REMARKS ON THE OPTIMUM SPACING OF UPPER-AIR OBSERVATIONS

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[Manuscript received July 22, 1959; revised March 9, 1960]

ABSTRACT

The optimum time and space distribution of upper-air observations is considered in connection with the detection and prediction of instability lines—a mesoscale phenomenon. It is shown that the optimum spacing of observational stations is not only a function of the scale of the phenomena to be detected but of observational and analytical errors as well. When these errors are considered the optimum network spacing can be determined with regard to the dimensions of the atmospheric feature requiring detection. Because actual data are lacking, inferences concerning the maximum time and space distribution of upper-air sounding stations suitable for the detection of instability lines are drawn from a theoretical atmospheric model.

1. INTRODUCTION

The true dimensions of the instability line[1] have, for the most part, escaped detection except for its horizontal dimensions at the earth's surface [1] during the active stage. Some theoretical estimates of the dimensions of the instability line in the incipient stage can be inferred from [2]. However, the problem of detection of these lines by upper-air observations is also complicated by their relatively short life cycle. When one considers that the average dimensions of the instability line are of the order of 125 miles long, moving a distance of 175 miles in 5 hours, it can be seen that the chances of synoptically measuring the vertical distribution of horizontal gradients associated therewith are indeed small. The network spacing within the United States of 220 nautical miles (average) and time spacing of 12 hours provide a very low probability of the detection of the atmospheric processes involved in the production, propagation, and dissipation of such phenomena. On the other hand, the time and space distribution of surface weather reporting stations provides a much better probability of the detection and the prediction of the line. Unfortunately, without a knowledge of the vertical extent and relative strength of the dynamic processes involved in the production of intense vertical motion, the forecaster stands little chance of gaging correctly the intensity of activity associated with the thunderstorms. Likewise, the prediction of the formation and dissipation of this activity is handicapped.

Since the prediction of such phenomena is of economic importance, and the successful prediction is dependent upon initial observations of the meteorological processes involved, the purpose of this paper is to discuss the upper-air observational network needed to improve the detection and prediction of these phenomena.

[1] The instability line has been designated by the World Meteorological Organization as a line of incipient, active, or dissipating nonfrontal instability conditions. It is primarily an analytical tool for indicating the incipient and dissipating stages of squall line phenomena.

2. OPTIMUM NETWORK SPACING

The map analysis of any quantity derived from synoptic weather observations is subject to two principal errors. One error is the error of measurement inherent in the observations themselves, and the other is the truncation error due to smoothing of the isolines of the observed values. The design of an observational network must take both of these errors into account. The optimum space and time distribution of observations is dependent upon the relation between the errors of observation and the gradient of the quantity being analyzed. If the gradients involved in the system being analyzed are smaller than the gradient of errors, the analysis becomes one of errors rather than of the system itself.

The optimum spatial distribution of radiosonde stations is that distance between adjacent stations which minimizes the variance about the true horizontal gradient of the analyzed gradient (true gradient plus gradient of errors). Consider a situation in which two observation stations a distance d apart lie along the x-axis, one at $x=d/2$ and the other at $x=-d/2$. One error arises from observational errors and the other from truncation errors. Let E_1 be the root mean square of the observational errors and E_2 be the root mean square of the truncation errors that arise in computing the first derivative at the midpoint between stations of a quantity y from the observations at the two stations. The total error will then be

$$E_3=\sqrt{E_1^2+E_2^2}. \tag{1}$$

Let the root mean square error of observation of y be σ, then

$$E_1=\frac{\sqrt{2}\sigma}{d}. \tag{2}$$

The truncation error is found by forming Taylor's series involving y at $d/2$ and $-d/2$. Subtracting the second from the first series (after dropping terms of higher order than the third) and dividing by d gives a finite difference representation of the gradient that differs from $\partial y/\partial x$ by a third order term, the truncation error. The root mean square of the truncation error is found to be

$$E_2=\frac{d^2}{4!}\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{\partial^3 y}{\partial x^3}\right)_i^2}=\frac{d^2}{4!}\left|\frac{\partial^3 y}{\partial x^3}\right|. \tag{3}$$

The total error is then

$$E=\sqrt{E_1^2+E_2^2}=\left(\frac{2\sigma^2}{d^2}+\frac{d^4(y''')^2}{(4!)^2}\right)^{1/2} \tag{4}$$

where $y'''\equiv\partial^3 y/\partial x^3$.

Since it is desired that d be chosen such that $\bar{E}^2$ is minimized, the first derivative of the root mean square of the total error will be made zero.
Thus

$$\frac{dE}{dd}=0=-\frac{4\sigma^2}{d^3}+4d^3\left(\frac{y'''}{4!}\right)^2 \tag{5}$$

Solving for d gives the condition for optimum d,

$$d=2\left(\frac{3\sigma}{|y'''|}\right)^{1/3} \tag{6}$$

Upon substitution of the finite difference form of the third derivative for y''', equation (6) is identical to that contained in [3]. Under certain conditions a solution can be found by using the equation in the manner suggested in [3], i.e., as a first guess, the measuring interval is set to d and the values of the quantity are selected from the observational record. From this an approximate third derivative is computed and with σ given, a new d is found. The process is repeated until the successively derived values of d converge. However, for the very short waves of interest here, the values diverge; moreover, for short measuring intervals the evaluation of the third derivative by use of the approximate difference form becomes difficult to achieve with accuracy. Under such conditions it is necessary to utilize a more rigorous procedure for determination of the third derivative in seeking a solution to equation (6).

Given knowledge of the variation in time or space of any continuous function it is possible to approximate the time or space variation mathematically. The mathematical expression that describes the variation can then be differentiated three times to obtain the third derivative, the value of which can then be determined precisely for any point along the curve. This value can then be substituted into equation (6) and with σ given, d is found. This is then the d required within the limits of observational error to give the optimum finite difference representation of the gradient of a quantity y having a complexity described by the third derivative around the point of application. A greater spacing of stations would result in an analysis such that the real feature would be lost through smoothing of the isopleths. Any spacing less than the optimum would tend to become an analysis of observational errors. An inspection of equation (6) reveals that it has no practical application when y''' is zero or infinite since the optimum spacing would then be infinite or zero, respectively.

3. COMPUTATION OF NETWORK SPACING

As indicated above a knowledge of not only the dimensions of the atmospheric features but a knowledge of observational errors is required to determine the optimum observational network. Unfortunately, there is little published information concerning the standard error of upper-air observations. Data secured as a result of radiosonde compatibility tests [4] indicate that an error of ± 50 feet at the 500-mb. level is reasonable. A later study [5] using an indirect method estimated this error to be about 15 meters or approximately 49 feet. A value of 50 feet will be used subsequently for σ in evaluating equation (6).

The detection and prediction of instability lines are dependent, in the main, upon a knowledge of the time change of the existing thermal instability. The prediction of the intensities of weather phenomena associated with these lines is also dependent upon the time rate of change of the vertical thermal instability. Consequently, it appears desirable to specify the upper-air properties of the instability line in terms of a parameter that involves the horizontal as well as the vertical thermal structure. Since thickness permits us to interpret the horizontal and vertical temperature structure it can be utilized. The significance of thickness with respect to cyclonic development is discussed in [6], and its relation to instability line formation is indicated in [2]. Little is known about the true dimensions of thickness and thickness gradients with respect to the instability line; however, theoretical computations [2] can provide the basis for a beginning.

Data from figure 7 of [2], reproduced here as figure 1, were used to develop table 1, a tabulation of the thickness computed for the layer 1,000–500 mb. for points 6, 7, 8, 9, and 10 of that study, located a distance of 60 n. mi. apart. These data were utilized to construct the vertical section shown in figure 2. It was found that this curve is approximated closely by the equation

TABLE 1.—*Thickness values for layer 1,000–500 mb. through a space section normal to an instability line computed from the theoretical data of figure 1. Points 6, 7, 8, 9, and 10 are located 60 n. mi. apart.*

Location	6	7	8	9	10
Thickness	18,670	18,705	18,615	18,625	18,631

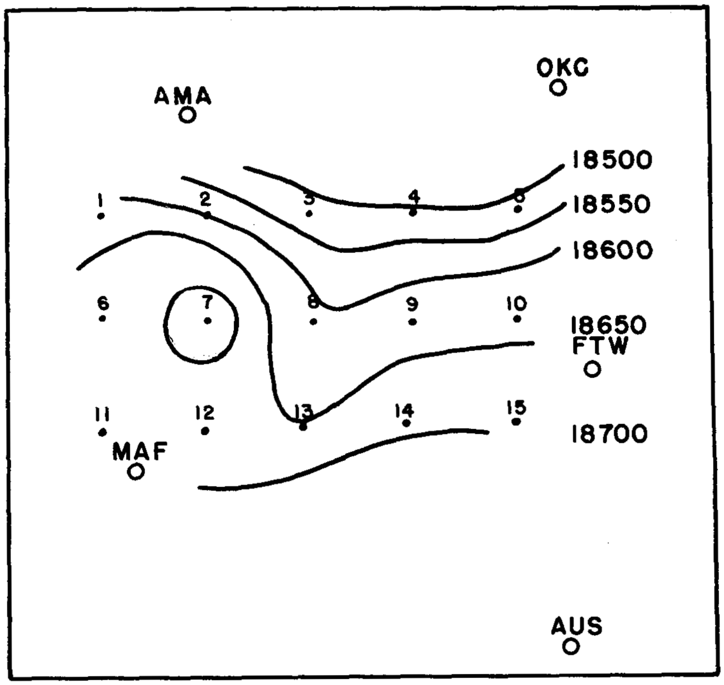

FIGURE 1.—Thickness field associated with a theoretically computed instability line. Computations were made at numbered grid points which are 60 n. mi. apart. (After [2].)

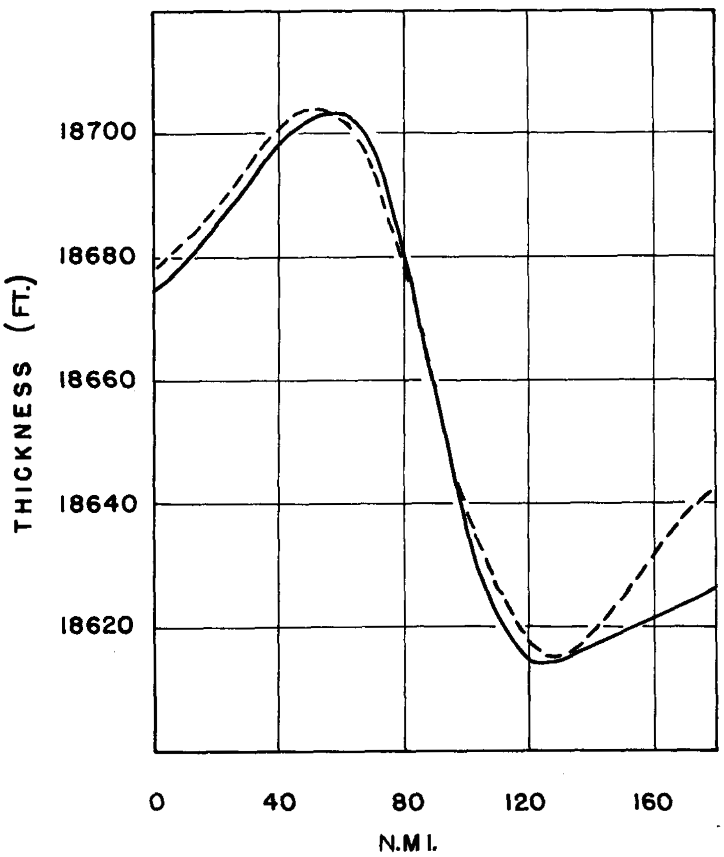

FIGURE 2.—Solid curve is vertical profile of thickness along sections 6–10 of figure 1. Dashed curve is plot of equation (7) given in the text.

$$y=y_0-A_1\cos\left(\frac{2\pi}{L_1}x\right)+A_2\cos\left(\frac{2\pi}{L_2}x\right) \tag{7}$$

where $y_0=18{,}660$ feet, $A_1=22.5$ feet, $L_1=120$ n. mi., $A_2=40$ feet, and $L_2=360$ n. mi. Differentiation of equation (7) three times gives

$$y'''=-A_1\left(\frac{2\pi}{L_1}\right)^3\sin\left(\frac{2\pi}{L_1}x\right)+A_2\left(\frac{2\pi}{L_2}\right)^3\sin\left(\frac{2\pi}{L_2}x\right). \tag{8}$$

y''' was evaluated from equation (8) at $x=90$ n. mi. (location of inflection point) and $x=130$ n. mi. (trough location) and its average value was computed for the interval $x=50$ to $x=130$ n. mi. Substitution of these values of y''' into equation (6) gave the optimum spacing of stations. The results are given in table 2.

It can be seen from equations (6) and (8) and table 2 that the optimum spacing of stations varies according to the choice of location at which the third derivative is computed. For this reason it appears that the spacing computed for the average value y''' average for the portion of the curve most significant to the analysis problem at hand should be considered the optimum. In the case at hand a spacing of stations 93 miles apart would assure the detection of the feature when both observational and analytical errors are considered. The corresponding optimum time interval for observations, on the assumption of a translation of the pattern at a speed of 30 m.p.h., is 3.1 hours.

At this writing knowledge of the dimensions of incipient instability lines is so meager there is no way of knowing the deviation of the above data from that averaged from a large number of cases. It is clearly evident, however, that establishment of such an average or mean dimension is dependent on a greater time and space density of stations.

Preliminary analyses of data secured from a few aircraft traverses through instability lines during the incipient stage indicate that important waves may exist whose ratio of wave length to amplitude is smaller than that used in the theoretical computation above. The significance of the theoretical study is that operational experience indicates that a feature of that size or larger is probably in existence in about one-half the cases a short time before the development of thunderstorms along the instability line.

TABLE 2.—*The optimum spacing (d) of stations corresponding to values of $|y'''|$ at the inflection point ($x=90$ n. mi.) and trough point ($x=130$ n. mi.), and its average over the interval $x=50$, 130 n. mi.*

	x n. mi.	$\lvert y'''\rvert$ ft. (n. mi.)$^{-3}$	d n. mi.
Inflection point	90	3.44×10^{-3}	70.4
Trough point	130	1.45×10^{-3}	93.8
Average	50–130	1.53×10^{-3}	92.2

4. CONCLUSIONS

The above considerations indicate clearly that the optimum time and space density of observations is a function not only of the observational errors but a function of the truncation error as well. Obviously, then, the time and space density of upper-air observations and analyses thereof must be such as to insure the capability of detecting the smallest atmospheric feature important to the particular forecast problem. When these errors are considered the optimum network spacing then can be determined with regard to the dimensions of the atmospheric features requiring detection.

From a research or forecasting standpoint, it is obvious that a greater density of upper-air sounding stations is a necessity if we are to determine with certainty the dynamic processes that result in the formation, propagation, and dissipation of the instability line. Although a network as dense as that suggested here by the computation of optimum spacing for incipient instability lines may never become economically feasible for routine forecasting, it is also obvious that the existing network is much too sparse to provide the basis for any appreciable improvement in the forecast. This is because the extent to which the forecaster is able to decrease his forecast errors is limited by the extent to which the observational network approaches an optimum design with respect to the scale of the phenomena which he must predict.

A recent study by Gleeson [7] suggests a method for computing the probabilities of observing an atmospheric feature of a given size. Using this approach and the dimensions of the theoretically derived instability line [2], it appears that the detection of the existence of such a feature by the existing time and space distribution of radiosonde stations does not exceed a probability of 10 percent. Doubling the number of stations would increase the probability of detection by about 20 percent.

The foregoing considerations indicate clearly that a network of rawinsonde stations at least double the existing number, and taking observations at 6-hourly intervals, supplemented by winds aloft stations, would result in a significant improvement in the detection of the phenomena in the incipient stage, and thus the prediction of the active stage.

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