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Identification of one independent shock in structural VARs

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Identification of one independent shock in structural VARs *

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Abstract

We establish the identification of a specific shock in a structural vector autoregressive model under the assumption that this shock is independent of the other shocks in the system, without requiring the latter shocks to be mutually independent, unlike the typical assumptions in the independent component analysis literature. The shock of interest can be either non-Gaussian or Gaussian, but, in the latter case, the other shocks must be jointly non-Gaussian. We formally prove the global identification of the shock and the associated column of the impact multiplier matrix, and discuss parameter estimation by maximum likelihood. We conduct a detailed Monte Carlo simulation to illustrate the finite sample behavior of our identification and estimation procedure. Finally, we estimate the dynamic effect of a contraction in economic activity on some measures of economic policy uncertainty.

Keywords: Independent component analysis, Non-Gaussian maximum likelihood, Impact multipliers, Economic policy uncertainty.

JEL: C32, C52

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1 Introduction

Structural vector autoregressive (SVAR) models are valuable tools for addressing both theoretical and practical economic questions. To recover the future effects of economic shocks on the variables in the system and their instantaneous relationships it is essential to identify the structural shocks. Common identification schemes use short- and long-run restrictions, sign restrictions or instrumental variables (see e.g. Sims, 1980; Blanchard and Quah, 1989; Faust, 1998; Mertens and Ravn, 2012). More agnostic statistical identification can be achieved through heteroskedasticity (Sentana and Fiorentini, 2001) or, more recently, through non-Gaussianity, as in Lanne et al. (2017) and Gouriéroux et al. (2017), based on independent component analysis (ICA). In the ICA setting, the Darmois-Skitovich theorem ensures identification of the matrix of impact multipliers up to column permutations and scaling (see Comon, 1994; Moneta et al., 2013). However, if the ICA assumptions fail, the model becomes underidentified. Lanne and Luoto (2021) consider a GMM estimator that achieves local identification under weaker assumptions, requiring only fourth moment independence. Guay (2021) derives sufficient conditions for local identification based on the third and fourth unconditional moments of reduced-form innovations, which allow determining prior to estimation which subsets of the structural parameters are identified. Mesters and Zwiernik (2024) establish identification by replacing the independence assumption with higher order moment or cumulant restrictions. Lee and Mesters (2024) propose a semiparametric estimator which remains robust even when the shocks are nearly Gaussian. Maxand (2020) explore the identification of a non-Gaussian subvector of shocks in the presence of multiple Gaussian components using independence-based approaches. For a complete review of the statistical identification approach we refer to Lewis (2024).

Researchers are often primarily interested in the effects of a single shock, such as the monetary policy shock. In this paper we consider weakening the full mutual independence assumption, which is often too strong and not innocuous, as suggested by Montiel Olea et al. (2022). Specifically, we focus on a setting where the effect of one shock is of interest and assume its independence from the other ones. Under our assumptions, we establish the identification of the shock of interest and the corresponding column of the impact multiplier matrix. Notably, the shock of interest can be statistically identified even if it is Gaussian. Furthermore, in order to identify the shock of interest, we do not need to impose a column-permutation of the estimated impact matrix, as it is typically done in applications of ICA to SVAR models. Related works on partial statistical identification of SVAR models include Lütkepohl et al. (2024), Anttonen et al. (2023), and Keweloh et al. (2023). In Section 2, we present the main theoretical results and discuss estimation methods. Section 3 provides the results of a simulation study and an empirical illustration is presented in Section 4. Section 5 concludes.

2 Svar model and identification

Consider the following N-variate stable Svar(p) model

$$\mathbf{X}_t = \boldsymbol{\mu} + \mathbf{A}_1\mathbf{X}_{t-1} + \ldots + \mathbf{A}_p\mathbf{X}_{t-p} + \mathbf{C}\boldsymbol{\varepsilon}_t, \tag{1}$$

where $\mathbf{u}_t = \mathbf{X}_t - P(\mathbf{X}_t|I_{t-1})$ denote the reduced-form innovations, corresponding to the residuals of the linear projection of $\mathbf{X}_t$ on past information, with $\mathbf{u}_t = \mathbf{C}\,\boldsymbol{\varepsilon}_t$, and $det(I_N - A_1 z - \cdots - A_p z^p) \neq 0$, for all $|z| \leq 1$ and $z \in \mathbb{C}$. Without loss of generality, the variance matrix of the structural errors can be normalized, so that $E[\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'] = \mathbf{I}_N$. Then, the reduced-form error variance matrix is $\boldsymbol{\Sigma} = \mathbf{C}\mathbf{C}'$. Assume the matrix of impact multipliers $\mathbf{C}$ is invertible, hence $\boldsymbol{\varepsilon}_t = \mathbf{C}^{-1}\mathbf{u}_t$. Knowledge of $\mathbf{C}$ allows tracing the effects of economic shocks on current and future values of the model variables and the instantaneous relationships among them. As is well-known, identification of $\mathbf{C}$ requires further statistical assumptions and/or prior economic information.

2.1 Identification with partially independent components

Consider partitions of the N structural shocks $\boldsymbol{\varepsilon}_t$ and of the $N \times N$ matrix $\mathbf{C}$

$$\begin{aligned}\boldsymbol{\varepsilon}_t &= \left(\varepsilon_{1t}, \boldsymbol{\varepsilon}_{2t}'\right)' \\ \mathbf{C} &= [\mathbf{c}_1\mathbf{C}_2]\,,\end{aligned} \tag{2}$$

where ε_{1t} is a scalar, and $\boldsymbol{\varepsilon}_{2t}$, $\mathbf{c}_1$, $\mathbf{C}_2$ have dimensions $(N-1)\times 1$, $N \times 1$, $N \times (N-1)$, respectively. The following proposition establishes identification of $\mathbf{c}_1$.

Proposition 1 *Consider the SVAR model in (1) and partitions in (2). Assume that*

i) $\mathbf{C}$ *is nonsingular;*

and the error process is such that

ii) ε_{1t} *is independent of* $\boldsymbol{\varepsilon}_{2t}$;

iii) one of the following is true:

(a) ε_{1t} *is non-Gaussian and* $\boldsymbol{\varepsilon}_{2t}$ *is Gaussian;*

(b) ε_{1t} *is Gaussian,* $\boldsymbol{\varepsilon}_{2t}$ *is jointly non-Gaussian, and* $\boldsymbol{\delta}'\boldsymbol{\varepsilon}_{2t}$ *is non-Gaussian* $\forall\boldsymbol{\delta} \in \Re^{N-1}$;

(c) ε_{1t} *is non-Gaussian and* $\boldsymbol{\varepsilon}_{2t}$ *is jointly non-Gaussian.*

Given two alternative representations $\mathbf{C}\boldsymbol{\varepsilon}_t = \mathbf{C}^*\boldsymbol{\varepsilon}_t^*$ *with* $\mathbf{C}\mathbf{C}' = \mathbf{C}^*\mathbf{C}^{*'}$ *and* $E[\boldsymbol{\varepsilon}_t^*\boldsymbol{\varepsilon}_t^{*\prime}] = \mathbf{I}_N$*, where* $\mathbf{C}^*$ *and* $\boldsymbol{\varepsilon}^*$ *also satisfy the same assumptions with analogous partitions, it holds that:*

$$\mathbf{c}_1^* = \pm\mathbf{c}_1; \qquad \mathbf{C}_2^* = \mathbf{C}_2\mathbf{Q} \ \textit{with}\ \mathbf{Q}\mathbf{Q}' = \mathbf{I}_{N-1}; \qquad \varepsilon_{1t}^* = \pm\varepsilon_{1t}; \qquad \boldsymbol{\varepsilon}_{2t}^* = \mathbf{Q}'\boldsymbol{\varepsilon}_{2t}.$$

Notice that case (iii)-(a) is just a special case of Maxand (2020), while cases (iii)-(b) and (iii)-(c) are new and allow identification of the shock of interest even when the other shocks are not independent (differently from the standard Ica case).

2.2 Maximum likelihood estimation

We estimate the model parameters by non-Gaussian maximum likelihood. To do so, define the vector of parameters $\mathbf{\Phi} = (\boldsymbol{\theta}', \boldsymbol{\eta}')'$, where $\boldsymbol{\theta} = (\boldsymbol{\mu}', \mathbf{a}', \mathbf{c}')'$, $\boldsymbol{\eta} = (\boldsymbol{\eta}_1', \boldsymbol{\eta}_2')'$, with $\mathbf{a} = vec(\mathbf{A}_1, \ldots, \mathbf{A}_p)$, $\mathbf{c} = vec(\mathbf{C})$, and $\boldsymbol{\eta}_1'$, $\boldsymbol{\eta}_2'$ are the vector of shape parameters of the distributions assumed for ε_{1t} and $\boldsymbol{\varepsilon}_{2t}$. The log-likelihood function for a sample of size T is

$$\mathcal{L}_T(\mathbf{X}; \mathbf{\Phi}) = -\frac{T}{2} \log |CC'| + \sum_{t=p+1}^{T} \log f(\varepsilon_{1,t}(\boldsymbol{\theta}), \boldsymbol{\eta}_1) + \sum_{t=p+1}^{T} \log g(\boldsymbol{\varepsilon}_{2t}(\boldsymbol{\theta}), \boldsymbol{\eta}_2), \tag{3}$$

where $f(\cdot)$ and $g(\cdot)$ are the assumed parametric densities. For example, if a standardized Student t distribution is assumed for the shocks, we have

$$\begin{aligned} \log g(\boldsymbol{\varepsilon}_{2t}(\boldsymbol{\theta}), \eta) \quad = \quad & \log \left[\Gamma \left(\frac{[N-1]\,\eta + 1}{2\eta} \right) \right] - \log \left[\Gamma \left(\frac{1}{2\eta} \right) \right] - \frac{N-1}{2} \log \left(\frac{1-2\eta}{\eta} \right) - \frac{N-1}{2} \log \pi \\ & - \frac{(N-1)\eta + 1}{2\eta} \log \left[1 + \frac{\eta}{1-2\eta} \boldsymbol{\varepsilon}_{2t}(\boldsymbol{\theta})' \boldsymbol{\varepsilon}_{2t}(\boldsymbol{\theta}) \right], \end{aligned}$$

where $\eta = 1/\nu$ with ν equal to degrees of freedom, and $\log f(\varepsilon_{1,t}(\boldsymbol{\theta}), \eta)$ is obtained by setting $N = 2$ in the equation above.

Since $\mathbf{C_2}$ is only identified up to rotations it is convenient to impose $(N-1)(N-2)/2$ arbitrary exact identification restrictions on $\mathbf{C}_2$ which amounts to fixing a particular rotation matrix $\mathbf{Q}$. Assumption *(ii)* and $\det(\mathbf{Q}) = 1$ imply that the estimator of $\mathbf{c}_1$, which identifies the shock of interest, is numerically invariant to the particular choice of the identification restrictions.

3 Monte Carlo simulations

To evaluate the finite sample behaviour of our estimator, we conduct a simulation exercise using a VAR(1) model with $N = 5$ with:

$$\mathbf{X}_t = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 0 & 0.8 \end{bmatrix} \mathbf{X}_{t-1} + \mathbf{C}\boldsymbol{\varepsilon}_t$$

We consider two different designs for $\mathbf{C}$:

$$\text{Design 1:} \quad \mathbf{C} = \begin{bmatrix} 4 & 1 & 1 & 2 & -1 \\ 0 & 4 & 1 & 3 & -2 \\ 1 & 2 & 2 & -1 & -4 \\ 2 & 1 & -1 & 6 & -1 \\ 1 & 2 & -1 & -2 & 6 \end{bmatrix}, \qquad \text{Design 2:} \quad \mathbf{C} = \begin{bmatrix} 1 & 1 & 2 & 4 & -1 \\ 4 & 1 & 3 & 0 & -2 \\ 2 & 2 & -1 & 1 & -4 \\ 1 & -1 & 6 & 2 & -1 \\ 2 & -1 & -2 & 1 & 6 \end{bmatrix}.$$

Notice the under Design 1, the columns of $\mathbf{C}$ satisfy the ordering proposed in Ilmonen and Paindaveine (2011), while under Design 2 they do not.

We assume three different data-generating processes (DGPs) for the structural shocks:

DGP A:	$\varepsilon_{1t} \sim \text{Student-}t(0, 1, 5);$	$\boldsymbol{\varepsilon}_{2t} \sim N(\mathbf{0}, \mathbf{I}_{N-1}).$
DGP B:	$\varepsilon_{1t} \sim N(0, 1);$	$\boldsymbol{\varepsilon}_{2t} \sim \text{Student-}t(\mathbf{0}, \mathbf{I}_{N-1}, 5).$
DGP C:	$\varepsilon_{1t} \sim \text{Student-}t(0, 1, 8);$	$\boldsymbol{\varepsilon}_{2t} \sim \text{Student-}t(\mathbf{0}, \mathbf{I}_{N-1}, 5).$

For each design, we generate 2000 samples of size $T = 1000$ and estimate the model by maximizing the likelihood under the assumption that ε_{1t} follows a Student-t distribution and $\boldsymbol{\varepsilon}_{2t}$ follows a multivariate Student-t distribution with degrees of freedom ν_1 and ν_2, respectively. We impose six arbitrary zero restrictions on the elements of $\mathbf{C}_2$. Additionally, we estimate the model using the standard ICA assumptions, where $\mathbf{C}$ is unrestricted and each shock is assumed to follow an independent Student-t distribution with degrees of freedom υ_i ($i = 1, \ldots, N$), as in Lanne et al. (2017). Table 1 presents the Monte Carlo means and standard deviations of our proposed estimator (ICA-1S), the standard independent components estimator (ICA), and the ICA estimator where, after each replication, the columns of $\hat{\mathbf{C}}$ are permuted according to the scheme proposed by Ilmonen and Paindaveine (2011) (ICA-P).

To save space, Table 1 presents results only for the estimator of $\mathbf{c}_1$, which measures the impact of the shock of interest. Our proposed estimator consistently shows superior precision across all designs. The ICA estimator, however, exhibits significant variability, primarily due to the underidentification of the correct column permutation. Under Design 1, when the columns of the ICA estimator of $\mathbf{C}$ are ordered using the Ilmonen and Paindaveine (2011) scheme, the ICA-P estimator performs reasonably well. This is expected since the true $\mathbf{C}$ in Design 1 adheres to the column ordering of Ilmonen and Paindaveine (2011). In contrast, Design 2 presents a different scenario, as shown in the bottom panel of Table 1, where both the ICA and ICA-P estimators perform poorly. In contrast, our proposed ICA-1S estimator continues to perform remarkably well, as it does not rely on any particular *a priori* assumptions about the column ordering of $\mathbf{C}$.

4 Empirical illustration

SVAR models have been extensively used in the empirical macroeconomic literature to investigate the relationship between uncertainty and economic activity (Fajgelbaum et al., 2017; Carriero et al., 2021; Ludvigson et al., 2021). However, most works have focused on measuring the effects of uncertainty on the business cycle, but it has been pointed out that causality can also run in the opposite direction (Angelini et al., 2019). Carriero et al. (2021) investigate this issue through a Bayesian estimated SVAR model with stochastic volatility that allows for both causal directions. Their findings point out that some macroeconomic variables have indeed a significant and contemporaneous feedback effect on uncertainty. In this illustrative application, we examine the dynamic impact that a contractionary shock to economic activity has on uncertainty. Thus, we focus on the backward direction of the causal relationship, and for this, we consider a SVAR model with four variables for the US: rate of growth of industrial production (IPI), economic policy uncertainty (EPU), monetary policy uncertainty (MPU) and fiscal policy uncertainty (FPU). Data are monthly and IPI data are obtained from the FRED database, while we refer to Baker et al. (2016) for the three uncertainty measures considered, downloadable at *https://www.policyuncertainty.com/*. An alternative definition of uncertainty is provided by Jurado et al. (2015), who define it as the common volatility in the unforecastable component of a number of economic indicators. Given the illustrative purpose of this application, we prefer to use the data from Baker et al. (2016), so that we have a setting with one shock (the shock to economic activity) independent of multiple uncertainty (possibly mutually dependent) shocks.

The sample period spans from 1985:1 to 2019:12. To clearly analyse the relationship between real economic activity and uncertainty, we do not include data sampled from the COVID-19 pandemic period. Lenza and Primiceri (2022) have shown that dropping these observations is not appropriate for forecasting, but it is an acceptable solution for the sake of parameter estimation, given the presence of extreme observations. We estimate the model with 4 lags using our method and the standard ICA maximum likelihood estimator, obtaining

$$\hat{\mathbf{C}}_{\text{ICA-1S}} = \begin{bmatrix} 0.487 & -0.174 & 0.071 & 0.065 \\ -0.087 & -0.092 & 0.088 & 0.174 \\ -0.125 & 0 & 0.378 & 0.219 \\ -0.061 & 0 & 0 & 0.318 \end{bmatrix}, \quad \hat{\mathbf{C}}_{\text{ICA}} = \begin{bmatrix} 0.166 & -0.082 & 0.474 & -0.061 \\ 0.214 & 0.015 & -0.083 & 0.018 \\ 0.326 & 0.240 & -0.126 & -0.153 \\ 0.233 & 0.091 & -0.057 & 0.195 \end{bmatrix}.$$

The results reveal that the first column of $\hat{\mathbf{C}}_{\text{ICA-1S}}$ (our method) aligns well with expectations. An (expansionary) shock on economic activity exerts a positive impact on IPI and negatively affects various measures of uncertainty. Interestingly, the third column of $\hat{\mathbf{C}}_{\text{ICA}}$ (standard ICA) closely resembles the first column $\hat{\mathbf{C}}_{\text{ICA-1S}}$, reinforcing the hypothesis that the economic-activity shock is independent of other shocks. However, the standard $\hat{\mathbf{C}}_{\text{ICA}}$ estimator fails to accurately spot the economic-activity shock, leaving the task of labelling to the researcher.

Figure 1 presents the impulse response functions to a negative shock to (IPI), which we interpret as a contractionary shock to economic activity, along with bootstrap-based confidence intervals. The MPU index shows a more pronounced response compared to the EPU and FPU indices, though the latter two exhibit greater persistence.

5 Concluding remarks

We examine a significant scenario in which economists aim to quantify the dynamic effects of a specific shock on system variables within a SVAR model. Our findings demonstrate that, under certain assumptions, the shock of interest is statistically identified without assuming a predetermined permutation of the impact multiplier matrix. Our Monte Carlo simulation study confirms the feasibility of our proposal, and the empirical exercise on the relationship between economic activity and uncertainty indices illustrates the potentiality of applications in contexts in which one is interested in identifying a single independent structural shock.

Our approach could be extended to relax the statistical independence assumption between the shock of interest and other shocks by considering only higher order moments independence. Importantly, the identifying assumptions are testable, and it would be valuable to adapt the testing procedures from Amengual et al. (2022, 2024), Matteson and Tsay (2017), and Maxand (2020) to our context. We are currently exploring these research avenues.

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Table 1: Monte Carlo results

Design 1		DGP A			DGP B			DGP C		
Parameter	True	ICA-1S	ICA	ICA-P	ICA-1S	ICA	ICA-P	ICA-1S	ICA	ICA-P
C_{11}	4.00	3.97	2.83	3.96	3.95	2.71	3.95	3.97	2.75	3.97
		(0.26)	(1.46)	(0.29)	(0.18)	(1.47)	(0.21)	(0.17)	(1.47)	(0.22)
C_{21}	0.00	0.01	1.16	0.01	0.01	1.08	0.00	0.01	1.09	0.02
		(0.41)	(1.64)	(0.44)	(0.31)	(1.65)	(0.38)	(0.26)	(1.63)	(0.32)
C_{31}	1.00	1.01	1.24	1.01	0.99	1.08	0.99	0.99	1.03	0.99
		(0.38)	(1.41)	(0.40)	(0.28)	(1.48)	(0.35)	(0.24)	(1.36)	(0.30)
C_{41}	2.00	1.98	2.03	1.98	1.96	1.99	1.97	2.00	2.08	2.00
		(0.49)	(1.62)	(0.52)	(0.36)	(1.61)	(0.44)	(0.31)	(1.59)	(0.37)
C_{51}	1.00	0.97	0.01	0.96	1.00	0.03	0.98	0.99	0.04	0.99
		(0.52)	(2.18)	(0.54)	(0.37)	(2.23)	(0.46)	(0.33)	(2.19)	(0.39)
Design 2		DGP A			DGP B			DGP C		
Parameter	True	ICA-1S	ICA	ICA-P	ICA-1S	ICA	ICA-P	ICA-1S	ICA	ICA-P
C_{11}	1.00	0.98	2.54	3.38	0.99	2.38	3.36	0.99	2.42	3.36
		(0.35)	(1.42)	(0.91)	(0.26)	(1.35)	(0.81)	(0.24)	(1.34)	(0.82)
C_{21}	4.00	3.92	1.58	0.78	3.95	1.52	0.70	3.96	1.60	0.72
		(0.61)	(1.57)	(1.26)	(0.23)	(1.65)	(1.22)	(0.22)	(1.60)	(1.23)
C_{31}	2.00	1.92	0.93	1.13	1.96	0.80	1.15	1.97	0.89	1.21
		(0.44)	(1.53)	(1.26)	(0.27)	(1.71)	(1.45)	(0.25)	(1.69)	(1.47)
C_{41}	1.00	1.01	2.47	2.14	1.00	2.41	2.12	1.00	2.45	2.06
		(0.49)	(2.58)	(2.06)	(0.36)	(2.56)	(2.01)	(0.33)	(2.52)	(1.99)
C_{51}	2.00	2.01	-0.52	-0.03	1.99	-0.40	-0.08	2.00	-0.52	-0.10
		(0.58)	(2.08)	(1.96)	(0.36)	(2.33)	(2.00)	(0.33)	(2.26)	(2.08)

Monte Carlo means and (standard deviations) of different estimators of $\mathbf{c}_1 = (C_{11}, \ldots, C_{N1})'$. Sample length=1000. Replications=2000.

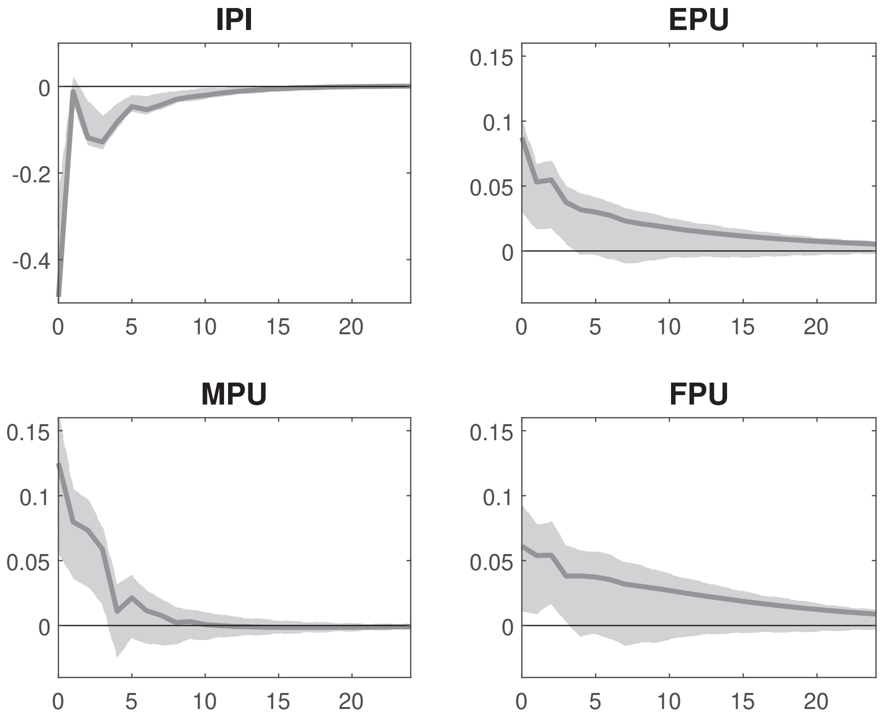

Figure 1: Impulse response functions to a contractionary shock to economic activity and 16% and 84% bootstrapped quantiles.

Appendix

A Proof of Proposition 1

Notice first that identification of $\boldsymbol{\mu}$ and the autoregressive matrix coefficients $\mathbf{A}_j$, $j = 1\ldots,p$ is trivially obtained from the reduced-form VAR and, thus, we focus on the identification of $\mathbf{c}_1$, the first column of $\mathbf{C}$.

Let $\mathbf{C}\boldsymbol{\varepsilon}_t = \mathbf{C}^*\boldsymbol{\varepsilon}_t^*$

$$\boldsymbol{\varepsilon}_t = \mathbf{C}^{-1}\mathbf{C}^*\boldsymbol{\varepsilon}_t^* = \mathbf{M}\boldsymbol{\varepsilon}_t^*.$$

Under Assumption (*ii*) ε_{1t}^* and $\boldsymbol{\varepsilon}_{2t}^*$ are mutually independent, and moreover, under Assumption (*iii*)-*(a)* ε_{1t}^* is non-Gaussian. Let

$$\begin{aligned}
\varepsilon_{1t} &= m_{11}\varepsilon_{1t}^* + \sum_{i=2}^{N} m_{1i}\varepsilon_{it}^* \\
\varepsilon_{\ell t} &= m_{\ell 1}\varepsilon_{1t}^* + \sum_{i=2}^{N} m_{\ell i}\varepsilon_{it}^* \quad \ell = 2,\ldots,N\,,
\end{aligned}$$

given that ε_{1t} and $\varepsilon_{\ell t}$ are mutually independent and ε_{1t}^* is non-Gaussian, $m_{11}m_{\ell 1} = 0$, $\forall l \geq 2$ (see Theorem 19 in Comon, 1994). In addition, Lemma 9 in Comon (1994) implies that $m_{11} \neq 0$, since ε_{1t} is non-Gaussian. Therefore, $m_{\ell 1} = 0$, $\forall \ell \geq 2$. Also, since ε_{1t} and $\boldsymbol{\varepsilon}_{2t}$ are mutually independent and $E(\varepsilon_{it}^{*2}) = 1$, we have

$$E(\varepsilon_{1t}\varepsilon_{\ell t}) = \sum_{i=2}^{N} m_{1i}m_{\ell i}E(\varepsilon_{it}^{*2}) = 0 \;\Rightarrow\; \sum_{i=2}^{N} m_{1i}m_{\ell i} = 0$$

which implies that $(m_{12},\ldots m_{1N})$ is orthogonal to $(m_{\ell 2},\ldots m_{\ell N})$, $\forall \ell.\geq 2$. Invertibility of $\mathbf{M}$ implies that $m_{1l} = 0$, $\forall \ell \geq 2$, and therefore

$$\mathbf{C}^* = \mathbf{C}\mathbf{M}, \quad \text{with} \quad \mathbf{M} = \begin{bmatrix} m_{11} & \mathbf{0}' \\ \mathbf{0} & \mathbf{Q} \end{bmatrix},$$

where $|m_{11}| = 1$ and $\mathbf{Q}$ is an orthonormal matrix of order $N-1$, which implies

$$\mathbf{c}_1^* = \pm\mathbf{c}_1 \quad \text{and} \quad \mathbf{C}_2^* = \mathbf{C}_2\mathbf{Q}.$$

Under Assumption (*ii*) ε_{1t}^* and $\boldsymbol{\varepsilon}_{2t}^*$ are mutually independent, while under Assumption (*iii*)-*(b)* $\boldsymbol{\varepsilon}_{2t}^*$ is jointly non-Gaussian and $\boldsymbol{\delta}'\boldsymbol{\varepsilon}_{2t}$ is non-Gaussian $\forall\boldsymbol{\delta} \in \Re^{N-1}$.

But then, for any real vector $\mathbf{q}$, Assumption (*ii*) implies ε_{1t}^* and $\mathbf{q}'\boldsymbol{\varepsilon}_{2t}^*$ are mutually independent. Let

$$\begin{aligned}
\varepsilon_{1t} &= m_{11}\varepsilon_{1t}^* + \sum_{i=2}^{N} m_{1i}\varepsilon_{it}^* = m_{11}\varepsilon_{1t}^* + \mathbf{q}_1'\boldsymbol{\varepsilon}_{2t}^* = m_{11}\varepsilon_{1t}^* + \alpha_1\mathbf{q}_1^{*\prime}\boldsymbol{\varepsilon}_{2t}^* \\
\varepsilon_{\ell t} &= m_{\ell 1}\varepsilon_{1t}^* + \sum_{i=2}^{N} m_{\ell i}\varepsilon_{it}^* = m_{\ell 1}\varepsilon_{1t}^* + \mathbf{q}_\ell'\boldsymbol{\varepsilon}_{2t}^* = m_{\ell 1}\varepsilon_{1t}^* + \alpha_\ell\mathbf{q}_\ell^{*\prime}\boldsymbol{\varepsilon}_{2t}^*
\end{aligned}$$

where $\mathbf{q}_j^* = (m_{j2}, \ldots, m_{jN})/\alpha_j$. Since ε_{1t} and $\varepsilon_{\ell t}$ are mutually independent, and $\mathbf{q}_1^{*\prime}\boldsymbol{\varepsilon}_{2t}^*$ and $\mathbf{q}_\ell^{*\prime}\boldsymbol{\varepsilon}_{2t}^*$ are non-Gaussian, $\alpha_1\alpha_\ell = 0, \forall\ell \geq 2$ If $\alpha_1 \neq 0$, it must be that $\alpha_\ell = 0$ for any $\forall\ell \geq 2$ which is impossible because $\mathbf{M}$ is full rank, and therefore the only nonzero element of the first row of $\mathbf{M}$ is m_{11}. We then finally have

$$E(\varepsilon_{1t}\varepsilon_{\ell t}) = \sum_{i=2}^{N} m_{1i}m_{\ell i}E(\varepsilon_{it}^{*2}) = m_{11}m_{\ell 1} = 0,$$

which implies that the only non zero elements of the first column is m_{11}. Under assumptions *(ii)* and *(iii)-(c)* identification is implied *a fortiori* by the previous two cases.

□