NDBC Technical Document 03-01

Nondirectional and Directional Wave Data Analysis Procedures

Stennis Space Center
January 2003

U.S. DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
National Data Buoy Center

NDBC Technical Document 03-01

Nondirectional and Directional Wave Data Analysis Procedures

Marshall D. Earle, Neptune Sciences, Inc.
40201 Highway 190 East, Slidell, LA 70461
January 2003

U.S. DEPARTMENT OF COMMERCE
Don Evans, Secretary

National Oceanic and Atmospheric Administration
Conrad C. Lautenbacher, Jr., Administrator

National Data Buoy Center
Paul F. Moersdorf, PhD, Director

List of Acronyms

AFOS	Automation of Field Operations and Services
ARES	Acquisition and Reporting Environmental System
AR/T	Angular Rate / Tilt Sensor
CERC	Coastal Engineering Research Center
CHL	Coastal and Hydraulics Laboratory
CSC	Computer Sciences Corporation
CSD	Cross-Spectral Density
DACT	Data Acquisition and Control Telemetry
DQA	Data Quality Assurance
DWA	Directional Wave Analyzer
DWA-MO	Magnetometer Only Directional Wave Analyzer
DWPM	Directional Wave Processing Module
EDF	Equivalent Degrees of Freedom,
FFT	Fast Fourier Transform
FWGP	Field Wave Gaging Program
GOES	Geostationary Operational Environmental Satellite
GSBP	General Service Buoy Payload
HIPPY	A wave sensor made by Datawell
MARS	Multi-functional Acquisition and Reporting System
MEM	Maximum Entropy Method
MLM	Maximum Likelihood Method
MO	Magnetometer Only
NCDC	National Climatic Data Center
NDBC	National Data Buoy Center
NDWPM	Nondirectional Wave Processing Module
NMC	National Meteorological Center
NOAA	National Oceanic and Atmospheric Administration
NODC	National Oceanographic Data Center
NSI	Neptune Sciences, Inc.
NWS	National Weather Service
PSD	Power Spectral Density
PTF	Power Transfer Function
RAO	Response Amplitude Operator
SAIC	Science Applications International Corporation
TSC	Technical Support Contractor (for NDBC)
U.S.	United States
USACE	U.S. Army Corps of Engineers
VEEP	Value Engineered Environmental Payload
WA	Wave Analyzer
WAVEOB	WMO Wave Observation Code
WDA	Wave Data Analyzer
WMO	World Meteorological Organization
WPM	Wave Processing Module.
WRA	Wave Record Analyzer

Table of Contents

1.0 INTRODUCTION

The National Data Buoy Center (NDBC), a component of the National Oceanic and Atmospheric Administration's National Weather Service (NWS) maintains a network of data buoys to monitor oceanographic and meteorological data off U.S. coasts including the Great Lakes. NDBC's non-directional and directional wave measurements are of significant interest to users because of the importance of waves for boating, fishing, shipping, and other marine operations. NDBC's wave data and climatological information derived from the data are also used for a variety of engineering and scientific applications.

NDBC began its wave measurement activities in the mid-1970's. Since then, it has developed, tested, and deployed a series of buoy-mounted and fixed platform-mounted wave measurement systems. Types of systems for which data analyses are described in this report have been deployed since 1979. These systems are generally recognized as representing the evolving state-of-the-art in buoy wave measurement systems. As NDBC's newer systems have been developed, they have provided increasingly more comprehensive wave data. NDBC's data buoys relay wave information to shore by satellite in real-time. Transmission of raw time series data is not feasible with today's satellite-limited message lengths. Thus, much of the data analysis is performed onboard using pre-programmed microprocessors. Data Quality Assurance (DQA) information as well as key wave parameters are relayed to shore. Additional data processing and quality control are performed onshore. Several aspects of NDBC's wave data are unique to NDBC because of onboard analysis methods and NDBC's detailed consideration of buoy hull-mooring response functions.

2.0 PURPOSE

This report's primary purpose is to document NDBC's wave data analysis procedures in a single publication for users of NDBC's wave data who wish to understand how these data are processed. NDBC's methods for analysis of non-directional and directional wave measurements have been described over many years in papers published in the refereed literature and in papers presented at professional society meetings, in reports, and in internal documents and memoranda. Although these descriptions were correct at the time of their preparation, the methods have evolved over time so that no one document describes NDBC's wave data analysis procedures. In addition, concise descriptions with mathematical background are needed to facilitate technical understanding by NDBC's wave information users. This report is an update of an earlier technical report (Earle, 1996a) that is now out-of-date and that should no longer be used.

The U.S. Army Corps of Engineers (USACE) Coastal Engineering Research Center (CERC), now the Coastal and Hydraulics Laboratory (CHL), has been and is a major sponsor of NDBC buoys and a major user of its wave data. CERC/CHL has documented (Earle et al., 1995) wave data analysis procedures of all organizations, other than NDBC, that provide wave data and analysis results for its Field Wave Gaging Program (FWGP). Partly because of onboard data analysis with satellite transmission of results and consideration of buoy hull-mooring response functions, NDBC's wave data processing is significantly different from those of other organizations participating in the FWGP. Also, the FWGP's other wave measurements are made with different types of sensors (e.g. commercially made buoys and fixed in-situ sensors such as pressure sensors and wave orbital velocity sensors). This report's secondary purpose is to accompany and parallel the CERC/CHL documentation (Earle et al., 1995).

Finally, this report may be very useful as a comprehensive reference for those who may be planning or developing wave measurement buoy instrumentation or buoy-based wave measurement programs.

3.0 DATA ANALYSIS PROCEDURES

3.1 DOCUMENTATION APPROACH

Previous papers, reports, memoranda, and other pertinent documentation dating from the mid-1970's to the present were identified, obtained, and reviewed. In addition, the earlier edition (Earle, 1996a) of this document was reviewed. This information was examined for completeness and correctness. Modifications and additions were made to the earlier document to produce this revised document.

The first part of this section provides the mathematical background and theory that serve as the scientific basis for NDBC's wave data analysis procedures. This information is described first because much of it applies to several NDBC wave measurement systems. Analysis of data from individual systems is described next. A later section briefly describes archived results that are analysis outputs. Key definitions and further information about technical details are provided in several appendices.

This report emphasizes NDBC's wave data analysis techniques, rather than wave data collection, instrumentation, or applications. Aspects of wave data collection are not described except as they directly relate to wave data analysis. The most pertinent references pertaining to analysis that are available in the open literature or through NDBC are referenced. Many of these references also provide further detail about NDBC's data collection and instrumentation. There are numerous references describing data sets (e.g. descriptions of wave conditions during particular events of meteorological or oceanographic interest) of NDBC's wave data.

Unless they provide additional information about analysis procedures beyond that provided in other references, these applications oriented references are not referenced in this report.

Because this report parallels CERC/CHL documentation of wave data analysis procedures used by the FWGP, this report's format is similar to the corresponding CERC/CHL report (Earle et al., 1995). Where wave analysis theory is the same, text for the two reports is identical or similar.

3.2 MATHEMATICAL BACKGROUND AND DATA ANALYSIS THEORY OVERVIEW

NDBC's wave data analysis involves application of accepted time series, and spectral, analysis techniques to measured records of buoy motion. Hull-mooring response function corrections, methods for obtaining buoy orientation angles from magnetic field measurements, corrections for use of hull-fixed accelerometers on some systems, and encoding-decoding procedures for relay of information by satellite involve techniques developed by NDBC. Following parts of this section provide the mathematical background and theory which applies to analysis of data from NDBC's wave measurement systems. Some techniques, such as calculation of confidence intervals, are not routinely applied by NDBC. But, users of NDBC data may wish to apply these techniques to NDBC data. Information in this section is written generally to apply to analysis of buoy-collected wave data as much as possible, so that it will have application to other buoy wave measurements. Thus, specific symbols and manners in which equations are written may be different from those of some noted references. In several cases, equations can be re-written or combined in different ways to yield the same results. Because this section is generally applicable to analysis of buoy-collected wave data, information in it does not have to be repeated in later descriptions of particular NDBC wave measurement systems. The style of presenting mathematical background and theory also parallels the corresponding CERC/CHL documentation (Earle et al., 1995).

3.2.1 Types of Data and Assumptions

Non-directional wave data measured by NDBC buoys that do not report directional wave data consist of digitized records from a single-axis accelerometer with its measurement axis perpendicular to the deck of the buoy. If NDBC's buoys were perfect slope-following buoys, the accelerometer axis would be nearly perpendicular to the wave surface at all times.

Measured directional wave time series consist of digitized data representing one of the following types of data sets:

(1) buoy near-vertical acceleration (in the earth frame of reference), pitch, and roll measured using a Datawell HIPPY 40 sensor incorporating a nearly-vertical stabilized platform, and buoy azimuth obtained from measurements of the earth's magnetic field with a hull-fixed triaxial magnetometer. For some systems, near-vertical displacement time series are obtained by analog double integration of near-vertical accelerometer analog output .

(2) buoy acceleration from a single-axis accelerometer with its measurement axis perpendicular to the deck of the buoy, as well as buoy pitch, roll, and azimuth obtained from measurements of the earth's magnetic field with a hull-fixed magnetometer. Systems that make use of this suite of sensors are sometimes called Magnetometer Only (MO) systems, as they do not use a Datawell HIPPY 40 sensor, but derive pitch and roll from the magnetometer only.

For non-directional wave measurement systems, spectra are calculated directly from measured acceleration time series. For directional wave measurement systems, buoy pitch and roll time series are resolved into earth-fixed east-west and north-south buoy slope time series using instantaneous buoy azimuth. Spectra and cross-spectra are calculated from these slope time series and acceleration (or displacement) time series. Parameters derived from the spectra and cross-spectra provide non-directional and directional wave information, including estimates of directional wave spectra.

Spectral analysis assumes that the measured time series represent stationary random processes. Ocean waves can be considered as a random process. For example, two wave records that are collected simultaneously a few wave lengths apart would produce similar, but not identical, results due to statistical variability. Similarly, two wave records that are collected at exactly the same place, but at slightly different times, even when wave conditions are stationary, would produce similar, but not identical, results. For the purpose of documenting NDBC's wave data analysis, a random process is assumed to be simply one that must be analyzed and described statistically. A stationary process is one for which actual (true) values of statistical information (e.g. significant wave height or wave spectra) are time invariant. Wave conditions are not truly stationary. Thus, wave data analysis is applied to time series with record lengths over which wave conditions usually change relatively little with time.

In applying statistical concepts to ocean waves, the sea surface is assumed to be represented by a superposition of small amplitude linear waves with different amplitudes, frequencies, and directions. Statistically, individual sinusoidal wave components are assumed to have random phase angles between 0° and 360°. To describe the frequency distribution of these wave components, wave spectra are assumed to be narrow. That is, the wave components have frequencies within a reasonably narrow range. Except for high wave conditions, the assumption of small amplitude linear wave theory is usually realistic for intermediate and deep water waves. It is not generally true for shallow water waves, but NDBC's buoys are seldom in shallow water from a wave theory viewpoint. Regardless, this assumption is well-known to provide suitable results for most purposes. The narrow spectrum assumption, except in some cases of swell, is almost never strictly true, but nevertheless also provides suitable results for most purposes. Even though these assumptions are not always valid, they provide a theoretical basis for wave data analysis procedures and help establish a framework for a consistent approach toward analysis.

Kinsman (1965) provides useful descriptions of these concepts. Earle and Bishop (1984) describe wave analysis procedures involving statistics in an introductory manner. Several papers by Longuet-Higgins (e.g. 1952, 1957, 1980) are among the most fundamental and useful papers that describe statistical aspects of ocean waves. Longuet-Higgins et al. (1963) provide a classic description of directional wave data analysis that is used for buoy directional wave measurements. Donelan and Pierson (1983) and Kuik et al. (1988) provide statistical results that are particularly useful for examining statistical variability effects. Dean and Dalrymple (1984) discuss theoretical and practical aspects of waves without emphasis on statistics. Borgman (1990) provides a comprehensive overview of wave theory and analysis. The corresponding CERC/CHL documentation (Earle et al., 1995) describes several basic aspects of wave analysis.

Time series analysis and spectral analysis techniques that are applied to measured buoy motion data are described in the following parts of this section.

3.2.2 Data Segmenting

A measured time series can be analyzed as a single record, or as a number of shorter records, or segments, that are part of the single record. Data segmenting with overlapping segments decreases statistical uncertainties (i.e. confidence intervals). Data segmenting also increases spectral leakage since, for the shorter record lengths of each segment, fewer Fourier frequencies are used to represent actual wave frequencies. However, for most wave data applications, spectral leakage effects are small compared to spectral confidence interval sizes.

Several older NDBC wave measurement systems use data segmenting, while NDBC's newer systems do not. These newer systems are the Wave Processing Module (WPM), the Directional Wave Data Processing Module (DWPM), and the Non-directional Wave Data Processing System (NDWPM). Data segmenting is based on ensemble averaging of J spectral estimates from 50% overlapping data segments following procedures adapted from Welch (1967). Estimates from 50% overlapping segments produce better statistical properties for a given frequency resolution than do estimates from the commonly used band-averaging method (Carter et al., 1973) without data segmenting. A measured time series, x(nΔt), with N data points digitized at a time interval, Δt, is divided into J segments of length L. Each segment is defined as x(j,nΔt), where j represents the segment number, and x indicates that values within each segment are the same as the ones that were in the equivalent part of the original time series. Following are definitions of the segments

$$1 \quad x(1,n\Delta t) = x(n\Delta t),\ n = 0\ \ldots\ ,L-1$$

$$2 \quad x(2,n\Delta t) = x(n\Delta t),\ n = \frac{L}{2},\ \ldots,\frac{3L}{2}-1$$

$$J \quad x(J,n\Delta t) = x(n\Delta t),\ n = (J-1)\ \frac{L}{2},\ldots,\ (J+1)\frac{L}{2}-1$$

where

$$J = \frac{2\left(N - \frac{L}{2}\right)}{L}$$

The number of segments affects confidence intervals which are discussed later. Not using segmenting is equivalent to using one segment containing all data points (L = N, J = 1).

3.2.3 Mean and Trend Removal

Measured buoy motion time series (acceleration, pitch, and roll) as well as calculated east-west and north-south buoy slopes (obtained from pitch and roll with consideration of buoy azimuth) have small, often near-zero, time-averaged means. Non-zero mean values of pitch and roll may occur due to wind and/or current effects on a buoy's hull and superstructure. Means of several parameters are calculated as DQA checks. Means are also removed from buoy acceleration and slope time series before further analysis (data segmenting and spectral analysis) to obtain wave spectra.

Seiches, tides, and other long-period water elevation changes produce negligibly small trends in the buoy motion time series that are measured. Thus, trends are not calculated or removed.

3.2.4 Spectral Leakage Reduction

Spectral leakage occurs when measured wave data time series, which represent contributions from wave components with a nearly continuous distribution of frequencies, are represented by the finite number of frequencies that are used for covariance calculations or Fourier transforms. Mathematically, a data record or segment has been convolved with a boxcar window. The data begin abruptly, extend for a number of samples and then end abruptly. The boxcar window has the advantage that all data samples have equal weight. However, the discontinuities at the beginning and the end introduce side lobes in later Fourier transforms. The side lobes allow energy to leak from the specific frequency at which a spectral estimate is being computed to higher and lower frequencies. How to reduce spectral leakage is discussed in many time series analysis textbooks (e.g. Bendat and Piersol, 1971, 1980, 1986; Otnes and Enochson, 1978) and leads to some disagreement in the wave measurement community.

Wave data are frequently analyzed without spectral leakage reduction. This approach is followed by the WPM. The rationale is that leakage effects are usually small for wave parameters, such as significant wave height and peak wave period, even though spectra may differ somewhat from those that would be obtained with use of leakage reduction techniques. Moreover, effects of leakage on spectra are generally far less than spectral confidence interval sizes. Not reducing leakage also eliminates the need for later variance corrections.

A measured wave data time series that has been processed by covariance methods or Fourier transforms provides estimates of wave component contributions at a finite number of frequencies. Different NDBC wave measurement systems that apply leakage reduction techniques do so in the time domain and the frequency domain. These systems use Hanning because it is the most commonly-used method for reducing spectral leakage (e.g. Bendat and Piersol, 1971, 1980, 1986; Otnes and Enochson, 1978).

For time domain Hanning, a cosine bell taper over each data segment is used. The cosine bell window is given by

where L is the record or data segment length.

$$W(n\Delta t) = \frac{1}{2}\left(1 - \cos\left(\frac{2\pi n}{L}\right)\right), 0 \le n \le L - 1$$

The data are multiplied by the tapering function

$$x_w(j,n\Delta t) = x(j,n\Delta t) * W(n\Delta t)$$

where the subscript w indicates that the data has been windowed.

Because of Hanning, the windowed data variance is less than the un-windowed data variance. Frequency domain corrections are made by multiplying spectral and cross-spectral densities by pairs of standard deviation ratios for each involved time series. A standard deviation ratio is given by

$$c = \frac{\sigma}{\sigma_w}$$

where

$$\sigma = \left[\frac{1}{L-1}\left(\sum_{n=0}^{L-1} x(j,n\Delta t)^2 - \frac{\left(\sum_{n=0}^{L-1} x(j,n\Delta t)\right)^2}{L}\right)\right]^{1/2}$$

$$\sigma_w = \left[\frac{1}{L-1}\left(\sum_{n=0}^{L-1} x_w(j,n\Delta t)^2 - \frac{\left(\sum_{n=0}^{L-1} x_w(j,n\Delta t)\right)^2}{L}\right)\right]^{1/2}$$

in which x and x_w indicate the time series before and after windowing respectively.

Leakage reduction in the frequency domain involves weighting frequency components within a window which is moved through all analysis frequencies. Because the variance of the data is reduced, a subsequent variance correction is also made.

For frequency domain Hanning, spectral estimates are smoothed using the following equations.

$$S_{w-xy}(m\Delta f) = 0.25\, S_{xy}((m-1)\Delta f) + 0.5\, S_{xy}(m\Delta f) + 0.25\, S_{xy}((m+1)\Delta f)$$

$$S_{w-xy}(M\Delta f) = 0.5\, S_{xy}(M - 1\Delta f) + 0.5\, S_{xy}(M\Delta f)$$

where S_{xy} is a Cross-Spectral Density (CSD) estimate for time series, x and y, M is the number of frequencies, Δf is the frequency interval, the subscript w indicates frequency domain windowing, and m = 1, 2, ... (M-1). Cross-spectral density estimates are defined in the Fourier transform section. If x=y, S_{xy} becomes a power spectral density estimate, S_{xx}.

The standard deviation ratios for frequency domain corrections can be written as

$$c = \frac{\sigma}{\sigma_w}$$

where

$$\sigma = \left[S_{xx}\right]^{1/2}$$

$$\sigma_w = \left[S_{xx-w}\right]^{1/2}.$$

As for corrections based on time domain calculations, windowed spectral and cross-spectral densities are multiplied by pairs of standard deviation ratios for each involved time series. For example, for buoy directional wave measurements, standard deviations may be for acceleration (or displacement), east-west slope, or north-south slope time series.

3.2.5 Covariance Calculations

The earliest system described in this report is the General Service Buoy Payload (GSBP) Wave Data Analyzer (WDA) system. This system is the only described system that determines wave spectra (non-directional) by covariance (autocorrelation) techniques rather than by use of Fast Fourier Transforms (FFT's). Spectral calculations via covariance techniques follow standard procedures (e.g. Bendat and Piersol, 1971, 1986).

For a buoy acceleration record, a covariance (autocorrelation) function for M lags is obtained from

$$R_j = \frac{1}{N-j}\sum_{n=1}^{N-j} a_n a_{n+j} - a_{mean}^2$$

where j is the lag number (1, 2, ... M), a_n indicates the nth acceleration value, N is the number of data points per record, and a_{mean} is the mean acceleration.

Acceleration spectra are computed from

$$S_{aa}(m\Delta f) = 2\Delta t\left(R_0 + 2\sum_{j=1}^{M-1} R_j \cos\left(\frac{\pi jm}{M}\right) + (-1)^k R_M\right)$$

Spectral estimates can be obtained at frequencies, mΔf, where m ranges from 1 to M and Δf is given by

$$\Delta f = \frac{f_{Nyquist}}{M}$$

and the Nyquist frequency is given by

$$f_{Nyquist} = \frac{1}{2\Delta t}$$

where Δt is the time between data points. The Nyquist frequency is the highest resolvable frequency in a digitized time series. Spectral energy at frequencies above $f_{Nyquist}$ incorrectly appears as spectral energy at lower frequencies.

The GSBP WDA calculates spectral estimates only up to m = (2/3)M so that spectra are cut-off at a frequency corresponding to the half-power frequency of the onboard low-pass anti-aliasing analog filter. Further GSBP WDA data analysis details are provided in a following section.

3.2.6 Fourier Transforms

Fourier transforms are calculated by fast Fourier transform (FFT) algorithms that were tested with known input data. Tests of system hardware also include tests with known analog inputs in place of actual sensor signals. Algorithms themselves were implemented in different computer languages, for different onboard microprocessors, and for different wave measurement systems. Thus, the algorithms themselves are not identical.

An FFT is a discrete Fourier transform that provides the following frequency domain representation, X, of a measured time series, x (or x_w with use of a window).

$$X(j,m\Delta f) = \Delta t\sum_{n=0}^{L-1} x(j,n\Delta t)\, e^{-i\frac{2\pi mn}{L}}$$

where

$$m = 0, 1, 2, \ldots, \frac{L}{2} \qquad L\ even$$

$$m = 0, 1, 2, \ldots, \frac{L-1}{2} \qquad L\ odd$$

The real and imaginary parts of X are given by

$$Re\left[X(j,m\Delta f)\right] = \Delta t\sum_{n=0}^{L-1} x(j,n\Delta t)\cos\left(\frac{2\pi mn}{L}\right)$$

$$Im\left[X(j,m\Delta f)\right] = \Delta t\sum_{n=0}^{L-1} x(j,n\Delta t)\sin\left(\frac{2\pi mn}{L}\right)$$

Spectral estimates are obtained at Fourier frequencies, mΔf, where the interval between frequencies is given by

$$\Delta f = \frac{1}{L\Delta t}$$

Utilized FFT algorithms require that the number of data points be a power of two for computational efficiency so that L is even. The frequency corresponding to m = L/2 is the Nyquist frequency given by

$$f_{Nyquist} = \frac{1}{2\Delta t}$$

3.2.7 Spectra and Cross-Spectra

Power Spectral Density (PSD) estimates for the jth segment are given by where X^* is the complex conjugate of X.

$$S_{xx}(j,m\Delta f) = \frac{X^*(j,m\Delta f)X(j,m\Delta f)}{L\Delta t} = \frac{|X(j,m\Delta f)|^2}{L\Delta t}$$

Cross-spectral density (CSD) estimates for the jth segment are given by

$$S_{xy}(j,m\Delta f) = \frac{X^*(j,m\Delta f)Y(j,m\Delta f)}{L\Delta t} = C_{xy}(j,m\Delta f) - iQ_{xy}(j,m\Delta f)$$

where C_{xy} is the co-spectral density (co-spectrum), Q_{xy} is the quadrature spectral density (quadrature spectrum), and X and Y are frequency domain representations of the time series, x and y. These equations have units of spectral density which are the multiplied units of each time series divided by Hz. C_{xy} and Q_{xy} can be written as

$$C_{xy}(j,m\Delta f) = \frac{Re[X]\,Re[Y] + Im[X]Im[Y]}{L\Delta t}$$

$$Q_{xy}(j,m\Delta f) = \frac{Im[X]\,Re[Y] - Re[X]Im[Y]}{L\Delta t}$$

where the arguments, (j,mΔf), of X and Y are not shown for brevity.

Final spectral estimates are obtained by averaging the results for all segments to obtain

$$S_{xx}(m\Delta f) = \frac{1}{J}\sum_{j=1}^{J} S_{xx}(j,m\Delta f)$$

$$C_{xy}(m\Delta f) = \frac{1}{J}\sum_{j=1}^{J} C_{xy}(j,m\Delta f)$$

$$Q_{xy}(m\Delta f) = \frac{1}{J}\sum_{j=1}^{J} Q_{xy}(j,m\Delta f)$$

With the definition of C_{xy}, the equation for S_{xx} is not needed. The co-spectra, C_{xx} and C_{yy}, are the same as the power spectra, S_{xx} and S_{yy}. In most following sections, the argument, mΔf, is dropped and f is used to indicate frequency.

If data segmenting is not used, one segment (J = 1) contains all of the data points in the original time series and spectral estimates are at individual Fourier frequencies, mΔf, with L = N (the total number of data points in the measured time series). Spectral estimates are then band-averaged over groups of consecutive Fourier frequencies to increase the degrees of freedom and the statistical confidence.

Conventional wave data analysis with band-averaging often uses frequency bandwidths within which one more or one less Fourier frequency may fall for adjacent bands. This situation occurs when bandwidths are not multiples of the Fourier frequency interval. Thus, adjacent bands may have slightly different degrees of freedom and confidence intervals. For NDBC wave measurement systems that employ band-averaging, Fourier frequencies fall precisely on boundaries between frequency bands. Spectral and cross-spectral estimates at boundaries are spread equally into adjacent frequency bands when band-averaging is performed.

Frequency-dependent effects caused by the measurement systems (e.g. hull-mooring, sensor, electronic filtering) are corrected for at this point in the analysis. Non-zero frequency-dependent phase shifts have no effect on calculation of non-directional wave spectra but may affect directional wave spectra. Non-unity frequency-dependent response amplitude operators affect both non-directional and directional wave spectra. A following section describes hull-mooring response function corrections which are an important and unique aspect of NDBC's wave data analysis.

3.2.8 Confidence Intervals

Calculated spectra are estimates of the actual spectra. Degrees of freedom describe the number of independent variables which determine the statistical uncertainty of the estimates.

Following standard wave data analysis practice, confidence intervals are defined for power spectra

(PSD's), but not for co-spectra and quadrature spectra (CSD's). Confidence intervals are not provided by NDBC, but may be calculated by users of NDBC's wave information.

There is 100 α% confidence that the actual value of a spectral estimate is within the following confidence interval

$$\left[\frac{S_{xx}(f)EDF}{\chi^2\left(EDF,\frac{(1.0-\alpha)}{2}\right)}, \frac{S_{xx}(f)EDF}{\chi^2\left(EDF,\frac{(1.0+\alpha)}{2}\right)}\right]$$

where X^2 are percentage points of a Chi-Square probability distribution and EDF are the equivalent degrees of freedom. As earlier noted, $S_{xx} = C_{xx}$. Ninety percent confidence intervals ($\alpha = 0.90$) are often used.

For data processed by Fourier transforms without data segmenting, the degrees of freedom for a frequency band equal two multiplied by the number of Fourier frequencies within the band. For consistency, degrees of freedom are referred to as equivalent degrees of freedom to correspond to terminology used for segmented data. Without segmenting, the equivalent degrees of freedom are given by

$$EDF = 2n_b$$

where n_b is the number of Fourier frequencies in the band. In general, n_b equals the bandwidth divided by the Fourier frequency interval.

For data processed by Fourier transforms with data segmenting, the equivalent degrees of freedom, EDF, for J segments, is given by
where J is the number of overlapping segments. The value 0.4 in the above equation is an approximation, but is adequate for segment lengths (≥100 data points) used by NDBC.

$$EDF = \frac{2J}{\left(1+\frac{0.4(J-1)}{J}\right)}$$

For data processed by covariance techniques, the equivalent degrees of freedom are given by

$$DF = \frac{2N}{M}$$

where N is the number of data points and M is the number of covariance lags. The degrees of freedom when data are processed by covariance techniques appears to be larger than the value based on the frequency separation interval because only every other spectral estimate is independent (e.g., Bendat and Piersol, 1971).

3.2.9 Directional and Non-Directional Wave Spectra

A directional wave spectrum provides the distribution of wave elevation variance as a function of both wave frequency, f, and wave direction, θ. Following the scientific convention used in many wave references, wave direction is the direction of wave propagation measured counterclockwise from east. As seen later, NDBC converts wave directions provided in products to correspond to directions from which waves come measured clockwise from north. A directional wave spectrum can be written as

$$S(f,\theta) = C_{11}(f)\, D(f,\theta)$$

where C_{11} is the non-directional wave spectrum (which could be determined from a wave elevation time series if such a time series were available) and D is a directional spreading function. Integration of a directional wave spectrum over all directions (0 to 2π) provides the corresponding non-directional spectrum given by

$$S(f) = \int_0^{2\pi} S(f,\theta) = C_{11}(f)$$

Following NDBC's conventions, subscripts are defined as follows:

1 =	wave elevation (also called displacement) which would correspond to buoy heave after sensor and buoy hull-mooring response corrections.
2 =	east-west wave slope which would correspond to buoy tilt in this direction after sensor and buoy hull-mooring response corrections.
3 =	north-south wave slope which would correspond to buoy tilt in this direction after sensor and buoy hull-mooring response corrections.

The CERC/CHL wave data analysis documentation (Earle et al., 1995) and some papers in the scientific literature use (x, y, and z) subscripts in place of (1, 2, and 3) subscripts. NDBC's wave measurement systems usually measure buoy acceleration rather than buoy heave. A superscript, m, indicates a spectral or cross-spectral estimate based on a measured time series that is not corrected for the effects of noise, hull-mooring responses, or sensor responses. Estimates based on buoy acceleration are converted to estimates based on wave elevation (displacement) during subsequent analysis steps as later described.

Directional wave spectra are estimated using a directional Fourier series approach originally developed by Longuet-Higgins et al. (1963). Since its development, this approach has been described and used by many others (e.g. Earle and Bishop, 1984; Steele at al., 1985; Steele et al., 1992). It yields directional analysis coefficients that are part of the World Meteorological Organization (WMO) WAVEOB code for reporting spectral wave information (World Meteorological Organization, 1988).

The directional Fourier series approach provides the directional Fourier coefficients, a_n and b_n, in the following truncated Fourier series

$$S(f,\theta) = \frac{a_o}{2} + \sum_{n=1}^{2}\left[a_n \cos(n\theta) + b_n \sin(n\theta)\right]$$

which can also be written as

$$S(f,\theta) = C_{11}(f)\, D(f,\theta)$$

in which

$$C_{11} = \pi a_o$$

and the directional spreading function is given by

$$D(f,\theta) = \frac{1}{\pi}\left(\frac{1}{2} + r_1 \cos[\theta - \theta_1] + r_2 \cos[2(\theta - \theta_2)]\right)$$

with

$$r_1 = \frac{1}{a_o}\left(a_1^2 + b_1^2\right)^{\frac{1}{2}}$$

$$r_2 = \frac{1}{a_o}\left(a_2^2 + b_2^2\right)^{\frac{1}{2}}$$

$$\theta_1 = \tan^{-1}(b_1, a_1)$$

$$2\,\theta_2 = \tan^{-1}(b_2, a_2)$$

Here, a comma separating numerator and denominator in the argument of the arc tangent means that the signs of the arguments are considered separately to place the angle in the correct quadrant. The ambiguity of π for θ_2 is resolved by choosing the value that is closest to θ_1. The parameter, θ_1 is called mean wave direction and the parameter, θ_2, is called principal wave direction. These directional parameters (r_1, θ_1, r_2, and θ_2) are more commonly considered as analysis results than the parameters (a_1, b_1, a_2, and b_2) originally developed by Longuet-Higgins et al. (1963). The latter parameters are often used as an intermediate calculation step.

Equations for calculation of the directional spectrum parameters from buoy wave measurements follow. Calculated directional spectra have units of wave elevation variance/(Hz- radians). Directional spectra are thus spectral densities in terms of both frequency, Hz, and direction, radians. Co-spectra and quadrature spectra estimates would be corrected for sensor and buoy hull-mooring responses before use of these equations. As seen when considerations that are specific to NDBC's data processing are described, NDBC combines response corrections, conversion from spectral estimates involving acceleration to those involving displacement, and conversion to the directional parameters (r_1, θ_1, r_2, and θ_2) into a few equations that can be applied without many intermediate steps.

In the following equations, wave number, k, is related to frequency, f, by the dispersion relationship for linear waves given by

$$(2\pi f)^2 = (gk)\tanh(kd)$$

where g is acceleration due to gravity and d is mean water depth during the measurements. For a given water depth, this equation can be solved for k by iterative methods or, as later described for NDBC's wave data analysis, k can be estimated from elevation and slope spectra.

For a buoy that measures heave (displacement), pitch, roll, and buoy azimuth (heading) the Longuet-Higgins directional parameters are given by

$$a_o = \frac{C_{11}}{\pi}$$

$$a_1 = \frac{Q_{12}}{k\pi}$$

Buoy azimuth is used to convert buoy pitch and roll values into buoy east-west and north-south slopes.

$$b_1 = \frac{Q_{13}}{k\pi}$$

$$a_2 = \frac{\left(C_{22} - C_{33}\right)}{k^2\pi}$$

$$b_2 = \frac{2\,C_{23}}{k^2\pi}$$

For a buoy that measures nearly-vertical acceleration, pitch, roll, and azimuth, the parameters are given by

$$a_o = \frac{C_{11}}{\left(2\pi f\right)^4\pi}$$

$$a_1 = -\frac{Q_{12}}{\left(2\pi f\right)^2 k\pi}$$

$$b_1 = -\frac{Q_{13}}{\left(2\pi f\right)^2 k\pi}$$

$$a_2 = \frac{\left(C_{22} - C_{33}\right)}{k^2\pi}$$

$$b_2 = \frac{2\,C_{23}}{k^2\pi}$$

where the subscript, 1, temporarily indicates acceleration which is considered positive upward and negative downward. Signs for the parameters involving quadrature spectra are often written as either positive or negative to consider outputs of particular accelerometers used to make the measurements. For these equations, wave slopes are positive if the slopes tilt upward toward the east (for east-west slopes) or toward the north (for north-south slopes).

The direction convention for these equations is the scientific convention (e.g. Longuet-Higgins et al., 1963) which is the direction toward which waves travel (propagation direction) measured counter-clockwise from the x axis (usually east, subscript 2, in NDBC's notation). Theoretically, this is the direction of the wave number vector for a particular wave component.

NDBC's direction convention is used by mariners and marine forecasters. It also corresponds to that of the World Meteorological Organization (WMO) WAVEOB code (World Meteorological Organization, 1988). An NDBC wave direction is the direction from which waves come measured clockwise from true north. During data processing, the local magnetic variation is used to convert from directions relative to magnetic north to directions relative to true north.. If the x axis is positive toward the east, an NDBC wave direction, α, is related to a scientific convention wave direction, θ, by

$$\alpha = \frac{3\pi}{2} - \theta$$

This method for estimating a directional wave spectrum is described mathematically as a directional convolution of a weighting function with the actual directional spectrum. For the utilized D(f,θ) parameters, the half-power width of the weighting function is 88°. This width is sometimes called the directional resolution. Longuet-Higgins et al. (1963) provide a weighting of the directional Fourier coefficients to prevent unrealistic negative values of D(f,θ) for directions far from θ_1, but this approach increases the half-power width to 130°. A directional spreading function that is determined by the described procedure is a smoothed version of the true directional spreading function. Even so, estimated directional spectra are useful. Separate directions of sea and swell can usually be identified since sea and swell often occur at different frequencies.

Higher resolution techniques, most notably Maximum Entropy Methods (MEM) and Maximum Likelihood Methods (MLM), have been developed. These techniques involve the same cross-spectral values and thus could be used. NDBC has investigated versions of these methods and has shown how NDBC's directional Fourier coefficients may be transformed to allow using these methods (Earle et al., 1999). However, because they are not as widely used, have several non-standardized versions, and may provide erroneous directional information unless they are carefully applied, they are not presently used by NDBC except for research purposes. Benoit (1994) summarizes and compares many higher resolution techniques.

3.2.10 Wave Parameters

NDBC calculates several widely-used wave parameters. As an example of their use, wave climatologies may be based on statistical analysis of wave parameters on a monthly, seasonal, or annual basis. As a second example, engineering calculations of extreme wave conditions may be based on statistical analysis of wave parameters near the times of highest waves during high wave events.

Significant wave height, H_{mo}, is calculated from the wave elevation variance which is also the zero moment, m_o, of a non-directional wave spectrum using

$$H_{mo} = 4.0\sqrt{m_o}$$

where m_o is computed from

$$m_o = \sum_{n=1}^{N} C_{11}\left(f_n\right) df_n$$

in which the summation is over all frequency bands (totaling N and centered at f_n) of the non-directional spectrum and df_n is the bandwidth of the nth band. For some NDBC wave measurement systems, frequency bands are more narrow at low frequencies than at high frequencies.

The theoretical significant wave height, $H_{1/3}$, is the average height of the highest one-third waves in a wave record. An assumption for approximating significant wave height by H_{mo} is that wave spectra are narrow-banded (e.g. Longuet-Higgins, 1952). Although this assumption is not strictly valid for actual waves, considerable wave data representative of different wave conditions shows that this method for determining significant wave height is suitable for nearly all purposes. Finite spectral width is the likely explanation for differences between $H_{1/3}$ and H_{mo} with H_{mo} values typically being about 5% to 10% greater than $H_{1/3}$ values (Longuet-Higgins, 1980).

NDBC does not calculate significant wave height confidence intervals, but these can be estimated by users based on statistical approaches for estimating time series variance confidence intervals (e.g. Bendat and Piersol, 1980; Donelan and Pierson, 1983). Confidence intervals depend on the total degrees of freedom, TDF, which in turn depend on spectral width. When a spectrum has been determined, the total degrees of freedom can be estimated from

$$TDF = \frac{2\left(\sum_{n=1}^{N} C_{11}\left(f_n\right)\right)^2}{\sum_{n=1}^{N}\left(C_{11}\left(f_n\right)\right)^2}$$

The 100 α% confidence interval for H_{mo} is given by

$$H_{mO}\left(\frac{TDF}{\chi^2\left(TDF, \frac{1.0-\alpha}{2}\right)}\right)^{\frac{1}{2}}, H_{mO}\left(\frac{TDF}{\chi^2\left(TDF, \frac{1.0+\alpha}{2}\right)}\right)^{\frac{1}{2}}$$

where χ^2 values are obtained from Chi-Square probability distribution tables. Ninety percent intervals (α = 0.90) are generally about -10% to +15% below and above calculated H_{mo} values. These confidence intervals are smaller than those for individual spectral density values because the variance is based on information over all frequencies. The following equations provide accurate results for 90% (α = 0.90) confidence intervals when TDF exceeds thirty which it usually does for waves.

$$\chi^2\left(TDF, 0.05\right) \cong TDF\left(1 - \frac{2}{9TDF} + 1.645\left(\frac{2}{(9TDF)}\right)^{\frac{1}{2}}\right)^3$$

$$\chi^2\left(TDF, 0.95\right) \cong TDF\left(1 - \frac{2}{9TDF} - 1.645\left(\frac{2}{(9TDF)}\right)^{\frac{1}{2}}\right)^3$$

Peak, or dominant, period is the period corresponding to the center frequency of the C_{11} (non-directional spectrum) spectral frequency band with maximum spectral density. That is, peak period is the reciprocal of the frequency, f_p (peak frequency), for which spectral wave energy density is a maximum. It is representative of the higher waves that occurred during the wave record. Confidence intervals for peak period are not usually estimated and there is no generally accepted method for doing so. Peak period, T_p, is given by

$$T_p = \frac{1}{f_p}$$

Average and zero-crossing periods are used less often than peak period, but provide important information for some applications. Average wave period is not calculated by NDBC, but can be calculated from non-directional spectra by the following equation

$$T_{av} = \frac{m_o}{m_1}$$

where T_{av} is average period. NDBC calculates zero-crossing wave period from the following equation

$$T_{zero} = \left(\frac{m_o}{m_2}\right)^{\frac{1}{2}}$$

where T_{zero} is zero-crossing period. The spectral moments (m_o, m_1, and m_2) are given by

$$m_i = \sum_{n=1}^{N} f_n^i C_{11}(f_n) df_n \quad i = 0, 1, 2$$

T_{zero} is called zero-crossing period because it closely approximates the time domain mean period which would be obtained from zero-crossing analysis of a wave elevation record. For this reason, NDBC and others often refer to T_{zero} as mean wave period. Average period and zero-crossing period each represent typical wave periods rather than periods of higher waves which are better represented by peak period.

Among those involved in wave data collection and analysis, there are differences of opinion about the best spectral width and directional width parameters. Calculation of these parameters is left to more specialized users of NDBC's wave information. Spectral and directional width parameters can be calculated from non-directional and directional spectra respectively.

Many applications of NDBC's data are based on a significant wave height, H_{mo}, peak wave period, T_p, and a representative wave direction for each data record rather than the spectrum. Mean wave direction, θ_1, using NDBC's direction convention, at the spectral peak is most often considered as the best direction to use as a representative direction. It is the mean wave direction of the non-directional spectrum frequency band with maximum spectral density, C_{11}.

A point of clarification to users is that NDBC's mean wave direction, as just defined, is sometimes called dominant wave direction by others. Some organizations also consider a mean wave direction to be representative of an entire wave spectrum by defining it as an energy weighted vector average of individual mean wave directions computed over all frequency bands. These distinctions may be important because directions associated with a single frequency band have considerably more statistical variability than directions computed over an entire spectrum.

NDBC also calculates a wave direction called peak, or dominant, wave direction. This direction is the direction with the most wave energy in a directional spectrum, $S(f,\alpha)$, where NDBC's direction convention is used. NDBC typically computes $S(f,\alpha)$ at 5° intervals to estimate peak, or dominant, wave direction. Dominant wave directions based on this definition are usually close to those based on mean wave direction associated with the maximum spectral density, C_{11}.

3.2.11 NDBC Buoy Considerations

3.2.11.1 Overview

This section documents aspects of NDBC's wave data analysis that are not simple applications of the provided mathematical background and theory. A key and unique aspect for determination of NDBC's non-directional spectra is use of a Power Transfer Function (PTF) to consider sensor, hull-mooring, and data collection filter responses. For directional spectra, methods for obtaining buoy azimuth, as well as pitch and roll for some wave measurement systems, from measurements of the earth's magnetic field have been developed and applied by NDBC. Directional spectra also require careful determination and application of both the PTF and phase responses associated with each measurement system. Accurate calculation of directional spectra require that frequency-dependent response information be determined for each measurement time period (i.e. set of simultaneous wave records) at each measurement location. To reduce quantities of information relayed to shore by satellite message, while preserving accuracies of relayed information, NDBC developed and uses special encoding-decoding procedures.

3.2.11.2 Non-Directional Power Transfer Function (PTF)

In buoy applications, a Power Transfer Function (PTF) converts acceleration spectra that are transmitted to shore to displacement spectra and corrects for frequency-dependent responses of the buoy hull and its mooring, the acceleration sensor, and any filters (analog or digital) that were applied to the data. The DACT DWA (defined and described later) now utilizes output from a nearly vertically-stabilized accelerometer, but formerly utilized an electronically double-integrated output from this accelerometer. In the latter case, the PTF considered the response of the double integration electronics and did not convert from acceleration to displacement spectra. The Power Transfer Function is given by

$$PTF = \left(R^{hH} R^{sH} R^{fa} R^{fn}\right)^2$$

where

R^{hH} = hull-mooring (h) Response Amplitude Opertator (RAO) for heave (H)
R^{sH} = sensor (s) RAO for heave (H)
R^{fa} = analog anti-aliasing filter (fa), if used, RAO
R^{fn} = digital (numerical) filter (fn), if used, RAO

R^{sH} considers [$(R^{sH})^2$ = radian frequency raised to the fourth power] conversion of acceleration spectra to displacement spectra, or the double integration electronics response. NDBC maintains tables of PTF component values for each wave measurement system. Steele et al. (1985, 1992) also discuss obtaining these values.

3.2.11.3 Hull-Mooring Heave Response Amplitude Operator, $R^{hH}(f)$

One of the elements of the Power Transfer Function (PTF) deserves particular discussion. While the other three elements of the PTF function are relatively easy to determine, determination of the hull-mooring heave RAO is usually complicated. Each hull type, and mooring-design, combination potentially has a different RAO. Since NDBC operates different types of hulls, and deploys them in various water depths requiring different mooring designs, it is a considerable task to determine all of the needed $R^{hH}(f)$ functions. This situation is compounded by effects of currents, particularly high speed currents.

For example, the heave RAO for a discus type hull depends on physical characteristics of the hull. The mass inertia of the hull causes the RAO to depend solely on frequency in cases where the wave length is much greater than the diameter of the hull. When the wave length is comparable to the diameter, the RAO depends on both frequency and the ratio of the hull diameter to the wave length. Ordinarily (in the absence of currents), wave length is uniquely related to frequency through the physics of linear wave theory, so that one may write the RAO either solely as a function of frequency or solely as a function of wave length. However, when significant currents are present, Doppler shifting can raise or lower the frequency seen by the hull, and the fixed relationship between frequency and wave length no longer holds. In this instance, precisely knowing the response of large hulls is virtually impossible, as existing theories do not allow the calculation of hull responses and, even if they did, calculations would require knowing current speed and direction at the station at the times of the measurements. For smaller hulls (3m or less), these complications cause little trouble, since these effects occur at frequencies so high that they do not affect the main part of the displacement spectrum.

During the 1970's, NDBC devoted significant resources toward determining heave RAO's for several relatively large hulls. The approach was to deploy a small buoy near the larger buoy and to determine the broad band displacement spectra from each The RAO of the small buoy was assumed to be unity. Then at each broad band frequency, f_c,

$$R^{hH}(f_c) = \sqrt{C_{11}(large\ buoy) / C_{11}(small\ buoy)}$$

Owing to Chi-Square probability distribution statistical variability of both C_{11} values, these estimates are scattered. So to achieve reasonable results, a month or so of data are needed. But, a simple least-squares fit of C_{11}(large) divided by C_{11}(small) is not appropriate because the scatter depends on the magnitude of the C_{11} values. A computer program (called BTFFIT) was developed to process the data in such a way that the program's user could make a reasonable estimate of R^{hH}. One example application was to data for a Datawell Waverider (small buoy) tethered to a NOMAD hull (large buoy) in the Gulf of Mexico. This analysis is the source of the RAO used for NOMAD hulls up to the present time.

In summary, determination of adequate RAO's for NDBC's hulls, particularly large hulls, is not easy because the effort to do it properly is substantial.

3.2.11.4 Noise Correction Functions for Acceleration Spectra

When buoy acceleration spectra are converted to displacement spectra, acceleration spectral density values are multiplied by the reciprocal of radian frequency to the fourth power. At low frequencies, this multiplier becomes very large. As a result, electronic noise, as well as noise that results from not vertically stabilizing or approximately stabilizing the accelerometer, is amplified (Earle and Bush, 1982; Earle et al., 1984). This noise is corrected for (removed) before acceleration spectra are converted to displacement spectra. Because the noise correction functions are determined empirically for different hull types from wave spectra with statistical variability, they are not exact for all wave conditions. In determining these functions, NDBC prefers to risk sometimes providing slightly reduced values of low frequency swell rather than to occasionally report low frequency swell that may not be real.

Considerable NDBC efforts have been made over the years to develop noise correction functions, NC(f), that remove the right amounts of noise. These functions depend on buoy hull type and their mathematical formulation includes sea state dependence. The equation for calculation of non-directional wave spectra is given by

$$\begin{array}{ll} C_{11} = \left(C_{11}^{m} - NC\right) / PTF & C_{11}^{m} \geq NC \\ C_{11} = 0 & C_{11}^{m} \leq NC \end{array}$$

where NC is a frequency-dependent noise correction function and PTF contains the multiplicative factor for converting acceleration spectra to displacement spectra. Empirical equations for values of NC were determined originally using low frequencies (e.g. 0.03 Hz) where real wave energy does not occur (Steele and Earle, 1979; Steele et al., 1985). Earle and Bush (1982) and Earle et al. (1984) showed that use of fixed accelerometers on some wave measurement systems was the main cause of noise and provided better formulations for empirical noise corrections when such systems are used. (Lang, 1987) further improved the noise correction functions that have been used up to the present time. For systems with nearly vertically-stabilized accelerometers (e.g. a HIPPY 40), Wang and Chaffin (1993) developed empirical NC values to correct for instrument-related low frequency acceleration spectra noise.

Older over corrections for noise associated with fixed accelerometer systems were reduced as refinements to the noise corrections functions were introduced into operational data processing from time to time after 1974. Thus, the quality of NDBC non-directional wave measurements in the low frequency, or swell, range of frequencies has improved with time so that all archived NDBC wave parameters are not of uniform quality. Some of the non-directional wave spectral estimates at low frequencies produced by NDBC in the 1970's and 1980's are undoubtedly biased low. Users of NDBC's older archived data should consider whether these changes affect their applications or interpretations of the data.

3.2.11.5 Directional Wave Spectra

Directional wave spectra involve the additional measurements of buoy azimuth, pitch, and roll. Buoy azimuth (heading of bow axis) is determined onboard a buoy from measurements of the earth's magnetic field along axes aligned in the buoy-fixed bow and starboard directions.

The bow (i=1) and starboard (i=2) components of the magnetic field vector measured by a magnetometer within a buoy hull are given by

$$\begin{aligned} B_i = b_{i0} + B_{ez}\left[b_{i1}\sin(P) - b_{i2}\cos(P)\sin(R)\right] + \\ B_{ey}\left\{\left[- b_{i2}\cos(R)\right]\sin(A) + \left[+ b_{i1}\cos(P) + b_{i2}\sin(P)\sin(R)\right]\cos(A)\right\} \end{aligned}$$

where A, P, and R are, respectively, buoy azimuth, buoy pitch, and buoy roll angles. NDBC's conventions are that azimuth of the buoy bow is positive clockwise from magnetic north, pitch is positive for upward bow motion, and roll is positive for downward motion of the starboard side of the buoy. These conventions are illustrated in Appendix D. B_{ey} and B_{ez} are, respectively, the horizontal and vertical components of the local earth magnetic field. The other constants correct for the residual (b_{10}, b_{20}) and induced (b_{11}, b_{22}, b_{12}, b_{21}) hull magnetic fields and can be determined for a particular hull by proven methods (Steele and Lau, 1986; Remond and Teng, 1990).

The equations for B_i (i=1, 2) are two equations in two unknowns, sin(A) and cos(A). Solutions are given by

$$\sin(A) = \frac{S}{D}$$

$$\cos(A) = \frac{C}{D}$$

in which

$$\begin{aligned} S = \left[b_{21}\cos(P) + b_{22}\sin(P)\sin(R)\right]\left[B_1 - b_{10}\right] - \\ \left[b_{11}\cos(P) - b_{12}\sin(P)\sin(R)\right]\left[B_2 - b_{20}\right] - B_{ez}\Delta\sin(R) \end{aligned}$$

$$C = \left[b_{22}\left(B_1 - b_{10}\right) - b_{12}\left(B_2 - b_{20}\right)\right]\cos(R) - B_{ez}\Delta\sin(P)\cos(R)$$

$$D = B_{ey}\Delta\cos(P)\cos(R)$$

$$\Delta = b_{11}b_{22} - b_{12}b_{21}$$

Azimuth relative to magnetic north is obtained from

$$A = \tan^{-1}\left(\frac{\sin(A)}{\cos(A)}\right)$$

A variation (Steele, 1990) of this approach (Steele and Earle, 1991) is used by NDBC for systems employing the Magnetometer Only (MO) method for obtaining buoy azimuth, pitch, and roll. Pitch and roll are obtained from high frequency parts of the measured magnetic field components. These in turn are used with the unfiltered (original) measured magnetic field components to provide azimuth as just described. The MO method is further described later in this section.

The azimuth quadrant is determined by the signs of sin(A) and cos(A). Magnetic azimuth is converted to true azimuth by use of the magnetic variation angle at the measurement location. Azimuth components relative to true north are obtained from

$$\sin\left(A_{true}\right) = \sin(A)\cos(VAR) + \cos(A)\sin(VAR)$$

$$\cos\left(A_{true}\right) = \cos(A)\cos(VAR) - \sin(A)\sin(VAR)$$

where A_{true} is bow heading relative to true north, A is bow heading relative to magnetic north, and VAR is the magnetic variation angle.

For each data point, azimuth is used to convert pitch and roll to buoy slopes in east-west and north-south directions, z_x and z_y respectively, relative to true north using

$$z_x = \left(\frac{\sin(A_{true})\sin(P)}{\cos(P)} - \frac{\cos(A_{true})\sin(R)}{\cos(P)\cos(R)} \right)$$

$$z_y = \left(\frac{\cos(A_{true})\sin(P)}{\cos(P)} + \frac{\sin(A_{true})\sin(R)}{\cos(P)\cos(R)} \right)$$

Derivations of these equations are provided in Appendix D.

NDBC's directional wave measurement systems obtain buoy pitch and roll in one of two ways. A Datawell HIPPY 40 sensor provides direct outputs of pitch [actually sin(P)] and roll [actually sin(R)cos(P)] relative to a nearly vertically-stabilized platform within the sensor. NDBC also developed the Magnetometer Only (MO) method to obtain pitch and roll from measurements of magnetic field components made with a magnetometer (Steele, 1990; Steele and Earle, 1991; Wang et al., 1994) that is rigidly mounted within a buoy hull. Because the second method utilizes magnetometer and fixed accelerometer data, sensors that are part of NDBC's directional wave measurement systems no matter what sensor suite is used, the second method is called the Magnetometer Only (MO) method.

Data show that hull inertia and a wind fin attached to a buoy measuring directional wave data inhibit buoy azimuth motions at wave frequencies. To obtain buoy pitch and roll, time series of B_1 and B_2 are first digitally high-pass filtered with the filter cutoff frequency just below the lowest frequency where non-negligible wave energy appears in the non-directional spectrum that is calculated from acceleration spectra, as earlier described. On the assumption that the variations in sin(A) and cos(A) above the cutoff frequency are negligible, and by making small angle approximations for P and R, it has been shown (Steele and Earle, 1991) that

$$b_{i1}\sin(p) - b_{i2}\sin(r) = \left(\frac{B'_i}{B_{ez}} \right)$$

where B'_i represents the high-pass filtered magnetic field data and p and r are pitch and roll angles, respectively, resulting from wave motion. Based on pitch and roll data from various buoys equipped with Datawell HIPPY sensors, the small angle approximations are generally realistic. These equations for i=1, 2 are two equations in two unknowns, sin(P) and sin(R). Solutions are given by

$$\sin(p) = \frac{b_{22}B'_1 - b_{12}B'_2}{\Delta B_{ez}}$$

$$\sin(r) = \frac{b_{21}B'_1 - b_{11}B'_2}{\Delta B_{ez}}$$

As noted by Steele and Earle (1991), these pitch and roll values could differ from true wave-produced pitch and roll values if forces due to winds or currents on the hull or mooring also cause non-negligible pitch and roll. However, it appears that this technique works well in practice even when non-wave-produced pitch and roll may occur.

NDBC's directional wave measurement systems determine wave direction information from cross-spectra between buoy acceleration (or displacement) and east-west and north-south buoy slopes. Depending on the system, accelerations may be measured by a nearly vertically-stabilized accelerometer or a fixed accelerometer with its measurement axis perpendicular to the buoy deck. Cross-spectra are corrected for sensor and buoy hull-mooring effects. Further mathematical detail is provided by Steele et al. (1985, 1992).

Effects of a buoy hull, its mooring, and sensors are considered to shift wave elevation and slope cross-spectra by a frequency-dependent phase angle, φ, to produce hull-measured cross-spectra. The phase angle, φ, is the sum of two angles,

$$\varphi = \varphi^{sH} + \varphi^{h}$$

where φ^{sH} = an acceleration (or displacement) sensor (s) phase angle for heave (H), and

$$\varphi^{h} = \varphi^{hH} - \varphi^{hS}$$

in which φ^{hH} = a hull-mooring (h) heave (H) phase angle, and φ^{hS} = a hull-mooring (h) slope (S) phase angle. Hull-mooring phase angle corrections are described more fully in Appendix B.

With a phase shift, φ, measured co-spectra and quadrature spectra are related to true quadrature spectra, Q_{12} and Q_{13}, by

$$Q_{12} = \left[\frac{\left(\frac{R^{sH}}{R^{h}} \right)}{PTF} \right] Q'_{12}$$

$$Q_{13} = \left[\frac{\left(\dfrac{R^{sH}}{R^{h}} \right)}{PTF} \right] Q'_{13}$$

where

$$R^{h} = \frac{R^{hS}}{R^{hH}}$$

in which R^{hS} = hull-mooring (h) amplitude response for buoy pitch and roll (same amplitude response as buoy slopes, S), and

$$Q'_{12} = Q^{m}_{12}\cos(\varphi) + C^{m}_{12}\sin(\varphi)$$
$$Q'_{13} = Q^{m}_{13}\cos(\varphi) + C^{m}_{13}\sin(\varphi)$$

Steele et al. (1985) use the following relationship from linear wave theory

$$k^{2}C_{11} = \left(C_{22} + C_{33}\right)$$

to show that

$$\frac{kR^{h}}{R^{sH}} = \left[\frac{\left(C^{m}_{22} + C^{m}_{33}\right)}{C^{m}_{11}} \right]^{\frac{1}{2}}$$

After considerable algebra, this equation combined with the quadrature spectra corrected for phase shifts and the Longuet-Higgins directional Fourier series formulation for directional spectra yields the following forms for the directional parameters used to describe directional wave spectra (Steele et al., 1985, 1992).

$$r_{1} = \left\{ \left[\left(Q'_{12}\right)^{2} + \left(Q'_{13}\right)^{2} \right] / \left[C^{m}_{11}\left(C^{m}_{22} + C^{m}_{33}\right) \right] \right\}^{\frac{1}{2}}$$

$$\alpha_{1} = \left(\frac{3\pi}{2}\right) - \tan^{-1}\left(Q'_{13}, Q'_{12}\right)$$

$$r_{2} = \left[\left(C^{m}_{22} - C^{m}_{33}\right)^{2} + \left(2C^{m}_{23}\right)^{2} \right]^{\frac{1}{2}} / \left(C^{m}_{22} + C^{m}_{33}\right)$$

whichever makes the angle between α_1 and α_2

$$\alpha_{2} = \left(\frac{3\pi}{2}\right) - \left[\frac{1}{2}\right] \tan^{-1}\left[2C^{m}_{23}, \left(C^{m}_{22} - C^{m}_{33}\right)\right] + \text{either } 0 \text{ or } \pi,$$

Directional spectra are then calculated from

$$S(f,\alpha) = C_{11}(f)D(f,\alpha)$$

where $C_{11}(f)$ is the non-directional spectrum, $D(f,\alpha)$ is a spreading function, f is frequency, and α is direction, clockwise relative to north, from which waves come. The spreading function can be written as a Fourier series,

The dispersion relationship is used to provide wave

$$D(f,\alpha) = \frac{\left[\frac{1}{2} + r_{1}\cos\left[\alpha - \alpha_{1}\right] + r_{2}\cos\left[2\left(\alpha - \alpha_{2}\right)\right] \right]}{\pi}$$

number, k, and the non-directional spectrum is also estimated from the wave slope spectra by

$$C_{11s} = \frac{\left(C_{22} + C_{33}\right)}{k^{2}}$$

This estimate, C_{11s}, is used as an ad-hoc DQA tool since it should be similar to C_{11} for parts of spectra with reasonable wave energy levels.

There are other equivalent ways that these calculations can be made and that provide identical results. For example, the Longuet-Higgins directional Fourier coefficients (a_1, b_1, a_2, and b_2) described in the background and theory section could be determined explicitly with consideration of response functions as an intermediate step and used in place of the directional parameters (r_1, α_1, r_2, and α_2). The forms of the equations provided by Steele et al. (1992) for the directional parameters are compact and well-suited for NDBC's operational use.

The angle, φ, is a critical parameter for obtaining proper wave directions. If φ was time-invariant, it could be determined once for each type of wave measurement system. However, as environmental conditions (e.g. winds, waves, and currents) change, NDBC's data show that phase angles change somewhat because of environment-induced forces and tensions placed on the hull-mooring system. Considerably better wave directions are obtained by calculating φ as a function of frequency for each wave record using linear wave theory.

According to linear wave theory, quadrature spectra between wave elevation and wave slopes are non-zero and co-spectra are zero for directions in which waves travel. NDBC's first approach for determining φ used this criteria with weighting based on quadrature spectra involving acceleration (or displacement) and both east-west and north-south slopes to consider that quadrature spectra involving slopes perpendicular to wave directions are small. The theory involves considerable algebra and is fully described by Steele et al. (1985). This approach generally worked well, but occasionally provided mean wave directions in the wrong compass quadrant.

During 1988-1989, an improved approach for determination of φ was implemented. This approach is based on the fact that quadrature spectra at each frequency have a maximum value for some direction. Steele et al. (1992) perform a complicated derivation that shows how φ is calculated with this criterion. This approach's advantage is that calculations are based on a direction in which substantial wave energy travels so that φ values are minimally contaminated by low signal to noise ratios. Steele et al. (1992) show that an ambiguity of π (180°) can be removed through use of an initial estimated φ value that is within π/2 (90°) of the correct value. For each buoy station, NDBC determines initial values as a function of frequency and adjusts the initial values if needed after the first data are acquired and processed. Procedures used since 1988-1989 for obtaining hull-mooring phase angles, φ, are described in Appendix B.

Because frequency dependent sensor responses (R^{sH}, φ^{sH}) are known and do not vary with time, these responses can be used to estimate hull-mooring effects (which implicitly include water depth effects on the mooring), separate from sensor effects. φ^h, which depends only on hull-mooring responses, is given by

$$\varphi^h = \varphi - \varphi^{sH}$$

which determines φ^h after φ is calculated. To separate hull-mooring and sensor effects for R^h, the parameter, q, is defined as

$$q = \tanh(kd) = \frac{\omega^2}{gk}$$

where k is wave number, d is mean water depth during a measurement time period, and g is acceleration due to gravity. From small amplitude wave theory, q = 1 in deep water and ranges from 0 to 1. Eliminating k between this equation and the equation for kR^h/R^{sH} yields

$$\frac{R^h}{q} = \left(\frac{gR^{sH}}{\omega^2}\right)\left[\frac{\left(C_{22}^m + C_{33}^m\right)}{\left(C_{11}^m\right)}\right]^{\frac{1}{2}}$$

Because q is usually 1 or nearly 1 for NDBC's measurement locations, and R^h is order of magnitude 1 for NDBC's buoys, $\frac{R^h}{q}$ has a convenient, predictable, range of values.

Estimating ($\frac{R^h}{q}$) from data was reasonably successful, but division of apparent noise in the slope co-spectra by small values of ω^2 led to unsatisfactory results at low frequencies. Better estimates are obtained by replacing (C_{22}^m,C_{33}^m) in the previous equation by noise-corrected values for frequencies ≤0.18 Hz. These values are obtained by subtraction of an empirical frequency-dependent noise correction function based on (C_{22}^m,C_{33}^m) at 0.03 Hz where there is not real wave energy (Steele et al., 1992).

Frequency-dependent values of φ^h and $\frac{R^h}{q}$ quantify hull-mooring phase and amplitude responses. As later noted, these parameters are included in information that is relayed to shore for some NDBC wave measurement systems.

3.2.12 Spectral Encoding and Decoding

Transmitting results by satellite requires minimizing transmitted record lengths. For systems that transmit spectral estimates, a nonlinear logarithmic encoding/decoding algorithm is used to do this, rather than a linear algorithm, so that small spectral values are accurately obtained in the presence of large values at other frequencies. Logarithmic encoding/decoding is also used for some other satellite relayed information that may have large ranges of values. Other values (e.g. angles) that have known ranges may be encoded and decoded linearly. Strictly speaking, encoding/decoding is not part of the wave analysis and it negligibly affects analysis results. Users who are interested in the considerable details of the encoding/decoding procedures should refer to Appendix E. This appendix also discusses the main reference (Steele, 1998) for encoding/decoding.

3.3 ANALYSIS OF DATA FROM INDIVIDUAL SYSTEMS TYPES OF SYSTEMS

NDBC applies aspects of the described mathematical background and theory to analyze data from the following wave measurement systems that have been deployed or are in advanced development stages at the time of this report (2001).

(1) GSBP Wave Data Analyzer (WDA), retired from service, large volumes of data archived
(2) DACT Wave Analyzer (WA)
(3) DACT Directional Wave Analyzer (DWA)
(4) DACT Magnetometer Only Directional Wave Analyzer (DWA-MO)
(5) VEEP Wave Analyzer (WA)
(6) Wave Processing Module (WPM)
(7) Directional Wave Processing Module (DWPM)
(8) Non-Directional Wave Processing Module (NDWPM)

where

GSBP = General Service Buoy Payload
DACT = Data Acquisition and Control Telemetry
VEEP = Value Engineered Environmental Payload

Table 1 summarizes the most important data collection parameters for these systems. Segment length is not applicable to the GSBP WDA because covariance calculations are made for entire data records. Segment length also is not applicable to the WPM because entire data records are Fourier transformed without segmenting. As noted with the table, systems that do not use analog and/or digital filters rely on hull filtering of high frequency waves to avoid aliasing. This approach works well because NDBC buoys are large compared to wave lengths of high frequency waves that could cause aliasing. These waves also have negligible energy for most applications of NDBC wave information. A 3 m diameter discus buoy has become NDBC's standard buoy. But, in severe environments, such as the Gulf of Alaska, non-directional wave data are collected by boat-shaped hulls approximately 6 m in length (NOMAD buoys). Some 10 m diameter discus buoys and 12 m diameter discus buoys also may be used at times for non-directional or directional wave measurements in severe environments. Over the years, different sampling rates have been used. After the GSBP WDA, sampling rates were selected to place (considering record lengths) Fourier frequencies precisely on selected broad frequency band boundaries for subsequent spectra and cross-spectra calculations. Doing this allows having broad frequency bands that have consistent widths and degrees of freedom. Differences in Table 1 result in negligible differences in analysis results for most

Table 1. Wave Measurement System Data Collection Parameters

System	GSBP WDA	DACT WA	DACT DWA	DACT DWA-MO	VEEP WA	WPM	DWPM	NDWPM
Sensor(s)	fixed acc.	fixed acc.	HIPPY 40 and 3-axis mag.	fixed acc. and 3-axis mag.	fixed acc.	HIPPY 40 and 3-axis mag. or fixed acc. and 3-axis mag.	Mast axis inclinometer, 3 angular rate sensors, triaxial magnetometer, 1 pitch tilt sensor, 1 roll tilt sensor	Mast axis inclinometer
A to D bits	8	12	12	12	4.5 digit decimal	12	12	12
Record length	20 min (1200 s)	20 min (1200 s)	20 min (1200 s)	20 min (1200 s)	20 min (1200 s)	40 min (2400 s)	20 min (1200 s)	20 min (1200 s)
Analog filter	0.50 Hz	0.50 Hz	none	none	0.50 Hz	none	none	none
Sampling rate	1.50 Hz	2.56 Hz	2.00 Hz	2.00 Hz	2.56 Hz	1.7066 Hz	1.7066 Hz	1.7066 Hz
Digital filter	none	none	0.39 Hz, 1 Hz subsampling	0.39 Hz, 1 Hz subsampling	none	none	none	none
Segment length	N/A	100 s	100 s	100 s	100 s	N/A	N/A	N/A

System abbreviations are defined in the text and the following tables.
A to D = analog to digital conversion
acc. = accelerometer
mag. = magnetometer. Although a 3-axis magnetometer is used, only bow and starboard components of the measured magnetic field are used during data analysis.
Half-power frequency is given for low-pass analog filters.
HIPPY 40 = Datawell HIPPY 40 that can provide: (1) nearly vertical acceleration, pitch, and roll, or (2) nearly vertical displacement (from electronic double integration of acceleration), pitch, and roll.
Systems that do not use analog and/or digital filters rely on hull filtering of high frequency waves to avoid aliasing.

users of NDBC's wave information.

3.3.1 GSBP Wave Data Analyzer (WDA)

Data from these non-directional wave measurement systems were first collected operationally in 1979. Although some systems are still used, these systems are obsolete and are being phased out as newer systems are deployed. Table 2 summarizes data analysis steps in the order that they are performed. Steele et al. (1976), Magnavox (1979), and Steele and Earle (1979) provide further detail about GSBP WDA wave measurement systems.

For each 20 minute acceleration record consisting of 1800 data points sampled at 1.50 Hz, an acceleration covariance (autocorrelation) function for M lags (M = 75 for normal data collection) is calculated by the onboard system. Covariance values are relayed to shore along with mean, minimum, and maximum acceleration values.

After DQA checks, acceleration spectra are computed onshore by direct summation of covariance values and cosine functions. Spectral estimates are obtained at frequencies, $m\Delta f$, where m ranges from 1 to 50 and $\Delta f = 0.01$ Hz. The time between samples, Δt, is 0.667 s and the Nyquist frequency is 0.75 Hz.

Spectral estimates are calculated to 0.50 Hz (M = 50) rather than to 0.75 Hz (M = 75) so that spectra are cut-off at the half-power frequency of the onboard low-pass anti-aliasing analog filter. Frequency domain Hanning with variance correction is used to reduce spectral leakage. Acceleration spectra are then corrected for electronic noise and low frequency noise due to use of a hull-fixed accelerometer. Buoy hull-mooring and onboard low-pass analog filter response functions are applied. Acceleration spectra are converted to displacement spectra by division by radian frequency raised to the fourth power. The number of degrees of freedom (also called equivalent degrees of freedom, EDF, to correspond to terminology when data segmenting is used) is 48 considering that adjacent spectral estimates are not independent as noted in the section describing spectral confidence intervals.

The most important difference between GSBP WDA's and later non-directional wave measurement systems is use of the covariance technique instead of direct Fourier transforms for onboard calculation of spectral estimates.

Table 2. General Service Buoy Payload (GSBP) Wave Data Analyzer (WDA) Data Analysis Steps

Calculation of mean, minimum, and maximum acceleration for entire 20 min (1200 s) record sampled at 1.5 Hz

Time domain calculation of acceleration covariance (autocorrelation) function for M lags incorporating mean removal. M = 75 (normal data collection), M = 150 (test data collection)

Transmission of acceleration covariances (normally 75), mean, minimum, and maximum values to shore

Data Quality Assurance (DQA)
- Parity checks
- Complete message checks
- Minimum or maximum acceleration out of range

Direct calculation of acceleration spectra from covariances using cosine functions. Nyquist frequency = 0.75 Hz. Spectra cutoff at 0.50 Hz corresponding to half-power frequency of onboard low-pass analog filter.

Spectral leakage reduction (Hanning) in frequency domain

Correction of acceleration spectra for electronic noise and noise due to use of fixed accelerometer by subtraction of empirical low frequency noise correction function. Before approximately 1984, low frequency noise correction described by Steele and Earle (1979) were used. From approximately 1984 -1987, low frequency noise correction described by Earle et al. (1984) were used. After approximately 1987, correction modifications described by Lang (1987) were used.

Conversion of acceleration spectra to displacement (elevation) spectra including use of frequency-dependent hull-mooring transfer function, correction for onboard low-pass analog filter, and division by radian frequency to the fourth power. Frequency bandwidth = 0.01 Hz, degrees of freedom = 48 for 75 covariances

Calculation of wave parameters from displacement spectra

Key References:	Steele et al. (1976) Steele and Earle (1979) Earle et al. (1984) Steele and Mettlach (1994) Lang (1987)

3.3.2 DACT Wave Analyzer (WA)

DACT Wave Analyzer (WA) data were first collected operationally in 1984. Table 3 summarizes data analysis steps in the order that they are performed. DACT WA's are non-directional wave measurement systems. DACT directional wave measurement systems are described in the next section.

Each 20 minute buoy acceleration record consisting of 3072 data points is sampled at 2.56 Hz. Data segmenting with 50% overlapping segments is used to decrease spectral estimate uncertainties (i.e. confidence intervals). As acceleration data are acquired, the data are stored in two groups of 256 data points each so that data points in the first half of the second group are the same as those in the second half of the first group. This approach permits analyzing one 256 data point group as data to complete the next group are acquired. Twenty-three 256 point records with 50% overlap and lengths of 100 s are analyzed.

For each segment, the pre-window variance is calculated, minimum and maximums are found, the mean is calculated and removed, Hanning is performed, and the post-window variance is calculated. Next, an FFT provides frequency domain information from 0.00 to 1.28 Hz (Nyquist frequency) with a Fourier frequency separation of 0.01 Hz, but only spectral estimates for frequencies to 0.40 Hz are calculated. These estimates are corrected for windowing and averaged over the segments. Considering segmenting, spectral estimates have 33 equivalent degrees of freedom. Minimum and maximum accelerations for the 20 minute record are determined from segment minimums and maximums.

Acceleration spectra are normalized to the maximum spectral density (i.e. at the peak frequency), logarithmically encoded, and relayed to shore. Minimum and maximum accelerations are also relayed and used onshore for DQA. The spectra are corrected for electronic noise and low frequency noise due to use of a hull-fixed accelerometer. Buoy hull-mooring and onboard low-pass analog filter response functions are applied. Acceleration spectra are converted to displacement spectra by division by radian frequency to the fourth power. Onshore processing is similar to that for GSBP WDA data. DACT WA systems are capable of performing onboard noise and response function corrections as well as conversion of acceleration spectra to displacement spectra if appropriate noise and power transfer functions are pre-stored in the systems, but these systems are not operated in this manner.

Table 3. Data Acquisition and Control Telemetry (DACT) Wave Analyzer (WA) Data Analysis Steps

Segmenting of record (as data acquired with analysis after each segment is complete) into 23 50% overlapping segments with lengths of 100 s (256 data points sampled at 2.56 Hz)

Calculation of mean, minimum, and maximum acceleration, mean removal for each segment

Spectral leakage reduction (Hanning) in time domain

Fast Fourier Transform (FFT)

Correction for window use and spectral calculations

Averaging of spectra over segments. Nyquist frequency = 1.28 Hz. Spectra cutoff at 0.40 Hz slightly less than half-power frequency, 0.5 Hz, of onboard low-pass analog filter.

Acceleration spectra logarithmic encoding, finding overall minimum and maximum acceleration

Transmission of acceleration spectra, minimum, and maximum values to shore

Data Quality Assurance (DQA)
- Parity checks
- Complete message checks
- Minimum or maximum acceleration out of range

Correction of acceleration spectra for electronic noise and noise due to use of fixed accelerometer by subtraction of empirical low frequency noise correction function described by Earle et al. (1984) and, after approximately 1987, by Lang (1987).

Conversion of acceleration spectra to displacement (elevation) spectra including use of frequency-dependent hull-mooring transfer function, correction for onboard low-pass analog filter, and division by radian frequency to the fourth power. Frequency bandwidth = 0.01 Hz, equivalent degrees of freedom considering segmenting = 33

Calculation of wave parameters from displacement spectra

Key References:	Magnavox (1986) Earle et al. (1984) Lang (1987) Steele and Mettlach (1994)

3.3.3 DACT Directional Wave Analyzer (DWA)

Operational DACT Directional Wave Analyzer (DWA) data were first collected in 1986. Table 4 summarizes data analysis steps in the order that they are performed. Steele et al. (1992) provide detailed descriptions of most aspects of DACT DWA data collection and processing.

Each 20 minute buoy acceleration (or displacement) record consisting of 2400 data points is sampled at 2.00 Hz. East-west and north-south buoy slopes are determined from buoy azimuth, pitch, and roll. Acceleration (or displacement) data as well as slope data are digitally low-pass filtered and subsampled at 1.00 Hz to provide 1200 data points. Mean, minimum, and maximum values of acceleration (or displacement) and the slopes are calculated and subtracted from each data point of each time series. Twenty-three 100 point records with 50% overlap and lengths of 100 s are analyzed. For each segment, the pre-window variance is calculated, the mean is calculated and removed, and Hanning is performed. An FFT then provides frequency domain information from 0.00 to 0.50 Hz (Nyquist frequency for subsampled data) with a Fourier frequency separation of 0.01 Hz. Post-window variances are calculated from the spectra and corrections for windowing are made. Spectral estimates are averaged over the segments so that there are 33 equivalent degrees of freedom.

Spectra and cross-spectra are normalized to their respective maximum absolute values, logarithmically encoded, and relayed to shore along with the absolute maximum value. Sign information also is relayed for cross-spectra that may have negative values. Non-directional spectra (C^m_{11}) are relayed to shore over the frequency range 0.03 Hz - 0.40 Hz. Directional spectra parameters (C^m_{22}, C^m_{33}, C^m_{23}, Q^m_{12}, C^m_{12}, Q^m_{13}, C^m_{13}) are relayed over the frequency range 0.03 Hz - 0.35 Hz. Q^m_{23}, which is theoretically zero, is not determined by DACT DWA systems. Non-directional acceleration spectra (C^m_{11}) are corrected for electronic noise and sensor-related noise using an empirical correction developed by Wang and Chaffin (1993). Buoy hull-mooring and onboard low-pass analog filter response functions are applied to spectra and cross-spectra through use of the Power Transfer Function (PTF) and phase angles described in the theory and background section. Acceleration spectra are converted to displacement spectra by division by radian frequency raised to the fourth power.

Various information referred to as "housekeeping" information pertains to system set-up and onshore DQA. This information includes mean, minimum, and maximum values of buoy acceleration (or displacement), buoy pitch, and buoy roll as well as, maximum buoy combined pitch and roll tilt angle, original (first data point) buoy bow azimuth (heading), and maximum clockwise and counter-clockwise deviations from original azimuth.

Table 4. Data Acquisition and Control Telemetry (DACT) Directional Wave Analyzer (DWA) Data Analysis Steps

Calculation of azimuth from pitch, roll, and measured bow and starboard magnetic field components. Acceleration, pitch, and roll provided by Datawell HIPPY 40 sensor.

Conversion of pitch and roll to earth-fixed east-west and north-south wave slopes

Digital low-pass filtering of acceleration (or displacement[1]) and slope data available at 2 Hz sampling rate and subsampling at 1 Hz

Calculation of mean, minimum, and maximum acceleration (or displacement[1]) and slope components for entire 20 min (1200 s) record

Segmenting of acceleration (or displacement[1]) and slope component data into 23 50% overlapping segments with lengths of 100 s (100 data points)

Mean removal for acceleration (or displacement[1]) and slope components for each segment

Spectral leakage reduction (Hanning) in time domain for acceleration (or displacement[1]) and slope components

Fast Fourier Transform (FFT)

Correction for window use (acceleration or displacement and slope components)

Spectral and cross-spectral calculations

Averaging of spectral estimates over segments. Nyquist frequency = 0.50 Hz. Directional spectra cutoff at 0.35 Hz slightly less than half-power frequency, 0.39 Hz, of onboard low-pass analog filter. Non-directional spectra cutoff at 0.40 Hz.

Transmission of cross-spectra and other information to shore. Other information includes mean, minimum, and maximum values of buoy acceleration (or displacement[1]), buoy pitch, and buoy roll as well as, maximum buoy combined pitch and roll tilt angle, original (first data point) buoy bow azimuth (heading), and maximum clockwise and counter-clockwise deviations from original azimuth.

Data Quality Assurance (DQA)
    Parity checks
    Complete message checks
    Parameter minimums or maximums out of range

Correction of acceleration spectra (usually transmitted rather than displacement spectra) for electronic and sensor-related noise using an empirical noise correction function (Wang and Chaffin, 1993).

Conversion of acceleration spectra to displacement (elevation) spectra (unless displacement originally used) by division by radian frequency to the fourth power, use of frequency-dependent hull-mooring transfer function, correction for onboard low-pass digital filter. Frequency bandwidth = 0.01 Hz, equivalent degrees of freedom considering segmenting = 33

Calculation of directional wave spectra including corrections for hull-mooring amplitude and phase responses that affect directional information. Techniques described by Steele et al. (1985) followed before 1988-1989. Techniques described by Steele et al. (1990) and Steele et al. (1992) followed after 1988-1989.

Calculation of wave parameters from displacement and directional spectra

Table 4. Data Acquisition and Control Telemetry (DACT) Directional Wave Analyzer (DWA) Data Analysis Steps (continued)

Key References:
- Steele et al. (1985)
- Tullock and Lau (1986)
- Steele et al. (1990)
- Steele et al. (1992)
- Wang and Chaffin (1993)
- Steele and Mettlach (1994)

---

[1] The Datawell HIPPY 40 onboard sensor can provide either nearly vertically-stabilized acceleration or displacement obtained by electronic double integration of acceleration. Either output has been used for particular buoy installations. Buoy pitch and roll are also provided.

3.3.4 DACT Magnetometer Only Directional Wave Analyzer (DWA-MO)

Operational DACT Magnetometer Only Directional Wave Analyzer (DWA-MO) data were first collected operationally in 1991. Table 5 summarizes data analysis steps in the order that they are performed.

These systems are essentially identical to DACT DWA systems except that a fixed accelerometer is used in place of a Datawell HIPPY 40 sensor and that pitch, roll, and azimuth are all calculated from the magnetometer data. Low frequency noise corrections described by Lang (1987) are used to correct for effects of using a fixed accelerometer. Steele (1990) and Steele et al. (1992) provide detailed descriptions of most aspects of DACT DWA-MO data collection and processing.

3.3.5 VEEP Wave Analyzer (WA)

Data from these non-directional wave measurement systems were first collected operationally in 1989. Table 6 summarizes data analysis steps in the order that they are performed.

From a data analysis point of view, these systems are essentially implementations of DACT WA procedures on VEEP hardware. DACT systems were developed in the early 1980's and VEEP systems were developed in the mid-to-late 1980's.

Table 5. Data Acquisition and Control Telemetry (DACT) Magnetometer Only Directional Wave Analyzer (DWA-MO) Data Analysis Steps

Calculation of pitch and roll from high frequency parts of measured bow and starboard magnetic field components

Calculation of azimuth from pitch, roll, and low frequency parts of measured bow and starboard magnetic field components

Conversion of pitch and roll to earth-fixed east-west and north-south wave slopes

Digital low-pass filtering of acceleration and slope data available at 2 Hz sampling rate and subsampling at 1 Hz. Acceleration provided by fixed accelerometer.

Calculation of mean, minimum, and maximum acceleration and slope components for entire 20 min (1200 s) record

Segmenting of acceleration and slope component data into 23 50% overlapping segments with lengths of 100 s (100 data points)

Mean removal for acceleration and slope components for each segment

Spectral leakage reduction (Hanning) in time domain for acceleration and slope components

Fast Fourier Transform (FFT)

Correction for window use (acceleration and slope components)

Spectral and cross-spectral calculations

Averaging of spectral estimates over segments. Nyquist frequency = 0.50 Hz. Directional spectra cutoff at 0.35 Hz slightly less than half-power frequency, 0.39 Hz, of onboard low-pass analog filter. Non-directional spectra cutoff at 0.40 Hz.

Transmission of cross-spectra and other information to shore. Other information includes mean, minimum, and maximum values of buoy acceleration, buoy pitch, and buoy roll as well as, maximum buoy combined pitch and roll tilt angle, original (first data point) buoy bow azimuth (heading), and maximum clockwise and counter-clockwise deviations from original azimuth.

Data Quality Assurance (DQA)
  Parity checks
  Complete message checks
  Parameter minimums or maximums out of range

Correction of acceleration, spectra for electronic noise using a noise threshold following approach of Steele et al. (1985). Low frequency noise correction functions described by Lang (1987) are also used.

Conversion of acceleration spectra to displacement (elevation) spectra by division by radian frequency to the fourth power, use of frequency-dependent hull-mooring transfer function, correction for onboard low-pass digital filter. Frequency bandwidth = 0.01 Hz, equivalent degrees of freedom considering segmenting = 33

Calculation of directional wave spectra including corrections for hull-mooring amplitude and phase responses that affect directional information. Techniques described by Steele et al. (1985) followed before 1988-1989. Techniques described by Steele et al. (1990) and Steele et al. (1992) followed after 1988-1989.

Calculation of wave parameters from displacement and directional spectra

Key References:	Lang (1987) Steele (1990) Steele et al. (1992) Steele and Mettlach (1994)

Table 6. Value Engineered Environmental Payload (VEEP) Wave Analyzer (WA) Data Analysis Steps

Segmenting of record (as data acquired with analysis after each segment is complete) into 23 50% overlapping segments with lengths of 100 s (256 data points sampled at 2.56 Hz)

Calculation of mean, minimum, and maximum acceleration, mean removal for each segment

Spectral leakage reduction (Hanning) in time domain

Fast Fourier Transform (FFT)

Correction for window use and spectral calculations

Averaging of spectra over segments. Nyquist frequency = 0.64 Hz. Spectra cutoff at 0.40 Hz slightly less than half-power frequency, 0.5 Hz, of onboard low-pass analog filter.

Acceleration spectra logarithmic encoding, finding overall minimum and maximum acceleration

Transmission of acceleration spectra, minimum, and maximum values to shore

Data Quality Assurance (DQA)
- Parity checks
- Complete message checks
- Minimum or maximum acceleration out of range

Correction of acceleration spectra for electronic noise and noise due to use of fixed accelerometer by subtraction of empirical low frequency noise correction function described by Lang (1987).

Conversion of acceleration spectra to displacement (elevation) spectra including use of frequency-dependent hull-mooring transfer function, correction for onboard low-pass analog filter, and division by radian frequency to the fourth power. Frequency bandwidth = 0.01 Hz, equivalent degrees of freedom considering segmenting = 33

Calculation of wave parameters from displacement spectra

Key References:	Lang (1987) Chaffin et al. (1992) Steele and Mettlach (1994)

3.3.6 Wave Processing Module (WPM)

The WPM represented a major advance in NDBC's wave data analysis capabilities. The WPM consists of onboard software and a powerful microprocessor dedicated to processing wave data. Results are passed to other onboard systems for relay via satellite. The WPM was initially used with VEEP systems. However, it is a software-hardware module that can also be used with other and future systems that control overall buoy functions including satellite data transmission. Earlier onboard software was programmed in languages that were not widely used and that were fairly specific to buoy payload microprocessors. To provide flexible onboard software that can be modified or expanded readily, WPM software was programmed in C language (Earle and Eckard, 1989). Some onboard WPM code contains various non-standard C aspects because of the particular C compilers that have been used (e.g. Chaffin et al., 1994).

Table 7 summarizes data analysis steps of the WPM in the order that they are performed. WPM data analysis aspects are further described by Earle and Eckard (1989), Chaffin et al. (1992), and Chaffin et al. (1994). In 1989, the prototype WPM software was called the Wave Record Analyzer (WRA) software. Data from a magnetometer and either a Datawell HIPPY 40 or a fixed accelerometer can be analyzed by the WPM. If a HIPPY 40 sensor is used, it provides buoy nearly-vertical acceleration, pitch, and roll. Buoy azimuth is obtained from the magnetometer data.

There are subtle aspects in the WPM design and in several subsequent derivative designs. In the WPM, DWPM, and NDWPM, Fourier band acceleration spectra are converted to Fourier band displacement spectra, and then broad band displacement spectra are determined by averaging Fourier band displacement spectra. Onboard a buoy, these broad band displacement spectra are then converted to broad band acceleration spectra using the center frequency, f_c, of the broad band. Resulting broad band acceleration spectral estimates are then encoded for transmission via GOES, transmitted, decoded, and converted once again back to broad band displacement spectra using the center frequency of the broad band. The onboard conversion of broad band displacement spectra to broad band acceleration spectra reduces the dynamic range of the transmitted spectra so that encoding and decoding is more accurate.

For the WPM, DWPM, and NDWPM, the Fourier band acceleration spectra could have been averaged over each broad band, and the resulting broad band acceleration spectra could have been encoded and sent to shore. There, the broad band acceleration spectra could have been decoded and multiplied by the reciprocal of center radian frequency raised to the fourth power to provide broad band displacement spectra. Although this procedure is a straightforward way to compute displacement spectra, the better approach noted above is utilized. The better approach considers that the multiplicative factor that coverts acceleration spectra to displacement spectra varies considerably over broad bands in the low frequency, or swell, range of frequencies. Using the center frequency may introduce significant errors.

When a fixed accelerometer is used in an NDBC buoy directional wave system, the Magnetometer Only (MO) technique (Steele and Earle, 1991) is utilized to obtain buoy azimuth, pitch, and roll. Pitch and roll are obtained from high frequency parts of measured magnetic field components and used with the total measured components to provide azimuth. As for other directional wave measurement systems, east-west and north-south buoy slopes are determined from buoy azimuth, pitch, and roll.

Forty minutes of data (4096 data points) are acquired at a sampling rate of 1.7066 Hz. Acceleration and slope data are Fourier transformed using FFT's for the full data record (4096 data points), the last twenty minutes (2048 data points) of the data record, and the last ten minutes (1024 data points) of the data record. Spectra and cross-spectra are calculated without use of a leakage reduction window. Results based on longer data records are used for lower frequencies to improve frequency resolution while maintaining a constant twenty-four equivalent degrees of freedom for each frequency band. Spectral estimates are obtained at resolutions of 0.005 Hz for frequency bands centered between 0.0325 Hz and 0.0925 Hz (based on 4096 data points), 0.01 Hz for frequency bands centered between 0.1000 Hz and 0.3500 Hz (based on 2048 data points), and 0.02 Hz for frequency bands centered between 0.3650 Hz and 0.4850 Hz (based on 1024 data points). Spectra and cross-spectra involving acceleration are converted to spectra and cross-spectra involving displacement at individual Fourier frequencies before band-averaging rather than at the more widely spaced band center frequencies. To reduce the dynamic range of spectral estimates, band-averaged spectra and cross-spectra are temporarily converted (later reverse conversion performed onshore) to acceleration spectra using band center frequencies.

Acceleration spectra are normalized to the maximum value, logarithmically encoded, and relayed to shore along with the maximum value. As presently configured, a WPM calculates directional spectra parameters (r_1, α_1, r_2, and α_2) onboard. Values of r_1 and r_2 are logarithmically encoded without normalization and relayed.

Values of α_1 and α_2 are linearly encoded and relayed. The response functions φ^h and $\frac{R^h}{q}$ are also calculated onboard for each frequency band. Values of φ^h are linearly encoded and relayed. Values of $\frac{R^h}{q}$ are logarithmically encoded and relayed.

Non-directional spectra (C^m_{11}) and directional spectra parameters (r_1, α_1, r_2, and α_2) relayed by WPM's are part of the World Meteorological Organization (WMO) WAVEOB code for reporting spectral wave information (World Meteorological Organization, 1988). The standard cross-spectral parameters between wave displacement and slopes can be derived from the relayed information (Earle et al., 1999). With this capability and the relayed information (including response information), it is possible to re-perform response calculations and/or re-calculate directional spectra onshore by other methods.

WPM's transmit considerable "housekeeping" information that pertains to system set-up and onshore DQA. Information includes mean, minimum, and maximum values as well as standard deviations of buoy pitch, buoy roll, buoy combined pitch and roll tilt angle, bow and starboard magnetic field components, buoy bow azimuth (heading), and buoy east-west and north-south slopes. Also relayed are original (first data point) buoy bow azimuth (heading), maximum clockwise and counter-clockwise deviations from original azimuth, and mean buoy bow azimuth.

Table 7. Wave Processing Module (WPM) Data Analysis Steps

In Magnetometer Only (MO) mode

Calculation of pitch and roll from high frequency parts of measured bow and starboard magnetic field components

In HIPPY 40 mode, pitch and roll provided by HIPPY 40 onboard sensor

Calculation of azimuth from pitch, roll, and measured bow and starboard magnetic field components

Calculation of mean, minimum, and maximum acceleration, pitch, and roll for entire 40 min (2400 s) record sampled at 1.7066 Hz

Conversion of pitch and roll to earth-fixed east-west and north-south wave slopes

Fast Fourier Transform (FFT) of data for 4096, 2048, and 1024 data point sections each ending with the last data point of the original 4096 data point record.

Cross-spectral calculations using longer record lengths for lower frequencies to obtain finer frequency resolution. Nyquist frequency = 0.8533 Hz.

Transmission to shore of frequency-dependent parameters for calculation of non-directional and directional spectra and other information. Other information includes mean, minimum, and maximum values as well as standard deviations of buoy pitch, buoy roll, buoy combined pitch and roll tilt angle, bow and starboard magnetic field components, buoy bow azimuth (heading), and buoy east-west and north-south slopes. Also relayed are original (first data point) buoy bow azimuth (heading), maximum clockwise and counter-clockwise deviations from original azimuth, and mean buoy bow azimuth.

Data Quality Assurance (DQA) is performed using the same procedures that are used for other NDBC directional wave measurement systems.

In Magnetometer Only (MO) mode

Corrections of acceleration spectra for electronic noise and noise due to use of fixed accelerometers by subtraction of empirical low frequency noise correction function are not made but may be implemented later.

In HIPPY 40 mode

Corrections of acceleration spectra for electronic noise and sensor-related noise are not made but may be implemented later.

Conversion of acceleration spectra to displacement (elevation) spectra by division by radian frequency to the fourth power and use of frequency-dependent hull-mooring transfer function. Frequency bandwidth = 0.005 Hz between 0.03250 and 0.0925 Hz, bandwidth = 0.01 Hz between 0.0100 and 0.3500 Hz, bandwidth = 0.02 Hz between 0.3650 and 0.4850 Hz, degrees of freedom = 24 for all frequency bands.

Calculation of directional wave spectra from transmitted parameters including corrections for hull-mooring amplitude and phase responses that affect directional information. Techniques described by Steele et al. (1992).

Calculation of wave parameters from displacement and directional spectra

Key References:	Earle and Eckard (1989) Steele and Earle (1991) Steele et al. (1992) Chaffin et al. (1992) Chaffin et al. (1994) Steele and Mettlach (1994) SAIC (2001b, 2001c, 2001d)

3.3.7 Directional Wave Processing Module (DWPM)

The Datawell HIPPY 40 sensor normally used by the WPM produces very good pitch and roll records, but it is expensive, large, heavy, and requires special handling. On the other hand, a magnetometer that provides pitch and roll is inexpensive, small, and rugged. Even with a HIPPY 40, a magnetometer is used to obtain azimuth, so that a HIPPY 40 represents an un-needed sensor. However, wave directions computed from data produced by a Magnetometer Only (MO) system (Steele and Earle, 1991) may infrequently be in error when conditions cause buoy azimuthal motion at wave frequencies. In this case, azimuthal motion cannot be separated unambiguously from pitch and roll morion.

NDBC developed the Directional Wave Processing Module (DWPM) as a system that is inexpensive and effective on all NDBC discus buoys. The DWPM onboard data processing is largely the same as that of the WPM and techniques used for transmitting data to shore are identical. A DWPM is intended to be used in the following three modes: (1) Datawell HIPPY 40, (2) Magnetometer Only (MO), and (3) Angular Rate / Tilt Sensor (AR/T). For mode (2), a single axis accelerometer with its axis parallel to the buoy deck provides heave acceleration. The MO method provides azimuth, pitch, and roll, and fluid tilt sensors provide mean pitch and mean roll for increased accuracies when a buoy does not have negligibly small mean tilts. Earle (1996b) showed that mean tilts can be obtained from the tilt sensors even though individual tilts may be in error due to forces caused by buoy wave motion. Mode (3) is similar, but angular rate sensors provide wave induced pitch and roll.

Most DWPM details are described in a system design document describing its development and testing (CSC, 1999). Because a HIPPY 40 provides pitch and roll (in which case DWPM analysis steps are similar to those of the WPM), and the MO method is described elsewhere in this report, the Angular Rate / Tilt Sensor (AR/T) mode of operation is described below. This mode, mode (3), is expected to be the one that is normally used.

For a pitch-roll buoy, total pitch, P, and total roll, R, each have two parts, a constant part, p_o and r_o, due to imperfect ballasting or quasi-steady winds acting on the buoy superstructure and a time dependent part, p and r, due to effects of waves. Thus,

$$P(t) = p_o + p(t)$$

$$R(t) = r_o + r(t)$$

where t is time. The constant values are measured using fluid tilt sensors and the wave frequency time dependent values are determined using three orthogonal angular rate sensors. After P and R are obtained, DWPM onboard calculations proceed as described for the WPM. The DWPM format for transmission of wave data to shore via GOES is identical to that for the WPM so that the shore software supporting WPM and DWPM systems is identical.

DWPM pitch and roll calculations are described next. More complete information and theory is provided by Steele et al. (1998). As for non-directional wave systems that use a buoy-fixed accelerometer, low frequency noise is introduced into acceleration spectra. For the DWPM, this noise is removed in the same manner as for non-directional systems. After noise corrections, each broad band acceleration spectrum has a number of low frequency bands with zero spectral energy. The onboard DWPM software examines the broad bands progressively from low frequencies to high frequencies until it locates the first non-zero energy value. The low frequency edge of this broad band is determined and saved. Above this cutoff frequency, data measured by the accelerometer, magnetometer, and angular rate sensors are assumed due to wave motion, plus small sensor related noise levels inherent in any measurement system. Below this frequency, data measured by the angular rate sensors are considered as non-wave noise and are excluded from further analysis.

Estimates of wave frequency pitch, p, and roll, r, are determined from the equations for their time derivatives given by

$$\dot{p} = \omega_2 \cos(R) - \omega_3 \sin(R)$$

$$\dot{r} = \omega_1 + \left[\omega_2 \sin(R) + \omega_3 \cos(R)\right] \tan(P)$$

where ω_1, ω_2, and ω_3 are the bow, starboard, and deck perpendicular angular velocity components. Because small angle approximations are reasonably valid for p and r, these equations become approximately

$$\dot{p} = \omega_2$$

$$\dot{r} = \omega_1$$

In the DWPM, FFT techniques are used to integrate the previous equations (Steele et al., 1998). Other integration methods could be used, but NDBC has shown that the FFT technique works well. An FFT is applied to the three angular velocity components. In the frequency domain, all

Fourier coefficient values below the previously determined low frequency edge of the acceleration spectrum are set equal to zero. Based on wave theory, known frequency dependent functions are used to transform Fourier coefficients for $\dot{p}$ and $\dot{r}$ to those for p and r. An inverse FFT than provides p and r.

These FFT procedures are performed in a sequence of iterations. First, the small angle equations for $\dot{p}$ and $\dot{r}$ are used to estimate p and r. These values are then used in the complete equations for $\dot{p}$ and $\dot{r}$ to obtain better estimates for p and r. The progressively better estimates are used iteratively three times with the complete equations for $\dot{p}$ and $\dot{r}$ to obtain p and r values that are quire accurate. Finally, total pitch and roll, P and R, are calculated by adding p_o and r_o to p and r, respectively. Details are provided by Steele et al. (1998).

Table 8. Directional Wave Processing Module (DWPM) Data Analysis Steps[1]

The DWPM may be operated in any one of the following three modes:

(1) Datawell HIPPY 40
(2) Magnetometer Only (MO)
(3) Angular Rate / Tilt Sensor (AR/T)

The DWPM is a more modern version of the WPM and data processing performed by these systems is very similar.

As for the WPM, 4096, 2048, and 1024 data point sections (sampling rate = 1.7066 Hz) each ending with the last data point of the original 4096 data point record are fast Fourier transformed so that spectral calculations can be made using longer record lengths for lower frequencies to obtain finer frequency resolution.

In each mode, noise corrections are applied to the acceleration spectra following Lang (1987) and acceleration spectra are converted to displacement (elevation) spectra by division by radian frequency to the fourth power. The frequency marking the low frequency edge of the spectrum is determined and saved for further use. Noise correction functions for MO and AR/T are same as for NDBC non-directional systems. Each of the three modes determines hull pitch, roll, and azimuth angles in a different way:

HIPPY 40: Heave acceleration and total pitch and total roll are sampled directly from the sensor. Total pitch and roll are used with unfiltered bow and starboard magnetometer data to compute instantaneous magnetic azimuth angle.

MO: Heave acceleration is provided by an accelerometer fixed perpendicular to the buoy deck. Bow and starboard magnetic component sample records are used to determine wave frequency pitch and roll with the MO method, after which magnetic azimuth angle is determined using unfiltered bow and starboard magnetic field components.

AR/T: Heave acceleration is provided by an accelerometer fixed perpendicular to the buoy deck. Bow, starboard, and mast components of angular velocity are sampled from three angular rate sensors and iteratively integrated to produce pitch and roll, using a low frequency at the edge of acceleration spectrum to eliminate noise (Steele et al.,1998). Then, magnetic azimuth angle is determined using pitch and roll with unfiltered bow and starboard magnetic field components

Once 40 minute, 4096 sample, records of azimuth, pitch, and roll have been determined, data analysis proceeds exactly as in the WPM, outlined in Table 7.

Key References: Earle and Eckard (1989)
Steele and Earle (1991)
Steele et al. (1992)
Chaffin et al. (1992)
Chaffin et al. (1994)
Steele and Mettlach (1994)
Steele et al. (1998)
CSC (1999)

---

[1] As of December 2001, development and testing of the DWPM is underway so that details of the design may change.

3.3.8 WPM AND DWPM Directional Wave Parameters Transmitted To Shore Via GOES

Onboard a buoy, WPM and DWPM systems compute, for each broadband, [C_{11m}, C_{22m}, C_{33m}, C_{23m}, Q_{12m}, C_{12m}, Q_{13m}, C_{13m}, Q_{23m}], where the subscript, m, indicates that these are measured co-spectra and quadrature spectra that have not been corrected for noise or hull response effects. When these systems are working correctly, these spectral parameters contain useful information about waves and system performance. For example, Q_{23m} is theoretically equal to zero.

However, one cannot easily form an opinion as to the validity of most data by taking a quick look at just these parameters. To produce directional wave parameters that are more easily interpreted and more convenient for Data Quality Assurance (DQA) evaluations, the nine spectral parameters are transformed to the ten frequency dependent parameters, [C_{11m}, r_1, α_1, r_2, α_2, $\frac{R^h}{q}$, ϕ_h, γ_2, γ_3, λ]. The first five of these parameters are the key parameters that a pitch-roll buoy produces for calculating directional wave spectra using the Longuet-Higgins et al. (1963) formulation described earlier. The last five are useful for DQA purposes, and for reconstructing the measured co-spectra and quadrature spectra onshore. Reconstruction is sometimes desirable. so that higher directional resolution directional spectra techniques can be employed as described by Earle et al. (1999).

The frequency dependent parameters $\frac{R^h}{q}$, ϕ^h), which depend on the hull type and water depth, approximately repeat themselves from hour to hour and provide useful information about the hull responses. They are used routinely to assure that an operational system is still working as it did when it was originally deployed (i.e., correctly). For a hull that follows the wave surface perfectly, R^h = 1 at all frequencies where significant wave energy is present and, when the buoy is moored in deep water, q = 1 at all such frequencies. When both of these conditions apply, $\frac{R^h}{q}$ reduces to the frequently used "check ratio". To the extent that linear wave theory holds, the check ratio should be unity for all broad bands where wave energy is present. A perfect wave following buoy should also report ϕ_h values near zero.

The last three parameters (γ_2,γ_3, λ) were developed by NDBC. These parameters are defined as follows:

$$\lambda = ATAN2\left(C_{23m}, Q_{23m}\right)$$

$$\gamma_2 = ATAN2\left(C'_{12}, Q'_{12}\right)$$

$$\gamma_3 = ATAN2\left(C'_{13}, Q'_{13}\right)$$

where the computer programming notation, ATAN2, is used to avoid potential quadrant errors if DQA methods involving these parameters are programmed by others. The onboard WPM and DWPM systems correct for amplitude and phase responses caused by the hull-mooring responses as well as by the sensors and electronics. Were these corrections (described in the earlier directional wave spectra section) perfect, C'_{12} and C'_{13} would be zero, but Q'_{12} and Q'_{13} would have finite values in all broad bands where energy is present. Thus, γ_2 and γ_3 as defined above would be either 0° or 180°, depending on the sign of Q'_{12} or Q'_{13}. Therefore, the values of γ_2 and γ_3 help determine to what extent (a) an operational system is functioning as designed, (b) the hull is behaving as a simple harmonic oscillator in heave and slope, and (c) linear wave theory is essentially valid under existing sea conditions. For the spectral peak band, either γ_2 or γ_3 (or both, depending on wave direction) will usually remain within 5° or so of either 0° or 180°. This behavior provides a good indication of the system's continued correct operation. As noted earlier, C_{23m} contains meaningful directional wave information, whereas Q_{23m} is theoretically zero. Thus, the angle, λ, should be nearly 0° or 180°, depending on the sign of C_{23m}. for a system that is operating correctly.

When buoys with WPM systems were first operated, synchronization was sometimes lost due to messages being a little too long for reliable GOES data transmission. When this situation occurred, the last data in the message were lost. To correct this problem, λ which was at the end of the message and was normally 0° or 180°, was deleted from the transmitted message. This change eliminated the synchronization problem even though λ could be a useful parameter for additional DQA.

The ten parameters, [C_{11m}, r_1, α_1, r_2, α_2, R^h/q, ϕ_h, γ_2, γ_3, λ], produced for transmission to shore via GOES are used onshore to reconstruct and archive the co-spectra and quadrature spectra that are useful to users interested in directional wave data analyses.

3.3.9 On Shore Data Processing Supporting WPM and DWPM Systems

A deployed WPM or DWPM system transmits to shore via GOES the coded parameters, [C_{11m}, r_1, α_1, r_2, α_2, R^h/q, ϕ^h, γ_2, γ_3, λ]. When these frequency dependent parameters are received onshore and decoded back into scientific units, the first five are treated as directional wave data that are distributed in near-real-time and archived for historical use. The remaining five are used as DQA information and to reconstruct the original, measured co-spectra and quadrature spectra. Making use of the definitions of the above parameters, it can be shown that

$$C_{22m} = -0.5 * R * \left[1 + r_2 * \cos\left(2 * \left((3 * \pi / 2) - \alpha_2\right)\right)\right] * C_{11m}$$

$$C_{33m} = -0.5 * R * \left[1 - r_2 * \cos\left(2 * \left((3 * \pi / 2) - \alpha_2\right)\right)\right] * C_{11m}$$

$$C_{23m} = -0.5 * R * r_2 * \sin\left[2 * \left((3 * \pi / 2) - \alpha_2\right)\right] * C_{11m}$$

$$C'_{12} = \sqrt{R} * r_1 * \cos\left((3 * \pi / 2) - \alpha_2\right) * \tan\left(\gamma_2\right) * C_{11m}$$

$$C'_{13} = \sqrt{R} * r_1 * \sin\left((3 * \pi / 2) - \alpha_1\right) * \tan\left(\gamma_3\right) * C_{11m}$$

$$Q'_{12} = \sqrt{R} * r_1 * \cos\left((3 * \pi / 2) - \alpha_1\right) * C_{11m}$$

$$Q'_{13} = \sqrt{R} * r_1 * \sin\left((3 * \pi / 2) - \alpha_1\right) * C_{11m},$$

in which

$$R = \left[\left(R^h / q\right) / g\right]^2$$

where g is the acceleration due to gravity. The frequency-dependent functions (C'_{12}, C'_{13}, Q'_{12}, Q'_{13}) produced just above are then used to compute

$$C_{12m} = Q'_{12} * \sin(\phi) + C'_{12} * \cos(\phi)$$

$$C_{13m} = Q'_{13} * \sin(\phi) + C'_{13} * \cos(\phi)$$

$$Q_{12m} = Q'_{12} * \cos(\phi) - C'_{12} * \sin(\phi)$$

$$Q_{13m} = Q'_{12} * \cos(\phi) - C'_{13} * \sin(\phi)$$

in which

$$\phi = \phi^h + \pi$$

Finally, C_{23m} is used to compute

$$Q_{23m} = C_{23m} * \tan(\lambda)$$

These nine co-spectra and quadrature spectra paramerters, [C_{11m}, C_{22m}, C_{33m}, C_{23m}, Q_{12m}, C_{12m}, Q_{13m}, C_{13m}, Q_{23m}], are then archived by NDBC for more advanced users. As noted earlier, the original message to shore was found to be so long that it lost synchronization when transmitted via GOES. To solve this situation, λ was deleted so that Q_{23m}, which is theoretically zero, is not now computed.

3.3.10 Non-Directional Wave Processing Module (NDWPM)

When NDBC began using data buoys, their payloads collected and processed only meteorological data. As requirements for wave data developed, wave measurement systems were designed to interface to existing operational payloads. This minimized impacts on the existing payloads largely because complex wave data acquisition and processing could be handled by the wave measurement system. This design philosophy has continued to the present day. Neither the Multi-functional Acquisition and Reporting System (MARS) nor the Acquisition and Reporting Environmental System (ARES) payloads have built in wave measurement capability. But, each of these systems is designed so that either a WPM or DWPM directional wave measurement system may be connected for a particular deployment.

So that MARS and ARES payloads could be deployed with only non-directional wave capability when directional wave data are not needed, NDBC developed the Non-directional Wave Processing Module (NDWPM). The NDWPM consists of a buoy-fixed accelerometer (measurement axis perpendicular to the buoy deck) connected to electronics that samples the data, calculates non-directional wave spectra, and passes the results to the main payload for transmission to shore via GOES. An NDWPM system design document (SAIC, 2001) provides many system details.

The NDWPM allows several configurations. One configuration, for use with buoys, uses a buoy-fixed accelerometer. Another, for use with fixed platforms, is designed to work in conjunction with a pressure transducer or laser wave sensor. The latter two systems report water level data in addition to non-directional wave data. Because spikes occur in laser wave sensor time series, the NDWPM software includes a spike removal algorithm for use with data from this type of sensor.

Table 9. Non-Directional Wave Processing Module (NDWPM) Data Analysis Steps[1]

The NDWPM may be operated in any one of the following two modes:

(1) Buoy with fixed accelerometer
(2) Fixed platform usually with laser wave sensor

The NDWPM is a non-directional version of the DWPM which itself is very similar to the WPM in its wave data processing. Tables 7 and 8 may be consulted considering that sensors and calculations related to wave directions are not used in the NDWPM.

As for these other systems, 4096, 2048, and 1024 data point sections (sampling rate = 1.7066 Hz) each ending with the last data point of the original 4096 data point record are fast Fourier transformed so that spectral calculations can be made using longer record lengths for lower frequencies to obtain finer frequency resolution.

When used with a buoy, noise corrections are applied to the acceleration spectra following Lang (1987) and acceleration spectra are converted to displacement (elevation) spectra by division by radian frequency to the fourth power. When used with a fixed platform, elevation time series are processed directly. Instrumentation related spikes are removed when a laser wave sensor is used before spectral analysis is performed.

Because the NDWPM is a non-directional wave measurement system, its data analysis is much simpler than that of a directional wave measurement system.

Key References:	Earle and Eckard (1989) Steele et al. (1992) Chaffin et al. (1992) Chaffin et al. (1994) SAIC (2001a)

---

[1] As of December 2001, development and testing of the NDWPM is underway so that details of the design may change.

3.3.11 Additional Information

Additional and more detailed information for NDBC wave measurement systems than that provided here is seldom needed by NDBC's wave information users. Considerable additional information is in the noted references, but information in a given reference may not always apply for times after the referenced document was written. Details in the references, especially those that are available in the published literature, may be of particular interest to those who are developing buoy wave measurement systems or wave monitoring programs that use buoys. As seen in this report, there are many considerations, and some complicated calculations, to obtain high quality wave data, especially directional wave data, from buoys. Very detailed NDBC information, such as values of specific parameters (e.g. for response corrections, encoding, decoding) would almost never be needed by users, and might have to be obtained by NDBC inspection of databases and data analysis computer codes.

4.0 ARCHIVED AND NEAR-REAL-TIME RESULTS

Results of NDBC's wave data analysis are archived and made available to users through NDBC, the National Oceanographic Data Center (NODC), and the National Climatic Data Center (NCDC). NODC and NCDC are also components of the National Oceanic and Atmospheric Administration (NOAA). The level of detail (e.g. complete results or basic wave parameters) from these sources varies. Today, many if not most users of NDBC's wave information, as well as other information (e.g. meteorological data) directly utilize NDBC's comprehensive website at http://www.ndbc.noaa.gov.

NDBC wave data are acquired and analyzed hourly. At each measurement location for each hour that data were collected and passed DQA checks, the following information is generally available. Similar information is also provided on a regular basis to other major users such as the US Army Crops of Engineers FWGP and the Minerals Management Service (Department of Interior).

- Significant wave height, H_{mo}
- Mean (average) wave period, T_{zero}
- Peak (dominant) wave direction (mean direction, α_1, at spectral peak)
- Peak (dominant) wave period, T_{peak}
- Center frequency of each spectral band
- Bandwidth of each spectral band
- Non-directional spectral density, C_{11}, for each band
- The following cross-spectra for each band

  C_{22}, C_{33}, C_{12}, Q_{12}, C_{13}, Q_{13}, C_{23}, Q_{23} (some systems), C_{22}-C_{33}
- The following directional Fourier coefficients for each band

  a_1, b_1, a_2, b_2
- The following directional parameters for each band

  r_1, α_1, r_2, α_2
- Non-directional spectral density, C_{11s}, estimated from wave slope spectra for each band

Although directional wave spectra are not directly available, directional spectra may be calculated from available frequency dependent parameters as described earlier.

Less comprehensive results are provided in near-real-time to National Weather Service (NWS) marine forecasters via NWS communication circuits after DQA and onshore processing by the NWS National Meteorological Center (NMC). Because of the almost real-time requirement to meet forecasting needs, NMC's DQA is not as extensive as that performed by NDBC for information that is permanently stored at NDBC, NODC, and NCDC. Overviews of near-real-time NDBC data distribution systems are provided in NDBC annual reports (NDBC, various years) and a description of NDBC's organization and operations (NDBC, 1999).

5.0 SUMMARY

Wave data analysis procedures used by NDBC are documented. Wave data analysis assumptions are noted and appropriate mathematical background and theory are provided. Procedures that can be applied by users of NDBC's wave information to obtain additional useful information (e.g. directional wave spectra and confidence intervals) are described. This report summarizes NDBC's wave data analysis procedures in one document. Earlier documentation is contained in numerous documents that were prepared as NDBC wave measurement systems and procedures evolved since the mid-1970's. This report was developed for users of NDBC's wave information, including the US Army Corps of Engineers, but it should also be a valuable scientific resource as a general guide for using buoys to measure waves.

6.0 ACKNOWLEDGMENTS

Dr. Chung-Chu Teng (with NDBC) contributed to several aspects of updating this document. James Elliott, Nik Fedoroff, and Carl Jackson [all with Science Applications International Corporation (SAIC)], NDBC's Technical Support Contractor (TSC), provided considerable information about recent NDBC systems and helped arrange access to needed NDBC technical documents. Kenneth E. Steele (formerly with NDBC) provided invaluable information needed to update this report from its earlier 1996 version. Cheryl Szydlik [with Neptune Sciences Inc (NSI)] performed technical editing of the updated document.

7.0 REFERENCES

Benoit, M., 1994. "Extensive Comparison of Directional Wave Analysis Methods from Gauge Array Data," Proceedings Ocean Wave Measurement and Analysis, WAVES 93 Symposium, New Orleans, LA, American Society of Civil Engineers, New York, NY, pp. 740-754.

Bendat, J.S., and A.G. Piersol, 1971, 1986. Random Data: Analysis and Measurement Procedures, John Wiley & Sons, New York, NY, 407 pp.

Bendat, J.S., A.G. Piersol, 1980. Engineering Applications of Correlation and Spectral Analysis, John Wiley & Sons, New York, NY.

Borgman, L.E., 1990. "Irregular Ocean Waves: Kinematics and Forces," Ocean Engineering Science, the Sea, vol. 9, Wiley-Interscience, New York, NY, pp. 121-168.

Carter, G. C., C.H. Knapp, and A.H. Nuttall, 1973. "Estimation of the Magnitude-squared Coherence Function via Overlapped Fast Fourier Transform Processing," IEEE Transactions on Audio and Electroacoustics, vol. AU-21.

Chaffin, J.N., W. Bell, and C.C. Teng, 1992. "Development of NDBC's Wave Processing Module," Proceedings of MTS 92, MTS, Washington, D.C. pp. 966-970.

Chaffin, J.N., W. Bell, K. O'Neil, and C.C. Teng, 1994. "Design, Integration, and Testing of the NDBC Wave Processing Module," Proceedings Ocean Wave Measurement and Analysis, WAVES 93 Symposium, ASCE, New York, NY, pp. 277-286.

Childers, D., and A. Durling, 1975. Digital Filtering and Signal Processing, West Publishing Company, St. Paul, MN.

CSC, 1999. "System Design Document, Development and Testing of the Directional Wave Processing Module (DWPM)," D-106903-2 Revision 0, Computer Sciences Corporation, Stennis Space Center, MS, 112 pp. plus reference data and data sheets.

Dean, R.G., and R.A. Dalrymple, 1984. Water Wave Mechanics for Engineers and Scientists, Prentice-Hall, Englewood Cliffs, NJ.

Donelan, M., and W.J. Pierson, 1983. "The Sampling Variability of Estimates of Spectra of Wind-Generated Gravity Waves," Journal of Geophysical Research, vol. 88, pp. 4381-4392.

Earle, M.D., 1990. "NDBC Real-Time Directional Wave Information User's Guide," National Data Buoy Center, Stennis Space Center, MS, 24 pp and appendices.

Earle, M.D., 1996a. "Non-directional and Directional Wave Data Analysis Procedures," NDBC Technical Document 96-01, National Data Buoy Center (NDBC), Stennis Space Center, MS, 36 pp.

Earle, M.D., 1996b. "Development of a System Concept for a Standard Buoy Directional Wave Measurement System," Neptune Sciences Inc., report for the National Data Buoy Center (NDBC), Stennis Space Center, MS, ** pp.

Earle, M.D., and K.A. Bush, 1982. "Strapped-Down Accelerometer Effects on NDBO Wave Measurements," Proceedings of OCEANS 82, IEEE, New York, NY, pp. 838-848.

Earle, M.D., and J.M. Bishop, 1984. A Practical Guide to Ocean Wave Measurement and Analysis, Endeco Inc., Marion, MA.

Earle, M.D., K.E. Steele, and Y.H.L. Hsu, 1984. "Wave Spectra Corrections for Measurements with Hull-Fixed Accelerometers," Proceedings of OCEANS 84, IEEE, New York, NY, pp. 725-730.

Earle, M.D., and J.D. Eckard Jr., 1989. "Wave Record Analyzer (WRA) Software," MEC Systems Corp. report for the National Data Buoy Center (NDBC), Stennis Space Center, MS, 74 pp., appendices.

Earle, M.D., D. McGehee, and M. Tubman, 1995. "Field Wave gaging program, Wave Data Analysis Standard," Instruction Report CERC-95-1, US Army Corps of Engineers Waterways Experiment Station, Vicksburg, MS, 44 pp.

Earle, M.D., K.E. Steele, and D.W. Wang, 1999. "Use of Advanced Directional Wave Spectra Analysis Methods," Ocean Engineering, vol. 26, pp. 1421-1434.

Kinsman, B., 1965. Wind Waves Their Generation and Propagation on the Ocean Surface, Prentice- Hall, Englewood Cliffs, NJ.

Kuik, A.J., G.PH. Van Vledder, and L.H. Holthuijsen, 1988. "A Method for the Routine Analysis of Pitch and Roll Buoy Wave Data," Journal of Physical Oceanography, vol. 18, pp. 1020-1034.

Lang, N., 1987. "The Empirical Determination of a Noise Function for NDBC Buoys with Strapped-Down Accelerometers," Proceedings of OCEANS 87, IEEE, New York, NY, pp. 225-228.

Longuet-Higgins, M. S., 1952. "On the Statistical Distribution of the Heights of Sea Waves," Journal of Marine Research, vol. 11, pp. 245-266.

Longuet-Higgins, M.S., 1957. "The Statistical Analysis of a Random Moving Surface," Proceedings of the Royal Society of London, 249A, pp. 321-387.

Longuet-Higgins, M.S., 1980. "On the Distribution of the Heights of Sea Waves: Some Effects of Nonlinearity and Finite Band Width," Journal of Geophysical Research, 85, pp. 1519-1523.

Longuet-Higgins, M.S., D.E. Cartwright, and N.D. Smith, 1963. "Observations of the Directional Spectrum of Sea Waves Using the Motions of a Floating Buoy," Ocean Wave Spectra, Prentice-Hall, Englewood Cliffs, NJ.

Magnavox, 1976. "Technical Manual (Revised) for the General Service Buoy Payload (GSBP)," Magnavox Government & Industrial Electronics Co., Fort Wayne, IN, February 1979, revised August 1979.

Magnavox, 1986. "Amendment No. 1 to the Technical Manual for Data Acquisition and Telemetry (DACT)," Magnavox Electronics Systems Co., Fort Wayne, IN, 26 pp.

NDBC, various years. "National Data Buoy Center Annual Report," varying pages.

NDBC, 1999. "Overview of NDBC Organization and Operations," NDBC Program Document 99-03, 60 pp.

Otnes, R.K., and Enochson, L, 1978. Applied Time Series Analysis, Interscience, New York, NY.

Remond, F.X., and C.C. Teng, 1990. "Automatic Determination of Buoy Hull Magnetic Constants," Proceedings Marine Instrumentation 90, pp. 151-157.

SAIC, 2001a. "System Design Description for the Non-Directional Waves Processing Module (NDWPM)," version 2.2, Science Applications International Corporation, Stennis Space Center, MS, 39 pp.

SAIC, 2001b. "NDBC Technical Manual for Wave Processing Module (WPM)," draft, Science Applications International Corporation, Stennis Space Center, MS, 19 pp.

SAIC, 2001c. "NDBC Technical Manual for Wave Processing Module (WPM) Users," draft, Science Applications International Corporation, Stennis Space Center, MS, 26 pp.

SAIC, 2001d. "NDBC Technical Manual for Wave Processing Module (WPM) Software Maintenance," draft, Science Applications International Corporation, Stennis Space Center, MS, 18 pp. plus appendices.

Steele, K.E., 1990. "Measurement of Buoy Azimuth, Pitch, and Roll Angles Using Magnetic Field Components," National Data Buoy Center, MS, 16 pp.

Steele, K.E., 1998. "A Proposed Method for Minimizing the Number of Bits Needed to Encode Non-directional Wave Spectra for Satellite Transmission", WAVES 97, Proceedings of the Third International Symposium, Ocean Wave Measurement and Analysis, American Society of Civil Engineers, vol 2., pp. 1021-1032.

Steele, K.E., P.A. Wolfgram, A. Trampus, and B.S. Graham, 1976. "An Operational High Resolution Wave Data Analyzer System for Buoys," Proceedings of OCEANS 76, IEEE, New York, NY.

Steele, K.E., and M.D. Earle, 1979. "The Status of Data Produced by NDBO Wave Data Analyzer (WDA) Systems," Proceedings of OCEANS 79, IEEE, New York, NY.

Steele, K.E., J.C. Lau and Y.H.L. Hsu, 1985. "Theory and Application of Calibration Techniques for an NDBC Directional Wave Measurements Buoy," Journal of Oceanic Engineering, IEEE, vol. OE-10, pp. 382-396.

Steele, K.E., and J.C. Lau, 1986. "Buoy Azimuth Measurements-Corrections for Residual and Induced Magnetism," Proceedings of Marine Data Systems International Symposium, MDS 86, MTS, Washington, D.C., pp. 271-276.

Steele, K.E., D.W. Wang, C. Teng, and N.C. Lang, 1990. "Directional Wave Measurements with NDBC 3-Meter Discus Buoys," National Buoy Center, MS, 35 pp.

Steele, K.E., and M.D. Earle, 1991. "Directional Ocean Wave Spectra Using Buoy Azimuth, Pitch, and Roll Derived from Magnetic Field Components," Journal of Oceanic Engineering, IEEE, vol. 16., pp. 427-433.

Steele, K.E., C. Teng and D.W.C. Wang, 1992. "Wave Direction Measurements Using Pitch-Roll Buoys," Ocean Engineering, vol. 19, pp. 349-375.

Steele, K.E., and T. Mettlach, 1994. "NDBC Wave Data - Current and Planned," Proceedings Ocean Wave Measurement and Analysis, WAVES 93 Symposium, ASCE, New York, NY, pp. 198-207.

Steele, K.E., D.W. Wang, M.D. Earle, E.D. Michelena, and R.J. Dagnall, 1998. “Buoy Pitch and Roll Computed Using Three Angular Rate Sensors”, Coastal Engineering, vol. 35, pp.123-139.

Tullock, K.H., and J.C. Lau, 1986. "Directional Wave Data Acquisition and Preprocessing on Remote Buoy Platforms," Proceedings of Marine Data Systems International Symposium, MDS 86, MTS, Washington, D.C., pp. 43-48.

Wang, D., and J. Chaffin, 1993. "Development of a Noise-Correction Algorithm for the NDBC Buoy Directional Wave Measurement System," Proceedings of MTS 93, MTS, Washington, D.C., pp. 413-419.

Wang, D.W., C.C. Teng, and R. Ladner, 1994. "Buoy Directional Wave Measurements Using Magnetic Field Components," Proceedings Ocean Wave Measurement and Analysis, WAVES 93 Symposium, ASCE, New York, NY, pp. 316-329.

Welch, P. D., 1967. "The Use of Fast Fourier Transform for the Estimation of Power Spectra: A Method Based on Time Averaging Over Short, Modified Periodograms," IEEE Transactions on Audio and Electroacoustics, vol. AU-15, p. 70-73.

World Meteorological Organization, 1988. Manual on Codes, vol. I, International Codes, FM 65-IX, WAVEOB, Geneva, Switzerland.

APPENDIX A — DEFINITIONS

Azimuth: Buoys are considered to have two axes which form a plane parallel to the buoy deck. One axis points toward the "bow" and the other axis points toward the starboard (right hand) "side". Azimuth is the horizontal angular deviation of the bow axis measured clockwise from magnetic north (subsequently calculated relative to true north). Azimuth describes the horizontal orientation (rotation) of a buoy.

Confidence intervals: Spectra and wave parameters (e.g. significant wave height) have statistical uncertainties due to the random nature of waves. For a calculated value of a given spectral density or wave parameter, confidence intervals are values less than and greater than the calculated value within which there is a specific estimated probability for the actual value to occur.

Cross Spectral-Density (CSD): The cross spectral density represents the variance of the in-phase components of two time series (co-spectrum) and the variance of the out-of-phase components of two time series (quadrature spectrum). Phase information contained in cross-spectral density estimates helps determine wave directional information.

Deep water waves: Waves have different characteristics in different water depths. Water depth classifications are based on the ratio of d/L where d is water depth and L is wavelength. Waves are considered to be deep water waves when d/L > 1/2.

Directional spreading function: At a given frequency, the directional spreading function provides the distribution of wave elevation variance (often called wave energy) with direction.

Directional wave spectrum: A directional wave spectrum describes the distribution of wave elevation variance as a function of both wave frequency and wave direction. The distribution is for wave variance even though spectra are often called wave energy spectra. For a single sinusoidal wave, the variance is 1/2 multiplied by wave amplitude squared and is proportional to wave height squared. Units are those of energy density = amplitude^2 per unit frequency per unit direction. Units are usually m^2/(Hz-degree) but may be m^2/(Hz-radian). Integration of a directional wave spectrum over all directions provides the corresponding non-directional wave spectrum.

Intermediate (transitional) water depth waves: Waves have different characteristics in different water depths. Water depth classifications are based on the ratio of d/L where d is water depth and L is wavelength. Waves are considered to be intermediate, or transitional, waves when $1/25 < d/L < 1/2$.

Leakage: Leakage occurs during spectral analysis because a finite number of analysis frequencies are used while measured time series that are analyzed have contributions over a nearly continuous range of frequencies. Because a superposition of wave components at analysis frequencies is used to represent a time series with actual wave components at other frequencies, variance may appear at incorrect frequencies in a wave spectrum.

Mean wave direction: Mean wave direction is the average wave direction as a function of frequency. It is mathematically described by $\theta_1(f)$ or $\alpha_1(f)$.

Measured time series: A sequence of digitized measured values of a wave parameter is called a measured time series. For computational efficiency reasons, the number of data points is equal to 2 raised to an integer power (e.g. 1024, 2048, or 4096 data points). A measured time series is also called a wave record.

MO Method: An NDBC technique for obtaining buoy pitch, roll, and azimuth from measurements of the two components of the earth's magnetic field parallel to a buoy's deck without use of a pitch/roll sensor.

Non-directional wave spectrum: A non-directional wave spectrum provides the distribution of wave elevation variance as a function of wave frequency only. The distribution is for wave variance even though spectra are often called wave energy spectra. It can be calculated by integrating a directional wave spectrum over all directions for each frequency. Units are those of spectral density which are amplitude^2/Hz (e.g. m^2/Hz). Also see directional wave spectrum.

Nyquist frequency: The Nyquist frequency is the highest resolvable frequency in a digitized measured time series. Spectral energy at frequencies above the Nyquist frequency appears as spectral energy at lower frequencies. The

Nyquist frequency is also called the folding frequency.

Peak (dominant) wave period: Peak, or dominant, wave period is the wave period corresponding to the center frequency of the frequency band with the maximum non-directional spectral density. Peak wave period is also called the period of maximum wave energy.

Pitch: Buoys are considered to have two axes which form a plane parallel to the buoy deck. One axis points toward the "bow" and the other axis points toward the starboard (right hand) "side". Pitch is the angular deviation of the bow axis from the horizontal (tilt of the bow axis).

Principal wave direction: Principal wave direction is similar to mean wave direction, but it is calculated from other directional Fourier series coefficients. It is mathematically described by $\theta_2(f)$ or $\alpha_2(f)$.

Random process: Measured wave characteristics (e.g. wave elevation, pressure, orbital velocities) represent a random process in which their values vary in a non-deterministic manner over a continuous period of time. Thus, descriptions of wave characteristics must involve statistics.

Roll: Buoys are considered to have two axes which form a plane parallel to the buoy deck. One axis points toward the "bow" and the other axis points toward the starboard (right hand) "side". Roll is the angular deviation of the starboard axis from the horizontal (tilt of the starboard axis).

Sea: Sea consists of waves which are observed or measured within the region where the waves are generated by local winds.

Shallow water waves: Waves have different characteristics in different water depths. Water depth classifications are based on the ratio of d/L where d is water depth and L is wavelength. Waves are considered to be shallow water waves when $d/L < 1/25$.

Significant wave height, H_{mo}: Significant wave height, H_{mo}, can be estimated from the variance of a wave elevation record assuming that the non-directional spectrum is narrow. The variance can be calculated directly from the record or by integration of the spectrum as a function of frequency. Using the latter approach, H_{mo} is given by $4(m_o)1/2$ where m_o is the zero moment of non-directional spectrum. During analysis, pressure spectra are converted to equivalent sea surface (elevation) spectra so that these calculations can be made. Due to the narrow spectrum assumption, H_{mo} is usually slightly larger than significant wave height, $H_{1/3}$, calculated by zero-crossing analysis.

Significant wave height, $H_{1/3}$: Significant wave height, $H_{1/3}$, is the average crest-to-trough height of the highest one-third waves in a wave record. Historically, $H_{1/3}$ approximately corresponds to wave heights that are visually observed. If a wave record is processed by zero-crossing analysis, $H_{1/3}$ is calculated by actual averaging of the highest-one heights. H_{mo} is generally used in place of $H_{1/3}$ since spectral analysis is more often performed than zero-crossing analysis and numerical wave models provide wave spectra rather than wave records.

Small amplitude linear wave theory: Several aspects of wave data analysis assume that waves satisfy small amplitude linear wave theory. This theory applies when ak, where a is wave amplitude and k is wave number (2π/wave length), is small. Wave amplitudes must also be small compared to water depth. Except for waves in very shallow water and very large waves in deep water, small amplitude linear wave theory is suitable for most practical applications.

Spectral density: For non-directional wave spectra, spectral density is the wave elevation variance per unit frequency interval. For directional wave spectra, spectral density is the wave elevation variance per unit frequency interval and per unit direction interval. Values of a directional or non-directional wave spectrum have units of spectral density. Integration of spectral density values over all frequencies (non-directional spectrum) or over all frequencies and directions (directional spectrum) provides the total variance of wave elevation. Also see directional wave spectrum and non-directional wave spectrum.

Spectral Moments: Spectral moments are used for calculations of several wave parameters from wave spectra. The rth moment of a wave elevation spectrum is given by

$$m_r = \sum_{n=1}^{N} \left(f_n\right)^r C_{11}\left(f_n\right)\Delta f_n$$

where Δf_n is the spectrum frequency band width, f_n is frequency, C_{11} is non-directional spectral density, and N is the number of frequency bands in the spectrum.

Stationary: Waves are considered stationary if actual (true) statistical results describing the waves (e.g. probability distributions, spectra, mean values, etc.) do not change over the time period during which a measured time series is collected. From a statistical viewpoint, this definition is one of strongly stationary. Actual waves are usually not truly stationary even though they are assumed to be so for analysis.

Swell: Swell consists of waves that have traveled out of the region where they were generated by the wind. Swell tends to have longer periods, more narrow spectra, and a more narrow spread of wave directions than sea.

Wave amplitude: Wave amplitude is the magnitude of the elevation of a wave from mean water level to a wave crest or trough. For a hypothetical sine wave, the crest and trough elevations are equal and wave amplitude is one-half of the crest to trough wave height. For actual waves, especially high waves and waves in shallow water, crests are further above mean water level than troughs are below mean water level.

Wave crest: A wave crest is the highest part of a wave. A wave trough occurs between each wave crest.

Wave energy: For a single wave, wave energy per unit area of the sea surface is $(1/2)\rho g a^2$ where ρ is water density, g is acceleration due to gravity, and a is wave amplitude. The constant factor, ρg, is usually not considered so that energy or wave variance is considered as $(1/2)\ a^2$. Since wave components have different frequencies and directions, wave energy can be determined as a function of frequency and direction.

Wave frequency: Wave frequency is 1/(wave period). Wave period has units of s so that wave frequency has units of Hertz (Hz) which is the same as cycles/s. Radian wave frequency (circular frequency) is $2\pi f$ where f is frequency in Hz.

Wave height: Wave height is the vertical distance between a wave crest and a wave trough. Wave height is approximately twice the wave amplitude. However, for large waves and waves in shallow water, wave crests may be considerably further above mean sea level during the time of the measurements than crests are below mean sea level. When using a spectral analysis approach, wave heights are calculated from spectra using well-established wave statistical theory instead of being calculated directly from a measured wave elevation time series.

Wave length: Wave length is the distance between corresponding points on a wave profile. It can be the distance between crests, between troughs, or between zero-crossings (mean water level crossings during measurement time period).

Wave number: Wave number is defined as 2π/wave length. Wave number, k, and wave frequency, f, are related by the dispersion relationship given by

$$\left(2\pi f\right)^2 = \left(gk\right)\tanh\ \left(kd\right)$$

where g is acceleration due to gravity and d is mean water depth during a wave record.

Wave period: Wave period is the time between corresponding points on a wave profile passing a measurement location. It can be the time between crests, between troughs, or between zero-crossings (mean sea level crossings). The distribution of wave variance as a function of wave frequency (1 / period) can be determined from spectral analysis so that tracking of individual wave periods from wave profiles is not necessary when wave spectra are calculated.

Wave trough: A wave trough is the lowest part of a wave. A wave crest occurs between each wave trough.

Zero-crossing wave period: Zero-crossing wave period is the average of the wave periods that occur in a wave height time series record where a wave period is defined as the time interval between consecutive crossings in the same direction of mean sea level during a wave measurement time period. It is also statistically the same as dividing the measurement time period by the number of waves. An estimate of zero-crossing wave period can be computed from a non-directional spectrum. This estimate is statistically the same as averaging all wave periods that occurred in the wave record.

APPENDIX B — HULL-MOORING PHASE ANGLES

Because corrections for hull-mooring phase angles are important for obtaining high quality directional wave spectra and because these corrections are unique to NDBC, hull-mooring phase angle mathematics is summarized in this appendix. Effects of a buoy hull, its mooring, and sensors are considered to shift wave elevation and slope cross-spectra by a frequency-dependent angle, φ, to produce hull-measured cross-spectra. The angle, φ, is the sum of two angles,

$$\varphi = \varphi^{sH} + \varphi^{h}$$

where φ^{sH} = an acceleration (or displacement) sensor(s) phase angle for heave (H), and

$$\varphi^{h} = \varphi^{hH} - \varphi^{hS}$$

in which φ^{hH} = a hull-mooring (h) heave (H) phase angle, and φ^{hS} = a hull-mooring (h) slope (S) phase angle. This appendix describes how φ is determined for each frequency from each wave measurement record following Steele et al. (1992). Described procedures were implemented for NDBC's directional wave measurement systems during 1988-1989. The main text notes references that describe earlier procedures and summarizes these procedures.

The time-varying hull slopes in an earth-fixed frame of reference that has been rotated clockwise by an angle, a, relative to the true [east($z_2(0)$) / north($z_3(0)$)] frame of reference can be written as

$$z_2(a) = z_2(0)\cos(a) - z_3(0)\sin(a)$$

$$z_3(a) = z_2(0)\sin(a) + z_3(0)\cos(a)$$

If both sides of these equations are transformed to the frequency domain, and heave/slope co-spectra and quadrature spectra in the rotated frame of reference are formed, the results are

$$C_{12}^{m}(a) = C_{12}^{m}(0)\cos(a) - C_{13}^{m}(0)\sin(a)$$

$$Q_{12}^{m}(a) = Q_{12}^{m}(0)\cos(a) - Q_{13}^{m}(0)\sin(a)$$

$$C_{13}^{m}(a) = C_{12}^{m}(0)\sin(a) + C_{13}^{m}(0)\cos(a)$$

$$Q_{13}^{m}(a) = Q_{12}^{m}(0)\sin(a) + Q_{13}^{m}(0)\cos(a)$$

in which spectra on the right hand sides are those reported by a buoy. The following definitions

$$\left[Q_{12}'(a)\right]^2 = \left[Q_{12}^{m}(a)\right]^2 + \left[C_{12}^{m}(a)\right]^2$$

$$\left[Q_{13}'(a)\right]^2 = \left[Q_{13}^{m}(a)\right]^2 + \left[C_{13}^{m}(a)\right]^2$$

and the previous four spectra equations provide

$$\left[Q_{12}'(a)\right]^2 = \left[\tfrac{1}{2}\right]\left[W - Z\cos(2a - b)\right]$$

$$\left[Q_{13}'(a)\right]^2 = \left[\tfrac{1}{2}\right]\left[W + Z\cos(2a - b)\right]$$

where

$$W = \left[Q_{13}'(0)\right]^2 + \left[Q_{12}'(0)\right]^2$$

$$Z = \left[X^2 + Y^2\right]^{\frac{1}{2}}$$

$$b = \tan^{-1}(Y, X)$$

Here, a comma separating numerator and denominator in the argument of the arc tangent means that the signs of Y and X are considered separately to place the angle b in the correct quadrant. When a slash (/) is used to separate numerator and denominator, the angle lies in the first or fourth quadrant.

and

$$X = \left[Q_{13}'(0)\right]^2 - \left[Q_{12}'(0)\right]^2$$

$$Y = 2\left[C_{12}^{m}(0)C_{13}^{m}(0) + Q_{12}^{m}(0)Q_{13}^{m}(0)\right]$$

$Q_{13}'(a)$ has its largest magnitude(s) at either of two angles, π apart, one of which is given by

$$a_0 = b/2.$$

Assuming that the best estimate of φ each hour for each frequency band can be obtained by using the values of $C_{13}^{m}(a)$ and $Q_{13}^{m}(a)$ corresponding to the earth-fixed frame of reference yielding the largest value of $Q'_{13}(a)$, these cross-spectra can be written as

$$C_{13}^{m}\left(a_{0}\right) = C_{12}^{m}(0)\sin\left(a_{0}\right) + C_{13}^{m}(0)\cos\left(a_{0}\right)$$

$$Q_{13}^{m}\left(a_{0}\right) = Q_{12}^{m}(0)\sin\left(a_{0}\right) + Q_{13}^{m}(0)\cos\left(a_{0}\right)$$

If $C_{13}^{m}(a_0)$ and $Q_{13}^{m}(a_0)$ are used with

$$C'_{13}\left(a_{0}\right) = 0 = -\,Q_{13}^{m}\left(a_{0}\right)\sin\left(\varphi\right) + C_{13}^{m}\left(a_{0}\right)\cos\left(\varphi\right)$$

to determine φ, the result is given by

$$\varphi = \tan^{-1}\left[C_{13}^{m}\left(a_{0}\right) / Q_{13}^{m}\left(a_{0}\right)\right] + \left(0 \; or \; \pi\right)$$

To remove the π ambiguity, the $\underline{\varphi}$ frame of reference is first rotated by an angle $\bar{\varphi}(f)$, which is the best available estimate of φ for each frequency. This estimate is determined from hull-mooring models, or from previously measured data that have been supported by hull-mooring models. From $C_{13}^{m}(a_0)$ and $Q_{13}^{m}(a_0)$, the new co-spectra and quadrature spectra are given by

$$y = -\,Q_{13}^{m}\left(a_{0}\right)\sin\left(\bar{\varphi}\right) + C_{13}^{m}\left(a_{0}\right)\cos\left(\bar{\varphi}\right)$$

$$x = +\,Q_{13}^{m}\left(a_{0}\right)\cos\left(\bar{\varphi}\right) + C_{13}^{m}\left(a_{0}\right)\sin\left(\bar{\varphi}\right)$$

With these values and if the angle $\bar{\varphi}$ is within (π/2) of the correct angle, the correct angle is given by

$$\varphi = \bar{\varphi} + \tan^{-1}\left(y / x\right)$$

The ratio (y/x) is computed first, so that φ is forced to lie within ±(π/2) degrees of $\bar{\varphi}$, the best estimate. For each station with a directional wave measurement system, a frequency dependent table of $\bar{\varphi}$ is maintained because of station to station differences. This approach is used to estimate φ for each wave record.

APPENDIX C — SUBROUTINE TO INTEGRATE A FUNCTION USING FAST FOURIER TRANSFORMS

The information presented here shows how NDBC integrates a function. The integrations are used to determine buoy pitch and roll from angular rate data as described in the report. Start with N samples of the time-derivative, ṗ(t), of the continuous function p(t).

$$\dot{p}(n), \left[0 \le n \le (N-1)\right], \text{ taken at times } t(n) = n / f_s$$

where f_s (samples/sec) = sampling rate.

Then set up the real, R_1, and imaginary, I_1, computer arrays:

$$R_1(n+1) = \dot{p}(n) \text{ for } \left[0 \le n \le (N-1)\right]$$

$$I_1(n+1) = 0 \text{ for } \left[0 \le n \le (N-1)\right]$$

Perform a standard[1] time domain to frequency domain Discrete Fourier Transform (DFT), using a proven Fast Fourier Transform (FFT) routine, on

$$\left[R_1(m), I_1(m)\right], \left[1 \le m \le N\right], \text{ to produce}$$

$$\left[R_s(k), I_2(k)\right], \left[1 \le k \le N\right].$$

Create arrays $[R_3(k), I_3(k)]$, $[1 \le k \le N]$, and set

$$R_3(1) = I_3(1) = R_3((N/2)+1) = I_3((N/2)+1) = 0$$

Compute the constant C

$$C = N / (2 * \pi * f_s)$$

For $[1 \le k \le ((N/2) - 1)]$, compute

$$R_3(k+1) = + C * I_2(k+1) / k$$

$$I_3(k+1) = - C * R_2(k+1) / k$$

$$R_3(N+1-k) = + R_3(k+1)$$

$$I_3(N+1-k) = - I_3(k+1)$$

Perform a standard[1] frequency domain to time domain inverse FFT using a proven routine on the arrays

$$\left[R_3(j), I_3(j)\right], \left[1 \le j \le N\right]$$

to obtain

$$\left[R_4(n), I_4(n)\right], \left[1 \le n \le N\right]$$

Then the integrated function time samples are given by

$$p(n) = R_4(n) \ for \left[1 \le n \le N\right]$$

where the time of the nth sample is

$$t(n) = (n-1) / f_s$$

Note that: $I_4(n) = 0$

---

[1] A standard FFT routine is one for which the time domain discrete values x(n) are converted to frequency domain with

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{\frac{-(i2\pi nk)}{N}}$$

and the standard form for a complex number is (a + i*b).

APPENDIX D — DERIVATION OF THE RELATIONSHIPS BETWEEN BUOY EARTH-FIXED SLOPES AND BUOY ANGULAR MOTIONS

Figure 1-D shows NDBC's conventions and definitions for azimuth, A, pitch, P, and roll, R, and how the buoy frame of reference is related to the earth frame of reference. Suppose that the plane of the deck of a discus buoy is oriented so that it makes an angle, X, with the x axis and an angle, Y, with the y axis. Then unit vectors, V_{dE} and V_{dN}, along the deck in the east and north directions respectively are given by

$$V_{dE} = \cos(X)*V_x + \sin(X)*V_z = V_x*\cos(X) + V_z*Z_x*\cos(X) = \cos(X)*\left(V_x + Z_x*V_z\right)$$

$$V_{dN} = \cos(Y)*V_y + \sin(Y)*V_z = V_y*\cos(Y) + V_z*Z_y*\cos(Y) = \cos(Y)*\left(V_y + Z_y*V_z\right)$$

in which V_x and V_y are unit vectors in the east and north directions respectively in the earth frame of reference. Earth-fixed directions are relative to magnetic north. Wave analysis results are later transformed from being relative to magnetic north to be relative to true north. If the vector V_{dE} is crossed into V_{dN}, a vector perpendicular to the deck, pointing up, results. This vector is not a unit vector since vectors V_{dE} and V_{dN} are not exactly perpendicular to each other. To create a unit vector perpendicular to the deck, V_d, the magnitude of the cross product of V_{dE} and V_{dN} is found from the above equations. It is divided into the cross product, leading to

$$V_d = U_x*V_x + U_y*V_y + U_z*V_z$$

in which

$$U_x = -\,Z_x \,/\, \sqrt{1 + Z_x*Z_x + Z_y*Z_y}$$

$$U_y = -\,Z_y \,/\, \sqrt{1 + Z_x*Z_x + Z_y*Z_y}$$

$$U_z = +\,1 \,/\, \sqrt{1 + Z_x*Z_x + Z_y*Z_y}$$

In the buoy frame of reference, the unit vector U has bow (subscript 1) , starboard (subscript 2) and mast-down (subscript 3) components of (0, 0, -1). From the matrix equation relating buoy-fixed and earth-fixed vectors in several NDBC papers (e.g.Steele and Lau, 1986), it can be written that

$$U_x = -\,M_{13}$$

$$U_z = -\,M_{33}$$

$$U_y = -\,M_{23}$$

where M_{13}, M_{23}, and M_{33} are matrix elements (e.g.Steele and Lau, 1986). If the right hand sides of the above pairs of equations for U_x, U_y, and U_z respectively are equated to each other, it can be derived that

$$Z_x = +\,M_{13} \,/\, M_{33}$$

$$Z_y = +\,M_{23} \,/\, M_{33}$$

Thus, earth-fixed buoy slopes, Zx and Zy, in the east and north directions respectively are given by

$$Z_x = \sin(A)*\tan(P) - \cos(A)*\tan(R)/\cos(P)$$

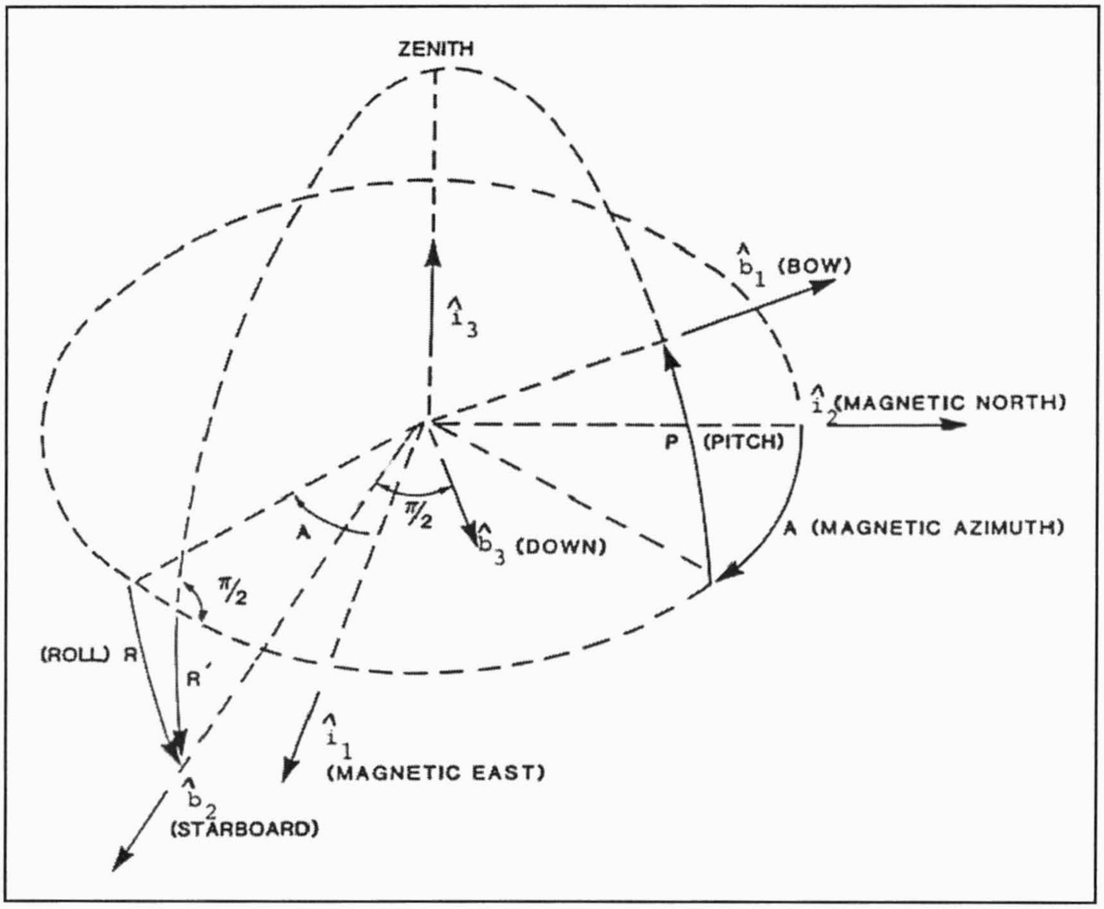
$$Z_y = \cos(A)*\tan(P) + \sin(A)*\tan(R)/\cos(P)$$

The last two equations above do not depend on small angle approximations.

A Datawell HIPPY 40 measures R', rather than R. For this sensor, R is computed from

$$\sin(R) = \sin(R')/\cos(P)$$

Figure 1-D. Conventions and Definitions for Azimuth, Pitch, and Roll.



APPENDIX E — ENCODING WAVE DATA FOR TRANSMISSION VIA GOES, AND DECODING ON SHORE

NDBC wave data are translated onboard a buoy into a binary form for transmission to shore via GOES satellite. This onboard encoding of a parameter's value and decoding by a computer program onshore unavoidably introduces a round-off error into the parameter's value. Although statistical biases are generally avoidable in this process, the magnitude of maximum encode/decode error increases as the number of binary bits used to encode the parameter is minimized. An objective of the encode/decode process is to use as few bits as possible to minimize transmitting time and power consumption while maintaining sufficient accuracy for the transmitted data.

Sometimes minimizing the maximum absolute error of a parameter transmitted to shore in binary form is desired. Mean bow azimuth angle in degrees is an example of such a parameter. Other times, as in the case of wave spectral density values that have a Chi-Square distribution (a scatter that depends on the magnitude of the parameter measured), a constant maximum percentage error for the encode/decode process might be desired. For the purpose of minimizing the message length while maintaining good quality data, methods for encoding in these two different ways were developed by NDBC.

The details of these encode/decode procedures have been thoroughly documented by Steele (1998). This published paper provides the many equations that are needed to encode and decode WPM, DWPM, and NDWPM wave data both in the straightforward, "absolute" way, and in the constant percentage, or "logarithmic" way. The paper's Table 1 (Steele, 1998) provides the maximum percentage errors that result from various combinations of numbers of bits used to encode and the dynamic range that is allowed. Appendix A, "WP9 Message Format", of the DWPM System Design Document (CSC, 1999) provides the number of bits used for encoding each parameter. It also provides the dynamic range covered, and the magnitude of the maximum encode/decode error (absolute or percentage) for each DWPM wave parameter transmitted to shore via GOES. Because the DWPM data transmission formats are identical to those of the WPM, the WP9 Message Format applies to both systems. NDWPM formats are similar to those for the non-directional part of the WPM and DWPM data.

Strictly speaking, encoding/decoding is not part of the wave data analysis. Typical users of NDBC's wave information will seldom be concerned with the details of the encoding/decoding procedures. However, some scientists may be concerned about all uncertainty sources, even if they are small, for particular research purposes.

APPENDIX F — NDBC PAYLOAD/WAVE MEASUREMENT SYSTEM COMBINATIONS

Table 1-F shows NDBC payloads that have been, or may be, used with wave measurement systems that have been developed, are under development, or may be developed. As noted in the main text with additional background information, NDBC's original data buoys were developed primarily to collect meteorological data. NDBC's wave measurement system design approach has been to perform complicated wave data processing on wave measurement systems that pass their results to the main payload.

Figure 1-F. NDBC Payload/Wave Measurement System Matrix.

	PAYLOADS				
	GSBP[1]	DACT	VEEP	MARS	ARES
Buoy Non-Directional Wave Systems	WDA	WA	WA	NDWPM	Future[2]
Buoy Directional Wave Systems	None	DWA	WPM/HIPPY WPM/MO DWPM/HIPPY[3] DWPM/MO[3] DWPM/RATE/TILT[3]	WPM/HIPPY WPM/MO DWPM/HIPPY[3] DWPM/MO[3] DWPM/RATE/TILT[3]	WPM/HIPPY WPM/MO DWPM/HIPPY[3] DWPM/MO[3] DWPM/RATE/TILT[3]
Fixed Platform Non-Directional Wave Systems	None	WA	NDWPM	NDWPM	NDWPM Future[2]

Notes:

[1] GSBP's have been retired from use, but large quantities of GSBP data are archived.
[2] At this date (December, 2001), a non-directional wave system built into ARES is under development.
[3] Possible system configurations that have not yet been deployed (December, 2001).
The DWPM is still under development at this date (December, 2001).