

## A Case Study of Extreme Turbulence Possibilities Based on Considerations of Buoyancy and Horizontal Divergence

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### ABSTRACT

A study into the possibility of extreme turbulence occurring in a particular synoptic situation is discussed here. The synoptic weather data are reviewed, and the magnitude of turbulence that might be expected is computed by two different methods. The limitations of each method are discussed. While it might be concluded that thunderstorm updraft velocities were of the order of those producing extreme turbulence, the assumptions are so restrictive that it is not possible to assign confidence limits to the computed gust velocities.

### 1. Introduction

Investigations into the precise nature of meteorological conditions in the vicinity (time and space) of a particular incident are nearly always hampered by the insufficiency of recorded meteorological information to determine with precision the factors involved. There is little doubt that the dimensions of many meteorological disturbances are so small in time and space that the usual observations from the existing surface and upper-air reporting network only record at best a fleeting glimpse of some of the factors. This study of the possibility of occurrence of extreme turbulence<sup>1</sup> over the Baltimore area on the particular date is described because it is felt that meteorologists should be aware of existing methods of estimating the magnitude of updrafts and gust velocities, in any given weather situation, from routinely observed meteorological parameters. The results, even though subject to a wide range of error, are the best available.

Because extreme vertical velocities are associated with convective cells small in diameter, com-

<sup>1</sup> WB Manual, Vol. III, Ch. B-20, p. 24, gives the following definitions of turbulence:

"Extreme—associated with the strongest forms of convective wind shear or standing wave action. Rarely encountered. May cause structural damage.

"The following turbulence definitions were drawn up by NACA Subcommittee on Meteorological Problems and are used by transport pilots in reporting turbulence:

"Extreme—a rarely encountered turbulent condition in which aircraft is violently tossed about and is practically impossible to control. May cause structural damage. (Effective and derived gust velocities: 30 fps and above; 48 fps and above.)"

pared with the distance between observing stations, it was recognized that the results of the study would probably not be definitive. Nevertheless, some quantitative estimate of the probability of extreme turbulence was required. Two methods of estimating the vertical velocities which could have occurred in this situation were used: (1) the so-called "parcel method," supplemented by qualitative considerations of the effect on the air mass of frontal lifting, and (2) a method involving the computation of horizontal divergence at the wind-shift associated with the passage of the front. The parcel method, together with its shortcomings, is discussed in detail. The divergence-convergence model, though in some ways the more attractive of the two methods, assumes an arbitrary vertical distribution of the horizontal divergence within a thunderstorm cell and depends on estimates of the surface divergence which are not without error.

### 2. Analyses and prognoses—general

At 1200GCT 12 May 1959, the midtropospheric weather pattern was characterized by an intense cyclonic disturbance centered over the southern shore of Hudson Bay. This disturbance had moved slowly eastward during the past twenty-four hours under the influence of a strong band of westerlies located over the central Great Lakes region. While the dominant disturbance was located over southern Canada, a small but quite intense minor trough had moved across eastern Texas becoming a cut-off low at 500 mb over northern Mississippi by 1200GCT. The major jet-stream flow was in connection with

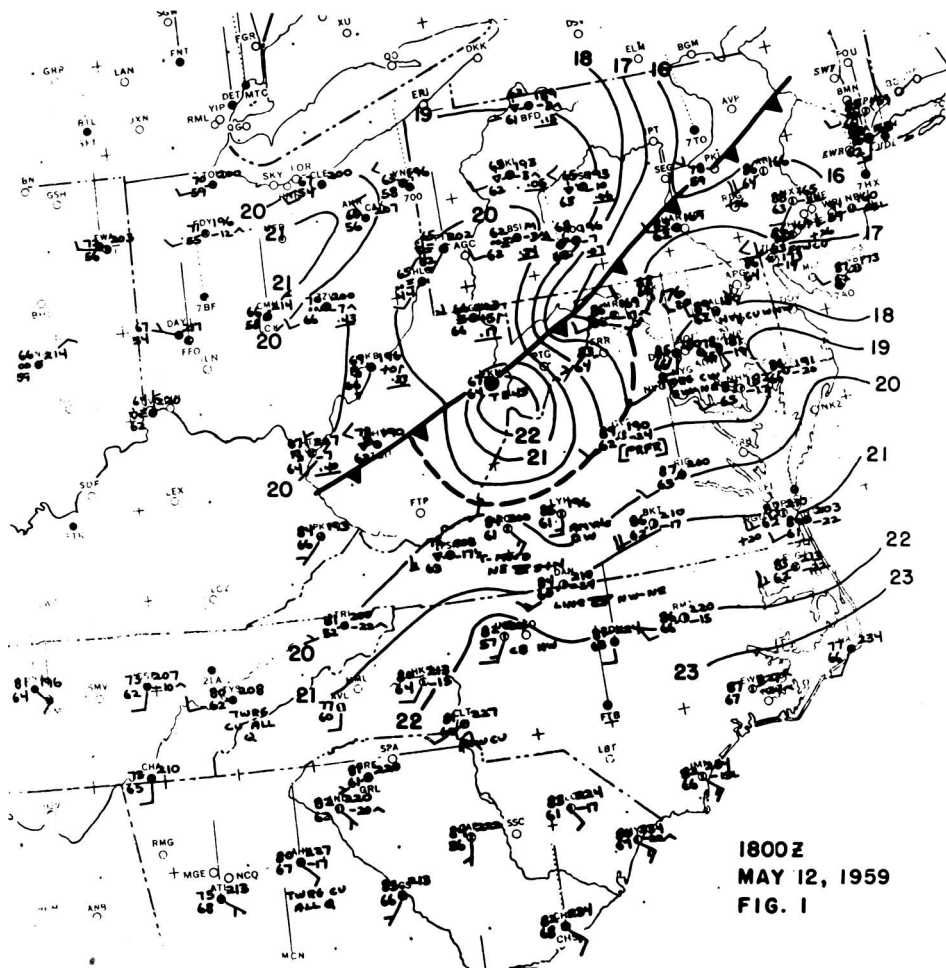


FIG. 1. 1800GCT surface mesoanalysis.

the major storm over southern Canada, and the axis of this flow at 1200GCT was along a line from Glasgow, Montana to Huron, South Dakota to LaCrosse, Wisconsin to Sault Ste. Marie, Michigan with speeds of 160 kn reported at Sault Ste. Marie. The surface cold front at 1200GCT was located in the vicinity of a line extending from Caribou, Maine to Albany, New York to Harrisburg, Pennsylvania to Elkins, West Virginia and extended southwestward to southern Louisiana.

Upon receipt of the 1200GCT upper-air data, a stability analysis of the air mass along and to the east of the cold front in New England and the Middle Atlantic States was accomplished. According to the usual methods, this analysis [1] showed that the possibility existed of thunderstorms producing hail in the size range of around  $\frac{1}{2}$  inch and moderate to severe turbulence. A prognosis of the change in air structure over the

area in question indicated little change in stability during the day.

### 3. Surface analysis

The following is a modified type of meso-analysis of the synoptic surface pattern. Isobars are drawn for 1-mb intervals, hourly surface charts were used, and the barograph or microbarograph records from Harrisburg, Pa. (Weather Bureau Airport Station); Reading, Pa. (City Office); Wilmington, Del.; Martinsburg, W. Va.; Baltimore, Md. (Friendship International Airport); Salisbury, Md.; Frederick, Md. (Weather Bureau Airport Station); Washington National Airport; Philadelphia, Pa. (Mustin Field); Anacostia, D. C. (Naval Air Station); Patuxent River Naval Air Station, Md.; Quantico, Va. (Marine Corps Air Station); Andrews Air Force

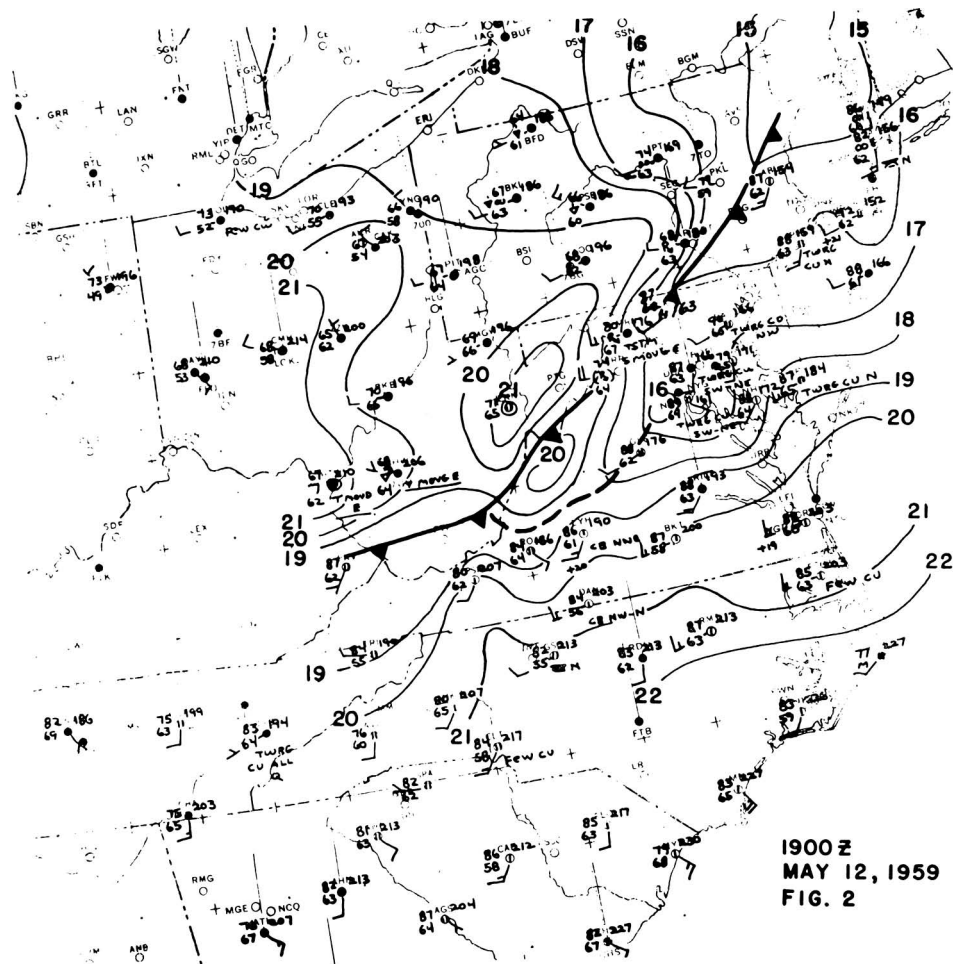


FIG. 2. 1900GCT surface mesoanalysis.

Base, Md.; and Bolling Air Force Base, D. C., evaluated in preparation of the charts.

As indicated in the preceding section, the cold front at 1200GCT was in a line from Park Place, Pennsylvania, to the west of Elkins, West Virginia to the east of Charleston, West Virginia and thence into Kentucky. This cold front continued a slow eastward movement along the northern portion with little, if any, movement in the southern portions. The surface map at 1800GCT (fig. 1) indicated a relatively inactive instability line from Martinsburg, West Virginia to a point just west of Gordonsville, Virginia to twenty miles north of Roanoke, Virginia and thence northward. A small high-pressure cell was centered southeast of Elkins, West Virginia. Several small low-pressure areas were in evidence on the map—one to the southwest of Quantico, Virginia, and one in the vicinity of Martinsburg, West Virginia.

As can be seen from maps at 1900GCT and 2000GCT (figs. 2 and 3), the cold front progressed slowly eastward with the low pressure in the vicinity of Martinsburg moving northeastward and generally losing its identity. The low pressure southwest of Quantico became more prominent, although only slight deepening was in evidence. By 2000GCT, the high-pressure cell in the vicinity of Elkins, West Virginia had become a rather general area of pressure of 1017 mb or more.

The 2100GCT map (fig. 4) indicates the cold front had progressed to a line from the vicinity of Philadelphia, Pennsylvania to six miles east of Baltimore to a point between Washington National Airport and Andrews Air Force Base to ten miles east of Gordonsville, Virginia and thence southwestward. A small meso-high about fifty miles in diameter had formed east of Frederick, Maryland in an extension of the general high-

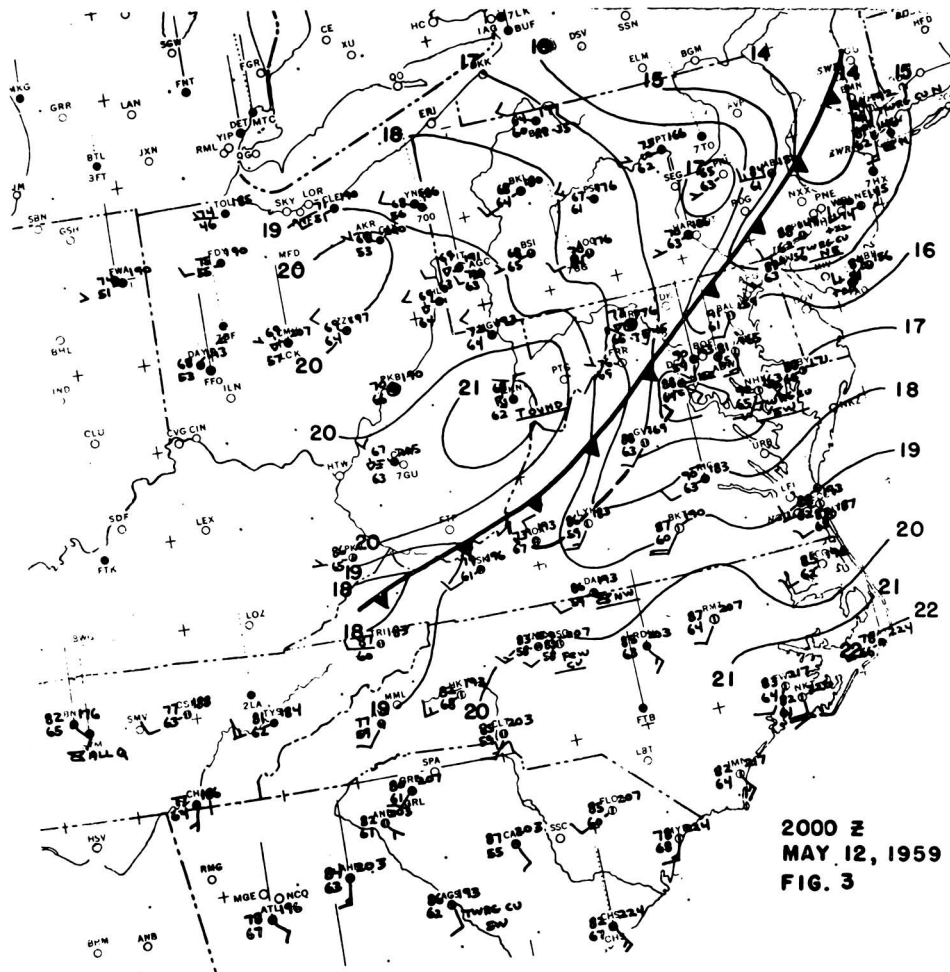


FIG. 3. 2000GCT surface mesoanalysis.

pressure area behind the front. The low-pressure area in the vicinity of Quantico, Virginia had moved northeastward. This resulted in a pressure gradient of about 2 mb in ten miles normal to the front in the segment of the front between Washington, D. C. and a point northeast of Aberdeen, Maryland. The front had passed Baltimore at 2048GCT, was in the vicinity of Washington National Airport at 2045GCT and passed Andrews Air Force Base at 2110GCT. The front passed Wilmington, Delaware just before 2100GCT, and it passed Philadelphia between 2055GCT and 2114GCT. At 2200GCT (fig. 5), observational data indicate a relaxation of the the pressure gradient in the vicinity of the front and a spreading our of the meso-scale high. The 2300GCT surface data (fig. 6) further indicate the dissipation of the meso-high. The duration of a strong meso-high was therefore less than three hours.

Computation of the speed of movement on the front from the vicinity of Martinsburg, West Virginia and Front Royal, Virginia to the Baltimore-Washington line shows the front to be moving east-southeastward at an average speed of 30 kn.

Observational data indicate that the pressure rose upon passage of the front as follows:

- 1.69 mb in 15 min at Harrisburg, Pennsylvania,
- 1.02 mb in 12 min at Wilmington, Delaware,
- 1.02 mb in 25 min at Martinsburg, West Virginia,
- 2.71 mb in 20 min at Baltimore, Maryland, and
- 2.37 mb in 33 min at Washington, D. C.

The thunderstorm activity in most cases lagged about five to ten minutes behind the surface wind shift and beginning of the pressure rise.

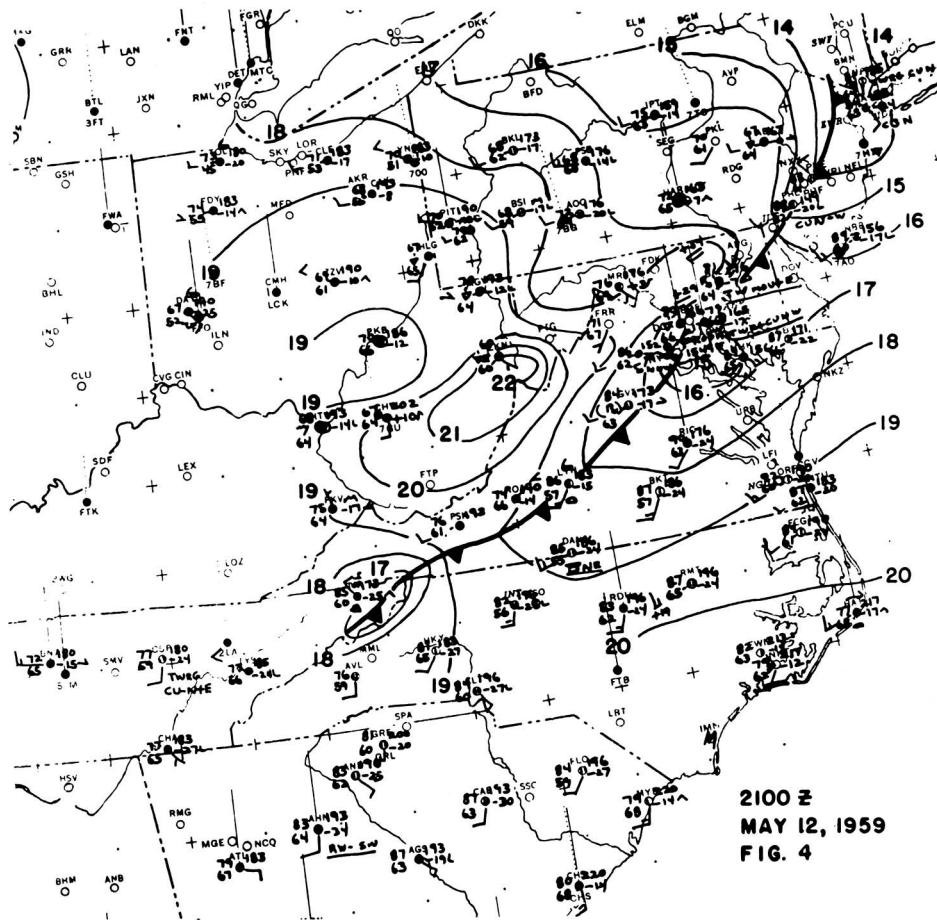


FIG. 4. 2100GCT surface mesoanalysis.

**4. Estimated thunderstorm updraft velocities in the vicinity of Baltimore, Maryland on 12 May 1959**

The 1200GCT DCA sounding is shown in fig. 7. The air mass over the Potomac and Chesapeake Bay area, as characterized by this sounding, is convectively unstable (lapse rate of pseudo wet-bulb temperatures in excess of the pseudo adiabatic rate) in the layer 900 to 680 mb. This layer is potentially unstable and will become actually unstable if subjected to sufficient lifting. The amount of instability that can be realized in terms of a thunderstorm updraft is a function of the environmental temperature of the air mass in which the thunderstorm occurs and the thunderstorm updraft temperature.

There are several known processes whereby air-mass modification can be achieved in the atmosphere as follows:

1. evaporation-transpiration processes,
2. addition or subtraction of heat from sources other than adiabatic processes,
3. adiabatic lifting of the air mass, and
4. horizontal advective processes.

Of these processes, 1 and 4 can be ruled out in this case as possible contributors to air-mass modification prior to the passage of the cold front. Process 2 acted in the main to increase temperatures in the lowest portion of the sounding as a result of solar insolation prior to the cold-frontal passage. The most probable process contributing to modification of the air mass in the middle levels (above the level of low-level warming) is that in 3 above. Adiabatic lifting of the air mass at the leading edge of the cold front may come about as a result of low-level convergence surmounted by high-level divergence. This convergence-divergence couple acts to produce positive (upward)

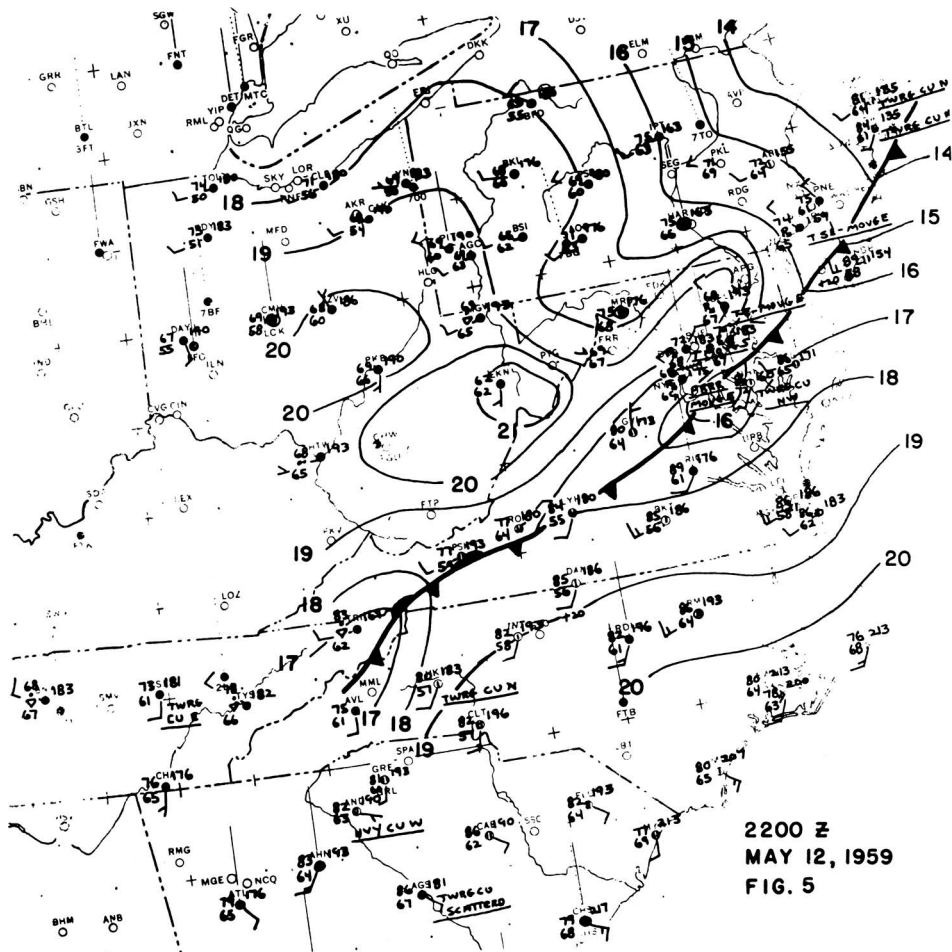


FIG. 5. 2200GCT surface mesoanalysis.

vertical motion that acts to cool upper-level temperatures, and thus increases thermal instability and, at the same time, increases the temperature difference between thunderstorm updraft and environmental air temperature.

It should be noted that for one to two observations prior to the passage of the cold front, convective cloud bases (also convective condensation level (CCL)) at DCA and BAL were reported at 6500 ft, and the reported surface temperatures were 90F and 91F, respectively. By referring to fig. 7, it can be seen that the CCL of a surface temperature of 90F corresponds to a potential temperature of 304K, and since saturation is reached at 6500 ft above the surface we can establish the mean properties of the air mass in the frictional layer which combined to establish the CCL. It should be noted that in this case the CCL is the same as the level of free convection (LFC).

A first approximation of the thunderstorm updraft temperature can be obtained by assuming that an air parcel as it is lifted above the condensation level cools at the moist adiabatic rate. This is the assumption used in the parcel method of estimating the energy available for continued acceleration of the rising air. It is generally considered, however, that the temperature decrease of the rising air is normally greater than that indicated by the moist adiabatic lapse rate. The usual assumption is that, because of horizontal pressure gradients and turbulent mixing near the cloud wall, the cooler, drier air of the environment is drawn into the cloud; the rising air of the cell, mixed with the entrained air, cools at a rate—lying between the dry and moist adiabatic rates—called the *entrainment lapse rate*. However, investigators do not always agree on these principles. The parcel method also assumes that the air of



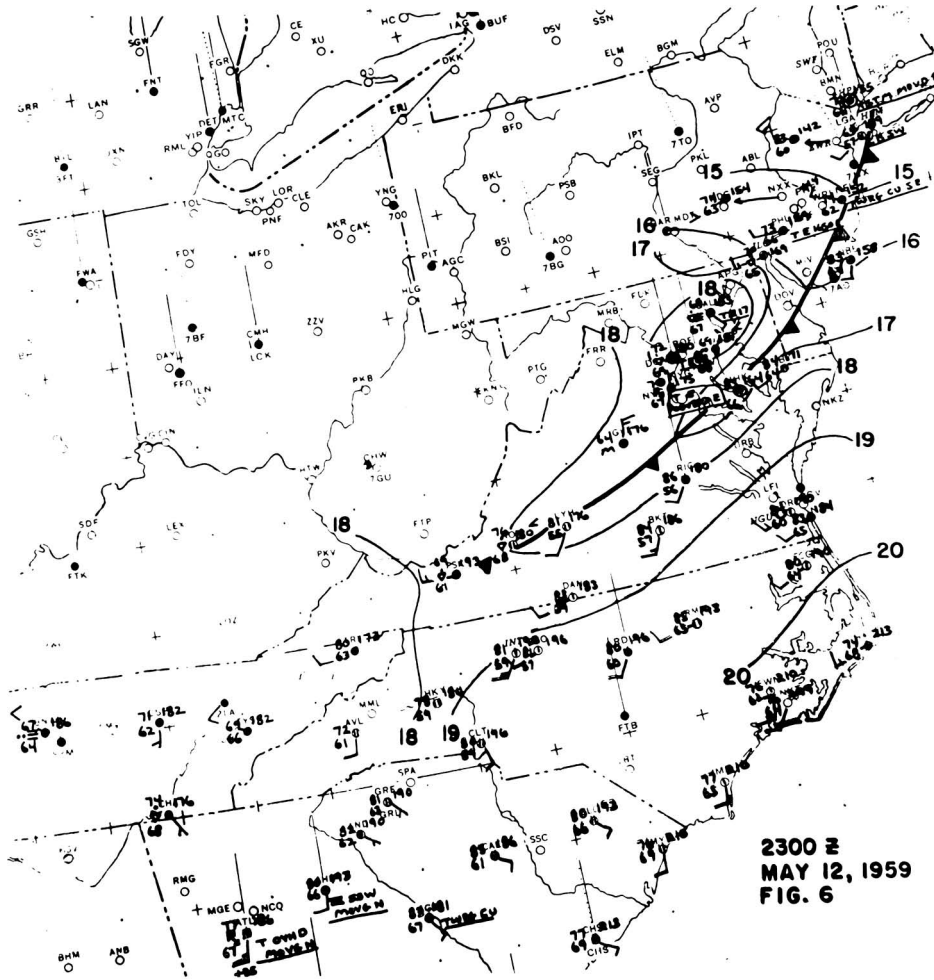


FIG. 6. 2300GCT surface mesoanalysis.

the environment remains unmodified. However, since continuity considerations require at least some descent of the surrounding air to maintain the thunderstorm updraft, the environmental air is normally warmed slightly. For these reasons, the parcel method may overestimate the thermal contrast between the updraft and the environment, in most circumstances indicating an *upper limit* on the energy available for lifting.

In this particular case, the lifted parcel temperature curve is defined by the 20C equivalent potential wet-bulb curve shown in fig. 7. This curve crosses the temperature curve of the sounding at 35,000 ft msl, above the highest level shown in fig. 7; at this point, deceleration of the parcel would begin due to negative buoyancy.

The velocity of the thunderstorm updraft, based on simple buoyancy considerations, is a function

of the difference between the virtual temperature of the updraft and the environmental air and the height above the LFC. In actuality, momentum exchange with the environment and frictional drag exerted by the liquid water in the cloud both act to reduce the velocity of the updraft computed from the buoyancy formula. An example suggests the importance of water droplets in decelerating the rising current. If 6.5 gm m<sup>-3</sup> of liquid water are present in an updraft at 400 mb, the temperature of the upward current must be at least 3C higher than that of the surroundings for rising motion to continue [3]. On the other hand, this effect may be compensated for by exchange of heat between water droplets and the free air in the updraft. Thus, the speed of the updraft computed from the buoyancy formula may represent an upper limit on the actual motion.

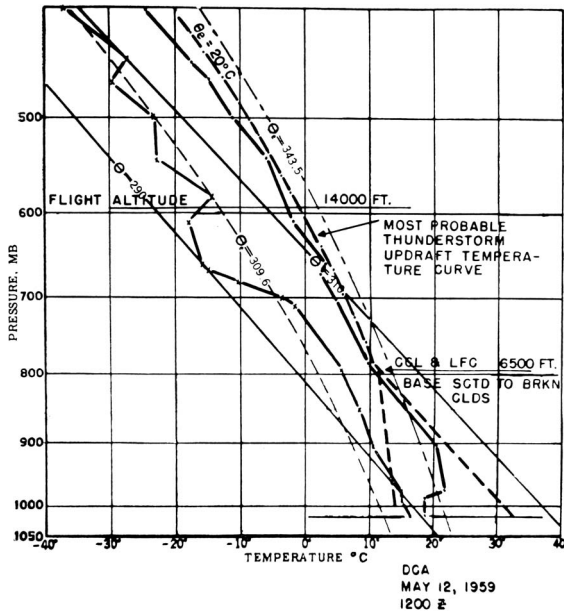


FIG. 7. 1200GCT DCA upper-air sounding. Solid line is temperature curve; dashed-plus (-+) is dew point; dash-dotted curve is moist adiabat from convective condensation level determined from intersection of the mean moisture in the lower three thousand feet and the dry adiabatic lapse rate from maximum afternoon surface temperature.

An integration of the buoyancy equation (see for example [2]) yields the formula:

$$W_H = \left( \frac{g}{T} \Delta T \Delta h \right)^{\frac{1}{2}} \quad (1)$$

where  $W_H$  = vertical velocity at level H in cm sec<sup>-1</sup>,  $g$  = acceleration of gravity,  $T$  = mean temperature of the environment in the layer concerned,  $\Delta T$  = difference in temperature at level H between environmental temperature and thunderstorm updraft temperature, and  $\Delta h$  = height difference in cm between LFC and level H.

The Thunderstorm Project [3] found a correlation between thunderstorm updraft velocities and maximum effective gust velocities. By using these data, Bates [2] found the relationship

$$U_e = 0.3W + 6 \text{ (ft sec}^{-1}\text{)} \quad (2)$$

where  $U_e$  is a maximum effective gust velocity and  $W$  is updraft velocity. An effective gust velocity of 30 ft sec<sup>-1</sup> or more is considered indicative of the possibility of extreme turbulence. From eq (2), it can be seen that a thunderstorm updraft velocity of 80 ft sec<sup>-1</sup> would be required to equal an effective gust of 30 ft sec<sup>-1</sup>. Since

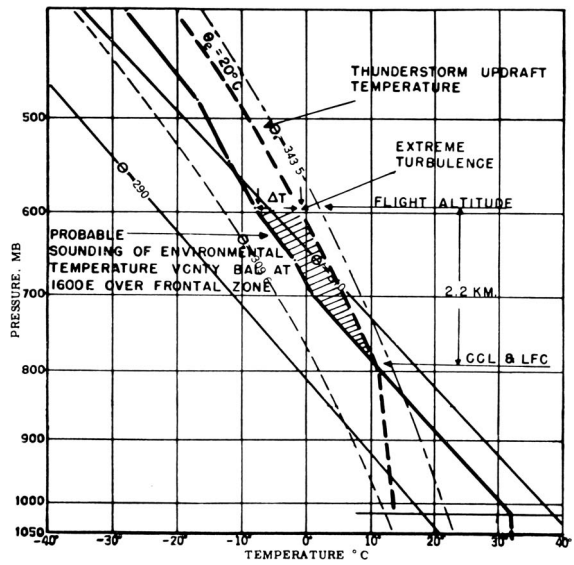


FIG. 8. Temperature sounding of fig. 7 (dashed-dotted curve) modified by lifting all points above 780 mb a distance of 80 mb. Dashed curve is moist adiabat from lifting condensation level as determined in fig. 7.

the actual relationship between updraft speeds and gust velocities may be nonlinear, there is considerable uncertainty in the effective gusts for high values of  $W$ .

Without any air-mass modification, other than surface heating, eq (1) applied to the DCA radiosonde observation yields an updraft of 17 ft sec<sup>-1</sup> at 10,000 ft, 33 ft sec<sup>-1</sup> at 14,000 ft, and 51 ft sec<sup>-1</sup> at 18,000 ft (table 1). These computed values represent an upper limit on the updrafts which would have occurred without any other modification of the air mass than that caused by surface heating. A more-accurate estimate of the updraft speeds would require data on the rate of entrainment and on the liquid-water content of the cloud. Since neither of these quantities is routinely available, it is clear that the use of the parcel method in determining the degree of turbulence is deficient for the reasons indicated.

TABLE 1. Buoyancy consideration.

Height (feet)	Unmodified sounding		Modified sounding—80-mb lift	
	Vertical velocity ( $W$ =ft sec <sup>-1</sup> )	Effective gusts ( $U_e$ =ft sec <sup>-1</sup> )	Vertical velocity ( $W$ =ft sec <sup>-1</sup> )	Effective gusts ( $U_e$ =ft sec <sup>-1</sup> )
10,000	17	11	46	20
14,000	33	16	80	30
18,000	51	21	92	34



The air mass in the vicinity of Baltimore could have been modified by vertical motion associated with the intensification of the cold front. The observations at BAL taken during the period 1550 EST to 1610EST show that during this period the wind shifted from SW 10 to NNW 45 and the pressure rose 2.71 mb in 20 min. These observations indicate intense low-level convergence, the exact magnitude of which remains unknown, but qualitative considerations leave little doubt that vertical motion was sufficient to modify the 1200 GCT data sounding, by at least an 80-mb lift of the air mass, to that shown in fig. 8.

From eq (1) and fig. 8, and by using a value of 7C for  $\Delta T$ ,  $\Delta h = 2.2$  km and  $T = 275$ A, we find that the updraft velocity is  $W \cong 24$  m sec<sup>-1</sup> or  $W \cong 80$  ft sec<sup>-1</sup> at 14,000 ft (table 1) and 92 ft sec<sup>-1</sup> at 18,000 ft.

Thus, if the air mass were modified by lifting due to low-level convergence at the front, to the extent outlined above, a possibility of extreme turbulence could be anticipated. The effective gusts computed from the updraft speeds and eq (2) would then be 30 ft sec<sup>-1</sup> at 14,000 ft and 33.6 ft sec<sup>-1</sup> at 18,000 ft. Greater lifting and modification of the air mass than that assumed would increase the probability of extreme turbulence.

According to George [4], whenever the cloud temperature above 9000 ft (judged by the moist adiabat through the CCL) exceeds the temperature of the environment by 7C or more, extreme turbulence is indicated on the modified sounding (fig. 7) between 14,000 and 16,000 ft.

During the Thunderstorm Project [3], of 812 measured maximum gusts, only 9, or about 1 per cent, exceeded a velocity of 30 ft sec<sup>-1</sup>. In order to encounter gusts exceeding 24 ft sec<sup>-1</sup> at the 14,000-ft level, pilots flew on the average about 70 mi within thunderstorms over the Ohio area of operations. These figures, though serving to indicate the likelihood of an airplane encountering extreme turbulence in storms over the east-central United States, unfortunately shed no light on the probable existence of such a hazard in the meteorological situation under study. Correlation of updrafts and gust velocities encountered in the Thunderstorm Project with routinely observed meteorological parameters, both preceding and during thunderstorm conditions, might conceivably suggest a means of determining the probability of extreme turbulence in a given set of conditions.

In addition to the parcel method, a technique devised by Williams [5] was used to compute

updraft speeds associated with the thunderstorm and frontal passage at Baltimore. With this method, the vertical velocity is indirectly ascertained from surface observational evidence and radar data on thunderstorm tops. It is assumed the the following conditions apply:

1. Divergence at the surface is compensated by divergence of the opposite sign aloft (Dines Compensation Principle).

2. Mass divergence at the level of thunderstorm tops ( $H$ ) is of equal magnitude and of opposite sign to that at the surface.

3. The distribution of mass divergence from the surface to the level of thunderstorm tops ( $H$ ) conforms to a cosine curve in the interval 0 to  $\pi$ , *i.e.*,

$$(\text{Div}_2 \rho \mathbf{V})_z = (\text{Div}_2 \rho \mathbf{V})_0 \cos \left( \frac{\pi z}{H} \right).$$

4. The vertical-motion field depends completely upon the divergence field regardless of the conditions that cause the vertical motion.

5. In the mass-continuity equation, the local change in density with respect to time is negligibly small, and

6. The advection of density at the surface of the earth and the vertical motion at the surface of the earth are negligibly small.

Then the vertical velocity at any level  $z$  between the surface of the earth and the top of the thunderstorm ( $H$ ) is expressed as:

$$W_z \cong \frac{H\rho_0}{\pi\rho_z} (\text{Div}_2 \mathbf{V})_0 \sin \left( \frac{\pi z}{H} \right) \quad (3)$$

where  $\rho_0$  = density of air at the surface,  $\rho_z$  = density of air at level  $z$ , and  $(\text{Div}_2 \mathbf{V})_0$  = low-level horizontal velocity divergence.

The weakest assumption among those listed as the basis for the model is the requirement for an arbitrary vertical distribution of the horizontal divergence. While the distribution assumed may be the most reasonable one for frontal thunderstorms, it is nevertheless an idealization which may or may not be approximated in an individual thunderstorm at a given stage of development. The computation of surface divergence presents a further difficulty in the absence of a close network of observing stations. Williams [6] has also developed a method for computing small-scale divergence by utilizing the sequence of changes at a wind-shift line and the speed of the line, but in practice the necessary parameters may be difficult

to estimate accurately from the wind records. Williams' [6] method is based on the relationship

$$\text{Div}_2 \mathbf{V} = \frac{V_1 \cos(\phi - \theta_1) - V_2 \cos(\phi - \theta_2)}{c \Delta t} \quad (4)$$

where  $\text{Div}_2 \mathbf{V}$  = horizontal velocity divergence,  $V_1$  = wind speed prior to shift,  $V_2$  = wind speed after shift (usually peak gust),  $\phi$  = direction from which wind-shift line moves,  $\theta_1$  = wind direction prior to shift,  $\theta_2$  = wind direction after shift,  $c$  = speed of movement of wind shift line, and  $\Delta t$  = time required to accomplish the wind shift.

Values of  $\theta_1$ ,  $\theta_2$ ,  $V_1$  and  $V_2$  can be obtained from surface observations at Baltimore;  $\phi$  and  $c$  can be computed from past positions of the cold front.

From the wind records at Baltimore the value of  $\Delta t$  lies somewhere between 2 and 4 min. By using a  $\Delta t$  value of 4 min and the other values determined by observational data, which are

$$\begin{aligned} \theta_1 &= 220 \text{ deg,} \\ \theta_2 &= 335 \text{ deg,} \\ V_1 &= 10 \text{ kn,} \\ V_2 &= 45 \text{ kn,} \\ \phi &= 300 \text{ deg, and} \\ c &= 30 \text{ kn,} \end{aligned}$$

and by solving eq (4) for  $\text{Div}_2 \mathbf{V}$ , we find that  $\text{Div}_2 \mathbf{V} = -18 \text{ hr}^{-1}$ .

By using this value of  $\text{Div}_2 \mathbf{V}$ ,  $H = 12 \text{ km}$  (tops of thunderstorms reported by radar), and by solving eq (3), we have at the 4.2-km level (14,000 ft) a value of  $24.6 \text{ m sec}^{-1}$  (or  $80 \text{ ft sec}^{-1}$ ) (table 2), which is to say that the assumed surface velocity convergence (negative divergence) is in agreement with the computations based upon the parcel method.

If a shorter time interval of two minutes were used, a value of  $161 \text{ ft sec}^{-1}$  ( $U_e = 54.3 \text{ ft sec}^{-1}$ ) at 14,000 ft (table 2) would be indicated. The two-minute time interval is in close agreement

TABLE 2. Convergence-divergence couple.

Height (feet)	Wind-shift time interval ( $\Delta T$ ) = 4 min		Wind-shift time interval ( $\Delta T$ ) = 2 min	
	Vertical velocity ( $W = \text{ft sec}^{-1}$ )	Effective gusts ( $U_e = \text{ft sec}^{-1}$ )	Vertical velocity ( $W = \text{ft sec}^{-1}$ )	Effective gusts ( $U_e = \text{ft sec}^{-1}$ )
6,000	34	16	68	26
10,000	59	24	118	41
14,000	80	30	160	54

with the wind-record at the Martin Company Airport at Baltimore.

While it is possible thus to assume conditions which could have existed in this weather situation, and which could have resulted in updrafts and gusts of dangerous intensity, we have no means of knowing whether or not these conditions were actually fulfilled. There is no indication, in the observed surface or upper-air data, that the situation was more intense than those accompanying the more severe storms investigated by the Thunderstorm Project. On that project, only one maximum effective gust in excess of  $40 \text{ ft sec}^{-1}$  was observed out of a total of 2976 measured at the 15,000-ft level and out of the total of 10,446 measured at 5 levels varying from 5000 to 25,000 ft. However, the storms investigated by the Thunderstorm Project may not have constituted a representative sample, and the frequency of occurrence of extreme turbulence may possibly be greater than these figures indicate.

## 5. Summary

Two models were used to estimate the speed of updrafts and gusts associated with a thunderstorm and cold-front passage near Baltimore, Maryland on 12 May 1959. The assumptions necessitated by these models are so restrictive that it is not possible to assign confidence limits to the computed gust velocities. These circumstances point up a grave need for further research on turbulence hazards to aircraft and for the development of a method whereby meteorologists can determine, from ordinarily observed weather conditions, the probability of occurrence of extreme turbulence.

## REFERENCES

- U. S. Weather Bureau, 1956: *Forecasting tornadoes and severe local storms*. Forecasting Guide No. 1, Staff Members, SELS Center, Kansas City, Mo.
- Bates, F. C., 1955: A technique for predicting extreme turbulence related to thunderstorms. *Bull. Amer. meteor. Soc.*, **36**, 379-383.
- U. S. Weather Bureau, 1949: *The thunderstorm*. Rep. of the Thunderstorm Project.
- George, J. J., 1947: *Forecasting turbulence in thunderstorms*. Eastern Airlines Meteor. Bull. No. 47-38B & Supplements.
- Williams, D. T., 1959: A theoretical estimate of draft velocities in a severe thunderstorm. *Mon. Wca. Rev.*, **87**, 65-68.
- Williams, D. T., 1958: *A method for computing small-scale divergence along a wind-shift line*. Unpubl. ms., U. S. Weather Bureau, Kansas City, Mo.