NOAA Technical Memorandum NWS WR-133

SPECTRAL TECHNIQUES IN OCEAN WAVE FORECASTING

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October 1978

UNITED STATES
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ATMOSPHERIC ADMINISTRATION
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Date : March 19, 1979

Reply to Attn. of: OA/WFW3

To : Recipients of Western Region NOAA TM NWS WR-133

From : OA/WFW3 - L. W. Snellman

Subject: Corrections to NOAA Technical Memorandum NWS WR-133

Careful scrutiny of subject Technical Memorandum has disclosed some errors in the text. Please make the following pen and ink corrections on your copy(ies) of this publication.

Page 3 — Expression (2) for total average energy in the sea should be per-unit area, as the expression for (1) is.

Page 11 — In the last paragraph, second line, the word "spand" should be written "span".

Page 11 — Bottom of page. E should be equal to 58 ft^2, not 51 ft^2.

Page 13 — Third paragraph, last word in sixth line, should be "or", not "of".

Page 24 — The following sentence should be added to the first paragraph:

> "Figures 56 and 57 give the amplitudes of the fully arisen spectrum as a function of wind speed and wave frequency for reference with the observed values of the buoy."

Page 31 — Figure 2, the expression for s(f) should read:

$$s(f) = \text{constant} \cdot f^{-5}.$$

The negative sign before the number 5 was omitted.

Page 37 — Caption for Figure 13. The time reference of t_1 and t_2 should be t_a and t_b, respectively.

Page 38	The ordinate should be labeled "f", rather than "v". Also, decimal points should precede each number along the ordinate.
Page 40	The ordinate should be labeled "f", rather than "v".

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SPECTRAL TECHNIQUES IN OCEAN WAVE FORECASTING

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PREFACE

Weather Bureau Technical Memorandum (WBTM) WR-51 "Western Region Sea State and Surf Forecaster's Manual" was prepared as a ready-reference for operational techniques of wind wave, swell, and breaker forecasting. The "significant wave" forecast technique developed by Sverdrup-Munk-Bretschneider (S-M-B) was used. This method was described, rather than the "Wave Spectra" method of Pierson-Neumann-James (P-N-J), as the computations are much simpler making the method operationally feasible for use by forecasters who did not have access to computer facilities. With the advent of Automation of Field Operations and Services (AFOS) and minicomputers available to each Weather Service Forecast Office, the cumbersome computations of the P-N-J method no longer deter its use.

This Technical Memorandum is a brief outline to help forecasters understand the principles involved in the "Wave Spectral" method. Examples of the method, describing its usefulness are worked out. It is not intended to replace TM WR-51, but rather complement it as another tool available to the forecaster. Upon finishing this paper, one will not be an expert regarding the P-N-J spectral approach to wave forecasting. One should, however, have a better understanding of the method enabling better utilization of spectral wave observations, as available from some coastal recorders and data buoys, and spectral wave forecasting techniques.

I. DEFINITION OF TERMS

Saturation: point in time at which energy can no longer be added to the sea at that particular wind speed. The sea is sometimes said to be fully arisen or fully developed.

Spectrum: the energy distribution of the sea with respect to wave frequency.

Fetch: the distance over which the wind blows in approximately the same wind speed and direction.

Fetch limited: the area of the sea where saturation cannot be attained because the fetch length is too short.

Duration: length of time the wind blows at approximately the same wind speed and direction.

Duration limited: the area of the sea where saturation cannot be attained because the wind duration over the fetch is too short.

Wave velocity (celerity): the speed at which a wave of a given frequency travels.

Group velocity: the speed at which the energy of a given frequency component of the sea travels; in deep water this is one-half the wave celerity.

Co-cumulative Spectrum: the sum of all the energy in the sea from the highest frequency component to the lowest frequency component.

Band width: the range of wave frequencies present at a location at a particular time.

Dispersion: the narrowing of band widths as the wave trains travel farther from the generating area (the fetch).

Average wave height: the height of waves as determined by measuring all the wave heights on a wave record and dividing by the total number of waves.

Significant wave height: the average of the heights of the highest third of the waves. This was originally thought to be the wave height that an observer would report as most typical of a given sea.

II. FUNDAMENTALS OF SEA AND ITS CHARACTERISTICS

Just as erosion of the land is a consequence of frictional effects between the air and land, the sea state is the result of frictional effects between the air and sea. As air passes over the water surface, the air does work on the water surface. A resonance mechanism results in a linear growth rate for waves having a phase speed equal to the speed of eddies which are advected in the lower atmosphere. Continued growth is due to an instability mechanism which results in an expotential growth rate. This second growth process becomes dominant over the resonance mechanism shortly after the waves have been formed. Hence, energy is transferred from the air to the sea and is stored in the amplitude of the waves. The total average energy in the sea is

(1) $1/2\ \rho g A^2$/unit area

or

(2) $1/8\ \rho g H^2$ where

A = wave amplitude
H = wave height (2A)
g = gravity force
ρ = water density

In addition to direct input from the wind, wave energy of a particular frequency may grow (or diminish) at the expense of adjacent frequencies by virtue of non-linear interchange among wave components. Losses of energy may also occur by turbulence (white capping) to high frequency waves.

The growth of the sea begins with waves of smaller wave length (higher frequency) and works up toward longer waves (lower frequency) (see Figure (1)). The Energy Density Spectrum (S(f,t)), Figure (2), shows how the energy is distributed by frequency (f) and demonstrates the growth of the sea with time (t). The total energy in the sea, E, is the summation of all the various frequency wave amplitudes. As the wind of constant speed, V, blows over the area (the fetch), waves through frequency f_1 have formed at t_1. With increased time, t_2 and t_3, waves through frequency f_2 and f_3 are formed. Eventually the fully arisen sea condition is attained.

Another means of displaying the energy distribution is shown in Figure (3). This plot for any given wind speed is the sum of all the energy from the highest frequency waves to the desired lowest frequency component. This is called the Co-Cumulative Spectrum (C.C.S.). Figure (4) is the same as (3) except three different wind speed curves are superimposed on the graph.

The energy graphs shown have been for fully developed seas or developing seas which haven't yet reached the fully developed state. If the wind doesn't blow over the area long enough, the sea is duration limited in that the lower frequency waves will not form. If the distance over which the wind blows is too short, the sea is fetch limited and again the lower frequency waves will not form. Figure (5) shows the energy graph when the sea is fetch or duration limited. If the sea is not fetch or duration limited, it will reach saturation; no more energy can be added to the sea and all wave frequencies for that wind speed have been formed.

When the sea is not saturated (energy is still being added), the higher frequency waves will become saturated before the lower frequency waves. The higher frequency waves will lose any excess through turbulence (white capping). The wind will continue to add energy to new lower frequency waves until the sea is saturated (fully developed). If the wind speed increases, energy will be added to waves of all frequencies, but most of the energy will go to new lower frequency waves being created. Figure (6) shows the spectrum under various wind speeds. Note the shift of the maximum energy toward lower frequencies with increased wind. The frequency of maximum energy for a fully developed sea is:

(3) $$f_{max} = \frac{2.476}{V} \text{ where V is in knots.}$$

The Energy Saturation Limit is:

(4) $$E_s(V) = \text{constant} \cdot V^4.$$

This shows that a small increase in wind speed will significantly increase the energy in the sea. The constant in equation (4) depends only on the level at which the "surface" wind is measured (2 meter height, 10 meter height, geostrophic wind, etc.).

Once the sea has been described by its energy/frequency characteristics, statistics can be used to find an average state of the sea with respect to wave height and period. The table below shows some of these statistical relationships. In addition, these frequency components can be projected outside of the generating area.

III. TABLE OF WAVE-HEIGHT STATISTICS

Most Frequent Wave Height	$1.41 \sqrt{E}$
Average Height	$1.77 \sqrt{E}$
Significant Height ($H_{1/3}$)	$2.83 \sqrt{E}$
Average of the Heights of the 1/10 Highest Waves ($H_{1/10}$)	$3.60 \sqrt{E}$

Ten-Percent Ranges (10% of the Waves will have Crest-To-Trough Heights Between the Given Range of Values as follows:

10%	between	0.00	and	$0.64 \sqrt{E}$	feet
10%	"	$0.64 \sqrt{E}$	"	$0.94 \sqrt{E}$	"
10%	"	$0.94 \sqrt{E}$	"	$1.20 \sqrt{E}$	"
10%	"	$1.20 \sqrt{E}$	"	$1.42 \sqrt{E}$	"
10%	"	$1.42 \sqrt{E}$	"	$1.66 \sqrt{E}$	"
10%	"	$1.66 \sqrt{E}$	"	$1.92 \sqrt{E}$	"
10%	"	$1.92 \sqrt{E}$	"	$2.20 \sqrt{E}$	"
10%	"	$2.20 \sqrt{E}$	"	$2.54 \sqrt{E}$	"
10%	"	$2.54 \sqrt{E}$	"	$3.04 \sqrt{E}$	"
10%	greater than	$3.04 \sqrt{E}$	feet		

A typical wave has a height of about 10 feet and wave length of about 591 feet.

In summary, waves grow by energy transfer from the wind to the water until the wave reaches maximum height for its wave length before it becomes unstable resulting in breaking and white capping (Stokes has shown that the wave is stable if its height is about 1/7th or less of its wave length, steepness: $\sigma = H/L \leq 1/7$). Wave growth or decay may occur by interference and/or nonlinear interaction between waves. The sum total of these processes results in a chaotic superposition of various frequency waves. (Figure (7)).

IV. FUNDAMENTALS OF SWELL

After waves have formed in a fetch area, the collection of deep water waves of each frequency travels outward at the group velocity:

(5) $C_g = 1.515 \cdot T$.

where C_g is in knots and the period, T, is in seconds. Although the individual speed of each wave is:

(6) $V = 3.03 \cdot T$.

the energy in deep water does not propogate at that rate, but half of that speed, Cg. In shallow water ($d/L < 1/20$) wave energy is propagated at the phase speed ($Cg = V$). Once the waves leave the generating area they become swell waves. Swell waves are waves whose spectral components (frequency and direction) appear in a new region only negligibly changed during their travel from the primary source (the fetch).

To an observer, swell differs from the random character of sea waves by its uniformity of wave period and height. This is due to dispersion, which is a sorting out of waves as they leave the generating area. The longer period (low frequency) waves travel faster than do the shorter period waves, a consequence of equation (5). The travel time for energy of frequency (f) over a distance, R, is:

(7) $$t = \frac{R}{C_g(f)}$$

or

(8) $t = 0.66R\ f$ where R is in nautical miles.

The dependence on frequency acts as a filter on the waves; the low-frequency waves run out ahead of the high-frequency waves. With time, the high-frequency and low-frequency waves get farther apart. Thus, it takes longer for a certain frequency range of waves to pass a more distant point than a point closer to the generating area. Figure (8) demonstrates this graphically. Since the different waves are separated more at the farther distance from the fetch, the waves passing the farther point are in a narrower frequency range, hence are more uniform at any given time. Figure (9) demonstrates this.

The amount of energy received in each frequency band also varies dependent on the forecast points distance and orientation with respect to the fetch area. (See Figure (10)). As the waves move out radially, the same energy is being spread out over a larger area, hence the wave height (energy per unit area) decreases. Figure (11) summarizes this effect. Figure (11a) is the energy density spectrum vs. time at a point close to the fixed source of the swell. Figure (11b) is the energy spectrum at some point ΔR farther than the first point. The farther point receives energy at a lower level and in a narrower frequency band than does the closer point. The energy in each frequency band is:

(9) $E_{swell} = \int_{\theta_1}^{\theta_2} \int_{f_1}^{f_2} F(f,\theta)\,df\,d\theta$ where the θ's are the angles shown in Figure 10.

(10) $F(f,\theta)d\theta = S(f)q(f,\theta)d\theta$ and $\int S(f)df = \int_\theta F(f,\theta)d\theta$.

therefore $q(f,\theta)d\theta$, is that portion of the total directional energy in the $d\theta$ bandwith. From experimental measurements it has been found that

(11) $q(f,\theta) \simeq \cos^n\theta$ where $2 \leq n \leq 4$ and $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.

For higher frequency waves n=2 and for lower frequency waves n=4. As a close approximation for operational use

(12) $q(\theta) = \frac{2}{\pi}\cos^2\theta$. Now

(13) $$E_{swell} = \underbrace{\int_{f_1}^{f_2} S(f)\, df}_{A} \underbrace{\int_{\theta_1}^{\theta_2} \frac{2}{\pi} \cos^2\theta d\theta}_{B}$$

which can be evaluated quite easily graphically.

Tables of $\Delta r = \int_{\theta_1}^{\theta_2} q\ (\theta)d\theta$ have been calculated so that term B of equation (13) is the difference between the two r values at θ_2 and θ_1. Term A of equation (13) can be found by taking the difference $E_2 - E_1$ on the co-cumulative spectrum (C.C.S. graph) for the particular wind speed.

The real advantage to using spectral methods, rather than the significant wave height method, is when changes take place in the fetch area or when waves will be arriving at the forecast point from more than one fetch area. The energies from all sources can be added together from various waves and a resultant wave condition can be derived. Wave heights <u>cannot</u> be simply added together to derive the resultant wave condition (energy is proportional to H^2).

Suppose the wind speed changes in a stationary fetch area. A gradual increasing wind can be handled as a piecewise continuous increase as in Figure (12). The energy for each step is determined and this is used as the starting amount of energy for the next time step increase of wind speed (Figure (13)). In this case, the energy for the first step is:

(14) $E_1 = E\ (V_1, t_b) = E(V_2, t_a)$

and the energy for the second step is:

(15) $E_2 = E\ (V_2, t_b)$ where $t_b = t_a + \Delta t$.

A similar approach is used for decreasing winds. In a decreasing situation the sea may have enough energy in it already from the higher wind so that the lower wind speed may not be able to add any more energy to the sea. A rapid decrease will cause the waves to gradually lose energy by working on the atmosphere (a slow process though) and by waves moving out of the generating area; thus, the sea will eventually reach the saturation level for the new wind speed.

V. FORECASTING EXAMPLES

These examples will show how the previously discussed spectral equations are used in graphical form to make wave forecasts. These equations could just as easily be programmed into a computer, thus eliminating the need to use the various graphs. Each problem has been carried out in detail for days into the future, well beyond the period of operational forecasts.

Obviously, changes in the synoptic patterns and in fetches will take place during the period which have not been taken into account. This long forecast period was used for exemplary purposes only, neglecting other practical considerations.

EXAMPLE I - Sea Conditions.

A storm is developing in an area. In the last 6 hours, the wind has increased to 20 knots in the area with near uniform wind direction. The fetch is 400 nm wide and 600 nm long. Assuming that the wind continues at 20 knots, what is the wave condition in the area 9 hours later?

t_d = wind duration. Since the wind increased from calm to 20 knots in 6 hours, the midpoint of this period will be the starting point (3 hours ago) for the beginning of the 20-knot wind. This plus the 9 hours until the forecast time gives $t_d = (3+9) = 12$.

V = wind speed, which is 20 knots.

Looking at the co-cumulative spectra duration graph (Figure (27)), it is seen that it is not duration limited as full energy for a 20-knot wind is reached after 10 hours ($E = 8\ ft^2$). The co-cumulative spectra fetch limit graph (Figure (28)) shows that full energy for a 20 knot wind is reached with a minimum fetch length of 75 miles, so this fetch is not fetch limited either. The energy then, in this fetch is the maximum energy of about 8 ft^2.

From statistical relationships (see page 5) it is known that:

$$f_{max} = \frac{2.476}{V} \quad \text{where V is in knots}$$

$$H_{1/3} = 2.83\sqrt{E} \quad \text{where E is in ft}^2$$

$$H_{1/2} = 1.77\sqrt{E}$$

and

$$H_{1/10} = 3.60\sqrt{E}.$$

Therefore:

$$f_{max} = \frac{2.476}{20} = 0.1238\ \text{sec} \simeq .12/\text{sec}$$

$$T = T\ \text{average} = \frac{1}{.1238} \simeq 8\ \text{sec}$$

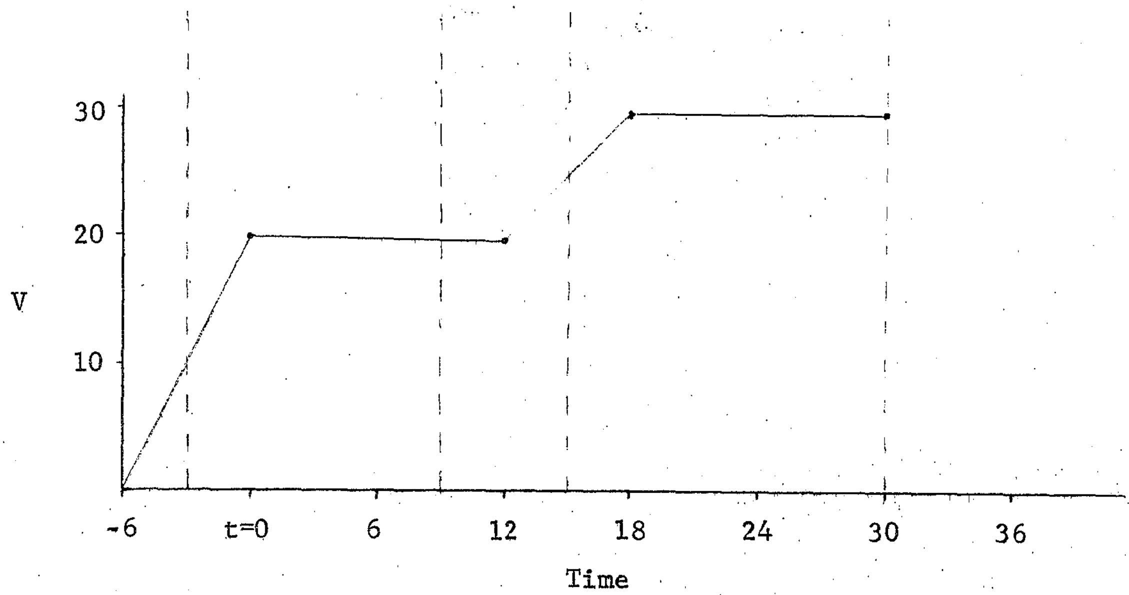
$$H_{1/10} = 3.60\sqrt{8} = 10\ \text{ft}$$

$$H_{1/3} = 2.83\sqrt{8} = 8\ \text{ft}$$

$$H_{1/2} = 1.77\sqrt{8} = 5\ \text{ft}$$

EXAMPLE II - Fetch With Increasing Wind Speed

Now suppose a surface prognosis is available which shows that the fetch area of Example I will remain in the same location and configuration, but further development of the storm increases the pressure gradient so that the wind will be 30 knots over the area from 18 to 24 hours from now. What will the wave condition be after 24 hours from now? (see diagram below for wind speed history.



Again the first task is to inspect the C.C.S. duration graph. The energy from t-3 to t+9 hours was found to be 8 ft^2. From t+9 to t+15 the wind remains at 20 knots, and since $E = E_{max} = 8\ ft^2$, no new energy is added. From t+15 to t+24 hours (another 9 hours duration) the wind is at 30 knots. Energy of 8 ft^2 is accumulated before this time period, and this energy is equivalent to a 30 knot wind blowing over the same area for 8 1/2 hours. The new total will be the equivalent of 8 1/2 + 9 = 17 1/2 hours at 30 knots. This is duration limited as the sea reaches an energy of about 33 ft^2 in 17 1/2 hours, where E_{max} is about 58 ft^2 (reached after about 23 hours).

Similarly, the same reasoning must be applied using the fetch graphs. During this whole time spand, the fetch length has been 600 nm. Even at the higher wind speed of 30 knots, the minimum fetch length is 280 nm, so the area is not fetch limited. Since duration, not the fetch length, is the limiting factor in determining the energy in the fetch area, the calculations above for the duration limited case will be used. With

$$E = 51\ ft^2 \qquad f_{max} = \frac{2.476}{30} = .0825,$$

but since it is duration limited, the lower frequency waves will not yet be formed and the f_{max} will actually be skewed toward the higher frequency end of the spectrum. For an E_{max} of 33 ft the wind would be about 26 or 27 (26.5) knots and the

$$f_{max} = \frac{2.476}{26.5} = .0934.$$

This can be used as an approximate f_{max} for this case, so the forecast results are;

$$T_{aver} = \frac{1}{.0934} \approx 11 \text{ sec}$$

$$H_{1/10} = 3.60 \sqrt{33} = 21 \text{ ft}$$

$$H_{1/3} = 2.83 \sqrt{33} = 16 \text{ ft}$$

$$H_{1/2} = 1.77 \sqrt{33} = 10 \text{ ft.}$$

EXAMPLE III - Swell From A Distant Fetch.

This is an example of a stationary fetch which is generating swell waves toward a forecast point which is outside of the fetch area. The fetch configuration is shown in Figure (14) and the specifications are:

Ro	=	1000 nm
V	=	20 Kts
td	=	24 hrs
Fetch width	=	400 nm
Fetch length	=	600 nm
θ_1	=	-11^{o}
θ_2	=	11^{o}

After looking at surface prognoses it was determined that between now (to) and 18 hours from now (to + 18) the above fetch conditions are essentially unchanged.

Equation (10)
$$E\ swell = \underbrace{\int_{f_1}^{f_2} S(f)df}_{A} \ \underbrace{\int_{\theta}^{\theta_2} \frac{2}{\pi} \cos^2 \theta d\theta}_{B}$$

must be evaluated for the forecast. Term A is evaluated in the graphs of Figures (29), (30), and (31).

The C.C.S. fetch limit graph (Figure (28)) shows that it is not fetch limited as it reached E max of 8 ft^2 in only 75 nm. The C.C.S. duration limit graph (Figure (27)) shows that it is not duration limited either as the E_{max} of 8 ft^2 is attained after 10 hours. Further inspection of the C.C.S. graph shows that very little energy is contained in the high frequency waves (less than 1 ft^2 in waves of frequency f = .25 of higher). Due to this, only waves of frequency f = .25 or lower will be considered. In a developing sea of 20 knots, after 4 hours waves up to f = .21 will form, after 6 hours waves up to f = .16 will form, after 8 hours waves of f = .12 will form, and by the minimum duration of 10 hours for maximum energy, waves up to f = .08 will form.

Now that it is known when the various frequency waves formed, the time it takes the waves to travel to the forecast point must be determined. Figure (30) portrays the travel time of various frequency waves as a function of distance (Ro). This can also be computed by using equation

(5) t = .66 Ro f. The actual time that the waves arrive at the forecast point is the travel time (t) less the number of hours ago that these waves formed (duration of the fetch, td = 24, plus the time it takes the various frequency waves to form). Using Ro of 1000 nm and the above selected frequencies the arrival times are:

f	t (travel time)	arrival at time to +
.25	165 hours	165 - 24 + 3 = 144
.21	139	139 - 24 + 4 = 119
.16	106	106 - 24 + 6 = 88
.12	79	79 - 24 + 8 = 63
.08	53	53 - 24 +10 = 39

These times vs. frequency have been plotted in Figure (15).

Now that it is known what frequencies are present, the energy in the fetch must be determined to forecast the energy received in the swell waves (term B in equation (10)). Figure (15) shows that the first frequency waves received at the forecast point are actually the last ones formed (lowest frequency waves) in the fetch. This means that the energy, E, is the maximum, $E_{max} = 8\ ft^2$.

The angular spreading factor from Figure (32) is the graphical evaluation of term B. It tells what percentage of the energy from the fetch will be arriving at the forecast point. The values are:

$\theta_2 = 11^o$ 62
$\theta_1 = -11^o$ 38
$\theta_4 - \theta_3$ 24

The forecast point will receive 24% of the energy in the fetch.

at t_o + 20 none of the energy will have yet arrived at the forecast point

at t_o + 40 waves of frequency .08 will arrive.
Little energy is contained in waves of f = .08 only (< 1 ft^2) E < 1 ft^2 • .24 = .24 ≃ 0.2

at t_o + 60 ($.08 \le f \le .115$) fetch energy between f = .08 and .115 is 8 - 6 = 2 . E = 2ft^2 • .24 ≃ 0.5

at t_o + 80 ($.08 \le f \le .145$) fetch energy between f = .08 and .145 is 8 - 4 = 4 . E = 4 ft^2 • .24 = 1.0

at t_o + 100 (.08 and .18) fetch energy is 8 - 2 = 6
E = 6 ft^2 • .24 = 1.4 ft^2

at t_o + 120 ($.08 \le f \le .210$) fetch energy is 8 - 1 = 7
E = 7 ft^2 • .24 = 1.7 ft^2

at t_o + 140 ($.08 \le f \le .245$) fetch energy is 8 - 1 = 7
E = 7 ft^2 • .24 = 1.7 ft^2

at t_o + 160 ($.08 \le f \le .25$) fetch energy is 8 - 0 = 8
E = 8 ft^2 • .24 = 1.9 ft^2

The following table sums up the results with time at the forecast point.

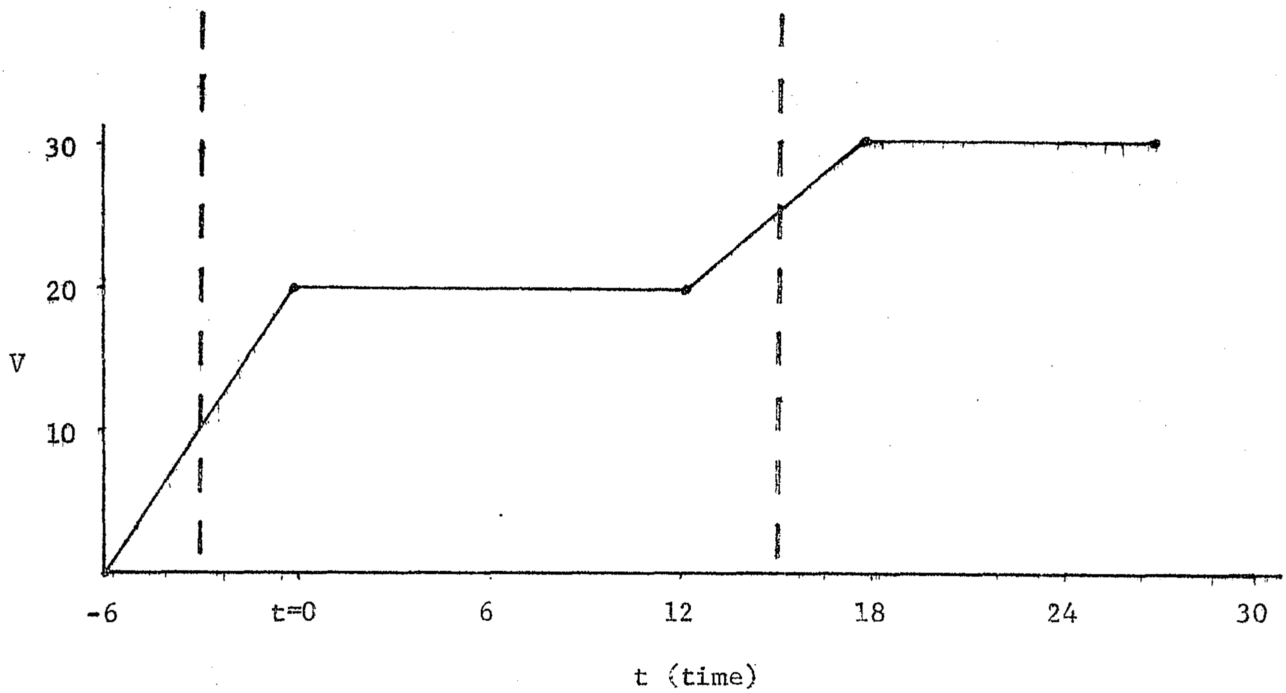
t (time)	f	E	$H_{1/3} = 2.83\sqrt{E}$
20	0	0	0
40	f = .08	0.2	1 ft
60	$.08 \leq f \leq .115$	0.5	2 ft
80	$.08 \leq f \leq .145$	1.0	3 ft
100	$.08 \leq f \leq .18$	1.4	3 ft
120	$.08 \leq f \leq .210$	1.7	4 ft
140	$.08 \leq f \leq .245$	1.7	4 ft
160	$.08 \leq f \leq .25$	1.9	4 ft

EXAMPLE IV - Swell From a Fetch With Increasing Winds.

This fetch is part of a developing storm. The wind increases to 20 knot in the area over a 6 hour period and has continued. Then surface progs show that by 18 hours from now the wind is 30 knots. What are the wave conditions at the forecast point? The area is described below:

$$Ro = 750 \text{ nm}$$
$$\text{fetch width} = 400 \text{ nm}$$
$$\text{fetch length} = 600 \text{ nm}$$
$$\theta_1 = -10^o$$
$$\theta_2 = 22^o$$
$$V \text{ at } t_o = 20 \text{ knots}$$
$$V \text{ at } t_o + 18 = 30 \text{ knots}$$

The wind history diagram is shown below.



The first graphs to check are the C.C.S. graphs in Figures (27) and (28) to see if the fetch is duration or fetch limited. Since 700 nm is not fetch limited for either wind speed and 18 hours is not duration limited for the 20 knot wind, E_{max} will eventually be attained at the 20-knot speed, and it will be assumed that it will also be attained for the 30-knot wind speed.

It must now be determined which frequency waves are formed under the two different wind speeds. The arrival time of these waves at the forecast point will be the time it takes for waves to develop, plus the travel time of the waves, less the difference between when the wind started and the initial time (t_o). The table below is the computation for the 20 knot wind which began at time t_o - 3.

f	created in X no. of hours	travel time	arrival time
.25	3	124	124 + 3 -3 = 124
.20	4.2	99	99 + 4.2 -3 = 100.2
.17	5.7	84	84 + 5.7 -3 = 86.7
.15	6.7	74	74 + 6.7 -3 = 77.7
.13	7.5	64	64 + 7.5 -3 = 68.5
.10	9	50	50 + 9 -3 = 56
.08	10	40	40 + 10 -3 = 47

When the 20-knot wind suddenly changes to become a 30-knot wind, the waves that are already in existence (frequencies between .25 and .08) will begin gaining energy until they reach the saturation amount for the new wind speed. In addition the 30-knot wind will begin forming new lower frequency waves that the 20-knot wind could not create (frequencies as low as .06). Information concerning when these waves will reach saturation can be found in the C.C.S. duration limit graph. This time can be added to the travel time of the wave to get the arrival time of the waves from the 30 knot wind. This is done in the table below.

f	Created in X no. of hours	Travel time (distance Ro)	Arrival Time(t)
.25	3 hours	124	124 + 3 + 15 = 142
.20	4	99	99 + 4 + 15 = 118
.17	6	84	84 + 6 + 15 = 105
.15	8.2	74	74 + 8.2 + 15 = 97.2
.13	11	64	64 + 11 + 15 = 90
.10	16	50	50 + 16 + 15 = 81
.08	19.5	40	40 + 19.5+ 15 = 74.5
.06	22	30	30 + 22 + 15 = 67

Now that it is known what frequency waves are arriving at the forecast point and whether they were generated by the 20-knot or 30- knot wind, (plotted in Figure (16)) the wave heights at the forecast point can be computed. The total energy will be the energy in the waves from the 20 knot wind plus the energy in waves from the 30 knot wind. For each particular forecast time, Figure (16) can be checked to see what frequency will be there and then reference made to the C.C.S. graph to find the energy. The results of this are in the table below.

t	Δ f 20 kts	Δ E 20 kts	Δ f 30 kts	ΔE 30 kts	$E=E_{20}+E_{30}$
40	0	0	0	0	0 + 0 = 0
60	.08 -.11	1.5	0	0	1.5 + 0 = 1.5
80	.095-.155	4	.06 -.095	27	4 + 27 = 31
100	.16 -.20	1.5	.06 -.16	52	1.5 + 52 = 53.5
120	.205-.24	.75	.06 -.205	56	.75+ 56 = 56.75
140	.245-.25	0	.06 -.245	57	0 + 57 = 57
160	0	0	.06 -.25	57	0 + 57 = 57

etc.

The energy values above are the values in the fetch. At the forecast point the energy is reduced by the amount given by the angular spreading factor. Using $\theta_3 = -10^o$ and $\theta_4 = 22^o$ in Figure (32) the spreading factors are:

$\theta_2 = 22^o$ 73%
$\theta_1 = -10^o$ 39%
$\theta_2 - \theta_1$ = 34% (.34)

The energy at the forecast point and the wave forecast is computed below:

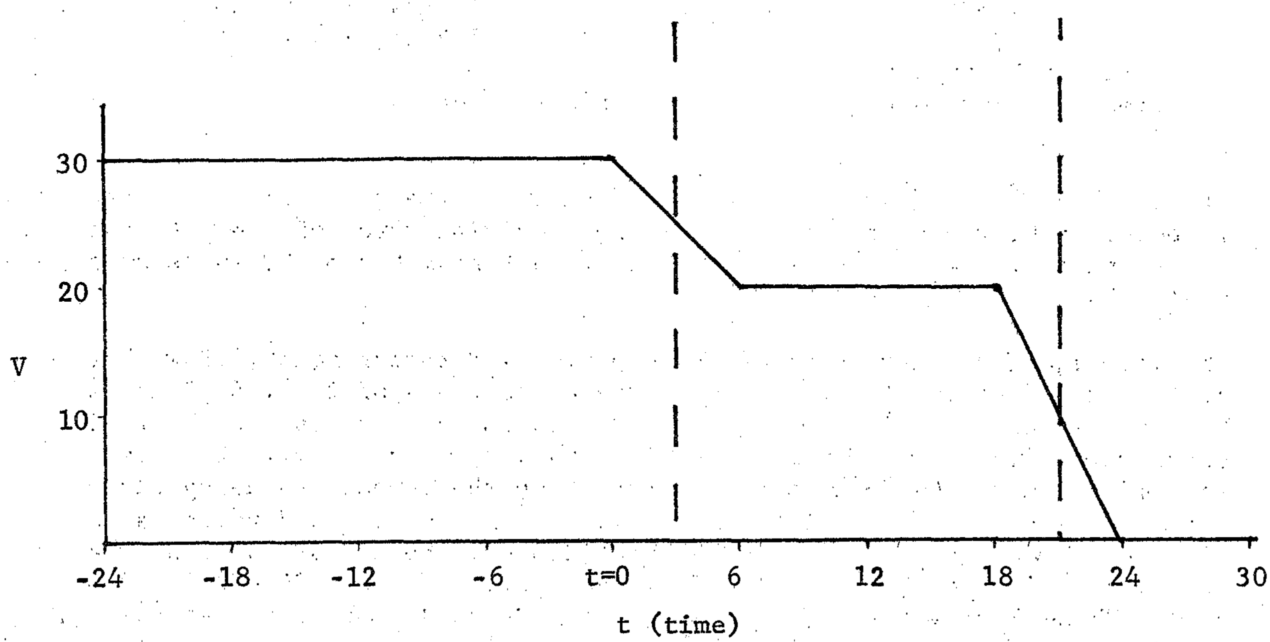
t	E forecast point					$H_{1/3}$ (= $2.83\sqrt{E}$)
40	0	•	.34	=	.0	0
60	1.5	•	.34	=	.5	2 ft
80	31	•	.34	=	10.5	9 ft
100	53.5	•	.34	=	18.2	12 ft
120	56.75	•	.34	=	19.3	12 ft
140	57	•	.34	=	19.4	12 ft
160	57	•	.34	=	19.4	12 ft

etc.

The most striking features of this example are the rapid rise in wave heights (between t=60 and t=80) and the large difference in the amount of energy in the sea generated by 30-knot and 20-knot winds. The sudden increase in wind speed from 20 knots to 30 knots is somewhat unrealistic, but it is a good approximation to the actual, gradual increase. When computing facilities are available, the single increase can be broken down into two smaller increases for a better approximation without significantly increasing the workload to the forecaster.

EXAMPLE V - A Dying Fetch.

This is an example similar to the fetch described in Example II except that the trend of the wind is reversed as diagramed below.



The fetch is oriented such that:

Ro = 1000 nm   width 400 nm
V = 30 kts   length 600 nm

$\theta_1 = 80.5^o$
$\theta_2 = 59^o$
$\theta_1 - \theta_2 = 21.5\%$

Examination of the C.C.S. graphs show that the fetch is neither duration or fetch limited. The C.C.S. graphs are examined to find the frequency of the waves traveling out of the fetch.

At t_o the fetch is fully developed and frequencies of .06 or higher are present. Travel time (t) for these frequencies are computed below the same way as was done in Example IV:

f	t	arrival time
		* **
.06	40 hrs	40-24 +22 = 38
.08	53 hrs	53-24 +20 = 49
.10	66 hrs	66-24 +16 = 58
.12	79 hrs	79-24 +12 = 67
.15	99 hrs	99-24 + 8 = 83
.18	119 hrs	119-24 + 5 = 100
.20	132 hrs	132-24 + 4 = 112

* t_o-24 is beginning of 30 knot wind.
** time it takes to develop waves of this frequency.

If this fetch had had 30 knot winds prior to the t_o - 24 time, the forecast point would probably already be receiving waves of f = .06, but for simplicity sake it is assumed that it instantly became 30 knots at t = t_o - 24.

The wind speed history diagram and the C.C.S. duration graph, show that it took 22 hours for the fetch to generate the waves of f = .06, 2 hours before t_o. At t_o + 3 hours, the wind decreases to 20 knots (in the wind speed step function). This means that waves of f = .06 will be formed for only 5 hours (t_o - 2 to t_o + 3) and then the lowest frequency waves formed will be f = .08. Since the minimum fetch length to develop waves of f = .06 with a 30-knot wind is about 280 nm, these last waves of f = .06 will be 320 nm behind the leading edge of the fetch (600 - 280 - 320 nm). These last waves will travel then 1320 nm to the forecast point and Figure (30) shows that it will take them 52 hours to get there (t = 55, as last formed at t = 3 hours).

Similarly, waves of f = .07 will no longer be formed after t_o + 3 hours. The minimum fetch length for them is 240 nm. These last waves will travel 1360 nm (360 + 1000) and Figure (30) shows that travel time to be 63 hours (t = 66 hours as last formed at t = 3 hours). Similarly, the last waves of f = .08 from the 30-knot wind will arrive at t = 77.

The same reasoning applies after the end of the 20-knot wind at t = 21 hours, when no more waves will be generated. The results are listed below:

f	min fetch	travel distance	travel time
.08	80 nm	1520	80 hrs
.10	70 nm	1530	101 hrs
.12	60 nm	1540	122 hrs
.15	45 nm	1555	154 hrs
.18	32 nm	1568	186 hrs
.20	24 nm	1576	208 hrs

Figure (17) is a plot of the arrival times of the various frequency waves.

The energy in the waves must now be considered. This gets somewhat complicated as at certain times some of the waves arriving at the forecast point were generated by the 20-knot wind (less energy) while others were generated by the 30-knot wind (more energy).

The cut-off times of the various frequency waves resulting from the end of the 30 knot wind must be determined. This is done similarly to the above times calculated for the 20-knot wind. The results are in the table below:

f	min fetch	travel distance	travel time
.08	210	1390	73
.10	160	1440	95
.12	110	1490	118
.15	70	1530	151
.18	28	1572	187
.20	19	1581	209

The graph of frequencies with respect to time (Figure (17)) now has the various frequency waves segregated by wind speed and the energy can be determined as in the table below.

	Δ f 20 kts	Δ E fetch 20 kts	Δ f 30 kts	Δ E fetch 30 kts
40	0	0	.06 - .065	2 ft^2
60	0	0	.065- .105	30 ft^2
80	~0	~0	.08 - .145	33 ft^2
100	.08 - .10	1 ft^2	.10 - .18	23 ft^2
120	.10 - .12	2 ft^2	.12 - .20	13 ft^2
160	.135- .155	1 ft^2	.155- .20	4 ft^2
200	.17 - .19	1 ft^2	.19 - .20	0.5 ft^2
240	0	0	0	0

The forecast for the point is then the total energy $(\Delta E_{30\ kt} + \Delta E20_{kt})$ times the angular spreading factor (21.5%). The forecast for various times is listed below:

t	E fcst point	$H_{1/3} = 2.83\ E$	f
40	.4 ft^2	2 ft	.06 - .065
60	6.5	7 ft	.065 - .105
80	7.1	8 ft	.08 - .145
100	5.2	6 ft	.08 - .18
120	3.2	5 ft	.10 - .20
160	1.1	3 ft	.135 - .20
200	.3	2 ft	.17 - .20
240	0	0 ft	0

This example was carried out to 240 hours for the purpose of noting the arrival of the various waves and the increase and decrease of wave height with time. Obviously, by 240 hours (10 days) from the observation time (t_o), changes will take place in the synoptic situation and new fetches (probably even closer than 1000 nm) will be creating swell waves for the forecast point.

EXAMPLE VI - Swell from Two Separate Fetches.

One of the real advantages to using spectral forecast techniques is when energy is arriving at the forecast point from more than one fetch. The energies can simply be added together and the wave heights computed from this total energy. To show this, suppose energy is coming from the fetches in examples III and IV.

Figure (18) shows a possible synoptic situation for this.

The energies in the forecast tables from examples III and IV are added together below to get the total energy received with respect to time.

The resultant wave height forecast is in the last column.

t (hours)	Δf Fetch 1	Δf Fetch 2	ΔE Fetch 1	+	ΔE Fetch 2	=	E	$H_{1/3} = 2.83\sqrt{E}$
20	0	0	0	+	0	=	0	0 ft
40	.08	0	0.2	+	0	=	0.2	1 ft
60	.08-.115	.08-.11	0.5	+	.5	=	1.0	3 ft
80	.08-.145	.06-.155	1.0	+	10.5	=	11.5	10 ft
100	.08-.18	.06-.20	1.4	+	18.2	=	19.6	13 ft
120	.08-.210	.06-.24	1.7	+	19.3	=	21.0	13 ft
140	.08-.245	.06-.25	1.7	+	19.4	=	21.1	13 ft
160	.08-.25	.06-.25	1.9	+	19.4	=	21.3	13 ft

By the 80 hour time most of the energy is coming from the second fetch, so the major portion of the waves will be coming from the southwest in the frequency band of Δf fetch 2 in the table above.

This is a simple calculation which shows that the wave heights cannot be added together but the energy can.

USE OF SPECTRAL WAVE OBSERVATIONS

Spectral wave observations give much more detailed information about the sea condition than do simple estimates of wave height and period. The input of the forecaster into Spectral Wave Forecasting is his assessment of the fetch area; its wind speed and direction, its length and width, and the length of its existence (past and future). Spectral observations can tell us if the forecast of all these parameters is correct and can single out those parameters which are in error. The next forecast can then be modified accordingly. When data buoys lie between the fetch and the forecast point, there is additional lead time as to the oncoming wave conditions. Suppose a forecast for the buoy location had been made as well as for the forecast point. If the swell arriving at the buoy is of different height and period than expected, the forecast for the forecast point can be modified accordingly. Details of this modification are discussed below after some evaluation of buoy reports.

Figure (19) is a buoy report and its plotted spectrum taken from the fetch shown in Figure (20) (see appendix for key to decoding buoy reports). The wind speed as reported from the buoy was 16 meters per second or 31 knots, which fits the gradient rather well. A fully developed sea with a wind of 31 knots will have an $f_{max} = \frac{2.476}{31} = .08$ sec^{-1} and needs a minimum fetch length of 310 nautical miles and a minimum duration of 25 hours. The significant wave height will be about 23 feet. The report states that the observed significant wave height is only 9 half meters or about 15 feet. The plot shows that the wave energy at the buoy is much less than that for a fully developed 31 knot wind and the f_{max} much higher at .12 sec^{-1}. This indicates that the fetch is either duration or fetch limited.

Figure (27) or (28) show that a 31-knot wind, which has energy accumulated up to waves of f = .10 should have a significant wave height of 15 feet. If it is fetch limited, the fetch length is about 170 nm. Figure (20) shows that it is indeed fetch limited as the wind upstream becomes more easterly due to the sharp curvature of the flow around the low center. The upstream winds are from the NNE for about 170 nm as expected. If this fetch area was not fetch limited, then the report would have told us that it was a developing system as winds of 31 knots had only been in existance for 14 hours.

Figure (21) is another example of a buoy report and a plot of its spectrum. Without looking at a surface map, information concerning its location with respect to synoptic features can still be obtained. The wind is only 5 meters/second or 10 knots, yet there is quite a bit of energy centered near f = .105. The energy is confined to a narrow band around this frequency and is much higher than could be produced by a 10 knot wind. The waves must be swell waves arriving at this point, and with time, the energy will be shifting toward higher frequencies (unless the fetch is in a developing situation).

This information can be useful in forecasting waves as well. Suppose it is known that swell waves are arriving at a buoy and will later be arriving at the forecast point. If they arrive at the buoy sooner than expected, the wind must have started sooner than was thought. If the first low frequency waves arriving at the buoy are of lower (or higher) frequency than expected, the wind in the fetch was stronger (or weaker) than assessed when the forecast was made. The wave forecast for the forecast point will have to be modified to bring in the lower (or higher) frequency waves and the energy and wave height will have to be increased (or decreased) for all of the frequency waves. The same information can be attained by observing the first waves reaching the forecast point, but some of the lead time, that was available when a buoy was between the fetch and the forecast point, is lost. The following is an example applying this information.

EXAMPLE VII - Use of Spectral Wave Observations.

Figure (22) shows a surface low-pressure system and its orientation to a forecast point that has spectral observations. The fetch has been stationary for the last 24 hours. The forecaster has drawn in the fetch orientation and has determined that the mean wind is 25 knots. As done in previous forecast examples in this text, the arrival time of the various frequency waves have been calculated by determining when the waves formed and how long it will take them to travel the distance, R . This has been done in the table below and the arrival times plotted in Figure (23), as a solid line.

f	Formation Time	Travel Time	Arrival Time
.25	3.0	107	107 - 24 + 3 = 86
.22	3.5	94	93 - 24 + 4 = 74
.20	4.2	86	86 - 24 + 4 = 48
.16	7.2	69	69 - 24 + 7 = 52
.12	10.6	51	51 - 24 + 11 = 38
.10	13	43	43 - 24 + 13 = 32
.08	15	34	34 - 24 + 15 = 25
.07	16	30	30 - 24 + 16 = 22

Forecast Information

Time	Δf	E_{fetch}	$.46 \times E_{fetch}$	$H_{1/3} = 2.83\sqrt{E}$
24	.07 - .075	1 ft^2	.5	2 ft
30	.07 - .095	7	3.2	5 ft
36	.07 - .115	12	5.5	7 ft
42	.07 - .13	16	7.4	8 ft

By using this table and the angular spreading factor of .46, the forecaster made the wave forecast for the next 48 hours as shown in the last column in the above table. The wave energy integrated with respect to time is shown as curve A in Figure (24).

Now the next day, the forecaster again assesses the situation and determines that his wave forecast for today still looks good but decides to check the spectral wave observation first. The observation time is

t_o + 27 hours. He notes that the significant wave height has recently increased to 5 feet and the energy is increasing between f = .06 and f = .07 with energy density values of about 17 m^2 sec and 30 m^2 sec, respectively. These energy levels and observed waves are higher and at lower frequencies than expected. The wind in the fetch must have been stronger than analyzed. To check it, he refigures the above table using V = 30 knots instead of V = 25 knots. This is shown in the table below, and the arrival times plotted in Figure (23) as a dashed line.

f	Formation Time	Travel Time	Arrival Time
.25	3	107	107 - 24 + 3 = 86
.22	3.5	94	94 - 24 + 4 = 74
.20	4.0	86	86 - 24 + 4 = 48
.16	7.0	69	69 - 24 + 7 = 52
.12	13	51	51 - 24 + 13 = 40
.10	16	43	43 - 24 + 16 = 35
.08	20	34	34 - 24 + 20 = 30
.07	21	30	30 - 24 + 21 = 27
.06	23	26	26 - 24 + 23 = 25

Forecast Information

Time	Δf	E_{fetch}	.46 x E_{fetch}	$H_{1/3} = 2.83 \sqrt{E}$
24	0	0 ft^2	0	0 ft
30	.06 - .08	16	7.4	8
36	.06 - .105	34	15.6	11
42	.06 - .125	43	19.8	13

By using the same angular spreading factor of .46, the wave forecast is determined (last column in above table). The new calculated spectrum is superimposed on Figure (24) as curve B, and the energy levels conform to those already observed in the first low frequency waves arriving at the forecast point.

If the frequencies which arrived were as expected, except that the energy levels were higher (lower), then the forecaster would have known that the wind speed was correctly determined but that the fetch was wider (narrower) than analyzed, and the angular spreading factor would have to be modified accordingly.

Up until now, the examples have been hypothetical to demonstrate a particular point. To promote confidence using spectral observations, this last example deals with an actual situation which occurred at 18Z on February 5, 1978.

Figure (25) shows the surface analysis at this time. The spectral wave observation for the buoy located at 130°W and 42.5°N is used for demonstration. The report has the wind speed at 10 m/sec or 20 knots. The buoy spectrum is plotted along with the fully arisen sea spectrum for a 20-knot wind in Figure (26). Although both plots are similar at the high frequency of the spectrum (slightly lower energy at the buoy at the high-frequency end), the buoy has much more energy at the lower frequency end. This means that it must be receiving swell energy from another fetch. Examination of Figure (25) shows that the buoy is near the lead edge of a long band of west-southwesterly flow. The winds near the buoy are 20 knots but are stronger upstream as the gradient gets tighter (especially behind the next cold front). Hence, the report is of a sea with a 20-knot wind not fully developed superimposed with swell waves from the long upstream fetch of 25 or 30 knots.

With the fetch situation, a forecast could be made for the Oregon Coast and for the buoy for the next 48 hours. Then the expected trends can be compared with the observed trend at the buoy. Any discrepancies can be adjusted, and this information used to update later forecasts for the Oregon Coast.

(Note: Obviously, this buoy location would not be suitable for the Oregon Coast if the waves were coming from the northwest rather than the southwest.)

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2. H. O. Pub. No. 603, Practical Methods for Observing and Forecasting Ocean Waves by means of Wave Spectra and Statistics by Willard J. Pierson, Jr., Gerhard Neumann, and Richard W. James. U. S. Navy Hydrographic Office, 1955.

3. Wind Waves their generation and propagation on the ocean surface, Blair Kinsman, 1965, Pentice-Hall, Inc., Englewood Cliffs, New Jersey.

4. Notes and handouts from "A Short Course in Marine Forecasting" at U. S. Naval Postgraduate School, Monterey, California (funded by the National Weather Service).

DEVELOPING SEA

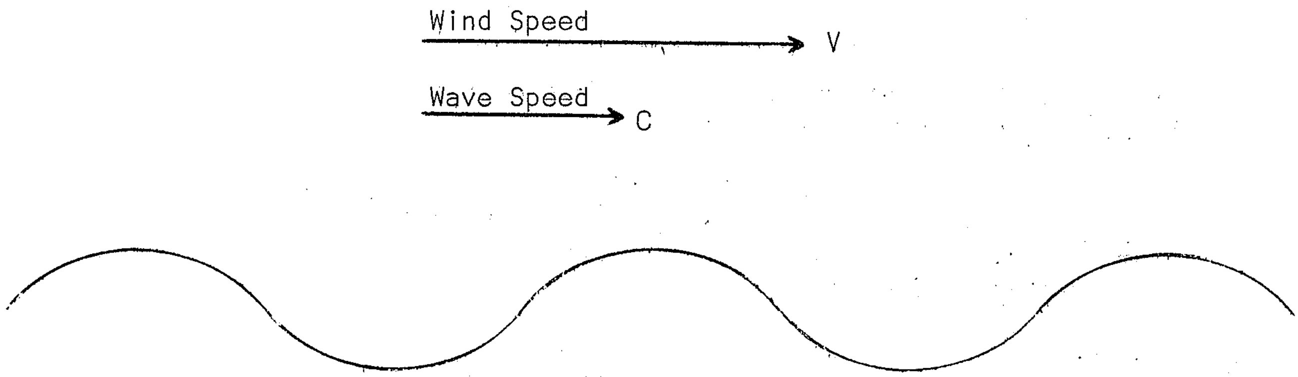


Figure Ia. Longer period waves will develop as the present waves are moving slower than the wind speed (C<V). As the wind continues, energy will be added to longer period waves until the waves move as fast as the wind speed.

FULLY DEVELOPED SEA

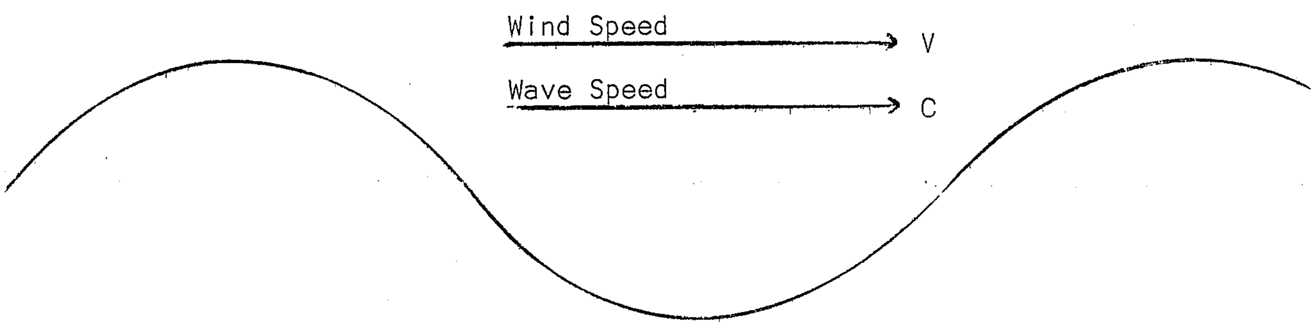
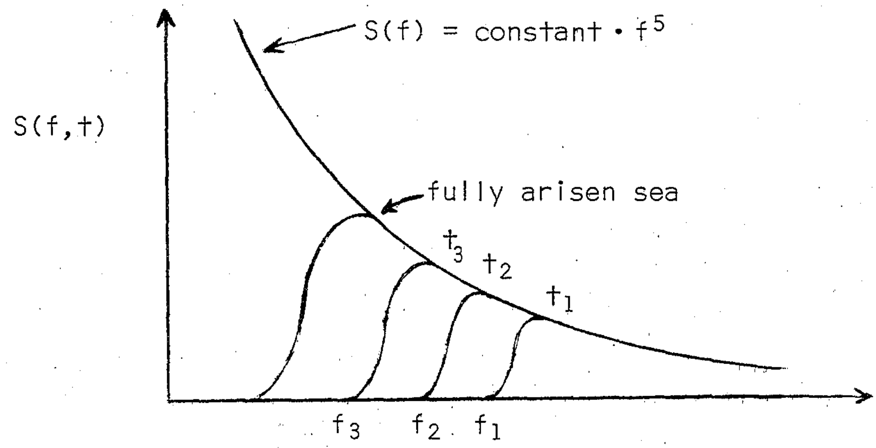


Figure Ib. Longer period waves will not grow as the waves are moving as fast as the wind. Longer period waves would move faster than the wind, which is not possible as they are being created by the eddies in that wind. Since no more energy can be added to the sea, the sea is considered saturated at this wind speed.



$$E(f) = S(f)\ f$$

Figure 2. Growth of the spectrum at fixed wind speed, V, as a function of time (or distance). At a particular time the energy in the sea is the area under the curve and to the right of the cut-off frequency, f.

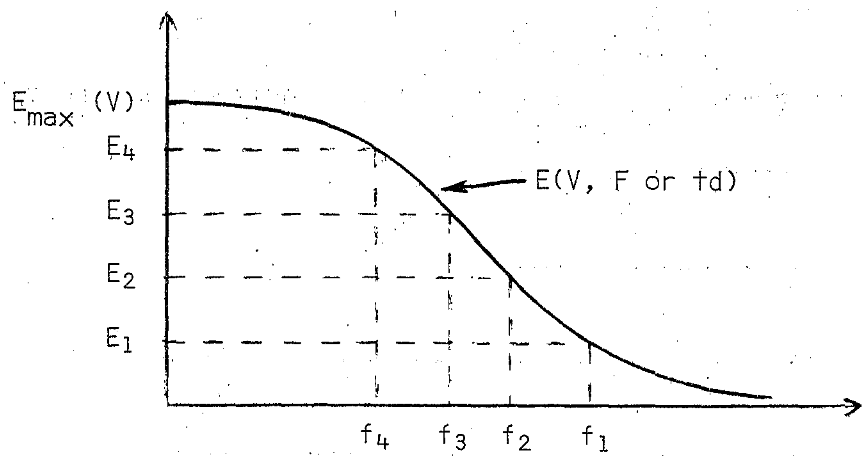


Figure 3. The energy in the sea can be read off the ordinate using the cut-off frequency on the abscissa. The energy between two frequencies, such as f_3 and f_2, is simply $E_3 - E_2$. The energy is also a function of the wind speed, V, and the fetch length, F, or wind duration, td.

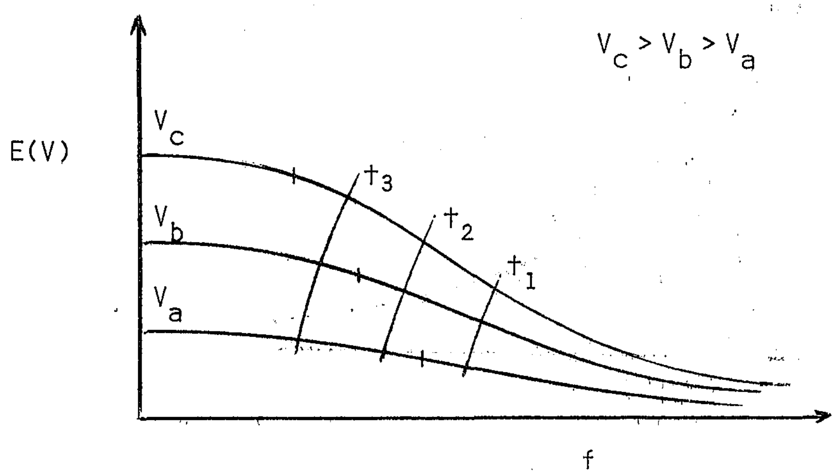


Figure 4. Co-cumulative spectrum, as in Figure 3, except for three different wind speeds.

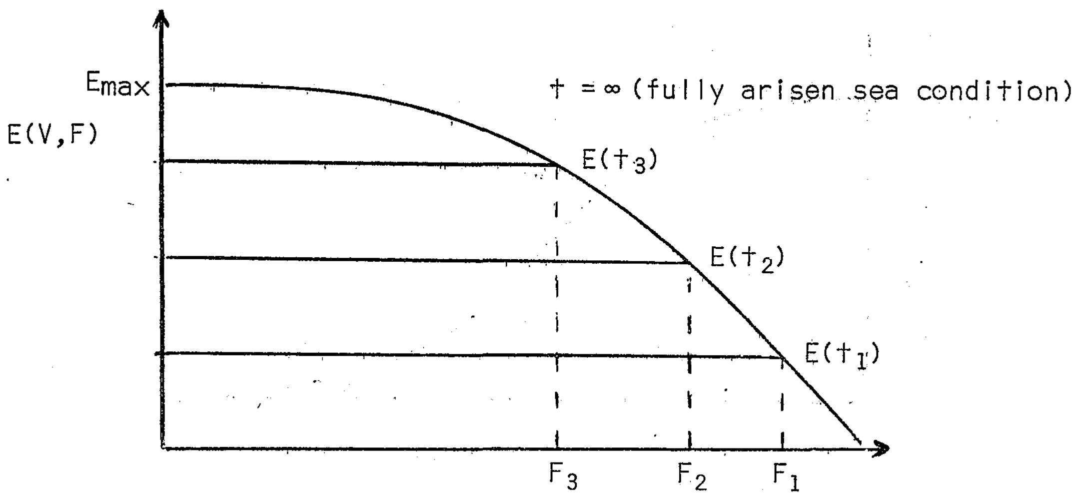


Figure 5. The co-cumulative spectrum when fetch limitations are imposed. When the fetch length is F_1, energy up to $E(t_1)$ is all that can be attained. If the fetch length is increased to F_2, energy will increase to $E(t_2)$.

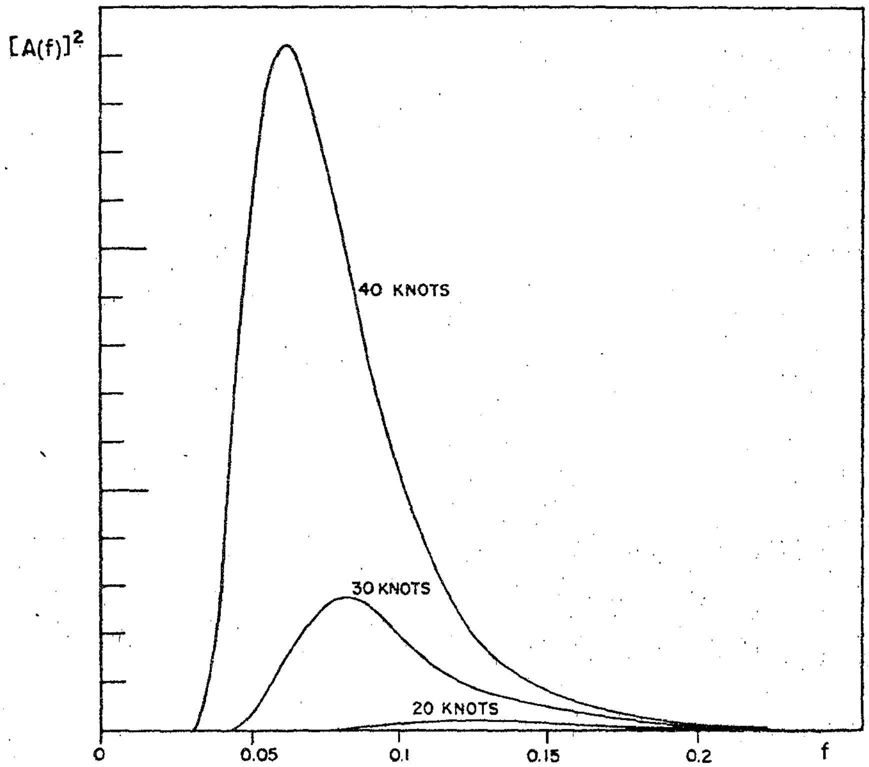


Figure 6. The comparative spectrum of 20-, 30-, and 40-knot wind speeds. Note the reduction of the maximum frequency with increased wind speed, V.

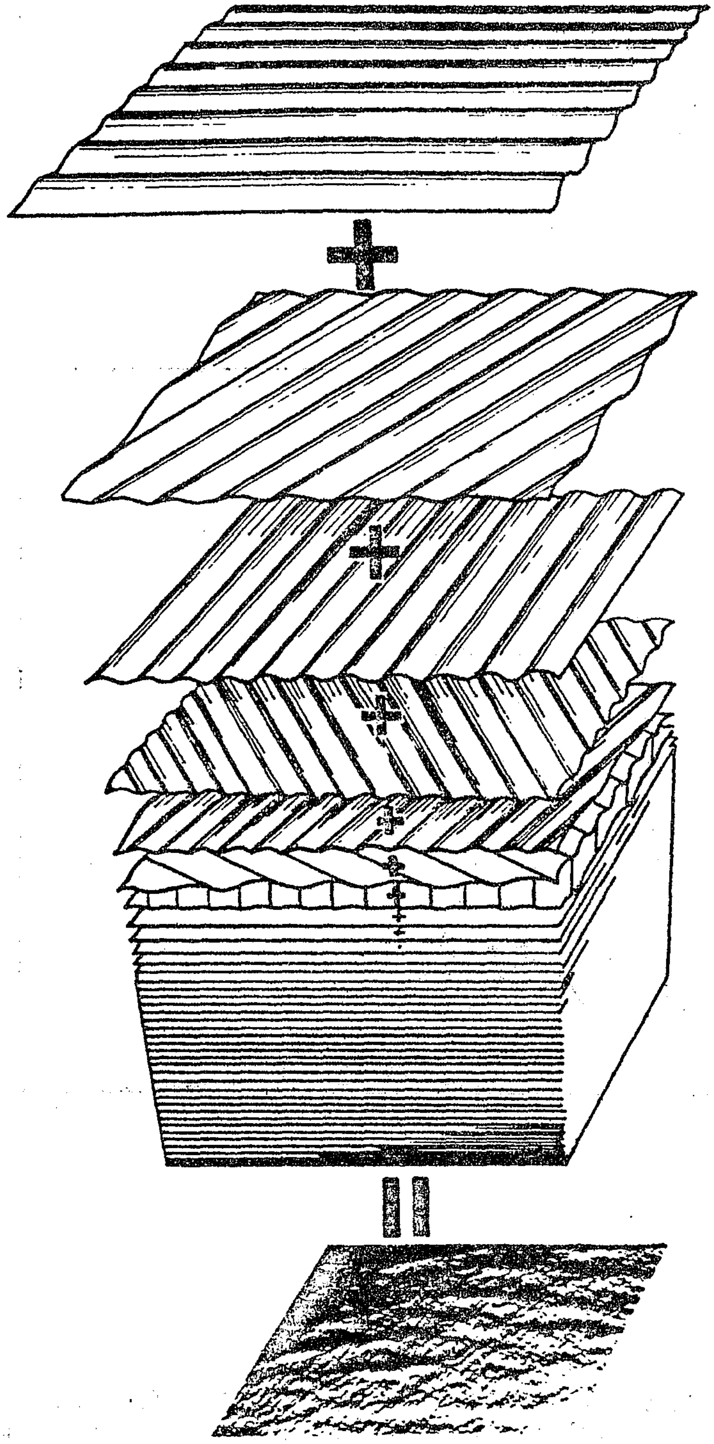


Figure 7. The sea can be approximated by many simple sine waves of various frequencies and amplitudes in superposition.

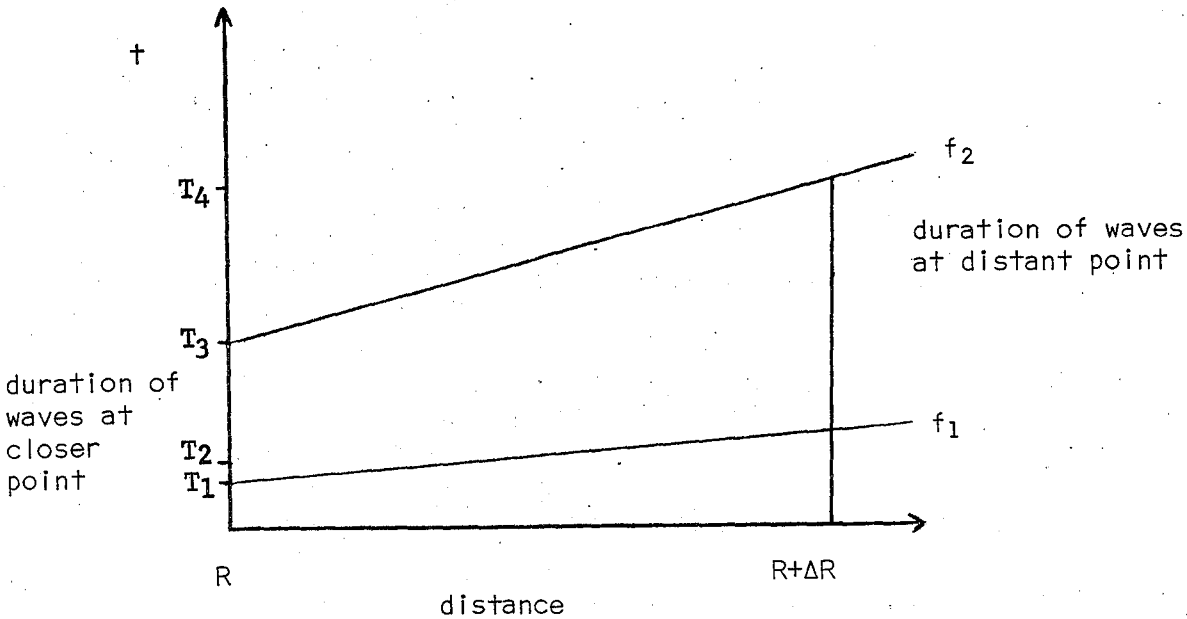


Figure 8. The duration of the swell waves increases as the distance from the storm increases. At point R the duration is only $t_3 - t_1$, while at R and ΔR the duration is $t_4 - t_2$.

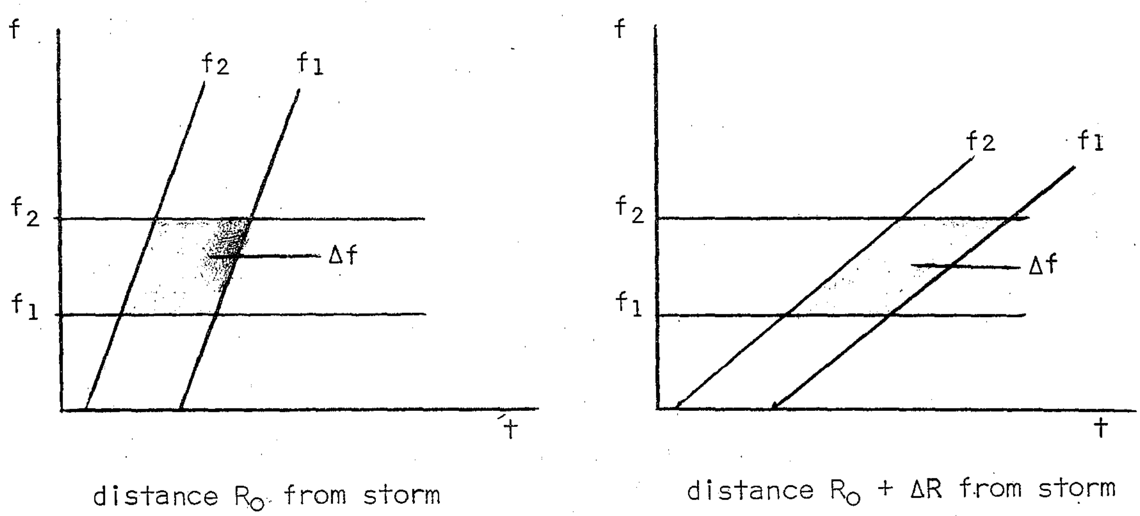


Figure 9. The bandwidth, Δf, at the forecast point decreases as the storm distance increases. At any time (any vertical line on the graphs), t, the waves are in a narrower frequency band at the more distant point, $R_o + \Delta R$.

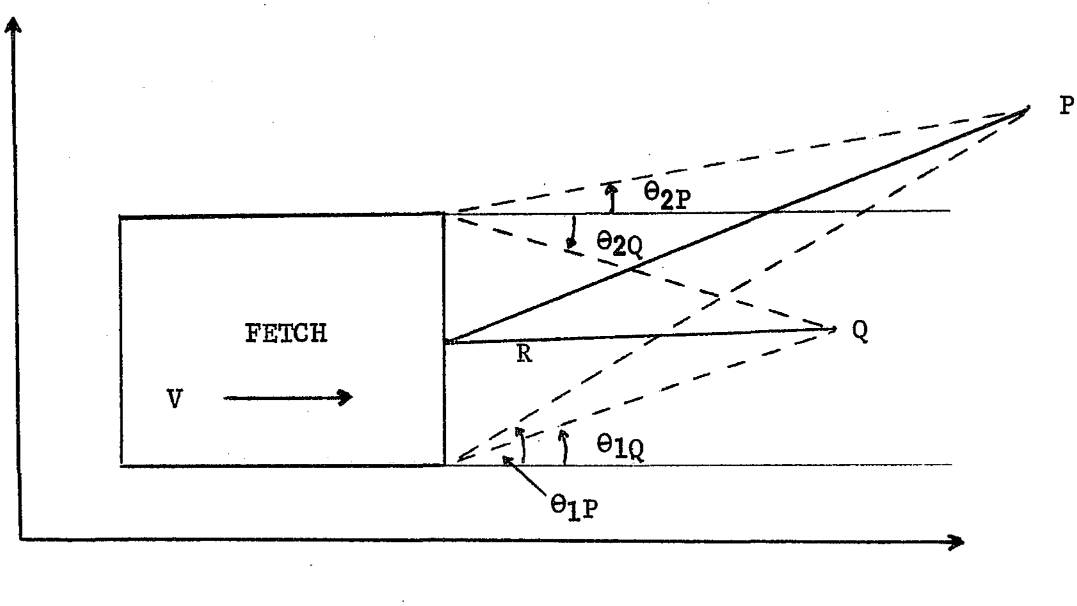


Figure 10. Wave energy is sorted out (dispersed) more for the distant point, P, than for the closer point, Q. Point Q received much more energy than does P.

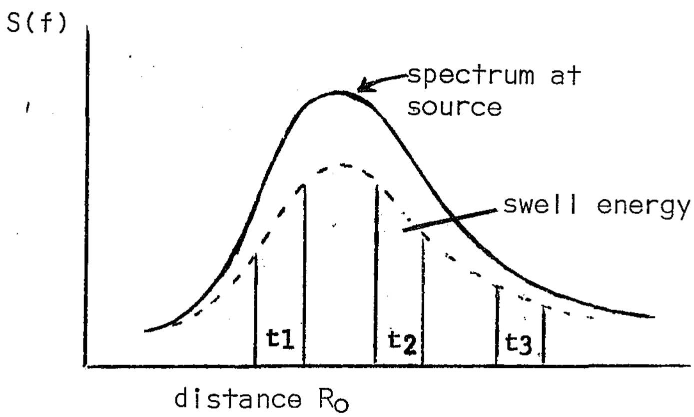


Figure 11a.

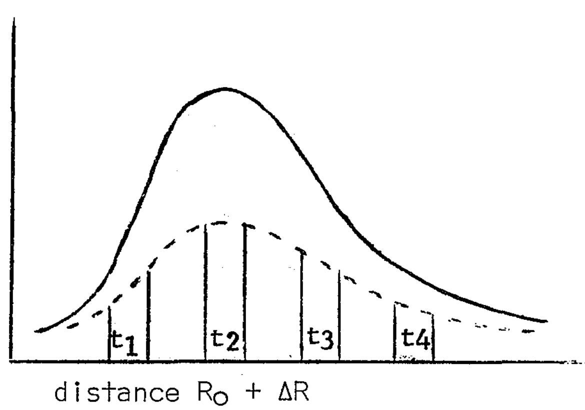


Figure 11b.

Figure 11. Swell from a fixed source arriving at 2 different first points.

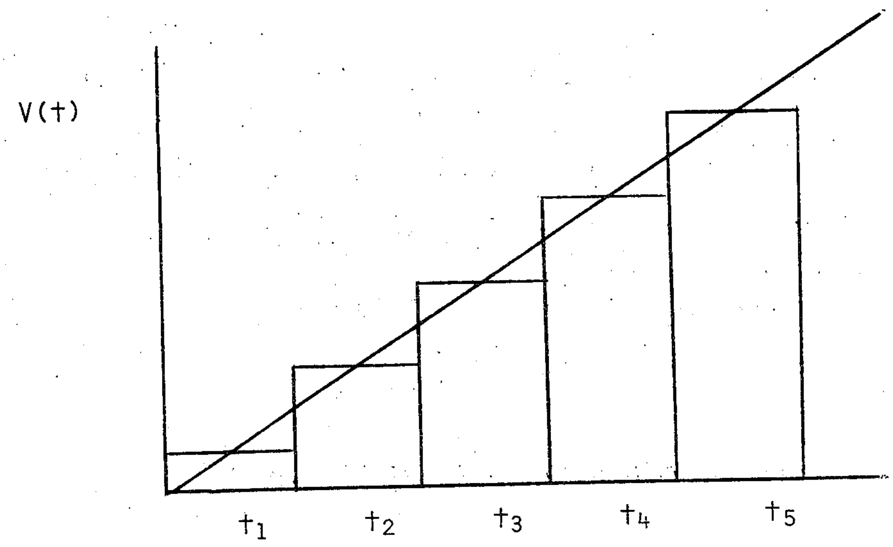


Figure 12. A step function of time increments is used to approximate the gradual increase in wind speed.

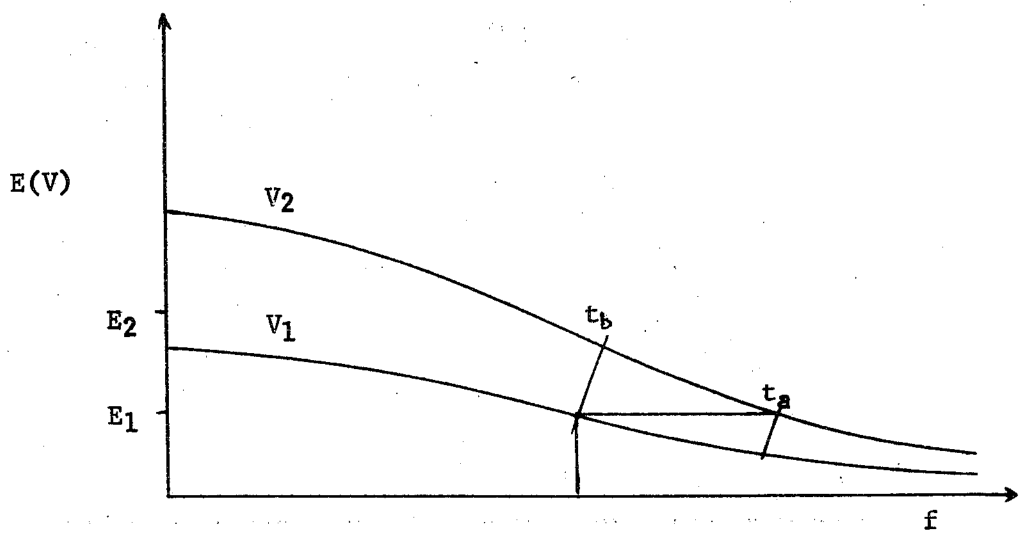


Figure 13. The accumulated energy in the sea by the wind speed V_1 up to time t_1 is used as the starting energy for the new wind speed of V_2 (point A projected on the V_2 graph which is point B). The duration of the V_2 wind is then added to this until the final energy is reached at the cut-off time of t_2.

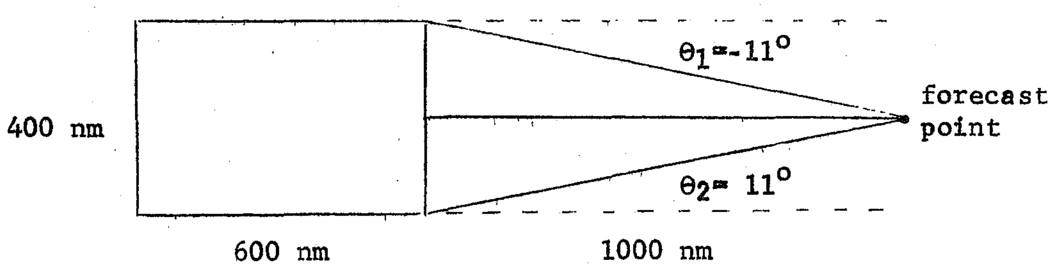


Figure 14. Fetch orientation to the forecast point for Example III.

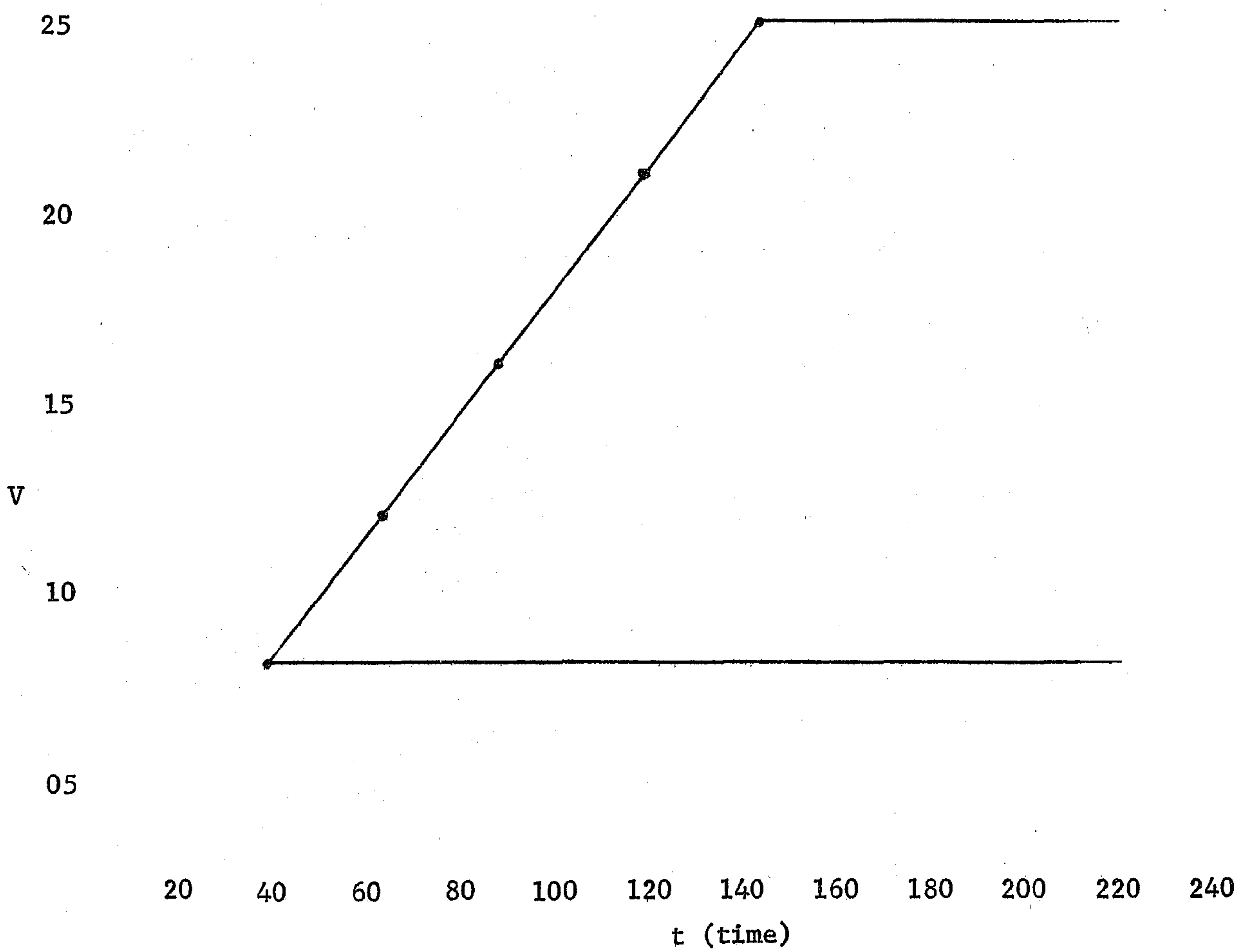


Figure 15. Plot of the wave frequencies (1/sec) present with respect to time (hours) for Example III.

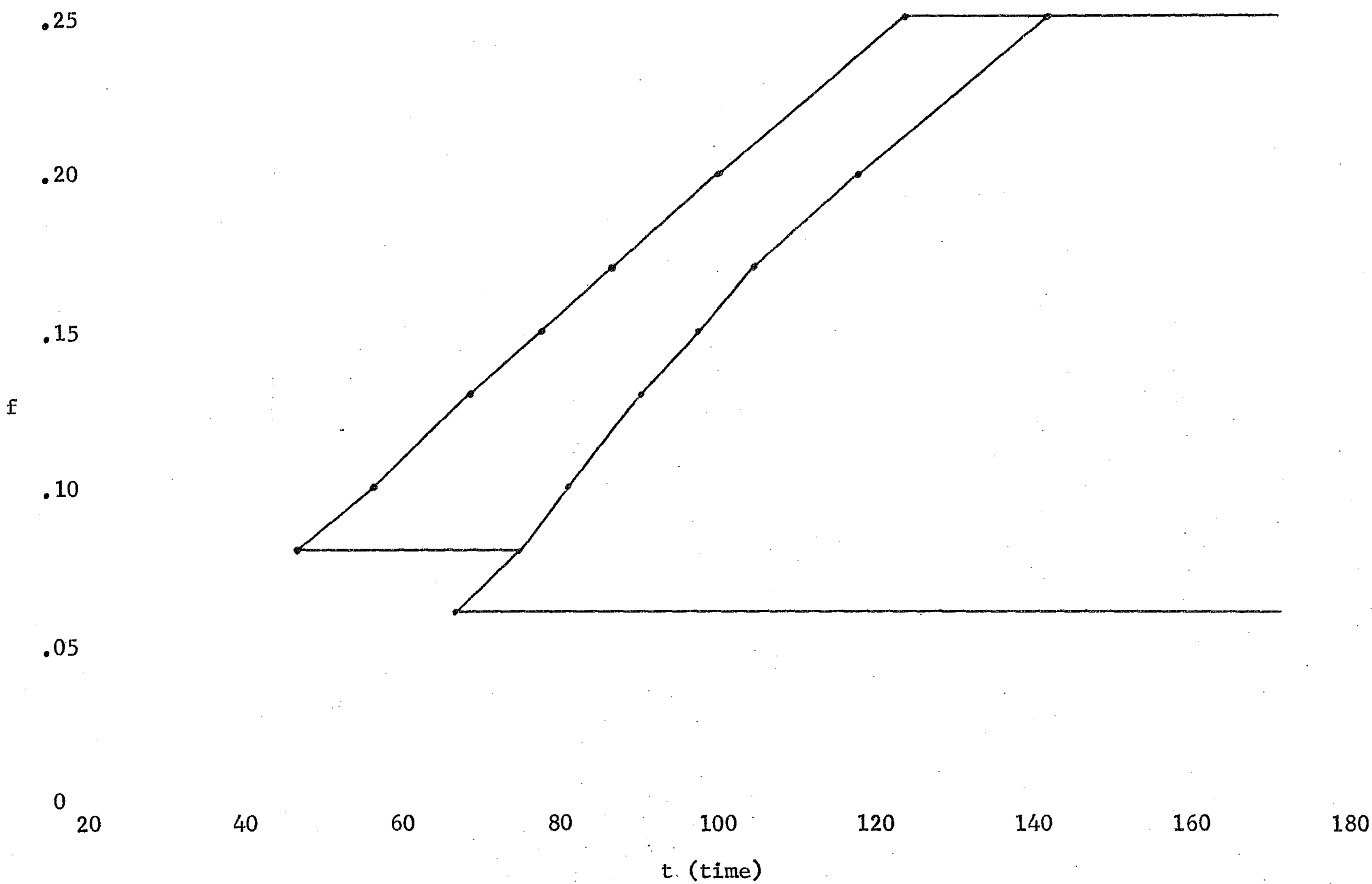


Figure 16. Plot of the wave frequencies (1/sec) present with respect to time (hours) for Example IV.

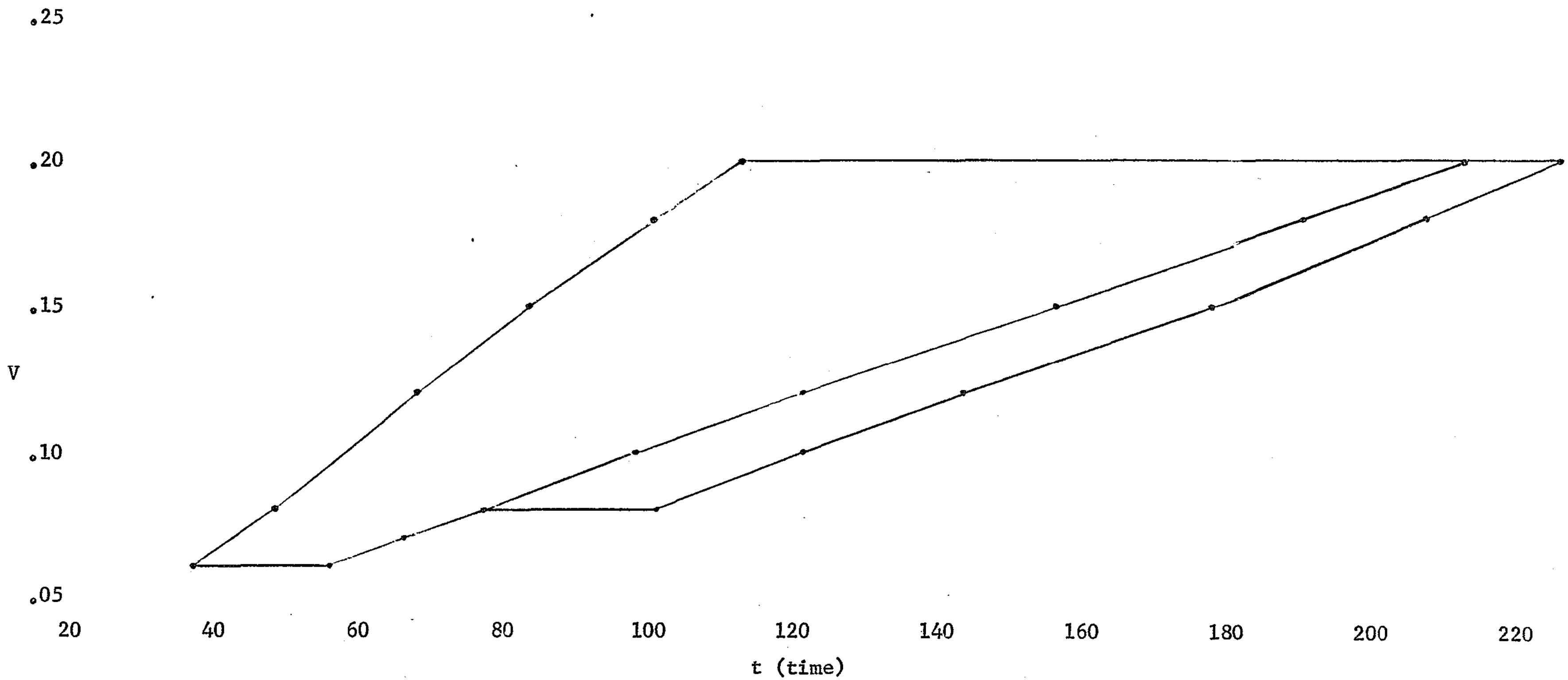


Figure 17. Plot of the wave frequencies (1/sec) present with respect to time (hours) for Example V.

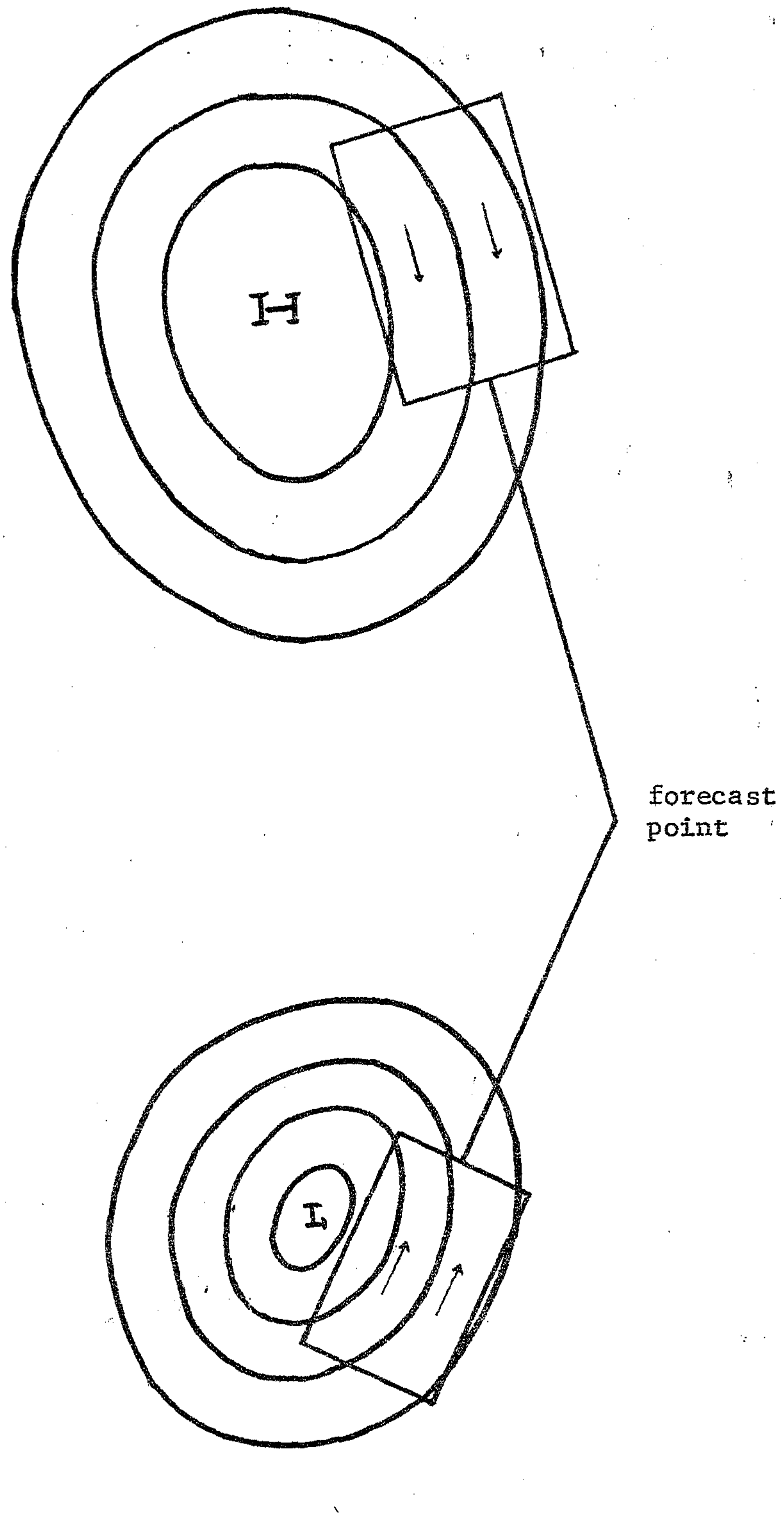


Figure 18. The fetch orientation for Example VI. The forecast point is getting swell energy from two different directions; from two different weather systems.

```
ZCZC
SLC
SXVD10 PANC 141800
EB72  //571 71517 14181 /0216 30609
99100 88030 10000 27380 30000 42010 52120 67950 72491 81092
92002 02052 11772 21372 38391 44381 52891 62651 72201 81591
91551 01911 11691 21251 31081 48940 56580 66330 77650 87320
96330 05770 15570 25630 36250
WAVE SPECTRAL DATA
```

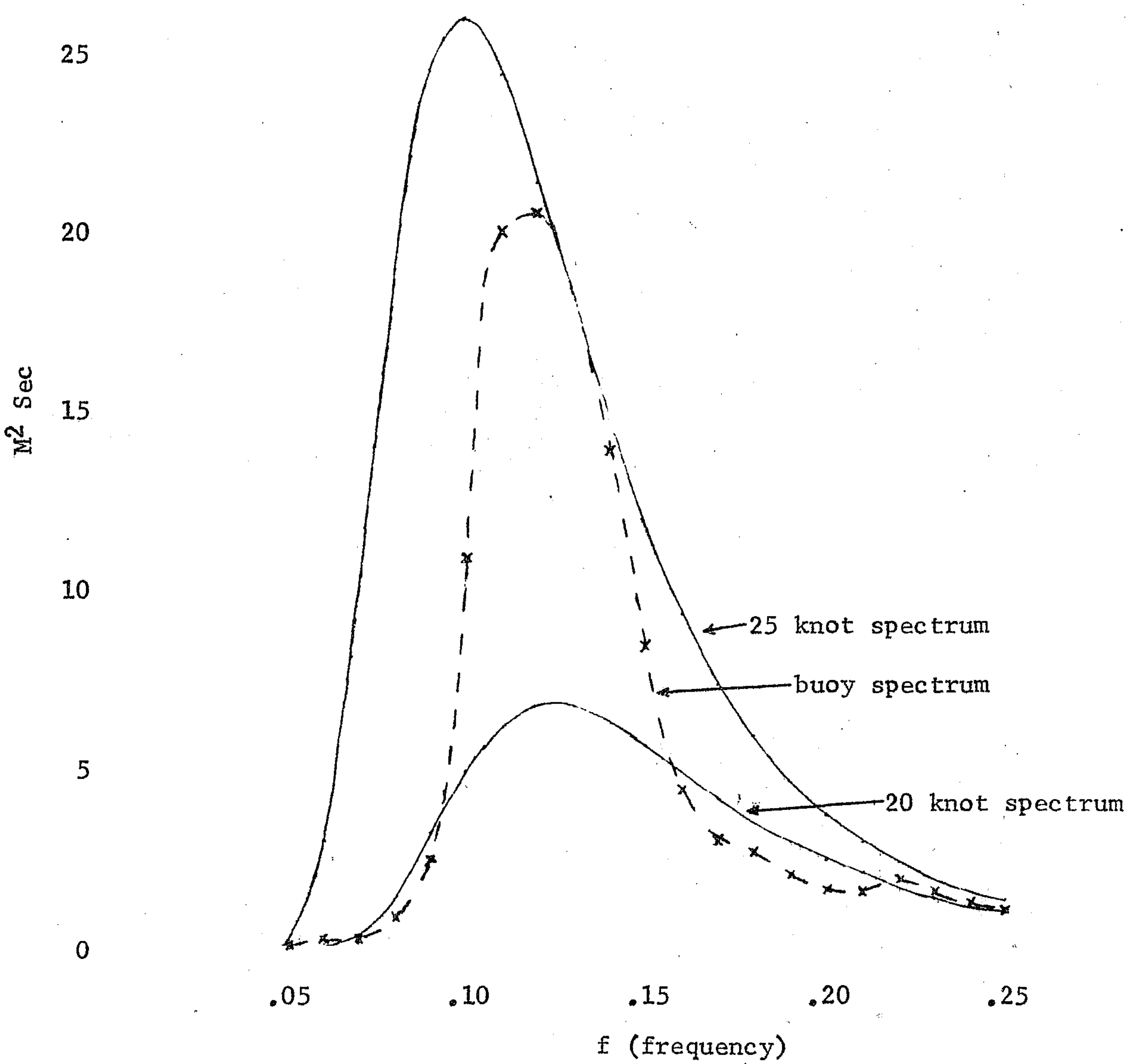


Figure 19. Buoy report and plotted spectrum. The decoded report from the buoy is plotted along with the fully developed spectrum generated by a 20-knot and 25-knot wind.

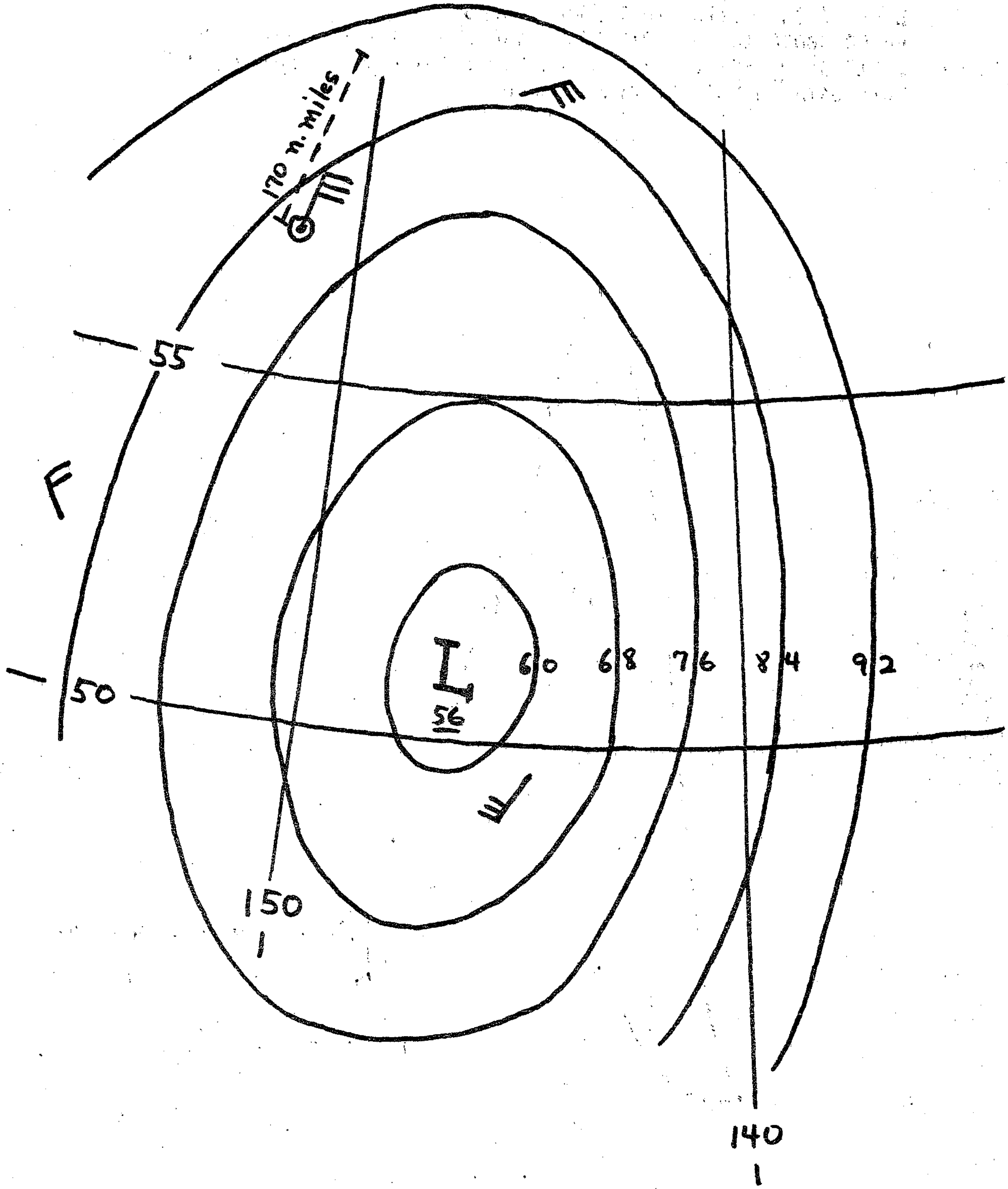


Figure 20. Surface map for buoy report of Figure 19. The buoy location is circled in the northwest quadrant of the low-pressure area.

```
ZCZC
SXVD10 PANC 110327
EB42  //387 71276 11031 /3205 30603
99100 88050 100/0 200/0 378/0 43641 58131 61632 99/// 88123
11012 99400 88160 11781 21241 31051 487/0 99/// 88330 132/0
THIS DATA UNRELIABLE BELOW .08 HZ
```

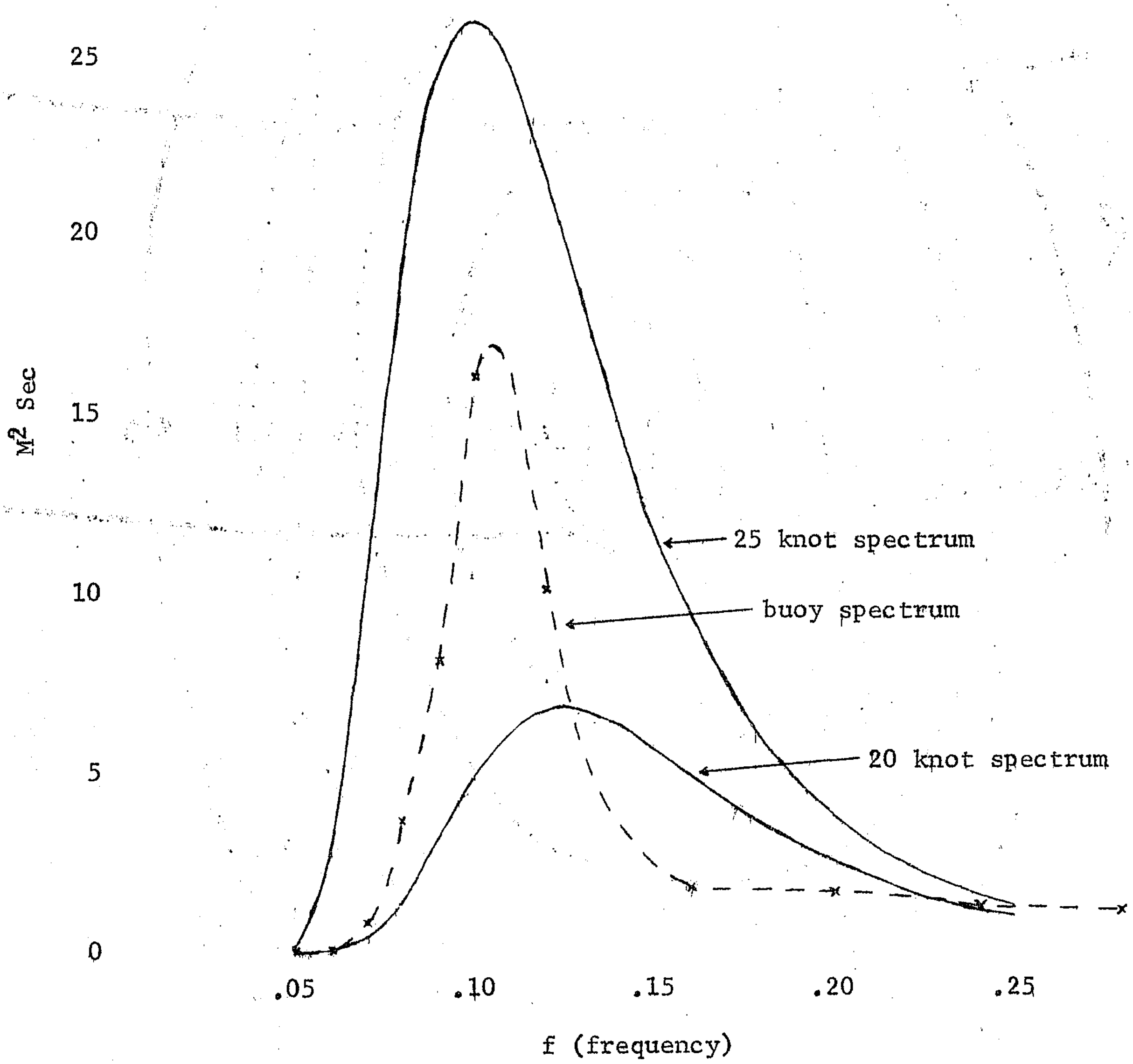
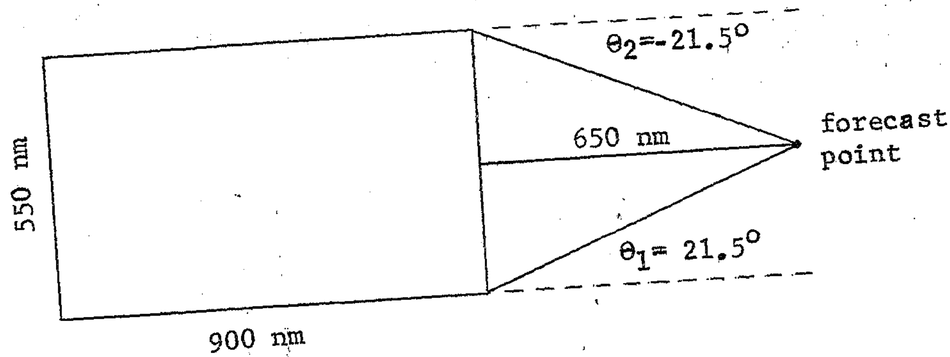


Figure 21. Buoy report and plotted spectrum. The decoded report from the buoy is plotted along with the fully developed spectrum generated by a 20-knot and 25-knot wind.



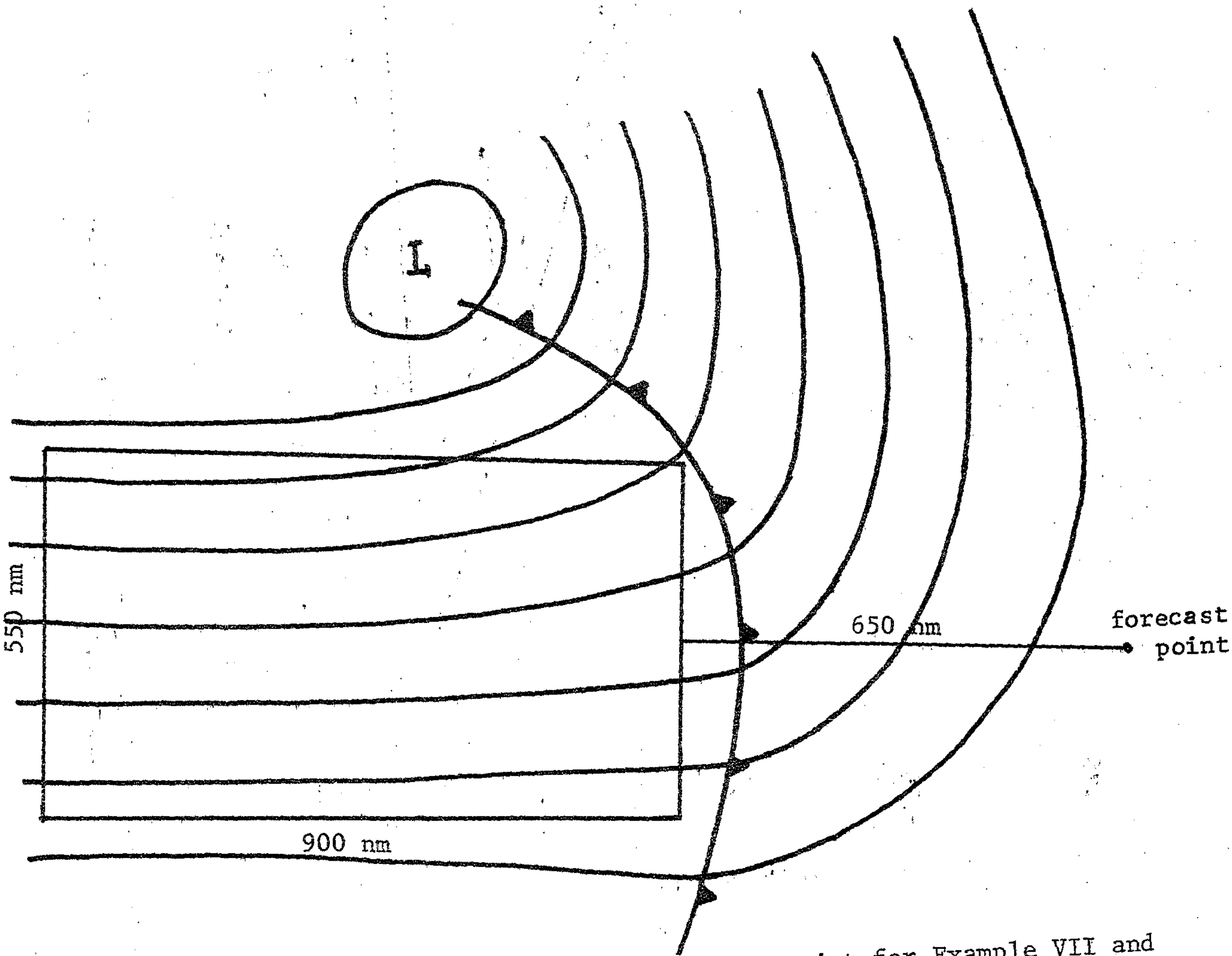


Figure 22. Orientation of the fetch to the forecast point for Example VII and surface analysis for this case.

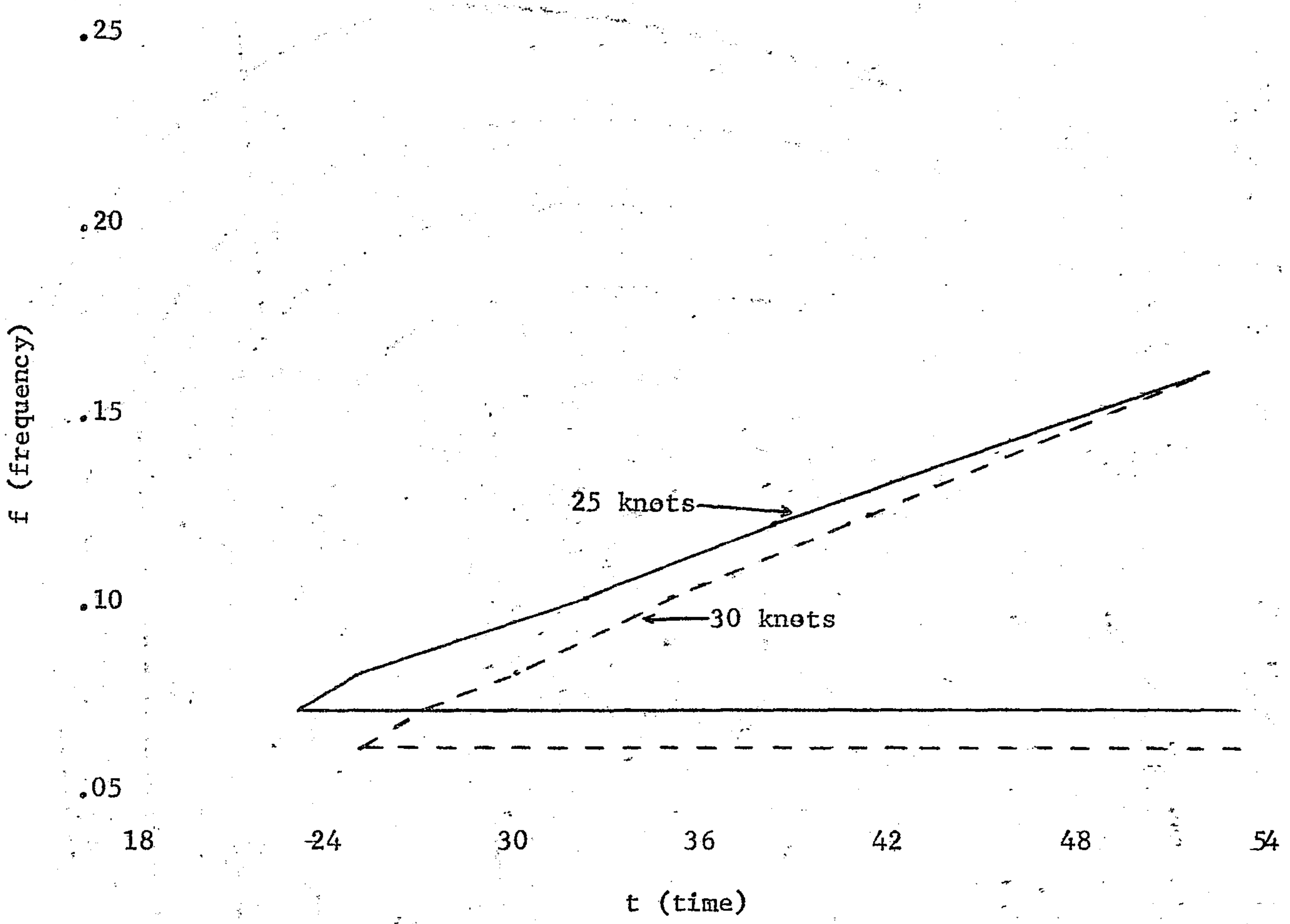


Figure 23. Plot of the wave frequencies (1/sec) present with respect to time (hours) for Example VII.

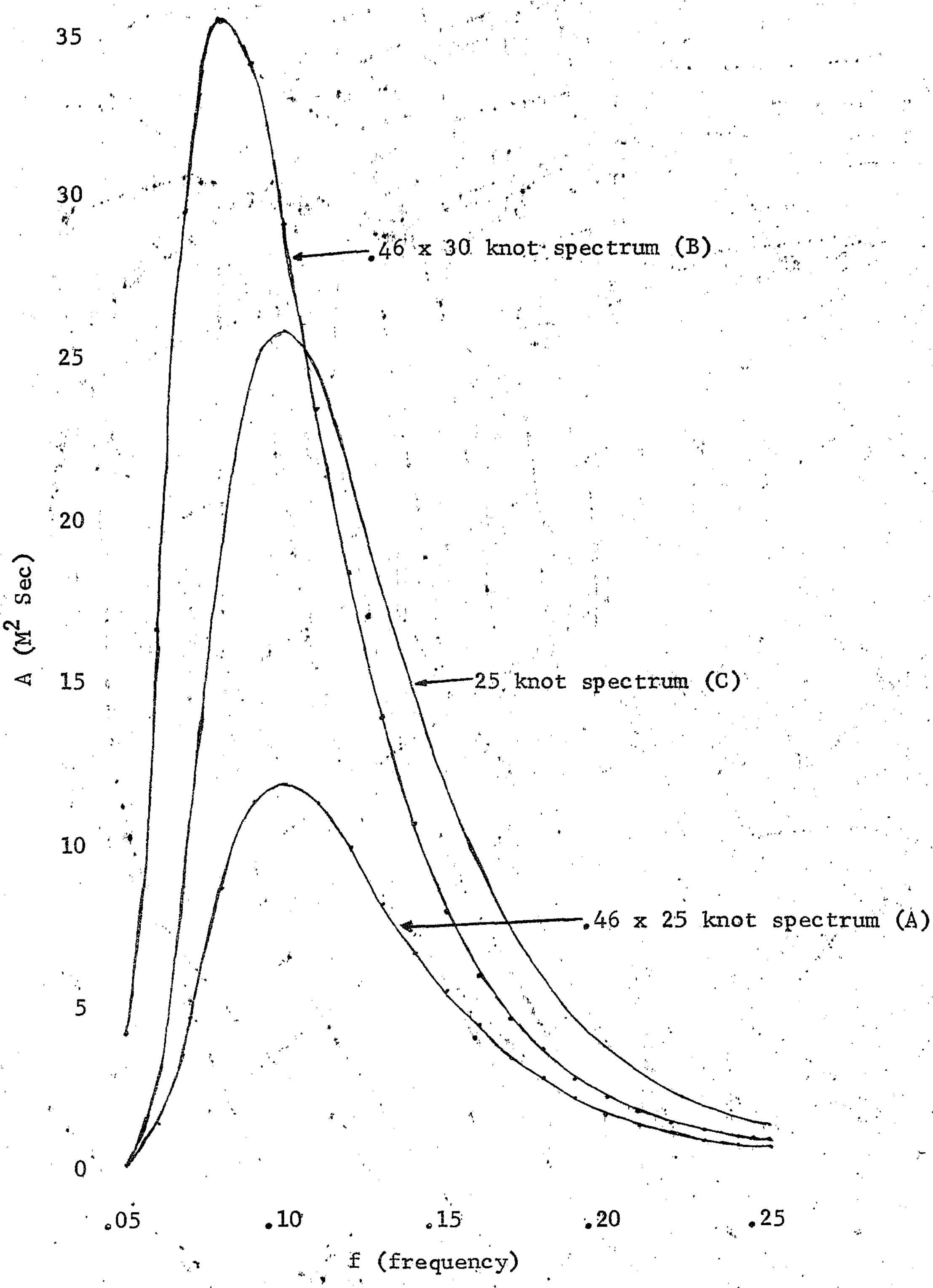


Figure 24. Forecast wave spectrum for Example VII.

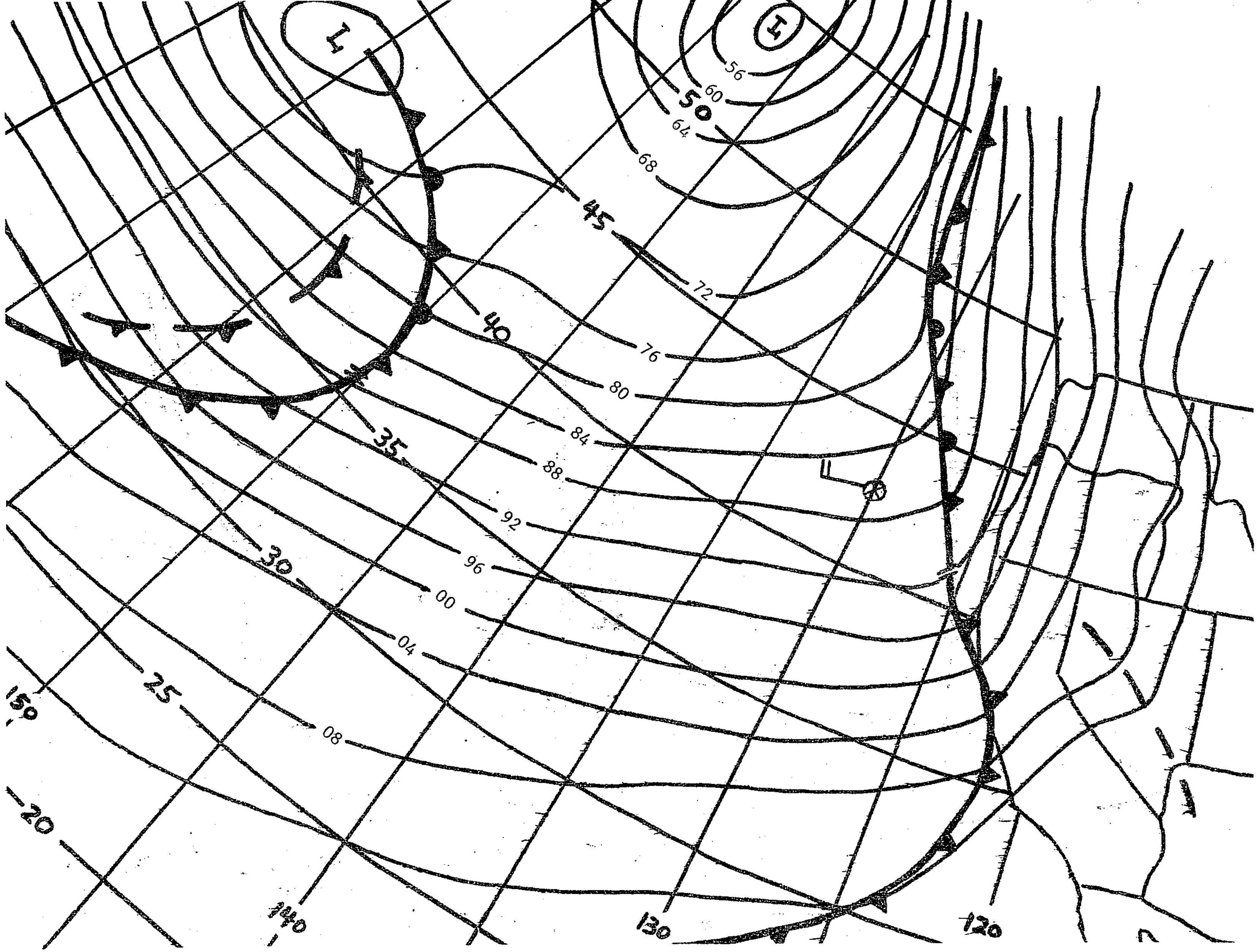


Figure 25. Surface map for 18Z February 5, 1978. There is a long fetch of west-southwesterly flow that gets stronger with increasing longitude (westward).

SXVD10 PANC 051800
EB06 //425 71300 05181 /2510 30809
99100 88050 187/0 215/1 332/1 474/1 522/2 620/2 99400 88120
111/2 216/1 323/1 425/0 527/0 99/// 88330 123/0
WAVE SPECTRAL DATA®

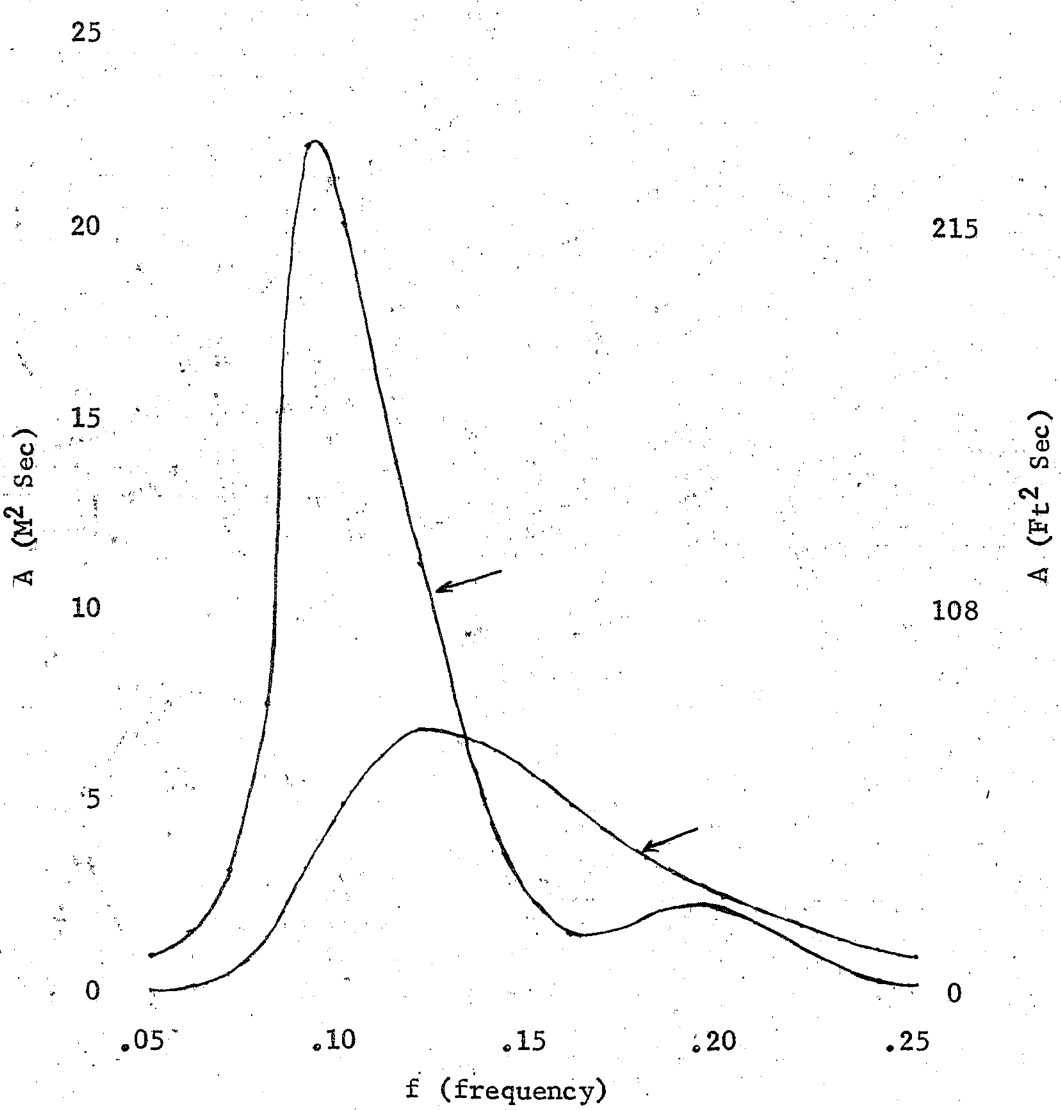


Figure 26. Buoy report and plotted spectrum. The decoded report is plotted along with a fully developed spectrum generated by a 20-knot wind.

DURATION GRAPH

DISTORTED CO-CUMULATIVE SPECTRA FOR WIND SPEEDS 10 TO 44 KNOTS AS A FUNCTION OF DURATION

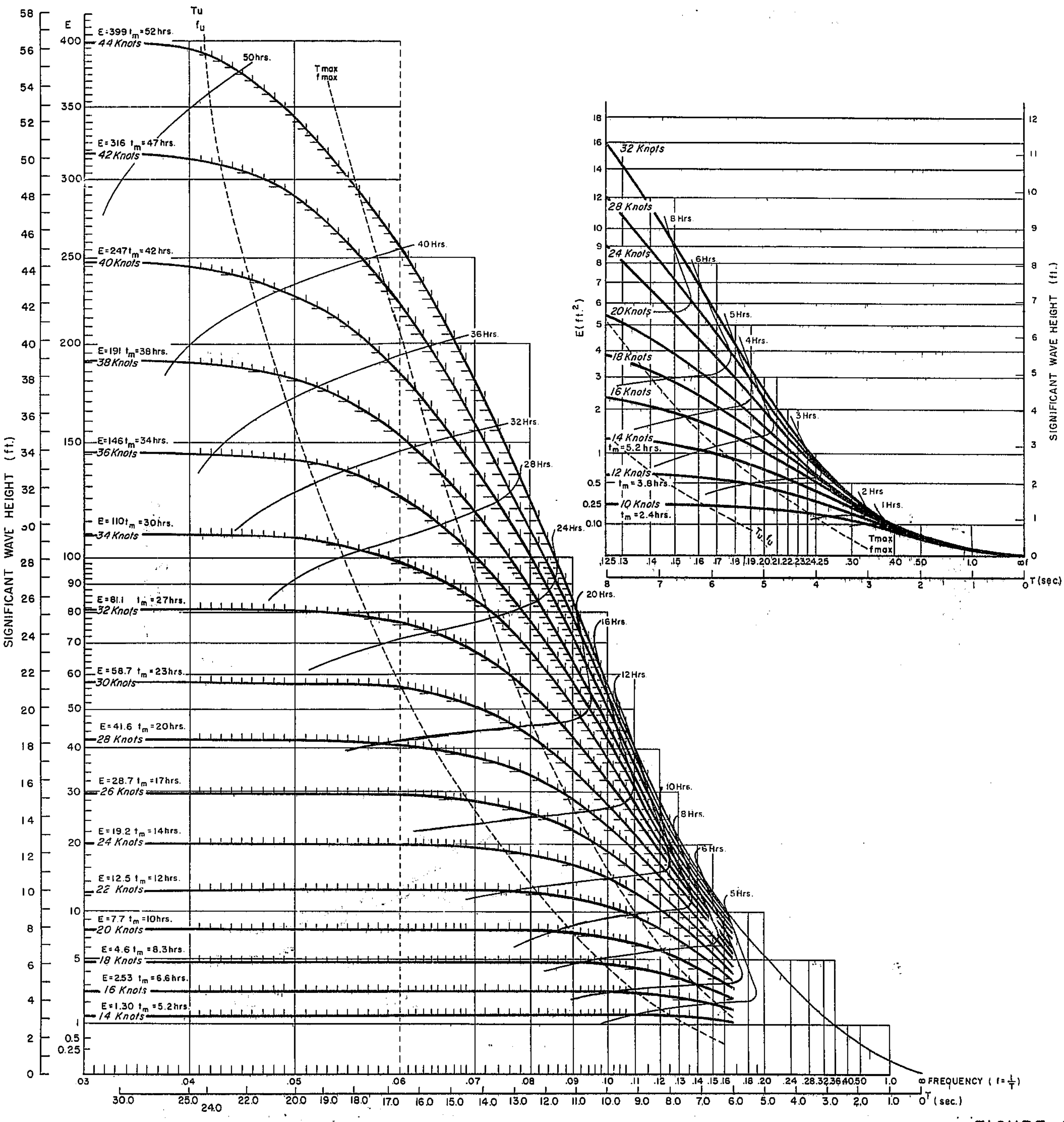


FIGURE 27

FETCH GRAPH

DISTORTED CO-CUMULATIVE SPECTRA FOR WIND SPEEDS 10 TO 44 KNOTS AS A FUNCTION OF FETCH

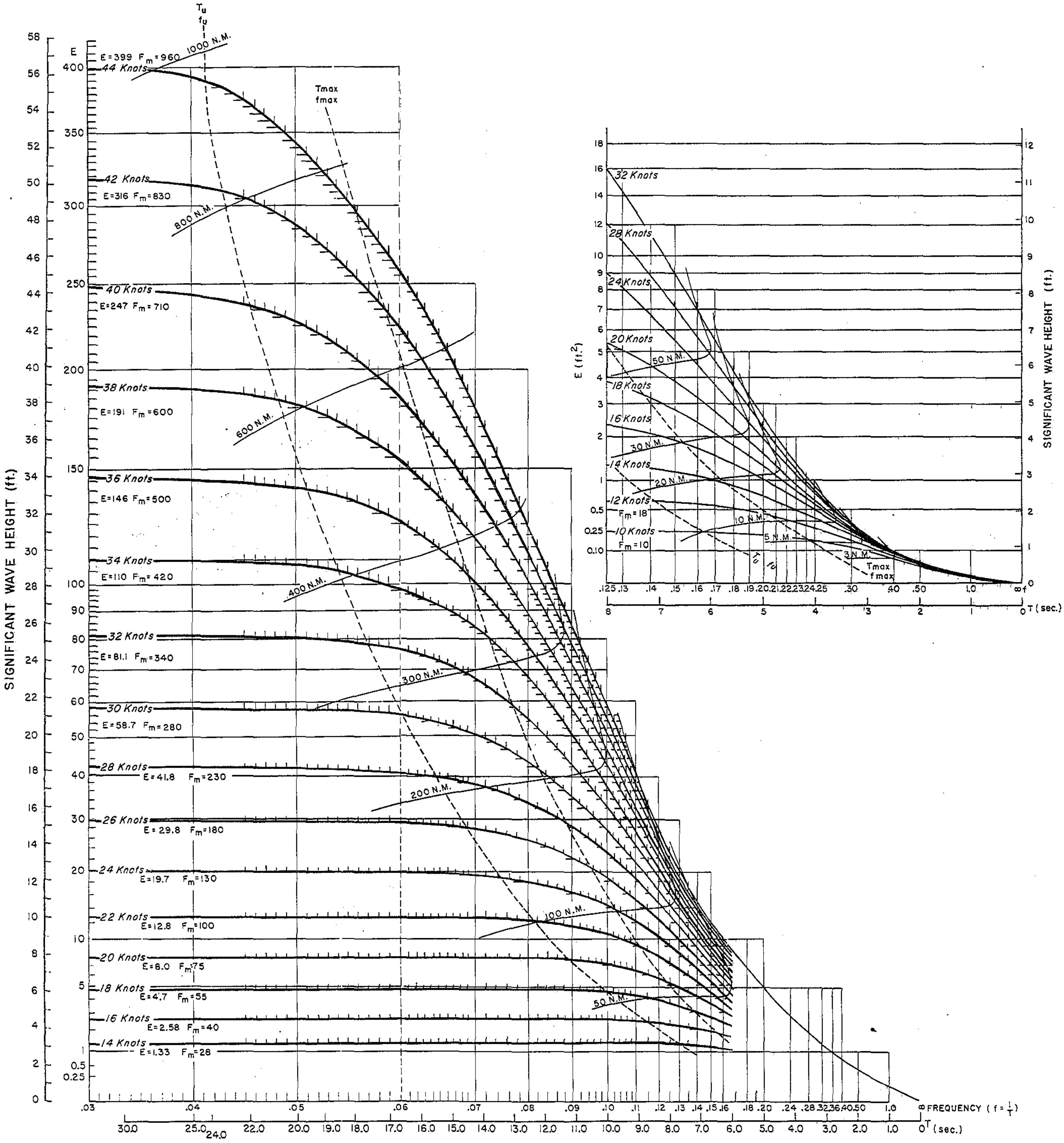


FIGURE 28

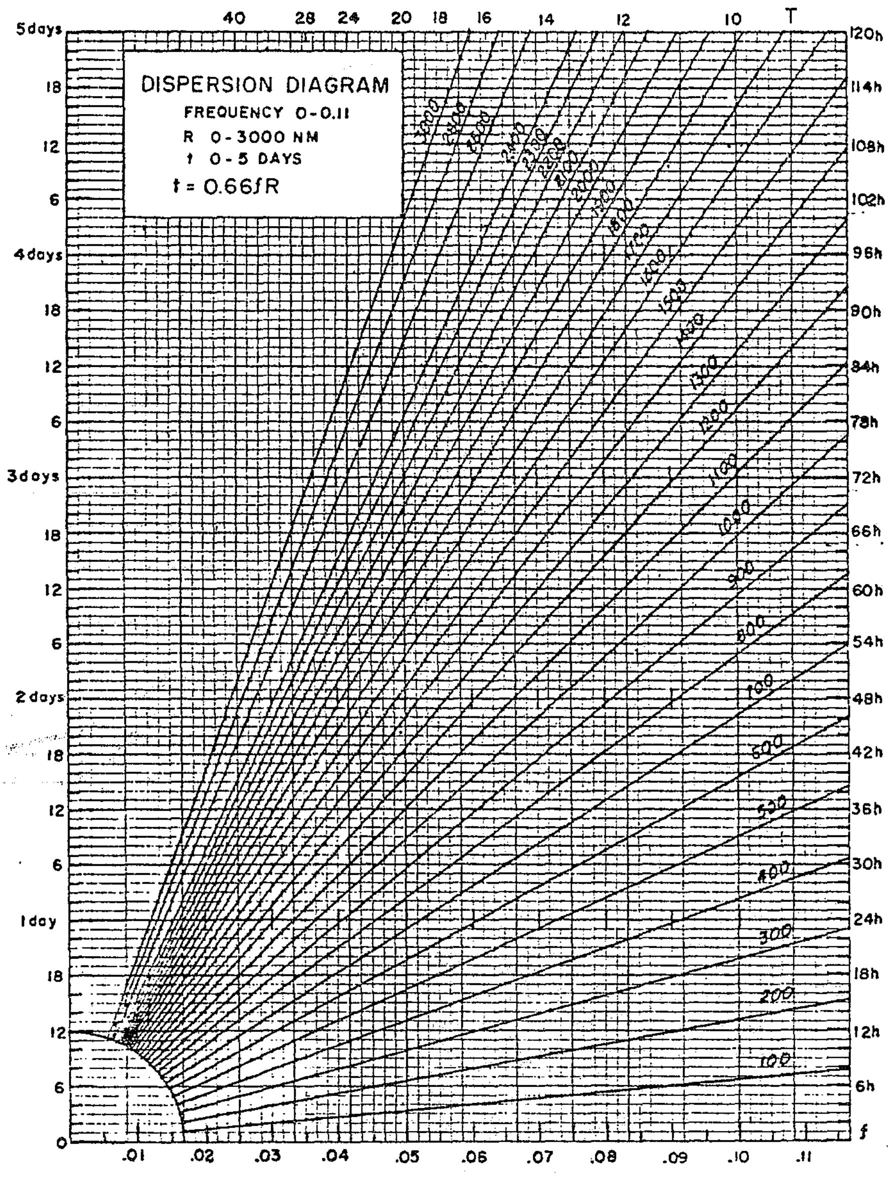


Figure 29. Dispersion diagram for a distance to the forecast point between 0 and 3000 nautical miles.

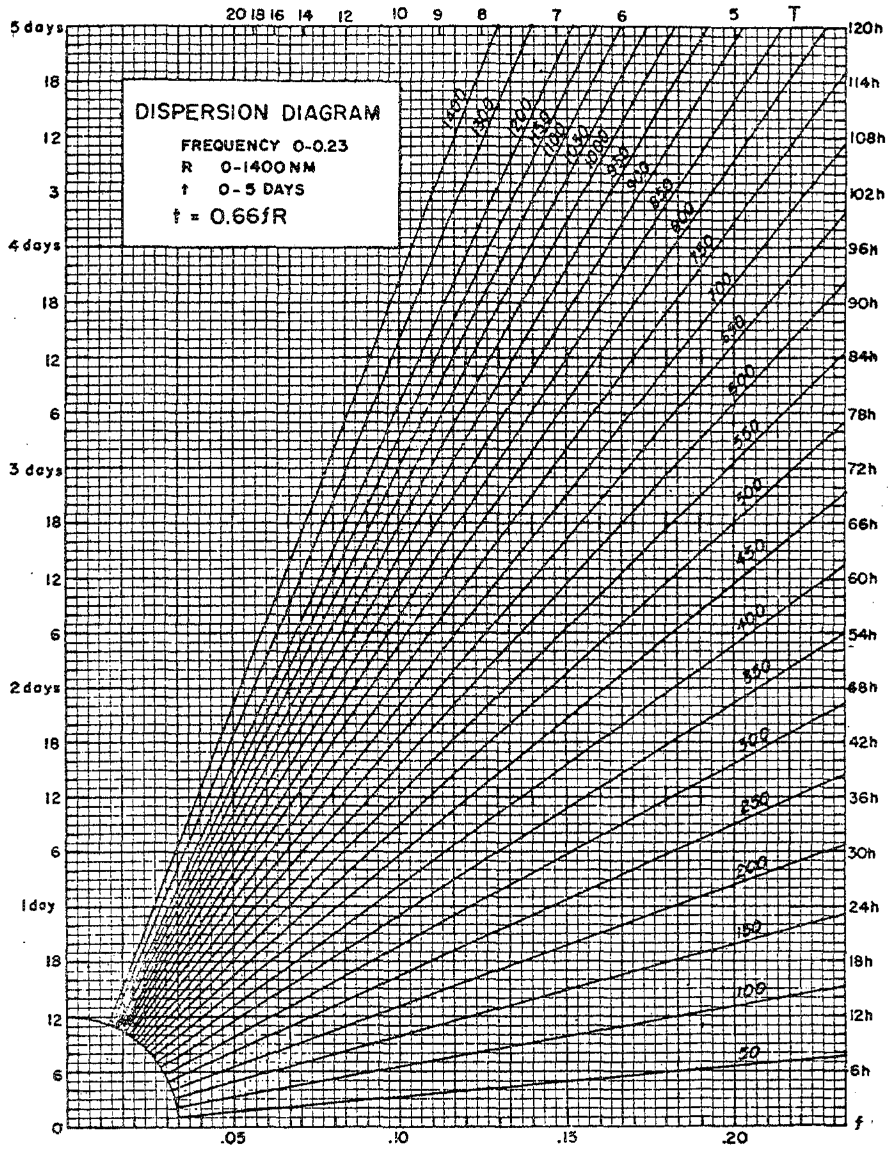


Figure 30. Dispersion diagram for a distance to the forecast point between 0 and 1400 nautical miles.

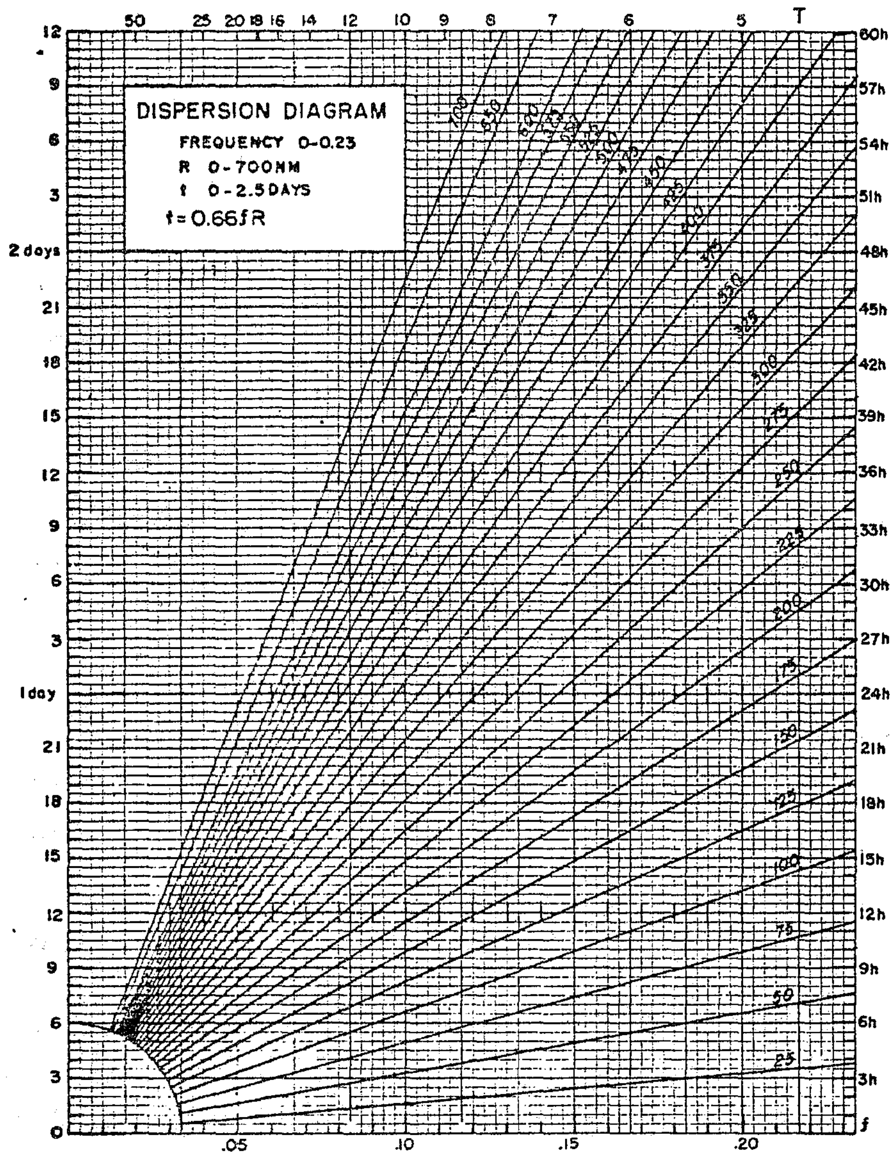
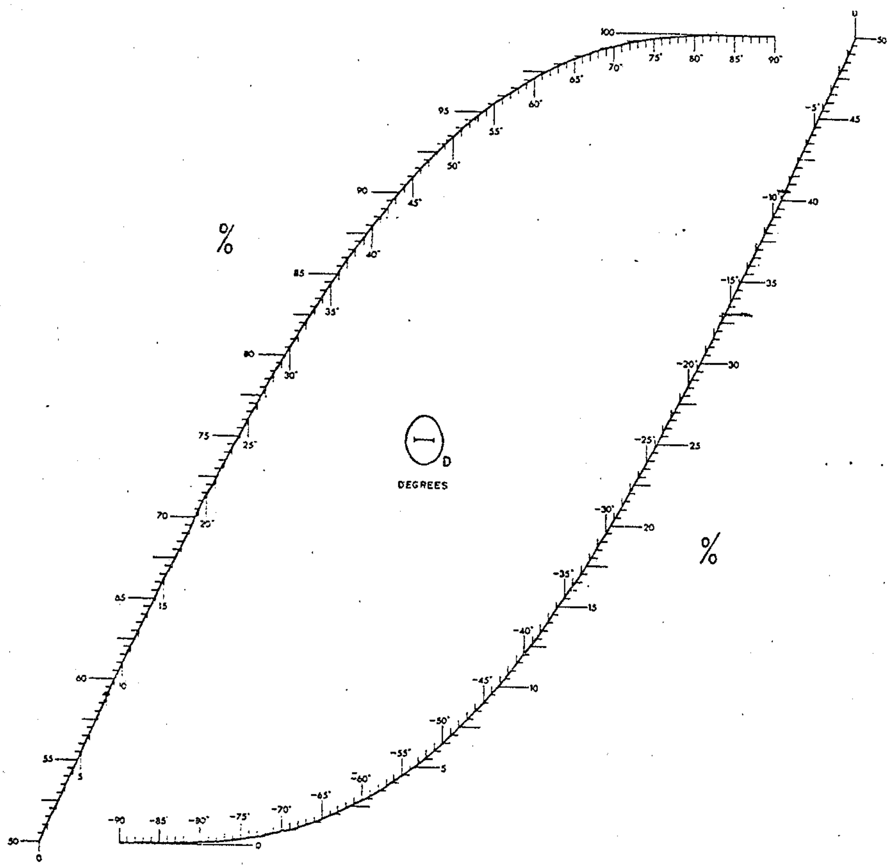


Figure 31. Dispersion diagram for a distance to the forecast point between 0 and 700 nautical miles.



$$\% = 100\left[\frac{1}{2} + \frac{\theta}{180^\circ} + \frac{\sin 2\theta}{2\pi}\right]$$

(θ in degrees)

Figure 32. Nomogram for determining the angular spreading factor.

	20	25	30	35	40	45	50	55	60
f=.04	.00000	0.0012	0.3468	10.21	91.74	413.3	1213.	2690.	4930.
.05	.00033	0.2470	8.953	78.01	317.9	833.1	1659.	2763.	4072.
.06	.03044	2.998	36.28	163.1	432.8	845.0	1364.	1943.	2543.
.07	0.3555	10.36	64.70	195.2	399.9	653.7	929.1	1205.	1469.
.08	1.433	18.95	77.04	179.5	310.7	452.6	592.4	723.0	841.2
.09	3.185	24.49	74.19	144.7	223.3	300.6	371.8	435.2	490.5
.10	4.968	25.93	63.62	109.3	155.3	197.6	234.7	266.6	293.8
.11	6.220	24.37	51.17	80.03	107.0	130.5	150.5	167.2	181.2
.12	6.763	21.31	39.74	57.87	73.85	87.30	98.39	107.5	115.0
.13	6.704	17.82	30.31	41.76	51.40	59.28	65.64	70.78	74.96
.14	6.248	14.52	22.95	30.25	36.18	40.91	44.67	47.67	50.09
.15	5.585	11.64	17.35	22.06	25.79	28.71	30.99	32.80	34.24
.16	4.855	9.257	13.14	16.24	18.63	20.46	21.89	23.01	23.89
.17	4.141	7.335	10.01	12.07	13.63	14.81	15.72	16.43	16.99
.18	3.489	5.809	7.664	9.057	10.09	10.87	11.47	11.93	12.29
.19	2.916	4.609	5.910	6.866	7.568	8.090	8.485	8.790	9.029
.20	2.427	3.668	4.590	5.256	5.738	6.094	6.362	6.568	6.729
.21	2.015	2.930	3.592	4.061	4.397	4.644	4.829	4.971	5.081
.22	1.672	2.352	2.831	3.166	3.404	3.578	3.708	3.807	3.884
.23	1.388	1.897	2.247	2.490	2.660	2.784	2.876	2.947	3.001
.24	1.154	1.537	1.797	1.974	2.098	2.187	2.254	2.304	2.343
.25	0.9615	1.252	1.446	1.577	1.668	1.733	1.782	1.818	1.847

AMPLITUDE OF THE WAVE SPECTRUM AT VARIOUS WIND SPEEDS AND FREQUENCIES (M^2 SEC).

$$[A(\sigma)]^2 = C \frac{\pi}{2} \sigma^{-6} e^{(-2g^2 \sigma^{-2} u^{-2})}$$

$$\sigma = 2 \pi f$$

FIGURE 33

WIND (KNOTS)	20	25	30	35	40	45	50	55	60
f=.04	.00000	.01366	3.731	109.9	987.1	4447.	13050.	28940.	53050.
.05	.00358	2.658	96.33	839.3	3421.	8964.	17860.	29730.	43810.
.06	0.3276	32.26	390.4	1755.	4657.	9092.	14670.	20900.	27360.
.07	3.825	111.5	696.2	2101.	4303.	7034.	9997.	12970.	15800.
.08	15.42	203.9	828.9	1931.	3343.	4870.	6375.	7779.	9052.
.09	34.27	263.6	798.2	1557.	2402.	3234.	4001.	4682.	5278.
.10	53.45	279.0	684.5	1176.	1671.	2126.	2526.	2869.	3161.
.11	66.92	262.2	550.6	861.1	1151.	1405.	1620.	1799.	1950.
.12	72.77	229.2	427.6	622.6	794.7	939.3	1059.	1157.	1237.
.13	74.14	191.8	326.2	449.3	553.1	637.8	706.2	761.6	806.5
.14	67.23	156.2	246.9	325.4	389.3	440.2	480.7	513.0	539.0
.15	60.10	125.3	186.7	237.4	277.5	308.9	333.5	352.9	368.4
.16	52.23	99.60	141.4	174.7	200.4	220.2	235.5	247.5	257.1
.17	44.56	78.92	107.7	129.8	146.6	159.4	169.2	176.8	182.8
.18	37.54	62.51	82.46	97.46	108.6	117.0	123.4	128.3	132.2
.19	31.38	49.59	63.59	73.88	81.43	87.04	91.30	94.58	97.15
.20	26.11	39.47	49.39	56.55	61.74	65.57	68.46	70.67	72.41
.21	21.68	31.53	38.65	43.69	47.32	49.97	51.96	53.49	54.68
.22	17.99	25.31	30.46	34.07	36.63	38.50	39.89	40.96	41.79
.23	14.93	20.41	24.18	26.79	28.63	29.96	30.95	31.71	32.29
.24	12.42	16.54	19.33	21.24	22.57	23.53	24.25	24.79	25.21
.25	10.35	13.48	15.56	16.96	17.95	18.65	19.17	19.57	19.87

AMPLITUDE OF THE WAVE SPECTRUM AT VARIOUS WIND SPEEDS AND FREQUENCIES (FT^2 SEC).

$$[A(\sigma)]^2 = C \frac{\pi}{2} \sigma^{-6} e^{(-2g^2 \sigma^{-2} u^{-2})}$$

FIGURE 34

$$\sigma = 2\pi f$$

SUPPLEMENT

CODED MESSAGE FORMAT FOR SPECTRAL WAVE DATA

Every three hours, values of power spectral density of vertical displacement at specified frequency intervals are coded and transmitted from Miami SCS. These messages provide users with the means to construct real-time wave spectral curves. They are coded in the following form:

A. Header Format

SXVDI0 (sp) SSSS (sp) YYGGMM

(1) The first six characters define the particular wave data report.

(2) SSSS = the originating station.

(a) SSSS = KMMA for buoys in the Gulf of Mexico and on the East Coast.

(b) SSSS = PANC for buoys in the area of the Pacific Ocean and Alaska.

(3) YYGGMM - defines the time the report was filed at SCS.

(a) YY is the day of the month (01 through 31)

(b) GG is the hour of the day (00 through 23) CUT (Coordinated Universal Time).

(c) MM is the minute (00 through 59).

(4) Example: SXVDI0 KMMA 031825 or SXVDI0 PANC 031827.

B. Data Heading Format

EBMN (2sp)//LAT (sp) QLCNG (sp) YYGGU (sp) /DDFF (sp) 3PPHH

(1) MN = two-digit buoy identifying number.

(2) LAT = Latitude of buoy to the nearest 10th degree.

(3) Q = Quadrant of the Globe = 7.

(4) LONG = Longitude to the nearest 10th degree.

(5) YY = Day of month the data were collected by the buoy (01 - 31).

(6) GG = Frame time - the CUT hour that the data were collected by the buoy (01 - 23).

(7) U = Units of wind magnitude = 1 for meters per second.

(8) DD = Wind direction to the nearest 10 degrees.

(9) FF = Wind magnitude in meters per second.

(10) PP = Wave period in seconds.

(11) HH = Wave height in half meters.

NOTE: (sp) = indicates space character.

C. Data Format (a series of 5 character fields)

a	b	c	d
99BBB	88FFF	1VVVE	2VVVE .... NVVVE

(1) A decimal point is implied at the left of the BBB, FFF, and VVV subfields.

(2) FFF is the first center frequency (units = hertz) in this sequence. BBB is the increment (units - deci hertz) required to be added to obtain each additional center frequency in this sequence. VVVE contain the encoded values of the displacement spectrum at the frequencies derived using the BBB and FFF in the sequence. The leading integer N indicates the Nth frequency in the sequence. The 10th frequency (if required, will have 0 as the leading integer after which 1, 2, etc., are used for the 11th, 12th, etc.). VVV represent the mantissa (the three most significant digits) of the displacement spectrum. This may be given as VV/ to indicate two-place accuracy. The E character is the encoded exponent (base 10)--positive exponents 0 through 4 are direct reading while negative exponents -1 through -5 are represented as 5 through 9, respectively.

(3) If the frequency spacing (BBB) changes, a new sequence is established with another set of two-sequence identifier fields 99BBB 88FFF.

(4) Missing data values VVVE are replaced with slashes.

(5) The BBB in a sequence may be replaced with slashes if only one frequency is represented in the sequence.

(6) The last sequence may be followed by a comment field for a data qualifying statement.

ZCZC

SXVD10 PANC 110327

EB42 //387 71276 11031 13205 30603

99100 88050 100/0 200/0 378/0 43641 58131 61632 99/// 88123

11012 99400 88160 11781 21241 31051 487/0 99/// 88330 132/0

THIS DATA UNRELIABLE BELOW .08 HZ:

NNNN

This is a hypothetical example of a spectral-density wave-data report. It was filed at 0327Z on the 11th day of the month. Data were obtained by buoy EB42 (located in quadrant 7 at 38.7N, 127.6W) at 0300Z on the 11th day of the month. The wind and wave data at the time were:

Wind direction = 320 deg.

Wind magnitude = 5 m/sec.

Wave period = 6 sec.

Wave height = 1.5m (3 half meters).

The spectral density values were as follows:

freq	.05	.06	.07	.08	.09	.10	.123	.16	.20	.24	.28	.33
S.D.	0.00	0.00	0.76	3.64	8.13	16.3	10.1	1.78	1.24	1.05	0.87	0.32

NOAA Technical Memoranda NWSWR: (Continued)

92 Smoke Management in the Willamette Valley. Earl M. Bates, May 1974. (COM-74-11277/AS)
93 An Operational Evaluation of 500-mb Type Stratified Regression Equations. Alexander E. MacDonald, June 1974. (COM-74-11407/AS)
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97 Eastern Pacific Cut-off Low of April 21-28, 1974. William J. Alder and George R. Miller, January 1976. (PB-250-711/AS)
98 Study on a Significant Precipitation Episode in the Western United States. Ira S. Brenner, April 1975. (COM-75-10719/AS)
99 A Study of Flash Flood Susceptibility--A Basin in Southern Arizona. Gerald Williams, August 1975. (COM-75-11360/AS)
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119 Relative Frequency of Occurrence of Warm Season Echo Activity as a Function of Stability Indices Computed from the Yucca Flat, Nevada, Rawinsonde. Darryl Randerson, June 1977. (PB-271-290/AS)
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132 Estimates of the Effects of Terrain Blocking on the Los Angeles WSR-74C Weather Radar. R. G. Pappas, R. Y. Lee, and B. W. Finke, October 1978

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