left[\Pi^{net}_{c,t}>0\right]} \delta^C \Pi^{net}_{c,t} \tag{A.50}$$

Where δ^C is the dividend rate, which is assumed to be constant and homogeneous across C-firms.

A.4.8 Failure

C-Firms in the model go bankrupt if they are unable to meet a payment obligation or if their net worth is negative. When this is the case they exit the market and are replaced by new firms (see Section A.5).

Note that since C-Firms scale back their productive activity and investment if they cannot (fully) finance them, C-Firms never fail due to inability to pay for wages or investment. This is because credit demand is computed when the wage rate and the prices charged by suppliers of capital goods are already known. As discussed above, if a C-Firm is so constrained on the credit market that it cannot finance any production, it exits without producing any output and hence does not have any payment obligations towards households, K-Firms, the government or the energy sector.

Once production and sales of consumption goods have taken place, C-Firms have a number of other payment obligations on which they can potentially default:

1. **Energy payments**: The first claimant in line is the energy sector, which demands payment for the energy input used in production by C-Firms. Since energy demand is computed before the current price of energy is known, a C-Firm may be unable to (fully) pay for the energy input it used. If this is the case, the C-Firm in question pays as much as it can and then exits.
2. **Principal and interest payments on loans**: Recall that in addition to paying interest on loans, C-Firms must also repay a fraction ξ_C of loans outstanding at the end of period t. If a C-Firm has insufficient liquidity to make both interest and principal payments, it pays as much as it can and then exits

3. **Emission and profit tax**: If a firm has insufficient liquidity to make tax payments, it pays as much as it can (beginning with the emission tax). However, we assume that a C-Firm which cannot meet a tax payment obligation does not exit.

The reasons for exiting given above all arise due to illiquidity. In addition, a C-Firm also exits if, at the end of a period, its net worth is negative, even when it has been able to meet all payment obligations in that period.

Finally, recall that a C-Firm also exits if its market share falls below a threshold $\mathfrak{f}$. This happens even if the firm has been able to meet all payment obligations and if its net worth is positive.

A.5 Firm exit and entry

As described above, both K-Firms and C-Firms may exit the model economy for a variety of reasons such as having zero customers (K-Firms), a very low market share (C-Firms) or being unable to meet a payment obligation (both). In all cases, exiting firms are replaced one for one with new firms of the same type, meaning that the numbers of both K-Firms and C-Firms are constant throughout a simulation. We begin by describing the exit and replacement process for K-Firms and subsequently turn to C-Firms.

A.5.1 Capital good firm replacement

Due to the simple balance sheet structure of K-Firms (see Table 1), the exit process for K-Firms is very straightforward. K-Firms hold bank deposits as their only asset and have no liabilities. This also implies that illiquidity and insolvency always coincide in the case of K-Firms.

If a K-Firm k exits, it loses any customers it may still have. However, any capital goods ordered by customers of k in t are still delivered at the beginning of $t+1$. Any deposits which k still holds are transferred to the household sector. Next, a random surviving K-Firm i is drawn. The initial production technique and capital good vintage produced by the new K-Firm j replacing k is copied from the randomly drawn i. Similarly, j's initial selling price is copied from i.

The new K-Firm j receives a transfer of deposits from households in order to provide it with an initial stock of liquidity. This transfer is given by

$$T_{j,t} = \mathfrak{d}_{j,t}\overline{D}_{k,t} \tag{A.51}$$

where $\mathfrak{d}_{j,t}$ is a uniform random variable drawn from the interval $\left(\mathfrak{d}^1_K, \mathfrak{d}^2_K\right)$ and $\overline{D}_{k,t}$ is the average stock of deposits held by surviving K-Firms. The bank serving the new K-Firm j is the same which was serving the exiting firm k.

If households are unable to fully cover the injection of liquidity for entering K-Firms from their accumulated deposits, one of two routines is activated depending on the setting of an exogenous indicator variable:[15]

1. The entry cost which cannot be covered by households is instead covered by the government.
2. The banks serving the newly entering K-Firms create the initial deposits of these firms ex nihilo and the net worth of the banks in question is reduced by the corresponding amount.

The number of brochures which an entering K-Firm j will send to potential customers in the following period is initialised to $\lfloor \Gamma\mathfrak{n} \rceil$, where $\mathfrak{n}$ is homogeneous across K-Firms. The sales of j, which are needed to determine its initial R&D spending, are initialised to $p_{j,t}\mathfrak{n}$.

A.5.2 Consumption good firm replacement

Since C-Firms have multiple assets (bank deposits, capital goods and inventories) as well as liabilities (loans) their exit process is somewhat more complicated than that of K-Firms.

The simplest case obtains if, at the time of exit, a C-Firm c's deposits exceed its outstanding loans (this may happen if it exits due to low market share). In this case, c's deposits are used in order to pay off the outstanding loans, with the remainder being transferred to households. Any remaining inventories are scrapped. The link between c and its current capital goods supplier in the C-Firm-K-Firm network is deleted.

If, instead, c's outstanding loans exceed its deposits at the time of exit, the difference between loans and deposits is initially recorded as a loss for the bank serving c. In this case, too, c's remaining inventories are scrapped and the link between c and its current capital goods supplier in the C-Firm-K-Firm network is severed.

Recall from the above description of C-Firm bankruptcy that C-Firms can never fail due to an inability to pay for investment in capital goods. However, when a C-Firm fails, all capital goods which had been ordered and paid for by that firm in t to be delivered in $t+1$ are scrapped. What happens to any capital goods already held by c is determined by a stylised second hand market for capital goods. The routine of this market begins with the determination of the overall number of machine tools needed by newly entering C-Firms. This is given by

$$mach^{entry}_t = max\left(N2^{exit}_t, ceil\left(f^{entry}_t \frac{Dem^{\Sigma}_t}{u\mathfrak{Q}}\right)\right) \tag{A.52}$$

where

- $N2^{exit}_t$ is the number of C-Firms which exit in t and which hence have to be replaced (this ensures that each newly entering C-Firm will enter with at least one machine).
- Dem^{Σ}_t is the sum of demand for consumption goods experienced by C-Firms in t, i.e. the sum of $Dem_{c,t}$ described above, summed across all C-Firms.

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[15] When changing the model calibration, users should ensure that this situation never or only very rarely arises in simulation runs (and indeed it does not happen under the calibration presented in this paper). Nevertheless, routines for the case in which households cannot finance firm entry were incorporated to preserve stock-flow consistency while still enabling the replacement of failed firms.

- u is the fixed and exogenous desired capacity utilisation of C-Firms.
- $\mathfrak{Q}$ is the uniform and constant maximum amount of output which can be produced using one machine.
- f_t^{entry} is the overall initial market share of entering C-Firms. If the sum of the market shares of exiting C-Firms, f_t^{exit}, is positive, we set $f_t^{entry} = f_t^{exit}$. If f_t^{exit} is zero, we instead set $f_t^{entry} = N2_t^{exit}\mathfrak{f}^{entry}$ where $\mathfrak{f}^{entry}$ is a parameter with a small positive value.

Next, $mach_t^{entry}$ is compared to $mach_t^{exit}$, the overall number of machines still held by exiting C-Firms. If $mach_t^{entry} > mach_t^{exit}$, the model sets $mach_t^{entry} = mach_t^{exit}$. Having determined the number of machine tools needed/available for newly entering C-Firms, the remaining capital goods of exiting C-Firms are first ordered according to their cost-efficiency (i.e. the unit cost implied by using them in the production of consumption goods). Next, the model iterates over these remaining machines, starting from the most cost-efficient one, until $mach_t^{entry}$ is reached (any remaining machines beyond $mach_t^{entry}$ are scrapped). For each machine m reached by this iteration process, the following operations take place:

1. The nominal value of m is multiplied by $1 - \frac{age_m}{\aleph^K}$, where age_m is the age of m and $\aleph^K$ is the uniform maximum age of machine tools. Machines on the second hand market for capital goods are hence re-valued according to their remaining lifespan.
2. If the exiting C-Firm c which owns m has paid off all outstanding loans using its remaining deposits, m is transferred to the household sector at no cost.
3. If c still has outstanding loans from its bank, the bank takes possession of m in order to subsequently sell m to the household sector. The outstanding loans of c are reduced by the updated nominal value of m.

Following this iteration, all capital goods taken into possession by banks are purchased by the household sector at their new marked-down value. Losses on loans taken by the banks are reduced by the amount they were able to recover through this process. If households are unable to (fully) finance the purchase of second-hand capital goods using accumulated deposits, the remaining cost is either covered by the government or booked as a loss by the banking sector, depending on the setting of an exogenous indicator variable.[16]

Once the second hand market for capital goods has closed, the initialisation of newly entering C-Firms begins. First, the number of machines which will be assigned to each newly entering C-Firm is determined. Initially, each entering firm is assigned one machine. Any remaining machines from the pool of second-hand capital goods are then assigned randomly, with each entering C-Firm receiving $\left\lfloor \left(mach_t^{entry} - N2_t^{exit}\right) \frac{\varepsilon_{c,t}^{entry}}{\sum_{i=1}^{N2} \varepsilon_{i,t}^{entry}} \right\rfloor$ where $\varepsilon_{i,t}^{entry}$ is given by a draw from a uniform distribution on the interval $[0, 1]$ for entering C-Firms and set to 0 for surviving ones. Any second-hand machines still remaining after this process are assigned one by one to randomly drawn entering firms. Having thus determined the number of machines which each entering C-Firm will receive, the actual machines assigned to each individual entering C-Firm are drawn randomly from the pool of second hand capital goods available and transferred to the balance sheets of entering firms.

Next, each entering C-Firm receives a transfer of bank deposits from the household sector. Similarly to the case of K-Firms, the transfer received by an entering C-Firm i is given by

$$T_{i,t} = \mathfrak{d}_{i,t}\overline{D}_{c,t} \tag{A.53}$$

where $\mathfrak{d}_{i,t}$ is a uniform random variable drawn from the interval $\left(\mathfrak{d}_C^1, \mathfrak{d}_C^2\right)$ and $\overline{D}_{c,t}$ is the average stock of deposits held by surviving C-Firms. As in the case of K-Firms, if households are unable to (fully) finance this transfer, it is either covered by the government or by losses taken by the banking sector depending on the setting of an exogenous indicator variable.[17] The bank serving the new C-Firm i is the same which was serving the exiting firm c which i replaces. In addition, each entering C-Firm is assigned a randomly drawn initial supplier of capital goods.

Based on the initial stock of capital goods received through the second hand market, an entering C-Firm i computes its unit cost. Its mark-up is initialised to an exogenous value μ^{entry}. It then sets its initial price using this unit cost and mark-up

$$\begin{aligned} \mu_{i,t} &= \mu^{entry} \\ p_{i,t} &= \left(1 + \mu_{i,t}\right) uc_{i,t} \end{aligned} \tag{A.54}$$

Recall that f_t^{entry} is the overall market share which will be assigned to entering C-Firms. To allocate this share among individual entering firms the model uses a simplified form of the quasi-replicator dynamics described in Section A.4. In particular, the competitiveness of an entering C-Firm i is given by

$$E_{i,t}^{entry} = -\frac{p_{i,t}}{\hat{p}_t^{entry}} \tag{A.55}$$

i.e., it is a function of its price relative to the average price across entering C-Firms, $\hat{p}_t^{entry}$. The share of f_t^{entry} which i will receive is computed as

$$share_{i,t}^{entry} = \frac{1}{N2_t^{exit}} \left((1 + \chi) \frac{E_{i,t}^{entry} - \overline{E}_t^{entry}}{\overline{E}_t^{entry}} \right) \tag{A.56}$$

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[16] See footnote 15

[17] See footnote 15.

which is then normalised. The initial consumption good market share of the entering C-Firm i is given by

$$\tilde{f}_{i,t} = f_t^{entry} share_{i,t}^{entry} \tag{A.57}$$

$\tilde{f}_{i,t}$ is then used to initialise the entering C-Firm's expected demand, ability to satisfy demand, sales and net revenue:

$$\begin{aligned} Dem_{i,t} &= min\left(\mathfrak{K}_{i,t}, \tilde{f}_{i,t} Dem_t^{\Sigma}\right) \\ Dem_{i,t}^{e} &= Dem_{i,t} \\ l_{i,t} &= 1 + \tilde{f}_{i,t} Dem_t^{\Sigma} - Dem_{i,t} \\ Sales_{i,t} &= p_{i,t} Dem_{i,t} \\ NR_{i,t} &= Sales_{i,t} - uc_{i,t} Dem_{i,t} \end{aligned} \tag{A.58}$$

where $\mathfrak{K}_{i,t}$ is the productive capacity of i based on the capital goods it received from the second-hand market and Dem_t^{Σ} is the sum of consumption demand received by all C-Firms in t. Once all entering C-Firms have been assigned a market share, the market shares of all C-Firms (i.e. both entering and surviving ones) are normalised to ensure that they sum to one.

A.6 Banks

The banking sector consists of NB individual banks. We use the index b, where $b = 1, ..., NB$ to denote individual banks. All banks are functionally identical, but banks differ in the number of individual firm customers that are assigned to them at the beginning of a simulation. Since each bank serves a different set of customers, both the size and composition of individual banks' balance sheets are heterogeneous.

A.6.1 Distribution of customers

At the beginning of a simulation, individual K-Firms and C-Firms are allocated to the banks as customers. The initial distribution of the number of C-Firm customers per bank is assumed to follow a truncated Pareto distribution with lower bound $\mathfrak{p}_1^C$, upper bound $\mathfrak{p}_2^C$, and shape parameter $\mathfrak{p}$. Similarly, the initial distribution of the number of K-Firm customers per bank is assumed to follow a truncated Pareto distribution with lower bound $\mathfrak{p}_1^K$, upper bound $\mathfrak{p}_2^K$, and shape parameter $\mathfrak{p}$. Banks' balance sheets are initialised using this distribution of firm customers. Aggregate stocks such as household deposits are initially distributed in line with the share of firm customers of each bank (i.e. each bank receives a share $\frac{clients_b}{N1+N2}$, where $clients_b$ is the number of K-Firm and C-Firm customers of b). The distribution of firm clients subsequently remains fixed, that is, the model currently features a static firm-bank network. If a firm exits the model, the new firm replacing it becomes a customer of the same bank. The only circumstance under which a bank's number of clients may change is if that bank fails. If the simulation setting is such that a failing bank is purchased by a surviving one (see below), the purchasing bank takes over all customers of the failing one.

A.6.2 Deposits

As indicated by Table 1, the main liability of the banking sector are deposits, which are held by firms, households and the energy sector. Changes in the deposits of a firm are reflected in a corresponding change in the deposits on the liability side of the balance sheet of that firm's bank. Changes in aggregate deposit stocks (households and energy sector) are distributed among individual banks using their previous market share in the respective deposit market. For instance, if a change occurs in the stock of deposits held by households (such as when households receive wage payments), the stock of household deposits on the balance sheet of bank b changes by $\Delta D_h \frac{D_{h,b}}{\sum_{b=1}^{NB} D_{h,b}}$.[18] If and when banks pay interest on deposits (including on household and energy sector deposits), the corresponding stock of deposits is augmented by the interest payment. The interest rate on deposits is identical across banks and is given by

$$r_{b,t}^{d} = \left(1 - \upsilon_B\right) r_{CB,t}^{d} \tag{A.59}$$

where $r_{CB,t}^{d}$ is the central bank deposit rate and υ_B is an homogeneous and exogenous markdown.

A.6.3 Loans

On the asset side, the main activity of banks consists in lending to the C-Firm sector. C-Firms' loan demand was described in Section A.4.4. On the supply side, every bank sets a maximum overall amount of loans it is prepared to hold on its balance sheet in t, which is given by a multiple of its net worth at the end of $t-1$. Depending on the parameter setting of the simulation, this multiple may be endogenous and heterogeneous across banks:

$$buffer_{b,t} = cm\left(1 + \mathfrak{v} fragility_{b,t}\right) \tag{A.60}$$

Where cm is an exogenous and homogeneous credit multiplier set by the regulator (i.e. the central bank in the case of this model), $\mathfrak{v}$ an exogenous parameter and $fragility_{b,t}$ a measure of the financial fragility of bank b (cf. Tasca and Battiston, 2011), defined as:

$$fragility_{b,t} = \frac{BD_{b,t-1} + LE_{b,t-1}}{NW_{b,t-1}} \tag{A.61}$$

Where $BD_{b,t-1}$ is the value of defaulted loans which b incurred in the previous period and $LE_{b,t-1}$ are losses from firm entry taken by banks (these are only relevant if households were unable to finance firm entry and if the simulation setting is such that in this case, firm entry is financed by banks; see Section A.5). Note that in the simulations presented below, we set $\mathfrak{v} = 0$, so that $buffer_{b,t} = cm$.

The maximum potential credit supply of each bank is given by (see also Section 3.5):

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[18] If the stock of aggregate household or energy sector deposits should become zero, the market shares are re-initialised using each bank's number of firm customers.

$$\mathfrak{C}^{s}_{b,t} = \frac{NW_{b,t}}{buffer_{b,t}} \tag{A.62}$$

Typically, the model would be calibrated such that $buffer_{b,t}$ is always much smaller than 1. Once banks determine the maximum amount of credit they are willing to extend, they decide on credit applicants. The first choice to be made regards the interest rate to be charged. For this purpose, each bank ranks all of its C-Firm customers in ascending order according to their debt service-to-revenue ratio. C-Firms with lower ratios are considered more credit-worthy than firms with higher ratios. The more credit-worthy a C-Firm is perceived to be, the lower the loan interest rate that its bank will charge:

$$r^{l}_{b,c,t} = r^{l}_{b,t}\left(1 + \left(rank_{c,t} - 1\right)\mathfrak{M}\right) \tag{A.63}$$

Where:

- $r^{l}_{b,c,t}$ ≡ the interest rate on loans charged by bank b, to firm c, at time t. Note that since the credit network is static, i.e. firms do not change banks except in the case in which failing banks are purchased by surviving ones, we usually omit the b subscript.

- $r^{l}_{b,t}$ ≡ bank b's baseline loan rate, defined as:

  $$r^{l}_{b,t} = \left(1 + \mu^{B}\right)r^{l}_{CB,t-1} \tag{A.64}$$

  Where $r^{l}_{CB,t-1}$ is the lending rate set by the central bank and μ^{B} is a constant and homogeneous mark-up, meaning that $r^{l}_{b,t}$ is identical across banks

- $\mathfrak{M}$ ≡ an exogenous penalizing factor

- $rank_{c,t}$ ≡ the quartile of the distribution of debt service-to-revenue ratios among b's customers to which c belongs

In addition to interest rate discrimination, bank b will also engage in credit rationing whenever the total demand for credit exceeds the maximum it is willing to extend, $\mathfrak{C}^{s}_{b,t}$. For this purpose, banks again use the debt service-to-revenue ranking to determine the order in which credit demand is satisfied. First, the most credit-worthy customer, c^{*}, is served. The amount of credit extended to c^{*} is the minimum between c^{*}'s credit demand and b's maximum credit supply, i.e. $max\left(\mathfrak{C}^{s}_{b,t}, \mathfrak{l}^{d}_{c^{*},t}\right)$. If $\mathfrak{C}^{s}_{b,t} \geq \mathfrak{l}^{d}_{c^{*},t}$, c^{*} is served in full, b reduces the remaining amount of credit it is willing to extend by the amount given to c^{*}, and moves to the next customer in the ranking. If $\mathfrak{C}^{s}_{b,t} < \mathfrak{l}^{d}_{c^{*},t}$, c^{*}'s credit demand is reduced by cutting investment expenditure and possibly planned production, until the credit required by c^{*} can be provided by b (see Section A.4.4). All subsequent customers of b are then denied credit. The procedure continues up to the point at which either all applicants have been given credit or b's credit supply is exhausted.

A.6.4 *Demand for government bonds*

Bank b's demand for government bonds is set as:

$$\Delta^{d}_{GB_{b,t}} = max\left(0, \mathfrak{G}\sum_{c\in\Phi_{b,c}} \mathfrak{l}_{c,t} - GB_{b,t-1}\right) \tag{A.65}$$

Where $\Delta^{d}_{GB_{b,t}}$ is the desired change in the stock of bonds held by b and $\sum_{c\in\Phi_{b,c}} \mathfrak{l}_{c,t}$ is the loan stock held by b, with $\Phi_{b,c}$ being the set of C-Firms who are customers of b. $GB_{b,t-1}$ is the stock of government bonds accumulated up to the previous period and $\mathfrak{G}$ is an exogenous parameter, which can be interpreted as the bank's desired government bond to loans ratio. The supply side of the government bond market is described in Section A.7.

A.6.5 *Profits and dividends*

Once all bank decisions and market interactions have taken place, gross profits, on which taxes are paid, can be computed: interest payments on loans, government bonds, and reserves enter the profit calculation with a positive sign; interest payments on deposits and central bank advances, losses stemming from bad debt (net of recovered collateral), and possible losses from the financing of firm entry enter the profit calculation with a negative sign:

$$\begin{aligned}\Pi_{b,t} = &\sum_{c\in\Phi_{b,c}} r_{b,c,t}\mathfrak{l}_{c,t} + r_{GB,t-1}GB_{b,t-1}\\ &+ r^{d}_{CB,t-1}R_{b,t-1} - r^{d}_{b,t-1}D_{b,t-1}\\ &- r^{l}_{CB,t-1}A_{b,t-1} - (BD_{b,t} - CR_{b,t}) - LE_{b,t}\end{aligned} \tag{A.66}$$

Where:

- $\Phi_{b,c}$ ≡ subset of consumption firm clients of b; $\mathfrak{r}_{b,c,t}$ ≡ loan interest rate charged by bank b to firm c; $\mathfrak{l}_{c,t}$ ≡ outstanding loans to c.[19]

- $r_{GB,t-1}$ ≡ interest rate on government bonds; $GB_{b,t-1}$ ≡ public debt held by bank b.

- $r^{d}_{CB,t-1}$ ≡ central bank interest rate on reserves; $R_{b,t-1}$ ≡ stock of reserves held by bank b.

- $r^{d}_{b,t-1}$ ≡ deposit rate offered by bank b; $D_{b,t-1}$ ≡ deposits on the liability side of b's balance sheet.

- $r^{l}_{CB,t-1}$ ≡ central bank lending rate; $A_{b,t-1}$ ≡ stock of central bank advances to b.

- $BD_{b,t}$ ≡ value of defaulted debt; $CR_{b,t}$ ≡ recovered collateral from failed firms (see Section A.5).

- $LE_{b,t}$ ≡ losses from firm entry taken by b (see Section A.5).

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[19] Note that for exiting C-Firms $\mathfrak{l}_{c,t}$ has already been set to 0.

If profits are positive, banks pay a fraction τ^B of them in taxes. In addition, if profits are positive, dividends are paid:

$$Div_{b,t} = \mathbf{1}^{[\Pi_{b,t}>0]} \delta^B \Pi_{b,t} \tag{A.67}$$

Where $\mathbf{1}^{[\Pi_{b,t}>0]}$ is an indicator variable taking the value 1 if $\Pi_{b,t} > 0$ and 0 otherwise, and δ^B is the exogenous and homogeneous dividend rate.

A.6.6 Net worth and bankruptcy

Banks' net worth is updated in each period according to:

$$NW_{b,t} = NW_{b,t-1} + \Pi_{b,t} - Div_{b,t} - Tax_{b,t} \tag{A.68}$$

Where $Tax_{b,t}$ are taxes paid by bank b (see also Section A.7).

A bank fails if $NW_{b,t} < 0$. At present, the model includes two ways to deal with bank failures between which the user can switch through the setting of an indicator variable. Under the first option, failing banks are always bailed out by the government. Under the second option, a failed bank will be purchased by the surviving bank with the largest net worth and the government only provides a bailout if that latter bank is unable to purchase the failing one.

1. Under the first option, the government determines a specific bailout for a failed bank b which re-sets its net worth to:

$$NW_{b,t} = Bail_{b,t} \tag{A.69}$$

$Bail_{b,t}$, in turn, is determined as:

$$Bail_{b,t} = max\left(-NW_{b,t} + cm \sum_{c\in\Phi_{b,c}} \mathfrak{l}_{c,t}, \right. \\ \left. - NW_{b,t} + mb_{b,t} NW^*_{b,t}\right) \tag{A.70}$$

Where $NW_{b,t}$ is to be understood as the (negative) net worth of b *prior* to being bailed out, cm is the exogenous credit multiplier (see Equation (A.60)) and $\sum_{c\in\Phi_{b,c}} \mathfrak{l}_{c,t}$ is b's existing stock of loans. $mb_{b,t}$ is an individual bailout multiplier, given by a random draw from a uniform distribution on the support $\left(\mathfrak{d}^1_B, \mathfrak{d}^2_B\right)$. Finally, $NW^*_{b,t}$ is calculated as follows: Let υ denote the bank among the set of surviving banks which has the highest net worth *per customer* in t (i.e. the bank for which $\frac{NW_{\upsilon,t}}{clients_\upsilon}$ takes the highest value). $NW^*_{b,t}$ is determined by taking this maximum net worth per customer and multiplying it by the number of firm customers served by the failing bank b.[20]

2. Under the second option, the surviving bank with the highest net worth (total, not per customer), υ, purchases the failed bank, b: if $NW_{\upsilon,t} > 0$, i.e. υ is not itself also failed, and $NW_{\upsilon,t} > |NW_{b,t}|$, υ is able to purchase b. In this case, all assets and liabilities of b are transferred to υ, as are all of b's customers. b subsequently becomes inactive for the rest of the simulation. $NW_{\upsilon,t}$ is reduced by the amount $|NW_{b,t}|$. If, however, υ is unable to purchase b, b is bailed out by the government according to the mechanism described under option one.

A.7 Government

Depending on the scenario being simulated and the calibration used, the government may collect taxes from households (on wage income), firms (on profits and emissions), banks (on profits), as well as the energy sector (on emissions).

Taxes on wage income and profits are assumed to be fixed at a flat rate, while taxes on emissions may change over the course of a simulation. Accordingly, government tax income from households is given by

$$Tax^H_t = \tau^H W_t \tag{A.71}$$

where τ^H is the tax rate and W_t is nominal wage income in t. Taxes on C-Firms are given by

$$Tax^C_t = \sum_{c=1}^{N2} \tau^C \mathbf{1^\Pi_c} \Pi_{c,t} + \tau^{Em,C}_t Em_{c,t} \tag{A.72}$$

summing across all $N2$ C-Firms. $\Pi_{c,t}$ is the profit of C-Firm c in period t. $\mathbf{1^\Pi_c}$ is an indicator function taking the value 1 if $\Pi_{c,t} > 0$ and 0 otherwise. $\tau^{Em,C}_t$ is the current tax rate on emissions from C-Firms and $Em_{c,t}$ are the emissions produced by C-Firm c in t. Similarly, taxes paid by K-Firms are given by

$$Tax^K_t = \sum_{k=1}^{N1} \tau^K \mathbf{1^\Pi_k} \Pi_{k,t} + \tau^{Em,K}_t Em_{k,t} \tag{A.73}$$

Finally, banks pay taxes on positive profits, while the energy sector is assumed to pay taxes only on emissions.

$$Tax^B_t = \sum_{b=1}^{NB} \tau^B \mathbf{1^\Pi_b} \Pi_{b,t} \tag{A.74}$$

$$Tax^E_t = \tau^{Em,E}_t Em_{e,t} \tag{A.75}$$

Total tax revenue is then given by the sum of the tax revenue received from the different sectors:

$$Tax_t = Tax^H_t + Tax^C_t + Tax^K_t + Tax^B_t + Tax^E_t \tag{A.76}$$

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[20] If all banks fail in t, $NW^*_{b,t}$ is replaced with $NW_{b,t-1}$ in Equation (A.70). Note that $NW_{b,t-1}$ is always positive since it is calculated after bailouts occur in $t-1$. While this ensures that the model can continue to run even if all banks fail in a single period, the model should ideally be calibrated such that this does not happen (and indeed it does not happen in the runs shown below).

In addition, any profits made by the central bank, $\Pi_{cb,t}$ (described below) are paid to the government as a transfer $T_{cb,t}$. Importantly, this also applies if the central bank makes a loss, i.e. central bank losses are compensated by the government.

At present, the model includes the possibility to set separate emission tax rates for the energy and firm sectors. Regarding the trajectory of the emission tax over time, a range of scenarios can be activated through the setting of an indicator variable.[21] In all cases, the emission tax is activated in the first period in which the climate module is called and is subsequently adjusted at the same frequency at which the climate module is called.

- A constant real emission tax, whereby the nominal tax rate increases with consumer price inflation: $\tau_t^{Em} = \tau_0^{Em}\frac{cpi_{t-1}}{cpi_1}$
- An emission tax rate which increases linearly with time: $\tau_t^{Em} = \tau_0^{Em}+\frac{t-t_0^{clim}}{g_{\tau,Em}^1}$, where t_0^{clim} is the first period in which the climate module is called and $g_{\tau,Em}^1$ is an exogenous parameter.
- An emission tax rate which increases exponentially with time and is corrected for inflation: $\tau_t^{Em} = \tau_0^{Em}\frac{cpi_{t-1}}{cpi_1}\left(1+g_{\tau,Em}^2\right)^{t-t_0^{clim}}$, where $g_{\tau,Em}^2$ is an exogenous parameter.
- An emission tax rate which increases with nominal GDP: $\tau_t^{Em} = \tau_0^{Em}\frac{GDP_{t-1}^n}{GDP_1^n}$.

Note that the model as presented here does not include a dedicated 'recycling' mechanism for carbon tax revenue (though such a policy can easily be implemented). Carbon tax revenue is hence not redistributed to households or firms but instead enters the public sector budget in the same way as any other tax revenue.

The main expenditure item of the government are unemployment benefits paid to households. As explained above, in any given period t, households will supply any amount of labour demanded at the current wage rate up to a maximum LS_t, which represents the current labour force. If total labour demand from C-Firms, K-Firms and the Energy sector exceeds LS_t, the labour demand for production purposes of each K-Firm and C-Firm is scaled back by a uniform percentage. Production is then adjusted downward accordingly. With L_t being the amount of labour actually employed in t, unemployment is given by $LS_t - L_t$. The unemployment benefit is given by a fraction ζ of the current nominal wage, making total unemployment benefit payments

$$UB_t = \zeta w_t\left(LS_t - L_t\right) \tag{A.77}$$

In addition, the government may have expenditures to finance the entry of new firms and for the bailout of failing banks. As explained in Section A.5, depending on the specific model setting being simulated, the government may finance the entry of new K-Firms and/or C-Firms if the deposits of households are insufficient to purchase second-hand capital and/or to provide newly entering firms with an initial stock of liquidity. If called upon, the government will in this situation provide a transfer T_g to newly entering firms to cover any financing which cannot be provided by transfers of household deposits.

Similarly, depending on the specific model setting being simulated, the government may provide bailouts $Bail$ to failing banks according to the routine outlined in Section A.6.

Finally, the government makes interest payments on the stock of outstanding government bonds, GB_{t-1}, given by

$$iGB_t = r_{GB,t-1}GB_{t-1} \tag{A.78}$$

where $r_{GB,t-1}$ is the nominal interest rate on government bonds.

The overall budget balance of the government is hence given by

$$Sav_{g,t} = Tax_t + T_{cb,t} - UB_t - iGB_t - T_{g,t} - Bail_t \tag{A.79}$$

In addition to expenditures, the government also needs to finance bond repayments; in particular, it is assumed that in every period, the government must repay a share ξ_{GB} of outstanding government bonds (but can repay more if it has a sufficiently large surplus). The 'public sector borrowing requirement' hence becomes:

$$\begin{aligned} PSBR_t = & UB_t + iGB_t + T_{g,t} + Bail_t + \xi_{GB}GB_{t-1} \\ & - Tax_t - T_{cb,t} \end{aligned} \tag{A.80}$$

If $PSBR_t > 0$, the government issues new bonds. New government bonds are in the first instance offered to banks, which demand bonds according to the rule set out in Section A.6. Any new bonds which are not acquired by banks are assumed to be purchased by the central bank. The current interest rate on bonds is assumed to apply to all outstanding and newly issued bonds and is given by a markdown on the central bank lending rate (the determination of which is described below):

$$r_{GB,t} = \left(1 - \upsilon_{GB}\right)r_{CB,t}^l \tag{A.81}$$

If $PSBR_t < 0$, the government is able to repay bonds beyond the required amount $\xi_{GB}GB_{t-1}$. In this case, the government first repays bonds held by commercial banks (distributing the total amount to be repaid among individual banks according to each bank's share of total bonds held by the banking sector). If all bonds held by banks have been

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[21] Additional scenarios can easily be implemented, as was done for the example shown in the main text.

repaid, the government repays bonds held by the central bank. If all outstanding bonds have been repaid, the government accumulates any remaining surplus as a deposit with the central bank, which is remunerated at the central bank deposit rate (the determination of which is described below). Note that in this latter case, interest payments on government bonds, iGB_t, will become negative and hence represent a revenue for the government.

A.8 Central bank

The central bank in the model is tasked with maintaining the payments system and setting the base interest rate. In setting its lending rate, the central bank follows a Taylor-type rule given by

$$\begin{aligned} r^l_{CB,t} = max\Big(& \underline{r}, \iota_1 r^l_{CB,t-1} + \left(1 - \iota_1\right) \\ & \times \left(r + \iota_2\left(\pi^a_t - \pi^*\right) + \iota_3(U^* - U_t)\right)\Big) \end{aligned} \tag{A.82}$$

where ι_1 is an interest rate smoothing parameter, r is a fixed intercept, π^a_t is the current year-on-year inflation rate with π^* being the year-on-year inflation target, U_t being the current unemployment rate and U^* the central bank's target unemployment rate. $\underline{r}$ is a fixed lower bound close to 0. If the model is calibrated to be simulated at quarterly frequency (as is the case in the calibration shown below), this annual lending rate is subsequently converted to a quarterly one. The central bank deposit rate is given by marking down the lending rate:

$$r^d_{CB,t} = \left(1 - \upsilon_{CB}\right) r^l_{CB,t} \tag{A.83}$$

As indicated in the previous sections, the other interest rates in the model are closely tied to these central bank rates, being determined by applying exogenous (e.g. government bonds, bank deposits) or endogenous (bank lending rates to individual customers) mark-ups or mark-downs to the central bank lending or deposit rate.

The central bank maintains the payments system in the model by supplying reserves required to settle interbank transactions. For simplicity, the model currently does not include an interbank market. Instead, all transactions implying flows of reserves from one bank to another are recorded over a period. At the end of every period, a net in- or outflow of reserves is calculated for each individual bank. If a bank has experienced a net outflow of reserves, it first makes use of any existing reserve balances to cover this outflow. If the stock of reserves it currently holds is insufficient, the central bank provides advances on demand at the current central bank lending rate. The bank then uses these reserves borrowed from the central bank to cover its net outflow. Conversely, the reserve balance of every bank experiencing a net inflow of reserves is augmented by the size of that net position. If a bank experiencing a net inflow of reserves has outstanding advances from the central bank, it uses the inflow of reserves to repay as much of these advances as possible and accumulates any remaining reserves on its balance sheet.

The present version of the model also includes a fossil fuel sector, described below. In order to enable the stylised modeling of an 'external' fossil fuel supplier, the fossil fuel sector is not directly linked to the commercial banking system but instead holds a reserve account with the central bank. When the energy sector makes a payment to the fossil fuel sector, this hence implies a net outflow of reserves for the commercial banking system as a whole, which is accumulated in the reserve account of the fossil fuel sector. In contrast to commercial banks, the fossil fuel sector is not able to borrow reserves from the central bank and its holdings of reserves are not remunerated.

A.9 Energy Sector

The energy sector consists of a single representative agent which sells energy as an input to K-Firms and C-Firms. Energy is produced using both 'green' and 'brown' technologies, possibly with multiple plants of each technology and of different vintages operational at any given time.

A.9.1 Capacity expansion

The total amount of energy produced is determined by the model's overall demand for energy, which at present comes only from C-Firms and K-Firms. Based on the amount of consumption goods and capital goods produced in t and the energy efficiency of the capital vintages and production techniques utilised to do so, a total demand for energy, En^d_t is calculated.

The existing productive capacity of the energy sector is given by $\mathfrak{K}_{e,t-1}$, expressed in units of energy producible. This productive capacity can in turn be divided into a capacity for producing 'brown/dirty' $\left(\mathfrak{K}^{de}_{t-1}\right)$ and 'green/clean' $\left(\mathfrak{K}^{ge}_{t-1}\right)$ energy. At present, the modelling of green and brown energy technologies is strongly stylised and simplified; in particular, green and brown energy plants differ in the following respects:

- Green energy production does not give rise to greenhouse gas emissions, while the emission intensity of brown energy production is positive.
- The production of energy from existing green energy plants is assumed to be costless, whereas the production of energy from brown energy plants requires a costly fossil fuel input.
- The expansion of productive capacity is assumed to be costless for brown energy plants, while additions to the productive capacity of green energy have a positive cost.

if $En^d_t > \left(\mathfrak{K}^{de}_{t-1} + \mathfrak{K}^{ge}_{t-1}\right)$, the energy sector must expand its productive capacity to meet the model's current demand for energy. In order to avoid situations in which current production of output is constrained by the availability of energy, it is assumed that the energy sector can expand its capacity instantaneously. Depending on the setting of an indicator variable, the expansion of productive capacity in the energy sector may take four forms:

1. Considering only the most advanced vintages of green and brown energy production technologies, the energy sector compares the per-unit production cost of brown energy to the per-unit, per-period amortised investment cost of green energy production capacity, to determine which technology is more cost-effective. All investment then takes place in the more cost-effective technology.
2. As above, but an upper bound given by $\varsigma^e \mathfrak{K}^{ge}_{t-1} + scrap^{ge}_{t-1}$, where $scrap^{ge}_{t-1}$ is the amount of green energy capacity scrapped in the previous period, is placed on the per-period expansion of green energy capacity. Hence, even if the green technology is more cost-effective, the energy sector can only replace scrapped green capacity and expand it at most by $\varsigma^e \mathfrak{K}^{ge}_{t-1}$, while any remaining investment goes into the brown technology.
3. Same as 2, but the share of green capacity in total productive capacity cannot fall below a minimum given by the initial share.
4. The shares of green and brown capacity in total capacity are exogenously given and constant, and expansion investment in both technologies is made according to these shares.

For every vintage κ^{de} of brown energy technologies, the per-unit production cost of energy is given by:

$$c_{\kappa^{de},t} = \frac{p_{f,t-1}}{TE_{\kappa^{de}}} + \tau^{Em,E}_t EF_{\kappa^{de}} \tag{A.84}$$

where $p_{f,t-1}$ is the price of the fossil fuel input to be paid in the current period, $TE_{\kappa^{de}}$ denotes the thermal efficiency of vintage κ^{de}, $\tau^{Em,E}_t$ is the current value of the tax on emissions applied to the energy sector, and $EF_{\kappa^{de}}$ is the emission intensity of vintage κ^{de}.

As indicated above, the production of green energy is assumed to be costless. However, the expansion of green energy production capacity (which is assumed costless for brown energy) carries a positive cost. For every vintage κ^{ge}, the expansion/investment cost per unit of productive capacity is given by $c_{\kappa^{ge},t}$.[22]

In cases 1), 2) and 3) described above, the energy sector determines the minimum $c_{\kappa^{ge},t}$ and $c_{\kappa^{de},t}$ among all vintages κ^{de} and κ^{ge}. It then checks the condition

$$c^{min}_{\kappa^{de},t} < \frac{c^{min}_{\kappa^{ge},t}}{b^e} \tag{A.85}$$

where b^e is a payback period parameter. If this condition holds, all expansion investment takes the form of brown energy capacity in settings 1) and 2) described above. In case 3), the energy sector will invest as much as possible in brown capacity given the constraint that the share of green capacity cannot fall below its initial value. If the condition does not hold, all investment will be in green energy in setting 1). In settings 2) and 3), the energy sector will invest as much as possible in green energy as described above, and carry out any necessary additional capacity expansion in brown energy.

While expansion of brown energy capacity is costless, green capacity expansion incurs the per-unit cost $c^{min}_{\kappa^{ge},t}$, making the total cost of green energy investment $c^{min}_{\kappa^{ge},t} EI^{ge}_t$, where EI^{ge}_t is the additional capacity for green energy production installed in t. It is assumed that this cost is staggered over the payback period b^e of the investment. This means that if the energy sector invests in green energy capacity in t, it will incur a cost $IC_{e,t} = \frac{c^{min}_{\kappa^{ge},t} EI^{ge}_t}{b^e}$ in t as well as in the following $b^e - 1$ periods. This cost is transformed into an associated demand for labour by dividing it by the current nominal wage rate, $\frac{IC^e_t}{w_t}$.

For accounting purposes, the productive capacity of the energy sector is valued at installation cost. This implies that the nominal value of brown capacity is zero, while the nominal value of a unit of existing green capacity is given by the construction cost incurred. All energy production plants are assumed to have a fixed lifetime of $\aleph^E$ periods after which they are written off and scrapped.

A.9.2 Production and sales

Having expanded capacity if necessary, the energy sector satisfies the model's demand for energy by activating plants in the order of their cost-effectiveness (Sensfuß et al., 2008). Since the production cost for green energy is assumed to be zero, green plants are always activated first. If the existing green capacity is insufficient to satisfy all energy demand, brown plants are activated starting from the one with the lowest unit cost of production.

The uniform price of energy to be paid by all firms is then given by

$$p_{e,t} = \mu_{e,t} + mc_{e,t} \tag{A.86}$$

$\mu_{e,t}$ is a mark-up, while $mc_{e,t}$ denotes the marginal cost of energy production, i.e. the unit cost of production of the last (and hence least cost-effective) plant activated to satisfy energy demand in t. If no brown energy is produced in t,

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[22] Depending on the setting being simulated, the expansion cost for green energy of a given vintage may be constant or grow over time. In the simulation setting shown below, both the carbon tax and the fossil fuel price grow over time (the former with nominal GDP and the latter with the nominal wage, see below). To keep the cost of green energy expansion comparable to the production cost of brown energy, it is hence assumed that the expansion cost for each green energy technology vintage κ^{ge} grows with the nominal wage. To assume otherwise would imply that the expansion cost of green energy capacity could only decline over time through R&D as described below but never increase. Meanwhile the production cost of brown energy would grow with the carbon tax and fossil fuel price and only decline if the pace of technological progress in brown energy technology described below exceeded the the pace of the increase in the carbon tax and fossil fuel price in some period(s).

$mc_{e,t} = 0$. The mark-up $\mu_{e,t}$ is assumed to change over time according to:

$$\mu_{e,t} = \mu_{e,t-1} * \overline{\Delta}_{w,t} \tag{A.87}$$

where $\overline{\Delta_{w,t}}$ is a weighted average of current and past changes in the nominal wage rate:

$$\overline{\Delta}_{w,t} = \eta \overline{\Delta}_{w,t-1} + (1-\eta)\frac{w_t}{w_{t-1}} \tag{A.88}$$

This assumption is made to ensure that in the absence of shocks, the price of energy grows roughly in line with the nominal size of the overall economy. This is important in particular since, as discussed below, the baseline calibration of the model is designed to lead to a roughly constant real energy use.[23]

If brown energy is produced in t, the energy sector also calculates the fossil fuel input required by the activated vintages, as well as the emissions resulting from production. Energy sector emissions, $Em_{e,t}$ are calculated taking into account the heterogeneous emission intensities of the different vintages of brown energy plants activated in t. Section A.12.5 explains how these emissions and those arising from K-Firms and C-Firms are aggregated and transformed into an input for the climate module. Having received revenue from the sale of energy, the energy sector makes a payment for the fuel inputs to the fossil fuel sector (described below) and pays the emission tax.

A.9.3 R&D

It is assumed that the energy sector wishes to devote a share $\mathfrak{o}^e$ of its revenue to R&D activities. R&D expenditure is given by

$$RD_{e,t} = \mathfrak{o}^e p_{e,t} En_t^d \tag{A.89}$$

if $\mathfrak{o}^e p_{e,t} En_t^d < p_{e,t} En_t^d - IC_{e,t} - PC_{e,t}$ and

$$RD_{e,t} = \mathfrak{o}^e p_{e,t}\left(En_t^d - IC_{e,t} - PC_{e,t}\right) \tag{A.90}$$

otherwise, where $IC_{e,t}$ is the cost paid in t for capacity expansion as described above and $PC_{e,t}$ denotes the total cost of energy production, including costs for fossil fuel inputs and the emissions tax.

Depending on the setting of an indicator variable, the division of R&D expenditure between green and brown technology, $RD_{ge,t}$ and $RD_{de,t}$ is either:

- Exogenous and given by $\mathfrak{x}^E$
- Endogenous, with the share devoted to brown technologies corresponding to the share of brown energy capacity in total capacity, $\frac{\mathfrak{K}_t^{de}}{\mathfrak{K}_{e,t}}$
- Endogenous, with the share devoted to brown technologies corresponding to the share of brown energy in total energy produced in t

As in the case of K-Firms, R&D is carried out using labour as an input:

$$\begin{aligned} L_{de,t}^{rd} &= \frac{RD_{de,t}}{w_t} \\ L_{ge,t}^{rd} &= \frac{RD_{ge,t}}{w_t} \end{aligned} \tag{A.91}$$

Since the energy sector only contains a single representative agent, R&D activities are fully devoted to innovation (recall that, by contrast, K-Firms may also imitate the technology of a competitor). The probability of an innovation taking place in green/brown energy technology is a function of the amount of labour devoted to R&D to each technology:

$$\begin{aligned} P(Innovation\ Brown) &= 1 - exp\left(-\mathfrak{b}_1^E L_{de,t}^{rd}\right) \\ P(Innovation\ Green) &= 1 - exp\left(-\mathfrak{b}_1^E L_{ge,t}^{rd}\right) \end{aligned} \tag{A.92}$$

The determination of the characteristics of the innovated technologies then proceeds in the same fashion as in the case of K-Firms. If innovation in brown technology takes place, a random draw is made from a beta distribution with shape parameters $\mathfrak{b}_2^E$ and $\mathfrak{b}_3^E$ over the support $\left(\mathfrak{b}_4^E, \mathfrak{b}_5^E\right)$ where, depending on the calibration, $\mathfrak{b}_4^E$ may be smaller than 0 such that innovation can also result in an inferior technology. The random number $\mathfrak{I}_{de,t}$ thus drawn is used to determine the thermal efficiency and emission intensity of the new technology based on the characteristics of the current most efficient technology:

$$\begin{aligned} TE_{inn} &= TE_{\kappa^{de}}\left(1 + \mathfrak{I}_{de,t}\right) \\ EF_{inn} &= EF_{\kappa^{de}}\left(1 - \mathfrak{I}_{de,t}\right) \end{aligned} \tag{A.93}$$

where $TE_{\kappa^{de}}$ is the thermal efficiency of the most recent vintage of brown energy technology, and $EF_{\kappa^{de}}$ is the emission intensity of that vintage. The innovated brown technology is adopted if the unit cost of producing energy using this technology (taking into account both fuel cost and tax payments) is lower than that of the current vintage κ^{de}. Otherwise, the current vintage remains unchanged.

If innovation in green technology takes place, a draw is made from the same beta distribution described above. The new random number $\mathfrak{I}_{ge,t}$ thus drawn is used to determine the per-unit expansion investment cost of the innovated green technology on the basis of the characteristics of the current vintage, κ^{ge}

$$c_{ge}^{inn} = c_{\kappa^{ge}}\left(1 - \mathfrak{I}_{ge,t}\right) \tag{A.94}$$

As in the case of brown technology, the innovation is only adopted if the resulting expansion investment cost is lower than that of the current vintage.

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[23] In calibrations with growing or declining energy use, this assumption may of course have to be modified.

A.9.4 Profit and dividends

Having determined its labour demand for R&D activities as well as expansion investment (as described above), the energy sector hires workers and pays the corresponding wages to the household sector. Similarly to the case of labour employed for R&D purposes in the K-Firm sector, we assume that the energy sector is never rationed on the labour market when seeking to hire workers.

The energy sector then calculates its current profit as:

$$\begin{aligned}\Pi^{gross}_{e,t} =& Sales_{e,t} + iD_{e,t} - W_{e,t} + \left(K_{e,t} - K_{e,t-1}\right) \\ & - CTAX_{e,t} - FF_t\end{aligned} \tag{A.95}$$

where:

$$\begin{aligned} Sales_{e,t} &= p_{e,t} En_t \\ iD_{e,t} &= r^d_{t-1} Dep_{e,t-1} \\ W_{e,t} &= w_t L_{e,t} \\ CTAX_{e,t} &= \tau^{Em,E}_t Em_{e,t} \\ FF_t &= p_{f,t} ff^d_t \end{aligned} \tag{A.96}$$

- $Sales_{e,t}$ ≡ nominal sales; $p_{e,t}$ ≡ price; En_t ≡ quantity of energy sold;
- $iD_{e,t}$ ≡ interest payments on deposit; r^d_{t-1} ≡ interest rate on deposits; $Dep_{e,t-1}$ ≡ stock of deposits;
- $W_{e,t}$ ≡ wage bill; w_t ≡ nominal wage; $L_{e,t}$ ≡ number of employed workers (for R&D and expansion investment);
- $K_{e,t} - K_{e,t-1}$ ≡ change in the nominal value of the energy sector's capital stock;
- $CTAX_{e,t}$ ≡ emission tax paid; $\tau^{Em,E}$ ≡ tax rate per unit of emission charged to the energy sector; $Em_{e,t}$ ≡ emissions;
- FF_t ≡ cost of fossil fuel input; $p_{f,t}$ ≡ price of fossil fuel; ff^d_t ≡ quantity of fossil fuel demanded;

If $\Pi^{gross}_{e,t}$ is positive, the energy sector pays a constant share δ^E of that profit as dividend to the household sector. As in the case of firms, all retained earnings of the energy sector are held in the form of bank deposits, $D_{e,t}$. In order to distribute (changes in) energy sector deposits among individual banks, the same rule as that applied for households is used (see Section A.6).

A.10 Fossil fuel sector

The current version of the model features a stylised aggregated fossil fuel sector which operates separately from the energy sector and sells fossil fuels as inputs to the latter.

The fossil fuel sector has purposely been modelled in such a way that it can be separated to a large degree from the rest of the model. This was done to enable the mimicking of an external fossil fuel supplier when the model is simulated under a calibration intended to depict a particular geographical region (e.g., the European Union). In particular, as can be gathered from Table 1, the fossil fuel sector is not linked to the domestic commercial banking system but instead holds a non-remunerated reserve account at the central bank, which it uses to receive payments from the energy sector and to make payments to households. The sector is able to 'produce' fossil fuels without the use of any inputs and at zero cost.

The price of fossil fuels is re-set in every period and is given by:

$$p_{f,t} = p_{f,t-1} * \overline{\Delta}_{w,t} \tag{A.97}$$

where $\overline{\Delta_{w,t}}$ is the same weighted average of past changes in the nominal wage which is also used to update the mark-up of the energy sector. This specification is chosen to assure that, unless otherwise specified, the fossil fuel price does not exhibit a secular trend.

The energy sector's demand for fossil fuel, ff^d_t, is determined by the model's current demand for energy and the fossil fuel input necessary to produce this energy given the characteristics of currently existing brown energy plants in the energy sector. The revenue of the fossil fuel sector is hence given by

$$FF_t = p_{f,t} ff^d_t \tag{A.98}$$

Given that the fossil fuel sector has zero cost, its revenue coincides with its profit. In order to ensure that the sector does not continuously drain financial wealth from the rest of the model, we assume that it pays a share of its accumulated wealth to the household sector in each period as a 'dividend':

$$Div_{f,t} = \delta^F \left(R_{f,t-1} + FF_t\right) \tag{A.99}$$

By setting the parameter δ^F to a small value, this setup can be used as a stylised depiction of an external fossil fuel producer which receives revenue from the 'domestic' economy but makes only very small payments to the rest of the system.

A.11 Climate

At present, the model allows the user to choose between two climate modules using an indicator variable. Both climate modules run at annual frequency, such that if, for instance, the economic model is simulated at quarterly frequency, the active climate module is called every four periods. t^{clim}_0, the first period in which the climate module is called, is currently set to be equal to the first post-transient period of the economic component of the model. Depending on whether the calibration being simulated is intended to represent the global economy or a particular geographical region, emissions may either be partly exogenous or fully endogenous. Exogenous emissions are assumed to grow at a fixed rate g_{Em} starting from some initial value (implying that when this initial value is set to 0, exogenous emissions will be 0 throughout). Endogenous emissions are the sum of current emissions from the capital and consumption good sectors and the energy sector. Section A.12.5 outlines how emissions are transformed prior to being passed as an input to either climate module.

A.11.1 Cumulative emissions climate module

The simpler of the two climate modules is based on cumulative emissions (cf. Matthews et al., 2009, 2012), deriving a simulated global temperature anomaly as a function of cumulative emissions. Every time the climate module is called, all exogenous and endogenous emissions which have taken place since the last call to the module are transformed as described in Section A.12.5 and used to update cumulative emissions.

The global temperature anomaly in t is then given by

$$Temp_t = \Upsilon_1 + \Upsilon_2 \mathscr{E}^{\Sigma}_t \tag{A.100}$$

where $\mathscr{E}^{\Sigma}_t$ are cumulative emissions up to period t (including, of course, periods in which the climate module is not called). The resulting temperature anomaly $Temp_t$ then remains constant until the next period in which the climate module is called.

A.11.2 Carbon cycle climate module

The somewhat more complex climate module similarly receives transformed exogenous and endogenous emissions as an input. It uses them to update the atmospheric carbon content and calculate a global temperature anomaly. In particular, it represents a simplified version of the C-Roads model proposed by Sterman et al. (2013).

It depicts a carbon cycle, in which the atmospheric carbon content (measured in GtC) depends on anthropogenic emissions (i.e. exogenous and endogenous model emissions fed into the climate module after transformation as described in Section A.12.5) as well as on carbon exchange between the atmosphere, the oceans and biomass. In the following, we describe which steps occur in a period t in which the climate module is called. In all periods in which it is not called, endogenous state variables (such as the atmospheric carbon stock) are held constant.

Net primary production, i.e. the amount of carbon taken up by biomass, is given by

$$\begin{aligned} NPP_t = & NPP_0 \left(1 + \text{Ж} ln\left(\frac{\mathscr{C}^{at}_{t-1}}{\mathscr{C}^{at}_0}\right)\right) \\ & \times \left(1 + \text{Б} Temp_{t-1}\right) \end{aligned} \tag{A.101}$$

where NPP_0 is an initial value, Ж is an exogenous parameter denoting the carbon fertilisation effect on net primary production, $\mathscr{C}^{at}_{t-1}$ is the atmospheric carbon stock in GtC in $t-1$ (with $\mathscr{C}^{at}_0$ being the initial value), $\text{Б} < 0$ is an exogenous parameter describing the heat stress effect of global warming on net primary production, and $Temp_{t-1}$ is the global temperature anomaly in $t-1$. Net primary production hence increases with atmospheric carbon, but decreases with the temperature anomaly since $\text{Б} < 0$.

Next, a preliminary value for the atmospheric carbon stock in t, excluding ocean uptake, is computed as

$$\widetilde{\mathscr{C}^{at}_t} = \mathscr{C}^{at}_{t-1} + \widehat{\mathscr{E}_t} + \frac{\mathscr{C}^{hum}_{t-1}}{\text{Я}_1} + \frac{\mathscr{C}^{bio}_{t-1}}{\text{Я}_2}\left(1 - \text{Я}_3\right) \tag{A.102}$$

$\widehat{\mathscr{E}_t}$ is the sum of all (transformed) exogenous and endogenous emissions which have occurred since the last period in which the climate module has been called. $\mathscr{C}^{hum}_{t-1}$ is the stock of carbon stored in humus, and $\mathscr{C}^{bio}_{t-1}$ is the stock of carbon stored in biomass. Я_1 denotes the decaying time of humus (expressed in climate module time-units, i.e. years in the calibration shown in this paper), while Я_2 denotes the decaying time of biomass (expressed in climate module time-units), and Я_3 the fraction of decaying biomass carbon which ends up in humus.

The stocks of carbon stored in biomass and humus are updated following

$$\begin{aligned} \mathscr{C}^{bio}_t &= \mathscr{C}^{bio}_{t-1} + NPP_t - \frac{\mathscr{C}^{bio}_{t-1}}{\text{Я}_2} \\ \mathscr{C}^{hum}_t &= \mathscr{C}^{hum}_{t-1} + \frac{\mathscr{C}^{bio}_{t-1}}{\text{Я}_2}\text{Я}_3 - \frac{\mathscr{C}^{hum}_{t-1}}{\text{Я}_1} \end{aligned} \tag{A.103}$$

The flux of carbon between ocean layers is given by

$$\begin{aligned} \mathscr{F}^{C}_{d,t} = & eddy \frac{\frac{\mathscr{C}^{oc}_{d,t-1}}{depth_d} - \frac{\mathscr{C}^{oc}_{d+1,t-1}}{depth_{d+1}}}{0.5\left(depth_d + depth_{d+1}\right)} \\ & \forall\, d \in (1, (ND-1)) \end{aligned} \tag{A.104}$$

where $eddy$ is an Eddy diffusion coefficient, $\mathscr{C}^{oc}_{d,t-1}$ is the carbon content of ocean layer d (in GtC) and $depth_d$ is the depth of layer d. ND is the total number of ocean layers depicted, with $d = 1$ denoting the top and $d = ND$ being the deepest ocean layer. The fluxes thus calculated are then used to update the carbon contents in all ocean layers except for the top one:

$$\begin{aligned} \mathscr{C}^{oc}_{d,t} &= \mathscr{C}^{oc}_{d,t-1} + \mathscr{F}^{C}_{d-1,t} - \mathscr{F}^{C}_{d,t} \\ & \forall\, d \in (2, (ND-1)) \\ \mathscr{C}^{oc}_{ND,t} &= \mathscr{C}^{oc}_{ND,t-1} + \mathscr{F}^{C}_{ND-1,t} \end{aligned} \tag{A.105}$$

For the top ocean layer, a preliminary carbon content prior to carbon exchange with the atmosphere is calculated as

$$\widetilde{\mathscr{C}}^{oc}_{1,t} = \mathscr{C}^{oc}_{1,t-1} - \mathscr{F}^{C}_{1,t} \tag{A.106}$$

Next, the carbon exchange between the atmosphere and the top ocean layer takes place in the form of a loop used to solve for an equilibrium atmospheric content. $\widetilde{\mathscr{C}}_t$ denotes the sum of the preliminary carbon contents in the upper ocean layer and the atmosphere:

$$\widetilde{\mathscr{C}}_t = \widetilde{\mathscr{C}}^{at}_t + \widetilde{\mathscr{C}}^{oc}_{1,t} \tag{A.107}$$

Keeping $\widetilde{\mathscr{C}}_t$ constant, the loop redistributes carbon between the atmosphere and the upper ocean layer until an equilibrium atmospheric stock is reached. Before entering the loop, several temporary variables are initialised.

$$\mathscr{C}^* = \widetilde{\mathscr{C}}^{at}_t \tag{A.108}$$

$$\begin{aligned}\mathscr{C}^\dagger =& \widetilde{\mathscr{C}}_t - \mathscr{C}^* - \text{Ш}_1\big(1 - \text{Ш}_2 Temp_{t-1}\big)\\ &\times \left(\frac{\mathscr{C}^*}{\mathscr{C}^{at}_0}\right)^{\frac{1}{\text{Ц}_1 + \text{Ц}_2 ln\left(\frac{\mathscr{C}^*}{\mathscr{C}^{at}_0}\right)}}\end{aligned} \tag{A.109}$$

$$\mathscr{C}^{**} = \text{Д}_1 \mathscr{C}^* \tag{A.110}$$

$$\begin{aligned}\mathscr{C}^{\dagger\dagger} =& \widetilde{\mathscr{C}}_t - \mathscr{C}^{**} - \text{Ш}_1\big(1 - \text{Ш}_2 Temp_{t-1}\big)\\ &\times \left(\frac{\mathscr{C}^{**}}{\mathscr{C}^{at}_0}\right)^{\frac{1}{\text{Ц}_1 + \text{Ц}_2 ln\left(\frac{\mathscr{C}^{**}}{\mathscr{C}^{at}_0}\right)}}\end{aligned} \tag{A.111}$$

$$\mathscr{C}' = \frac{\mathscr{C}^{\dagger\dagger} - \mathscr{C}^\dagger}{\mathscr{C}^{**} - \mathscr{C}^*} \tag{A.112}$$

where:

- Ш_1 is a pre-industrial reference carbon content in the upper ocean layer
- Ш_2 describes the influence of the temperature anomaly on the effect of Ш_1
- Ц_1 gives a value for the Revelle factor
- Ц_2 denotes the influence of carbon content on the Revelle factor
- Д_1 is a multiplicative factor used to generate a value of $\mathscr{C}^{**}$ close to $\mathscr{C}^*$

$\mathscr{C}^*$ represents the initial guess for the atmospheric carbon stock. $\mathscr{C}^\dagger$ is a residual. It denotes the difference between $\widetilde{\mathscr{C}}_t$ on the one hand, and $\mathscr{C}^*$, as well as the theoretical carbon content of the upper ocean layer if the atmospheric content were equal to $\mathscr{C}^*$ on the other. The goal of the loop is to bring this residual (close) to zero. $\mathscr{C}'$ denotes the slope of the relationship between $\mathscr{C}^*$ and $\mathscr{C}^\dagger$; it is estimated using nearby values of $\mathscr{C}^*$ and $\mathscr{C}^\dagger$, namely $\mathscr{C}^{**}$ and $\mathscr{C}^{\dagger\dagger}$. The model then enters the loop, which continues until either a termination condition is met, or the maximum number of iterations, n^{clim} is reached.[24] At each iteration, the temporary variables are updated as follows:

$$\mathscr{C}^* = \mathscr{C}^* - \frac{\mathscr{C}^{**}}{\mathscr{C}'} \tag{A.113}$$

$$\begin{aligned}\mathscr{C}^\dagger =& \widetilde{\mathscr{C}}_t - \mathscr{C}^* - \text{Ш}_1\big(1 - \text{Ш}_2 Temp_{t-1}\big)\\ &\times \left(\frac{\mathscr{C}^*}{\mathscr{C}^{at}_0}\right)^{\frac{1}{\text{Ц}_1 + \text{Ц}_2 ln\left(\frac{\mathscr{C}^*}{\mathscr{C}^{at}_0}\right)}}\end{aligned} \tag{A.114}$$

if

$$|\mathscr{C}^\dagger| < \epsilon^{clim} \quad \rightarrow \quad exit \tag{A.115}$$

else

$$\mathscr{C}^{**} = \mathscr{C}^* - \text{Д}_2 \frac{\mathscr{C}^{**}}{\mathscr{C}'} \tag{A.116}$$

$$\begin{aligned}\mathscr{C}^{\dagger\dagger} =& \widetilde{\mathscr{C}}_t - \mathscr{C}^{**} - \text{Ш}_1\big(1 - \text{Ш}_2 Temp_{t-1}\big)\\ &\times \left(\frac{\mathscr{C}^{**}}{\mathscr{C}^{at}_0}\right)^{\frac{1}{\text{Ц}_1 + \text{Ц}_2 ln\left(\frac{\mathscr{C}^{**}}{\mathscr{C}^{at}_0}\right)}}\end{aligned} \tag{A.117}$$

$$\mathscr{C}' = \frac{\mathscr{C}^{\dagger\dagger} - \mathscr{C}^\dagger}{\mathscr{C}^{**} - \mathscr{C}^*} \tag{A.118}$$

where Д_2 is a multiplicative factor used to generate a new guess for $\mathscr{C}^{**}$. The final atmospheric carbon content in t is then given by the equilibrium value of $\mathscr{C}^*$

$$\mathscr{C}^{at}_t = \mathscr{C}^* \tag{A.119}$$

such that the carbon content in the upper ocean layer is given by the residual

$$\mathscr{C}^{oc}_{1,t} = \widetilde{\mathscr{C}}_t - \mathscr{C}^{at}_t \tag{A.120}$$

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[24] We ensure that in the simulations shown below, the algorithm always converges within n^{clim} iterations.

Total radiative forcing is determined by

$$\mathscr{R}_t = \text{Л}_1\text{Л}_2 ln\left(\frac{\mathscr{C}^{at}_t}{\mathscr{C}^{at}_0}\right) \tag{A.121}$$

where Л_1 and Л_2 are exogenous parameters. Л_1 is a factor by which CO_2-induced radiative forcing is multiplied to roughly account for non-CO_2 radiative forcing. Depending on the setting of an exogenous indicator variable, Л_1 can either be equal to one (no non-CO_2 forcing) or greater than one. Л_2 denotes radiative forcing from e-folding CO_2.

Additional out-radiated energy due to global warming is calculated as

$$\mathscr{R}^{out}_t = \text{Ю}Temp_{t-1} \tag{A.122}$$

where Ю is an exogenous parameter denoting the amount of outgoing radiation per degree of warming. The flux of heat between ocean layers is determined by

$$\begin{aligned}\mathscr{F}^H_{d,t} =& eddy\frac{\frac{\mathscr{H}^{oc}_{d,t-1}}{depth_d} - \frac{\mathscr{H}^{oc}_{d+1,t-1}}{depth_{d+1}}}{0.5\left(depth_d + depth_{d+1}\right)}\\ &\forall\, d \in (1,(ND-1))\end{aligned} \tag{A.123}$$

where $\mathscr{H}^{oc}_{d,t-1}$ is the heat content of ocean layer d in $t-1$. The heat content of all ocean layers below the top one is then updated as follows:

$$\begin{aligned}\mathscr{H}^{oc}_{d,t} =& \mathscr{H}^{oc}_{d,t-1} + \mathscr{F}^H_{d-1,t} - \mathscr{F}^H_{d,t}\\ &\forall\, d \in (2,(ND-1))\\ \mathscr{H}^{oc}_{ND,t} =& \mathscr{H}^{oc}_{ND,t-1} + \mathscr{F}^H_{ND-1,t}\end{aligned} \tag{A.124}$$

The heat content of the top ocean layer is also influenced by radiative forcing and out-radiated energy, being given by

$$\mathscr{H}^{oc}_{1,t} = \mathscr{H}^{oc}_{1,t-1} - \mathscr{F}^H_{1,t} + (\mathscr{R}_t - \mathscr{R}^{out}_t)\frac{\text{И}}{\text{З}} \tag{A.125}$$

И is an exogenous parameter the value of which is equal to the climate module time-interval expressed in seconds (e.g. if the climate module is called once per year, its value is given by the number of seconds in one year). З is the fraction of the planetary surface covered by oceans.

Finally, the temperature anomaly in each ocean layer is calculated as

$$\mathscr{T}^{oc}_{d,t} = \frac{\mathscr{H}^{oc}_{d,t}}{depth_d\text{Ч}} \tag{A.126}$$

where Ч is an exogenous parameter giving the heat capacity of water. The global temperature anomaly is then assumed to be equal to that of the top ocean layer, i.e. we set

$$Temp_t = \mathscr{T}^{oc}_{1,t} \tag{A.127}$$

A.11.3 Climate shocks

Both climate modules pass only a single result to the rest of the model, namely the current value of the global temperature anomaly. This value is in turn used to calculate the size of any climate shocks affecting the model economy. Climate shocks may enter the economic model through a variety of channels at both the micro- and macroeconomic levels. Individual channels are switched on or off through the setting of a set of indicator variables. In the cases in which microeconomic shocks for individual agents are required, these are determined by drawing from a beta distribution. The shape parameters of this distribution evolve as a function of the temperature anomaly and may be specific to the particular shock channel to account for the fact that the range of conceivable or realistic shocks for a given temperature anomaly may differ between shock channels.

Formally, we posit that shocks through some impact channel s follow:

$$shock^s_t \sim Beta\left(S^s_{1,t}, S^s_{2,t}\right), \tag{A.128}$$

and that

$$\begin{aligned}S^s_{1,t} &= S^s_{1,0}\left(1 + ln\left(\frac{Temp_{t-1}}{Temp_0}\right)\right)^{\Upsilon^s_3}\\ S^s_{2,t} &= S^s_{2,0}\left(\frac{Temp_0}{Temp_{t-1}}\right)^{\Upsilon^s_4}\end{aligned} \tag{A.129}$$

where $\Upsilon^s_3 > 1$ and $\Upsilon^s_4 > 1$. This implies that as global surface temperature increases, the mode of the distribution will shift upwards and the right tail of the distribution will become thicker. Both the intensity of 'ordinary'/common shocks and the frequency of extreme events will hence increase with global surface temperature.

If a shock takes place directly at the microeconomic level, an independent draw is made for each agent (e.g. for each C-Firm and each K-Firm). Microeconomic shock channels hence allow for a non-uniform distribution of impacts across individual agents. If the economic variable to be shocked is instead an aggregate one (e.g. labour supply or the aggregate capital stock, see below), the size of the shock is given by the current mean value of the beta distribution, given by $\frac{S^s_{1,t}}{S^s_{1,t}+S^s_{2,t}}$.

In addition to allowing for endogenous climate shocks, the model also allows for simulation settings in which climate shocks are instead supplied exogenously. This may be useful for diagnostic purposes, or to allow for an investigation of climate damages of a given intensity and distribution.

As previously indicated, the model currently includes a range of possible impact channels which can be activated/deactivated separately or jointly using exogenous indicator variables in order to simulate different impact scenarios:

1. **Current output (specification 1):** Following production, but prior to being able to sell on the consumption

goods market, each C-Firm c loses a fraction $shock_{c,t}^{y}$ of its current output. Similarly, after production but prior to delivery (and payment) of capital goods ordered by C-Firms, each K-Firm k loses a fraction $shock_{k,t}^{y}$ of its current output of capital goods.

2. **Current output (specification 2):** An aggregate percentage loss of current output of consumption goods, $shock_{2,t}^{Y}$ and capital goods, $shock_{1,t}^{Y}$, is determined by the two shape parameters described above. These aggregate percentage losses are multiplied by the aggregate outputs of consumption and capital goods, respectively, to arrive at aggregate absolute losses, $Loss_{2,t}^{Y}$ and $Loss_{1,t}^{Y}$. Next, individual firms are drawn at random with uniform probability. A randomly drawn C-Firm (K-Firm) loses all of the output it has produced in t and $Loss_{2,t}^{Y}$ $\left(Loss_{1,t}^{Y}\right)$ is reduced by the amount of output thus destroyed. This process continues until $Loss_{2,t}^{Y}$ $\left(Loss_{1,t}^{Y}\right)$ has become zero.[25] As with specification 1, damages take place prior to sales, delivery and payment.
3. **Current output (specification 3):** Same as specification 2, but firms' probability of being affected by shocks is not homogeneous. Instead, each firm has an individual and constant probability of being drawn each time a shock occurs. Across C-Firms/K-Firms, these probabilities follow a normal distribution, normalised on the interval $\left(\epsilon_1^s, 1\right)$, where ϵ_1^s is a small positive number.[26]
4. **Capital stock (specification 1):** Prior to carrying out current production, each C-Firm c loses a fraction $shock_{c,t}^{cap}$ of its existing capital stock (with the precise vintage(s) affected being determined randomly). If planned production cannot be carried out with the reduced capital stock, it is scaled back but affected firms still have to make wage payments based on initially planned production.
5. **Capital stock (specification 2):** An aggregate percentage loss of existing capital stocks $shock_{t}^{Cap}$, is determined by the two shape parameters described above. This aggregate percentage loss is multiplied by the aggregate capital stock of all C-Firms to arrive at an aggregate loss, $Loss_{t}^{Cap}$. Next, individual firms are drawn at random with uniform probability. A randomly drawn C-Firm loses all of its capital stock except for a single machine. $Loss_{t}^{Cap}$ is reduced by the amount of capital thus destroyed. This process continues until $Loss_{t}^{Cap}$ has become zero. As with specification 1, damages take place prior to production.
6. **Capital stock (specification 3):** Same as specification 2, but firms' probability of being affected by shocks is not homogeneous. Instead, each firm has an individual and constant probability of being drawn each time a shock occurs. Across C-Firms, these probabilities follow a normal distribution, normalised on the interval $\left(\epsilon_1^s, 1\right)$, where ϵ_1^s is a small positive number.
7. **Inventories (specification 1):** Following production, but prior to being able to sell on the consumption goods market, each C-Firm c loses a fraction $shock_{c,t}^{inv}$ of its current stock of inventories.
8. **Inventories (specification 2):** An aggregate percentage loss of existing stocks of inventories, $shock_{t}^{Inv}$, is determined by the two shape parameters described above. This aggregate percentage loss is multiplied by the aggregate stock of inventories all C-Firms to arrive at an aggregate loss, $Loss_{t}^{Inv}$. Next, individual firms are drawn at random with uniform probability. A randomly drawn C-Firm loses all of its inventories. $Loss_{t}^{Inv}$ is reduced by the amount of inventories thus destroyed. This process continues until $Loss_{t}^{Inv}$ has become zero. As with specification 1, damages take place prior to sales.
9. **Inventories (specification 3):** Same as specification 2, but firms' probability of being affected by shocks is not homogeneous. Instead, each firm has an individual and constant probability of being drawn. Across C-Firms, these probabilities follow a normal distribution, normalised on the interval $\left(\epsilon_1^s, 1\right)$, where ϵ_1^s is a small positive number.
10. **Capital stock (Energy sector):** At the end of a period (i.e. after energy production for the current period), the energy sector loses a share $shock_{t}^{De}$ of its productive capacity for brown energy, and a share $shock_{t}^{Ge}$ of its productive capacity for green energy. The two shocks are determined by the two shape parameters described above.
11. **Productivity (specification 1):** Regardless of the mix of technology vintages installed, and leaving the inherent characteristics of capital good vintages unaffected, the labour productivity of every C-Firm c decreases by a percentage given by $shock_{c,t}^{lp}$. Similarly, leaving the inherent characteristics of production techniques for capital goods unaffected, the labour productivity of every K-Firm k decreases by a percentage given by $shock_{k,t}^{lp}$.
12. **Productivity (specification 2):** In this alternative specification, productivity shocks enter the model by affecting the characteristics of capital good vintages and production techniques for capital goods. As each vintage of capital good and each production technique for capital goods is associated with a particular K-Firm k, two shocks, $shock_{k,t}^{tp}$ and $shock_{k,t}^{mp}$, are drawn for each K-Firm. The labour productivity associated with the current vintage of capital good produced by k is then reduced by the percentage given by $shock_{k,t}^{mp}$. Similarly, the labour productivity associated with the

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[25] For the last C-Firm (K-Firm) drawn, the loss will be given by the residual value of $Loss_{2,t}^{Y}$ $\left(Loss_{1,t}^{Y}\right)$.

[26] Specifically, the probabilities are determined the first time a shock occurs as elaborated here for the example of C-Firms: make a draw $P(shock)_c$ from a standard normal distribution for each C-Firm. Next, transform them as follows: $P(shock)_c = P(shock)_c - min(P(shock)_c) + \epsilon_2^s$, where ϵ_2^s is a fixed and homogeneous parameter. Finally, normalise the probabilities by calculating $P(shock)_c = \frac{P(shock)_c}{\sum_{i=1}^{N2} P(shock)_i}$.

production technique used by k to produce the current vintage is reduced by the percentage $shock_{k,t}^{tp}$. The effect of these shocks differs from specification 1 in two important respects. Firstly, the shock is inherently persistent as all machines of the current vintage which currently exist and will be installed in the future will be less productive than they would otherwise be. Secondly, as described above, endogenous technological progress is applied as a percentage improvement over the characteristics of current vintages and production techniques. By reducing the productivity of current vintages and production techniques, technological progress will hence also be slowed down compared to a situation without shocks. This latter characteristic makes these shocks somewhat similar to the 'Total Factor Productivity' damages which have been explored by some works in the climate economics literature (e.g. Dietz and Stern, 2015; Letta and Tol, 2019).

13. **Productivity (specification 3):** Same as specification 2, but shocks to the productivity of vintages of capital goods affect not only the current vintage produced by each K-Firm k, but also all vintages which k has produced in the past and which are still in use at one or more C-Firms.
14. **Energy efficiency (specification 1):** Same as Productivity (specification 1), but shocks instead increase the amount of energy required per unit of output.
15. **Energy efficiency (specification 2):** Same as Productivity (specification 2), but shocks instead affect the energy efficiency of current vintages of capital goods and production techniques used to produce capital goods.
16. **Energy efficiency (specification 3):** Same as specification 2, but shocks to the energy efficiency of vintages affect all vintages produced in the past which are still in use at one or more C-Firms.
17. **R&D:** Instead of affecting the characteristics of existing technologies, climate shocks can also be set to affect the R&D process of K-Firms. For each K-Firm k, a shock $shock_{k,t}^{rd}$ is drawn. Depending on the setting of an indicator variable, the shock then affects the effectiveness of the R&D process by
    - reducing the value of the first shape parameter of the beta distribution from which technological innovations are drawn by a percentage given by $shock_{k,t}^{rd}$, or
    - shifting the support of the distribution from which technological innovations are drawn downward by a percentage given by $shock_{k,t}^{rd}$, or
    - reducing the (otherwise endogenous) probability that K-Firm k innovates or imitates a technology at all in the current period by a percentage given by $shock_{k,t}^{rd}$, or
    - reducing the amount of effective resources devoted to R&D activities by K-Firm k (without reducing the corresponding expenditure) by a percentage given by $shock_{k,t}^{rd}$.
18. **Consumption demand:** Prior to the allocation of consumption expenditure to C-Firms, Households' desired consumption expenditure is reduced by a percentage $shock_t^{Cons}$, determined by the two shape parameters described above.
19. **Labour supply:** In addition to changing at the exogenous rate g_L as discussed in the description of the household sector, the aggregate labour force is altered by a percentage $shock_t^{Ls}$, determined by the two shape parameters described above.

A.12 Aggregation and stock-flow consistency

In this section we describe how a range of important macroeconomic aggregates and averages are computed in the model and how checks for stock-flow consistency are performed.

A.12.1 GDP

Recall that the maximum quantity of consumption goods which a single unit of capital good in the model can produce is uniform and constant, and is denoted by $\mathfrak{Q}$. To render output of consumption and capital goods comparable, we measure real GDP as

$$GDP_t = \sum_{c=1}^{N2} Q_{c,t} + \mathfrak{Q} \sum_{k=1}^{N1} Q_{k,t} \tag{A.130}$$

i.e. it is given by the sum of output of all C-Firms plus that of K-Firms, where the latter is expressed as the amount of productive capacity embodied by the number of machines produced in t.

Similarly, nominal GDP is measured as

$$GDP_t^n = \sum_{c=1}^{N2} \left(p_{c,t} Q_{c,t}\right) + \mathfrak{a}\mathfrak{Q} \sum_{k=1}^{N1} \left(p_{k,t} Q_{k,t}\right) \tag{A.131}$$

where $\mathfrak{a}$ is a uniform scaling parameter applied to the labour productivity of K-Firms.

A.12.2 Labour demand and employment

Total labour demand in a period is given by the amount of labour demanded by C-Firms and K-Firms for the production of output, plus the labour required by K-Firms and the energy sector for R&D ($L_{k,t}^{rd}$ and $L_{de,t}^{rd} + L_{ge,t}^{rd}$, respectively) and that demanded by the energy sector for capacity expansion $L_{ge,t}^{EI}$:

$$\begin{aligned} LD_t = &\sum_{c=1}^{N2} L_{c,t}^{d} + \sum_{k=1}^{N1} \left(L_{k,t}^{d} + L_{k,t}^{rd}\right) \\ &+ L_{de,t}^{rd} + L_{ge,t}^{rd} + L_{ge,t}^{EI} \end{aligned} \tag{A.132}$$

Recall that if labour demand exceeds the current value of the aggregate labour force, LS_t, the labour input available for production of consumption and capital goods is scaled back until $LD_t = LS_t$. Otherwise, aggregate employment L_t is given by LD_t and unemployment by $LS_t - LD_t$.

A.12.3 Price level

The price level used in the model (e.g. to compute inflation for use in the wage equation and the Taylor rule described above) is a consumption price index. As described in Section A.4, prior to the consumption goods market interaction, this is computed using C-Firms' market shares as determined by the quasi-replicator equation as weights:

$$cpi_t = \sum_{c=1}^{N2} f_{c,t} p_{c,t} \tag{A.133}$$

Recall that since individual consumption good firms may not be able to satisfy all the demand they receive, in which case demand is redistributed to other firms, the ex-post distribution of consumption expenditure does not necessarily correspond to the $f_{c,t}$'s. After the consumption goods market has closed, the consumption price index is therefore recomputed as

$$cpi_t = \frac{\sum_{c=1}^{N2} Sales_{c,t}}{Q_t^s} \tag{A.134}$$

i.e., it is given by the sum of C-Firm sales divided by the aggregate quantity of consumption goods sold, Q_t^s.

A.12.4 Energy demand

At present, demand for energy comes purely from the C-Firm and K-Firm sectors, such that total energy demand is given by

$$En_t^d = \sum_{c=1}^{N2} En_{c,t}^d + \sum_{k=1}^{N1} En_{k,t}^d \tag{A.135}$$

Recall from Section A.9 that the energy sector is able to instantaneously increase its productive capacity to meet any level of demand for energy. This simplifying assumption implies that the quantity of energy sold is always equal to that demanded, $En_t = En_t^d$.

A.12.5 Emissions

Endogenous emissions arise from the production of consumption goods, capital goods and energy. Total emissions are given by the sum of these endogenous emissions and exogenous 'rest of the world' emissions:

$$Em_t = \sum_{k=1}^{N1} Em_{k,t} + \sum_{c=1}^{N2} Em_{c,t} + Em_{e,t} + Em_{row,t} \tag{A.136}$$

where $Em_{row,t} = (1 + g_{Em})Em_{row,t-1}$ since, as set out above, exogenous emissions grow at a constant rate g_{Em}. Exogenous emissions can hence be switched of by simply setting $Em_{row,0} = 0$. Importantly, emissions in the model are currently not measured in some meaningful quantity such as gigatonnes of carbon. In order to convert them into a quantity which can serve as a useful input for the climate modules, emissions are transformed prior to being fed into the climate module chosen for a given simulation.

In the first period in which a climate module is called, the model calculates an initial quantity of emissions $\mathfrak{E}_0$, given by the sum of endogenous and exogenous emissions over the past number of periods corresponding to one time interval in the climate module. For instance, if the climate module runs at annual frequency and the economic model at quarterly frequency, $\mathfrak{E}_0$ is given by $\sum_{i=0}^{3} Em_{t-1}$, as is $\mathfrak{E}_t$. The value of emissions which is then supplied to the active climate module in any period in which the climate module is called is given by

$$\mathscr{E}_t = \mathscr{E}_0 \frac{\mathfrak{E}_t}{\mathfrak{E}_0} \tag{A.137}$$

where $\mathscr{E}_0$ is an initial value denoted in gigatonnes of carbon. In the case of the cumulative emissions climate module, $\mathscr{E}_t$ is used to update the cumulative amount of emissions, which at the beginning of a simulation is similarly initialised to a meaningful quantity. In the case of the carbon cycle climate module, $\mathscr{E}_t$ is used to update the atmospheric carbon content.

A.12.6 Productivity

Recall that the determination of the nominal wage rate depends partly on a weighted average of current and past changes in average labour productivity across firms. This average productivity is calculated as

$$\overline{Pr}_t = \frac{\sum_{c=1}^{N2} Pr_{c,t} + \sum_{k=1}^{N1} \left(Pr_{k,t}\mathfrak{a}\right)}{N1 + N2} \tag{A.138}$$

where $Pr_{c,t}$ is the labour productivity of C-Firm c, computed as a weighted average of the labour productivities of the machines owned by c, with the weights being given by the weight of each vintage owned by c in the overall capital stock of c.

A.12.7 Stock-flow consistency

To ensure that any inconsistencies in the accounting system of the model can be detected, the following set of stock-flow consistency checks is performed at the end of every period:

1. Calculate the sectoral financial balance (i.e. revenues minus expenditures calculated at the sectoral level) for each sector. Check that when added together across sectors, these balances sum to zero.
2. Check that the sum of sectoral net worths is equal to the nominal value of all tangible assets (capital stock and inventories) summed together.

3. Calculate net worth for all agents/sectors using the stock approach (assets minus liabilities at the end of the period). Re-calculate net worth for all agents/sectors using the flow approach (net worth at the end of the *previous* period plus the sum of all flows affecting net worth which have taken place in the current period). Check that both methods always produce the same result.
4. Check that balance sheet items which are tracked both at the agent and aggregate/sectoral level are consistent. For instance, the stock of bank deposits on the asset side of the household sectoral balance sheet (an aggregate quantity) should be identical to the sum of household deposits on the liability sides of individual banks' balance sheets, and so on.

A.13 Initialisation

At present, the initialisation protocol of the model is strongly simplified. While we ensure that the initial values we set do not violate stock-flow consistency, we do not yet make use of the SFC structure to attempt to initialise (an aggregated version of) the model to a steady state as is proposed by Caiani et al. (2016). This task, which may be quite challenging for models which were not initially written with this objective in mind, is left for future work. In addition, future initialisation procedures of the model will make use of sectoral balance sheet and flow of funds data where possible, as is sometimes done with aggregate SFC models (e.g. Burgess et al., 2016). For the moment, our approach is closer to that used to initialise other well-known macroeconomic ABMs in the literature, such as Assenza et al. (2015) or earlier incarnations of the K+S/DSK model family. We initialise model variables to fairly arbitrary values and subsequently allow the model to undergo a 'burn-in' period, during which it converges to its eventual trajectory. This transient period is subsequently discarded from simulated data.

As discussed in Section A.6, the number of K-Firm and C-Firm customers of each bank is stochastically determined at the beginning of a simulation through draws from a truncated Pareto distribution. The actual firms which are assigned to a given bank are then drawn at random. Each K-Firm is assigned an identical initial stock of deposits $D_{k,0}$. Similarly, each C-Firm is given an identical initial stock of deposits $D_{c,0}$. In addition, each C-Firm begins the simulation with an identical initial stock of loans $L_{c,0}$. These loans and deposits are also entered on the balance sheets of the banks to which the corresponding firms are assigned.

The remainder of the bank balance sheets is initialised in a similar fashion. Both households and the energy sector are given an initial stock of deposits, which are distributed among individual banks in proportion to the number of firm clients of each bank as described in Section A.6. Each bank is assigned an initial stock of government bond holdings given by a percentage of its initial stock of loans, in line with banks' behavioural rule for investing in government bonds described in Section A.6. The initial aggregate net worth of the banking sector is given exogenously and then distributed among banks according to each banks' share of firm customers. Central bank advances are initialised to zero. With all other balance sheet items of banks hence being given, the initial stocks of reserves are given as a residual. Since, in the absence of outstanding central bank advances, this initial stock of reserves must be identical to the central bank's initial holdings of government bonds, the latter are thereby also given, as is, consequently, the overall initial stock of government bonds. The initial central bank lending rate is supplied exogenously. All other interest rates which need to be initialised are defined either as mark-downs or mark-ups over this rate and are set accordingly.

Initial capital good vintages and K-Firm production techniques are in the first instance initialised homogeneously, meaning that both K-Firms and C-Firms begin the simulation with homogeneous unit costs of production. Together with initial values for the mark-ups, the nominal wage, the energy price and the emission tax rates on firms, this allows to set the initial prices of consumption and capital goods.

Every C-Firm is given an identical initial stock of capital goods, valued at the uniform initial price of capital goods. The initial age of each individual machine is drawn randomly from a uniform distribution over the support $(0, \aleph^K + 1)$. C-Firms are initially assigned as customers to K-Firms uniformly, such that every K-Firm begins the simulation with the same number of customers. The initial investment of C-Firms (which in turn implies the initial sales of K-Firms which are needed for the initial run of the endogenous R&D process described below) is set to the average value needed to replace those machines out of the initial stock which have reached their maximum lifespan.

The aggregate initial expected demand of C-Firms is set equal to the level consistent with an unemployment rate equal to the central bank's target and then uniformly distributed among C-Firms. C-Firms' initial sales, inventories, net revenue and dividend payments are initialised in line with this level of demand, and each C-Firms' initial ability to satisfy demand, $l_{c,0}$ is set to 1. C-Firms' initial market shares are set to the value $\frac{1}{N2}$.

At the end of the initialisation procedure of the economic model components, the endogenous R&D process described in Section A.3 is called one time in order to initialise K-Firms' wage bill for R&D labour input and produce an initial degree of heterogeneity in technology.

The initial productive capacity of the energy sector is set once the energy demand for the first simulation period has been determined. The energy sector is provided with a productive capacity equal to this energy demand, a given share of which is represented by green energy production capacity of the initial green energy vintage, with the rest being brown energy capacity of the initial brown energy vintage. In order to ensure that this initial capacity depreciates uniformly rather than all at once after $\aleph^E$ periods, a share $\frac{1}{\aleph^E}$ of the energy sector's initial capacity is scrapped in each of the first $\aleph^E$ periods of a simulation (which form part of the transient phase).

The cumulative emissions climate module is initialised by setting an initial stock of cumulative emissions. For the more complex climate module, a range of initial values such as the initial atmospheric carbon content, the initial average surface temperature, and oceanic carbon content, heat content and temperature are set. A full list of all initial values required to simulate the model along with the values used for the simulations shown in this paper is given in appendix B.

A.14 Calibration

To calibrate the model, we begin by manually exploring the parameter space in order to identify a region in which the model gives rise to reasonable business cycle dynamics. We then manually calibrate the major parameters driving the long-run growth rates of labour productivity, energy efficiency and emission intensity (in particular the shapes and supports of the probability distributions from which the characteristics of innovated technologies are drawn) in order to bring the long-run average growth rates of real GDP, endogenous emissions and energy use produced by the model in line with SSP2 scenarios and projections for Europe (see Riahi et al., 2021; Byers et al., 2022; Koch and Leimbach, 2023).

As outlined in the model description above, the model code contains a number of indicator variables allowing the user to simulate different specifications of certain parts of the model. In the calibration shown here, these indicator variables are set as follows:

- C-Firm inventory dynamics are deactivated. Any unsold C-Firm output is scrapped immediately.
- If households cannot finance firm entry, entry is financed by the government.[27]
- All failing banks are bailed out by the government rather than purchased by a surviving bank.
- R&D expenditure of the energy sector is allocated between green and brown technologies according to the share of each technology in total energy sector productive capacity.
- There is an upper bound on the per-period expansion of green energy production capacity. In addition, there is a lower bound on the share of green capacity set equal to the initial share.
- There is a carbon tax on emissions of the *energy sector only*. The tax rate per unit of emission grows with nominal GDP.
- The climate module used is the more detailed one rather than the one based on cumulative emissions. Non-CO_2 radiative forcing is activated.
- All climate shocks are deactivated in the baseline runs.

As mentioned above we aim to arrive at a calibration in which the growth rates of real GDP, emissions and energy use in industry are close to SSP2 scenarios for the European Union with current policy, implying fairly low GDP growth compared to historical experience together with roughly constant energy use and emissions. Under roughly constant energy demand, the sizes of the energy and fossil fuel sectors as a share of nominal GDP would eventually tend to zero (one) if the energy and fossil fuel prices grew more slowly (faster) than nominal GDP. As already indicated in the model description, we hence impose that the price of fossil fuel as well as the mark-up of the energy sector grow following a weighted average of past changes in the nominal wage (which in turn endogenously grows roughly in line with nominal GDP). To keep the production cost of brown energy and the expansion cost of green energy capacity comparable, we assume that the expansion cost of each available vintage of green energy technology also grows at this rate. If the model were to be used to simulate a different setting (e.g. with growing or declining energy demand), these assumptions would have to be modified.

Adding to the information provided in Figure 2 in the main body of the paper, Figure 10 plots the cross-correlation functions of two additional variables with real GDP. It shows that both aggregate R&D spending and the stock of outstanding loans to C-Firms are pro-cyclical. In addition, the figure shows that ’bad debt’ (i.e. the aggregate value of C-Firm loans which are in default) is significantly positively correlated with the lagged stock of outstanding loans.

Figure 11 shows the autocorrelations of the *annualised* and filtered simulated series for carbon emissions and energy demand from firms. In addition, it shows the cross-correlation functions of these variables and annualised real GDP, showing that both emissions and energy demand are pro-cyclical.

Making use of the new comprehensive accounting structure of the model, Figure 12 plots the ratios of sectoral net worths to annualised nominal GDP, taking averages across seeds.[28] It can be seen that under the calibration shown here, none of these ratios exhibits an upward or downward trend over the course of a simulation.

Similarly taking advantage of the stock-flow consistent framework underlying the model, Figure 13 plots the ratios of the sectoral financial balances[29] to quarterly nominal GDP. As with the previous figure, it can be seen that none of the series exhibits an upward or downward trend.

A secular trend in any of the variables plotted in the two aforementioned figures would indicate that one or more balance sheet items and/or transaction flows persistently grow faster or more slowly than nominal GDP. Unless it has been intentionally introduced to examine its consequences, such a dynamic, (if persisting over the entirety of a run representing 100 years and beyond if the simulation length were extended) should be carefully investigated, as it may

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[27] This situation never arises in the simulations presented in this paper.

[28] Note that the net worth of the government is negative as it consists solely of government bonds (i.e. liabilities).

[29] I.e., the difference between revenues and expenditures for each sector.

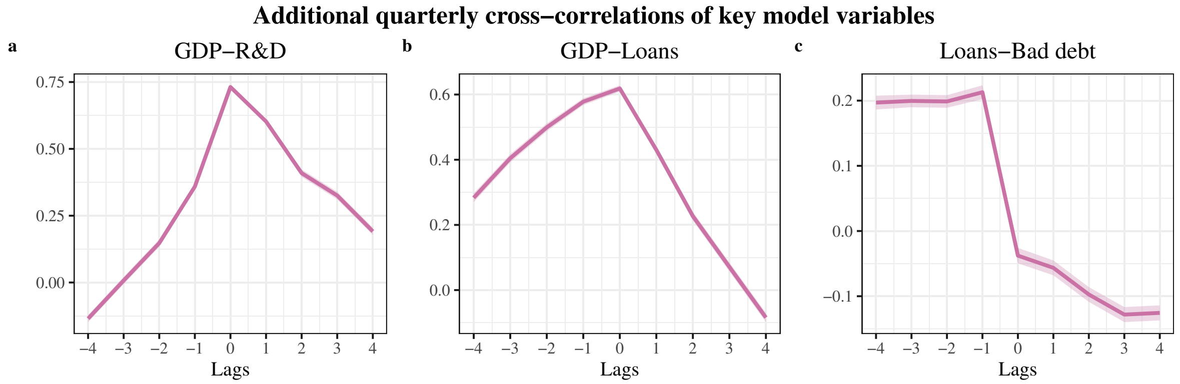

Figure 10: Panel (a): Cross-correlation of simulated real GDP and K-Firm R&D expenditure; Panel (b): Cross-correlation of simulated real GDP and stock of bank loans; Panel (c): Cross-correlation of stock of bank loans and bad debt. Cross-correlations are calculated on filtered quarterly simulated time-series. Bold lines represent averages across 108 simulations with different seeds. Shaded bands represent 95% confidence intervals.

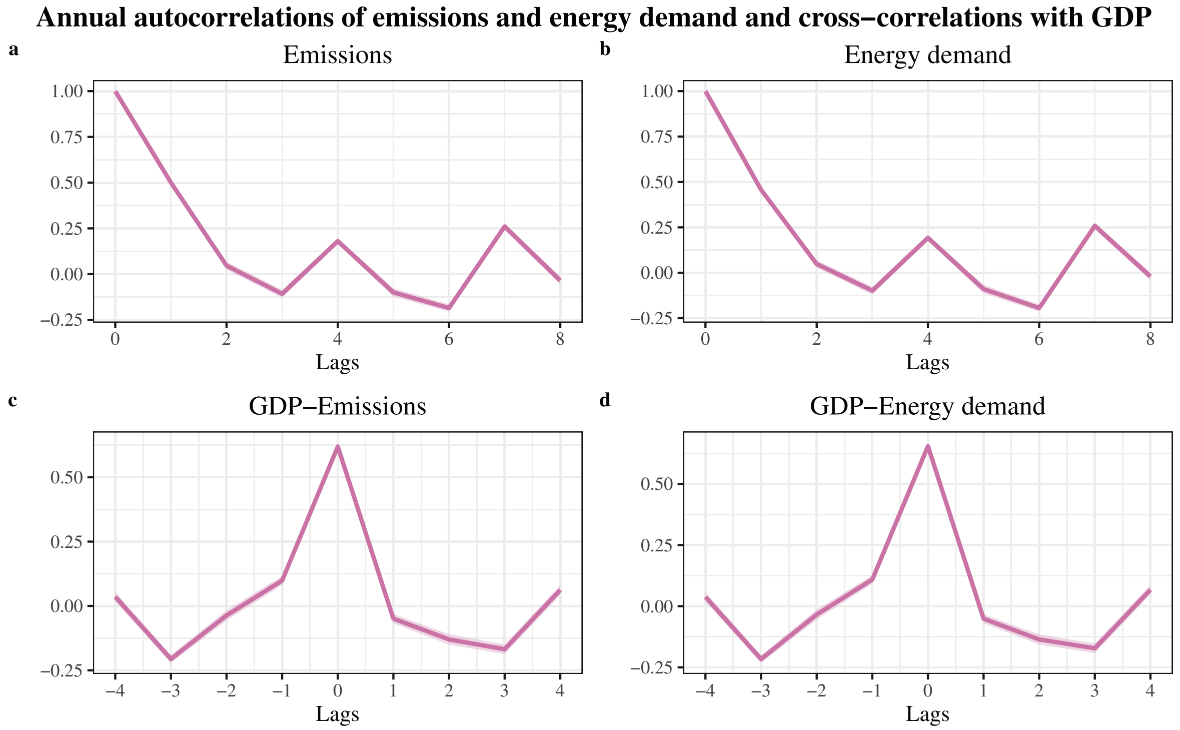

Figure 11: Panel (a): Autocorrelation of simulated annualised emissions; Panel (b): Autocorrelation of simulated annualised energy demand; Panel (c): Cross-correlation of annualised real GDP and emissions; Panel (d): Cross-correlation of annualised real GDP and energy demand. Auto- and Cross-correlations are calculated on filtered annualised simulated time-series. Bold lines represent averages across 108 simulations with different seeds. Shaded bands represent 95% confidence intervals.

eventually lead to an unrealistic and possibly unsustainable configuration of sectoral balance sheets (cf. Godley, 2012). It may indicate either a problem with the model calibration being used, or with the specification of one or more behavioural assumptions.

Figure 14 gives some further examples of time-series which can be tracked more accurately using the newly added stock-flow consistent structure of the model, and which are likely to play an important role particularly in applications of the model to macro-financial research questions. It shows the averages across seeds of the number of C-Firm bankruptcies per period, loans in default as a ratio of annualised nominal GDP, and the number of bank failures per period.

Table A1 lists a range of qualitative empirical stylised facts reproduced by the DSK-SFC model under the calibration used in this paper, alongside corresponding references to the literature.

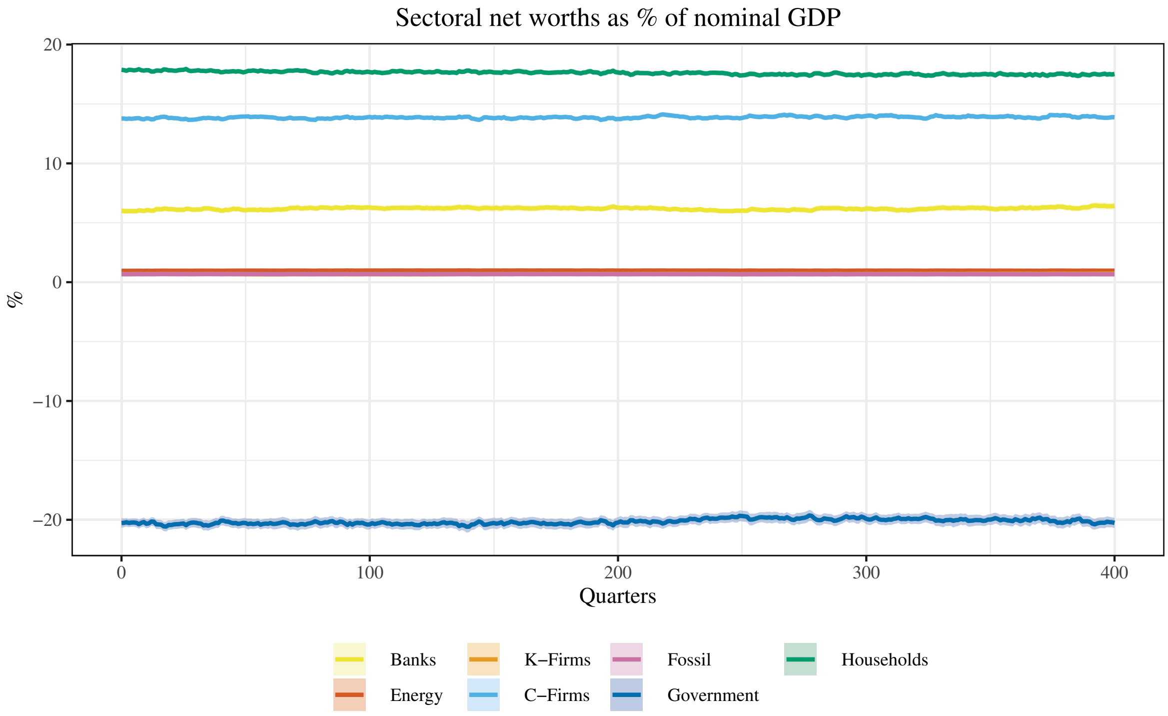

Figure 12: Simulated sectoral net worths as percentage of annualised nominal GDP. Bold lines represent averages across 108 simulations with different seeds. Shaded bands represent 95% confidence intervals.

Appendix B Additional tables

Table B2 provides a full list of all economic model parameters with descriptions. It also gives the values used for the simulations shown in this paper. Table B3 provides the same information for parameters related to the climate modules. Tables B4 contains a list of all initial values needed to simulate the economic model, along with descriptions. Table B5 provides the same information for the climate modules.

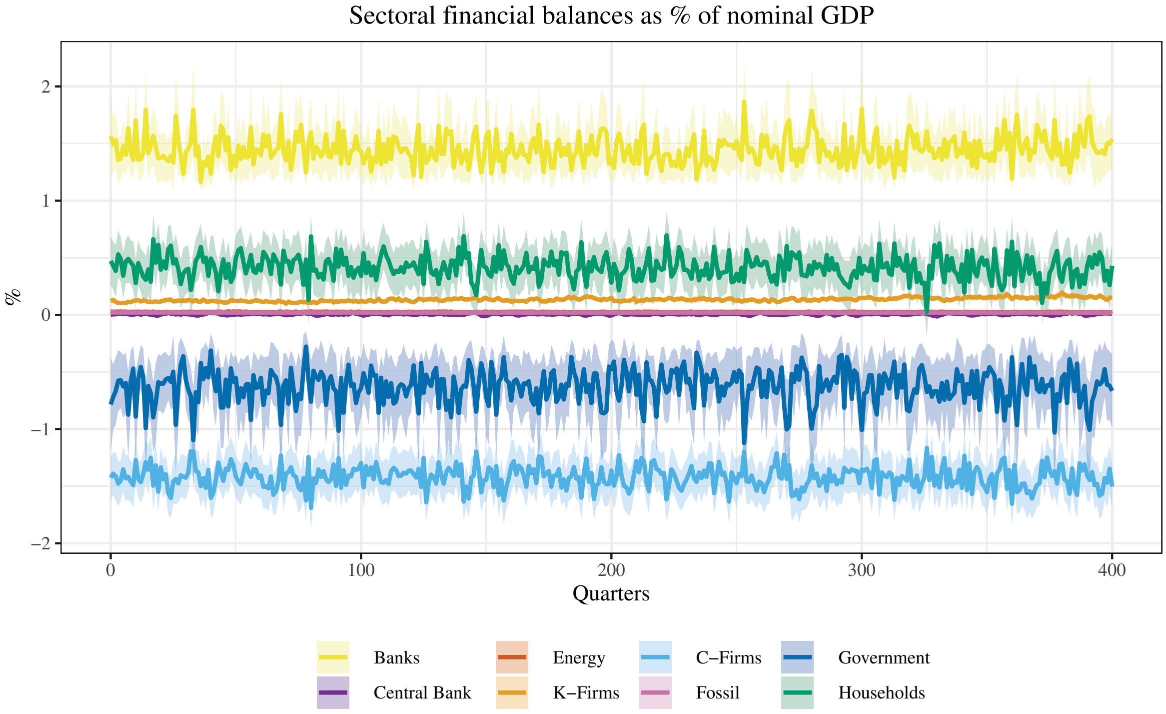

Figure 13: Simulated sectoral financial balances as percentage of quarterly nominal GDP. Bold lines represent averages across 108 simulations with different seeds. Shaded bands represent 95% confidence intervals.

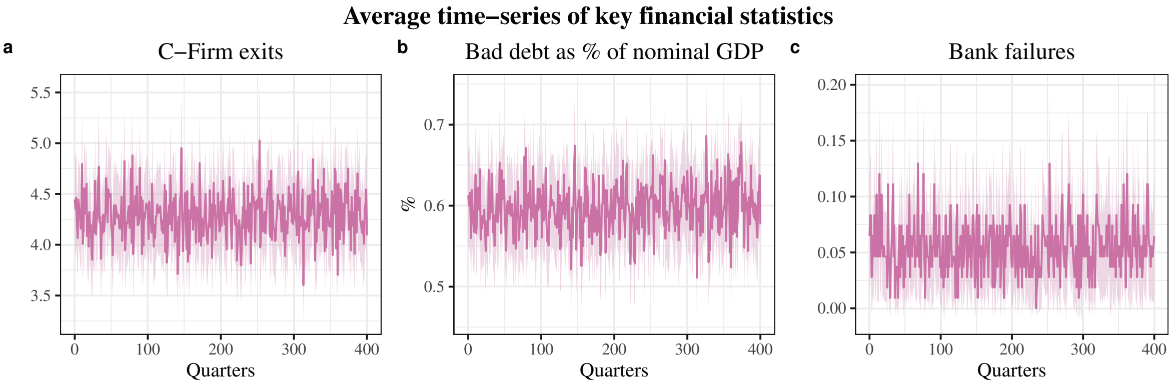

Figure 14: Panel (a): Simulated number of C-Firm failures per period; Panel (b): Simulated bad debt as percentage of annualised nominal GDP; Panel (c) Simulated number of bank failures per period. Bold lines represent averages across 108 simulations with different seeds. Shaded bands represent 95% confidence intervals.

Table A1
Qualitative stylised facts reproduced by DSK-SFC

Stylised fact	Reference(s)
Endogenous growth with persistent fluctuations	Burns and Mitchell (1946); Kuznets (1966); Zarnowitz (1985); Stock and Watson (1999)
Fat-tailed GDP growth-rate distribution	Fagiolo et al. (2008); Castaldi and Dosi (2009); Lamperti and Mattei (2018)
Relative volatility of main macroeconomic aggregates	Stock and Watson (1999); Napoletano et al. (2006)
Cross-correlations of main macroeconomic aggregates	Stock and Watson (1999); Napoletano et al. (2006)
Pro-cyclical private debt	Lown and Morgan (2006)
Pro-cyclical R&D investment	Wälde and Woitek (2004)
Pro-cyclical energy demand	Moosa (2000)
Pro-cyclical emissions	Doda (2014)
Cross-correlation between private debt and loan losses	Foos et al. (2010); Mendoza and Terrones (2012)
Fat-tailed firm growth-rate distribution	Bottazzi and Secchi (2003, 2006)
Lumpy investment rates at firm level	Doms and Dunne (1998)
Persistent productivity heterogeneity across firms	Bartelsman and Doms (2000); Dosi (2007)
Persistent energy efficiency heterogeneity across firms	DeCanio and Watkins (1998); Petrick (2013)
Persistent emission intensity heterogeneity across firms	Petrick (2013)

Table B2: Full list of economic model parameters

Symbol	Description	Value
$N1$	Number of K-Firms	20
$N2$	Number of C-Firms	200
NB	Number of banks	10
g_L	Growth rate of labour force	-1.15e-5
ζ	Unemployment benefit ratio	0.4
τ_h	Tax rate on labour income	0
α_1	Propensity to consume out of wage & benefit income	0.965
α_2	Propensity to consume out of profit & interest income	0.3
α_3	Propensity to consume out of wealth	0.1
$\overline{\mathfrak{w}}$	Maximum per-period % change in the wage rate	0.025
ψ_1	Sensitivity of nominal wage to inflation gap	0.4
ψ_2	Sensitivity of wage to productivity	1
ψ_3	Sensitivity of nominal wage to unemployment	0.26
η	Parameter used for calculating weighted averages	0.76
$\aleph^K$	Maximum lifespan of machine tools	19
$\mathfrak{a}$	K-Firm productivity scaling parameter	0.1
μ^K	K-Firm mark-up	0.1
Γ	# brochures sent by K-Firms (fraction of current customers)	0.32
$\mathfrak{o}$	Share of K-Firm revenue dedicated to R&D	0.04
$\mathfrak{x}^K$	Share of K-Firm R&D dedicated to innovation	0.5
$\mathfrak{b}_1^K$	Parameter governing K-Firm probability of innovating	0.3
$\mathfrak{b}_2^K$	Parameter governing K-Firm probability of imitating	0.3
$\mathfrak{b}_3^K$	Shape parameter of beta distribution for capital vintage labour productivity innovation	1.5
$\mathfrak{b}_4^K$	Shape parameter of beta distribution for capital vintage labour productivity innovation	3
$\mathfrak{b}_5^K$	Lower bound for random capital vintage labour productivity innovation	-0.015
$\mathfrak{b}_6^K$	Upper bound for random capital vintage labour productivity innovation	0.015
$\mathfrak{b}_7^K$	Shape parameter of beta distribution for capital vintage energy efficiency innovation	1.5
$\mathfrak{b}_8^K$	Shape parameter of beta distribution for capital vintage energy efficiency innovation	3
$\mathfrak{b}_9^K$	Lower bound for random capital vintage energy efficiency innovation	-0.01
$\mathfrak{b}_{10}^K$	Upper bound for random capital vintage energy efficiency innovation	0.035
$\mathfrak{b}_{11}^K$	Shape parameter of beta distribution for capital vintage environmental friendliness innovation	1.5
$\mathfrak{b}_{12}^K$	Shape parameter of beta distribution for capital vintage environmental friendliness innovation	3
$\mathfrak{b}_{13}^K$	Lower bound for random capital vintage environmental friendliness innovation	-0.01
$\mathfrak{b}_{14}^K$	Upper bound for random capital vintage environmental friendliness innovation	0.02
$\mathfrak{b}_{15}^K$	Shape parameter of beta distribution for labour productivity of K-Firm production technique	1.5
$\mathfrak{b}_{16}^K$	Shape parameter of beta distribution for labour productivity of K-Firm production technique	3
$\mathfrak{b}_{17}^K$	Lower bound for random K-Firm production technique labour productivity innovation	-0.015
$\mathfrak{b}_{18}^K$	Upper bound for random K-Firm production technique labour productivity innovation	0.03
$\mathfrak{b}_{19}^K$	Shape parameter of beta distribution for energy efficiency of K-Firm production technique	1.5
$\mathfrak{b}_{20}^K$	Shape parameter of beta distribution for energy efficiency of K-Firm production technique	3
$\mathfrak{b}_{21}^K$	Lower bound for random K-Firm production technique energy efficiency innovation	-0.01
$\mathfrak{b}_{22}^K$	Upper bound for random K-Firm production technique energy efficiency innovation	0.05
$\mathfrak{b}_{23}^K$	Shape parameter of beta distribution for environmental friendliness of K-Firm production technique	1.5
$\mathfrak{b}_{24}^K$	Shape parameter of beta distribution for environmental friendliness of K-Firm production technique	3
$\mathfrak{b}_{25}^K$	Lower bound for random K-Firm production technique environmental friendliness innovation	-0.005
$\mathfrak{b}_{26}^K$	Upper bound for random K-Firm production technique environmental friendliness innovation	0.0025
b	Payback parameter	160
τ^K	Tax rate on K-Firm profit	0.1
δ^K	K-Firm dividend payout rate	0.75
φ	C-Firm desired inventory ratio	0
σ	C-Firm adaptive demand expectations parameter	0.16
$\mathfrak{Q}$	Maximum output producible with one unit of capital good	40
u	C-Firms' desired capacity utilization	0.8
λ	C-Firm maximum capacity growth	0.25
θ	C-Firm price updating probability	1
Δ^μ	C-Firm mark-up adjustment coefficient	0.01

Table B2 – continued from previous page

Symbol	Description	Value
ϕ	C-Firm maximum borrowing coefficient	10
ω_1	Weight of relative price in C-Firm competitiveness	20
ω_2	Weight of relative ability to satisfy demand in C-Firm competitiveness	1
ω_3	Parameter limiting size of period-to-period change in C-Firm market share	0.8
χ	Sensitivity of C-Firm market share to competitiveness	-1.39
τ^C	Tax rate on C-Firm profits	0.1
ξ_C	Share of loans C-Firms must repay at the end of a period	0.15
Δ	C-Firm dividend payout rate	0.75
$\mathfrak{d}_K^1$	Lower bound of distribution for entering K-Firm deposits	0.5
$\mathfrak{d}_K^2$	Upper bound of distribution for entering K-Firm deposits	0.5
$\mathfrak{n}$	Parameter used to initialise brochures and revenues of entering K-Firms	10
$\mathfrak{f}$	Lower bound for market share below which a C-Firm exits	1e-5
$\mathfrak{f}^{entry}$	Parameter used to initialise market shares of entering C-Firms	0.0005
$\mathfrak{d}_C^1$	Lower bound of distribution for entering C-Firm deposits	0.1
$\mathfrak{d}_C^2$	Upper bound of distribution for entering C-Firm deposits	0.9
μ^{entry}	Initial mark-up of entering C-Firms	0.2
$\mathfrak{p}$	Shape parameter of Pareto distribution for initialisation of bank-firm network	0.8
$\mathfrak{p}_1^C$	Lower bound of Pareto distribution for initialisation of bank-C-Firm network	10
$\mathfrak{p}_2^C$	Upper bound of Pareto distribution for initialisation of bank-C-Firm network	35
$\mathfrak{p}_1^K$	Lower bound of Pareto distribution for initialisation of bank-K-Firm network	1
$\mathfrak{p}_2^K$	Upper bound of Pareto distribution for initialisation of bank-K-Firm network	4
υ_B	Bank deposit interest rate markdown	1
cm	Credit multiplier	0.05
$\mathfrak{v}$	Sensitivity of credit multiplier to bank fragility	0
$\mathfrak{M}$	Individual bank lending rate mark-up parameter	0.1
μ^B	Bank baseline loan rate mark-up	0.7
$\mathfrak{G}$	Banks' desired holdings of government Bonds as a fraction of outstanding loans	0.1
τ^B	Tax rate on bank profits	0.1
δ^B	Bank dividend payout rate	0.5
$\mathfrak{d}_B^1$	Lower bound for distribution of net worth of bailed out banks	1
$\mathfrak{d}_B^2$	Upper bound for distribution of net worth of bailed out banks	1
$g_{\tau,Em}^1$	Emission tax rate parameter	140
$g_{\tau,Em}^2$	Emission tax rate parameter	0.0346
ξ_{GB}	Share of government bonds which must be repaid in each period	1
υ_{GB}	Markdown parameter for government bond interest rate	0
r	Central bank lending rate intercept[30]	0.04
$\underline{r}$	Central bank rate lower bound	1e-6
ι_1	Taylor rule smoothing parameter	0.74
ι_1	Taylor rule inflation sensitivity	1.23
ι_1	Taylor rule unemployment sensitivity	0.17
$\pi *$	Central bank target inflation rate	0.02015
$U *$	Central bank target unemployment rate	0.05
υ_{CB}	Markdown parameter for central bank deposit rate	1
ς^e	Upper bound on per-period expansion of green energy capacity	0.015
b^e	Energy sector payback period parameter	10
$\aleph^E$	Maximum lifespan of energy production plants	80
$\mathfrak{o}^e$	Fraction of energy sector revenue devoted to R&D	0.01
$\mathfrak{x}^E$	Share of energy sector R&D expenditure devoted to brown energy[31]	0.6
$\mathfrak{b}_1^E$	Parameter governing probability of innovation in energy technology	0.01
$\mathfrak{b}_2^E$	Shape parameter of beta distribution for energy technology innovation	3
$\mathfrak{b}_3^E$	Shape parameter of beta distribution for energy technology innovation	1.5
$\mathfrak{b}_4^E$	Lower bound for random energy technology innovation	-0.01
$\mathfrak{b}_5^E$	Lower bound for random energy technology innovation	0.005
δ^E	Energy sector dividend payout rate	0.99
δ^F	Fossil fuel sector dividend payout rate	0.01

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[30] Annual

[31] As set out in Section A.14, the simulation settings used in this paper are such that this parameter is not used.

Table B3: Full list of climate module parameters

Symbol	Description	Value
Υ_1	Temperature anomaly intercept (cumulative emission module)	-0.2674
Υ_2	Temperature anomaly emission sensitivity (cumulative emission module)	0.0018
g_{Em}	Growth rate of exogenous emissions[32]	0.0045
Ж	Carbon fertilisation effect of atmospheric carbon on net primary production	0.42
Б	Heat stress effect of global warming on net primary production	-0.01
$Я_1$	Decaying time of humus (years)	27.8
$Я_2$	Decaying time of biomass (years)	10.6
$Я_3$	Fraction of decaying biomass carbon ending up in humus	0.428
$eddy$	Eddy diffusion coefficient $\left(\frac{m^2}{year}\right)$	4400
ND	Number of ocean layers depicted	5
$depth$	Depth of ocean layers (meters; additive)	100 300 300 1300 1800
$Ш_1$	Pre-industrial carbon content of the upper ocean layer (GtC)	1023.73
$Ш_2$	Impact of global warming on the effect of $Ш_1$	0.003
$Ц_1$	Revelle factor	9.7
Ц2	Effect of carbon content on the Revelle factor	3.92
$Д_1$	Multiplicative factor used in carbon content loop	1.5
$Д_2$	Multiplicative factor used in carbon content loop	2
n^{clim}	Maximum number of carbon content loop interations	5
ϵ^{clim}	Termination criterion for carbon content loop	1e-10
$Л_1$	Multiplicative factor to account for non-CO_2 radiative forcing	1.12
$Л_2$	Radiative forcing from e-folding CO_2 $\left(\frac{W}{m^2}\right)$	5.35
Ю	Outgoing radiation per degree of warming $\left(\frac{W}{m^2 K}\right)$	1.23
И	Number of seconds per climate module time-unit (1 year here)	31557600
З	Fraction of planetary surface covered by oceans	0.708
Ч	Hear capacity of water$\left(\frac{J}{m^3}\right)$	4230000
ϵ_2^s	Determines lower bound of the probability that a firm will be affected by a climate shock	0.01

Table B4: Full list of economic model initial values

Symbol	Description	Value
LS_0	Initial labour force	25000
Pr_{κ_0}	Labour productivity of initial capital good vintages	1
EE_{κ_0}	Energy efficiency of initial capital good vintages	1
EF_{κ_0}	Environmental friendliness of initial capital good vintages	60
$Pr_{k,0}$	Labour productivity of initial K-Firm production techniques	0.275
$EE_{k,0}$	Energy efficiency of initial K-Firm production techniques	1
$EF_{k,0}$	Environmental friendliness of initial K-Firm production techniques	60
$TE\kappa_0^{de}$	Thermal efficiency of initial brown energy vintage	0.01
$c_{\kappa_0^{ge}}$	Per-unit expansion cost of initial green energy vintage	0.05
$TE\kappa_0^{de}$	Emission intensity of initial brown energy vintage	110
$D_{h,0}$	Initial household deposits	250000
$D_{e,0}$	Initial energy sector deposits	10000
$D_{k,0}$	Initial individual K-Firm deposits	500
$D_{c,0}$	Initial individual C-Firm deposits	320
$NW_{B,0}$	Initial aggregate banking sector net worth	70000
A_0	Initial central bank advances	0
$\mathfrak{l}_{c,0}$	Initial individual C-Firm loans	470
w_0	Initial nominal wage rate	1
$\mathfrak{K}_{c,0}$	Initial individual C-Firm productive capacity	1280

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[32] Annual

Table B4 – continued from previous page

Symbol	Description	Value
$\frac{\mathfrak{K}^{ge}_0}{\mathfrak{K}^{ge}_0+\mathfrak{K}^{ge}_0}$	Initial share of green energy productive capacity	0.2
$p_{f,0}$	Initial fossil fuel price	1e-05
$\mu_{e,0}$	Initial energy sector mark-up	0.05
$\mu_{c,0}$	Initial C-Firm mark-up	0.2
$\tau_0^{Em,C}$	Initial emission tax rate on C-Firms	0
$\tau_0^{Em,K}$	Initial emission tax rate on K-Firms	0
$\tau_0^{Em,E}$	Initial emission tax rate on the energy sector	0.000025
$r^l_{CB,0}$	Initial central bank policy rate[33]	0.04

Table B5: Full list of climate module initial values

Symbol	Description	Value
$\mathscr{E}_0^{\Sigma}$	Initial cumulative emissions (GtC)	600
$\mathscr{E}_0$	Initial emissions per climate module time-unit (1 year here; GtC)	12
NPP_0	Initial net primary production in GtC per climate module time-unit (1 year here)	85.1771
$\mathscr{C}_0^{at}$	Pre-industrial stock of atmospheric carbon (GtC)	864.6616
$Temp_0$	Initial temperature anomaly	1.0856
$\mathscr{C}_0^{hum}$	Initial carbon stock in humus (GtC)	1095
$\mathscr{C}_0^{bio}$	Initial carbon stock in biosphere (GtC)	1014.7
$\mathscr{C}_{d,0}^{oc}$	Initial carbon content of each ocean layer (GtC)	1056
		3136
		3110
		13334
		18429
$\mathscr{H}_{d,0}^{oc}$	Initial heat content of each ocean layer per square meter of ocean surface $\left(\frac{J}{m^2}\right)$	4.5922e8
		8.9026e8
		5.1279e8
		3.2073e8
		0.2518e8

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[33] Annual